STRAIGHT LINES

Exercise – 1(A)

Q.1

Slope of line is $\Longrightarrow \dfrac{\sqrt{3}-0}{-2-1} = \dfrac{-1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \dfrac{-1}{\sqrt{3}}$, $\theta = 150^{o}$

Q.2

Mid-point of B & C is (4 , 2):

slope is $\dfrac{3-2}{2-4} = -\dfrac{1}{2}$

Q.3

Vertices of integral $\Longrightarrow$ Δ never equilateral

Because equate Areas

$$\Rightarrow \frac{\sqrt{3}}{2}\left((x_1-x_2)^2+(y_1-y_2)^2\right)=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

This is irrational & $R\pi S$ is Rational

It is a contradiction $\therefore$ be equilateral

Q.4

Slope is zero : 4^2

$\Rightarrow$ y = 2

Q.5

use slope – intercept form

y = 2x – y

Q.6

$\Rightarrow$ m = $\frac{3}{5}$ & c = – 3

$\therefore$ slope intercept form y = $\frac{3}{5}$x – 3

Q.7

$m = \dfrac{4-(-5)}{2-3} = -9$

line is $y = -9x + c$ put (3 , 4) to get c

Q.8

The information c = – 2 & m = $\sqrt{3}$

Slope intercept $y = \sqrt{3x} - 2$

Q.9

form $\dfrac{y-0}{x-0} = \dfrac{a\sin\theta - 0}{a\cos\theta - 0} \Rightarrow \boxed{y = x\tan\theta}$

Q.10

Both a = b

$\therefore$ intercept form $\frac{x}{a} + \frac{y}{a} = 1$

$\Rightarrow$ $\boxed{x + y = a}$ ………slope – 1

Q.11

$x\cos\alpha + y\sin\alpha = a$

$\therefore$ for y intercept put x = 0

$\therefore \quad y = a \operatorname{cosec} \alpha$

Q.12

$$\Rightarrow \frac{x}{a} + \frac{y}{a} = 1 \quad \text{or} \quad x + y = a$$

Put (1, – 2) to get ‘a’

$\Rightarrow 1 - 2 = a \Rightarrow a = -1$

Line is x + y + 1 = 0

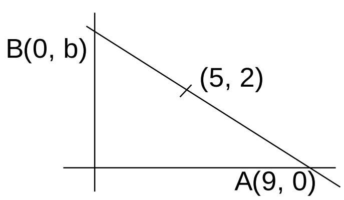
Q.13

Intercept form $\frac{x}{a} + \frac{y}{b} = 1$; here b = 2a

(1, 2) satisfies $\frac{1}{a} + \frac{2}{b} = 1$

$$\Rightarrow \frac{1}{a} + \frac{1}{a} = 1 \quad \Rightarrow \begin{pmatrix} a = 2 \\ b = 4 \end{pmatrix}$$

Q.14

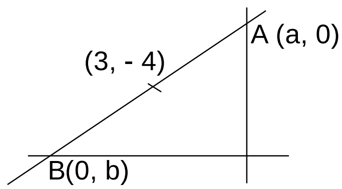


$$\Rightarrow \therefore \frac{a}{2} = 5 \ \& \ \frac{b}{2} = 2$$

$\Rightarrow a = 10, b = 4$

$$\Rightarrow \therefore \text{line } \frac{x}{10} + \frac{y}{4} = 1$$

Q.15



By section formula

$\Rightarrow \frac{2a}{5} = 3$ & $\frac{3b}{5} = -4$

$\Rightarrow a = \frac{15}{2}, b = -\frac{20}{3}$

Line is $\frac{2x}{15} - \frac{3y}{20} = 1$

Q.16

$\Rightarrow a + b = -2$

Let $\frac{x}{a} + \frac{y}{b} = 1$

put (3 – 3)

$\Rightarrow \frac{2}{a} - \frac{3}{b} = 1$

$\Rightarrow 2b - 3a - ab = 0$

$\Rightarrow 2b + 3(b + 2) + b(b + 2) = 0$

$\Rightarrow b^2 + 7b + 6 = 6$

$\Rightarrow b = -6$, $b = -1$

$\Rightarrow a = 4$, $b = -1$

Q.17

$$\Rightarrow \frac{x}{3a} + \frac{y}{3b} = 1$$

$A(3a, 0)$; $B(0, 3b)$; $0(0, 0)$

Centroid (a, b)

1(A)

18. $m_1 = 1$, $m_2 = \sqrt{3}$ $\therefore \tan\theta = \left|\frac{\sqrt{3}-1}{1+\sqrt{3}}\right| \Rightarrow \theta = 15^\circ$ — (A)

19. $m_1 = -2/3$, $m_2 = 3/2$ Since $m_1 m_2 = -1 \Rightarrow \theta = 90^\circ$ — (D)

20. $m_1 = 2$, $m_2 = -3$

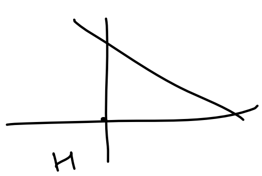
$\therefore \tan\theta = \left|\frac{2+3}{1-6}\right| = 1$, $\theta = 45^\circ$ — (C)

21. $m_1 = 1$, $m_2 = 0$ $\therefore \tan\theta = 1 \Rightarrow \theta = 45^\circ$

$\therefore$ obtuse $= 135^\circ$ — (B)

22. $m_1 = 0$, $m_2 = \sqrt{3}$ $\therefore \tan\theta = \sqrt{3}$, $\theta = 60^\circ$

23. slope of $L_1 = 4/7$, slope of $L_3 = -7/4$ Since $m_1 m_2 = -1 \Rightarrow$ right $\triangle$



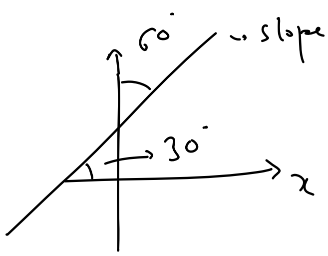
H is right angled vertex

i.e. intersection of $4x - 7y + 10 = 0$

& $7x + 4y - 15 = 0$

solve $H(1, 2)$ — (D)

24.



$\therefore$ slope $\frac{1}{\sqrt{3}}$

$\therefore$ angle $= 60^\circ$ — (B)

(25) $\perp r \Rightarrow m_1 m_2 = -1$

$$-\frac{m}{2} \times -\frac{2}{3} = -1 \Rightarrow \boxed{m = -3}$$

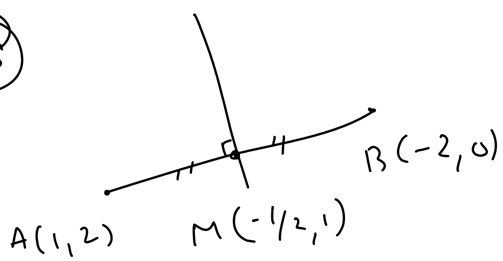
(26) $m_1 = 2$, $m_2 = \frac{3-\lambda}{2}$

Since $m_1 m_2 = -1 \Longrightarrow \frac{3-\lambda}{2} = -\frac{1}{2} \Rightarrow \lambda = 4$ — (A)

(27) slope of line joining $(-5, 6)$ & $(-6, 5)$ is $m = 1$

$\therefore$ Eqn is $\frac{y-3}{x-4} = -1 \Rightarrow x + y - 5 = 0$ — (D)

(28)



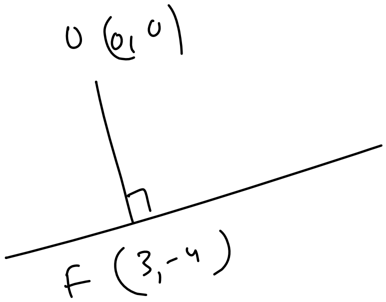
$m_{AB} = \frac{2}{3}$ $\therefore \perp^r$ bisector

$$\frac{y-1}{x+\frac{1}{2}} = -3/2$$

$\Rightarrow 2y - 2 = -3x - 3/2$

$\Rightarrow 3x + 2y = 1/2$ — (C)

(29)



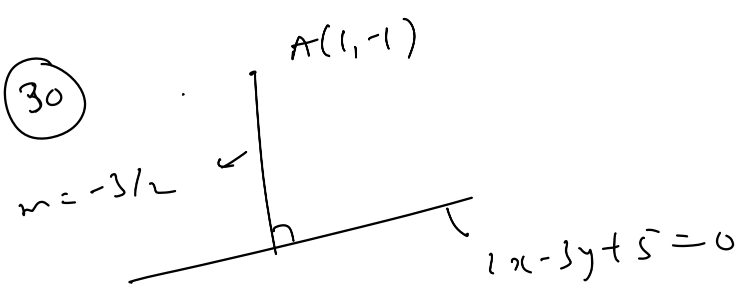
$m_{OF} = -4/3$

line slope $3/4$

Eqn of line $\frac{y+4}{x-3} = \frac{3}{4}$

or $3x - 4y = 25$ — (A)

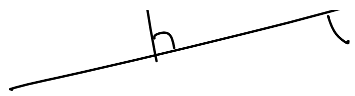
(30)



slope $= -3/2$

$\therefore$ Eqn $\frac{y+1}{x-1} = -3/2$

m^-



$2x-3y+5=0$

$\therefore$ Eqⁿ $\dfrac{y+1}{x-1} = -3/2$

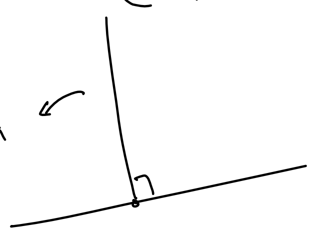
$-3x+3 = 2y+2$

$3x+2y-1=0$ — (A)

(31) Eqⁿ $\dfrac{y-d}{x-c} = -\dfrac{a}{b} \Rightarrow a(x-c)+b(y-d)=0$ — (C)

(32) (a,b)

$m = b/a$



$ax+by+c=0$

Eqⁿ is $\dfrac{y-b}{x-a} = \dfrac{b}{a}$

$\boxed{bx-ay=0}$ — (C)

(33) parallel to x-axis $y=-2$

(34) let line be $\dfrac{x}{a}+\dfrac{y}{b}=\lambda$ put (a,b)

$\Rightarrow \lambda = 2$

$\therefore$ line $\dfrac{x}{a}+\dfrac{y}{b}=2$ — (B)

(35) Eqⁿ of line is $\dfrac{y-2}{x-2} = \dfrac{1}{3} \Rightarrow x-3y+4=0$

$\therefore$ y-intercept $4/3$

— (D)

(36) $\perp^r$ distance $= \left|\dfrac{3(2)-4(1)+8}{5}\right| = 2$

(37) $\perp^r$ distance $= \left|\dfrac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right| = \dfrac{ab}{\sqrt{a^2+b^2}}$

(38) $\perp^r = \left| \dfrac{c_1 - c_2}{\sqrt{a^2+b^2}} \right| = \left| \dfrac{9-(-13)}{\sqrt{5^2+12^2}} \right| = \dfrac{22}{13}$

(39) $2x - y + 4 = 0$

$2x - y - 5/3 = 0$

$\therefore \perp^r = \left| \dfrac{4 + 5/3}{\sqrt{2^2+1^2}} \right| = \dfrac{17}{3\sqrt{5}}$ — (D)

(40) By formula

$\dfrac{h-7}{2} = \dfrac{k-8}{3} = -\left(\dfrac{14+24-4}{13} \right)$

$\Rightarrow h = \dfrac{23}{13}, \quad k = 2/13$

(41) By formula of Image

$\dfrac{h+5}{2} = \dfrac{k-13}{-3} = -2\left(\dfrac{-52}{13} \right)$ $\therefore (h, k) = (11, -11)$

(42) let the line be $(2x-y+1) + \lambda(x+y-2) = 0$

put $O(0,0)$ $\therefore \lambda = 1/2$

Eqn is $(2x-y+1) + \frac{1}{2}(x+y-2) = 0$

$\Rightarrow 5x - y = 0$

(43) Intersecn point is $(-3, 0)$ & slope $= 2$

$\therefore$ Eqn $\dfrac{y}{x+3} = 2 \Rightarrow 2x - y + 6 = 0$

(44) upon Solving two given lines

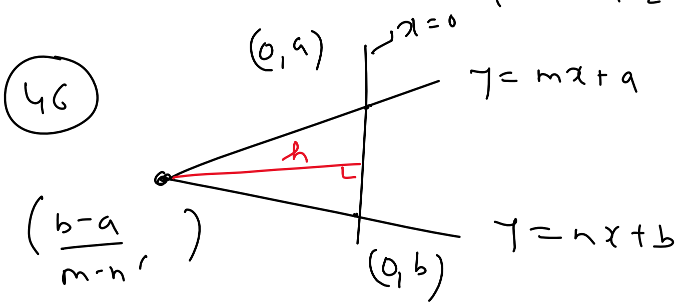
(44) Upon solving two given lines intersection point is (1,1)

& slope $= 2/3$

Hence eqⁿ $\frac{y-1}{x-1} = \frac{2}{3} \Rightarrow 2x-3y+1=0$ — (c)

(45) By formula $\left|\frac{(c_1-c_2)(d_1-d_2)}{a_1b_2-a_2b_1}\right| = \left|\frac{2\times 1}{16-9}\right| = \frac{2}{7}$

(46)



Since $x=0$ is vertical line

$\therefore$ Area $= \frac{1}{2}\times B\times H$

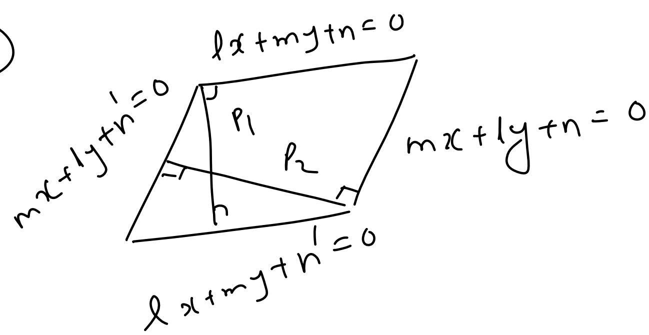
$= \frac{1}{2}|a-b|\times\left|\frac{b-a}{m-n}\right|$

(47) let line be $\frac{x}{a}+\frac{y}{b}=1$ — (i)

$\therefore$ given $\frac{1}{a}+\frac{1}{b}=\frac{1}{p} \Rightarrow \frac{p}{a}+\frac{p}{b}=1$

$\Rightarrow (p,p)$ satisfies eqⁿ(i)

(48)



observe $P_1 = \left|\frac{n-n'}{\sqrt{l^2+m^2}}\right|$

$P_2 = \left|\frac{n-n'}{\sqrt{m^2+l^2}}\right|$

Since $P_1=P_2 \Rightarrow$ Rhombus

$\Rightarrow$ diagonals are $\perp$r

(49) $L_1: \frac{x}{\cos\theta} + \frac{y}{\sin\theta} = a$ OR $x\sin\theta + y\cos\theta - \frac{a\sin 2\theta}{2} = 0$

$$p_1 = \left|\frac{a\sin 2\theta}{2}\right|$$

$L_2: x\cos\theta - y\sin\theta - a\cos 2\theta = 0 \quad \therefore p_2 = \left|\frac{a\cos 2\theta}{1}\right|$

$$\therefore 4p_1^2 + p_2^2 = a^2\sin^2 2\theta + a^2\cos^2 2\theta = a^2$$

(50) By condition of concurrency $\begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_3 & q_3 & 1 \end{vmatrix} = 0$

Area of $\Delta = \frac{1}{2}\begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_3 & q_3 & 1 \end{vmatrix} = 0 \quad \therefore$ Collinear pts

(51) By condition of concurrency $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

$\Rightarrow a+b+c = 0$ or $\cancel{\boxed{a = b = c}}$ → rejected as line will be coincident

(52) rearrange $(2x+3y-5)\cos\theta + (3x-5y+2)\sin\theta = 0$

or $(2x+3y-5) + \tan\theta(3x-5y+2) = 0$

Since line in form $L_1 + \lambda L_2 = 0$

or $(2x+3y-5)+\tan\theta(3x-5y+2)=0$

Since line in form $L_1+\lambda L_2=0$

$\Rightarrow$ it passes through pt of intersection

of $2x+3y=5$
& $3x-5y=-2$ i.e. $(1,1)$

(53) let the no's be $2n-1,\ 2n+1,\ 2n+3$

$\therefore$ line is $(2n-1)x+(2n+1)y+2n+3=0$

re-arrange $2n[x+y+1]+(-x+y+3)=0$

By family of lines it passes through

pt of intersection of $x+y+1=0$
& $-x+y+3=0$

i.e. $(1,-2)$

(54) If $a+b+c=0$ $\therefore$ line L_1 becomes $ax-ay=p$ or $x-y=p/a$

L_2 becomes $bx-by=p$ or $x-y=p/b$

L_3 becomes $cx-cy=p$ or $x-y=p/c$

which are clearly parallel

(55) let slope be m & $4m$

$\therefore$ $m+4m=-10$ $\Rightarrow$ $m=-2$

$m(4m)=a$ & $a=16$

(56) By formula $d=\left|2\sqrt{\dfrac{g^2-ac}{\ldots}}\right|$

(56) By formula $d = \left| 2\sqrt{\dfrac{g^2 - ac}{a(a+b)}} \right|$

$$d = \left| 2\sqrt{\frac{4-1}{1(1+2)}} \right| = 2$$

(57) By Homogenijation the line pair is given by

$$3x^2 + 4xy - 4x(2x+y) + (2x+y)^2 = 0$$

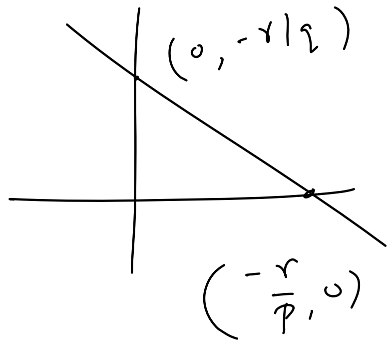
OR

$$-x^2 + 4xy + y^2 = 0 \qquad \therefore \quad a+b=0$$

$\Rightarrow \perp^r$ lines

1(B)

(1)



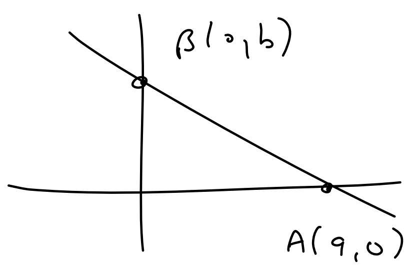
$\therefore$ Area $= \frac{1}{2}\left|\frac{r^2}{pq}\right|$

But given $r^2 = pq$

$\Rightarrow$ Area $= \frac{1}{2}$ = Constant

(2) All point lie on line $y = \frac{bx}{a}$ $\Rightarrow$ Colli

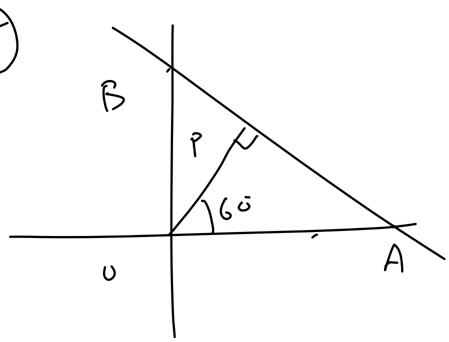
(3)



Area $= \frac{1}{2} \times a \times b$ — (B)

(4) Same as Q5

(5)



let perpendicular distance be p

$\therefore$ Eqn of line

$x\cos 60° + y\sin 60° = p$

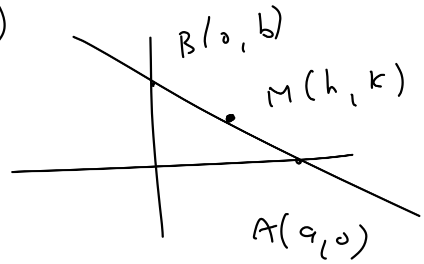
$OA = \frac{p}{\cos 60°}$, $OB = \frac{p}{\sin 60°}$

Area $= \frac{1}{2} \cdot \frac{p^2}{\sin 60° \cos 60°} = 54\sqrt{3}$

$p^2 = 81 \Rightarrow p = 9$ — (A)

$p^2 = 81 \Rightarrow p = 9$ — (A)

(6)



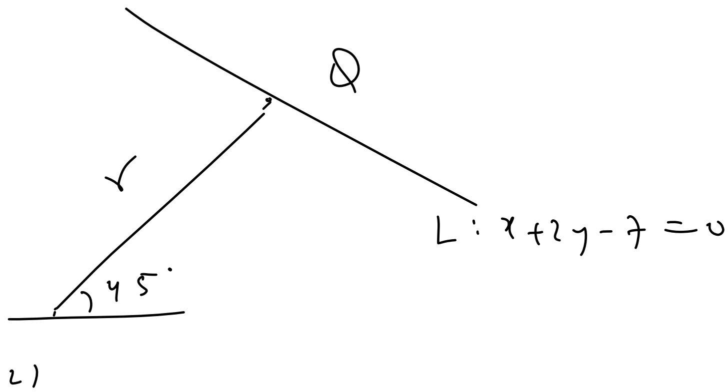
$\therefore\ h = a/2, \quad b = k/2$

given $a + b = 10$

$\Rightarrow h + k = 5$

or $x + y = 5$

(7)

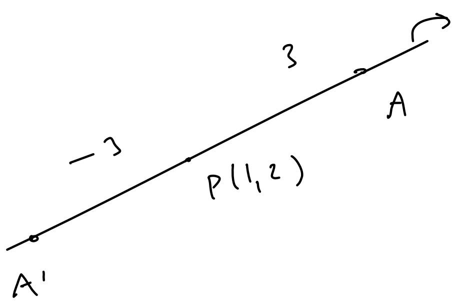


Let $PQ = r$ $\therefore$ coordinates of $Q(1 + r\cos 45°, 2 + r\sin 45°)$

put Q on line $L = 0$ $\therefore$ $1 + \frac{r}{\sqrt{2}} + 4 + \sqrt{2}\,r - 7 = 0$

$r = \sqrt{2}$

(8)

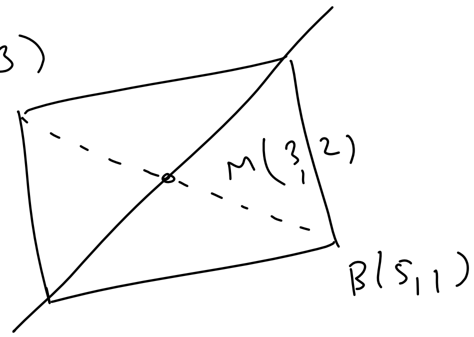


$\tan\theta = 60°$

By parametric Eqn of line

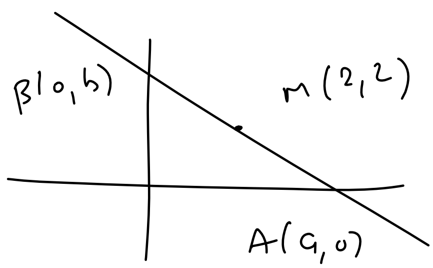
A & A' $(1 \pm 3\cos 60°, 2 \pm 3\sin 60°)$

or $\left(1 \pm 3/2, 2 \pm \frac{3\sqrt{3}}{2}\right)$ — (C)

(9) 

put M(3, 2) on line

$2 = 6 + c \Rightarrow c = -4$

(10) 

By Section formula

$\therefore$ $a = 4$, $b = 4$

$\therefore$ line $\frac{x}{4} + \frac{y}{4} = 1$

(11) Conceptual

(12) Let the pt on line $x + y = 4$ be $(t, 4 - t)$

$\therefore$ $\left| \frac{4(t) + 3(4 - t) - 10}{5} \right| = 1 \Rightarrow t = 3, -7$

Ans $(3, 1)$ OR $(-7, 11)$

(13) $3a + 2b = 13$

& $-a + 4b = 5$

solve a & b

$a = 3$, $b = 2$

$\therefore$ $P(3, 2)$ $Q(2, 3)$ $\therefore$ line PQ : $x + y = 5$

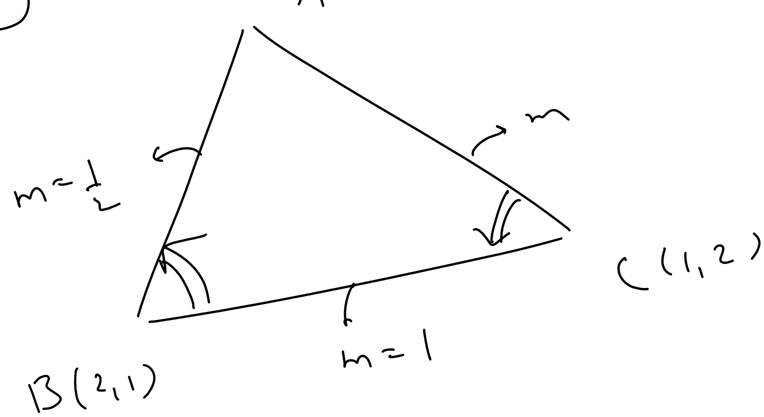
(14) $4x + y = 1$ & $7x - 3y = 35$ Solve $x = 2$, $y = -7$

The Eqn of line is $\dfrac{y+7}{x-2} = \dfrac{12}{1}$

$\Rightarrow \boxed{12x - y = 31}$ which passes through mid pt of $(0,0)$ & $(8,34)$ i.e. $(4,17)$

$\Rightarrow$ Equidistant from $(0,0)$ & $(8,34)$

(15) AB: $y = x/2$



let slope of AC be m

$$\frac{\frac{1}{2} - 1}{1 + \frac{1}{2}} = \frac{1-m}{1+m}$$

$$-\frac{1}{3} = \frac{1-m}{1+m}$$

$\Rightarrow -1 - m = 3 - 3m$

$\Rightarrow m = 2$

$\therefore$ Eqn $\dfrac{y-2}{x-1} = 2$ — (B)

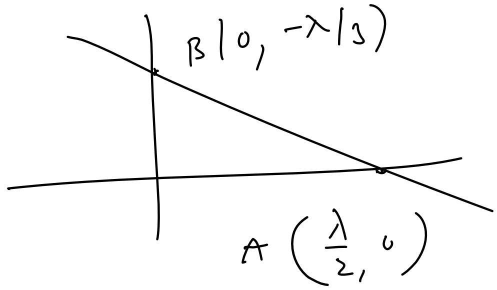
(16) $-x - y = 7/3$

$$\Rightarrow -\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = \frac{7}{3\sqrt{2}} \Rightarrow x\cos 225^\circ + y\sin 225^\circ = \frac{7}{3\sqrt{2}}$$

$$P = \frac{7}{3\sqrt{2}}$$

$$p = \frac{7}{3\sqrt{2}}$$

(17) let line be $2x - 3y = \lambda$



$$\therefore \text{Area} = \left| \frac{1}{2} \cdot \frac{\lambda}{2} \cdot \frac{-\lambda}{3} \right| = 12$$

$$\Rightarrow \lambda = \pm 12$$

Ans $2x - 3y \pm 12 = 0$

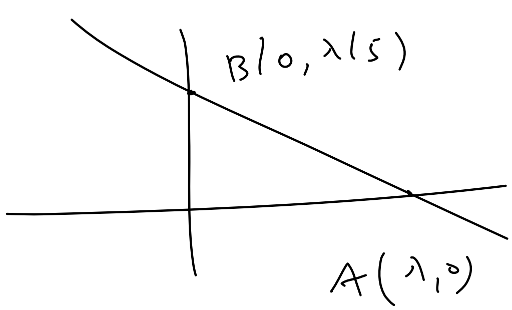
(18) solve $x - y = 4$, $3x + y = 7$ $\quad x = \frac{11}{4}, \; y = -5/4$

$\therefore$ eqn of line is $\quad y + 5/4 = -\frac{1}{2}(x - 11/4)$

$$8y + 10 = -4x + 11$$

$$\Rightarrow 4x + 8y = 1$$

(19) let the line be $x + 5y = \lambda$

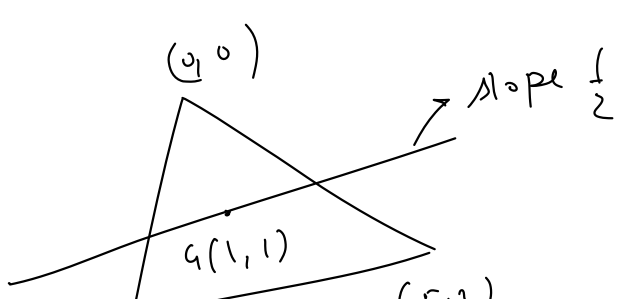


$$\text{Area} = \frac{1}{2}\left(\frac{\lambda^2}{5}\right) = 5$$

$$\therefore \lambda = \pm 5\sqrt{2}$$

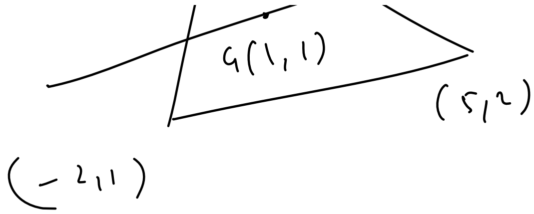
Ans $x + 5y \pm 5\sqrt{2} = 0$

(20)



Eqn $\quad y - 1 = \frac{1}{2}(x - 1)$

$$2y - 2 = x - 1$$



$$2y - 2 = x - 1$$
$$x - 2y + 1 = 0$$

(21) let the line be $x \operatorname{cosec}\theta - y \sec\theta = \lambda$

$\therefore$ put $(a\cos^3\theta, a\sin^3\theta)$

$$\Rightarrow \frac{a\cos^3\theta}{\sin\theta} - \frac{a\sin^3\theta}{\cos\theta} = \lambda$$

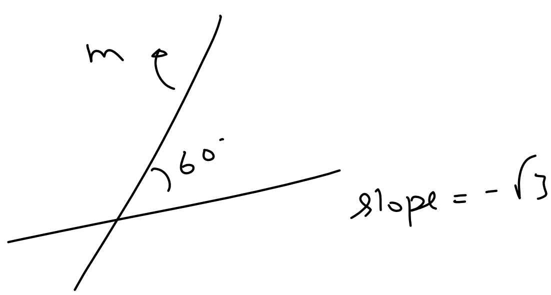
$$\Rightarrow \frac{2a\cos 2\theta}{\sin 2\theta} = \lambda$$

$\therefore$ Ans

$$\frac{x}{\sin\theta} - \frac{y}{\cos\theta} = \frac{2a\cos 2\theta}{\sin 2\theta}$$

OR $x\cos\theta - y\sin\theta = a\cos 2\theta$

(22)



$$\therefore \tan 60^\circ = \left| \frac{m+\sqrt{3}}{1-\sqrt{3}m} \right|$$

OR $\frac{m+\sqrt{3}}{1-\sqrt{3}m} = \sqrt{3}$ or $-\sqrt{3}$

$m + \sqrt{3} = \sqrt{3} - 3m$ or $-\sqrt{3} + 3m$

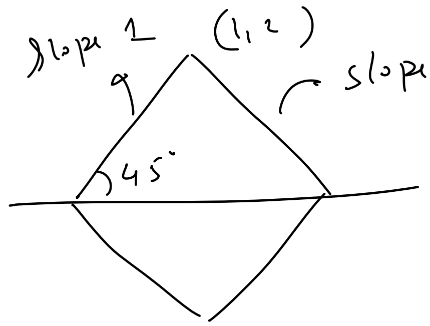
$m = 0$ or $m = \sqrt{3}$

$\therefore$ Eqn

$y = -2$ or $\frac{y+2}{x-3} = \sqrt{3}$

OR $y = -2$ or $\sqrt{3}x - y = 2 + 3\sqrt{3}$

(23)



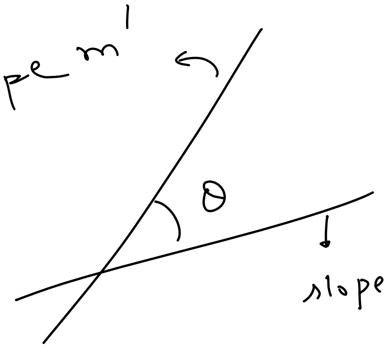
$\therefore$ Eqn of sides

$$\frac{y-2}{x-1} = 1 \text{ or } -1$$

Ans $\quad x - y + 1 = 0$ or $x + y = 3$

(24)

let slope m'



$$\therefore \left| \frac{m' - m}{1 + mm'} \right| = \tan\theta = m$$

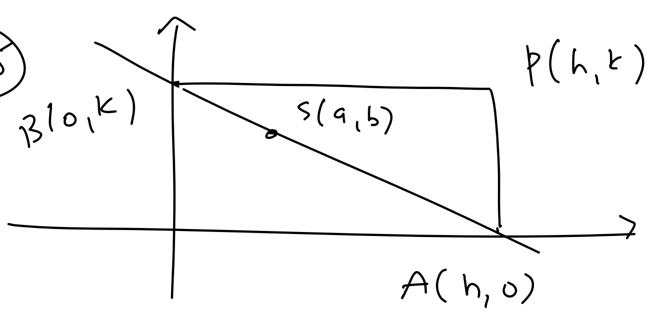
$$\frac{m' - m}{1 + mm'} = m \text{ or } -m$$

$$\Rightarrow m' - m = m + m^2 m' \text{ or } -m - m^2 m'$$

$$\Rightarrow m' = 0 \text{ or } m' = \frac{2m}{1 - m^2}$$

$$\therefore \text{Eqn of line } y = 0 \text{ or } \frac{y}{x} = \frac{2m}{1 - x^2}$$

(25)



Eqn of line AB

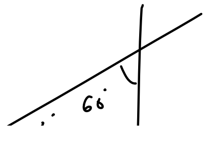
$$\frac{x}{h} + \frac{y}{k} = 1$$

Put (a, b)

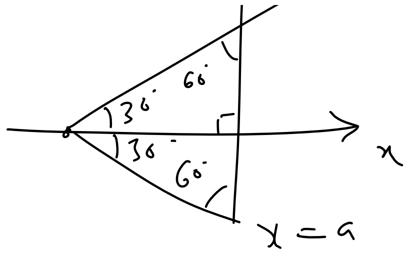
$$\frac{a}{h} + \frac{b}{k} = 1$$

OR $\quad \dfrac{a}{x} + \dfrac{b}{y} = 1$

(26) lines are $x = a$, $y = \pm \dfrac{1}{\sqrt{3}} x$

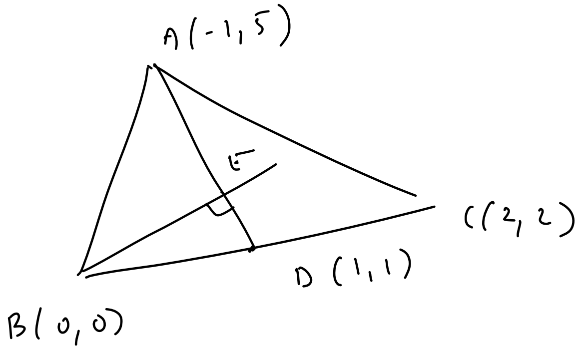


Equilateral Δ



Equilateral Δ

(27)



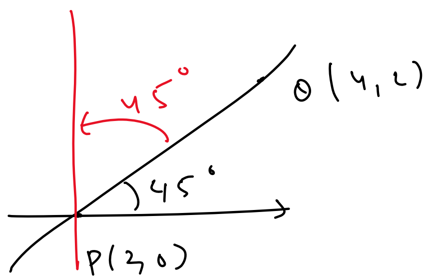
$m_{AD} = -2$

$\therefore m_{BE} = 1/2$

$\therefore$ Eqn of $\perp$r is

$\frac{y}{x} = \frac{1}{2}$

(28)



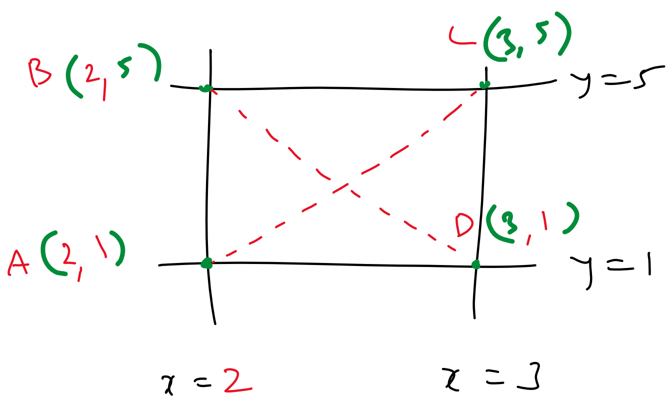
final line PQ is $\perp$r to x-axis $\therefore$ Eqn $x = 2$

(29) Slope of diagonal $= \frac{1}{2}$

$\therefore$ slope of other diagonal $= -2$ $\therefore$ option (c)

(30) $x^2 - 5x + 6 = 0 \Rightarrow x = 2$ or $x = 3$

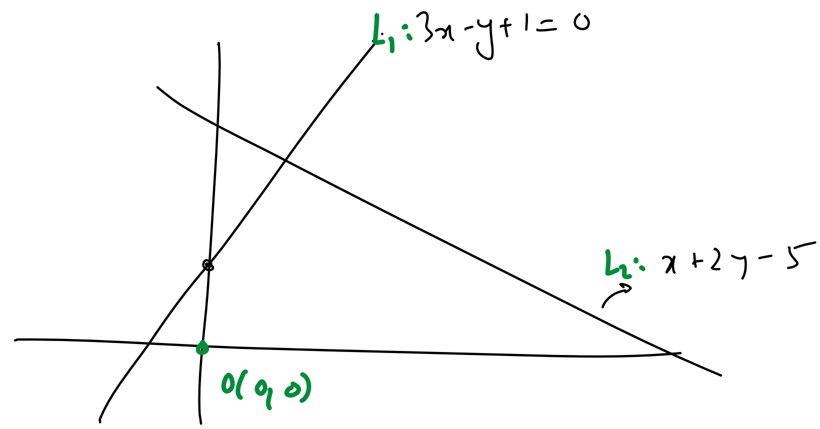
$y^2 - 6y + 5 = 0 \Rightarrow y = 1$ or $y = 5$



$\therefore$ AC: $\frac{y-3}{x-2} = 4$

BD: $\frac{y-1}{x-3} = -4$

(31) Draw accurate diagram



$O(0,0)$ gives (+)ve on L_1 $\Rightarrow$ $(a^2, a+1)$ should follow the same sign

gives (−)ve on L_2 (i) $3a^2-a>0$ & (ii) $a^2+2a-3<0$

upon solving $a \in (-3,0) \cup (\frac{1}{3},1)$

(32) $\perp r \Rightarrow m_1 m_2 = -1$

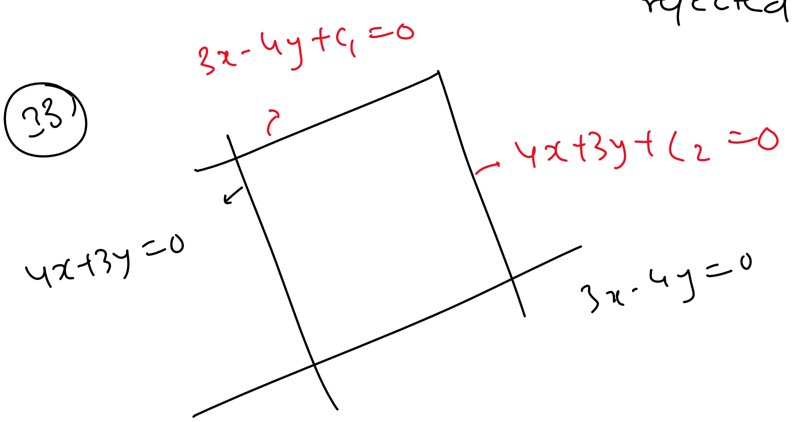
$$-\frac{1}{a-1} \times \frac{-2}{a^2} = -1$$

$$\Rightarrow 0 = a^3 - a^2 + 2$$

$$(a+1)\cancel{(a^2-2a+2)} = 0$$

rejected $\therefore a = -1$ — (D)

(33)



Since area = 25

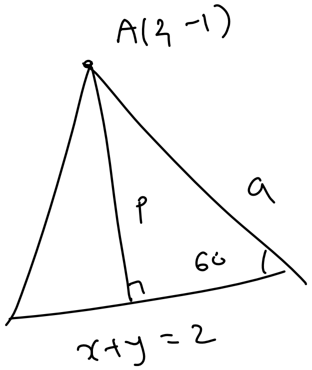
$\Rightarrow$ side length = 5

distance between // lines

$$\left|\frac{c_1}{5}\right| = 5 \quad \& \quad \left|\frac{c_2}{5}\right| = 5 \Rightarrow c_1, c_2 = \pm 25$$

lines are $3x-4y\pm 25=0$ & $4x+3y\pm 25=0$

(34)

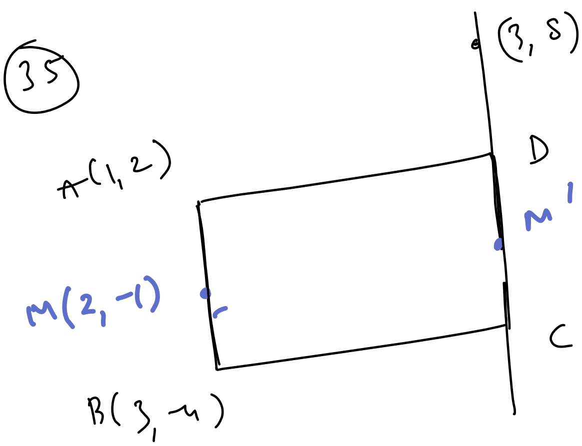


$$p = \left|\frac{2-1-2}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$$

Also $\frac{p}{a} = \sin 60^\circ \Rightarrow a = \sqrt{\frac{2}{3}}$

$\therefore$ Area of $\triangle = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{6}$

(35)



slope AB = -3 = slope CD

$\therefore$ Eqn of line CD $y-8=-3(x-3)$

$\Rightarrow 3x+y=17$

$$BC = MM' = \left|\frac{12}{\sqrt{10}}\right|$$

$m_{BC} = \frac{1}{3}$ $\therefore \sin\theta = \frac{1}{\sqrt{10}}$

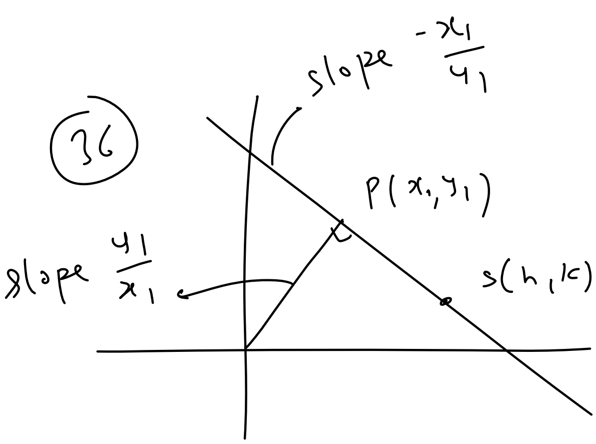
$\cos\theta = \frac{3}{\sqrt{10}}$

By parametric Eqn of line (MM')

$$M'\left(2+\frac{12}{\sqrt{10}}\cdot\frac{3}{\sqrt{10}},\ -1+\frac{12}{\sqrt{10}}\cdot\frac{1}{\sqrt{10}}\right)$$

OR $\left(\frac{28}{5}, \frac{1}{5}\right)$ — (D)

(36)



let the foot of ⊥r be (x_1, y_1)

$\therefore$ Eqn of line PS is

$$\frac{y-y_1}{x-x_1} = -\frac{x_1}{y_1}$$

or $xx_1+yy_1 = x_1^2+y_1^2$

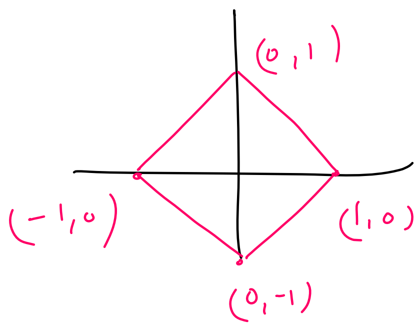
put $S(h,k)$ $hx_1+ky_1 = x_1^2+y_1^2$

put $S(h,k)$ $\quad hx_1 + ky_1 = x_1^2 + y_1^2$

now $\quad x_1 \to x$ $\quad \therefore \; x^2 + y^2 = hx + ky$

$y_1 \to y$

(37) The curves when plotted together form a square



$\therefore \; a = \sqrt{2} \quad \therefore \;$ Area $= 2$

(38) lines are $\quad \sqrt{3}x - y = 4\sqrt{3}k$

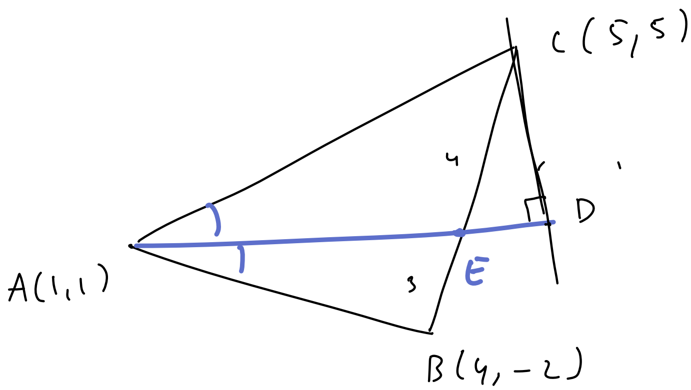
$$\sqrt{3}x + y = \frac{4\sqrt{3}}{k}$$

Multiply Eqns to eliminate variable 'k'

$\Rightarrow \quad 3x^2 - y^2 = 48$ (which is a hyperbola)

(39) Conceptual

(40)



By angle bisector property

$$\frac{AB}{AC} = \frac{BE}{EC} = \frac{3}{4}$$

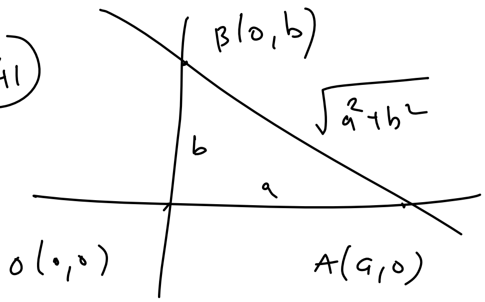
$\therefore$ By section formula get E

$E\left(\frac{31}{7}, 1\right)$

$\therefore \; m_{AE} = 0 \quad \therefore$ CD is a vertical line $\therefore$ Eqn $x = 5$

$\therefore m_{AE} = 0 \quad \therefore$ CD is a vertical line $\therefore$ Eqⁿ $x = 5$

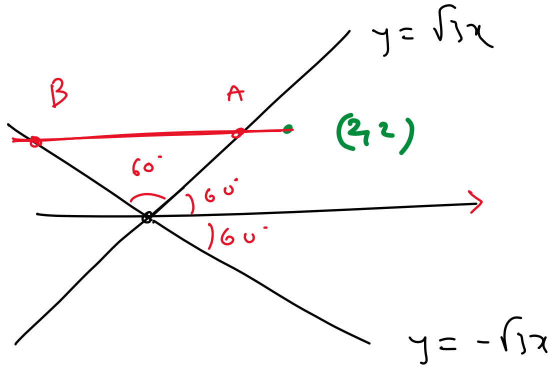
(41)



By formula of Incenter

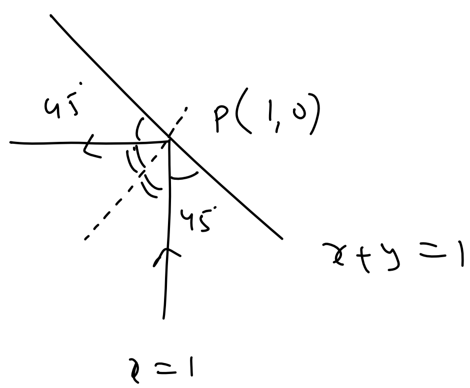
$$\left(\frac{ab}{\sqrt{a^2+b^2}+a+b} \, , \, \frac{ab}{a+b+\sqrt{a^2+b^2}} \right)$$

(42)



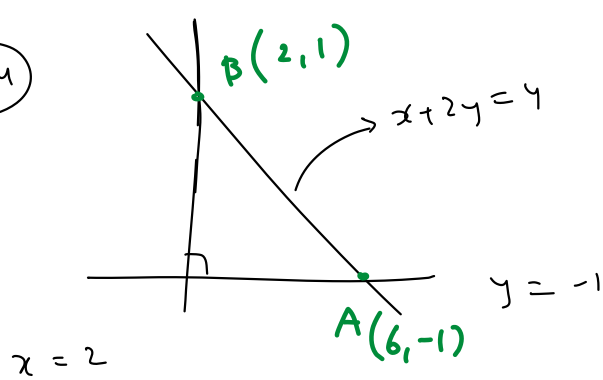
clearly it must be horizontal line i.e. $y = 2$

(43)



the reflected ray is $y = 0$

(44)



Since Right $\Delta \Rightarrow$ mid pt of hypotenuse is circumcenter

$S(4, 0)$

(45)

$$16a^2 - 40ab + 25b^2 = c^2$$

$$(4a - 5b)^2 = c^2$$

(45) $16a^2 - 40ab + 25b^2 = c^2$

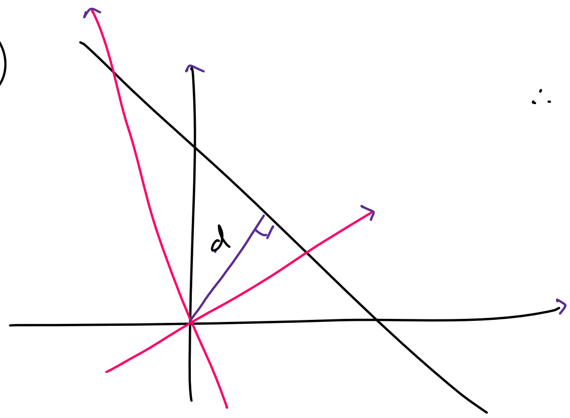
$(4a-5b)^2 = c^2$

$\Rightarrow \quad 4a - 5b + c = 0 \quad$ or $\quad -4a + 5b + c = 0$

$\Rightarrow$ pts $(4,-5)$ & $(-4,5)$

Satisfy the line $ax + by + c = 0$

(46)



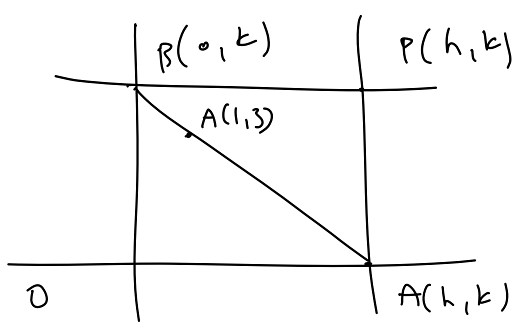
$\therefore$ Since $\perp$r distance of line from $(0,0)$ doesn't change

(i) $\dfrac{x}{a} + \dfrac{y}{b} = 1$ $\qquad$ (ii) $\dfrac{x}{p} + \dfrac{y}{q} = 1$

$\therefore d = \dfrac{1}{\sqrt{\dfrac{1}{a^2} + \dfrac{1}{b^2}}}$ $\qquad$ $d = \dfrac{1}{\sqrt{\dfrac{1}{p^2} + \dfrac{1}{q^2}}}$

Equating $\quad \dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{p^2} + \dfrac{1}{q^2}$

(47)



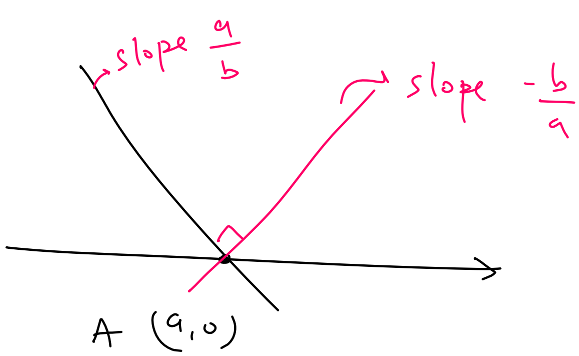
clearly line AB is $\dfrac{x}{h} + \dfrac{y}{k} = 1$

put point $A(1,3)$

$\dfrac{1}{h} + \dfrac{3}{k} = 1$

OR $\quad \dfrac{1}{x} + \dfrac{3}{y} = 1$

(48)



$\therefore$ Eqn of required line

$\dfrac{y}{x-a} = \dfrac{-b}{a}$

$\Rightarrow \boxed{bx + ay = ab}$ — (B)

$\Rightarrow \boxed{bx + ay = ab}$ — (B)

(49) Solving $y = x + 7$

$x + 2y + 1 = 0$ we get $(-5, 2)$

$\therefore$ Required line $\dfrac{y-2}{x-5} = -\dfrac{2}{5}$

$\Rightarrow 2x + 5y = 0$

(50) Solve $x - y + 1 = 0$

$3x + y - 5 = 0 \Longrightarrow (1, 2)$

$\therefore$ Eqn of line can be $\dfrac{y-2}{x-1} = -1$ or $\dfrac{y-2}{x-1} = \dfrac{1}{3}$ — (A)

(51) If a, b, c in A.P $\Longrightarrow a + c = 2b$

or $a - 2b + c = 0$

$\Rightarrow (1, -2)$ lies on $ax + by + c = 0 \Longrightarrow$ (A)

(52) $3a + 2b + 4c = 0 \Rightarrow \frac{3}{4}a + \frac{1}{2}b + c = 0$

clearly $\left(\frac{3}{4}, \frac{1}{2}\right)$ satisfies $ax + by + c = 0$

(53) By condition of concurrency $\begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix} = 0$

$\Rightarrow a(-1) - b(-2) + c(-1) = 0$

$\Rightarrow a - 2b + c = 0 \longrightarrow$ (A)

(54) $x(a+2b)+y(a+3b)=a+b$

$$\Rightarrow \quad a(x+y-1)+b(2x-3y-1)=0$$

By family of lines it passes through pt of intersection of

$x+y-1=0$

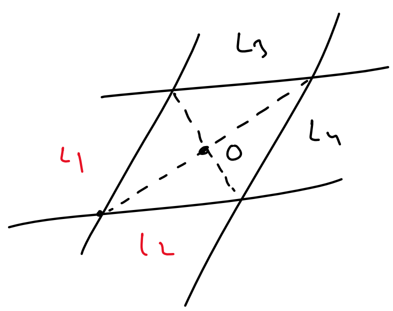
$2x-3y-1=0$ Solve $(-2,1)$

(55) We know line pair through $O(0,0)$ & $\perp^r$ to pair

$ax^2+2hxy+by^2+2gx+2fy+c=0$ is $bx^2-2hxy+ay^2=0$

$\therefore$ Ans is $+3x^2+xy=0$ — (B)

(56)



Given Combined Eqn of L_1L_2

$$6x^2-xy-y^2+x+12y-35=0$$

L_3L_4 is image of L_1L_2 about origin

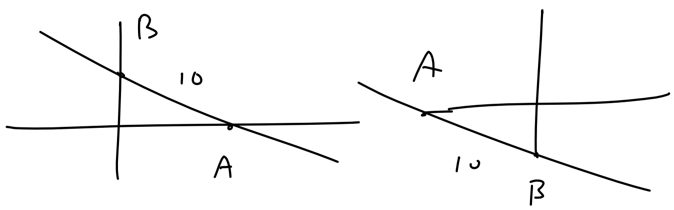
$\therefore$ replace $x\to -x$ & $y\to -y$ we get L_3L_4 as

$6x^2-xy-y^2-x-12y-35=0$ — (A)

1(c)

(1) By distance formula $\perp^r = \left| \dfrac{0-0+2}{\sqrt{3+1}} \right| = 1$

(2) let line be $x+3y=\lambda$



$\therefore$ Just be geometry 2 lines possible (as slope fixed)

$\boxed{\text{Ans } 2}$

(3) for line pair $\Delta = 0$

$$\begin{vmatrix} 12 & -5 & \frac{11}{2} \\ -5 & 2 & -5/2 \\ 11/2 & -5/2 & \lambda \end{vmatrix} = 0$$

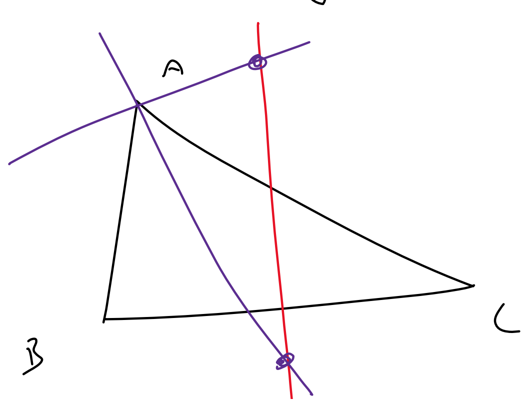
$$= 12\left[2\lambda - \frac{25}{4}\right] + 5\left[-5\lambda + \frac{55}{4}\right] + \frac{11}{2}\left[\frac{25}{2} - 11\right]$$

$$= 24\lambda - 75 - 25\lambda + \frac{275}{4} + \frac{33}{4} = 0$$

$$\Rightarrow -\lambda + \frac{41}{4} = 0 \Rightarrow \boxed{\lambda = \frac{41}{4}}$$

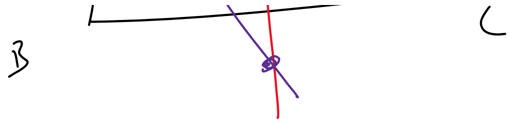
(4) Equidistant from B & C $\Longrightarrow$ pts on $\perp^r$ bisector of BC

Equidistant from AB & AC $\Rightarrow$ pts on angle bisector of $\angle A$



2 pts of intersection

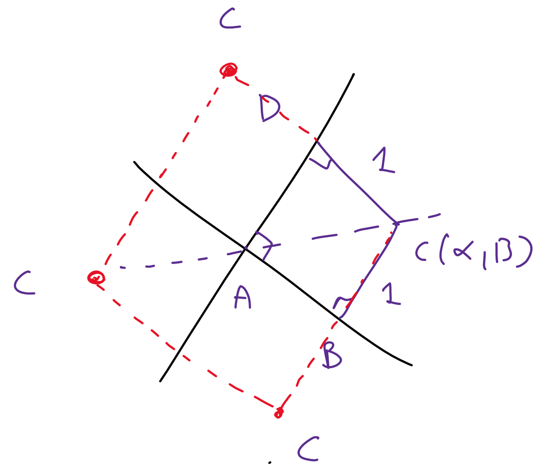
c



(5) 2 lines are $\perp^r$

$\Rightarrow$ ABCD square with side length 1

There can be 4 such pts C



$\therefore$ solving 2 lines we get $A\left(\frac{-3}{5}, \frac{1}{10}\right)$

$\therefore$ Pretty obvious that mean of x coordinates of all 4 possible C's is also pt A $\therefore$ Sum of all possible α's $= 4 \times (-3/5)$

$\therefore$ Ans 12

(6) By formula $\perp^r = \left| 2\sqrt{\frac{g^2 - ac}{a(a+b)}} \right|$

$$= \left| \frac{2 \cdot 5a}{\sqrt{2}} \right| = 25\sqrt{2}$$

$\Rightarrow \quad |5a| = 25 \qquad \therefore |a| = 5$

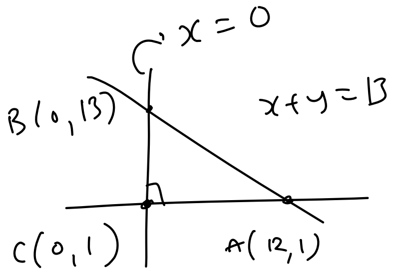
(7) $x^2 - y^2 + 2y = 1$

$x^2 - (y-1)^2 = 0 \Rightarrow$ lines are $x - y + 1 = 0$ & $x + y - 1 = 0$

$\therefore$ Angle bisector are

$$\frac{x-y+1}{\cancel{\sqrt{2}}} = \pm\left(\frac{x+y-1}{\cancel{\sqrt{2}}}\right)$$

$x = 0 \qquad \Rightarrow \qquad y = 1 \qquad x = 0$

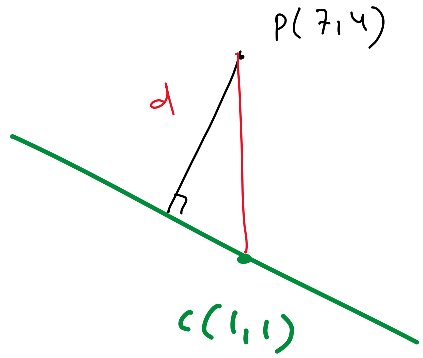


$\Rightarrow$ $y = 1$, $x = 0$

Area $= \frac{1}{2} \times 12 \times 12 = 72$

(8) $2x+3y = 5$

$x+y = 2$ Solve $\therefore$ pt $(1,1)$



Pretty obvious that max 'd' Can be PC i.e. hypotenuse

$\therefore$ The required line is the one $\perp$r to PC & through C(1,1)

i.e. slope $= -2 = -\frac{(2+\lambda)}{3+\lambda}$ $\Rightarrow \lambda = -4$

(9) $(p+q)x + (2p+q)y = p+2q$

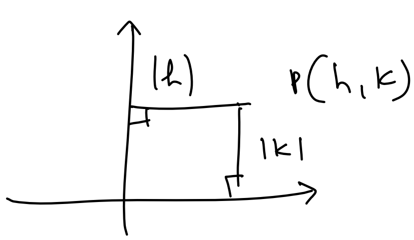
$p(x+2y-1) + q(x+y-2) = 0$

By family $p(x_1, y_1)$ is the pt of intersection of

$x+2y = 1$

$x+y = 2$ $\therefore x_1 + y_1 = 2$

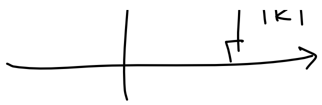
(18) let the 2 lines be axes (Area won't change)



$\therefore$ Given $|h|+|k| = 3$

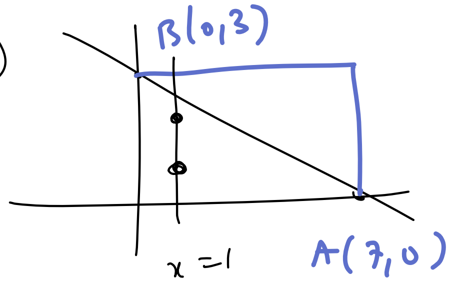
OR $|x|+|y| = 3$

Which forms square of area '18'



Which forms square of area '18'

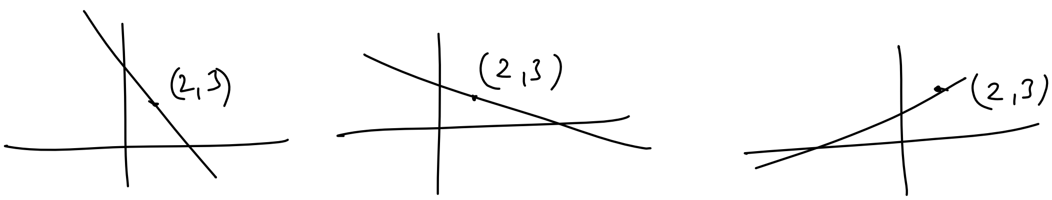
(11)



2 lattice pts on each line $x = 1, 2, 3, \ldots 6$

$\therefore$ total lattice pts inside $= 12$

(12) Conceptual draw diagram & see that 3 lines



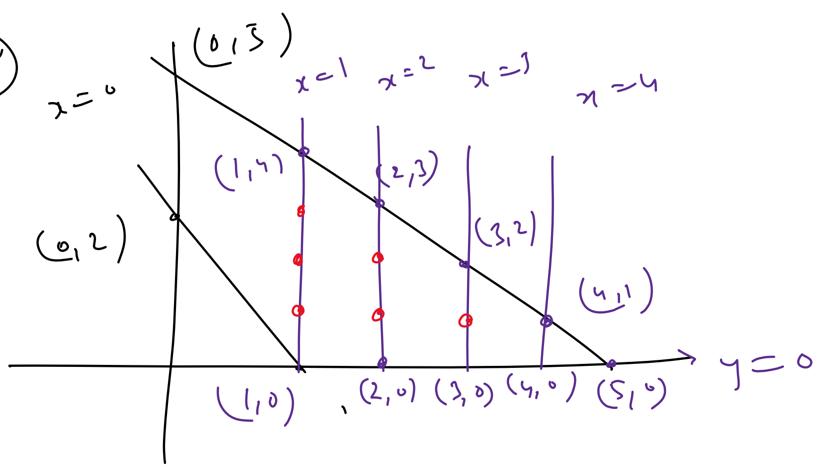
(13) Conceptual

(14) $|x-1| + |y-3| = 1$ Shift the origin to $(1,3)$

$\therefore$ New Eqn $|x| + |y| = 1 \implies$ Square of side length $\sqrt{2}$

$\Rightarrow$ Area 2

(15)



6 pts inside

(16) Homogenize line with give 2-degree curve

$5x^2 - 24xy - 6y^2 + (4x - 2y)(x + ky) + 3(x + ky)^2 = 0$

$$5x^2 - 24xy - 6y^2 + (4x-2y)(x+ky) + 3(x+ky)^2 = 0$$

This Eqn represents required line pair.

Given $m_1 + m_2 = 0 \Rightarrow h = 0$

or Coeff $xy = 0$

$$-24 + 4k - 2 + 3(2k) = 0$$

$$k = 26/10$$

Correction

Q3 $\rightarrow$ 10.25

Q5 $\rightarrow$ 12

Q8 $\rightarrow$ 4

Q9 $\rightarrow$ 2

1. (D)

Let $(1, \sqrt{3})$, (0, 0) and (2, 0) are the coordinates of vertices A, O, B of ΔABC.

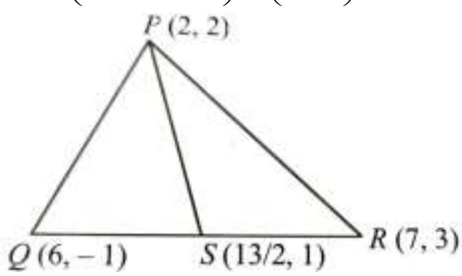
$\therefore$ AO = OB = AB. So, it is an equilaterial triangle and the incentre coincides with centroid.

$$\therefore \quad \text{Incentre} = \left(\frac{0+1+2}{3}, \frac{0+0+\sqrt{3}}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$$

2. (D)

S is the midpoint of Q and R

$$\therefore \ S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$$



Now slope of PS $= \dfrac{2-1}{2-13/2} = -\dfrac{2}{9}$

Now equation of the line passing through (1, -1) and parallel to PS is

$$y+1 = -\frac{2}{9}(x-1) \Rightarrow 2x+9y+7=0$$

3. (A)

$3x + 4y = 9$ and $y = mx + 1$ are two lines.

On equating the value of y from both equations to get the x co-ordinate of the point of intersection,

$$3x + 4(mx+1) = 9 \Rightarrow (3+4m)x = 5$$

$$\Rightarrow \quad x = \frac{5}{3+4m}$$

For x to be an integer $3 + 4m$ should be a divisor of 5 i.e., $1, -1, 5$ or -5.

$3 + 4m = 1 \Rightarrow m = -1/2$ (not integer)

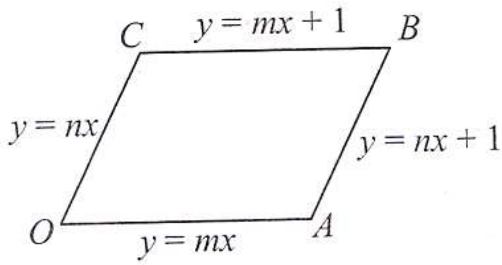
$3 + 4m = -1 \Rightarrow m = -1$ (integer)

$3 + 4m = 5 \Rightarrow m = 1/2$ (not an integer)

$3 + 4m = -5 \Rightarrow m = -2$ (integer)

$\therefore$ There are 2 integral values of m.

4. (D)



The vertices, $O(0,0), A\left(\frac{1}{m-n}, \frac{m}{m-n}\right), B(0,1)$

Area (parallelogram $OABC$) = 2 area (ΔOAB)

$$= 2 \times \frac{1}{2}\left|\left[0\left(\frac{m}{m-n} - 1\right) + \frac{1}{m-n}(1-0) + 0\left(0 - \frac{m}{m-n}\right)\right]\right|$$

$$= \frac{1}{|m-n|}$$

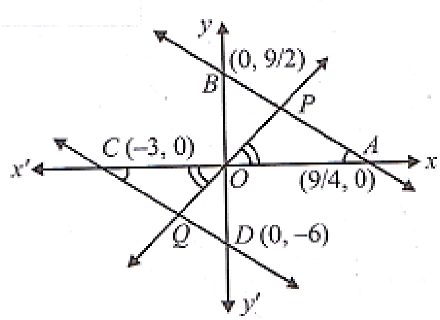
5. (B)

The given lines are

$2x + y = 9/2$ ......(i)

And $2x + y = -6$ ......(ii)

Signs of constants on R.H.S. show that two lines lie
On opposite sides of origin. Let a line through origin
Meets these lines in P and Q respectively then required
Ratio is OP : OQ



In ΔOPA and ΔOQC

$\angle POA = \angle QOC$ (Ver. Opp.angles)

$\angle PAO = \angle OCQ$ (alt.int.angles)

$\therefore$ $\Delta OPA \sim \Delta OCQ$ (By AA similarly)

$\therefore$ $\frac{OP}{OQ} = \frac{OA}{OC} = \frac{9/4}{3} = \frac{3}{4}$

$\therefore$ Required ratio = 3 : 4

6. (A)

7. (A)

8. (D)

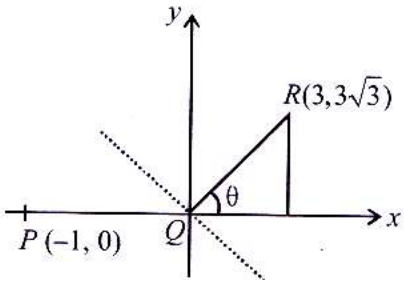
9. (A)

10. (C)

$\tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ \Rightarrow \angle PQR = 120^\circ$

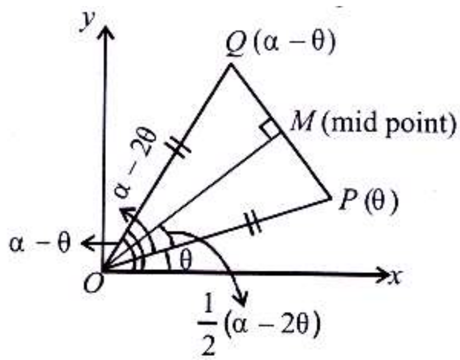
$\therefore$ Slope of bisector of $\angle PQR = \tan 120^\circ$

Hence, equation of bisector is $\sqrt{3}x + y = 0$



11. (D)

Clearly $OP = OQ = 1$ and $\angle QOP = \alpha - \theta - \theta = \alpha - 2\theta$

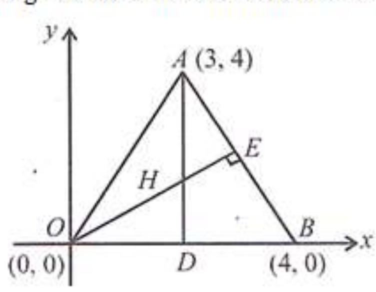


The bisector of $\angle QOP$ will be a perpendicular bisector of PQ also. Hence Q is reflection of P in the line OM which makes an angle $\angle MOP + \angle POX$ with x-axis, i.e., $\frac{1}{2}(\alpha - 2\theta) + \theta = \alpha/2$

So that slope of OM is $\tan\alpha/2$.

12. (C)

We know that point of intersection of altitudes of a triangle is the orthocentre of the triangle



Equation of altitude AD

i.e., line parallel to y-axis through (3, 4) is

$x = 3$ .... (i)

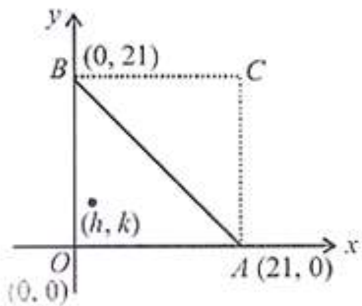
Now, equation of $OE \perp AB$ is

$y = -\frac{3-4}{4-0}x \Rightarrow y = x/4$

Solving (i) and (ii), we get orthocentre as (3, 3/4)

13. (B)

Total number of points within the square OACB
$= 20 \times 20 = 400$



Points line AB = 20 {(1, 20)(2, 19), (3, 18)…….
(10, 11)(11, 10)….(20, 1)
Points within $\Delta OBC = 400 - 20 = 380$
By symmetry, points within $\Delta OBC = \frac{380}{2} = 190$

14. (C)

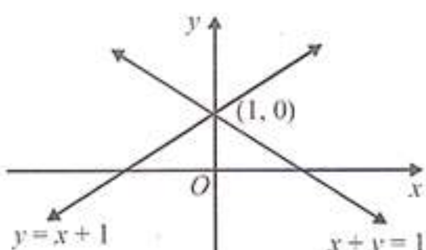
15. (C)

16. (A)

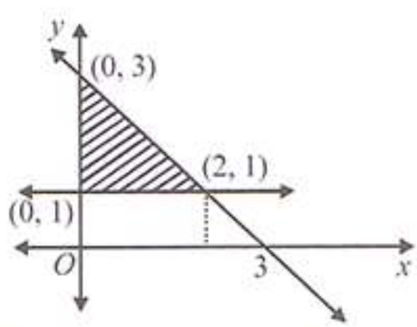
17. (D)

18. (A)

$$x^2 - y^2 + 2y = 1 \quad \Rightarrow \quad x = \pm(y-1)$$



Bisectors of above lines are x = 0 and y = 1



$\therefore$ Area between x = 0, y = 1 and x + y = 3 is the shaded region shown in figure.

$\therefore$ Area $= \frac{1}{2} \times 2 \times 2 = 2$ sq.units.

19. (C)

20. (D)

21. (A)

22. (D)

23. (C)

24. (B)

25. (D)

26. (D)

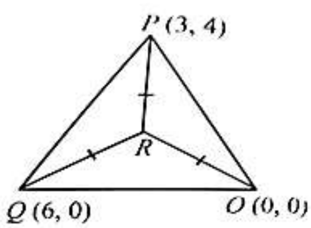
27. (C)

28. (C)

29. (C)

30. (C)

$\because\ Ar(\Delta OPR) = Ar(\Delta PQR) = Ar(OQR)$



$\therefore$ By simply geometry, R should be the centroid of ΔPQO

$\Rightarrow$ coordinate of R $= \left(\frac{3+6+0}{3}, \frac{4+0+0}{3}\right) = \left(3, \frac{4}{3}\right)$

31. (A)

32. (A)

33. (A)

34. (A)

35. (D)

36. (A)

37. (C)

38. (C)

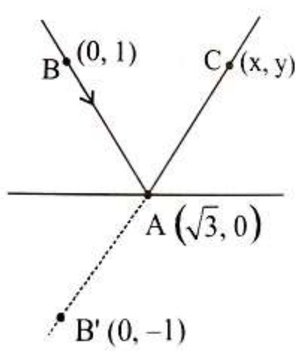
39. (B)

40. (B)

41. (B)

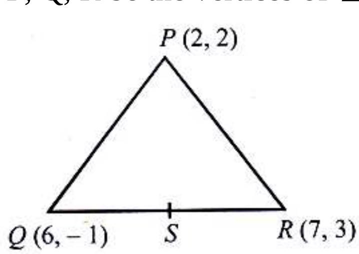
42. (A)
Since three lines $x-3y=p$,
$ax+2y=q$ and $ax+y=r$
Form a right angled triangle
$\therefore$ product of slopes of any two lines $=-1$
Suppose $ax+2y=q$ and $x-3y=p$ are $\perp$ to each other.
$\therefore \frac{-a}{2}\times\frac{1}{3}=-1 \Rightarrow a=6$
Now, consider option one by one
$a=6$ satisfies only option(A)
$\therefore$ Required answer is $a^2-9a+18=0$

43. (B)
Suppose B (0, 1) be any point on given line and co-ordinate of $A\left(\sqrt{3},0\right)$ is . So, equation of



Reflected ray is $\frac{-1-0}{0-\sqrt{3}}=\frac{y-0}{x-\sqrt{3}}$
$\Rightarrow \sqrt{3}y=x-\sqrt{3}$

44. (D)
Let P, Q, R be the vertices of ΔPQR



Since PS is the median
S is mid-point of QR
So, $S=\left(\frac{7+6}{2},\frac{3-1}{2}\right)=\left(\frac{13}{2},1\right)$

Now, slope of $PS = \dfrac{2-1}{2-\dfrac{13}{2}} = -\dfrac{2}{9}$

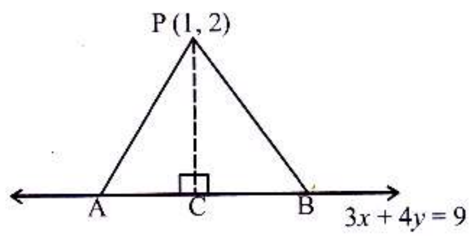
Since, required line is parallel to PS therefore slope of required line $=$ slope of PS

Now, equation of line passing through $(1,-1)$ and having slope $-\dfrac{2}{9}$ is

$$y-(-1) = -\frac{2}{9}(x-1)$$

$$9y+9 = -2x+2 \Rightarrow 2x+9y+7=0$$

45. (B)



Shortest distance of a point (x_1, y_1) from line

$ax+by=c$ is $d = \left|\dfrac{ax_1+by_1-c}{\sqrt{a^2+b^2}}\right|$

Now shortest distance of $P(1,2)$ from $3x+4y=9$ is

$$PC = d = \left|\frac{3(1)+4(2)-9}{\sqrt{3^2+4^2}}\right| = \frac{2}{5}$$

Given that ΔAPB is an equilateral triangle

Let $'a'$ be its side

Then $PB = a, CB = \dfrac{a}{2}$

Now, In $\Delta PCB, (PB)^2 = (PC)^2 + (CB)^2$ (By Pythagoras theorems)

$$a^2 = \left(\frac{2}{5}\right)^2 + \frac{a^2}{4}$$

$$a^2 - \frac{a^4}{4} = \frac{4}{25} \Rightarrow \frac{3a^2}{4} = \frac{4}{25}$$

$$a^2 = \frac{16}{25} \Rightarrow a = \sqrt{\frac{16}{75}} = \frac{4}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{15}$$

$\therefore$ Length of Equilateral triangle $(a) = \dfrac{4\sqrt{3}}{15}$

46. (D)

Circumference = (0, 0)

Centroid = $\left(\dfrac{(a+1)^2}{2}, \dfrac{(a-1)^2}{2}\right)$

We know the circumcentre (O),

Centroid (G) and orthocentre (H) of a triangle lie on the line joining the O and G.

Also, $\frac{HG}{GO} = \frac{2}{1}$

$\Rightarrow$ Coordinate of othocentre = $\frac{3(a+1)^2}{2}, \frac{3(a-1)^2}{2}$

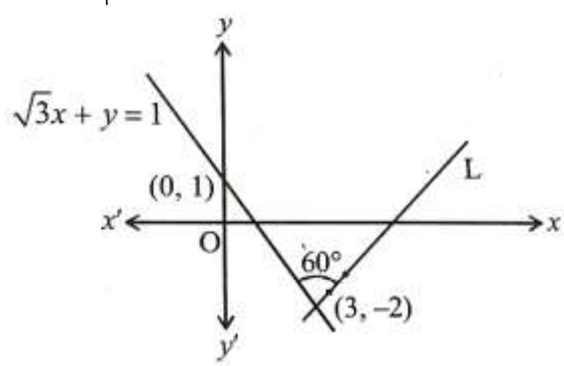
Now, these coordinates satisfies eqn given option (d) Hemce, required eqn. of line is

$(a-1)^2 x - (a+1)^2 y = 0$

47. (B)

Let the slope of line L be m. Then

$\left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$



$\Rightarrow m + \sqrt{3} = \pm\left(\sqrt{3} - 3m\right)$

$\Rightarrow 4m = 0$ or $2m = 2\sqrt{3} \Rightarrow m = 0$ or $m = \sqrt{3}$

$\because$ L intersect x – axis $\quad \therefore m = \sqrt{3}$

$\therefore$ Equation of L is $y + 2 = \sqrt{3}(x-3)$

$\Rightarrow \sqrt{3}x - y - \left(2 + 3\sqrt{3}\right) = 0$

48. (C)

Given eqn of line is $y + \sqrt{3}x - 1 = 0$

$\Rightarrow y = -\sqrt{3}x + 1$

$\Rightarrow (\text{slope})\, m_2 = -\sqrt{3}$

Let the other slope be m_1

$\therefore \tan 60^\circ = \left|\frac{m_1 - \left(-\sqrt{3}\right)}{1 + \left(-\sqrt{3}m_1\right)}\right|$

$\Rightarrow m_1 = 0, m_2 = \sqrt{3}$

Since line L is passing through (3, -2)

$\therefore y - (-2) = +\sqrt{3}(x-3)$

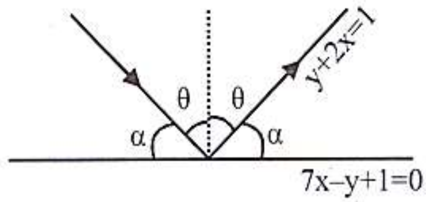
$\Rightarrow y + 2 = \sqrt{3}(x-3) = 0$

$y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

49. (C)

Let slope of incident ray be m.

$\therefore$ angle of incidence = angle of reflection



$$\therefore \quad \left|\frac{m-7}{1+7m}\right| = \left|\frac{-2-7}{1-14}\right| = \frac{9}{13}$$

$$\Rightarrow \frac{m-7}{1+7m} = \frac{9}{13} \text{ or } \frac{m-7}{1+7m} = -\frac{9}{13}$$

$$\Rightarrow 13m - 91 = 9 + 63m \text{ or } 13m - 91 = -9 - 63m$$

$$\Rightarrow 50m = -100 \text{ or } 76m = 82$$

$$\Rightarrow m = -\frac{1}{2} \quad \text{or} \quad m = \frac{41}{38}$$

$$\Rightarrow y - 1 = -\frac{1}{2}(x-0) \text{ and } y - 1 = \frac{41}{38}(x-0)$$

i.e., $x + 2y - 2 = 0$ or $38y - 38 - 41x = 0$

$\Rightarrow 41x - 38y + 38 = 0$

50. (A)

$L_1 : 4x + 3y - 12 = 0$

$L_2 : 3x + 4y - 12 = 0$

$L_1 + \lambda L_2 = 0$

$(4x + 3y - 12) + \lambda(3x + 4y - 12) = 0$

$x(4 + 3\lambda) + y(3 + 4\lambda) - 12(1 + \lambda) = 0$

Point A $\left(\frac{12(1+\lambda)}{4+3\lambda}, 0\right)$

Point B $\left(0, \frac{12(1+\lambda)}{3+3\lambda}\right)$

Mid point $\Rightarrow$ $h = \frac{6(1+\lambda)}{4+3\lambda}$

$k = \frac{6(1+\lambda)}{3+4\lambda}$

51. (B)

Since Orthocentre of the triangle is A(–3, 5) and centriod of the triangle is B(3, 3), then

$AB = \sqrt{40} = 2\sqrt{10}$

Centroid divides orthocentre and circumcentre of the triangle in ratio 2 : 1

$\therefore AB : BC = 2 : 1$

Now, $AB = \frac{2}{3}AC$

$\Rightarrow \quad AC = \frac{3}{2}AB = \frac{3}{2}(2\sqrt{10}) \quad \Rightarrow AC = 3\sqrt{10}$

$\therefore$ Radius of circle with AC as diametre

$= \frac{AC}{2} = \frac{3}{2}\sqrt{10} = 3\sqrt{\frac{5}{2}}$

52. (D)

Equation of the line, which is perpendicular to the line, $3x + y = \lambda(\lambda \neq 0)$ and passing through origin, is given by

For foot of perpendicular

$r = \frac{-(3\times0)+(1\times0)-\lambda)}{3^2+1^2} = \frac{\lambda}{10}$

So, foot of perpendicular $P = \left(\frac{3\lambda}{10}, \frac{\lambda}{10}\right)$

So, foot of perpendicular $P = \left(\frac{3\lambda}{10}, \frac{\lambda}{10}\right)$

Given the line meets $X-$axis at $A = \left(\frac{\lambda}{3}, 0\right)$ and meets Y-axis at $B = (0, \lambda)$

So, $BP = \sqrt{\left(\frac{3\lambda}{10}\right)^2 + \left(\frac{\lambda}{10} - \lambda\right)^2} \Rightarrow BP = \sqrt{\frac{9\lambda^2}{100} + \frac{81\lambda^2}{100}}$

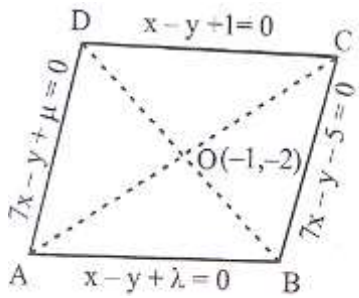
$\Rightarrow BP = \sqrt{\frac{90\lambda^2}{100}}$

Now, $PA = \sqrt{\left(\frac{\lambda}{3} - \frac{3\lambda}{10}\right)^2 + \left(0 - \frac{\lambda}{10}\right)^2}$

$\Rightarrow PA\sqrt{\frac{\lambda^2}{900} + \frac{\lambda^2}{100}} \Rightarrow PA = \sqrt{\frac{10\lambda^2}{900}}$

Therefore $BP : PA = 3 : 1$

53. (A)



Let other two sides of rhombus are

$x-y+\lambda=0$

And $7x-y+\mu=0$

Then O is equidistant from AB and DC and from AD and BC

$\therefore\ |-1+2+1|=|-1+2+\lambda| \Rightarrow \lambda=-3$

And $|-7+2-5|=|-7+2+\mu| \Rightarrow \mu=15$

$\therefore$ Other two sides are x – y – 3 = 0 and 7x – y + 15 = 0

$\therefore$ On solving the equations of sides pairwise, we get the vertices as

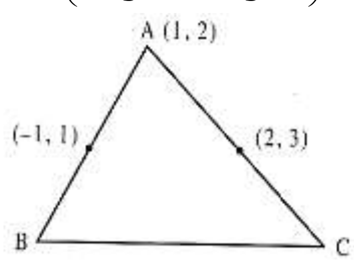
$\left(\frac{1}{3},\frac{-8}{3}\right),(1,2),\left(\frac{-7}{3},\frac{-4}{3}\right),(-3,-6)$

54. (B)

From the mid-point formula co-ordinates of vertex B and C are B(–3, 0) and C(, 4).

Now, centroid of the triangle

$G=\left(\frac{3-3+1}{3},\frac{0+4+2}{3}\right) \Rightarrow G\equiv\left(\frac{1}{3},2\right)$



55. (A)

The given equations of the set of all line

$px+qy+r=0$ ……(i)

And given condition is :

$3p+2q+4r=0$

$\Rightarrow \frac{3}{4}p+\frac{2}{4}q+r=0$ …….(ii)

From (i) & (ii) we get:

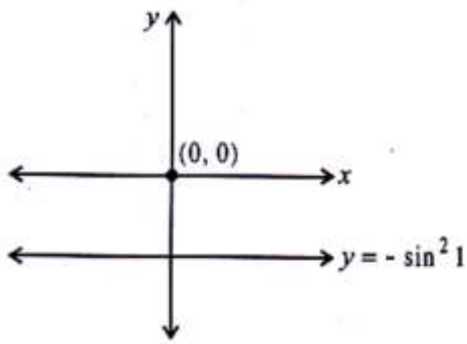
$\therefore x=\frac{3}{4}, y=\frac{1}{2}$

56. (D)

Consider the equation,

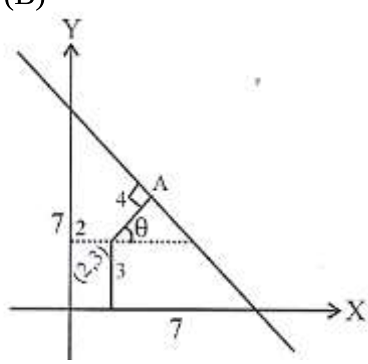
$y=\sin x.\sin(x+2)-\sin^2(x+1)$

$$\frac{1}{2}\left[2\sin x.\sin(x+2)-2\sin^2(x+1)\right]$$
$$=\frac{1}{2}\cos(-2)-\frac{\cos(2x+2)}{2}-\left[\frac{1-\cos(2x+2)}{2}\right]$$
$$=\frac{(\cos 2)-1}{2}=-\sin^2 1$$



By the graph y lies in III and IV quadrant.

57. (B)



Since point at 4 units from P (2, 3) will be

$A(4\cos\theta+2, 4\sin\theta+3)$ and this point will satisfy the equation of line x + y = 7

Since x + y = 7

$2+4\cos\theta+3+4\sin\theta=2$

$\Rightarrow \cos\theta+\sin\theta=\frac{1}{2}$

On squaring

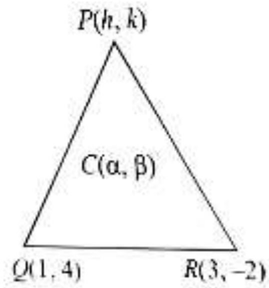
$\Rightarrow \sin 2\theta-\frac{3}{4} \Rightarrow \frac{2\tan\theta}{1+\tan^2\theta}=-\frac{3}{4}$

$\Rightarrow 3\tan^2 \Rightarrow +8\tan\theta+3=0$

$\Rightarrow \tan\theta=\frac{-8\pm 2\sqrt{7}}{6}$ (ignoring –ve sign)

$\Rightarrow \tan\theta=\frac{-8\pm 2\sqrt{7}}{6}=\frac{1-\sqrt{7}}{1+\sqrt{7}}$

58. (C)



Let centroid C be (α, β)

We have $\alpha = \frac{1+3+h}{3} \Rightarrow h = 3\alpha - 4$

$\beta = \frac{4-2+k}{3} \Rightarrow k = 3\beta - 2$

But P (h, k) lies on 2x -3y + 4 =0

$\Rightarrow 2(3\alpha - 4) - 3(3\beta - 2) + 4 = 0$

$\Rightarrow 6\alpha - 9\beta - 8 + 6 + 4 = 0$

$\Rightarrow 6\alpha - 9\beta + 2 = 0$

Locus : $6x - 9y + 2 = 0$

$\Rightarrow y = \frac{6}{9}x + \frac{2}{9} \quad \therefore$ its slope $= \frac{6}{9} = \frac{2}{3}$

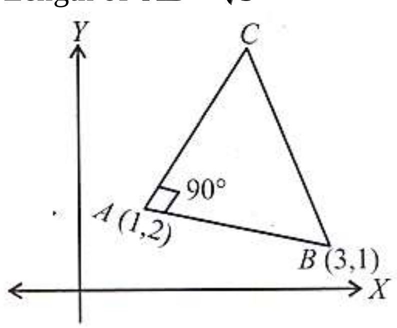
59. (B)

Let ΔABC be in the first quadrant

Slope of line $AB = -\frac{1}{2}$

Slope of line AC = 2

Length of $AB = \sqrt{5}$



It is given that $ar(\Delta ABC) = 5\sqrt{5}$

$\therefore \frac{1}{2} AB \cdot AC = 5\sqrt{5} \Rightarrow AC = 10$

$\therefore$ Coordinate of vertex $C = (1 + 10\cos\theta, 2 + 10\sin\theta)$

$\because \tan\theta = 2 \Rightarrow \cos\theta = \frac{1}{\sqrt{5}}, \sin\theta = \frac{2}{\sqrt{5}}$

$\therefore$ Coordinate of $C = (1 + 2\sqrt{5}, 2 + 4\sqrt{5})$

$\therefore$ Abscissa of vertex C is $1 + 2\sqrt{5}$.

60. (A)

The line in xy-plane is,

$$\frac{x}{3}+y=1 \Rightarrow x+3y-3=0$$

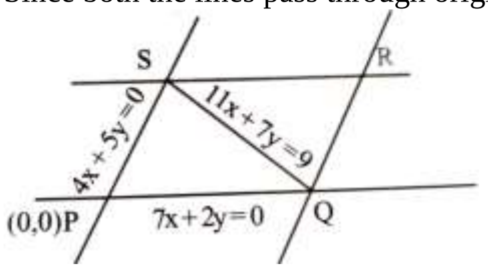
Let image of the point $(-1,-4)$ be (α,β), then

$$\frac{\alpha+1}{a}=\frac{\beta+\gamma}{3}=-\frac{2(-1-12-3)}{10}$$

$$\Rightarrow \alpha+1=\frac{\beta+4}{3}=\frac{16}{5} \Rightarrow \alpha=\frac{11}{5}, \beta=\frac{28}{5}$$

61. (B)

Since both the lines pass through origin. Then



Point S will be point of intersection of 4x + 5y = 0 and 11x + 7y = 9

So, coordinates of point S = $\left(\frac{5}{3}, -\frac{4}{3}\right)$

Also, point Q is point of intersection of 7x + 2y = 9 and 11x + 7y = 9

So, coordinates of point Q = $\left(-\frac{2}{3}, \frac{7}{3}\right)$

Since, diagonals of parallelogram intersect at middle, then the middle point of SQ is

$$\left(\frac{\frac{5}{3}-\frac{2}{3}}{2}, \frac{\frac{-4}{3}+\frac{7}{3}}{2}\right)=\left(\frac{1}{2}, \frac{1}{2}\right)$$

So, equation of diagonal PR is, (y – 0) = $\dfrac{\frac{1}{2}-0}{\frac{1}{2}-0}(x-0)$

y = x

62. (A)

Coordinates of Centroides

$$C=\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$=\left(\frac{3+1+2}{3}, \frac{-1+3+4}{3}\right)=(2,2)$$

The given equation of lines are

$x+3y-1=0$

$3x-y+1=0$

Then, from (i) and (ii)

Point of intersection P $\left(-\frac{1}{5}, \frac{2}{5}\right)$

Equation of line DP

$8x - 11y + 6 = 0$

63. (C)

If equation of pair of straight line are $ax^2 + 2hxy + by^2 = 0$ then pair of angle bisector are $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$

Here $a = 1, b = -5$ and $h = -2$

$\therefore$ Pair of angle bisector are:

$\frac{x^2 - y^2}{1 + 5} = \frac{xy}{-2} \Rightarrow x^2 - y^2 + 3xy = 0$

64. (C)

Let m be the slope of line

$\therefore \left|\frac{m - 3\sqrt{2}}{1 + 3\sqrt{2}m}\right| = \sqrt{2}$

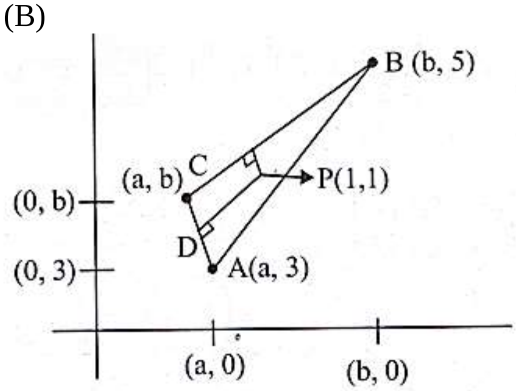
$\Rightarrow m - 3\sqrt{2} = \pm\sqrt{2} \pm 6m \Rightarrow m \mp 6m = \pm\sqrt{2} + 3\sqrt{2}$

$\Rightarrow m = -\frac{4\sqrt{2}}{5}$ or $-\frac{2\sqrt{2}}{7}$

So, the equation of line passing through the point $(1, 3)$ and slope $-\frac{4\sqrt{2}}{5}$ is

$y - 3 = \frac{-4\sqrt{2}}{5}(x - 1) \Rightarrow 4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) = 0$

65. (B)



Slope of AC = ∞

Slope of PD = 0

$D\left(\frac{a + a}{2}, \frac{b + 3}{2}\right) = D\left(a, \frac{b + 3}{2}\right)$

$\frac{b + 3}{2} - 1 = 0; b + 3 - 2 = 0 \Rightarrow b = -1$

$\boxed{b = -1}$

$$E\left(\frac{b+a}{2},\frac{5+b}{2}\right)=\left(\frac{af}{2};2\right)$$

$$\text{Slope of } BC \times \text{Slope of } EP = -1$$

$$\left(\frac{5-b}{b-a}\right)=\left(\frac{2-1}{\frac{a-1}{2}-1}\right)=-1$$

$$\Rightarrow\left(\frac{6}{-1-a}\right)=\left(\frac{2}{a-3}\right)=-1\Rightarrow 12=(1+a)(a-3)$$

$$\Rightarrow 12=a^2-3a+a-3\Rightarrow a^2-2a-15=0$$

$$(a-5)(a+3)=0$$

Given $ab>0\Rightarrow a(-1)>0;-a>0;a<0$

$\boxed{a=-3}$ Accept

Equation of AP (-3, 3), P (1, 1)

$$y-1=\left(\frac{3-1}{-3-1}\right)(x-1)$$

$$-2y+2=x-1$$

$$\Rightarrow \boxed{x+2y=3}$$

$B(-1,5)$

$C(-3,-1)$

Equation of BC

$$(y-5)=\frac{6}{2}$$

$$y-5=3x+3$$

$$\boxed{y=3x+8}$$

Solving (i) and (ii)

$$x+2(3x+8)=3$$

$$\Rightarrow 7x+16=3\Rightarrow 7x=-13\Rightarrow x=-\frac{13}{7}$$

$$y=3\left(-\frac{13}{7}\right)+8=\frac{-39+56}{7}:y=\frac{17}{7}$$

$$x+y=\frac{-13+17}{7}=\frac{4}{7}$$

66. (B)

Given coordinates of ΔABC are

$$A(\alpha,-2):B(\alpha,6):C\left(\frac{\alpha}{4},-2\right)$$

AC is perpendicular to AB

So, ΔABC is right angles at A.

Circumcentre = mid point of BC = $\left(\frac{5\alpha}{8},2\right)$