

PACE-IIT & MEDICAL

MUMBAI/DELHI-NCR/PUNE/NASHIK/AKOLA/GOA/JALGAON/BOKARO/AMRAVATI/DHULE

IIT - JEE: 2025

TW TEST (ADV)

DATE: 11/02/24

TOPIC: HEAT & THERMODYNAMICS
ROTATIONAL DYNAMICS

SOLUTIONS

1. (B)

Apparent coefficient of volume expansion

$$Y_{app} = Y_L - Y_s = 7Y_s - Y_s = 6Y_s \quad (\text{given } Y_L = 7Y_s)$$

Ratio of absolute and apparent expansion of liquid

$$\frac{Y_L}{Y_{app}} = \frac{7Y_s}{6Y_s} = \frac{7}{6}$$

2. (A)

As with the rise in temperature, the liquid undergoes volume expansion therefore the fraction of solid submerged in liquid increases.

Fraction of solid submerged at $t_1^\circ\text{C} = f_1$ = volume of displaced liquid

$$= V_0(1 + Yt_1) \quad \dots(\text{i})$$

And fraction of solid submerged at $t_2^\circ\text{C} = f_2$ = Volume of displaced liquid

$$= V_0(1 + Yt_2) \quad \dots(\text{ii})$$

$$\text{From Eqs. (i) \& (ii), } \frac{f_1}{f_2} = \frac{1 + Yt_1}{1 + Yt_2}$$

$$\Rightarrow Y = \frac{f_1 - f_2}{f_2 t_1 - f_1 t_2}$$

3. (C)

$$PV = \frac{1}{3}mv_{rms}^2 = RT$$

On lower curve, we have $V = 1\text{m}^3$, corresponding to $P = 1 \times 10^3 \text{ Pa} = 1 \text{ atmosphere}$. At T_1 , we have

$$1 \times 1 = \frac{1}{3}M(v_{rms})_1^2 = RT_1$$

Similarly, for upper curve we have

$$V = 2\text{m}^3, P = 2 \text{ atmosphere}$$

$$2 \times 2 = \frac{1}{3}M(v_{rms})_2^2 = RT_2$$

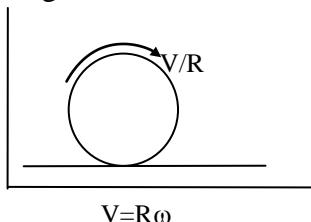
$$\text{Required ratio} = \sqrt{\frac{12/M}{3/M}} = 2$$

4. (A)

Conceptual

5. (D)

From figure,



$$V = R\omega$$

$$V_{\text{net}} (\text{for lowest point}) = v - R\omega = v - v = 0$$

$$\text{and Acceleration} = \frac{v^2}{R} + \alpha = \frac{v^2}{R} \text{ (Since linear speed is constant)}$$

Hence (D).

6. (BCD)

$$W = ms \text{ or, } m = \frac{W}{s} = \frac{4.5}{0.09} = 50 \text{ g}$$

The thermal capacity and the water equivalent of a body have the same numerical value.

$$\text{Also, } Q = 4.5 \times 8 = 36 \text{ cal}$$

Since, the temperature remains constant, during the process of melting no heat is exchanged with the calorimeter and hence,

$$Q = 15 \times 80 = 1200 \text{ cal}$$

Hence, the correct choices are (B,C) and (D)

7. (ACD)

$$\frac{\Delta A}{A} \times 100 = 2 \left(\frac{\Delta l}{l} \right) \times 100$$

$$\% \text{ increase in area} = 2 \times 0.2 = 0.4$$

$$\frac{\Delta V}{V} \times 100 = 3 \times 0.2 = 0.6\%$$

$$\text{Since } \Delta l = l\alpha\Delta T$$

$$\frac{\Delta l}{l} \times 100 = \alpha \Delta T \times 100 = 0.2$$

$$\alpha = 0.25 \times 10^{-4} / {}^\circ\text{C}$$

8. (AD)

$$\Delta V_L = \Delta V_V$$

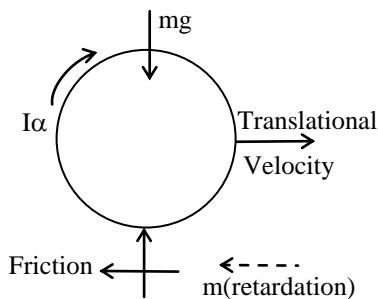
$$Y_L V_L = Y_V V_V \text{ or } \frac{Y_L}{Y_V} = \frac{V_V}{V_L}$$

$$V_V > V_L \Rightarrow Y_L > Y_V$$

9. (AB)

10. (BD)

11. (ACD)



Since, the impulse is applied along a horizontal diameter, therefore, due to that impulse the sphere starts to move translationally without any rotational motion. Since the floor is rough, therefore, friction comes into existence and that opposes forward sliding of the sphere as shown in figure. Hence, friction acts along backward direction which not only provides a retarding force but produces an accelerating torque also. Due to that torque the sphere experiences an angular acceleration. Hence, initially translational velocity of the sphere decreases but angular velocity increases. Therefore, KE of the sphere initially decreases. Rotational motion is accelerated till angular velocity (ω) becomes equal to $\frac{v}{r}$ and then the friction disappears. Hence, then total energy of sphere remains constant.

Therefore, option (B) alone is correct and rest options are incorrect

12. (AC)

$$L_O = mv \times 3R - \frac{1}{2} mR^2\omega$$

$$L_A = mv \times 0 + \frac{1}{2} mR^2\omega$$

13. (BCD)

Let rolling velocity is v and angular velocity is ω then,

$$v = v_0 - \mu gt \quad \dots (1)$$

$$\text{and } \omega = \frac{\mu g}{r} t \quad \dots (2)$$

Also, $v = r\omega$

$$\therefore \mu gt = v_0 - \mu gt \Rightarrow t = \frac{v_0}{2\mu g} \text{ and } v = \frac{v_0}{2}$$

14. [ABCD]

15. (AC)

16. (3)

PT = constant

Hence $P^2V = \text{constant}$ or $PV^{1/2} = \text{constant}$

Hence $x = 1/2$

$$C = \frac{R}{\gamma - 1} + \frac{R}{1-x} \quad \therefore \text{degree of freedom} = 3$$

17. (6.00)

Energy with 5 kg of H₂O at 20°C to become ice at 0°C

$$E_1 = 5000 \times 1 \times 20 = 100000 \text{ cal}$$

Energy to raise the temperature of 2kg ice from -20°C to 0°C

$$E_1 = 2000 \times 0.5 \times 20 = 20000 \text{ cal}$$

$$(E_1 - E_2) = 80000 \text{ cal is available to melt ice at } 0^{\circ}\text{C}$$

So only 1000 g or 1kg of ice would have melt.

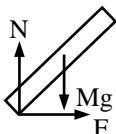
So, the amount of water available $1 + 5 = 6 \text{ kg}$

18. (7)

$$N = mg$$

$$F \frac{L}{2} \sin \theta = \frac{N^2}{2} \cos \theta$$

$$\Rightarrow \tan \theta = \frac{3}{4}$$



19. (4)

$$\text{K.E.} = \frac{1}{2} I \omega^2 = \frac{1}{2} (I\omega)(\omega)$$

$$\text{Or K.E.} = \frac{1}{2} L \omega$$

$$\text{Or } L = \frac{2 \text{K.E.}}{\omega}$$

$$\text{Now } L' = \frac{2(2 \text{K.E.})}{(\omega / 2)} = 4 L$$

20. (4)

$$L_H = M_2 v \frac{L}{2} = \frac{M_1 L^2}{3} \omega$$

$$\omega = \frac{3M_2 v}{2M_1 L} \quad \dots(1)$$

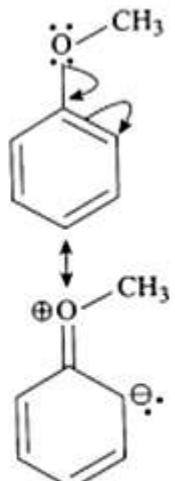
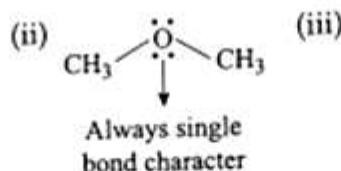
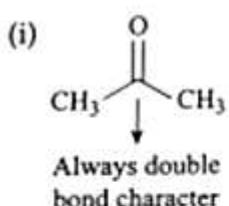
$$\text{for } e = 1, v_{CM} = v = \frac{\omega L}{2} \quad \dots(2)$$

$$\Rightarrow \frac{2v}{L} 1 = \frac{3M_2 v}{2M_1 L}$$

$$\Rightarrow \frac{M_1}{M_2} = \frac{3}{4}$$

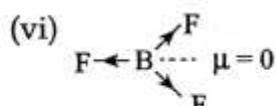
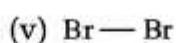
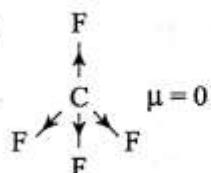
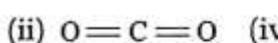
SOLUTIONS

21. (B)



Single and double bond character due to resonance

22. (A)



23. (B)

For the precipitation of $Fe(OH)_2$

$$K_{sp} = [0.04] [OH^-]^2 = 16 \times 10^{-6}$$

$$[OH^-] = 2 \times 10^{-2} \text{ and for } Fe(OH)_3$$

$$K_{sp} = 0.02 \times [OH^-]^3 = 8 \times 10^{-6}$$

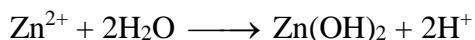
$$\therefore [OH^-] = 2 \times 10^{-8}$$

For $Fe(OH)_3$ the $[OH^-]$ ions are required less

$$pOH = 8 - \log 2$$

$$pH = 6 + \log 2 = 6.3$$

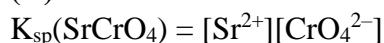
24. (C)



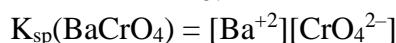
1	0	0
(1 - h)C	hC	2hC

$$K_H = 4C^3h^3, h = \left(\frac{K_H}{4 \times 10^{-6}} \right)^{\frac{1}{3}}$$

25. (A)



$$[\text{CrO}_4^{2-}] = \frac{3.5 \times 10^{-5}}{0.1} = 3.5 \times 10^{-4}$$

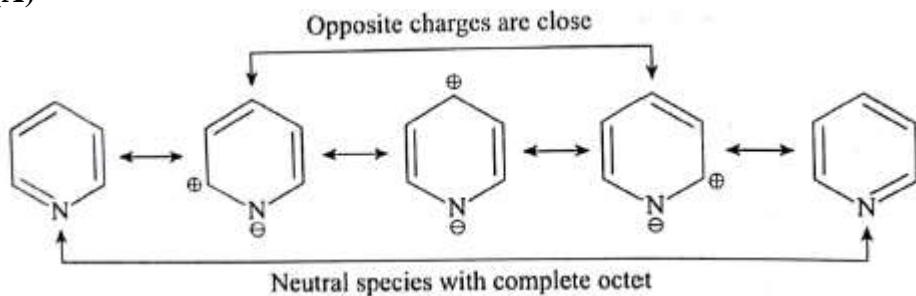


$$[\text{CrO}_4^{2-}]_{\text{total}} \approx [\text{CrO}_4^{2-}] \text{ from SrCrO}_4$$

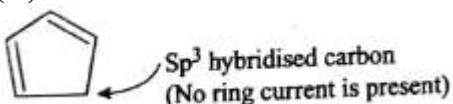
$$[\text{Ba}^{2+}] = \frac{1.2 \times 10^{-10}}{3.5 \times 10^{-4}} = 3.4 \times 10^{-7}$$

26. (ABD)

27. (A)



28. (D)



29. (A)

30. (D)

Resonance energy \propto extent of resonance.

31. (AC)

In case of polyprotic acids, divalent anion concentration is equal to K_{a_2}

32. (BC)



Initial : 2 1 0

R & F : 1 1 1

Left : 1 0 1

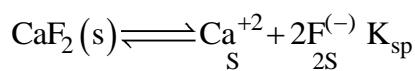
Hence basic buffer

33. (BC)

For acidic buffer, $pH = pK_a + \log \frac{s}{a}$, As $[s] \uparrow \Rightarrow pH \uparrow$

For basic buffer, $pOH = pK_b + \log \frac{s}{b} \Rightarrow As [s] \uparrow \Rightarrow pOH \uparrow \Rightarrow pH \downarrow$

34. (AB)



$$K_{sp} = [Ca^{+2}][F^{-}]^2 = 4s^3$$

$$[Ca^{+2}] = s \quad [F^{-}] = 2s$$

35. (AC)

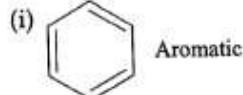
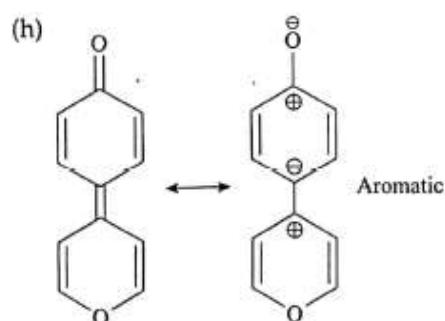
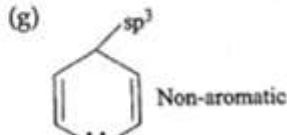
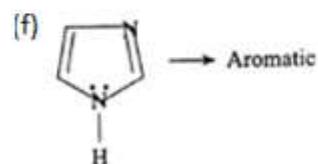
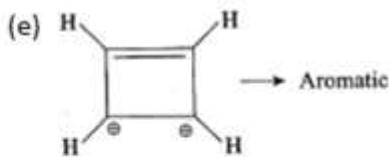
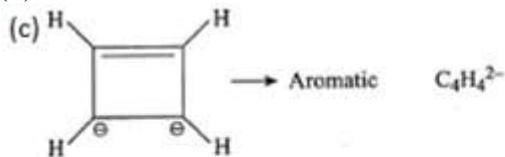
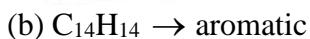
$$[H^{+}] = C\alpha$$

36. (8)

Electrophiles $CH_3, BF_3, AlCl_3, CaCl_3, SbCl_5, PCl_5, PCl_3CH_3-CL$

In CH_3-Cl , Cl-C bond is polar bond thus carbon have low energy anti-bonding orbitals

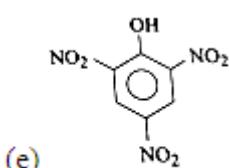
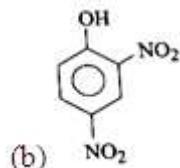
37. (6)



38. (8)

$$X = 6$$

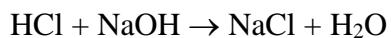
Number of phenol derivative gives +ve test with $NaHCO_3$.



$$Y = 2$$

$$X + Y = 8$$

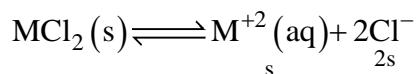
39. (2)



Solution will be acidic only when NaOH is limiting reagent & HCl is excessive reagent.
Hence, (ii) & (iii)

40. (2)

In pure water,

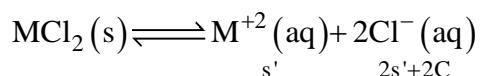


$$K_{sp} = 4s^3$$

$$4 \times 10^{-12} = 4s^3$$

$$s = 10^{-4} \text{ M}$$

Presence of CaCl_2



common ion effect

$$K_{sp} = s'(2s' + 2C)^2$$

$$4 \times 10^{-12} = s' \times 4C^2$$

$$s' = \frac{10^{-12}}{C^2}$$

$$\text{It is given that } \frac{s'}{s} = C^2 \times 10^8 = 4 \times 10^8$$

$$C = 2 \text{ M}$$

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TOPIC: LCD

SOLUTION

41. (D)

$$\text{Clearly, } \lim_{h \rightarrow 0} \frac{f(h^3 + 3h + 2) - f(2)}{(2h - 2h^2 + 1) - f(1)} = \lim_{h \rightarrow 0} \frac{f'(h^3 + 3h + 2) \cdot (3h^2 + 3)}{f'(2h - 2h^2 + 1)(2 - 4h)} = \frac{6 \cdot 3}{4 \cdot 2} = \frac{9}{4}$$

(Using L'Hospital Rule)

42. (B)

$$\begin{aligned} l &= \lim_{x \rightarrow 0} \frac{(1+P(x))^{1/n} - 1}{x} \quad (\text{Using binomial expansion}) \\ &= \lim_{x \rightarrow 0} \frac{\left(1 + \frac{1}{n}P(x) + \dots\right) - 1}{x} = \lim_{x \rightarrow 0} \frac{1}{n} \left[\frac{a_1 x + a_2 x^2 + a_3 x^3 + \dots}{x} \right] = \frac{a_1}{n} \end{aligned}$$

43. (B)

Since, $f(x)$ is continuous $\forall x \in R$.

$$\therefore \log(x^2 + kx + k + 1) \geq 0 \quad \forall x \in R$$

$$\text{And } x^2 + k \neq 0 \quad \forall x \in R$$

$$\Rightarrow x^2 + kx + k + 1 \geq 1 \quad \forall x \in R$$

$$\text{And } k > 0$$

$$\therefore x^2 + kx + k \geq 0 \quad \forall x \in R$$

$$D \leq 0 \Rightarrow k^2 - 4k \leq 0$$

$$k \in [0, 4]$$

$$\text{But } k > 0$$

$$\therefore (0, 4]$$

44. (D)

$$\lim_{x \rightarrow 0} \frac{\{(a-n)nx - \tan x\} \sin nx}{x^2} \frac{1}{nx} xn = 0$$

$$\Rightarrow \{(a-n)n - 1\}n = 0 \Rightarrow a = n + \frac{1}{n}$$

45. (A)

$$y = \lim_{n \rightarrow \infty} \left(\frac{n!}{n^n} \right)^{\frac{3n^3 + 4}{4n^4 - 1}}$$

$$\ln y = \lim_{n \rightarrow \infty} \frac{3n^3 + 4}{4n^4 - 1} \sum_{r=1}^n \ln \left(\frac{r}{n} \right) = \frac{3}{4} \int_0^1 \ln x \, dx = \frac{-3}{4} \Rightarrow y = e^{-3/4}$$

46. (AB)

$$\sin \alpha + \sin \beta = -\frac{\sin \beta}{\sin \alpha} \Rightarrow \sin \alpha = \sin \beta = -\frac{1}{2}$$

47. (BD)

$$\lim_{x \rightarrow 2^+} [5 - 2x] = 0$$

$$\lim_{x \rightarrow 2^-} [|x - 2| + a^2 - 6a + 9] = 0 \Rightarrow (a - 3)^2 < 1$$

48. (BD)

$$(A) \lim_{x \rightarrow 1^+} g(f(x)) = \lim_{x \rightarrow 0^-} g(x) = \text{D.N.E}$$

$$(B) \lim_{x \rightarrow \frac{\pi}{2}^-} g(f(g(x))) = \lim_{x \rightarrow 0^-} g(f(x)) = \lim_{x \rightarrow 2^-} g(x) = 0$$

$$(C) \lim_{x \rightarrow 2^+} \frac{f(g(x))}{f(x) - 2} = 0$$

$$(D) \lim_{x \rightarrow 0^+} \frac{g(f(x))}{(f(x) - 2)^2} = \lim_{x \rightarrow 0^+} \frac{g(2-x)}{(2-x-x)^2} \\ = \lim_{x \rightarrow 0^+} \frac{[2-x] - \cos(2-x-2)}{x^2} = \lim_{x \rightarrow 0^+} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

49. (BCD)

$$f(x) = \frac{1 - \cos \{x\}}{x^2 (x^2 + ax^2 + bx + c)^2}$$

$$x^3 + ax^2 + bx + c = (x-1)(x-2)(x-3) = x^3 - 6x^2 + 11x - 6$$

$$\therefore a+b+c = -1$$

$$f(x) = \frac{1 - \cos \{x\}}{x^2 ((x-1)(x-2)(x-3))^2}$$

$$\text{Now, } l = \frac{1}{8}, m = \frac{1}{8} \text{ and } n = \frac{1}{72}$$

$$\therefore l + m + n = \frac{19}{72}$$

50. (ACD)

$$\text{Given, } f(x) = \lim_{n \rightarrow \infty} \frac{\ln(2+x^2) - x^{2n} \sin(x^2)}{1+x^{2n}}$$

$$\text{Now, } f(x) = \begin{cases} \ln(2+x^2), & \text{for } 0 \leq x^2 < 1 \\ \frac{\ln 3 - \sin 1}{2}, & \text{for } x^2 = 1 \\ -\sin(x^2), & \text{for } x^2 > 1 \end{cases} = \begin{cases} -\sin(x^2), & -\infty < x < -1 \\ \frac{\ln 3 - \sin 1}{2}, & x = -1 \\ \ln(2+x^2), & -1 < x < 1 \\ \frac{\ln 3 - \sin 1}{2}, & x = 1 \\ -\sin(x^2), & 1 < x < \infty \end{cases}$$

Now, verify alternatives.

Note: $f(x)$ is an even function also.

51. (BD)

(A) As $\lim_{x \rightarrow 0^+} x^x = 1$

$$\therefore \lim_{x \rightarrow 0^+} (x^{x^x} - x^x) = (0^1 - 1) = -1$$

(B) $\lim_{x \rightarrow 0^+} x^2 \ln \sqrt{\frac{1}{x}} = \frac{-1}{2} \lim_{x \rightarrow 0^+} x^2 \ln x \quad (0 \times \infty) \text{ form}$

$$\frac{-1}{2} \lim_{x \rightarrow 0^+} \frac{\ln x}{1/x^2} \quad \left(\frac{\infty}{\infty}\right) \text{ form}$$

$$= \frac{-1}{2} \lim_{x \rightarrow 0^+} \frac{1/x}{-2/x^3} = \frac{1}{4} \lim_{x \rightarrow 0^+} x^2 = 0 \Rightarrow (\text{B}) \text{ vanishes.}$$

(C) Let $l = \lim_{x \rightarrow 0^+} x^{\ln(x+1)} \Rightarrow \ln l = \lim_{x \rightarrow 0^+} \frac{\ln x}{(\ln(x+1))^{-1}} \left(\frac{\infty}{\infty}\right) \text{ form}$

$$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{-1}{\ln^2(x+1)} \times \frac{1}{(x+1)}} = \lim_{x \rightarrow 0^+} \frac{(x+1)\ln^2(x+1)}{-x}$$

$$= -\lim_{x \rightarrow 0^+} \left(\frac{\ln(x+1)}{x} \right)^2 \times x(x+1)$$

$$= -(1)^2 \times 0 \times (0+1) = 0$$

Hence, $l = 1$.

(D) $\lim_{x \rightarrow 0} \frac{10^x - 2^x - 5^x + 1^x}{x + \tan x} = \lim_{x \rightarrow 0} \frac{\left(\frac{5^x - 1}{x} \right) \left(\frac{2^x - 1}{x} \right) x}{\left(1 + \frac{\tan x}{x} \right)} = 0 \Rightarrow (\text{D}) \text{ vanishes.}$

Solution for Que. No. 52 & 53

52. (C)

53. (B)

$$P(x) = (x^2 - 4)((x^2 - 1)(x^2 - 9) + 1) = (x^2 - 4)(x^4 - 10x^2 + 10)$$

$$\lim_{x \rightarrow -2} \frac{\sin P(x)}{P(x)} \cdot \frac{P(x)}{(x^2 - 4)} = \lim_{x \rightarrow -2} \frac{(x^2 - 4)(x^4 - 10x^2 + 10)}{x^2 - 4} = -14$$

Solution for Que. No. 54 & 55

$$\underbrace{\log_2 \left[(\alpha^3 - 8)^2 + 2 \right]}_{\geq 1} + \underbrace{\sqrt{4(\beta^2 - 1)^2 + 9}}_{\geq 3} + \underbrace{\left[\frac{\gamma}{3} - 2 \right]}_{\geq 0} = 4$$

The equation will satisfy if $\alpha = 2, \beta = \pm 1$.

$$0 \leq \frac{\gamma}{3} - 2 < 1 \Rightarrow 6 \leq \gamma < 9$$

54. (C)

$$\alpha = 2, \beta = \pm 1, \gamma = 6, 7, 8$$

Number of ordered triplets (α, β, γ) is 6.

55. (B)

$$\lim_{x \rightarrow 0} \left(\left[\frac{\sin 6x}{x} \right] + \left[\frac{\sin 7x}{x} \right] + \left[\frac{\sin 8x}{x} \right] \right) = 5 + 6 + 7 = 18$$

56. (4)

$$\begin{aligned} f(x) &= \lim_{n \rightarrow \infty} \ln \left(e^{\frac{\cos x}{2}} \cdot e^{\frac{3\cos x}{2^2}} \cdot e^{\frac{5\cos x}{2^3}} \cdots e^{\frac{(2n+1)\cos x}{2^n}} \right) \\ &= \lim_{x \rightarrow n} \left(\frac{\cos x}{2} + \frac{3\cos x}{2^2} + \frac{5\cos x}{2^3} + \cdots + \frac{(2n+1)\cos x}{2^n} \right) \\ f(x) &= \frac{\cos x}{2} + \frac{3\cos x}{2^2} + \frac{5\cos x}{2^3} + \cdots \infty \\ \frac{1}{2} f(x) &= \frac{\cos x}{2^2} + \frac{3\cos x}{2^3} + \cdots + \infty \\ \frac{1}{2} f(x) &= \frac{\cos x}{2} + \frac{2\cos x}{2^2} + \frac{2\cos x}{2^3} + \cdots \infty \\ f(x) &= \cos x + \cos x + \frac{\cos x}{2} + \frac{\cos x}{2^2} + \cdots \infty \\ &= \cos x + \frac{\cos x}{1 - \frac{1}{2}} = 3\cos x \\ g(x) &= \left[\frac{1}{3} f(x) \right] = [\cos x] \end{aligned}$$

Hence, number of values of x in $[0, 2\pi]$ where $[\cos x]$ is discontinuous is 4, i.e., $0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$.

57. (1)

For function to be differentiable at $x = 0$

It should be continuous at $x = 0$

$$\text{LHL } \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0} b \sin^{-1} \left(\frac{x+c}{2} \right) = b \sin^{-1} \left(\frac{c}{2} \right)$$

$$\text{RHL } \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0} \frac{(e^{ax/2} - 1)}{x} = \frac{a}{2}$$

For continuity

$$\text{LHL} = \text{RHL} = f(0)$$

$$\Rightarrow b \sin^{-1} \left(\frac{c}{2} \right) = \frac{a}{2} = \frac{1}{2}$$

$$\Rightarrow a = 1$$

58. (1)

$$F(x) = \begin{cases} f(x), & -1 < x < 1 \\ g(x), & x \in (-\infty, -1) \cup (1, \infty) \\ \frac{f(1) + g(1)}{2}, & x = 1 \\ \frac{f(-1) + g(-1)}{2}, & x = -1 \end{cases}$$

$$\begin{aligned} \text{Continuous at } x = 1 &\Rightarrow F(1^-) = F(1^+) = F(1) \\ &\Rightarrow f(1^-) = g(1^+) = \frac{f(1) + g(1)}{2} \\ &\Rightarrow 4 + a = 1 + b = \frac{(4+a)+(1+b)}{2} = \frac{5+a+b}{2} \end{aligned}$$

$$\therefore b - a = 3 \quad \dots(i)$$

$$\begin{aligned} \text{Continuous at } x = -1 &\Rightarrow F(-1^-) = F(-1^+) = F(-1) \\ &\Rightarrow b - 1 = 4 - a = \frac{(4-a)+(b-1)}{2} \\ &\Rightarrow b - 1 = 4 - a = \frac{3+b-a}{2} \end{aligned}$$

$$\therefore a + b = 5 \quad \dots(ii)$$

From (i) and (ii),

$$\Rightarrow a = 1, b = 4 \Rightarrow a^2 + b^2 = 17$$

59. (5)

$$\text{Put } n = \frac{1}{y}$$

$$\begin{aligned} \lim_{y \rightarrow 0} \frac{\left(\frac{1+y}{1-y}\right)^{\frac{1}{y}} - e^2}{y^2} &= \lim_{y \rightarrow 0} \frac{\frac{1}{y} \ln\left(\frac{1+y}{1-y}\right) - e^2}{y^2} = e^2 \lim_{y \rightarrow 0} \frac{\frac{\ln\left(\frac{1+y}{1-y}\right) - 2}{y}}{y^2} - 1 \\ &= e^2 \lim_{y \rightarrow 0} \frac{\frac{\ln\left(\frac{1+y}{1-y}\right) - 2}{y}}{y^2} - 1 \\ &= e^2 \lim_{y \rightarrow 0} \frac{\ln\left(\frac{1+y}{1-y}\right) - 2}{y^3} = e^2 \left(\frac{2}{3}\right) = \frac{2e^2}{3} \Rightarrow a = 2; b = 3 \\ \therefore a + b &= 5 \end{aligned}$$

60. (5)

We have

$$f(x) = \begin{cases} 0, & x \in (-\infty, -1) \\ 1+x, & x \in [-1, 0] \\ 1-x, & x \in (0, 1] \\ 0, & x \in (1, \infty) \end{cases} \Rightarrow f(x-1) = \begin{cases} 0, & x-1 \in (-\infty, -1) \\ 1+(x-1), & x-1 \in [-1, 0] \\ 1-(x-1), & x-1 \in (0, 1] \\ 0, & x-1 \in (1, \infty) \end{cases}$$

or $f(x-1) = \begin{cases} 0, & x < 0 \\ x, & 0 \leq x \leq 1 \\ 2-x, & 1 < x \leq 2 \\ 0, & x \in (1, \infty) \end{cases}$

Also, $f(x+1) = \begin{cases} 0, & x+1 \in (-\infty, -1) \\ 1+(x+1), & x+1 \in [-1, 0] \\ 1-(x+1), & x+1 \in (0, 1] \\ 0, & x+1 \in (1, \infty) \end{cases}$ or $f(x+1) = \begin{cases} 0, & x < -2 \\ 2+x, & -2 \leq x \leq -1 \\ -x, & -1 < x \leq 0 \\ 0, & 0 < x < \infty \end{cases}$

Now, $g(x) = f(x-1) + f(x+1) = \begin{cases} 0, & x < -2 \\ 2+x, & -2 \leq x \leq -1 \\ -x, & -1 < x \leq 1 \\ x, & 0 < x \leq 1 \\ 2-x, & 1 < x \leq 2 \\ 0, & x > 2 \end{cases}$

Clearly, It is easy to check that $g(x)$ is continuous $\forall x \in \mathbb{R}$ and non-differentiable at $x = -2, -1, 0, 1, 2$ and differentiable elsewhere. Hence, number of points of non-differentiability of $g(x)$ are 5.