STRAIGHT LINES

Exercise – 1(A)

Q.1

Slope of line is $\Longrightarrow \dfrac{\sqrt{3}-0}{-2-1} = \dfrac{-1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \dfrac{-1}{\sqrt{3}}$, $\theta = 150^o$

Q.2

Mid-point of B & C is (4 , 2):

slope is $\dfrac{3-2}{2-4} = -\dfrac{1}{2}$

Q.3

Vertices of integral $\Longrightarrow$ Δ never equilateral

Because equate Areas

$$\Rightarrow \frac{\sqrt{3}}{2}\left((x_1-x_2)^2+(y_1-y_2)^2\right)=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

This is irrational & $R\pi S$ is Rational

It is a contradiction $\therefore$be equilateral

Q.4

Slope is zero : 4^2

$\Rightarrow$ y = 2

Q.5

use slope – intercept form

y = 2x – y

Q.6

$\Rightarrow$ m = $\frac{3}{5}$ & c = – 3

$\therefore$ slope intercept form y = $\frac{3}{5}$x – 3

Q.7

$$m = \frac{4-(-5)}{2-3} = -9$$

line is $y = -9x + c$ put (3 , 4) to get c

Q.8

The information c = – 2 & m = $\sqrt{3}$

Slope intercept $y = \sqrt{3x} - 2$

Q.9

form $\frac{y-0}{x-0} = \frac{a\sin\theta - 0}{a\cos\theta - 0} \Rightarrow \boxed{y = x\tan\theta}$

Q.10

Both a = b

$\therefore$ intercept form $\frac{x}{a} + \frac{y}{a} = 1$

$\Rightarrow \boxed{x + y = a}$ ………slope – 1

Q.11

$x\cos\alpha + y\sin\alpha = a$

$\therefore$ for y intercept put x = 0

$\therefore \quad y = a\,\mathrm{cosec}\,\alpha$

Q.12

$\Rightarrow \dfrac{x}{a} + \dfrac{y}{a} = 1$ or $x + y = a$

Put (1, – 2) to get ‘a’

$\Rightarrow 1 - 2 = a \Rightarrow a = -1$

Line is x + y + 1 = 0

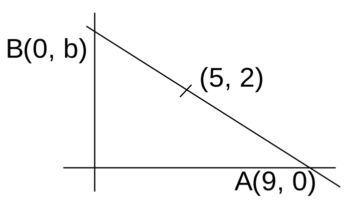
Q.13

Intercept form $\dfrac{x}{a} + \dfrac{y}{b} = 1$; here b = 2a

(1, 2) satisfies $\dfrac{1}{a} + \dfrac{2}{b} = 1$

$\Rightarrow \dfrac{1}{a} + \dfrac{1}{a} = 1 \;\Rightarrow \begin{pmatrix} a = 2 \\ b = 4 \end{pmatrix}$

Q.14

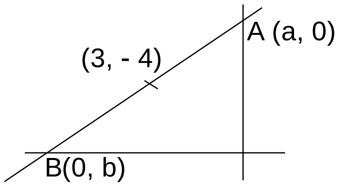


$\Rightarrow \therefore \dfrac{a}{2} = 5 \;\&\; \dfrac{b}{2} = 2$

$\Rightarrow$ a = 10, b = 4

$\Rightarrow \therefore$ line $\dfrac{x}{10} + \dfrac{y}{4} = 1$

Q.15



By section formula

$$\Rightarrow \frac{2a}{5} = 3 \;\; \& \;\; \frac{3b}{5} = -4$$

$$\Rightarrow a = \frac{15}{2}, b = -\frac{20}{3}$$

Line is $\frac{2x}{15} - \frac{3y}{20} = 1$

Q.16

$$\Rightarrow a + b = -2$$

Let $\frac{x}{a} + \frac{y}{b} = 1$

put (3 – 3)

$$\Rightarrow \frac{2}{a} - \frac{3}{b} = 1$$

$$\Rightarrow 2b - 3a - ab = 0$$

$$\Rightarrow 2b + 3(b + 2) + b(b + 2) = 0$$

$$\Rightarrow b^2 + 7b + 6 = 6$$

$$\Rightarrow b = -6 \;\; , \;\; b = -1$$

$$\Rightarrow a = 4 \;\;\; , \;\; b = -1$$

Q.17

$$\Rightarrow \frac{x}{3a} + \frac{y}{3b} = 1$$

$A(3a, 0)$; $B(0, 3b)$; $0(0 , 0)$

Centroid (a, b)

(18) $m_1 = 1$ $\therefore \tan\theta = \left|\frac{\sqrt{3}-1}{1+\sqrt{3}}\right| \Rightarrow \theta = 15^\circ$ — (A)

$m_2 = \sqrt{3}$

(19) $m_1 = -2/3$ Since $m_1 m_2 = -1 \Rightarrow \theta = 90^\circ$ — (D)

$m_2 = 3/2$

(20) $m_1 = 2,\ m_2 = -3$

$\therefore \tan\theta = \left|\frac{2+3}{1-6}\right| = 1,\ \theta = 45^\circ$ — (C)

(21) $m_1 = 1,\ m_2 = 0$ $\therefore \tan\theta = 1 \Rightarrow \theta = 45^\circ$

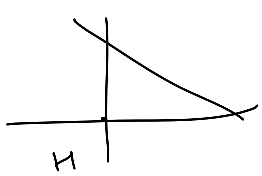
$\therefore$ obtuse $= 135^\circ$ — (B)

(22) $m_1 = 0$ $\therefore \tan\theta = \sqrt{3},\ \theta = 60^\circ$

$m_2 = \sqrt{3}$

(23) slope of $L_1 = 4/7$ Since $m_1 m_2 = -1 \Rightarrow$ right $\triangle$

slope of $L_3 = -7/4$



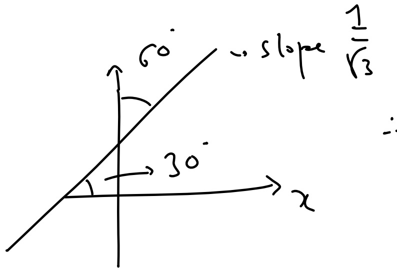
H is right angled vertex

i.e. intersection of $4x - 7y + 10 = 0$

& $7x + 4y - 15 = 0$

solve $H(1, 2)$ — (D)

(24)



$\therefore$ angle $= 60^\circ$ — (B)

(25) $\perp^r \Rightarrow m_1 m_2 = -1$

$$-\frac{m}{2} \times -\frac{2}{3} = -1 \Rightarrow \boxed{m = -3}$$

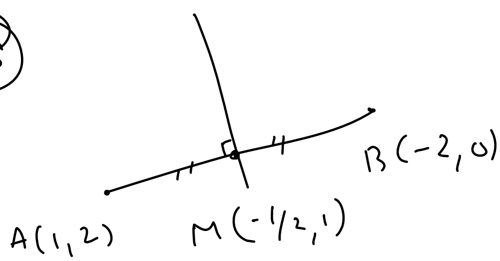
(26) $m_1 = 2 \;,\; m_2 = \frac{3-\lambda}{2}$

Since $m_1 m_2 = -1 \Longrightarrow \frac{3-\lambda}{2} = -\frac{1}{2} \Rightarrow \lambda = 4$ — (A)

(27) slope of line Joining $(-5, 6)$ & $(-6, 5)$ is $m = 1$

$\therefore$ Eqn is $\frac{y-3}{x-4} = -1 \Rightarrow x + y - 5 = 0$ — (D)

(28)



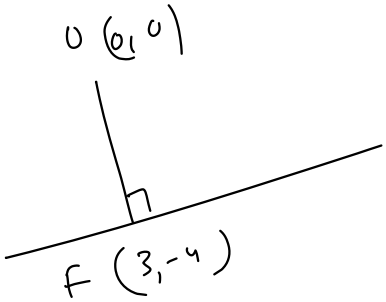
$m_{AB} = \frac{2}{3}$ $\therefore$ $\perp^r$ bisector

$$\frac{y-1}{x+\frac{1}{2}} = -3/2$$

$\Rightarrow 2y - 2 = -3x - 3/2$

$\Rightarrow 3x + 2y = 1/2$ — (C)

(29)



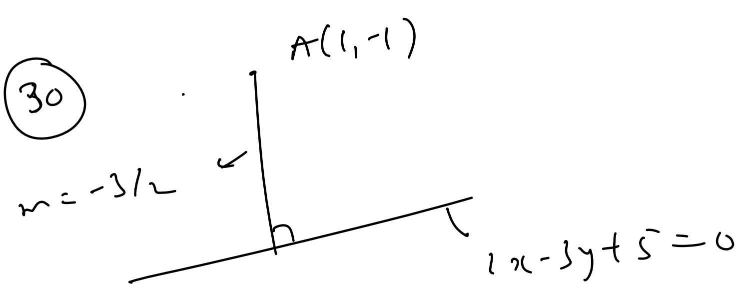
$m_{OF} = -4/3$

line slope $3/4$

Eqn of line $\frac{y+4}{x-3} = \frac{3}{4}$

or $3x - 4y = 25$ — (A)

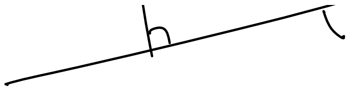
(30)



slope $= -3/2$

$\therefore$ Eqn $\frac{y+1}{x-1} = -3/2$

$m =$



$2x - 3y + 5 = 0$

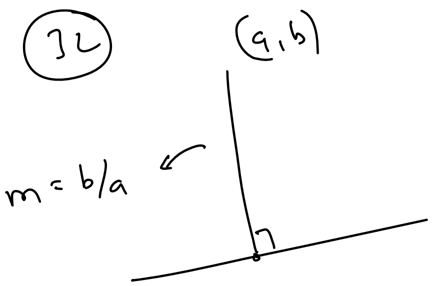
$\therefore$ Eqn $\dfrac{y+1}{x-1} = -3/2$

$-3x + 3 = 2y + 2$

$3x + 2y - 1 = 0$ — (A)

(31) Eqn $\dfrac{y-d}{x-c} = -\dfrac{a}{b} \Rightarrow a(x-c) + b(y-d) = 0$ — (C)

(32)



$ax + by + c = 0$

Eqn is $\dfrac{y-b}{x-a} = \dfrac{b}{a}$

$\boxed{bx - ay = 0}$ — (C)

(33) parallel to x-axis $y = -2$

(34) let line be $\dfrac{x}{a} + \dfrac{y}{b} = \lambda$ put (a,b)

$\Rightarrow \lambda = 2$

$\therefore$ line $\dfrac{x}{a} + \dfrac{y}{b} = 2$ — (B)

(35) Eqn of line is $\dfrac{y-2}{x-2} = \dfrac{1}{3} \Rightarrow x - 3y + 4 = 0$

$\therefore$ y-intercept $4/3$

— (D)

(36) $\perp^r$ distance $= \left| \dfrac{3(2) - 4(1) + 8}{5} \right| = 2$

(37) $\perp^r$ distance $= \left| \dfrac{-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \right| = \dfrac{ab}{\sqrt{a^2 + b^2}}$

(38) $\perp^r = \left|\dfrac{c_1 - c_2}{\sqrt{a^2+b^2}}\right| = \left|\dfrac{9-(-13)}{\sqrt{5^2+12^2}}\right| = \dfrac{22}{13}$

(39) $2x - y + 4 = 0$

$2x - y - 5/3 = 0$

$\therefore \perp^r = \left|\dfrac{4 + 5/3}{\sqrt{2^2+1^2}}\right| = \dfrac{17}{3\sqrt{5}}$ — (D)

(40) By formula

$\dfrac{h-7}{2} = \dfrac{k-8}{3} = -\left(\dfrac{14+24-4}{13}\right)$

$\Rightarrow \quad h = \dfrac{23}{13}, \quad k = 2/13$

(41) By formula of Image

$\dfrac{h+5}{2} = \dfrac{k-13}{-3} = -2\left(-\dfrac{52}{13}\right)$ $\quad \therefore (h, k) = (11, -11)$

(42) let the line be $(2x - y + 1) + \lambda(x + y - 2) = 0$

put $O(0, 0)$ $\quad \therefore \lambda = 1/2$

Eqn is $(2x - y + 1) + \frac{1}{2}(x + y - 2) = 0$

$\Rightarrow \quad 5x - y = 0$

(43) Intersecn point is $(-3, 0)$ & slope $= 2$

$\therefore$ Eqn $\dfrac{y}{x+3} = 2 \quad \Rightarrow \quad 2x - y + 6 = 0$

(44) upon Solving two given lines

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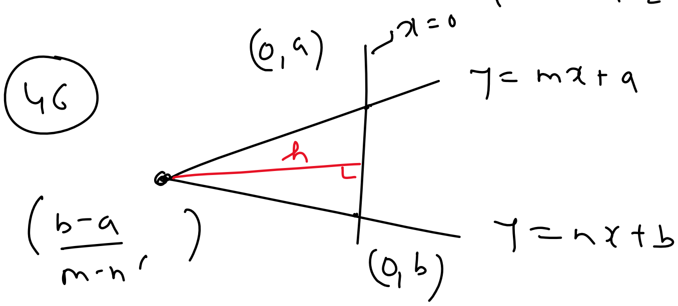
intersection point is (1, 1)

& slope $= 2/3$

Hence eq^n $\frac{y-1}{x-1} = \frac{2}{3} \Rightarrow 2x - 3y + 1 = 0$ — (c)

(45) By formula $\left|\frac{(c_1-c_2)(d_1-d_2)}{a_1b_2-a_2b_1}\right| = \left|\frac{2\times 1}{16-9}\right| = \frac{2}{7}$

(46)



Since $x=0$ is vertical line

$\therefore$ Area $= \frac{1}{2} \times B \times H$

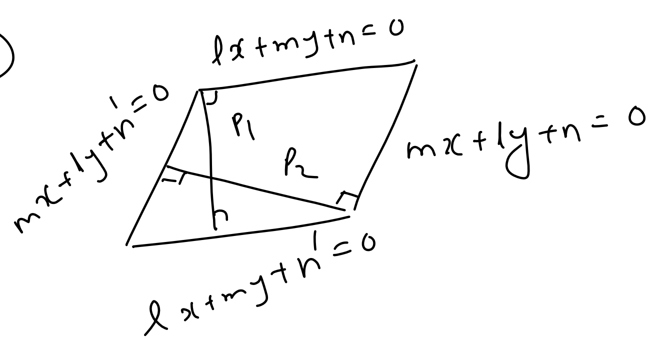
$= \frac{1}{2}\,|a-b| \times \left|\frac{b-a}{m-n}\right|$

(47) let line be $\frac{x}{a} + \frac{y}{b} = 1$ — (i)

$\because$ given $\frac{1}{a} + \frac{1}{b} = \frac{1}{p} \Rightarrow \frac{p}{a} + \frac{p}{b} = 1$

$\Rightarrow (p, p)$ satisfies eq^n(i)

(48)



observe $P_1 = \left|\frac{n-n'}{\sqrt{l^2+m^2}}\right|$

$P_2 = \left|\frac{n-n'}{\sqrt{m^2+l^2}}\right|$

Since $P_1 = P_2 \Rightarrow$ Rhombus

$\Rightarrow$ diagonals are $\perp r$

(49) $L_1: \frac{x}{\cos\theta} + \frac{y}{\sin\theta} = a$ OR $x\sin\theta + y\cos\theta - \frac{a\sin 2\theta}{2} = 0$

$p_1 = \left|\frac{a\sin 2\theta}{2}\right|$

$L_2: x\cos\theta - y\sin\theta - a\cos 2\theta = 0 \quad \therefore p_2 = \left|\frac{a\cos 2\theta}{1}\right|$

$\therefore 4p_1^2 + p_2^2 = a^2\sin^2 2\theta + a^2\cos^2 2\theta = a^2$

(50) By condition of concurrency $\begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_3 & q_3 & 1 \end{vmatrix} = 0$

. Area of $\Delta = \frac{1}{2}\begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_3 & q_3 & 1 \end{vmatrix} = 0$ $\therefore$ Collinear pts

(51) By condition of concurrency $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

$\Rightarrow a+b+c = 0$ or $\cancel{\boxed{a = b = c}}$ → rejected as line will be coincident

(52) rearrange

$(2x+3y-5)\cos\theta + (3x-5y+2)\sin\theta = 0$

or $(2x+3y-5) + \tan\theta(3x-5y+2) = 0$

Since line in form $L_1 + \lambda L_2 = 0$

or $(2x+3y-5) + \tan\theta(3x-5y+2) = 0$

Since line in form $L_1 + \lambda L_2 = 0$

$\Rightarrow$ it passes through pt of intersection

of $2x+3y=5$
& $3x-5y=-2$ i.e. $(1,1)$

(53) let the no.s be $2n-1,\ 2n+1,\ 2n+3$

$\therefore$ line is $(2n-1)x + (2n+1)y + 2n+3 = 0$

re-arrange $2n[x+y+1] + (-x+y+3) = 0$

By family of lines it passes through

pt of intersection of $x+y+1=0$
& $-x+y+3=0$

i.e. $(1,-2)$

(54) If $a+b+c=0$: line L_1 becomes $ax-ay=p$ or $x-y=p/a$

L_2 becomes $bx-by=p$ or $x-y=p/b$

L_3 becomes $cx-cy=p$ or $x-y=p/c$

which are clearly parallel

(55) let slope be m & $4m$

$\therefore$ $m+4m = -10$ $\Rightarrow$ $m=-2$

$m(4m) = a$ & $a = 16$

(56) By formula $d = \left| 2\sqrt{\dfrac{g^2-ac}{\ldots}} \right|$

(56) By formula $d = \left| 2\sqrt{\dfrac{g^2-ac}{a(a+b)}} \right|$

$$d = \left| 2\sqrt{\frac{4-1}{1(1+2)}} \right| = 2$$

(57) By Homogenijation the line pair is given by

$$3x^2 + 4xy - 4x(2x+y) + (2x+y)^2 = 0$$

OR

$$-x^2 + 4xy + y^2 = 0 \quad \therefore \quad a+b=0$$

$\Rightarrow \perp^r$ lines

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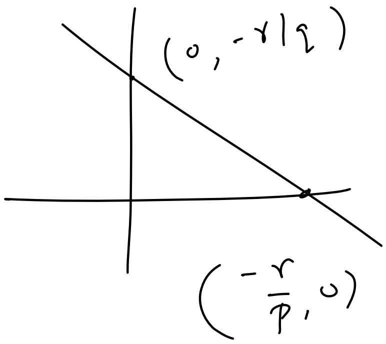
1 (B)

(1)



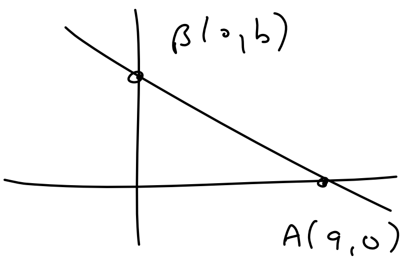
$\therefore$ Area $= \frac{1}{2}\left|\frac{r^2}{pq}\right|$

But given $r^2 = pq$

$\Rightarrow$ Area $= \frac{1}{2}$ = Constant

(2) All point lie on line $y = \frac{bx}{a}$ $\Rightarrow$ Colli

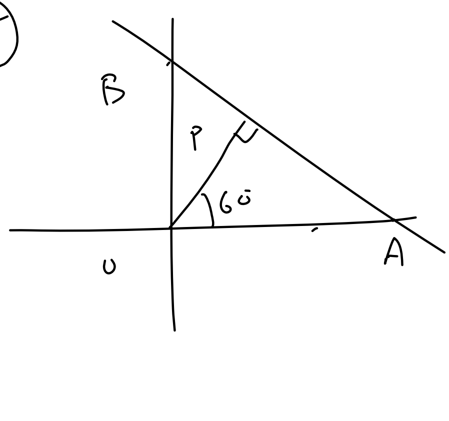
(3)



Area $= \frac{1}{2} \times a \times b$ — (B)

(4) Same as Q5

(5)



let perpendicular distance be p

$\therefore$ Eqn of line

$x\cos 60° + y\sin 60° = p$

$OA = \frac{p}{\cos 60°}$, $OB = \frac{p}{\sin 60°}$

Area $= \frac{1}{2} \cdot \frac{p^2}{\sin 60° \cos 60°} = 54\sqrt{3}$

$p^2 = 81 \Rightarrow p = 9$ — (A)

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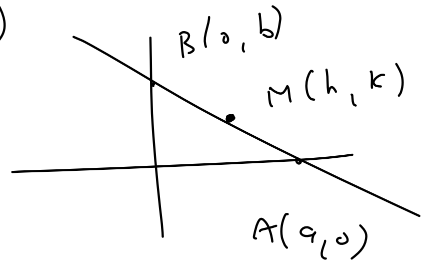
(6)



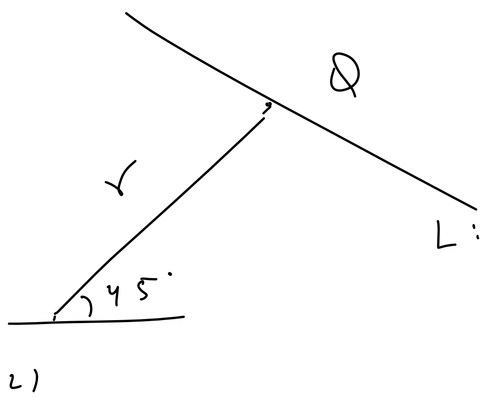
$\therefore h = a/2, \quad b = k/2$

given $a + b = 10$

$\Rightarrow h + k = 5$

or $x + y = 5$

(7)

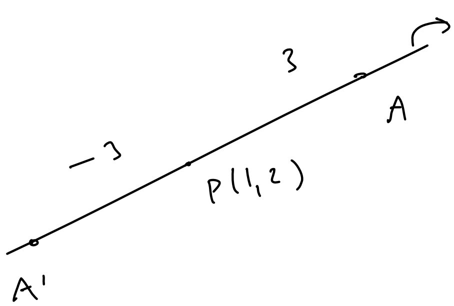


Let $PQ = r$ $\therefore$ coordinates of $Q(1 + r\cos 45°, 2 + r\sin 45°)$

put Q on line $L = 0$ $\therefore$ $1 + \frac{r}{\sqrt{2}} + 4 + \sqrt{2}r - 7 = 0$

$r = \sqrt{2}$

(8)



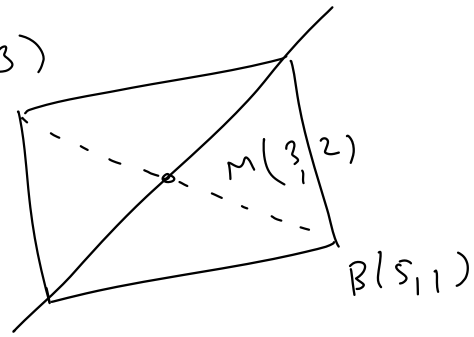
$\tan\theta = 60°$

By parametric Eqn of line

$A \;\&\; A' \; (1 \pm 3\cos 60°, \; 2 \pm 3\sin 60°)$

or $\left(1 \pm 3/2, \; 2 \pm \frac{3\sqrt{3}}{2}\right)$ — (C)

(9) $A(1,3)$

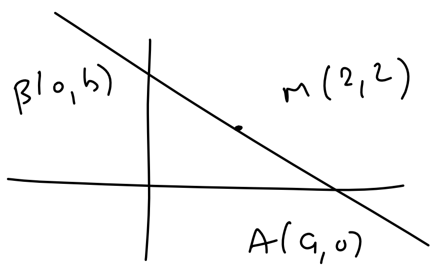


$y = 2x + c$

put $M(3,2)$ on line

$2 = 6 + c \Rightarrow c = -4$

(10)



By Section formula

$\therefore\ a = 4\ ,\ b = 4$

$\therefore$ line $\dfrac{x}{4} + \dfrac{y}{4} = 1$

(11) Conceptual

(12) Let the pt on line $x+y=4$ be $(t, 4-t)$

$\therefore \quad \left| \dfrac{4(t) + 3(4-t) - 10}{5} \right| = 1 \quad \Rightarrow \quad t = 3, -7$

Ans $(3, 1)$ OR $(-7, 11)$

(13) $3a + 2b = 13$

& $-a + 4b = 5$

solve a & b

$a = 3\ ,\ b = 2$

$\therefore\ P(3, 2) \quad Q(2, 3) \quad \therefore$ line PQ : $x + y = 5$

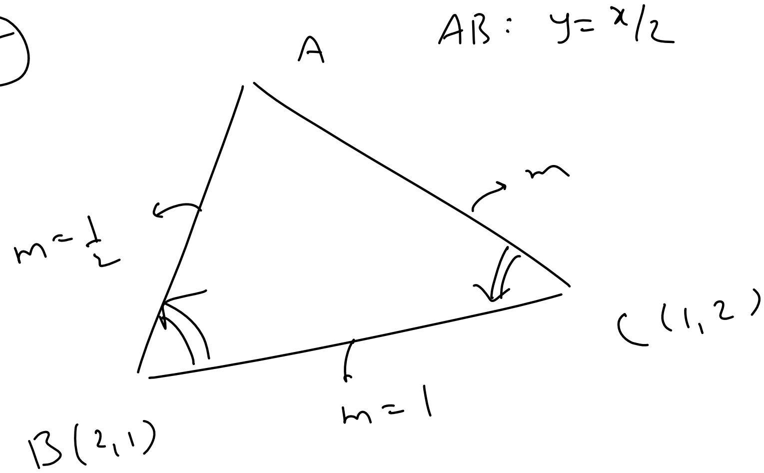
(14) $4x+y=1$ & $7x-3y=35$ Solve $x=2$, $y=-7$

The Eqn of line is $\frac{y+7}{x-2} = \frac{12}{1}$

$\Rightarrow \boxed{12x - y = 31}$ which passes through mid pt of $(0,0)$ & $(8,34)$ ie $(4,17)$

$\Rightarrow$ Equidistant from $(0,0)$ & $(8,34)$

(15) AB: $y = x/2$



let slope of AC be m

$$\frac{\frac{1}{2} - 1}{1 + \frac{1}{2}} = \frac{1-m}{1+m}$$

$$-\frac{1}{3} = \frac{1-m}{1+m}$$

$\Rightarrow -1-m = 3-3m$

$\Rightarrow m = 2$

$\therefore$ Eqn $\frac{y-2}{x-1} = 2$ — (B)

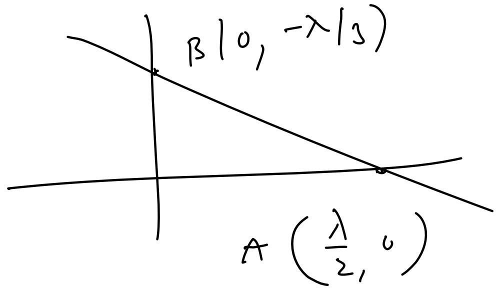
(16) $-x-y = 7/3$

$\Rightarrow -\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = \frac{7}{3\sqrt{2}} \Rightarrow x\cos 225^\circ + y\sin 225^\circ = \frac{7}{3\sqrt{2}}$

$p = \frac{7}{3\sqrt{2}}$

$p = \frac{7}{3\sqrt{2}}$

(17) let line be $2x - 3y = \lambda$



$\therefore$ Area $= \left| \frac{1}{2} \cdot \frac{\lambda}{2} \cdot \frac{-\lambda}{3} \right| = 12$

$\Rightarrow \lambda = \pm 12$

Ans $2x - 3y \pm 12 = 0$

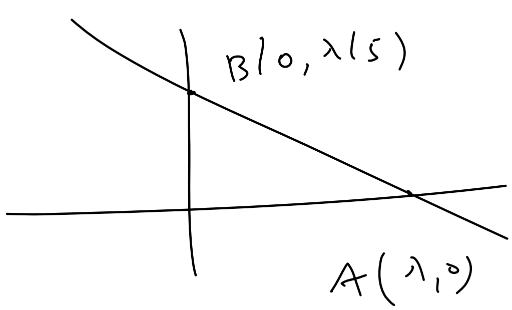
(18) Solve $x - y = 4$
$3x + y = 7$
$x = \frac{11}{4}, \ y = -5/4$

$\therefore$ eqn of line is $y + 5/4 = -\frac{1}{2}(x - 11/4)$

$8y + 10 = -4x + 11$

$\Rightarrow 4x + 8y = 1$

(19) let the line be $x + 5y = \lambda$

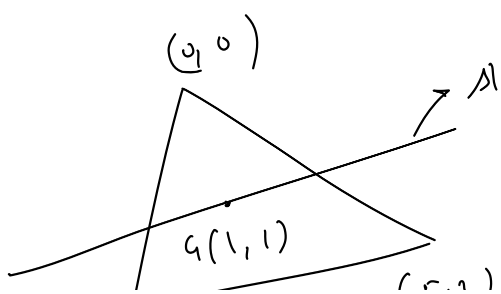


Area $= \frac{1}{2}\left(\frac{\lambda^2}{5}\right) = 5$

$\therefore \lambda = \pm 5\sqrt{2}$

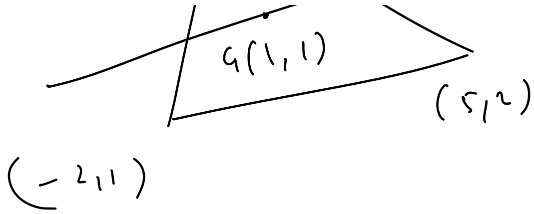
Ans $x + 5y \pm 5\sqrt{2} = 0$

(20)



Eqn $y - 1 = \frac{1}{2}(x - 1)$

$2y - 2 = x - 1$



$2y - 2 = x - 1$

$x - 2y + 1 = 0$

(21) let the line be $x \operatorname{cosec}\theta - y \sec\theta = \lambda$

$\therefore$ put $(a\cos^3\theta, a\sin^3\theta)$

$$\Rightarrow \frac{a\cos^3\theta}{\sin\theta} - \frac{a\sin^3\theta}{\cos\theta} = \lambda$$

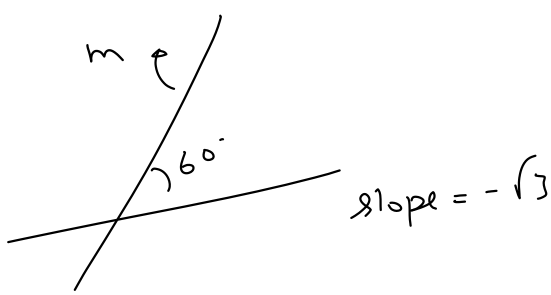
$$\Rightarrow \frac{2a\cos 2\theta}{\sin 2\theta} = \lambda$$

$\therefore$ Ans

$$\frac{x}{\sin\theta} - \frac{y}{\cos\theta} = \frac{2a\cos 2\theta}{\sin 2\theta}$$

OR $\quad x\cos\theta - y\sin\theta = a\cos 2\theta$

(22)



$$\therefore \tan 60^\circ = \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right|$$

OR $$\frac{m+\sqrt{3}}{1-\sqrt{3}m} = \sqrt{3} \text{ or } -\sqrt{3}$$

$$m+\sqrt{3} = \sqrt{3}-3m \text{ or } -\sqrt{3}+3m$$

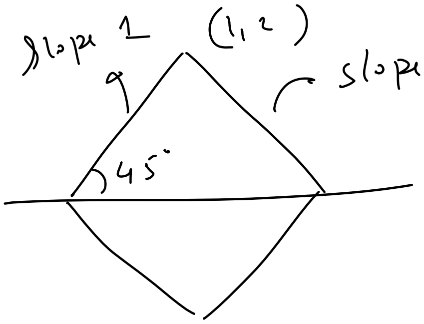
$$m = 0 \text{ or } m = \sqrt{3}$$

$\therefore$ Eqn

$$y = -2 \quad \text{or} \quad \frac{y+2}{x-3} = \sqrt{3}$$

OR $\quad y = -2 \quad$ or $\quad \sqrt{3}x - y = 2 + 3\sqrt{3}$

(23)

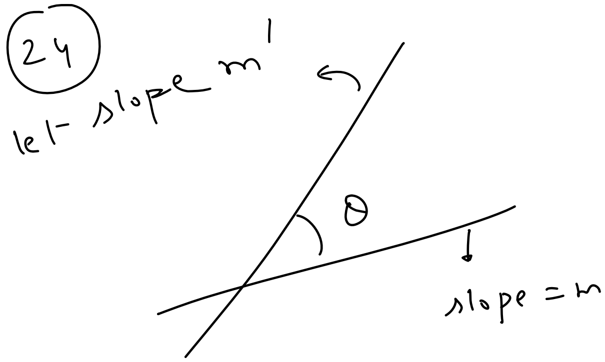


$\therefore$ Eqn of sides

$$\frac{y-2}{x-1} = 1 \text{ or } -1$$

Ans $\quad x - y + 1 = 0 \quad$ or $\quad x + y = 3$

(24)



$$\therefore \left|\frac{m'-m}{1+mm'}\right| = \tan\theta = m$$

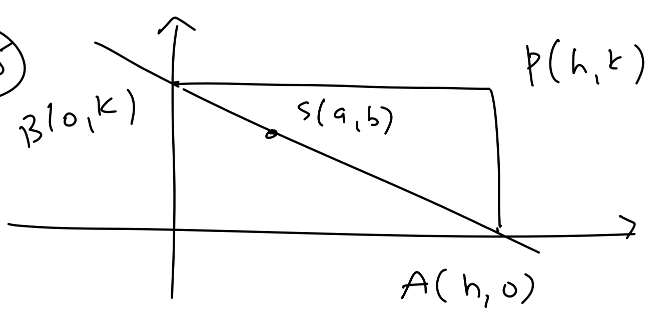
$$\frac{m'-m}{1+mm'} = m \text{ or } -m$$

$$\Rightarrow m'-m = m + m^2 m' \text{ or } -m - m^2 m'$$

$$\Rightarrow m' = 0 \text{ or } m' = \frac{2m}{1-m^2}$$

$\therefore$ Eqn of line $y = 0$ or $\dfrac{y}{x} = \dfrac{2m}{1-x^2}$

(25)



Eqn of line AB

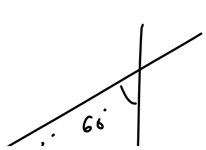
$$\frac{x}{h} + \frac{y}{k} = 1$$

Put (a,b)

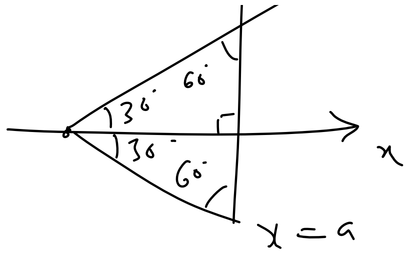
$$\frac{a}{h} + \frac{b}{k} = 1$$

OR $\quad \dfrac{a}{x} + \dfrac{b}{y} = 1$

(26) lines are $x = a$, $y = \pm \dfrac{1}{\sqrt{3}}x$

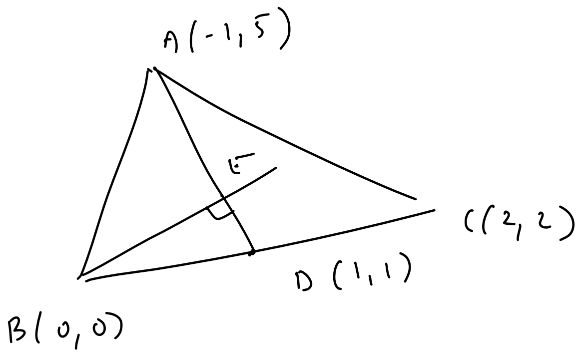


Equilateral Δ



Equilateral Δ

(27)



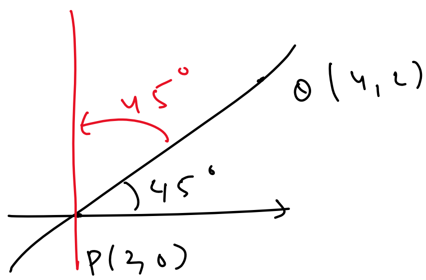
$m_{AD} = -2$

$\therefore m_{BE} = 1/2$

$\therefore$ Eqⁿ of $\perp^r$ is

$$\frac{y}{x} = \frac{1}{2}$$

(28)



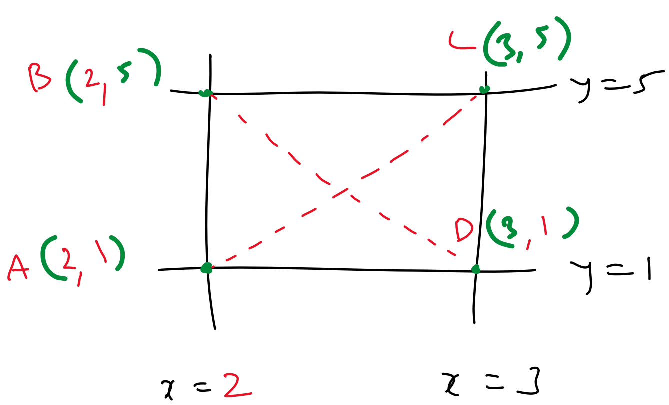
final line PQ is $\perp^r$ to x-axis $\therefore$ Eqⁿ $x = 2$

(29) Slope of diagonal $= \frac{1}{2}$

$\therefore$ slope of other diagonal $= -2$ $\quad \therefore$ option (c)

(30) $x^2 - 5x + 6 = 0 \Rightarrow x = 2$ or $x = 3$

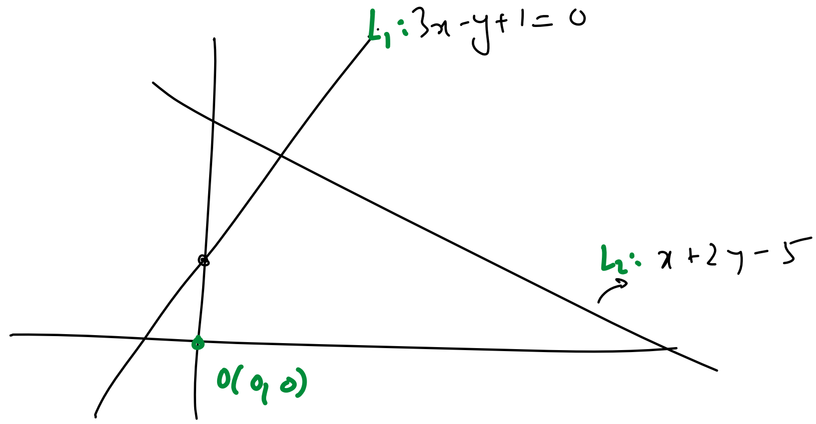
$y^2 - 6y + 5 = 0 \Rightarrow y = 1$ or $y = 5$



$\therefore$ AC: $\frac{y-3}{x-2} = 4$

BD: $\frac{y-1}{x-3} = -4$

(31) Draw accurate diagram



$O(0,0)$ gives (+)ve on L_1
gives (−)ve on L_2

$\Rightarrow$ $(a^2, a+1)$ should follow the same sign

(i) $3a^2 - a > 0$ & (ii) $a^2 + 2a - 3 < 0$

upon solving $a \in (-3, 0) \cup (\frac{1}{3}, 1)$

(32) $\perp^r \Rightarrow m_1 m_2 = -1$

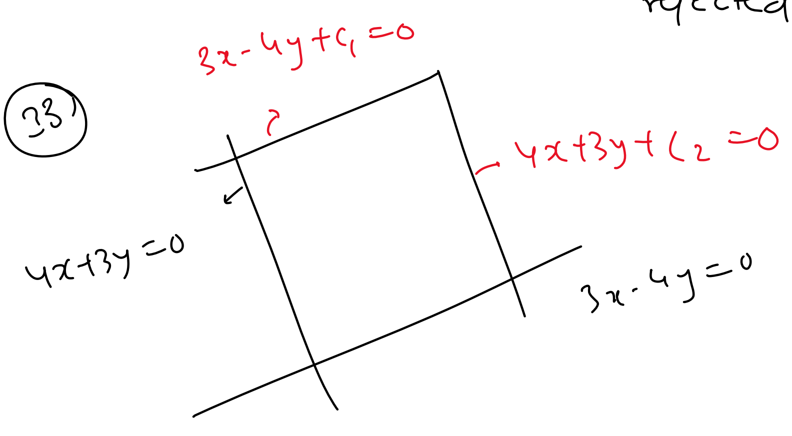
$$-\frac{1}{a-1} \times \frac{-2}{a^2} = -1$$

$$\Rightarrow 0 = a^3 - a^2 + 2$$

$$(a+1)\cancel{(a^2 - 2a + 2)} = 0$$

rejected $\therefore a = -1$ — (D)

(33)



Since area = 25

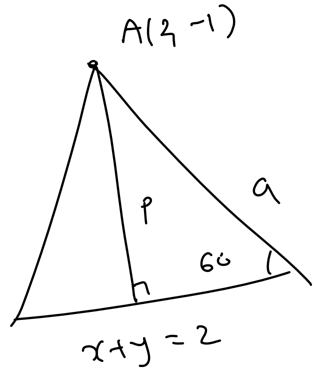
$\Rightarrow$ side length = 5

distance between // lines

$$\left|\frac{c_1}{5}\right| = 5 \quad \& \quad \left|\frac{c_2}{5}\right| = 5 \Rightarrow c_1, c_2 = \pm 25$$

lines are $3x-4y\pm25=0$ & $4x+3y\pm25=0$

(34)

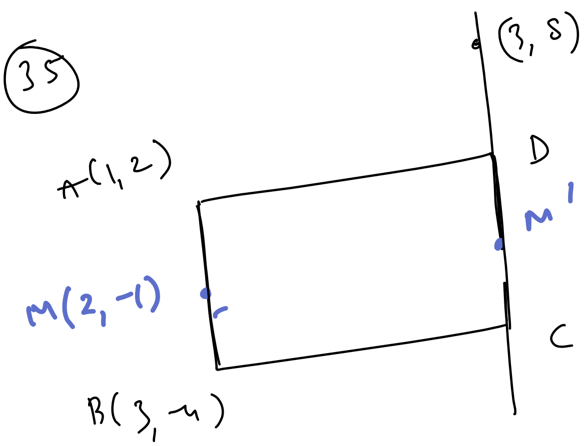


$p = \left|\frac{2-1-2}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$

Also $\frac{p}{a} = \sin 60^\circ \Rightarrow a = \sqrt{\frac{2}{3}}$

$\therefore$ Area of $\Delta = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{6}$

(35)



slope $AB = -3 =$ slope CD

$\therefore$ Eqn of line CD $\quad y-8 = -3(x-3)$

$\Rightarrow 3x+y=17$

$BC = MM' = \left|\frac{12}{\sqrt{10}}\right|$

$m_{BC} = \frac{1}{3} \quad \therefore \sin\theta = \frac{1}{\sqrt{10}}$

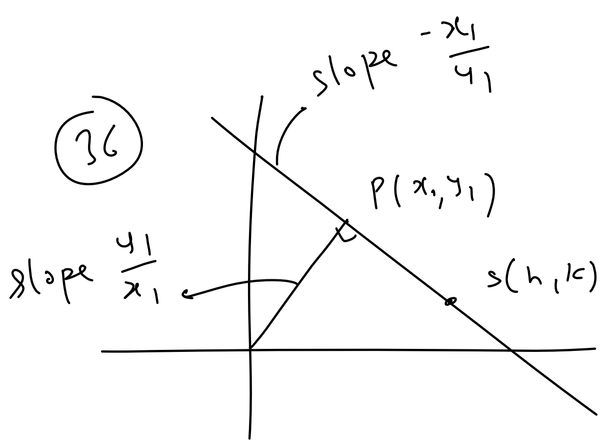
$\cos\theta = \frac{3}{\sqrt{10}}$

By parametric Eqn of line (MM')

$M'\left(2 + \frac{12}{\sqrt{10}}\cdot\frac{3}{\sqrt{10}},\ -1 + \frac{12}{\sqrt{10}}\cdot\frac{1}{\sqrt{10}}\right)$

OR $\left(\frac{28}{5}, \frac{1}{5}\right)$ — (D)

(36)



let the foot of $\perp^r$ be (x_1,y_1)

$\therefore$ Eqn of line PS is

$\frac{y-y_1}{x-x_1} = \frac{-x_1}{y_1}$

or $\quad x x_1 + y y_1 = x_1^2 + y_1^2$

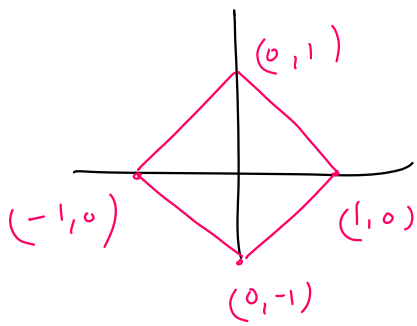
put $S(h,k) \quad h x_1 + k y_1 = x_1^2 + y_1^2$

put $S(h,k)$ $\quad hx_1 + ky_1 = x_1^2 + y_1^2$

now $\quad x_1 \to x \quad \therefore \; x^2 + y^2 = hx + ky$

$\quad\; y_1 \to y$

(37) The curves when plotted together form a square



$\therefore a = \sqrt{2} \quad \therefore$ Area $= 2$

(38) lines are $\quad \sqrt{3}x - y = 4\sqrt{3}k$

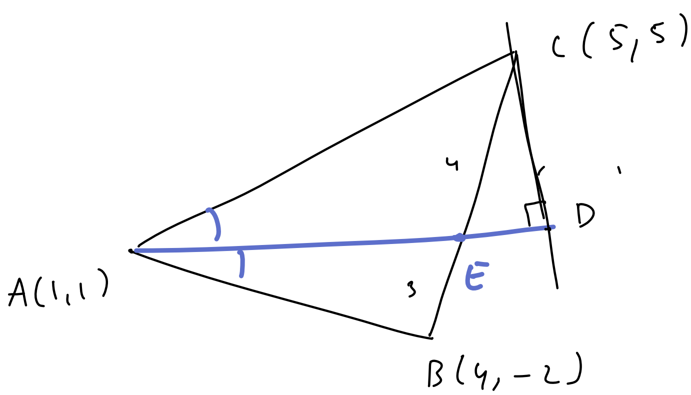
$$\sqrt{3}x + y = \frac{4\sqrt{3}}{k}$$

Multiply Eqns to eliminate variable 'k'

$\Rightarrow \quad 3x^2 - y^2 = 48$ (which is a hyperbola)

(39) Conceptual

(40)



By angle bisector property

$$\frac{AB}{AC} = \frac{BE}{EC} = \frac{3}{4}$$

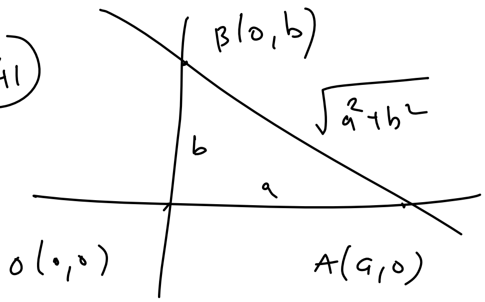
$\therefore$ By section formula get E

$E\left(\frac{31}{7}, 1\right)$

$\therefore m_{AE} = 0 \quad \therefore$ CD is a vertical line $\therefore$ Eqn $x = 5$

$\therefore m_{AE} = 0 \quad \therefore$ CD is a vertical line $\therefore$ Eq^n $x = 5$

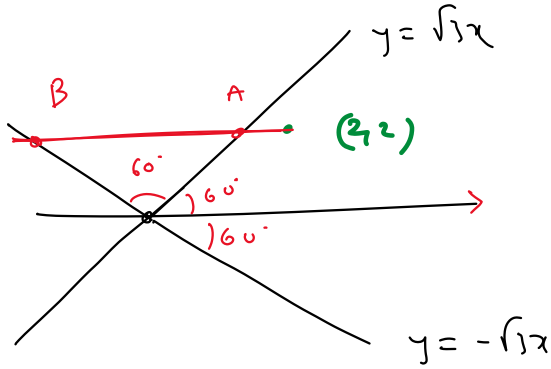
(41)



By formula of Incenter

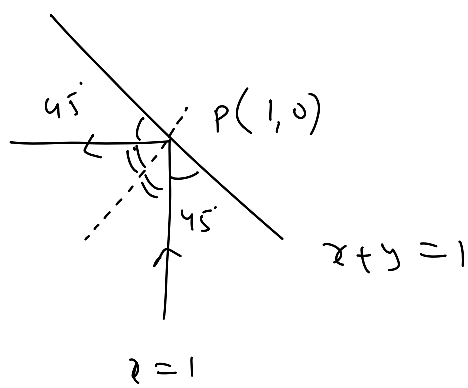
$$\left(\frac{ab}{\sqrt{a^2+b^2}+a+b} , \frac{ab}{a+b+\sqrt{a^2+b^2}} \right)$$

(42)



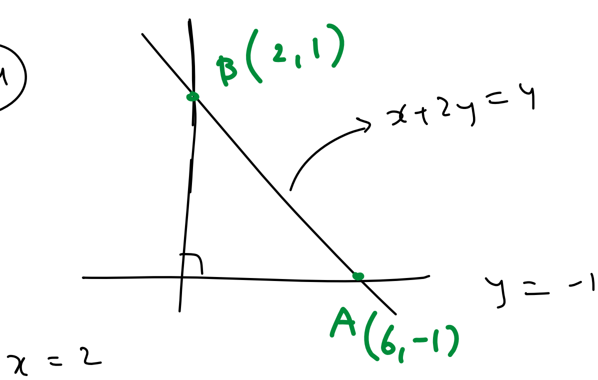
clearly it must be horizontal line i.e. $y = 2$

(43)



the reflected ray is $y = 0$

(44)



Since right $\Delta \Rightarrow$ mid pt of hypotenuse is circumcenter

S(4,0)

(45)

$16a^2 - 40ab + 25b^2 = c^2$

$(4a-5b)^2 = c^2$

(45) $16a^2 - 40ab + 25b^2 = c^2$

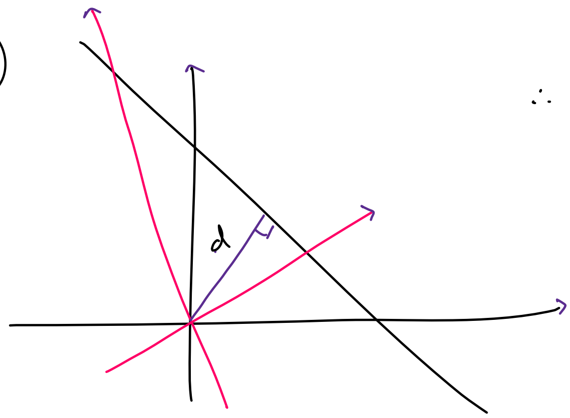
$(4a-5b)^2 = c^2$

$\Rightarrow \quad 4a - 5b + c = 0 \quad$ or $\quad -4a + 5b + c = 0$

$\Rightarrow$ pts $(4,-5)$ & $(-4,5)$

Satisfy the line $ax + by + c = 0$

(46)



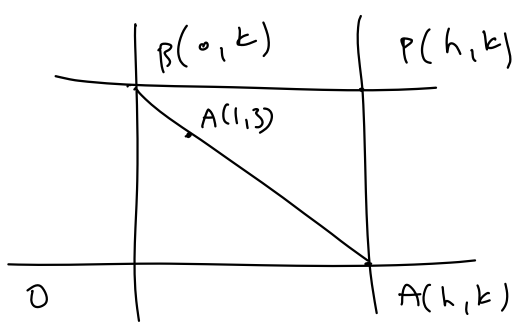
$\therefore$ Since $\perp$r distance of line from $(0,0)$ doesn't change

(i) $\dfrac{x}{a} + \dfrac{y}{b} = 1$ (ii) $\dfrac{x}{p} + \dfrac{y}{q} = 1$

$\therefore d = \dfrac{1}{\sqrt{\dfrac{1}{a^2} + \dfrac{1}{b^2}}}$ $\quad d = \dfrac{1}{\sqrt{\dfrac{1}{p^2} + \dfrac{1}{q^2}}}$

Equating $\dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{p^2} + \dfrac{1}{q^2}$

(47)



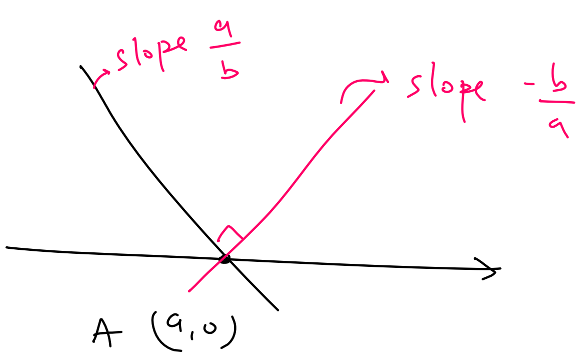
clearly line AB is $\dfrac{x}{h} + \dfrac{y}{k} = 1$

put point $A(1,3)$

$\dfrac{1}{h} + \dfrac{3}{k} = 1$

OR $\quad \dfrac{1}{x} + \dfrac{3}{y} = 1$

(48)



$\therefore$ Eqn of required line

$\dfrac{y}{x-a} = \dfrac{-b}{a}$

$\Rightarrow \boxed{bx + ay = ab}$ — (B)

$\Rightarrow \boxed{bx + ay = ab}$ — (B)

(49) Solving $y = x + 7$

$x + 2y + 1 = 0$ we get $(-5, 2)$

$\therefore$ Required line $\dfrac{y-2}{x-5} = -\dfrac{2}{5}$

$\Rightarrow 2x + 5y = 0$

(50) Solve $x - y + 1 = 0$

$3x + y - 5 = 0 \Longrightarrow (1, 2)$

$\therefore$ Eqn of line can be $\dfrac{y-2}{x-1} = -1$ or $\dfrac{y-2}{x-1} = \dfrac{1}{3}$ — (A)

(51) If a, b, c in A.P $\Longrightarrow a + c = 2b$

or $a - 2b + c = 0$

$\Rightarrow (1, -2)$ lies on $ax + by + c = 0 \Longrightarrow$ (A)

(52) $3a + 2b + 4c = 0 \Rightarrow \frac{3}{4}a + \frac{1}{2}b + c = 0$

clearly $\left(\frac{3}{4}, \frac{1}{2}\right)$ satisfies $ax + by + c = 0$

(53) By condition of concurrency $\begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix} = 0$

$\Rightarrow a(-1) - b(-2) + c(-1) = 0$

$\Rightarrow a - 2b + c = 0 \longrightarrow$ (A)

(54) $x(a+2b)+y(a+3b)=a+b$

$\Rightarrow \quad a(x+y-1)+b(2x-3y-1)=0$

By family of lines it passes through pt of intersection of

$x+y-1=0$

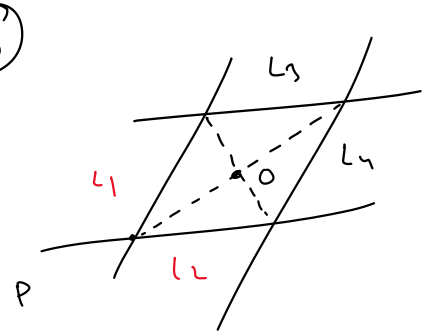
$2x-3y-1=0$ Solve $(-2,1)$

(55) We know line pair through $O(0,0)$ & $\perp^r$ to pair

$ax^2+2hxy+by^2+2gx+2fy+c=0$ is $bx^2-2hxy+ay^2=0$

$\therefore$ Ans is $+3x^2+xy=0$ — (B)

(56)



Given Combined Eqn of L_1L_2

$6x^2-xy-y^2+x+12y-35=0$

L_3L_4 is image of L_1L_2 about origin

$\therefore$ replace $x\to -x$ & $y\to -y$ we get L_3L_4 as

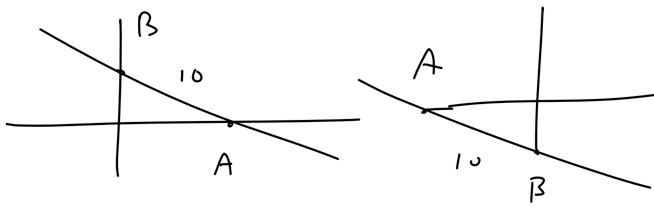
$6x^2-xy-y^2-x-12y-35=0$ — (A)

Sunday, February 4, 2024 6:42 PM

1(c)

(1) By distance formula $\perp^r = \left| \frac{0-0+2}{\sqrt{3+1}} \right| = 1$

(2) let line be $x+3y=\lambda$



$\therefore$ Just be geometry 2 lines possible (as slope fixed)

$\boxed{\text{Ans } 2}$

(3) for line pair $\Delta = 0$

$$\begin{vmatrix} 12 & -5 & \frac{11}{2} \\ -5 & 2 & -5/2 \\ 11/2 & -5/2 & \lambda \end{vmatrix} = 0$$

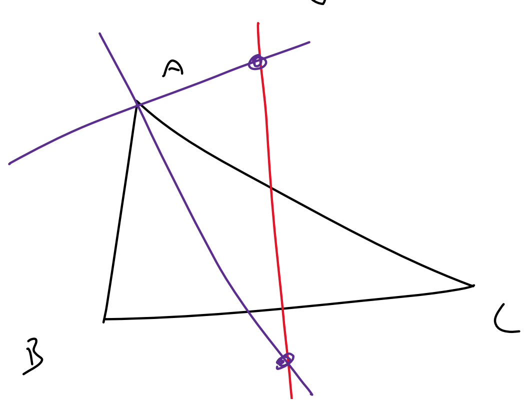
$$= 12\left[2\lambda - \frac{25}{4}\right] + 5\left[-5\lambda + \frac{55}{4}\right] + \frac{11}{2}\left[\frac{25}{2} - 11\right]$$

$$= 24\lambda - 75 - 25\lambda + \frac{275}{4} + \frac{33}{4} = 0$$

$$\Rightarrow -\lambda + \frac{41}{4} = 0 \Rightarrow \boxed{\lambda = \frac{41}{4}}$$

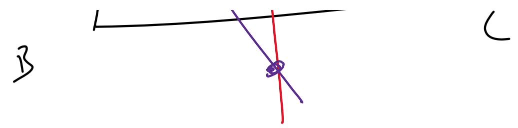
(4) Equidistant from B & C $\Longrightarrow$ pts on $\perp^r$ bisector of BC

Equidistant from AB & AC $\Rightarrow$ pts on angle bisector of $\angle A$



2 pts of intersection

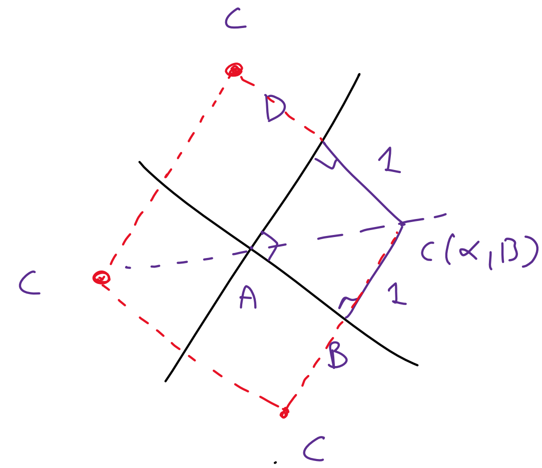
c



(5) 2 lines are $\perp^r$

$\Rightarrow$ ABCD Square with side length 1

There Can be 4 such pts C



$\therefore$ solving 2 lines we get $A\left(\frac{-3}{5}, \frac{1}{10}\right)$

$\therefore$ Pretty obvious that mean of x coordinates of all 4 possible C's is also pt A $\therefore$ Sum of all possible α's $= 4 \times (-3/5)$

$\therefore$ Ans 12

(6) By formula $\perp^r = \left| 2\sqrt{\frac{g^2 - ac}{a(a+b)}} \right|$

$$= \left| \frac{2 \cdot 5a}{\sqrt{2}} \right| = 25\sqrt{2}$$

$\Rightarrow |5a| = 25 \qquad \therefore |a| = 5$

(7) $x^2 - y^2 + 2y = 1$

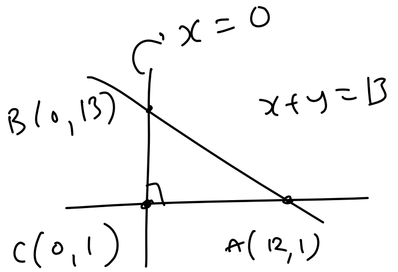
$x^2 - (y-1)^2 = 0 \Rightarrow$ lines are $x - y + 1 = 0$

& $x + y - 1 = 0$

$\therefore$ Angle bisector are

$$\frac{x - y + 1}{\cancel{\sqrt{2}}} = \pm\left(\frac{x + y - 1}{\cancel{\sqrt{2}}}\right)$$

$x = 0 \Rightarrow y = 1 \quad x = 0$

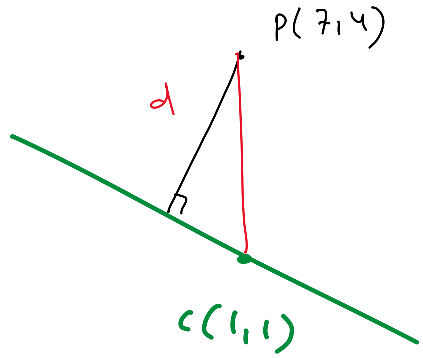


$\Rightarrow$ $y=1$, $x=0$

Area $= \frac{1}{2} \times 12 \times 12 = 72$

⑧ $2x+3y=5$

$x+y=2$ Solve $\therefore$ pt $(1,1)$



Pretty obvious that max 'd' Can be PC ie. hypotenuse

$\therefore$ The required line is the one $\perp$r to PC & through C(1,1)

ie slope $= -2 = -\frac{(2+\lambda)}{3+\lambda} \Rightarrow \lambda = -4$

⑨ $(p+q)x + (2p+q)y = p+2q$

$p(x+2y-1) + q(x+y-2) = 0$

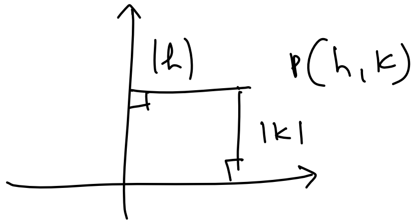
By family $p(x_1, y_1)$ is the pt of intersection of

$x+2y=1$

$x+y=2$

$\therefore x_1+y_1=2$

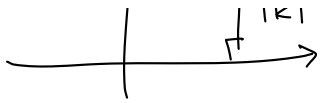
⑱ let the 2 lines be axes (Area won't change)



$\therefore$ Given $|h|+|k|=3$

OR $|x|+|y|=3$

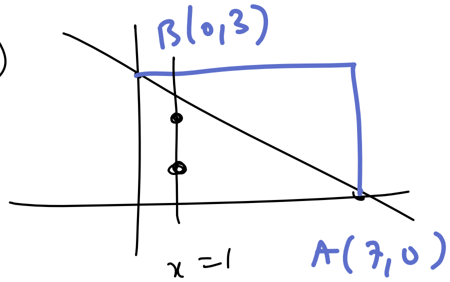
Which forms square of area '18'



$\ldots |x|+|y|=3$

Which forms square of area '18'

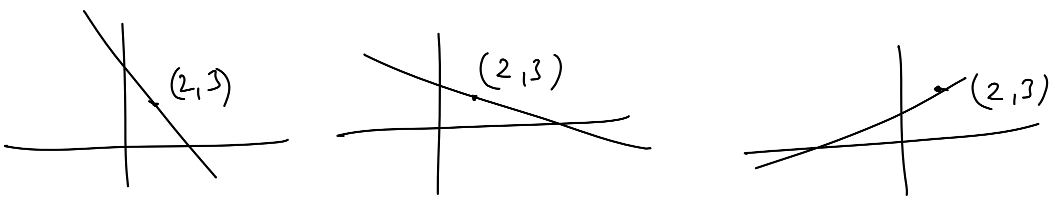
(11)



2 lattice pts on each line $x = 1, 2, 3, \ldots 6$

$\therefore$ total lattice pts inside $= 12$

(12) Conceptual draw diagram & see that 3 lines



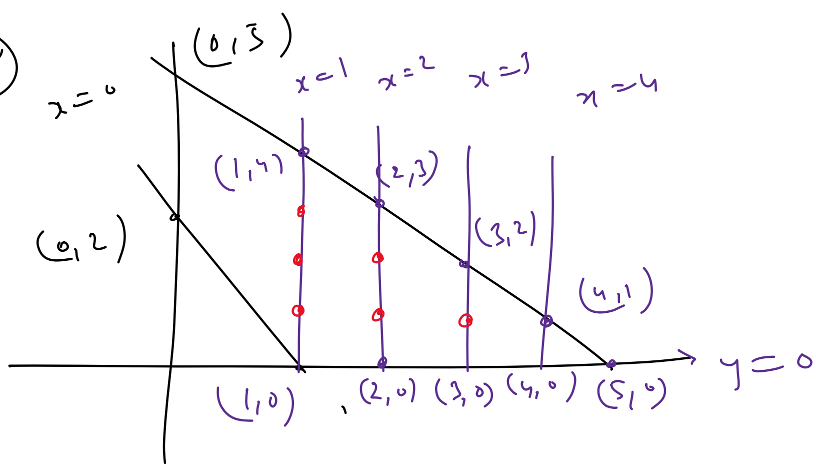
(13) Conceptual

(14) $|x-1|+|y-3|=1$ shift the origin to $(1,3)$

$\therefore$ New Eqn $|x|+|y|=1 \Rightarrow$ Square of side length $\sqrt{2}$

$\Rightarrow$ Area 2

(15)



6 pts inside

(16) Homogenize line with give 2-degree curve

$5x^2 - 24xy - 6y^2 + (4x - 2y)(x + ky) + 3(x + ky)^2 = 0$

$$5x^2 - 24xy - 6y^2 + (4x-2y)(x+ky) + 3(x+ky)^2 = 0$$

This Eq^n represents required line pair.

Given $m_1 + m_2 = 0 \Rightarrow h = 0$

or Coeff $xy = 0$

$$-24 + 4k - 2 + 3(2k) = 0$$

$$k = 26/10$$

Correction

Q3 $\rightarrow$ 10.25

Q5 $\rightarrow$ 12

Q8 $\rightarrow$ 4

Q9 $\rightarrow$ 2

1. (D)

Let $(1, \sqrt{3})$, $(0, 0)$ and $(2, 0)$ are the coordinates of vertices A, O, B of ΔABC.

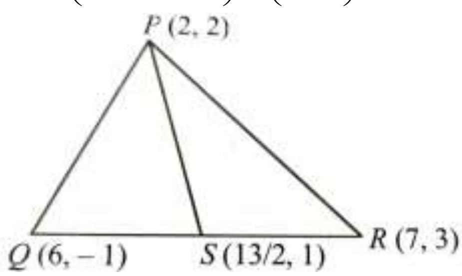
$\therefore$ AO = OB = AB. So, it is an equilaterial triangle and the incentre coincides with centroid.

$\therefore$ Incentre $= \left(\frac{0+1+2}{3}, \frac{0+0+\sqrt{3}}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$

2. (D)

S is the midpoint of Q and R

$\therefore\ S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$



Now slope of PS $= \frac{2-1}{2-13/2} = -\frac{2}{9}$

Now equation of the line passing through (1, -1) and parallel to PS is

$y+1 = -\frac{2}{9}(x-1) \Rightarrow 2x+9y+7=0$

3. (A)

$3x+4y=9$ and $y=mx+1$ are two lines.

On equating the value of y from both equations to get the x co-ordinate of the point of intersection,

$3x+4(mx+1)=9 \Rightarrow (3+4m)x=5$

$\Rightarrow\ x = \frac{5}{3+4m}$

For x to be an integer $3+4m$ should be a divisor of 5 i.e., $1, -1, 5$ or -5.

$3+4m=1 \Rightarrow m=-1/2$ (not integer)

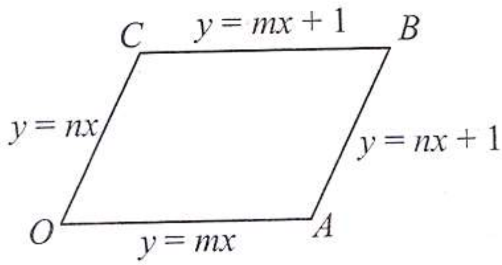
$3+4m=-1 \Rightarrow m=-1$ (integer)

$3+4m=5 \Rightarrow m=1/2$ (not an integer)

$3+4m=-5 \Rightarrow m=-2$ (integer)

$\therefore$ There are 2 integral values of m.

4. (D)



The vertices, $O(0,0), A\left(\frac{1}{m-n}, \frac{m}{m-n}\right), B(0,1)$

Area (parallelogram $OABC$) = 2 area (ΔOAB)

$$= 2 \times \frac{1}{2}\left|\left[0\left(\frac{m}{m-n} - 1\right) + \frac{1}{m-n}(1-0) + 0\left(0 - \frac{m}{m-n}\right)\right]\right|$$

$$= \frac{1}{|m-n|}$$

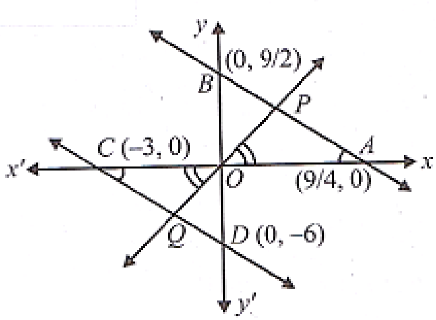
5. (B)

The given lines are

$2x + y = 9/2$ ......(i)

And $2x + y = -6$ ......(ii)

Signs of constants on R.H.S. show that two lines lie
On opposite sides of origin. Let a line through origin
Meets these lines in P and Q respectively then required
Ratio is OP : OQ



In ΔOPA and ΔOQC

$\angle POA = \angle QOC$ (Ver. Opp.angles)

$\angle PAO = \angle OCQ$ (alt.int.angles)

$\therefore$ $\Delta OPA \sim \Delta OCQ$ (By AA similarly)

$\therefore$ $\frac{OP}{OQ} = \frac{OA}{OC} = \frac{9/4}{3} = \frac{3}{4}$

$\therefore$ Required ratio = 3 : 4

6. (A)

7. (A)

8. (D)

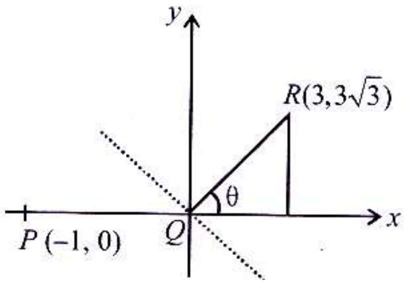
9. (A)

10. (C)

$\tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ \Rightarrow \angle PQR = 120^\circ$

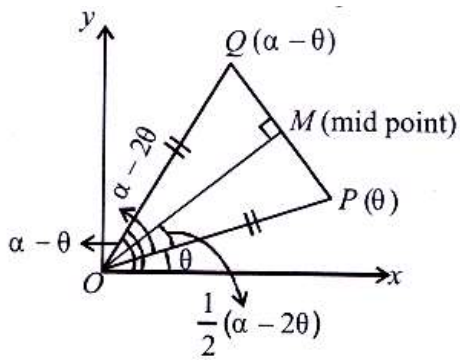
$\therefore$ Slope of bisector of $\angle PQR = \tan 120^\circ$

Hence, equation of bisector is $\sqrt{3}x + y = 0$



11. (D)

Clearly $OP = OQ = 1$ and $\angle QOP = \alpha - \theta - \theta = \alpha - 2\theta$

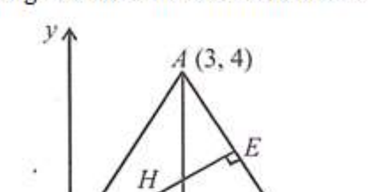


The bisector of $\angle QOP$ will be a perpendicular bisector of PQ also. Hence Q is reflection of P in the line OM which makes an angle $\angle MOP + \angle POX$ with x-axis, i.e., $\frac{1}{2}(\alpha - 2\theta) + \theta = \alpha / 2$

So that slope of OM is $\tan\alpha / 2$.

12. (C)

We know that point of intersection of altitudes of a triangle is the orthocentre of the triangle



Equation of altitude AD

i.e., line parallel to y-axis through (3, 4) is

$x = 3$ .... (i)

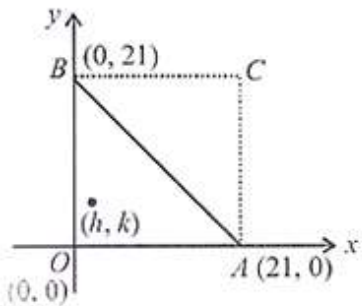
Now, equation of $OE \perp AB$ is

$y = -\frac{3-4}{4-0}x \Rightarrow y = x/4$

Solving (i) and (ii), we get orthocentre as (3, 3/4)

13. (B)

Total number of points within the square OACB
$= 20 \times 20 = 400$



Points line AB = 20 {(1, 20)(2, 19), (3, 18).......
(10, 11)(11, 10)....(20, 1)
Points within $\Delta OBC = 400 - 20 = 380$
By symmetry, points within $\Delta OBC = \frac{380}{2} = 190$

14. (C)

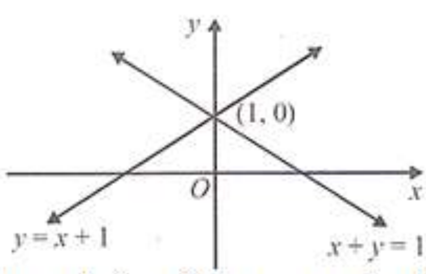
15. (C)

16. (A)

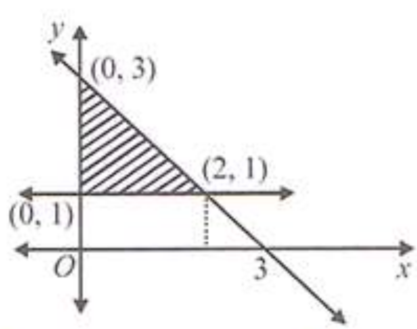
17. (D)

18. (A)

$$x^2 - y^2 + 2y = 1 \quad \Rightarrow \quad x = \pm(y-1)$$



Bisectors of above lines are x = 0 and y = 1



$\therefore$ Area between x = 0, y = 1 and x + y = 3 is the shaded region shown in figure.

$\therefore$ Area $= \frac{1}{2} \times 2 \times 2 = 2$ sq.units.

19. (C)

20. (D)

21. (A)

22. (D)

23. (C)

24. (B)

25. (D)

26. (D)

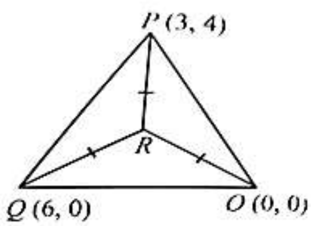
27. (C)

28. (C)

29. (C)

30. (C)

$\because\ Ar(\Delta OPR) = Ar(\Delta PQR) = Ar(OQR)$



$\therefore$ By simply geometry, R should be the centroid of ΔPQO

$\Rightarrow$ coordinate of R $= \left(\dfrac{3+6+0}{3}, \dfrac{4+0+0}{3}\right) = \left(3, \dfrac{4}{3}\right)$

31. (A)

32. (A)

33. (A)

34. (A)

35. (D)

36. (A)

37. (C)

38. (C)

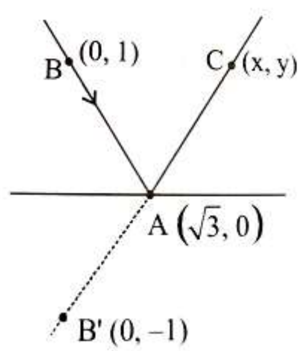
39. (B)

40. (B)

41. (B)

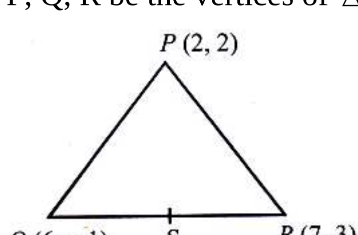
42. (A)
Since three lines $x-3y=p$,
$ax+2y=q$ and $ax+y=r$
Form a right angled triangle
$\therefore$ product of slopes of any two lines $=-1$
Suppose $ax+2y=q$ and $x-3y=p$ are $\perp$ to each other.
$\therefore \frac{-a}{2}\times\frac{1}{3}=-1 \Rightarrow a=6$
Now, consider option one by one
$a=6$ satisfies only option(A)
$\therefore$ Required answer is $a^2-9a+18=0$

43. (B)
Suppose B (0, 1) be any point on given line and co-ordinate of $A\left(\sqrt{3},0\right)$ is . So, equation of



Reflected ray is $\frac{-1-0}{0-\sqrt{3}}=\frac{y-0}{x-\sqrt{3}}$
$\Rightarrow \sqrt{3}y=x-\sqrt{3}$

44. (D)
Let P, Q, R be the vertices of ΔPQR



Since PS is the median
S is mid-point of QR
So, $S=\left(\frac{7+6}{2},\frac{3-1}{2}\right)=\left(\frac{13}{2},1\right)$

Now, slope of $PS=\frac{2-1}{2-\frac{13}{2}}=-\frac{2}{9}$

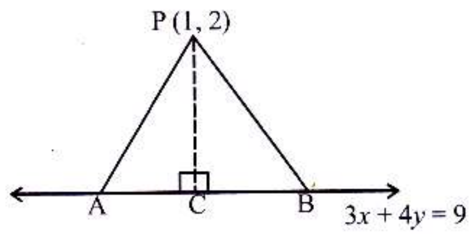
Since, required line is parallel to PS therefore slope of required line $=$ slope of PS

Now, equation of line passing through $(1,-1)$ and having slope $-\frac{2}{9}$ is

$y-(-1)=-\frac{2}{9}(x-1)$

$9y+9=-2x+2\Rightarrow 2x+9y+7=0$

45. (B)



Shortest distance of a point (x_1, y_1) from line

$ax+by=c$ is $d=\left|\frac{ax_1+by_1-c}{\sqrt{a^2+b^2}}\right|$

Now shortest distance of $P(1,2)$ from $3x+4y=9$ is

$PC=d=\left|\frac{3(1)+4(2)-9}{\sqrt{3^2+4^2}}\right|=\frac{2}{5}$

Given that ΔAPB is an equilateral triangle

Let 'a' be its side

Then $PB=a, CB=\frac{a}{2}$

Now, In $\Delta PCB, (PB)^2=(PC)^2+(CB)^2$ (By Pythagoras theorems)

$a^2=\left(\frac{2}{5}\right)^2+\frac{a^2}{4}$

$a^2-\frac{a^4}{4}=\frac{4}{25}\Rightarrow \frac{3a^2}{4}=\frac{4}{25}$

$a^2=\frac{16}{25}\Rightarrow a=\sqrt{\frac{16}{75}}=\frac{4}{5\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{4\sqrt{3}}{15}$

$\therefore$ Length of Equilateral triangle $(a)=\frac{4\sqrt{3}}{15}$

46. (D)

Circumference = (0, 0)

Centroid = $\left(\frac{(a+1)^2}{2},\frac{(a-1)^2}{2}\right)$

We know the circumcentre (O),

Centroid (G) and orthocentre (H) of a triangle lie on the line joining the O and G.

Also, $\frac{HG}{GO}=\frac{2}{1}$

$\Rightarrow$ Coordinate of othocentre = $\frac{3(a+1)^2}{2},\frac{3(a-1)^2}{2}$

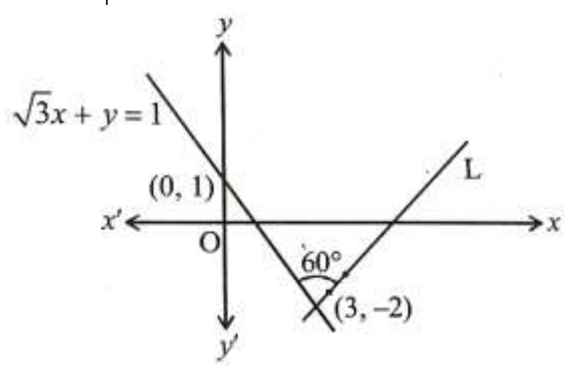
Now, these coordinates satisfies eqn given option (d) Hemce, required eqn. of line is

$(a-1)^2 x-(a+1)^2 y=0$

47. (B)

Let the slope of line L be m. Then

$\left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right|=\sqrt{3}$



$\Rightarrow\ m+\sqrt{3}=\pm\left(\sqrt{3}-3m\right)$

$\Rightarrow\ 4m=0 \text{ or } 2m=2\sqrt{3}\Rightarrow m=0 \text{ or } m=\sqrt{3}$

$\because$ L intersect x – axis $\quad\therefore\ m=\sqrt{3}$

$\therefore$ Equation of L is $y+2=\sqrt{3}(x-3)$

$\Rightarrow\ \sqrt{3}x-y-\left(2+3\sqrt{3}\right)=0$

48. (C)

Given eqn of line is $y+\sqrt{3}x-1=0$

$\Rightarrow y=-\sqrt{3}x+1$

$\Rightarrow (\text{slope})\, m_2=-\sqrt{3}$

Let the other slope be m_1

$\therefore\ \tan 60^\circ=\left|\frac{m_1-\left(-\sqrt{3}\right)}{1+\left(-\sqrt{3}m_1\right)}\right|$

$\Rightarrow\ m_1=0, m_2=\sqrt{3}$

Since line L is passing through (3, -2)

$\therefore\ y-(-2)=+\sqrt{3}(x-3)$

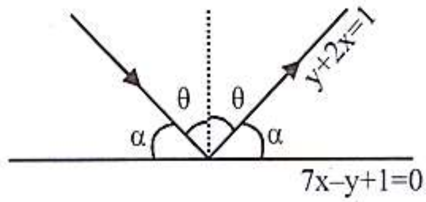
$\Rightarrow y+2=\sqrt{3}(x-3)=0$

$y-\sqrt{3}x+2+3\sqrt{3}=0$

49. (C)

Let slope of incident ray be m.

$\therefore$ angle of incidence = angle of reflection



$$\therefore \quad \left|\frac{m-7}{1+7m}\right| = \left|\frac{-2-7}{1-14}\right| = \frac{9}{13}$$

$$\Rightarrow \frac{m-7}{1+7m} = \frac{9}{13} \text{ or } \frac{m-7}{1+7m} = -\frac{9}{13}$$

$$\Rightarrow 13m - 91 = 9 + 63m \text{ or } 13m - 91 = -9 - 63m$$

$$\Rightarrow 50m = -100 \text{ or } 76m = 82$$

$$\Rightarrow m = -\frac{1}{2} \quad \text{or} \quad m = \frac{41}{38}$$

$$\Rightarrow y - 1 = -\frac{1}{2}(x - 0) \text{ and } y - 1 = \frac{41}{38}(x - 0)$$

i.e., $x + 2y - 2 = 0$ or $38y - 38 - 41x = 0$

$\Rightarrow 41x - 38y + 38 = 0$

50. (A)

$L_1 : 4x + 3y - 12 = 0$

$L_2 : 3x + 4y - 12 = 0$

$L_1 + \lambda L_2 = 0$

$(4x + 3y - 12) + \lambda(3x + 4y - 12) = 0$

$x(4 + 3\lambda) + y(3 + 4\lambda) - 12(1 + \lambda) = 0$

Point A $\left(\frac{12(1+\lambda)}{4+3\lambda}, 0\right)$

Point B $\left(0, \frac{12(1+\lambda)}{3+3\lambda}\right)$

Mid point $\Rightarrow$ $h = \frac{6(1+\lambda)}{4+3\lambda}$

$k = \frac{6(1+\lambda)}{3+4\lambda}$

51. (B)

Since Orthocentre of the triangle is A(–3, 5) and centriod of the triangle is B(3, 3), then

$AB = \sqrt{40} = 2\sqrt{10}$

Centroid divides orthocentre and circumcentre of the triangle in ratio 2 : 1

$\therefore AB : BC = 2 : 1$

Now, $AB = \dfrac{2}{3} AC$

$\Rightarrow \quad AC = \dfrac{3}{2} AB = \dfrac{3}{2}(2\sqrt{10}) \quad \Rightarrow AC = 3\sqrt{10}$

$\therefore$ Radius of circle with AC as diametre

$= \dfrac{AC}{2} = \dfrac{3}{2}\sqrt{10} = 3\sqrt{\dfrac{5}{2}}$

52. (D)

Equation of the line, which is perpendicular to the line, $3x + y = \lambda\left(\lambda \neq 0\right)$ and passing through origin, is given by

For foot of perpendicular

$$r = \frac{-\left(3 \times 0\right) + \left(1 \times 0\right) - \lambda\big)}{3^2 + 1^2} = \frac{\lambda}{10}$$

So, foot of perpendicular $P = \left(\dfrac{3\lambda}{10}, \dfrac{\lambda}{10}\right)$

So, foot of perpendicular $P = \left(\dfrac{3\lambda}{10}, \dfrac{\lambda}{10}\right)$

Given the line meets $X-$axis at $A = \left(\dfrac{\lambda}{3}, 0\right)$ and meets Y-axis at $B = \left(0, \lambda\right)$

So, $BP = \sqrt{\left(\dfrac{3\lambda}{10}\right)^2 + \left(\dfrac{\lambda}{10} - \lambda\right)^2} \Rightarrow BP = \sqrt{\dfrac{9\lambda^2}{100} + \dfrac{81\lambda^2}{100}}$

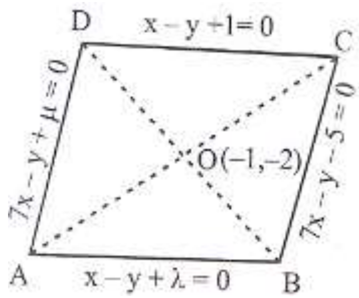
$\Rightarrow BP = \sqrt{\dfrac{90\lambda^2}{100}}$

Now, $PA = \sqrt{\left(\dfrac{\lambda}{3} - \dfrac{3\lambda}{10}\right)^2 + \left(0 - \dfrac{\lambda}{10}\right)^2}$

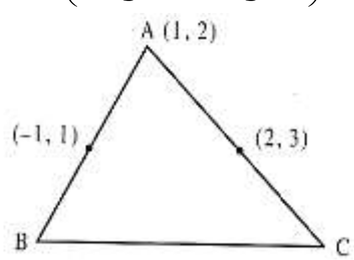
$\Rightarrow PA \sqrt{\dfrac{\lambda^2}{900} + \dfrac{\lambda^2}{100}} \Rightarrow PA = \sqrt{\dfrac{10\lambda^2}{900}}$

Therefore $BP : PA = 3 : 1$

53. (A)



Let other two sides of rhombus are
$x-y+\lambda=0$
And $7x-y+\mu=0$
Then O is equidistant from AB and DC and from AD and BC
$\therefore\ |-1+2+1|=|-1+2+\lambda| \Rightarrow \lambda=-3$
And $|-7+2-5|=|-7+2+\mu| \Rightarrow \mu=15$
$\therefore$ Other two sides are x – y – 3 = 0 and 7x – y + 15 = 0
$\therefore$ On solving the equations of sides pairwise, we get the vertices as
$\left(\frac{1}{3},\frac{-8}{3}\right),(1,2),\left(\frac{-7}{3},\frac{-4}{3}\right),(-3,-6)$

54. (B)
From the mid-point formula co-ordinates of vertex B and C are B(–3, 0) and C(, 4).
Now, centroid of the triangle
$$G=\left(\frac{3-3+1}{3},\frac{0+4+2}{3}\right) \Rightarrow G\equiv\left(\frac{1}{3},2\right)$$


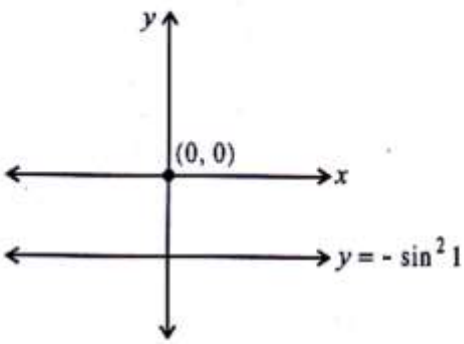
55. (A)
The given equations of the set of all line
$px+qy+r=0$ ……(i)
And given condition is :
$3p+2q+4r=0$
$\Rightarrow \frac{3}{4}p+\frac{2}{4}q+r=0$ …….(ii)
From (i) & (ii) we get:
$\therefore x=\frac{3}{4}, y=\frac{1}{2}$

56. (D)
Consider the equation,
$y=\sin x.\sin(x+2)-\sin^2(x+1)$

$$\frac{1}{2}\left[2\sin x.\sin(x+2)-2\sin^2(x+1)\right]$$

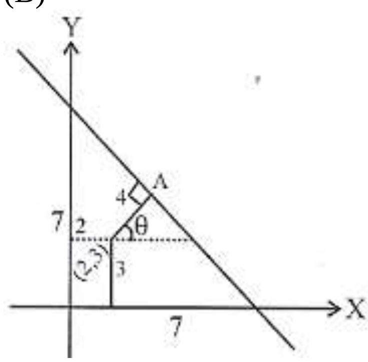
$$=\frac{1}{2}\cos(-2)-\frac{\cos(2x+2)}{2}-\left[\frac{1-\cos(2x+2)}{2}\right]$$

$$=\frac{(\cos 2)-1}{2}=-\sin^2 1$$



By the graph y lies in III and IV quadrant.

57. (B)



Since point at 4 units from P (2, 3) will be

$A(4\cos\theta+2, 4\sin\theta+3)$ and this point will satisfy the equation of line x + y = 7

Since x + y = 7

$2+4\cos\theta+3+4\sin\theta=2$

$\Rightarrow \cos\theta+\sin\theta=\frac{1}{2}$

On squaring

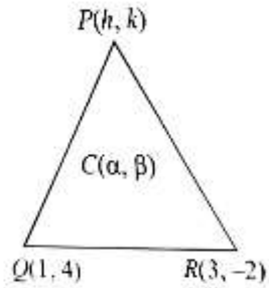
$\Rightarrow \sin 2\theta-\frac{3}{4}\Rightarrow \frac{2\tan\theta}{1+\tan^2\theta}=-\frac{3}{4}$

$\Rightarrow 3\tan^2 \Rightarrow \ +8\tan\theta+3=0$

$\Rightarrow \tan\theta=\frac{-8\pm 2\sqrt{7}}{6}$ (ignoring –ve sign)

$\Rightarrow \tan\theta=\frac{-8\pm 2\sqrt{7}}{6}=\frac{1-\sqrt{7}}{1+\sqrt{7}}$

58. (C)



Let centroid C be (α, β)

We have $\alpha = \dfrac{1+3+h}{3} \Rightarrow h = 3\alpha - 4$

$\beta = \dfrac{4-2+k}{3} \Rightarrow k = 3\beta - 2$

But P (h, k) lies on 2x -3y + 4 =0

$\Rightarrow 2(3\alpha-4)-3(3\beta-2)+4=0$

$\Rightarrow 6\alpha-9\beta-8+6+4=0$

$\Rightarrow 6\alpha-9\beta+2=0$

Locus : $6x-9y+2=0$

$\Rightarrow y=\dfrac{6}{9}x+\dfrac{2}{9} \quad \therefore \text{ its slope} = \dfrac{6}{9}=\dfrac{2}{3}$

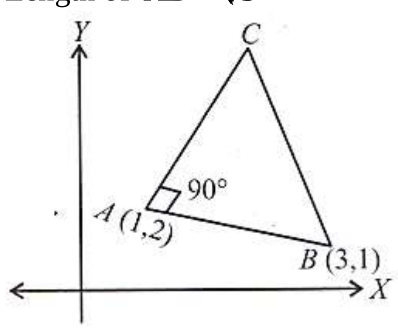
59. (B)

Let ΔABC be in the first quadrant

Slope of line $AB = -\dfrac{1}{2}$

Slope of line AC = 2

Length of $AB = \sqrt{5}$



It is given that $ar(\Delta ABC) = 5\sqrt{5}$

$\therefore \dfrac{1}{2} AB \cdot AC = 5\sqrt{5} \Rightarrow AC = 10$

$\therefore$ Coordinate of vertex $C = (1+10\cos\theta, 2+10\sin\theta)$

$\because \tan\theta = 2 \Rightarrow \cos\theta = \dfrac{1}{\sqrt{5}}, \sin\theta = \dfrac{2}{\sqrt{5}}$

$\therefore$ Coordinate of $C = (1+2\sqrt{5}, 2+4\sqrt{5})$

$\therefore$ Abscissa of vertex C is $1+2\sqrt{5}$.

60. (A)

The line in xy-plane is,

$\frac{x}{3}+y=1 \Rightarrow x+3y-3=0$

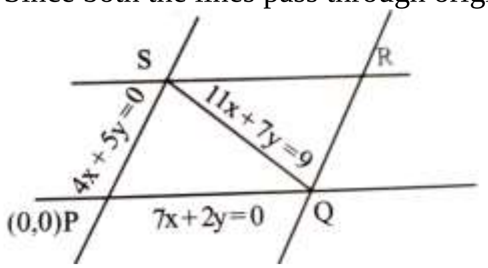
Let image of the point $(-1,-4)$ be (α,β), then

$\frac{\alpha+1}{a}=\frac{\beta+\gamma}{3}=-\frac{2(-1-12-3)}{10}$

$\Rightarrow \alpha+1=\frac{\beta+4}{3}=\frac{16}{5} \Rightarrow \alpha=\frac{11}{5},\beta=\frac{28}{5}$

61. (B)

Since both the lines pass through origin. Then



Point S will be point of intersection of 4x + 5y = 0 and 11x + 7y = 9

So, coordinates of point S = $\left(\frac{5}{3},-\frac{4}{3}\right)$

Also, point Q is point of intersection of 7x + 2y = 9 and 11x + 7y = 9

So, coordinates of point Q = $\left(-\frac{2}{3},\frac{7}{3}\right)$

Since, diagonals of parallelogram intersect at middle, then the middle point of SQ is

$\left(\frac{\frac{5}{3}-\frac{2}{3}}{2},\frac{\frac{-4}{3}+\frac{7}{3}}{2}\right)=\left(\frac{1}{2},\frac{1}{2}\right)$

So, equation of diagonal PR is, (y – 0) = $\frac{\frac{1}{2}-0}{\frac{1}{2}-0}(x-0)$

y = x

62. (A)

Coordinates of Centroides

$C=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

$=\left(\frac{3+1+2}{3},\frac{-1+3+4}{3}\right)=(2,2)$

The given equation of lines are

$x+3y-1=0$

$3x-y+1=0$

Then, from (i) and (ii)

Point of intersection P $\left(-\frac{1}{5},\frac{2}{5}\right)$

Equation of line DP

$8x-11y+6=0$

63. (C)

If equation of pair of straight line are $ax^2+2hxy+by^2=0$ then pair of angle bisector are $\frac{x^2-y^2}{a-b}=\frac{xy}{h}$

Here $a=1, b=-5$ and $h=-2$

$\therefore$ Pair of angle bisector are:

$\frac{x^2-y^2}{1+5}=\frac{xy}{-2}\Rightarrow x^2-y^2+3xy=0$

64. (C)

Let m be the slope of line

$\therefore\left|\frac{m-3\sqrt{2}}{1+3\sqrt{2}m}\right|=\sqrt{2}$

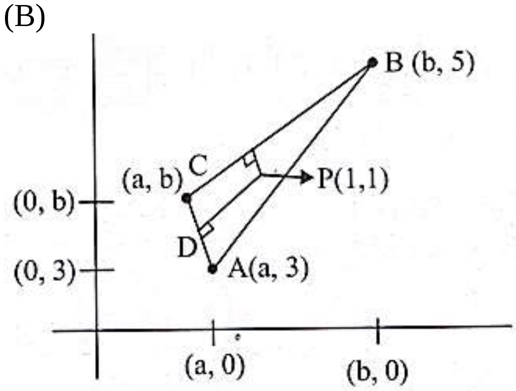
$\Rightarrow m-3\sqrt{2}=\pm\sqrt{2}\pm 6m\Rightarrow m\mp 6m=\pm\sqrt{2}+3\sqrt{2}$

$\Rightarrow m=-\frac{4\sqrt{2}}{5}$ or $-\frac{2\sqrt{2}}{7}$

So, the equation of line passing through the point $(1,3)$ and slope $-\frac{4\sqrt{2}}{5}$ is

$y-3=\frac{-4\sqrt{2}}{5}(x-1)\Rightarrow 4\sqrt{2}x+5y-\left(15+4\sqrt{2}\right)=0$

65. (B)



Slope of AC = ∞

Slope of PD = 0

$D\left(\frac{a+a}{2},\frac{b+3}{2}\right)=D\left(a,\frac{b+3}{2}\right)$

$\frac{b+3}{2}-1=0; b+3-2=0\Rightarrow b=-1$

$\boxed{b=-1}$

$$E\left(\frac{b+a}{2},\frac{5+b}{2}\right)=\left(\frac{af}{2};2\right)$$

$$\text{Slope of BC}\times\text{Slope of EP}=-1$$

$$\left(\frac{5-b}{b-a}\right)=\left(\frac{2-1}{\frac{a-1}{2}-1}\right)=-1$$

$$\Rightarrow\left(\frac{6}{-1-a}\right)=\left(\frac{2}{a-3}\right)=-1\Rightarrow 12=(1+a)(a-3)$$

$$\Rightarrow 12=a^2-3a+a-3\Rightarrow a^2-2a-15=0$$

$$(a-5)(a+3)=0$$

Given $ab>0\Rightarrow a(-1)>0;-a>0;a<0$

$\boxed{a=-3}$ Accept

Equation of AP (-3, 3), P (1, 1)

$$y-1=\left(\frac{3-1}{-3-1}\right)(x-1)$$

$$-2y+2=x-1$$

$$\Rightarrow\boxed{x+2y=3}$$

$B(-1,5)$

$C(-3,-1)$

Equation of BC

$$(y-5)=\frac{6}{2}$$

$$y-5=3x+3$$

$$\boxed{y=3x+8}$$

Solving (i) and (ii)

$$x+2(3x+8)=3$$

$$\Rightarrow 7x+16=3\Rightarrow 7x=-13\Rightarrow x=-\frac{13}{7}$$

$$y=3\left(-\frac{13}{7}\right)+8=\frac{-39+56}{7}:y=\frac{17}{7}$$

$$x+y=\frac{-13+17}{7}=\frac{4}{7}$$

66. (B)

Given coordinates of ΔABC are

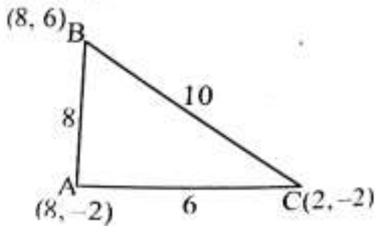
$$A(\alpha,-2):B(\alpha,6):C\left(\frac{\alpha}{4},-2\right)$$

AC is perpendicular to AB

So, ΔABC is right angles at A.

Circumcentre = mid point of BC = $\left(\frac{5\alpha}{8},2\right)$

$\therefore \frac{5\alpha}{8} = 5$ and $\frac{\alpha}{4} = 2 \Rightarrow \alpha = 8$



Now, find the area, perimeter, in radius and circumradius.

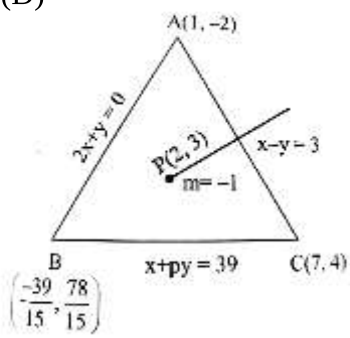
Then, Area = $\frac{1}{2}(6)(8) = 24$

Perimeter = 24

Circumradius = 5

Inradius = $\frac{\Delta}{8} = \frac{24}{12} = 2$

67. (D)



Perpendicular bisector of AB is x + y = 5

Take image of A

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{-2(-6)}{2} = 6$$

Here, (x, y) $\rightarrow$(7. 4)

Satisfy ppoint (7, 4) on equation x + py = 39

7 + 4p = 39

P = 8

Now, solve x + 8y = 39 and y = - 2x

$x = \frac{-39}{15}\, y = \frac{78}{15}$

$AC^2 = 72 = 9p$

$AC^2 + p^2 = 72 + 64 = 136$

Area of triangle $\Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & -2 & 1 \\ 7 & 4 & 1 \\ \frac{-39}{15} & \frac{78}{15} & 1 \end{vmatrix}$

$$= \frac{1}{2}\left[4 - \frac{78}{15} + 2\left(7 + \frac{78}{15}\right) + 7\left(\frac{78}{15}\right) + \frac{4\times 39}{15}\right]$$

$$=\frac{1}{2}\left[18+18\times\frac{13}{5}\right]=9\left[\frac{18}{5}\right]=\frac{162}{5}=32.4$$

68. (B)

According to question.

$(x-1)^2+(y-2)^2+(x+2)^2+(y-1)^2=14$

$\Rightarrow\ x^2+y^2+x-3y-2=0 \quad$ ....(i)

Put x = 0 in eq. (i)

$\Rightarrow\ y^2-3y-2=0$

Apply quadratic formula, $\Rightarrow\ y=\frac{3\pm\sqrt{17}}{2}$

Put y = 0, in eq. (i), $\Rightarrow\ x^2+x-2=0^2$

x = –2, 1.

Then, point A (-2, 0), B (1, 0) C $\left(0,\frac{3+\sqrt{17}}{2}\right)$ & D $\left(0,\frac{3-\sqrt{17}}{2}\right)$

Area of quadrilateral ABCD = $\frac{1}{2}.3.\sqrt{17}=\frac{3\sqrt{17}}{2}$

69. (B)

We have point $(\alpha,-3),(2,0)$ and $(1,\alpha)$

For collinearity, $\left(\frac{\alpha-0}{-1}\right)=\frac{\alpha+3}{1-\alpha}$

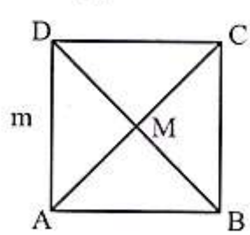
$\Rightarrow\ \alpha=-\frac{1}{3},3\ \Rightarrow\ \alpha_1=-1,\alpha_2=3\ \Rightarrow\ y-3=\sqrt{3}=\sqrt{3}(x+1)$

70. (B)

Given equation is $a^2+11a+3(m_1^2+m_2^2)=220$

$m_1m_2=-1$

$a^2+11a+3\left(m_1^2+\frac{1}{m_1^2}\right)=220$



Eq. of AC

$AC=(\cos\alpha-\sin\alpha)+(\sin\alpha+\cos\alpha)\,y=10$

$BD=(\sin\alpha-\cos\alpha)\,x+(\sin\alpha-\cos\alpha)\,y=0$

$(10(\cos\alpha-\sin\alpha),10(\sin\alpha-\cos\alpha)$

Slope of $AC=\left(\frac{\sin\alpha-\cos\alpha}{\sin\alpha-\cos\alpha}\right)=\tan\theta=M$

Eq. of line making an angle $\frac{\pi}{4}$ with AC

$$m_1.m_2=\frac{\frac{m\pm\tan\frac{\pi}{4}}{4}}{1\pm m\tan\frac{\pi}{4}}=\frac{m+1}{1-m}\text{ or }\frac{m-1}{1+m}$$

$$\Rightarrow \frac{\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}+1}{1-\left(\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}\right)}\cdot\frac{\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}-1}{1+\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}}$$

$m_1.m_2=\tan\alpha,\cot\alpha$

Mid-point of AC and BD

$=M(5(\cos\alpha-\sin\alpha),5(\cos\alpha+\sin\alpha))$

$B(10(\cos\alpha-\sin\alpha),10(\cos\alpha+\sin\alpha))$

$a=AB=\sqrt{2}BM=\sqrt{2}(5\sqrt{2})=10\,a=10$

$\because\ a^2+11a+3\left(m_1^2+\frac{1}{m_1^2}\right)=220$

$100+110+3(\tan^2\alpha+\cot^2\alpha)=220$

Hence $\tan^2\alpha=3,\tan^2\alpha=\frac{1}{3}\Rightarrow\alpha=\frac{\pi}{3}$ or $\frac{\pi}{6}$

Now $72(\sin^4\alpha=\cos^4\alpha)+a^2-3a+13$

$=72\left(\frac{9}{16}+\frac{1}{16}\right)+100-30+13$

$=72\left(\frac{8}{5}\right)+83=45+83=128$

71. (C)

Let B (h, h - 2)

$\sqrt{(h-4)^2+(h-2-3)^2}=\frac{\sqrt{29}}{3}$

Squaring on both side

$18h^2-162h+340=0$

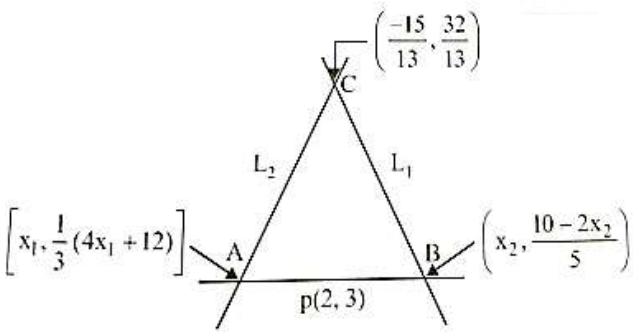
$h=\frac{51}{9}$ or $h=\frac{10}{3}$

$k=\frac{33}{9}$ or $k=\frac{4}{3}$

Option (C) will satisfy $\left(\frac{10}{3},\frac{4}{3}\right)$

72. (B)

Here solving L_1 and L_2 lines we get point C



Now from line L_2, point A is $\left[x_1, \frac{1}{3}(4x_1+12)\right]$ and from line L_1 we get point B is $\left[x_2, \frac{10-2x_2}{5}\right]$

And we are given that

$$\frac{1\times x_2+3\times x_1}{1+3}=2 \Rightarrow 3x_1+x_2=8$$

$$\frac{1\times\left(\frac{10-2x_2}{5}\right)+(4x_1+12)}{4}$$

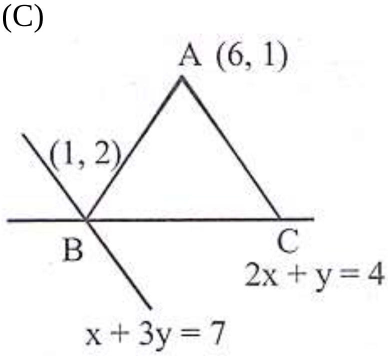
$$\Rightarrow \frac{10-2x_2+20x_1+60}{20} \Rightarrow \frac{20x_1-2x_2+70=3}{20}$$

$$\Rightarrow 10x_1-x_2=-5$$

$$\Rightarrow A\left(\frac{3}{13}, \frac{56}{13}\right) \text{ and } B\left(\frac{95}{13}, \frac{-12}{13}\right) \therefore \Delta ABC$$

$$=\frac{1}{2}\begin{vmatrix} \frac{3}{13} & \frac{56}{13} & 1 \\ \frac{95}{13} & \frac{-12}{13} & 1 \\ \frac{-15}{13} & \frac{32}{13} & 1 \end{vmatrix} \Delta ABC=\frac{132}{13} \text{sq.units}$$

73. (C)



Intersection of two lines is point B with coordinate (1, 2).
As C lies on 2x + y = 4, suppose x = h then k = 4 – 2h.
Δ is isosceles with base BC. Then, AB = AC

$$\sqrt{25+1}=\sqrt{(6-h)^2+(3-2h)^2}$$

$$\sqrt{26}=\left(36+h^2-12h+4h^2+9-12h\right)$$

Take square both sides

$$26=5h^2-24h+45 \Rightarrow 5h^2-24h+19=0$$

$\Rightarrow 5h^2 - 5h - 19h + 19 = 0$

$\Rightarrow 5h^2 - 24h + 19 = 0$

$h = \frac{19}{5}$ or $h = 1$. Thus $C\left(\frac{19}{5}, \frac{-18}{5}\right)$

Centroid (α, β) of triangle ABC = $\left(\frac{6+1+\frac{19}{5}}{3}, \frac{1+2-\frac{18}{5}}{3}\right)$

$= \left(\frac{35+19}{15}, \frac{15-18}{15}\right) = \left(\frac{54}{15}, \frac{-3}{15}\right)$; $\alpha = \frac{54}{15}; \beta = \frac{-3}{15}$

Put the value of α and β in $15(\alpha + \beta)$

$15(\alpha + \beta) = 51$

74. (D)

Given that $P(\alpha, \beta)$ lies below L_1 and above L_2

$\therefore 3\alpha - 4\beta + 12 > 0$

Now, $\left|\frac{3\alpha - 4\beta + 12}{\sqrt{3^2 + 4^2}}\right| = 1$

$\Rightarrow 3\alpha - 4\beta + 7 = 0$ ....(i)

For L_2 since $b > 0$

$\therefore 8\alpha + 6\beta + 11 > 0$

Now $\left|\frac{8\alpha + 6\beta + 11}{\sqrt{8^2 + 6^2}}\right| = 1$

$\Rightarrow 8\alpha + 6\beta + 1 = 0$ ......(i)

Solving (i) and (ii), we get

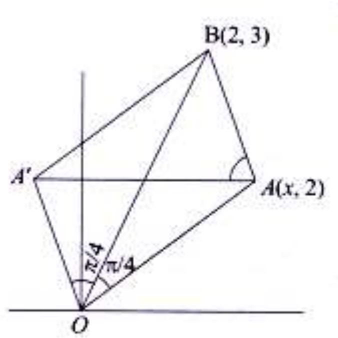
$\alpha = \frac{-23}{25} m\beta = \frac{106}{100}$

$\therefore 100(\alpha + \beta) = 106 - 92 = 14$

75. (C)

Lert origin O, slope $m_{OB} = \frac{3-0}{2-0} = \frac{3}{2}$

And $m_{OA} = \frac{2-0}{x-0} = \frac{2}{x}$



Then, $\tan\theta = \left|\frac{M_{OB} - M_{OA}}{1 + M_{OB}M_{OA}}\right|$

$\Rightarrow \tan\pi/4\left|\frac{3/2 - 2/x}{1 + 6/2x}\right| = 1$

$\Rightarrow x_1 = 10, x_2 = -2/5 \Rightarrow AA^1 = 52/5$

76. (C)

$AB \equiv x - 2y + 1 = 0$

$AC \equiv 2x - y = 0$

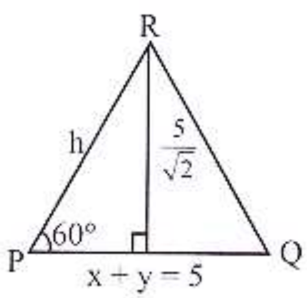
So $A(1,1)$

Altitude from B is $BH = x + 2y - 7 = 0 \Rightarrow B(3,2)$

Altitude from C is $CH = 2x + y - 7 = 0 \Rightarrow C(2,3)$

Centroid of $\Delta ABC = G(2,2), OG = 2\sqrt{2}$

77. (D)



New perpendicular distance from R to L is

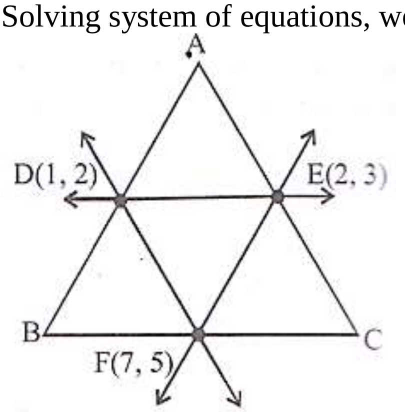
$P = \left|\frac{x + y - 5}{\sqrt{1+1}}\right| = \left|\frac{3 + 7 - 5}{\sqrt{2}}\right| = \frac{5}{\sqrt{2}}$

$\Rightarrow \sin 60^\circ = \frac{\frac{5}{\sqrt{2}}}{h} h = \frac{5\sqrt{2}}{3}$

Area of $\Delta PQR = \frac{\sqrt{3}}{4}h^2 = \frac{25}{2\sqrt{3}}$

78. (6)

Solving system of equations, we get point of intersections (1, 2), (7, 5), (2, 3)



$$\text{Area of }\Delta DEF = \frac{1}{2}\begin{vmatrix}1 & 2 & 1\\ 7 & 5 & 1\\ 2 & 3 & 1\end{vmatrix}$$

$$= \frac{1}{2}\left|1(5-3)-2(7-2)+1(21-10)\right|$$

$$= \frac{1}{2}\left|2-10+11\right| = \frac{1}{2}(3) = \frac{3}{2}$$

$$\text{ar}\Delta ABC = 4\text{ar}\Delta DEF = 4\left(\frac{3}{2}\right) = 6 \text{ sq.unit}$$

79. (3)

Given equations of AB is $2x + y = 0$,

BC is $x + py = 15a$ and CA is $x - y = 3$.

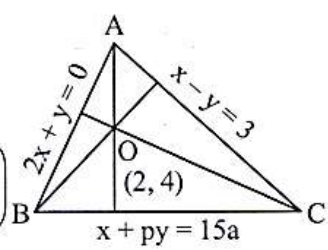
Intersection of lines AB and AC gives coordinate of A.

Then, $y = -2x$ and $x - y = 3$

$\Rightarrow x + 2x = 3 \Rightarrow 3x = 3 \Rightarrow x = 1$

Thus, $y = -2$, So, $A = (1, -2)$

similarly, B is $\left(\frac{15a}{1-2p}, \frac{-30a}{1-2p}\right)$



slope of AO = P

$a + 2 = p$ ...(i)

slope of BO = – 1

$$\frac{\frac{-30a}{1-2p} - a}{\frac{15a}{1-2p} - 2} = -1$$

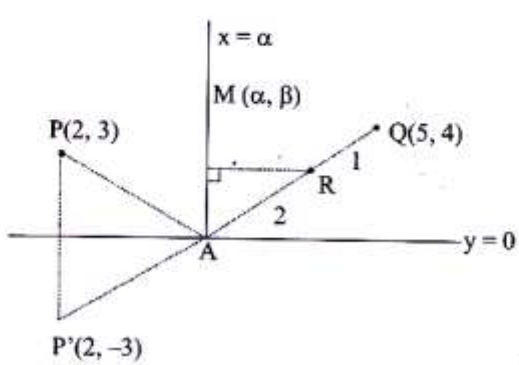
$-30a - a + 12\,pa = -15a + 2 - 4p$

$31a - 2ab = 15a + 4p - 2$ ...(ii)

put the value of p in (ii),

Then $a = 1$ and $p = 3$.

80. (31)



Here, x-coordinate of foot of perpendicular is α. Then, $A(\alpha)$

And $\beta =$ y-coordinate of R

$= \frac{2\times4+1\times0}{2+1} = \frac{8}{3}$ ……(i)

Now, P' is image of P in $y=0$ which will be $P'(2,-3)$

$\therefore$ Equation of $P'Q$ is $(y+3) = \frac{4+3}{5-2}(x-2)$

i.e. $3y+9 = 7x-14$

$7x-3y = 23$

Put $y=0$ in the above equation, then

$x = \frac{23}{7}$

So, coordinate of A is $\left(\frac{23}{7},0\right)$, Therefore , $\alpha = \frac{23}{7}$

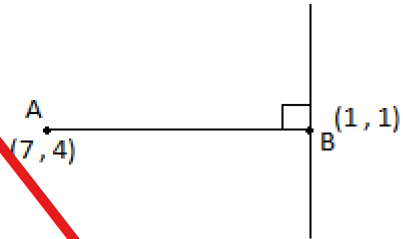
Put $\alpha = \frac{23}{7}$ & $\beta = \frac{8}{3}$ in equation

$7\alpha + 3\beta = 23 + 8 = 31$

STRAIGHT LINES

Exercise – 2(A)

Q.1.



$(2x+3y-5)+\lambda(x+y-2)=0$ represents family of straight lines passing through

intersection of

2x + 3y – 5 = 0 i.e. (1 , 1)

x + y – 2 = 0

the straight line furthest away from (7 , 4) with be perpendicular to the line

joining A (7 , 4) and B (1 , 1)

slope of line : $(2x+3y-5)+\lambda(x+y-2)=0$

i.e. $(2+\lambda)x+(3+\lambda)y-(5+2\lambda)=0$

is $\dfrac{-(2+\lambda)}{(3+\lambda)}$

slope of line is $\dfrac{-(2+\lambda)}{(3+\lambda)}$

slope of line joining (7 , 4) and (1 , 1) is $\dfrac{4-1}{7-1} = \dfrac{3}{6} = \dfrac{1}{2}$

$\therefore$ slope of line perpendicular to AB = – 2

$= \dfrac{-(2+\lambda)}{(3+\lambda)}$

$\therefore \dfrac{2+\lambda}{3+\lambda} = 2$

$$2+\lambda=6+2\lambda$$

$\lambda=-4$ (C)

1 Q.2. Line equidistant from 2x + y = 5 and x + 2y = 4 is the angle bisector of the 2 lines.

i.e.

$$\left(\frac{2x+y-5}{\sqrt{5}}\right)=\pm\left(\frac{x+2y-4}{\sqrt{5}}\right)$$

$$(2x+y-5)\pm(x+2y-4)=0$$

i.e. 3x + 3y – 9 = 0 or x – y – 1 = 0

i.e. x + y – 3 = 0 or x – y – 1 = 0 (A)

2 Q.3 3x + y + 2 = 0

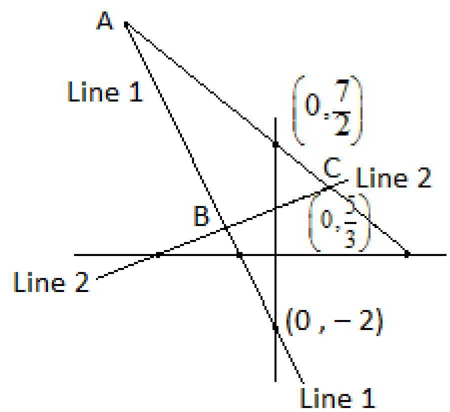
$$\frac{x}{-\frac{2}{3}}+\frac{y}{-2}=1$$

2x – 3y + 5 = 0

$$\frac{x}{-\frac{5}{2}}+\frac{y}{\frac{5}{3}}=1$$

x + 4y = 14

$$\frac{x}{14}+\frac{y}{\frac{7}{2}}=1$$

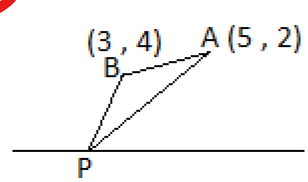


2y – intercepts $\left(0\ ,\ \frac{5}{3}\right)$ and $\left(0\ ,\ \frac{7}{2}\right)$

So point $(0\ ,\ \lambda)$

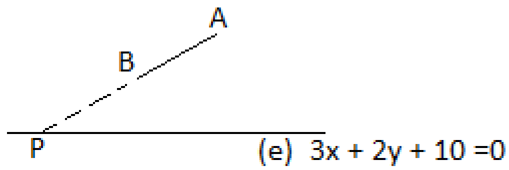
$$\frac{5}{3}<\lambda<\frac{7}{2}$$ (B)

3 Q.4 $|PA-PB|$



$AB > |PA - PB|$ (triangle law)

So PA – PB will be max if PBA are collinear, then PA = PB = AB



Slope of AB $= \frac{2-4}{5-3} = -1$

Equation of AB ≡ (y – 2) = (– 1) (x – 5)

y – 2 = 5 – x

y = 7 – x ………………….(1)

Intersection of 3x + 2y + 10 = 0 and (AB) x + y – 7 = 0

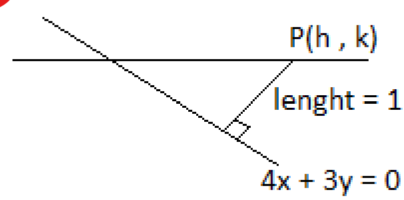
$$\frac{x}{\begin{vmatrix} 2 & 10 \\ 1 & -7 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 3 & 10 \\ 1 & -7 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}}$$

$$\frac{\lambda}{-24} = \frac{-y}{-31} = \frac{1}{1}$$

x = – 24 , y = 31

(x , y) ≡ (– 24 , 31) (A)

4 Q.5. Point P lies on x + y = 4



$$\left| \frac{4h+3k}{\sqrt{4^2+3^2}} \right| = 1 \qquad \begin{bmatrix} h+k=4 \because \text{point lies} \\ \text{on x + 4 = 4} \\ \therefore\ \text{k} = 4-\text{h} \end{bmatrix}$$

$$\left|\frac{4h+3(4-h)}{5}\right|=1$$

$$|4h+12-3h|=5$$

$$|12+h|=5$$

$$12+h=\pm 5$$

$$h=-12 \pm 5$$

h = − 7 , − 17

If h = −7 , h = −17

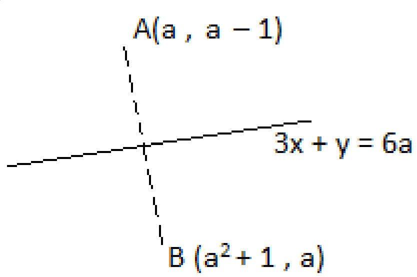
k = 4 −h , k = 21

k = 11

(−7 , 11) (−17,21)

Ans. : (D)

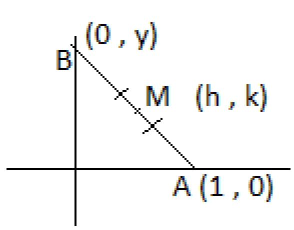
5 D Q.6.



Using formula for reflection about line

6 D Q.7. A (1 , 0)

B (0 , α)

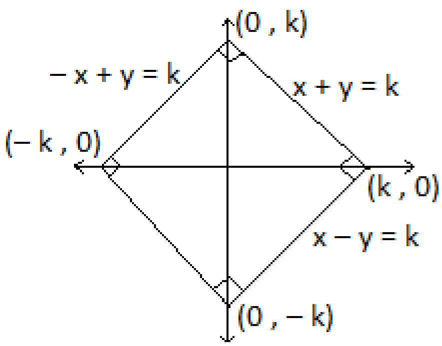


$M \equiv \left(\frac{1+0}{2}, \frac{\alpha+0}{2}\right)$ $\quad\quad$ $\therefore h = \frac{1}{2}$ and $k = \frac{\alpha}{2}$

$M \equiv \left(\frac{1}{2}, \frac{\alpha}{2}\right)$ $\quad\quad$ $\therefore x = \frac{1}{2}$ is the locus of M

Ans. : (A)

7 Q.8. $|x| + |y| = k$



Length of the side $= \sqrt{(k-0)^2 + (0-k)^2}$

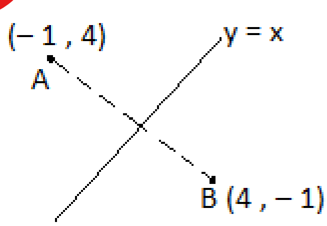
$= \sqrt{2}k$

So Area of square $= 2k^2$ = 8 sq. unit.

$\therefore k^2 = 4$

k = 2 $\quad\quad$ (C)

8 Q.9



A is (− 1 , 4)

Distance AB $= \sqrt{(-1-4)^2 + (4-(-1))^2}$

$$= \sqrt{25+25}$$

$$= 5\sqrt{2}$$ sq. unit. Ans. : (A)

9 Q.10. a + c = 2b c = 2b – a

ax + by + c = 0

ax + by + 2b – a = 0

a (x – 1) + b (y + 2) = 0

$$(x-1) + \frac{b}{a}(y+2) = 0$$

Represents family of straight lines passing through the intersection of the 2 lines

x – 1 and y = – 2

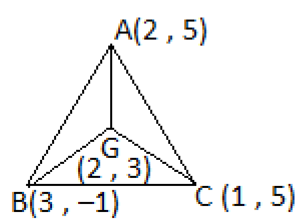
i.e. x = 1 and y = – 2

it of intersection is (1 , – 2)

So, ax + by + c = 0 always passes through (1 , – 2)

Ans. : (C)

10 Q.11. G is centroid of Δ ABC



Geometrically the areas will be equal but we will find the areas none the loss

$$S_1 = \Delta GBC = \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 & 2 \\ 3 & -1 & 5 & 3 \end{vmatrix}$$

$$= \frac{1}{2}\left|(-2+15+3)-(9-1+10)\right|$$

$$= \frac{1}{2}\left|16-(18)\right| = 1$$ sq. units.

$$S_3 = \Delta GBA = \frac{1}{2}\begin{vmatrix} 2 & 2 & 3 & 2 \\ 5 & 3 & -1 & 5 \end{vmatrix}$$

$$= \frac{1}{2} \left| (6-2+15)-(10+9-2) \right|$$

$$= \frac{1}{2} \left| 19-17 \right| = 1$$ sq. units.

$$S_2 = \Delta GAC = \frac{1}{2} \begin{vmatrix} 2 & 1 & 2 & 2 \\ 5 & 5 & 3 & 5 \end{vmatrix}$$

$$= \frac{1}{2} \left| (10+3+10)-(5+10+6) \right|$$

$$= \frac{1}{2} \left| 23-21 \right| = 1$$ sq. units.

$S_1 = S_2 = S_3$ Ans. : (D)

Q.12. (p + q) x + (2p + q) y = p + 2q

p (x + 2y – 1) + q (x + y – 2) = 0

This represents a family of straight lines passing through the intersection of

x + 2y – 1 = 0 &

x + y – 2 = 0

i.e.

$$\frac{x}{\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}}$$

Cancel

$$\frac{x}{-3} = \frac{-y}{-1} = \frac{1}{-1}$$

x = 3 , y = – 1

(3 , – 1)

$x_1 = 3$, $y_1 = -1$

11 $x_3 + y_1 = 2$ Ans. : (A)

Q.13. $2x^2 + 5x - 4 = 0 : \alpha, \beta$

Centroid of $\left(\alpha, \frac{1}{\alpha}\right), \left(\beta + \frac{1}{\beta}\right), (0,0)$

$$= \left(\frac{\alpha+\beta+0}{3}, \frac{\frac{1}{\alpha}+\frac{1}{\beta}+0}{3} \right)$$

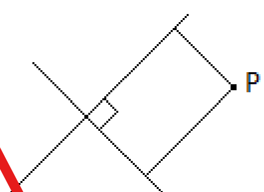
$$\alpha+\beta = \frac{-5}{2}$$

$$\alpha\beta = \frac{-4}{2}$$

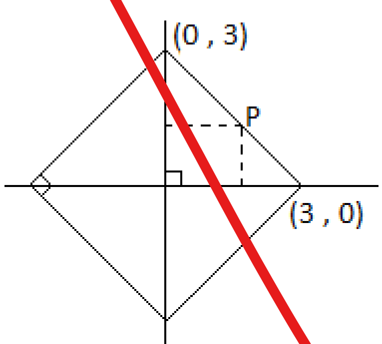
So, $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{-5}{2}}{\frac{-4}{2}} = \frac{5}{4}$

Centroid $\equiv \left(\frac{-5}{6}, \frac{5}{12}\right)$ Ans. : (B)

Q.14.



Without loss of generality we can assume the perpendicular lines as the x and y



So the condition becomes

$|x|+|y|=3$

Cancel

Area enclosed = Area of square

12

$= \left(3\sqrt{2}\right)^2$ = 18 sq. units. Ans. : (D)

Q.15. $x = -2 + \dfrac{r}{\sqrt{10}}$, $y = 1 + \dfrac{3r}{\sqrt{10}}$

$x = x_1 + r\cos\theta$, $y = y_1 + r\sin\theta$

Slope = $\tan\theta = \dfrac{3}{1}$

Point on the line is (– 2 , 1)

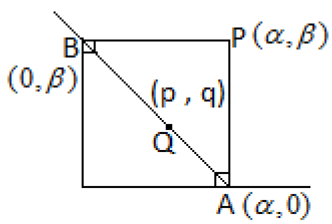
So, equation is

$$(y - 1) = 3 (x + 2)$$

$$y = 3x + 7$$

y intercept is 7 Ans. : (B)

13 Q.16.



Hence x – co – ordinate of P is α

y – co – ordinate of P is β

equation of the line (double intercept is)

$$\frac{x}{\alpha} + \frac{y}{\beta} = 1$$

Substituting co – ordinates of Q we get

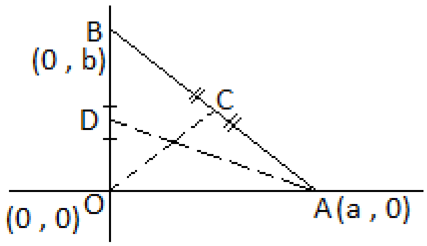
$$\frac{p}{\alpha} + \frac{q}{\beta} = 1$$

Replacing α , β by x and y to get the equation of P.

$\frac{p}{x}+\frac{q}{y}=1$ Ans. : (C)

Q.17.

141



Co – ordinates of D $\equiv\left(\frac{0+0}{2}, \frac{0+b}{2}\right)$

$\equiv\left(0, \frac{b}{2}\right)$

Co – ordinates of C $\equiv\left(\frac{0+a}{2}, \frac{b+0}{2}\right)$

$\equiv\left(\frac{a}{2}, \frac{b}{2}\right)$

AD perpendicular OC

Slope of AD $=\frac{0-\frac{b}{2}}{a-0}=\frac{-b}{2a}$

Slope of OC $=\frac{\frac{b}{2}-0}{\frac{a}{2}-0}=\frac{b}{a}$

slope AD × slope BC = – 1

$\frac{-b^2}{2a^2}=-1 \qquad \therefore\ b^2=2a^2$ Ans. : (B)

15 Q.18. $x\sec\theta + y\,\mathrm{cosec}\,\theta = a \quad (L_1)$

Perpendicular distance from origin

$$P_1 = \left| \frac{-a}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}} \right|$$

$$P_2 = \left| a\cos\theta \sin\theta \right| = \left| \frac{a\sin^2\theta}{2} \right|$$

$$(L_2) \quad x\cos\theta - y\sin\theta = a\cos 2\theta$$

Perpendicular distance of origin from line

$$P_2 = \left| \frac{-a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}} \right|$$

$$P_2 = \left| a\cos 2\theta \right|$$

$$4P_1^2 + P_2^2 = a^2\cos^2 2\theta + 4\left(\frac{a^2\sin^2 2\theta}{4}\right)$$

$$= a^2$$

Independent of θ Ans. : (A)

16 Q.19. y = mx (L_1) y – mx + 0 = 0

Equation of L_2 is (y – 3) = 2m (x – 2)

Y = 2mx – 4m + 3 , y – 2mx + 4m – 3 = 0 (L_2)

Intersection is $x = \frac{1}{m}$ and y = 1

$$\frac{x}{\begin{vmatrix} 1 & 0 \\ 1 & 4m-3 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -m & 0 \\ -2m & 4m-3 \end{vmatrix}} = \frac{1}{\begin{vmatrix} -m & 1 \\ -2m & 1 \end{vmatrix}}$$

$$\frac{x}{4m-3} = \frac{-y}{-m(4m-3)} = \frac{1}{m}$$

$$x = \frac{4m-3}{m} \quad , \quad y = 4m-3$$

$\therefore x = 4 - \frac{3}{m} \qquad \frac{y+3}{4} = m$ ....(ii)

$\frac{3}{m} = 4 - x$

$\frac{3}{4-x} = m$ ....(i)

Equating (i) and (ii) we get

$\frac{3}{4-x} = \frac{y+3}{4}$

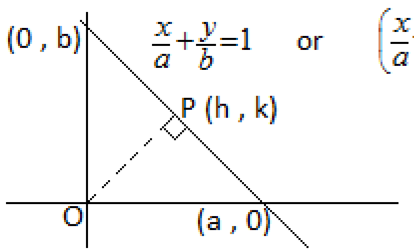
12= (y + 3) (4 – x)

$\cancel{12} = 4y - xy + \cancel{12} - 3x$

$\boxed{3x - 4y + xy = 0}$ Ans. : (C)

17 Q.20.

$\frac{x}{a} + \frac{y}{b} = 1$ or $\left(\frac{x}{a} + \frac{y}{b} - 1 = 0\right)$



$$\frac{h-0}{\frac{1}{a}} = \frac{k-0}{\frac{1}{b}} = -\frac{\left(\frac{0}{a} + \frac{0}{b} - 1\right)}{\frac{1}{a^2} + \frac{1}{b^2}}$$

$$h = \frac{\frac{1}{a}}{\left(\frac{1}{a^2} + \frac{1}{b^2}\right)} = \frac{c^2}{a}$$

$$k = \frac{\frac{1}{b}}{\left(\frac{1}{a^2} + \frac{1}{b^2}\right)} = \frac{c^2}{b}$$

$$\therefore\ h^2 + k^2 = c^4\left(\frac{1}{a^2} + \frac{1}{b^2}\right)$$

$$= c^4\left(\frac{1}{c^2}\right)$$

$$h^2 + k^2 = c^2$$

Locus is $\boxed{x^2 + y^2 = c^2}$ Ans. : (A)

18 @ Q.21. $x^2 + y^2 \leq 36$ and 3x – 4y = 25

Let $x = r\cos\theta$

$y = r\sin\theta$

So we get

$$r^2\cos^2\theta + r^2\sin^2\theta \leq 36$$

$$r^2 \leq 36$$

$$r \leq 6$$

and

$$3(r\cos\theta) - 4(r\sin\theta) = 25$$

$$r(3\cos\theta - 4\sin\theta) = 25$$

$$r = \frac{25}{3\cos\theta - 4\sin\theta}$$

$$r \leq 6$$

$$\therefore\ \frac{25}{3\cos\theta - 4\sin\theta} \leq 6$$

$$3\cos\theta - 4\sin\theta \geq \frac{25}{6}$$

$$\frac{3}{5}\cos\theta - \frac{4}{5}\sin\theta \geq \frac{5}{6}$$

$$\sin 37^\circ \, \underline{n} \, \frac{3}{5}$$

$\sin(37^\circ - \theta) \geq \dfrac{5}{6}$ no. of angles. Between θ and 360° are infinite.

So infinitely many solutions. Ans. : (D)

19 Q.22. Let A (x_1 , y_1) , B(x_2 , y_2) , C (x_3 , y_3) be the points.

ax + by + c = 0 be the line

Given

$$\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} + \frac{ax_2 + by_2 + c}{\sqrt{a^2 + b^2}} + \frac{ax_3 + by_3 + c}{\sqrt{a^2 + b^2}}$$

$$a(x_1 + x_2 + x_3) + b(y_1 + y_2 + y_3) + 3c = 0$$

Dividing by 3 we get

$$a\left(\frac{x_1 + x_2 + x_3}{3}\right) + b\left(\frac{y_1 + y_2 + y_3}{3}\right) + c = 0$$

Co – ordinates of the centroid satisfy

ax + by + c = 0

passes through the centroid Ans. : (B)

Q.23. $\dfrac{x}{a} + \dfrac{y}{b} = 1$

20

$$\frac{x}{b} + \frac{y}{a} = 1$$

Point of intersection is

$$\frac{x}{\begin{vmatrix} \frac{1}{a} & -1 \\ \frac{1}{b} & -1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} \frac{1}{b} & -1 \\ \frac{1}{a} & -1 \end{vmatrix}} = \frac{-1}{\begin{vmatrix} \frac{1}{a} & \frac{1}{b} \\ \frac{1}{b} & \frac{1}{a} \end{vmatrix}}$$

$$\frac{x}{\left[\frac{1}{b}-\frac{1}{a}\right]}=\frac{-y}{\left[\frac{1}{a}-\frac{1}{b}\right]}=\frac{-1}{\left[\frac{1}{a^2}-\frac{1}{b^2}\right]}$$

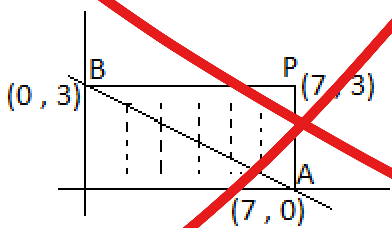
$$x = +\left(\frac{1}{a}+\frac{1}{b}\right)=y$$

So point $\left(\frac{1}{a}+\frac{1}{b}\ ,\ \frac{1}{a}+\frac{1}{b}\right)$ lies on the line y = x Ans. : (A)

~~Q.24. 3x + 7y = 21~~

$\cancel{\frac{x}{7}+\frac{y}{3}=1}$

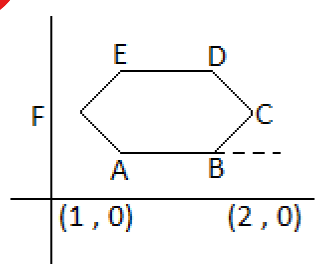
Cancel

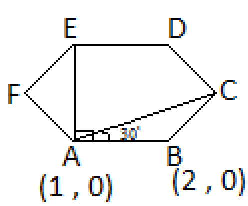


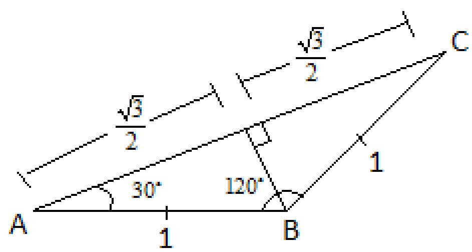
~~Number of lattize points is~~ $\cancel{=2\times 6}$

$= 12$ Ans. : (A)

21 11 Q.25.







AC = AE = $\sqrt{3}$ AC = $\sqrt{3}$

Rotate B about A by 30° and scale by $\sqrt{3}$ to get C i] $\left(2\ +\ 0\cdot\mathrm{i}\right)\ -\ \left(1\ +0\cdot\mathrm{i}\right)$ ……………(subtract)

= $1\ +\ 0\cdot\mathrm{i}$

ii] rotate/ scale

$$(1+0\cdot i)\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)\sqrt{3}$$

$$\equiv \frac{3}{2}+\frac{\sqrt{3}i}{2}$$

iii] Add co – ordinates of A

$$\frac{3}{2}+\frac{\sqrt{3}i}{2}+1+0\cdot i=\frac{5}{2}+\frac{\sqrt{3}i}{2}$$

$$C\equiv\left(\frac{5}{2}\ ,\ \frac{\sqrt{3}}{2}\right)$$

Get co – ordinates of E by rotating B 90° anticlockwise about A and scale by $\sqrt{3}$

1) Subtract

   $(2+0\cdot i)-(1+0\cdot i)$

   $=(1+0\cdot i)$

2) Scale/rotate

   $(1+0\cdot i)(0+i)(\sqrt{3})$

   $=\sqrt{3}i$

3) Add

   $(0+\sqrt{3}i)+(1+0\cdot i)$

   $\equiv 1+\sqrt{3}i$

   $E\equiv(1\ ,\ \sqrt{3})$

Slope of CE is $\dfrac{\sqrt{3}-\dfrac{\sqrt{3}}{2}}{1-\dfrac{5}{2}}\ =\ \dfrac{\dfrac{\sqrt{3}}{2}}{\dfrac{-3}{2}}\ =\ \dfrac{-1}{\sqrt{3}}$

Equation of CE

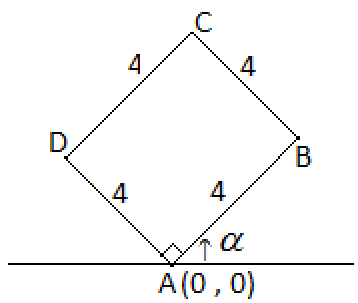
$$(y-\sqrt{3})=\frac{-1}{\sqrt{3}}(x-1)$$

$$\sqrt{3}y-3+x-1=0$$

$$\sqrt{3}+x=4$$ Ans. : (A)

Q.26.

22



Co – ordinates of B $\equiv$ $(4\cos\alpha, 4\sin\alpha)$

Co – ordinates of D $\equiv$ $(4\cos(90+\alpha), 4\sin(90+\alpha))$

$$\equiv(-4\sin\alpha, 4\cos\alpha)$$

Equation of BD = ?

Equation of BD $= \dfrac{4\cos\alpha-4\sin\alpha}{-4\sin\alpha-4\cos\alpha}$

$$=\left(\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}\right)$$

Equation is

$$(y-4\sin\alpha)=\left(\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}\right)(x-4\cos\alpha)$$

$$(y-4\sin\alpha)(\sin\alpha+\cos\alpha)=(\sin\alpha-\cos\alpha)(x-4\cos\alpha)$$

$$4\cos\alpha(\sin\alpha-\cos\alpha)-4\sin\alpha(\sin\alpha+\cos\alpha)=x(\sin\alpha-\cos\alpha)-y(\sin\alpha+\cos\alpha)$$

$$4\sin\alpha\cos\alpha-4\cos^2\alpha-4\sin^2\alpha-4\sin\alpha\cos\alpha=x(\sin\alpha-\cos\alpha)-y(\sin\alpha+\cos\alpha)$$

$$-4=x(\sin\alpha-\cos\alpha)-y(\sin\alpha+\cos\alpha)$$

$$4 = x(\cos\alpha - \sin\alpha) + y(\sin\alpha + \cos\alpha)$$ Ans. : (C)

Q.27 $\frac{2}{b} = \frac{1}{a} + \frac{1}{c} \quad \Rightarrow \quad \frac{1}{c} = \frac{2}{b} - \frac{1}{a}$

23

bcx + acy + ab = 0

$$\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$$

$$\frac{x}{a} + \frac{y}{b} + \frac{2}{b} - \frac{1}{a} = 0$$

$$(x-1)\left(\frac{1}{a}\right) + \left(\frac{1}{b}\right)(y+2) = 0$$

24

Family of straight line passing through (1 , − 2) Ans. : (D)

Q.28. Lines : 4x – 7y + 10 = 0 and 7x + 4y = 15 are perpendicular. So, it is a right angled triangle whose orthocenter is at the right angle. i.e. intersection of

4x – 7y + 10 = 0 and 7x + 4y – 15 = 0

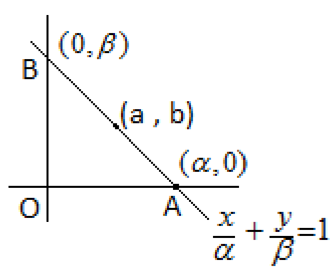
$$\frac{x}{\begin{vmatrix} 10 & -7 \\ -15 & 4 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 4 & 10 \\ 7 & -15 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 4 & -7 \\ 7 & 4 \end{vmatrix}}$$

$$\frac{x}{(-65)} = \frac{y}{(-130)} = \frac{1}{65}$$

$$x = -1 \quad , \quad y = -2$$

Q.29.

25



Equation of line is passing through (a , b). So,

$$\frac{a}{\alpha} + \frac{b}{\beta} = 1 \qquad \text{..................(i)}$$

Now centroid of $\Delta OAB = (h\ ,\ k)$

$$(h\ ,\ k) \equiv \left(\frac{0 + 0 + \alpha}{3}\ ,\ \frac{0\ +\ 0\ +\ \beta}{3} \right)$$

$$= \left(\frac{\alpha}{3}\ ,\ \frac{\beta}{3} \right)$$

$\alpha = 3h\ ,\ \ \beta = 3k$

So putting in equation (i) we get

$$\frac{a}{3h} + \frac{b}{3k} = 1$$

Replacing h by x and k by y we get

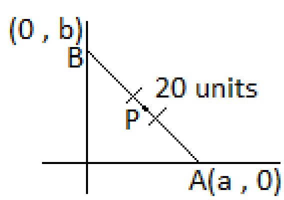
$$\frac{a}{3x} + \frac{b}{3y} = 1$$

ay + bx = 3xy

bx + ay – 3xy = 0 Ans. : (A)

Q.30.

26



Equation of AB is $\frac{x}{a} + \frac{y}{b} = 1$

Let co – ordinates of P be (h , k) mid – point of AB

$$(h\ ,\ k) \equiv \left(\frac{0 + a}{2}\ ,\ \frac{b + 0}{2} \right)$$

$$= \left(\frac{a}{2} \ , \ \frac{b}{2} \right)$$

$h = \frac{a}{2}$, $k = \frac{b}{2}$, a = 2h , b = 2k

$|AB| = 20$ units.

$$|AB| = 20 = \sqrt{(a-0)^2 + (0-b)^2}$$

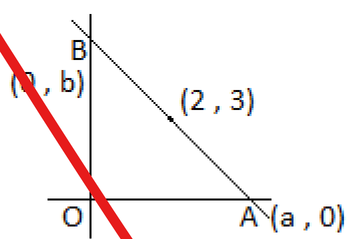
$$400 = a^2 + b^2$$

$$400 = 4h^2 + 4k^2$$

$\Rightarrow h^2 + k^2 = 100$

$\Rightarrow x^2 + y^2 = 100$ is the locus of P   Ans. : (D)

Q.31.



Line AB is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$\Delta\, AOB = 12$ sq. units = $\frac{1}{2}\,|\,ab\,|$

$\therefore |\,ab\,| = 24$

Line AB passes through (2 , 3). So,

$$\frac{2}{a} + \frac{3}{b} = 1$$

2b + 3a = ab ....(i)

Case 1 : ab = + 24

then $b = \frac{24}{a}$

$\therefore$ putting in (i) we get,

$$\frac{48}{a} + 3a = 24$$

$$\frac{16}{a} + a = 8$$

$$a^2 + 16 - 8a = 0$$

(a – 4) (a – 4) = 0

a = 4 $\therefore$ b = 6

$\frac{x}{4} + \frac{y}{6} = 1$ line is one

Case 2 : ab = – 24

$$b = \frac{-24}{a} \qquad \therefore \frac{2(-24)}{a} + 3a = -24$$

$$\frac{-48}{a} + 3a = -24$$

$$\frac{-16}{a} + a = -8$$

$$a^2 - 16 = -8a$$

$$a^2 + 8a - 16 = 0$$

$$a = \frac{-8 \pm \sqrt{64+64}}{2}$$

$$a = \frac{-8 \pm 8\sqrt{2}}{2}$$

$$a = -4 \pm 4\sqrt{2}$$

If $a = -4 + 4\sqrt{2} = 4 - 4\sqrt{2}$

$$b = \frac{-24}{-4 + 4\sqrt{2}} = \frac{-24(4 + 4\sqrt{2})}{16}$$

$b = -6(\sqrt{2} + 1)$

$a = 4(\sqrt{2} - 1)$

$b = -6(\sqrt{2} + 1)$

If $a = -4 - 4\sqrt{2}$

i.e. $a = -4(1+\sqrt{2})$

then $b = \dfrac{-24}{a}$

$b = 6(\sqrt{2} - 1)$

Lines can be

$$\frac{x}{4(\sqrt{2} - 1)} + \frac{y}{-6(\sqrt{2} - 1)} = 1$$

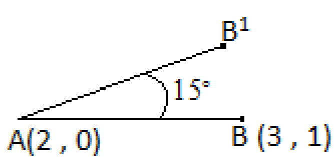
$$\frac{x}{-4(\sqrt{2} + 1)} + \frac{y}{6(\sqrt{2} - 1)} = 1$$

Number of possibilities = 3 Ans. : (C)

Cancel

Q.32.

27



Slope of AB $= \dfrac{1-0}{3-2} = 1$

Slope of $AB^1 = \dfrac{m + \tan\alpha}{1 - m\tan\alpha}$ $(\alpha = 15^\circ)$

$$= \frac{1 + 2 - \sqrt{3}}{1 - (1)(2 - \sqrt{3})}$$

$$= \frac{3-\sqrt{3}}{\sqrt{3}-1} = \sqrt{3}$$

$\therefore$ Equation of AB^1 is

$$y - 0 = \sqrt{3}\,(x-2)$$

$$y = \sqrt{3}x - 2\sqrt{3}$$

$$0 = \sqrt{3}x - y - 2\sqrt{3}$$

Ans. : (C)

Q.33

28

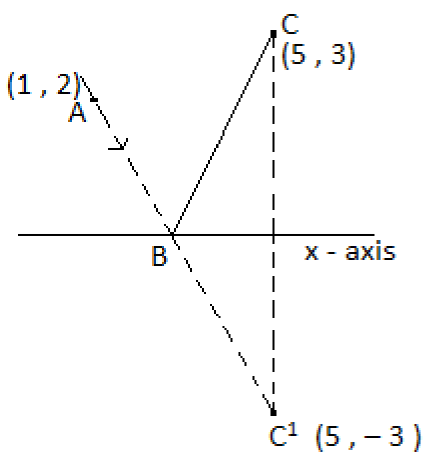


Image of C in x – axis is C^1

ABC^1 are collinear. So, equation of AB and AC^1 is the same

Slope of AC^1 is $\frac{-3-2}{5-1} = \frac{-5}{4}$

Equation is $y - 2 = \frac{-5}{4}(x-1)$

4 (y – 2) + 5 (x – 1) = 0

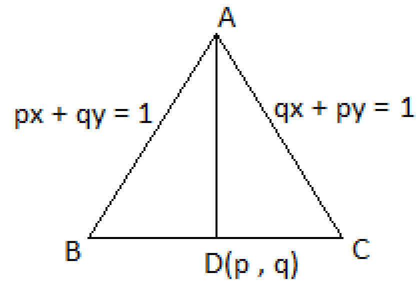
4y – 8 + 5x – 5 = 0

4y + 5x = 13

Ans. : (A)

29

Q.34 Family of straight lines passing through A is



$(px + qy - 1) + \lambda(qx + py - 1) = 0$

But this passing through (p , q)

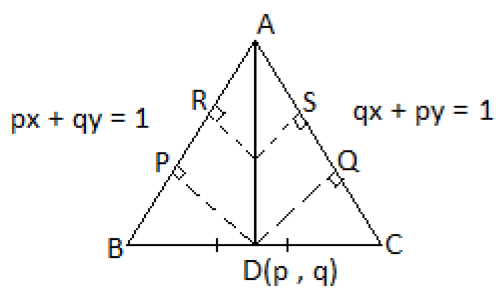
So $(p^2 + q - 1) + \lambda(pq + qp - 1) = 0$

$\therefore \lambda = \dfrac{-(p^2 + q^2 - 1)}{(2pq - 1)}$

$\therefore$ equation of AD is

$$(px + qy - 1) - \frac{(p^2 + q^2 - 1)}{(2pq)}(qx + py - 1) = 0$$

<u>Alternate way</u>



Let x be any point on AD

Using similarity we get

$$\frac{RX}{PD} = \frac{XS}{DQ} \qquad \therefore \qquad \frac{RX}{XS} = \frac{PD}{DQ}$$

$$PD = \left|\frac{p^2 + q^2 - 1}{\sqrt{p^2 + q^2}}\right| \qquad RX = \left|\frac{px + qy - 1}{\sqrt{p^2 + q^2}}\right|$$

$$DQ = \left|\frac{2pq - 1}{\sqrt{p^2 + q^2}}\right| \qquad XS = \left|\frac{qx + py - 1}{\sqrt{p^2 + q^2}}\right|$$

Substituting we get

$$\left|\left(\frac{px + qy - 1}{qx + py - 1}\right)\right| = \left|\frac{p^2 + q^2 - 1}{2pq - 1}\right|$$

$$\frac{px+qy-1}{qx+py-1} = +\left(\frac{p^2+q^2-1}{2pq-1}\right)$$

$\because$ x and (p , q) lie on the same side w.r.t. to the 2 lines.

$$(2pq-1)(px+pq-1)=(p^2+q^2-1)(qx+py-1)$$ Ans. : (C)

30

Q.35. $x^2 - 8x + 12 = 0$

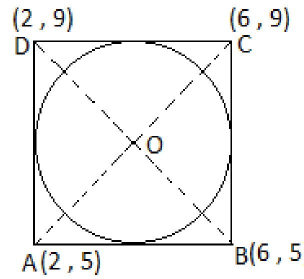
$\Rightarrow (x - 6)(x - 2) = 0$

$\Rightarrow x = 6 , x = 2$

$y^2 - 14y + 45 = 0$

$\Rightarrow (y - 9)(y - 5) = 0$

$\Rightarrow y = 9 , y = 5$



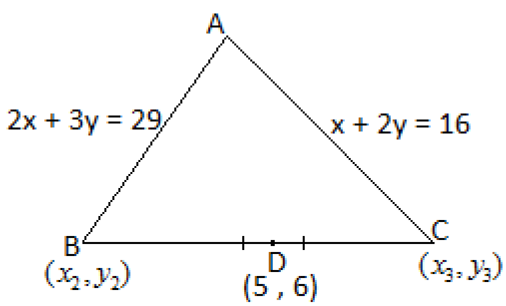
Center = mid – point of AC

$$= \left(\frac{2+6}{2}, \frac{5+9}{2}\right)$$

= (4 , 7) Ans. : (B)

31

Q.36.



$x_3 + 2y_3 = 16$ ....(i)

$2x_2 + 3y_2 = 29$ ....(ii)

$$\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right) \equiv (5,6)$$

$x_2 + x_3 = 10$, $y_2 + y_3 = 12$

$x_3 = 10 - x_2$, $y_3 = 12 - y_2$

Putting in (i) we get

$$(10 - x_2) + 2(12 - y_2) = 16$$

$$10 - x_2 + 24 - 2y_2 = 16$$

$$34 - x_2 - 2y_2 = 16$$

$$18 = x_2 + 2y_2$$

$36 = 2x_2 + 4y_2$ ....(iii)

(iii) – (ii)

$$y_2 = 7$$

And $\therefore x_2 = 18 - 2y_2$

$$x_2 = 18 - 14$$

$$x_2 = 4$$

B (4 , 7)

Equation of BC = Equation of BD

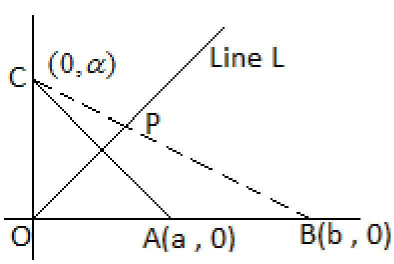
Slope BD $= \dfrac{7-6}{4-5} = -1$

(y – 6) = (– 1) (x – 5)

Y = 5 – x + 6 $\Rightarrow$ y = 11 – x   Ans. : (C)

11 Q.37

32



Equation of AC is

$\frac{x}{a}+\frac{y}{\alpha}=1$

So equation of OP is

$\frac{x}{\alpha}-\frac{y}{a}=0$ (perpendicular to AC passing through 0)

Or $ax=\alpha y$ ....(i)

Equation of BC is

$\frac{x}{a}+\frac{y}{\alpha}=1$ ....(ii)

Let $P\equiv(h\ ,\ k)$ is the intersection of (i) and (ii)

$$\frac{h}{\begin{vmatrix} 0 & \frac{-1}{a} \\ +1 & \frac{1}{\alpha} \end{vmatrix}}=\frac{k}{\begin{vmatrix} \frac{1}{\alpha} & 0 \\ \frac{1}{b} & +1 \end{vmatrix}}=\frac{1}{\begin{vmatrix} \frac{1}{\alpha} & \frac{-1}{a} \\ \frac{1}{b} & \frac{1}{\alpha} \end{vmatrix}}$$

$$\frac{h}{\frac{+1}{a}}=\frac{k}{\frac{+1}{\alpha}}=\frac{1}{\frac{1}{\alpha^2}+\frac{1}{ab}}=\frac{ab\alpha^2}{ab+\alpha^2}$$

$$\therefore\ h=\frac{+b\alpha^2}{ab+\alpha^2}$$

$$k=\frac{+ab\alpha}{ab+\alpha^2}$$

$$h-(ab+\alpha^2)=b\alpha^2$$

$$hab=\alpha^2\ (b-h)$$

$\alpha^2=\frac{hab}{b-h}$ ....(A)

$$ab\alpha=(ab+\alpha^2)k$$

$$= \left(ab + \frac{hab}{b-h} \right) k$$

$$ab\alpha = ab \left(\frac{b}{b-h} \right) k$$

$$\alpha = \frac{bk}{b-h} \qquad \text{..................(B)}$$

From (A) & (B) we get

$$\alpha^2 = \frac{hab}{b-h} = \frac{b^2k^2}{(b-h)^2}$$

$$ha\,(b-h) = bk^2$$

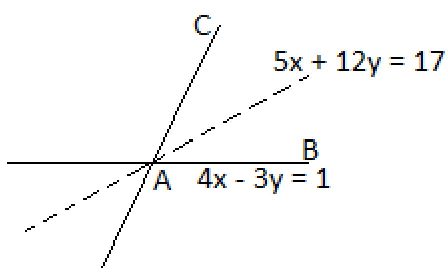
$$hab - h^2a = bk^2$$

$$hab = bk^2 + h^2a$$

$$\therefore \boxed{xab = ax^2 + by^2}$$

Ans. : (C)

Q.78.



AP is the angle bisector of AB and AC

$$\frac{(5x + 12y - 17)}{(\sqrt{5^2 + 12^2})} = \pm \left(\frac{4x - 3y - 1}{\sqrt{4^2 + 3^2}} \right)$$

$$\frac{5x + 12 - 17}{13} = \pm \left(\frac{4x - 3y - 1}{5} \right)$$

$$5(5x + 12y - 17) \pm 13(4x - 3y - 1) = 0$$

i] $(25x + 60y - 85) \pm (52x - 39y - 13) = 0$

1) 77x + 21y – 98 = 0 i.e.

OR 11x + 3y – 14 = 0

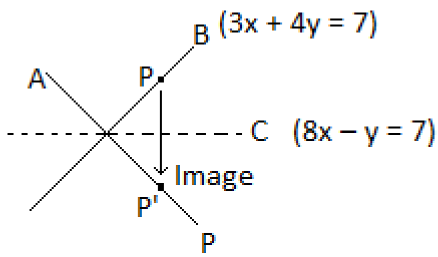
2) –27x + 99y – 72 = 0

 – 3x + 11y – 8 = 0 Ans. : (A) (B)

Q 39. 3x + 4y = 7 (AB)

74

8x – y = 7 (AC)



Any point online AB P can be written as

(1 – 4t , 1 + 3t) [parameter is t and it satisfies the equation]

Take the image of it write AC to get P' (h , k)

$$\frac{h - (1 - 4t)}{8} = \frac{k - (1 + 3t)}{-1} = \frac{-2(8(1-4t) - (1+3t) - 7)}{(8^2 + 1^2)}$$

$$\frac{h + 4t - 1}{8} = \frac{k - 3t - 1}{-1} = \frac{-2\,(8 - 1 - 7 - 32t - 3t)}{65}$$

$$= \frac{-2(-35t)}{65} = \frac{14t}{13}$$

$$\frac{h + 4t - 1}{8} = \frac{14t}{13}$$

$$h + 4t - 1 = 8\left(\frac{14}{13}\right)t$$

$$h = 1 + \left(\frac{112}{13} - 4\right)t$$

$$h = 1 + \left(\frac{60}{13}\right)t \quad \Rightarrow \quad \frac{t}{13} = \frac{h-1}{60}$$

$$k = 1 + 3t - \frac{14}{13}t$$

$$k = 1 + \left(\frac{25}{13}\right)t \quad \Rightarrow \quad \frac{t}{13} = \frac{k-1}{25}$$

$$\therefore \frac{h-1}{\underset{12}{\cancel{60}}} = \frac{k-1}{\underset{5}{\cancel{25}}}$$

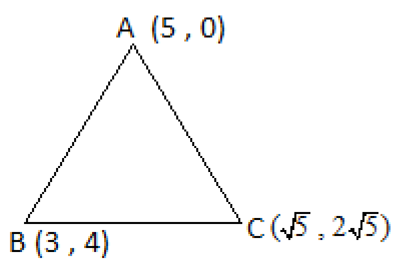
$$5(h-1) = 12(k-1)$$

$$5h - 5 = 12k - 12$$

$\therefore \boxed{5x+7=129}$ Ans. : (A)

Q.10.

35

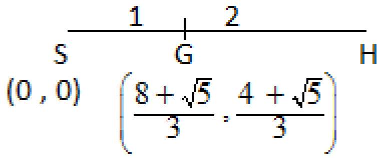


Circumcenter of ΔABC is (0 , 0)

OA = 5 units $\quad OC^2 \equiv \left((\sqrt{5})^2 + (2\sqrt{5})^2\right)$

OB = 5 units $\quad OC^2 = 25$ units.

Centroid $\equiv \left(\dfrac{8+\sqrt{5}}{3}, \dfrac{4+\sqrt{5}}{3}\right)$

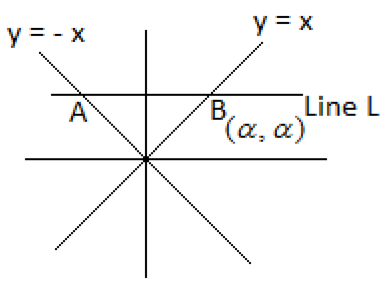


Co – ordinates of H

$\equiv (8 + \sqrt{5}\ , 4 + 2\sqrt{5})$ Ans. : (A)

Q.11.

36



$$\begin{pmatrix} x = \alpha + r\cos\theta \\ y = \alpha + r\sin\theta \end{pmatrix}$$

For A y = − x

$$\therefore\ \alpha + r\cos\theta = -(\alpha + r\sin\theta)$$

$$r(\cos\theta + \sin\theta) = -2\alpha$$

$$r = \left(\frac{-2\alpha}{\cos\theta + \sin\theta}\right)$$

$$|r| = \left|\frac{2\alpha}{\cos\theta + \sin\theta}\right| = \text{constant k}$$

Mid – point of AB is by taking

$$r = \frac{-\alpha}{\cos\theta + \sin\theta}$$

$$x = \alpha - \frac{\alpha}{\cos\theta + \sin\theta}$$

$$x = \frac{\alpha\sin\theta}{\cos\theta + \sin\theta}$$

$$x = \alpha - \frac{\alpha \sin\theta}{\cos\theta + \sin\theta}$$

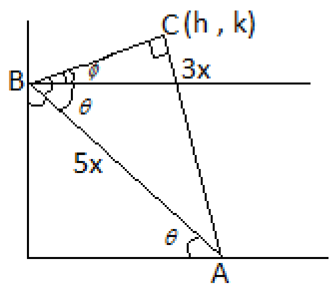
$$y = \frac{\alpha \cos\theta}{\cos\theta + \sin\theta}$$

$$x^2 + y^2 = \frac{\alpha^2}{(\sin\theta + \cos\theta)^2} = \frac{k^2}{4}$$

$4x^2 + 4y^2 = k^2$ is the locus of mid – point of AB

Q.4~~1~~.

37



$\angle CBA \; \underline{n} \; 37^\circ$

$\phi = 37^\circ - \theta$

$B \equiv (0 \, , \, 5x \sin\theta)$

Inclination of BC is $37^\circ - \theta$

$C \equiv (h \, , \, k)$ using parameter locus we get

$$\frac{h - 0}{\cos(37^\circ - \theta)} = \frac{k - 5x\sin\theta}{\sin(37^\circ - \theta)} = 4x$$

$$h = 4x\cos(37^\circ - \theta)$$

$$k = 4x\sin(37^\circ - \theta) + 5x\sin\theta$$

$$h = 4x\left(\frac{4}{5}\cos\theta + \frac{3}{5}\sin\theta\right) \qquad \text{.................(i)}$$

$$k = 4x\left(\frac{3}{5}\cos\theta - \frac{4}{5}\sin\theta\right) + 5x\sin\theta$$

$$k = \left(\frac{12}{5}\cos\theta + \frac{9}{5}\sin\theta\right)x$$

$$k = 3x\left(\frac{4\cos\theta}{5} + \frac{3\sin\theta}{5}\right) \qquad \text{..................(ii)}$$

$$\therefore \frac{h}{k} = \frac{4}{3}$$

3h = 4k

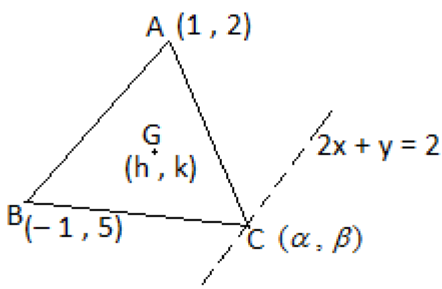
$\therefore$ Locus of P is

$\boxed{3x = 4y}$ Ans. : (A)

Q.43.

14

38



Co – ordinates of Centroid

$$(h\ ,\ k) \equiv \left(\frac{1-1+\alpha}{3}\ ,\ \frac{2+5+\beta}{3}\right)$$

$$(h\ ,\ k) \equiv \left(\frac{\alpha}{3}\ ,\ \frac{7+\beta}{3}\right)$$

C lines on 2x + y = 2

$$\therefore 2\alpha + \beta = 2$$

$$\therefore \beta = 2 - 2\alpha$$

$$(h, k) \equiv \left(\frac{\alpha}{3}, \frac{7+(2-2\alpha)}{3}\right)$$

$$\equiv \left(\frac{\alpha}{3}, \frac{9-2\alpha}{3}\right)$$

$$h = \frac{\alpha}{3} \qquad\qquad k = \frac{9-2\alpha}{3}$$

$\alpha = 3h$ $\qquad$ $3x = 9 - 2\alpha$

Now $\alpha = 3h$

$\therefore 3k = 9 - 6h$

$\therefore k = 3 - 2h$

$k + 2h = 3$

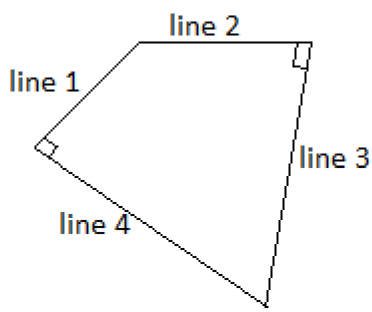
$\therefore$ locus of centroid is 2x + y = 3 $\qquad$ Ans. : (B)

Q.44. (a + b)x + (a – b)y = 2a ....line 1

39 (a – b)x – (a + b)y = 2b ....line 4 $\Big\}$ are perpendicular

(a + b)x – (a – b)y = 2b ....line 2

(a – b)x + (a + b)y = 2a ....line 3 $\Big\}$ lines are perpendicular



Whether line 1 & line 2 are perpendicular depends on the value of a & b $\qquad$ Ans. : (D)

Q.45. 2x – y = 0 $\quad L_1$

2x – y – 8 = 0 $\quad L_2$

40 2x + y – 4 = 0 $\quad L_3$

2x + y + 4 = 0 $\quad L_4$

L_1 & L_2 are parallel lines

L_3 & L_4 are parallel lines

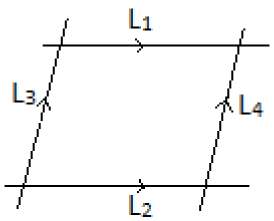
Angle between L_1 & L_3

$m_1 = 2$

$m_2 = -2$ $\qquad$ $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$= \left|\frac{2 - (-2)}{1 + 2(-2)}\right| = \frac{4}{3}$$

So it's not perpendicular



parallel distance between L_1 & L_2

$$\left|\frac{C - C^1}{\sqrt{a^2+b^2}}\right| = \left|\frac{0-(-8)}{\sqrt{2^2+1^2}}\right|$$

$$= \frac{8}{\sqrt{5}} \text{ units}$$

Perpendicular distance between L_3 & L_4 is

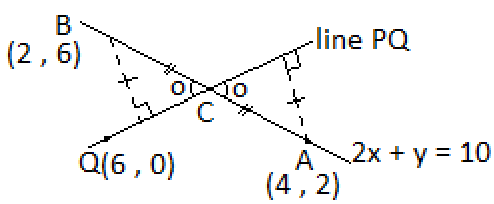
$$\left|\frac{4 - (-4)}{\sqrt{2^2 + 1^2}}\right| = \frac{8}{\sqrt{5}}$$

So, lines L_1 & L_2 and L_3 & L_4 are separated equally

$\therefore$ is is rhombus. $\qquad$ Ans. : (B)

41

Q.40.



Let equation of the line be ax + by + c = 0

By similarity mid – point of AB lies on PQ

So $C \equiv \left(\dfrac{2+4}{2}, \dfrac{6+2}{2}\right) \equiv (3, 4)$ lies on PQ

Equation of PQ = Equation of CQ

Slope CQ $= \dfrac{4-0}{3-6} = \dfrac{4}{-3}$

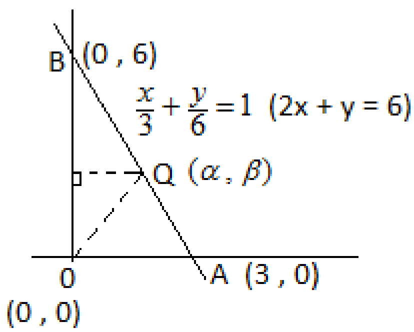
$\therefore$ Equation is y – 0 $= \dfrac{4}{-3}(x-6)$

– 3y = 4x – 2y

$\boxed{24 = 4x + 2y}$ Ans. : (C)

Q.47.

42



Area of $\Delta BOQ = \dfrac{1}{2} \times$ base $\times$ height

base = OB = 6 units.

height $= |\alpha|$

$\therefore \dfrac{1}{2} \times 6 \times |\alpha| = 12$ sq. units

$|\alpha| = 4$

$\alpha = \pm 4$

$\therefore$

If $\alpha = 4$	If $\alpha = -4$
$\beta = 6 - 2\alpha$	$\beta = 6 - 2\alpha$
$\beta = -2$	$\beta = 6 - 2(-4)$
Q (4 , – 2)	β = 14

Q (– 4 , 14)

Equation of OQ　　　　Equation of OQ

$y = \frac{-2}{4}x$　　　　$y = \frac{14}{-4}x$

$y = -\frac{x}{2}$　　　　$y = \frac{7x}{-2}$

2y + x = 0　　or　　2y + 7x = 0　　　　Ans. : (A)

Q.48. Equation of the line passing through the intersection of

2x – y – 4 = 0 and 3x + 2y – 13 = 0

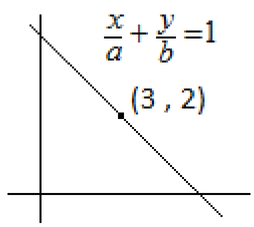
43 2x – y = 4

3x + 2y = 13

$$\frac{x}{\begin{vmatrix} 4 & -1 \\ 13 & 2 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 2 & 4 \\ 3 & 13 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}}$$

$\frac{x}{21} = \frac{y}{14} = \frac{1}{7}$

x = 3　,　y = 2



line passing through (3 , 2) such that a + b = 10

Let line be $\frac{x}{a} + \frac{y}{b} = 1$

A + b = 10, b = 10 – a

$\frac{x}{a} + \frac{y}{10 - a} = 1$

Point (3 , 2) satisfies this

$\frac{3}{a} + \frac{2}{10 - a} = 1$

3(10 – a) + 2a = a (10 – a)

$30-3a+2a=10a-a^2$

$a^2 - 11a + 30 = 0$

a = 5 , 6

∴ equation of the line is

$\frac{x}{5}+\frac{y}{10-5}=1$ & $\frac{x}{6}+\frac{y}{10-6}=1$

44

x + y = 5 & $\frac{x}{6}+\frac{y}{4}=1$ Ans. : (A)

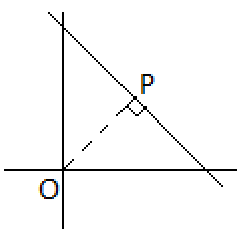
Q.49 sum number v49 is the same as sum number 48.

Q.50.

slope = – 2

45

slope of normal from origin is $\frac{1}{2}$



slope of OP = $\frac{1}{2}$

$\frac{1}{2}=\tan\alpha$ α ranges from $[0\ ,\ 2\pi]$

So $\sin\alpha=\frac{1}{\sqrt{5}}$ or $\sin\alpha=\frac{-1}{\sqrt{5}}$

And $\cos\alpha=\frac{2}{\sqrt{5}}$ or $\cos\alpha=\frac{-2}{\sqrt{5}}$

$\therefore\ x\cos\alpha+y\sin\alpha=p$

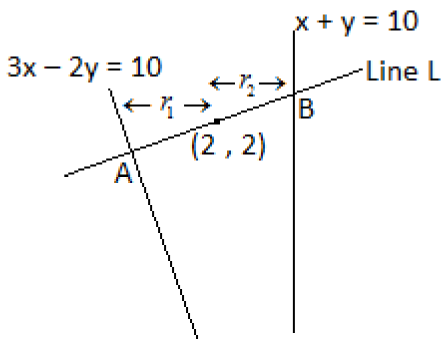
$\frac{2x}{\sqrt{5}}+\frac{y}{\sqrt{5}}=4\sqrt{5}$ $\frac{-2x}{\sqrt{5}}-\frac{y}{\sqrt{5}}=4\sqrt{5}$

2x + y = 20 – 2x – y = 20

$2x + y = -20$ Ans. : (A)

Q.51. 17

46



Parametric equation of line 1

$$\frac{x-2}{\cos\theta} = \frac{y-2}{\sin\theta} = 0$$

$$B \equiv (2 + r_1 \cos\theta\ ,\ 2 + r_2 \sin\theta)$$

$$A \equiv (2 + r_2 \cos\theta\ ,\ 2 + r_2 \sin\theta)$$

r_1 and r_2 are equal in magniside & opposite indirection $r_1 = -r_2$

A satisfies $3x - 2y = 0$

$$3(2 + r_2 \cos\theta) - 2(2 + r_2 \sin\theta)$$

$$6 - 4 + 3r_2 \cos\theta - 2r_2 \sin\theta = 0$$

$$2 = 2r_2 \sin\theta - 3r_2 \cos\theta$$

$$2 = r_2(2\sin\theta - 3\cos\theta) \qquad \text{......................(A)}$$

B satisfies $x + y = 10$

$$(2 + r_1 \cos\theta) + (2 + r_1 \sin\theta) = 0$$

$$r_1 (\cos\theta + \sin\theta) = -4 \qquad \text{......................(B)}$$

Dividing (A) by (B)

$$\frac{2}{-4} = \frac{r_2\ (2\sin\theta - 3\cos\theta)}{r_1(\cos\theta + \sin\theta)}$$

$$\frac{-1}{2} = (-1)\left(\frac{2\sin\theta - 3\cos\theta}{\cos\theta + \sin\theta}\right)$$

$$= -\left(\frac{2\tan\theta - 3}{1 + \tan\theta}\right)$$

$$(1 + \tan\theta) = 2(2\tan\theta - 3)$$

$$(1 + \tan\theta) = 4\tan\theta - 6$$

$$\tan\theta = \frac{7}{3}$$

$$y - 2 = \frac{7}{3}(x - 2)$$

3y – 14 = 7x – 14

3y = 7x

Ans. : (D)

6

Q.5. x + y = 4

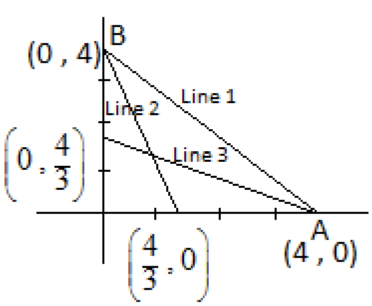
47

3x + y = 4

x + 3y = 4

None of the lines are perpendicular

So, it is not right angled



Line 1 : $\frac{x}{4} + \frac{y}{4} = 1$

Line 2 : $\frac{x}{\frac{4}{3}} + \frac{y}{4} = 1$

Line 3 : $\frac{x}{4} + \frac{y}{\frac{4}{3}} = 1$

Meeting of line 2 and 3

i.e. 3x + y = 4

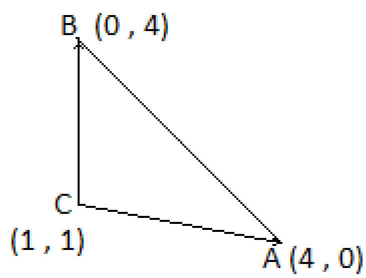
x + 3y = 4

$$\frac{x}{\begin{vmatrix} 4 & 1 \\ 4 & 3 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 3 & 4 \\ 1 & 4 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}}$$

$\frac{x}{8} = \frac{y}{8} = \frac{1}{8}$ x = y = 1

$C \equiv (1,1)$

C is equidistant from A (4 , 0) and B (0 , 4). So triangle is isosceles.



rector $\overline{CA} = (0-1)\hat{i} + (4-1)\,j$

$= -\hat{i} + 3\,j$

rector $\overline{CB} = (4-1)\hat{i} + (0-1)\,j$

$= 3\hat{i} - j$

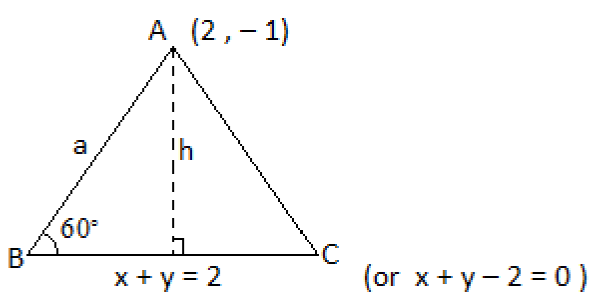
Dot product $\overline{CA} \cdot \overline{CB} = 3\,(-1) + 3\,(-1)$

= − 6 (negative)

So $\angle\, BCA$ is obtuse Ans. : (B)

Q.5.

48



$$\sin 60^\circ = \frac{h}{a}$$

$$a = \frac{h}{\frac{\sqrt{3}}{2}} = \frac{2h}{\sqrt{3}}$$

$$h = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|$$

$$= \left| \frac{2 + (-1) - 2}{\sqrt{1^2 + 1^2}} \right|$$

$$= \frac{1}{\sqrt{2}}$$

$$a = \frac{2h}{\sqrt{3}}$$

$$a = \frac{2 \times \frac{1}{\sqrt{2}}}{\sqrt{3}}$$

$$a = \sqrt{\frac{2}{3}}$$

Ans. : (A)

Q.54. $(-a, -b)$, $(0, 0)$, (a, b), (a^2, ab), (ab, b^2)

All these points lie on the line

$$\frac{y}{x} = \frac{a}{b} \quad \text{i.e. } \boxed{by = ax}$$

Collinear. Ans. : (C)

Cancel

Q.55. $ax + by + c = 0$ L_1

49

$bx + cy + a = 0$ L_2

$cx + ay + b = 0$ L_3

If a , b , c not all are same the L_1 , L_2 , L_3 cannot be co – incident lines.

If $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is zero the lines are concurrent

i.e. if $a^3 + b^3 + c^3 - 3abc = 0$ (value of the determinant) then lines are concurrent.

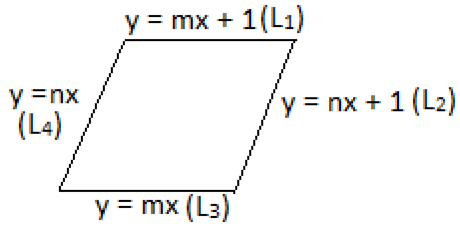
But it is given that,

$$a^3 + b^3 + c^3 - 3abc \neq 0$$

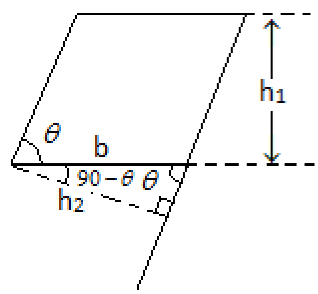
Lines from a triangle. Ans. : (A)

Q.56.

50



Area = b× h1



h_1 and h_2
are perpendicular disatnce between the parallel lines

$\cos(90 - \theta) = \frac{h_2}{b}$

$b = \dfrac{h_2}{\sin\theta}$ $\therefore$ Area = $= \dfrac{h_1 \times h_2}{\sin\theta}$

θ is the angle between the lines

h_1 = distance between L_1 and L_3

$$= \left| \frac{1}{\sqrt{1+m^2}} \right|$$

h_2 = distance between L_2 and L_4

$$= \left| \frac{1}{\sqrt{1+n^2}} \right|$$

$\tan\theta = \left| \dfrac{m-n}{1+mn} \right|$ ....(acute angle formula)

$$\Rightarrow \sin\theta = \frac{|m-n|}{\sqrt{(m-n)^2 + (1+mn)^2}}$$

$$= \frac{|m-n|}{\sqrt{m^2 + n^2 + 1 + m^2n^2}}$$

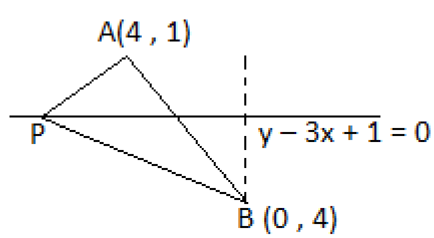
$$= \frac{|m-n|}{\sqrt{(1+m^2)(1+n^2)}}$$

Area $= \dfrac{h_1 h_2}{\sin\theta}$

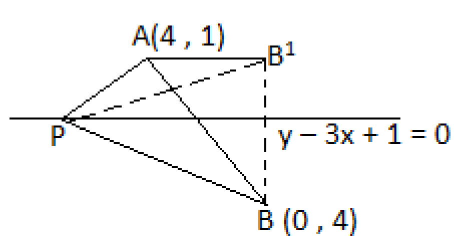
$$= \frac{\dfrac{1}{\sqrt{1+m^2}} \times \dfrac{1}{\sqrt{1+n^2}}}{\dfrac{|m-n|}{\sqrt{1+m^2}\sqrt{1+n^2}}}$$

$$A = \frac{1}{|m-n|}$$ Ans. : (A)

~~Q57~~ 51

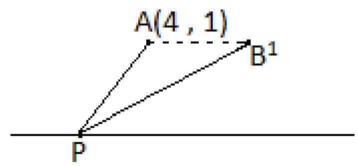


Construction : Reflect B about the line y – 3x + 1 = 0 to get B^1

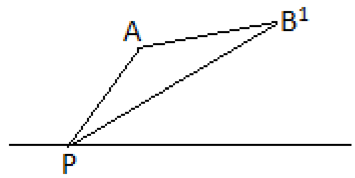


$PB = PB^1$

Symmetry so

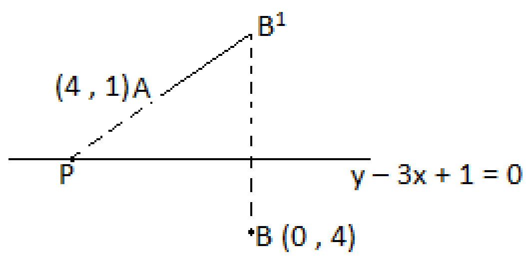


$$|PA - PB| = |PA - PB^1|$$



$AB^1 > |PA - PB^1|$ triangle inequality

So $|PA - PB^1| = AB^1$ if PAB^1 are collinear



$$\frac{x-0}{-3} = \frac{y-4}{1} = \frac{-2(4-3(0)+1)}{1^2+3^2}$$

$$= \frac{-10}{10} = -1$$

$\therefore\ x = 3$, $y = 4 - 1$,

y = 3

Equation of $AB^1 = (y - 3) = \left(\frac{3-1}{3-4}\right)(x - 3)$

$$y - 3 = \frac{2}{-1}(x-3)$$

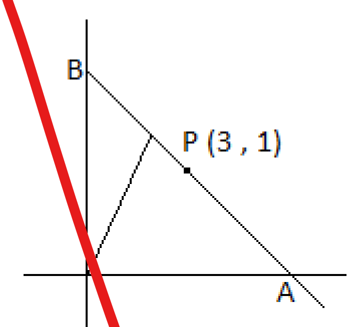
y – 2x + 3 = 0

Intersection of AB^1 y – 2x + 3 = 0 and y – 3x + 1 = 0 is

x = – 2 , y = – 7

P (– 2 , – 7) Ans. : (D)

Q.53.



Equation of the line is

$x\cos\alpha + y\sin\alpha = P$ (Normal from of line)

$P = x\cos\alpha + y\sin\alpha$

Point P (3 , 1) satisfies this so,

$P = 3\cos\alpha + \sin\alpha$. We have to choose α so, that P is maximum.

$-\sqrt{10} \le 3\cos\alpha + \sin\alpha \le \sqrt{10}$

So if $3\cos\alpha + \sin\alpha = \sqrt{10}$ then P is max.

$(P = \sqrt{10})$

i.e. if $\cos\alpha = \dfrac{3}{\sqrt{10}}$ and $\sin\alpha = \dfrac{1}{\sqrt{10}}$

i.e. $\tan\alpha = \dfrac{1}{3}$ and $P = \sqrt{10}$

So slope of line is – 3.

Cancel

Sol^n of 52 Missing

$\therefore$ Equation of line is

$$\frac{3x}{\sqrt{10}} + \frac{1}{\sqrt{10}} y = \sqrt{10}$$

$$\therefore \boxed{3x + y = 10}$$

$$\left[\begin{array}{l} a\cos\theta + b\sin\theta \text{ is max} \\ \text{when } \cos\theta = \dfrac{a}{\sqrt{a^2+b^2}} \text{ and} \\ \sin\theta = \dfrac{b}{\sqrt{a^2+b^2}} \end{array} \right]$$

Cancel

$$\frac{x}{\frac{10}{3}} + \frac{y}{10} = 1$$

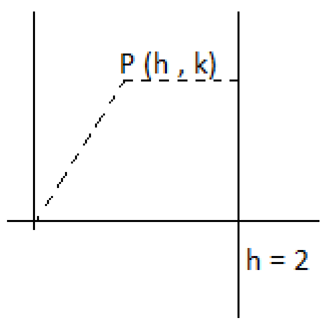
$$A \equiv \left(\frac{10}{3}, 0\right) \quad B \equiv (10, 0)$$

$\Delta\, ADB$ area $\equiv \frac{1}{2} \times 10 \times \frac{10}{3} = \frac{50}{3}$ sq. units

Ans. : (A)

Q.59.

53



Distance from origin = $\sqrt{h^2 + k^2}$

Distance from line x – 2 = 0 is

$$\left|\frac{h-2}{\sqrt{1}}\right| = |h-2|$$

$$\therefore \sqrt{h^2+k^2} + |h-2| = 4$$

<u>Case I</u> : $h \geq 2$ then

$$\sqrt{h^2+k^2} + h - 2 = 4$$

$$\sqrt{h^2+k^2} = 6 - h$$

$$h \le 6$$

Squaring we get

$h^2 + k^2 = h^2 + 36 - 12h$

$k^2 = 36 - 12h$

$k^2 = 12(3 - h) \ge 0$

$\therefore 3 - h \ge 0$

$h \le 3$ i.e. for $2 \le x \le 3$

Locus is $y^2 = 12(3 - x)$

<u>Case II</u> : $h \le 2$

Then,

$$\sqrt{h^2+k^2} + 2 - h = 4$$

$$\sqrt{h^2+k^2} = h + 2$$

Squaring we get

$h^2 + k^2 = h^2 + 4h + 4$

$k^2 = 4(h+1)$

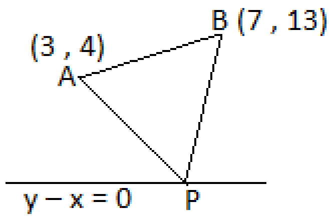
$k^2 \ge 0$

$\therefore h+1 \ge 0$

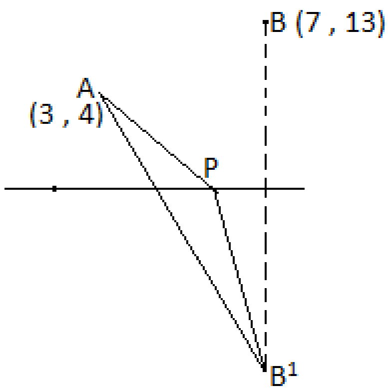
$h \ge -1$

So for $-1 \le x \le$ locus is $y^2 = 4(x+1)$ Ans. : (C)

~~Q.60.~~ 54



PA = PB > AB

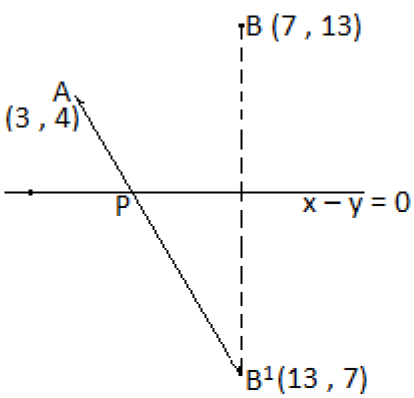


B^1 is the image of B in the line

$PB = PB^1$ (symmetry)
$PA + PB^1 > AB^1$
So
$PA + PB^1$ will be least

If APB^1 are collinear.

Least find image of B in line y = x



Equation of AB^1 is

$$y - 4 = \left(\frac{7-4}{13-3}\right)(x-3)$$

$$y - 4 = \frac{3}{10}(x-3)$$

10y – 40 = 3x – 9

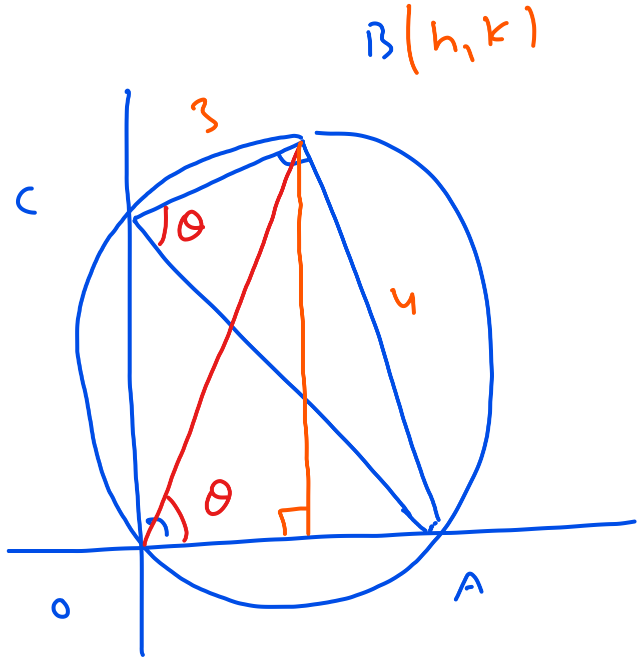
$10y = 3x + 31$ ……………Equation of AB^1

Finding the intersection of AB^1 and $x - y = 0$

We get,

$P \equiv \left(\frac{31}{7}, \frac{31}{7}\right)$ Ans. : (A)

55

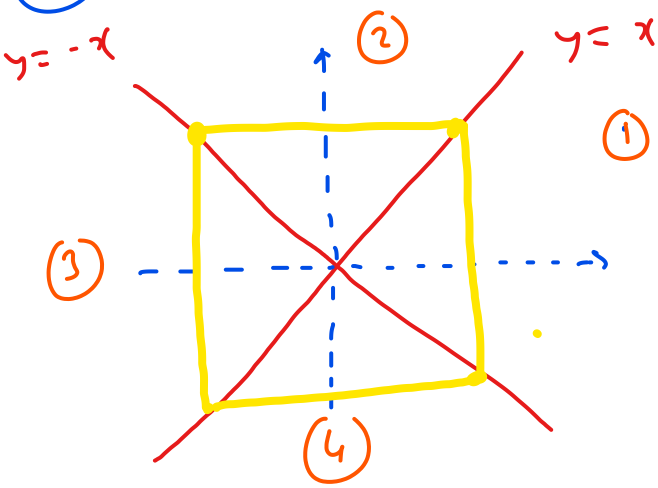


OABC cyclic quadrilatual

$\tan\theta = \frac{4}{3} = \frac{k}{h}$

$\Rightarrow 4x = 3y$

56 Given locus is $\max\{|x|, |y|\} = a$



(i) region 1 $|x| > |y|$

$\therefore$ locus $|x| = a$

or $x = a$

(ii) region 2 $|y| > |x|$

$\therefore$ locus $|y| = a$

or $y = a$

Similarly 3 & 4

locus is a square

EXERCISE 2 (A)

61. Compare with $Ax^2+2Hxy+By^2+2ax+2fy+C=0$

$A = 1, B = 1, H = 0, G = g, \quad F = f, \quad C = 1$

$\therefore \Delta = 0$

$$\Rightarrow \quad \begin{vmatrix} 1 & 0 & g \\ 0 & 1 & f \\ g & f & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad \left(1-f^2\right)+g\left(-g\right)=0$$

$$f^2+g^2=1$$

Cancel

62. $A=\lambda,\ B=2,\ H=-5/2,\ G=5/2,\ f=-7/2,\ C=3$

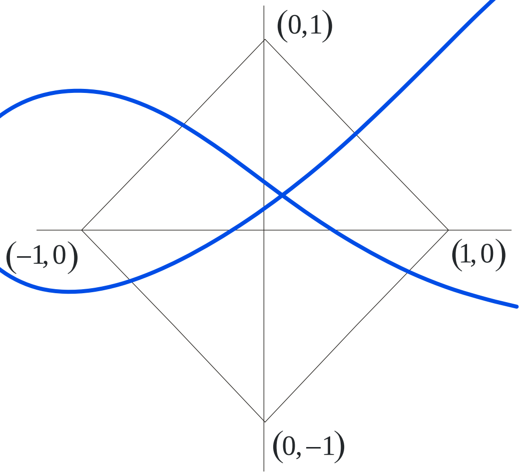
Put $\Delta=0$

$$\Rightarrow \quad \begin{vmatrix} \lambda & -5/2 & 5/2 \\ -5/2 & 2 & -7/2 \\ 5/2 & -7/2 & 3 \end{vmatrix} = 0$$

Solve to get $\lambda=2$

Cancel

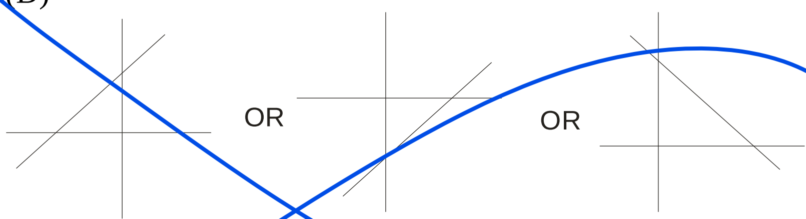
63. Area enclosed by $|x-1|+|y-3|=1$ is same as enclosed by $|x|+|y|=1$ (shift of origin)



Cancel

$\therefore \quad A=2$

64. (D)



Cancel

Here

$\left(\frac{-c}{a}\right)<0$ & $\frac{-c}{b}>0$ $\qquad$ $\frac{-c}{a}>0$, $\frac{-c}{b}<0$ $\qquad$ $\frac{-c}{a}>0$ & $\frac{-c}{b}>0$

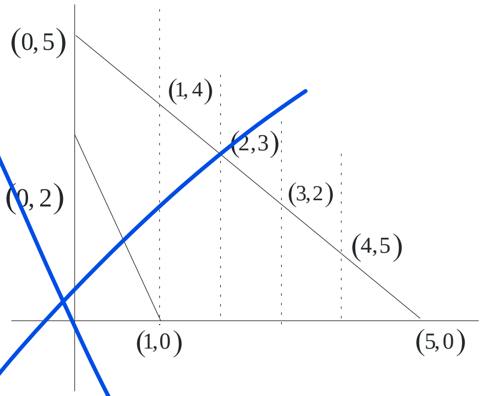
61 65. Equation of pair of bisection is $h\left(x^2-y^2\right)=xy/\left(a-b\right)$

57

$\therefore$ of live pair is coordinate Axes $\Rightarrow h=0$

So that equation is $xq=0$ as $x=0, y=0$

66.

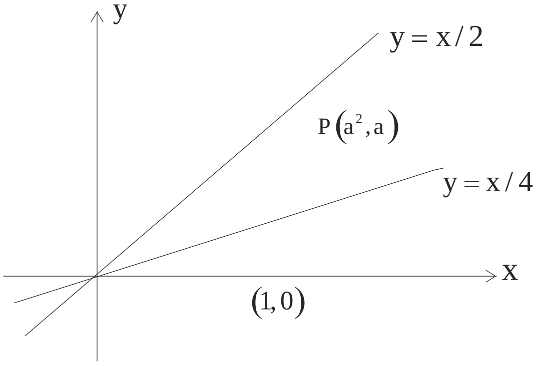


Cancel

Put x = 1, 3 pH here
Put n = 2, 2 pH
Put n = 3, 1 pt here
∴ Total 6 gourd point

67. 58

Plot lines



Let $L_1 = 2y - x, \quad L_2 = 4y - x$

∴ pt (1, 0) gives -lve sign to both lines

∴ P must give + lve to L_1 4∈ lve to L_2

∴ $2a - a^2 < 0 \quad \& \quad at(0, 4)$

$(-\infty, 0) \cup (3, \infty)$

A (2,4)

59

68. Let the live by y – mx – c = 0

∴ adol is algebraic distance

$$\frac{1-2m-c}{\sqrt{Hn2}} + \frac{2-3m-c}{\sqrt{Hn2}} + \frac{7+4m-c}{\sqrt{Hn2}}$$

10 – m – 3c = 0

$$\frac{10}{3} = m + c$$

60

∴ pares through (1, 10/3)

69. $\frac{ds}{dx} : ax + hg + g = 0$ if n eqn. put y = 0

$\frac{dg}{dy} ; hx + by + f = 0 \qquad \frac{-g}{a} = \frac{-f}{h}$

Hg = gf

~~70.~~ 61

Let P^2 (x, y)

$\therefore$ Alfa f_1 P^2 is (y, x)

Alfa f_2 is becomes $(y+3x, x_1)$

Alfa f_3 is becomes $\left(\frac{y_1+2x_1}{2}, \frac{y_1+4x_1}{2}\right)$

P^2 (A) becomes (0,0)

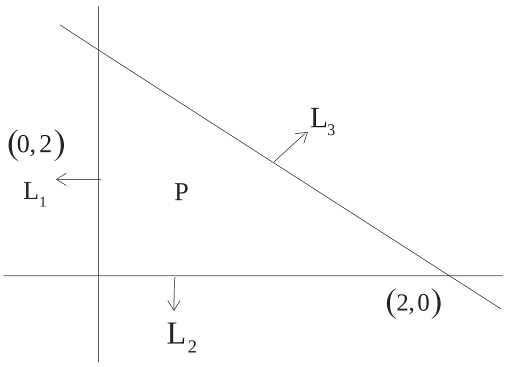
P^2 (B) (4,0) becomes (4, 8)

P^2 (C)(4,2) becomes (5, 9)

P^2 (D) (0,2) becomes (1, 1)

Now pb form 1Lgm

~~71.~~ 62



For P to be insides it must give signs (+) $\in$ lve & $\in$ lve w.r.t lives L_1, L_2, L_3 respectively

$\therefore$ $a > 0,\ a^2 > 0\ \&\ \text{at}\ a^2 - 2 < 0$

$a > 0\ \&\ \text{at}\ (-2,1)$

63

$\Rightarrow$ $\text{at}(0,1)$

~~72.~~

(B)

here $L_1 : x\cos\alpha + y\sin\theta = p$

$\Delta\ L_2 = x\sin\alpha - y\cos\alpha = 0$

$ax \perp r\ \&\ ax + by \perp p$ is @ $\frac{\pi}{4}$ with $L_1 \Rightarrow ax + by$ $\quad cp = 0$ is angle

_______ of $L_1 \& L_2$

$= (x\cos\alpha + y\sin\alpha = p) = 1(x\sin\alpha - y\cos\alpha)$

Take (+lve & y_n)

$x(\cos\alpha - \sin\alpha) + y(\sin\alpha + \cos\alpha) - p = 0$

Compare with

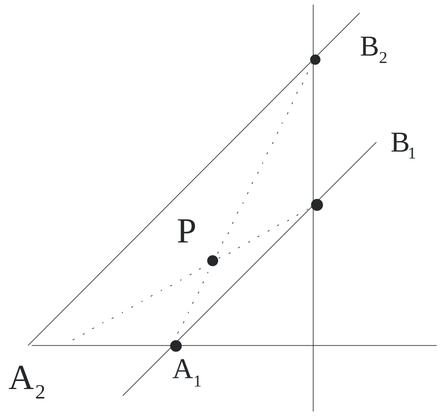
$ax + by + p = 0$

$\frac{a}{\cos\alpha - \sin\alpha} = \frac{b}{\sin\alpha + \cos\alpha} = \frac{-p}{p}$

$a^2 + b^2 = 2$

~~73.~~

64



$A_1 A_2 B_1 B_2$
Co cyclic
$= m_1 m_2 = 1$ or $m^2 = 1$
$\Rightarrow m = 1 \quad m \leftarrow R^{-1}$
$A_1\left(\frac{-C_1}{m}, 0\right), A_2\left(\frac{-C_2}{m}, 0\right)$
$B_1(0, C_1) \quad B_2(0, C_2)$ let p be (h, k)
$\therefore$ $A, P \,\&\, B_2$ collinear

$\frac{k}{h+C_1} = \frac{C_2}{C_1}$

$\rightarrow \frac{k}{C_2} = \frac{H}{C_1} + 1$ &

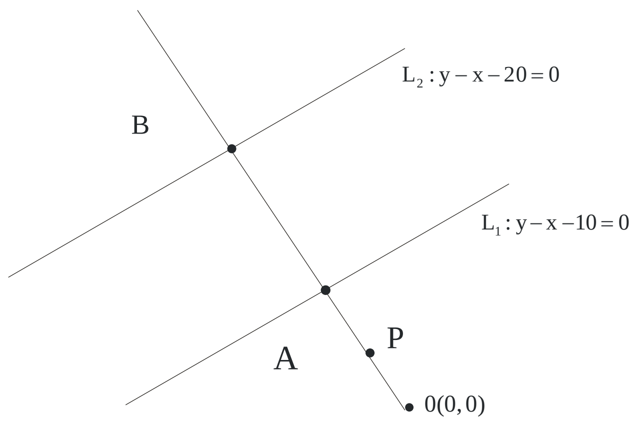
A_2 P & B_1 collinear

$\frac{k}{h+C_2} = \frac{C_1}{C_2}$

$\frac{k}{C_1} = \frac{H}{C_2} + 1$

Subtract to get
$k\left(\frac{1}{C_2} = \frac{1}{C_1}\right) = h\left(\frac{1}{C_1} = \frac{1}{C_2}\right)$
$k = -h$

Q 65 74.



Let $p(h, k)$ Now, parametric eqn. of live PAB can be taken as
$x = r\cos\theta, \quad y = r\sin\theta$
Let OP = r_1 $\therefore$ $h = r_1\cos\theta, \quad k = r\sin\theta$
OA = r_2 $A(r_2\cos\theta,\ r_2\sin\theta)$
OB = r_3 $B(r_3\cos\theta,\ r_3\sin\theta)$
Now put pts on L_1 & L_2
$r_2(\sin\theta - \cos\theta) = 10$ $r_3(\sin\theta - \cos\theta) = 20$

$$r_2 \frac{10}{\sin\theta - \cos\theta}$$

π is given the

$$\frac{2}{r_1} = \frac{1}{r_2} + \frac{1}{r_3}$$

$$\frac{2}{r_1} = \frac{3}{20}(\sin\theta - \cos\theta)$$

$$r_3 = \frac{20}{\sin\theta - \cos\theta}$$

$$\frac{40}{3} = r_1 \sin\theta - r_2 \cos\theta$$

$$\frac{40}{1} = y - x$$

66

75. From 74 put

$$r_1^2 = r_2 r_3$$

$$r_1^2 = \frac{200}{(\sin\theta - \cos\theta)^2}$$

$$x - (r_1 \sin\theta - r_1 \cos\theta)^2 = 200$$

$$(x - y)^2 = 200$$

76. From 74

67

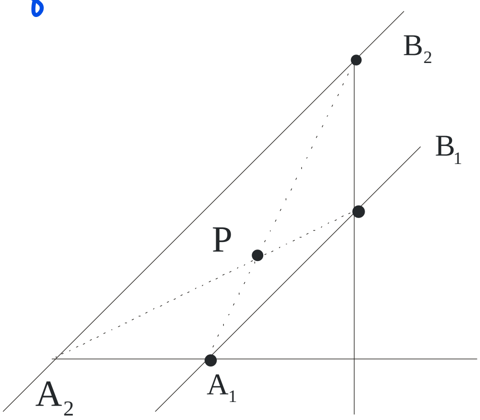
$$\frac{1}{r_1^2} = \frac{1}{r_2^2} + \frac{1}{r_3^2}$$

$$\frac{1}{r_1^2} = \frac{(\sin\theta - \cos\theta)^2}{400}$$

$$80 = (r_1 \sin\theta = r_1 \cos\theta)^2$$

$$(x - y)^2 = 80$$

77. 68



$A_1\left(\frac{-C_1}{2}, 0\right)$ $\qquad$ $B_1(0, C_1)$

$A_2\left(\frac{-C_2}{2}, 0\right)$ $\qquad$ $B_2(0, C_2)$

P(h, k) is collinear with A_1 B_2

$$\frac{k - C_2}{h} = \frac{2C_2}{C_1} \quad \text{or} \quad \frac{k}{C_2} - 1 = \frac{2h}{C_1}$$

Also p(h, k) collinear with A_2 & B_2

$$\Rightarrow \qquad \frac{k}{C_1} - 1 = \frac{2h}{C_2}$$

$\therefore$ Subtract

$$k\left(\cancel{\frac{1}{C_2}-\frac{1}{C_1}}\right)=2h\left(\cancel{\frac{1}{C_1}-\frac{1}{C_2}}\right)$$

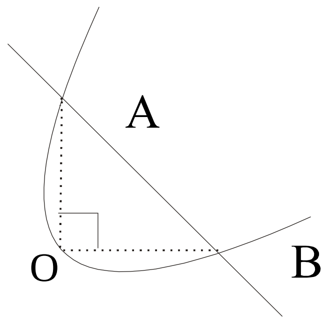
$k+2h=0$

78. Let live be $y = mx - c = 0$

69

Homogenize live with curve

$$3x^2-y^2+(4g-2n)\left(\frac{y-mx}{c}\right)=0$$



Sin OA & OB

$\therefore \operatorname{cosec} x^2+\operatorname{cosec} y^2=0$

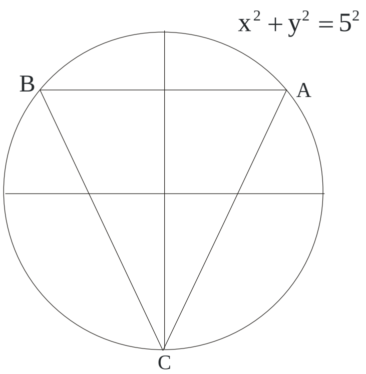
$(3c+2m)+(-c+4)=0$

$m+c=2 \qquad p^2$ is (1, -2)

79. (D)

70

A(3, 4) $\quad B(\cos\theta,\ 5\sin\theta) \quad C(5\sin\theta,\ -5\cos\theta)$



Now, 5(0,0) & centride is $C_1\left(\frac{3+5\cos\theta+5\sin\theta}{3},\frac{4+5\sin\theta-5\cos\theta}{3}\right)$

$\therefore\ H=(3+5\cos\theta+5\sin\theta),(4+5\sin\theta-5\cos\theta)$

$h-3=5\cos\theta+5\sin\theta$

$k-4=5\sin\theta=5\cos\theta$

$x+y-7=10\sin\theta$ also $x-y+1=10\cos\theta$

$(x+y-7)^2+(x-y+1)^2=100$

80.

~~By parametric live~~ ~~Also~~

$\cancel{k=0+4\sin\theta} \qquad \cancel{h=0+3\cos(\theta-90)}$

$\cancel{k=4\sin\theta} \qquad \cancel{h=3\sin\theta}$

$\cancel{\therefore \frac{K}{4}=\frac{H}{3}}$

Cancel

STRAIGHT LINES

Exercise – 2(B)

Q.1 (C)(D)

$y+1=\lambda^2 x$, $x+1=\lambda^2 y$, $x+y=\lambda^2$

are concurrent line. Let they intersect at $(x_\circ , y_\circ)$

Therefore,

$y_\circ+1=\lambda^2 x_\circ$ ....(1)

$x_\circ+1=\lambda^2 y_\circ$ ....(2)

$x_\circ+y_\circ=\lambda^2$ ....(3)

$\Rightarrow \lambda^2=-1 \quad or \quad x_\circ-y_\circ=0$

not possible $\quad x_\circ=y_\circ$

from (1) & (2)

$\Rightarrow 2x_\circ=2y_\circ=\lambda^2$

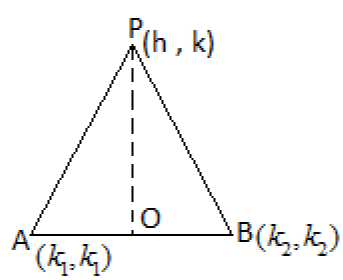
$\Rightarrow \frac{\lambda^2}{2}+1=\lambda^2\cdot\frac{\lambda^2}{2} \Rightarrow \lambda^4-\lambda^2-2=0$

$\Rightarrow (\lambda^2-2)(\lambda^2+1)=0 \Rightarrow \lambda^2=2 \Rightarrow \lambda=\pm\sqrt{2}$

Q.2 (A)(C)

$\mathrm{AB}=\sqrt{(\mathrm{k}_1-\mathrm{k}_2)^2+(\mathrm{k}_1-\mathrm{k}_2)^2}=\sqrt{2(\mathrm{k}_1-\mathrm{k}_2)^2}$

$=2\sqrt{2}$



$\text{Now Area}=\frac{1}{2}(\mathrm{AB})(\mathrm{PQ})$

$\Rightarrow 2=\frac{1}{2}\times 2\sqrt{2}\times PQ$

$\Rightarrow PQ=\sqrt{2}$

Equation of line AB is $y-k_1=\frac{k_2-k_1}{k_2k_1}(x-k_1)$

$y-x=0$.

Perpendicular distance from P to AB is $\sqrt{2}$.

$\Rightarrow \sqrt{2}=\left|\frac{h-k}{\sqrt{2}}\right| \Rightarrow h-k=\pm 2$

$\Rightarrow x-y=\pm 2$

and $(x-y-2)(x-y+2)=0$

Q.3 (B)(C)

(b + c) (b – c) = 4a (a + c)

$4a^2 + 4ac + (c^2 - b^2) = 0$

$\Rightarrow a=\frac{-c \pm b}{2}$

Substituting the value of a in ax + by + c = 0.

(b – c)x + 2by + 2c = 0	(b + c)x – 2by – 2ac = 0
b(x + 2y) + c (2 – x) = 0	b(x – 2y) + c (x – 2) = 0
$b+c\neq 0$	
∴ x = 2 y = – 1	x = 2 y = 1
(x , y) = (2 , – 1)	(x , y) = (2 , 1)

Q.4 (A)(B)

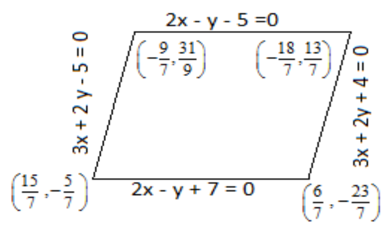
Let center of circle be M (a , b). Then the distance between M and x – axis, distance between M and y – axis and the distance between M and line 3x + 4y = 120 will be same.

$\Rightarrow k=h=\frac{|3h+4k-120|}{5}$

$5h = 3h + 4k - 120$	$-5h \neq 3h + 4k - 120$
$5h = 3h + 4k - 120$	$-5h = 3h + 4k - 120$
$h = 60$, $k = 60$	$h = 10$, $k = 10$

Q.5 (A)(B)(C)

Solving the equations get the vertices and then the equations of diagonals as 6x + 11y = 5 & 18x + 5y + 1 = 0.

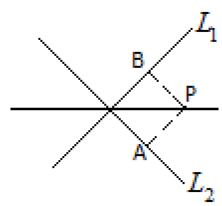


Area of parallelogram $= \left|\dfrac{(d_1 - d_2)\ (c_1 - c_2)}{a_1 b_2 - a_2 b_1}\right|$

$$= \left|\frac{(7+5)(4+5)}{4+3}\right| \quad = \frac{180}{7}$$

Q.6 (A)(B)(C)(D)

$L_1 : x \cos\alpha + y \sin\alpha - c$
$L_2 : x \sin\alpha - y \cos\alpha$
$L : ax + by + c = 0$ is the angle bisector of L_1 and L_2



Then, $P(x_\circ\ ,\ y_\circ)$ any point on L will be equidistance from L_1 and L_2

i.e. PA = PB { and $P(x_\circ\ ,\ y_\circ)$ lie on ax + by + c = 0 $\Rightarrow y_\circ = -\dfrac{ax_\circ - c}{b}$ }

$$PA = PB \Rightarrow |x_\circ \cos\alpha + y_\circ \sin\alpha - 0| = |x_\circ \sin\alpha - y_\circ \cos\alpha|$$

$$\Rightarrow \left|x_\circ \cos\alpha - \frac{(ax_\circ + c)}{b}\sin\alpha - c\right| = \left|x_\circ \sin\alpha - \frac{(ax_\circ + c)}{b}\cos\alpha - c\right|$$

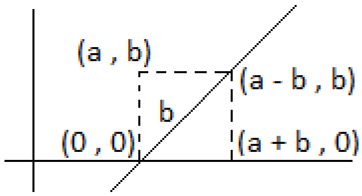
Solving we get $a^2 + b^2 = 2$

$$\Rightarrow -\sqrt{2} \ \le\ a\ \le\ \sqrt{2} \quad , \quad -\sqrt{2}\ \le\ b\ \le\ \sqrt{2}$$

Q.7 (A)(B)(C)

A is (a, b)

A cannot lie on y = x – a.



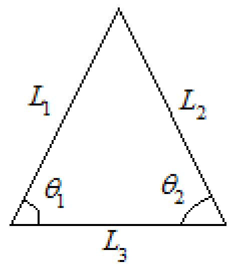
Q.8 (A)(B)

$L_1 : (a + b)x + (a – b)y = 2ab$

$L_2 : (a - b)x + (a + b)y = 2ab$

By solving them, we get x = y = b

Let the slope of third line be M



Then $L_3 : y – a + b = m (x – b + a)$

Isosceles triangle i.e. $\tan\theta_1 = \tan\theta_2$

$$\Rightarrow \left|\frac{m_1 - m}{1 + m_1 m}\right| = \left|\frac{m_2 - m}{1 + m_2 m}\right|$$

$$\Rightarrow \left|\frac{\frac{a+b}{b-c} - m}{1 + m\left(\frac{a+b}{b-a}\right)}\right| = \left|\frac{\left(\frac{b-c}{a+b}\right) - 1}{1 + m\left(\frac{b-a}{a+b}\right)}\right|$$

$$\Rightarrow \left|\frac{(a+b) - mb + ma}{b - a + ma + ma}\right| = \left|\frac{b - a - ma - mb}{a + b + mb - ma}\right|$$

$$(a+b)^2 - m^2(b-a)^2 = (b-a)^2 - m^2(a+b)^2$$

$$\Rightarrow 4ab = m^2 4ab \quad \Rightarrow \quad m = \pm 1$$

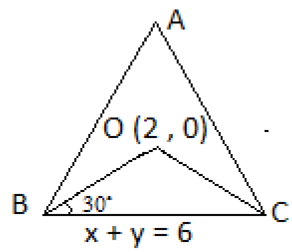
$$\therefore L_3 : x + y = 0 \qquad as \quad L_3 : x - y + 2(a-b) = 0$$

Q.9 (A)(B)(C)

Let M_1 = slope (OB)

M_2 = slope (OC)

M = slope (BC) = - 1



Therefore, $\left|\frac{M_1 - m}{1 + M_1 m}\right| = \left|\frac{M_2 - m}{1 + M_2 m}\right| = \tan 30^\circ$

$$\left|\frac{M_1 + 1}{1 - m}\right| = \frac{1}{\sqrt{3}} \quad \& \quad \left|\frac{M_2 + 1}{M_2 - 1}\right| = \frac{1}{\sqrt{3}}$$

$$M_1 = \frac{-(1+\sqrt{3})}{\sqrt{3}-1} \ \& \ M_2 = \frac{-(\sqrt{3}-1)}{\sqrt{3}+1}$$

OB : $(y - 2) = \frac{-(1+\sqrt{3})}{\sqrt{3}-1}(x-2)$

OC : $(y - 2) = \frac{-(\sqrt{3}-1)}{\sqrt{3}+1}(x-2)$

$\therefore B \equiv ((3-\sqrt{3}),\ (3+\sqrt{3})),\ C \equiv (3+\sqrt{3}),\ (3-\sqrt{3})$ and $A \equiv (0\ ,\ 0)$

$$a = \sqrt{(0-(3-\sqrt{3})^2)+(0-(3+\sqrt{3})^2)} = 2\sqrt{6}$$

Area $= \frac{3\sqrt{3}}{4}a^2 = 6\sqrt{3}$

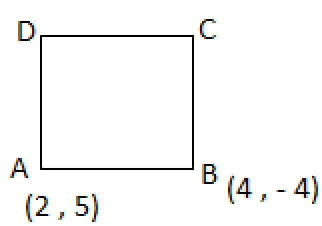
Q.10 (A)(B)

$A \equiv (2,3) \qquad B \equiv (4,-4)$

$\overrightarrow{AB} = (2,-9)$

By 90° Rotation

$\overrightarrow{AC} \equiv (9,2)$



$$\therefore (C_x - 2\ ,\ C_y - 5) = (9,2)$$

$$C_x = 11 \quad , \quad C_y = 7$$

$$C \equiv (11,7)$$

$(D_x - 4\ ,\ D_y + 4)\ =\ (4, 2)\quad D \equiv (13\ ,\ -2)$

$\overrightarrow{BA} = (-2, 9)$

$\overrightarrow{BC} = (-9, -2)$

$(C_x - 4, C_y + 4) = (-9, -2) \qquad\qquad C \equiv (-5, 0)$

$(D_x - 2, D_y - 5) = (-9, -2) \qquad\qquad D \equiv (-7, 3)$

Q.11 (A)(B)

Altitudes are y = m_1x , y = m_2x , y = m_3x.

$\Rightarrow$ orthocenter is origin.

And we know that orthocenter, centroid and circumcenter lie on the same line.

Y = hx is the equation of line passing through orthocenter and circumter and centroid will

lie on this line. But orthocenter will be fixed.

Q.12 (A)(B)

$A \equiv (a\cos\theta\ ,\ b\sin\theta) \qquad\qquad B \equiv (-a\sin\theta\ ,\ b\cos\theta)\backslash$

$C \equiv (-a\cos\theta\ ,\ -b\sin\theta) \qquad D \equiv (a\sin\theta\ ,\ -b\cos\theta)$

$$AB = \sqrt{a^2(\cos\theta + \sin\theta)^2 + b^2(\sin\theta - \cos\theta)^2}$$

$$= \sqrt{a^2\sin 2\theta - b^2\sin 2\theta}\ = CD$$

$$AD = \sqrt{a^2(\cos\theta - \sin\theta)^2 + b^2(\cos\theta + \sin\theta)^2}$$

$$= \sqrt{a^2 - a^2\sin 2\theta + b^2\sin 2\theta}\ = BC$$

$\therefore$ ABCD is parallelogram

And Area is function of θ

Q.13 (B)

$h_1 = |a\cos\theta - p|$ & $h_2 = |a\cos\theta - p|$

$$h_1h_2 = k^2 \Rightarrow (a\cos\theta + p)(a\cos\theta - p) = k^2$$

$$a^2\cos^2\theta - p^2 = k^2$$

Foot of perpendicular from A.

$$P_1 = \left(-a - \frac{\cos\theta(-a\cos\theta - p)}{1}, 0 - \frac{\sin\theta(-a\cos\theta - p)}{1}\right)$$

$$P_1 = (-a + a\cos^2\theta + p\cos\theta\ ,\ a\sin\theta\cos\theta + p\sin\theta)$$

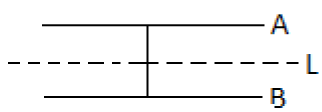
For locus of P_1

$$x^2 - y^2 = a^2 + a^2\cos^2\theta + p^2 - 2a^2\cos^2\theta - 2ap\cos\theta + 2ap\cos^3\theta + 2ap\sin^2\theta\cos\theta$$

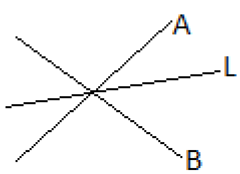
Similarly, locus of B : $x^2 + y^2 = a^2 + k^2$.

Q.14 (A)(B)(C)

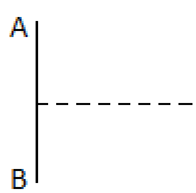
(a)



(b)



(c)

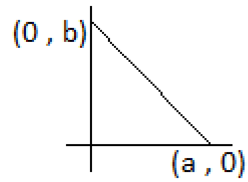


Q.15 (A)(B)(C)

$$L: \frac{x}{a} + \frac{y}{b} = 1$$

$$s = \left|\frac{-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}\right| \qquad \Rightarrow ab = 5\sqrt{a^2 + b^2}\ \ or\ \ a^2b^2 = 25(a^2 + b^2)$$

mid – point $p \equiv \left(\frac{a}{2}, \frac{b}{2}\right)$



$$\frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{a^2}+\frac{4}{b^2}=4\left(\frac{1}{25}\right)=\frac{4}{25}$$
$$\frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{25}$$

Foot of $\perp$ is at distance = 5 = constant

Locus is area of radius s and centre (0 , 0)

$\therefore x^2 + y^2 = 25$

Maximum area obtained when a = b

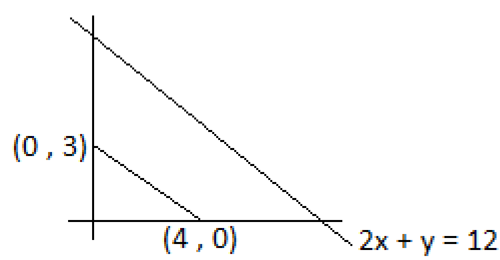
$\Rightarrow a^2 = 25(2a^2) \Rightarrow a = 5\sqrt{2}$

Area $= \frac{1}{2}a^2 = 25$

Q.16 (A)(B)

A lie on 2x + y = 12

A $\equiv (x, 12-2x)$



Centroid $\equiv \left(\frac{4+x}{3}, \frac{15-2x}{3}\right)$

$\frac{4+x}{3}=h$, $\frac{15-2x}{3}=k$

$\Rightarrow x = 3h-4$

Substituting this in $\frac{15-2x}{3}=k$

$\Rightarrow \frac{15-2(3h-4)}{3}=k \Rightarrow 3k = 15-64+8$

$\Rightarrow 6h + 3h = 23$

$\Rightarrow 6x + 3y - 23 = 0$

$$\text{Area } (\Delta ABC) = \left| \frac{1}{2}\begin{vmatrix} 4 & 0 & 1 \\ 0 & 3 & 1 \\ x & 12-3x & 1 \end{vmatrix} \right|$$

$$= \frac{1}{2}\left|4(3-(2+2x)+1(-3x))\right| = \frac{1}{2}\left|5x-36\right| \qquad \text{not constant.}$$

If Area = 7

$$\Rightarrow \left|5x-36\right| = 14$$

$$\Rightarrow 5x-36 = 14 \, or \qquad 5x-36 = -14$$

$$\Rightarrow x = 10 \quad y = -8 \qquad or \qquad x = \frac{22}{5} \quad y = \frac{10}{5}$$

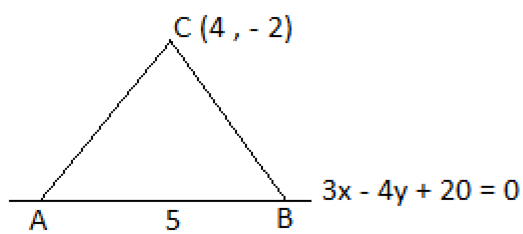
Q.17 (A)(B)(C)

Let A $\equiv (x, y)$

B $\equiv (x+5\times\frac{4}{5}, y+5\times\frac{3}{5})$

$\equiv (x+4, y+3)$

Centroid $\equiv \left(\frac{x+x+4+4}{3}, \frac{y+y+3-2}{3}\right)$



$$\equiv \left(\frac{2x+8}{3}, \frac{2y+1}{3}\right)$$

$$h = \frac{2x+8}{3}, \quad k = \frac{2y+1}{3}$$

$$3x - 4y = 20$$

$$\Rightarrow k = \frac{\frac{3x+20}{2}+1}{3} = \frac{9h-24+44}{12}$$

$$\Rightarrow 12k = 9h + 20$$

$$\Rightarrow 9x - 12y + 20 = 0$$

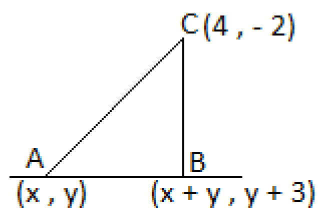
$$\text{Area} = \left| \frac{1}{2} \begin{vmatrix} 4 & -2 & 1 \\ x & y & 1 \\ x+4 & y+3 & 1 \end{vmatrix} \right|$$

$$= \left| \frac{1}{2} (4(y - y - 3) + 2(x - x - 4) + (xy + 3x - xy - 4y)) \right|$$

$$= \left| \frac{3x - 4y - 20}{2} \right| = \left| \frac{-20 - 20}{2} \right| = 20$$

Equation of CB : $y + 2 = \frac{-4}{3}(x - 4)$

$$3y + 6 = -4x + 16$$
$$4x + 3y = 10$$
$$3x - 4y = -20$$



We get,

$$x + 4 = \frac{-4}{5} \ \& \ y + 3 = \frac{22}{5}$$

$$A \in \left(\frac{-24}{5}, \frac{-4}{5} \right) \cup \left(\frac{7}{5}, \frac{22}{5} \right)$$

Sol^n from 18-35 in 2(A) - 2

~~Paper – 2(A)~~ 2(B)

18. (A, C)

Let P be (a_1, -39)

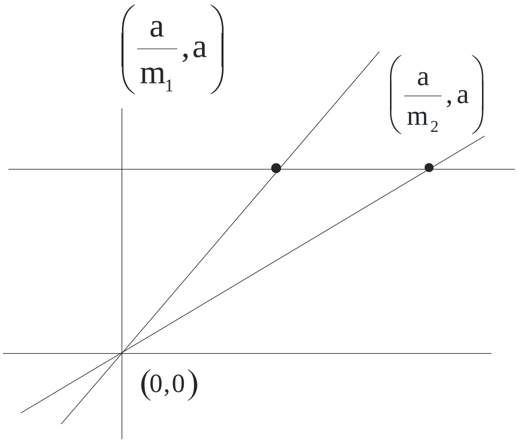
(3, 4) given (- lve sign with 3x – 4y – 8 = 0

$\therefore (a_1 - 3a)$ must give (+lve sign with line

$3a + 12a - 8 > 0$

$a > \frac{8}{15}$

19. (A, C, D)



$m_1 + m_2 = a$

$m_1 m_2 = -(a+1)$

$\therefore A = \frac{1}{2}\left|\frac{a^2}{m_1} - \frac{a^2}{m_2}\right| = \frac{a^2}{2}\frac{|m_1 - m_2|}{m_1 m_2}$

$A = \frac{a^2}{2}\frac{\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{|m_1m_2|} = \frac{a^2}{2}\frac{\left(\sqrt{a^2+4(a+1)}\right)}{|a+1|}$

$A = \frac{a^2}{2}\left|\frac{a+2}{a+1}\right|$

20. (A,B, C, D)

If they intersect @ 4 concylic points

$\therefore m_1 m_2 = 1$

$\left(\frac{a}{b}\right)\left(\frac{d}{c}\right) = 1$

$ac = bd$

Now, $(a-1)^2 = (b-d)^2 \Rightarrow (a-c) = 7(b-d)$

Now subtract lives

$x\left(\frac{1}{a} - \frac{1}{c}\right) + y\left(\frac{1}{b} - \frac{1}{d}\right) = 0$

$x\left(\frac{c-a}{ac}\right) + y\left(\frac{b-d}{bd}\right) = 0$

$\Rightarrow \quad x \pm y = 0$

21. (A, C)

Let line be y = mx + c

Put (1,0) $\Rightarrow$ c = - m

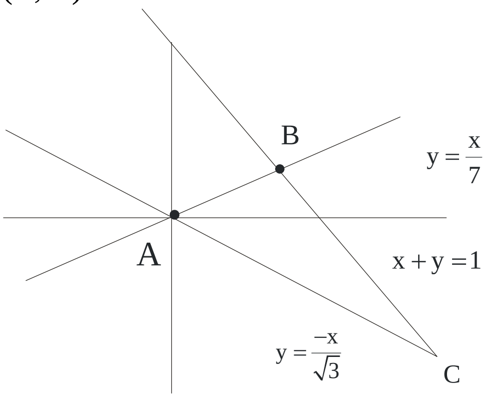
$\therefore \quad y = m(x-1)$ or $mx - y - 1 = 0$

$\therefore$ distance = $\frac{\sqrt{3}}{2}$

$$\frac{1}{\sqrt{Hm_2}} = \frac{\sqrt{3}}{2} \Rightarrow \frac{4}{3} = m^2 + 1$$

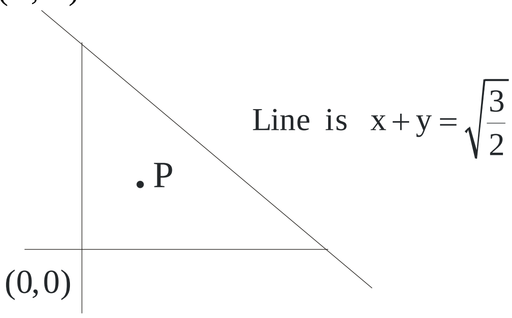
$$m = \pm\frac{1}{\sqrt{3}}$$

22. (B, C)



The Δ is obtuse $\Rightarrow$ interior are In centre & centroid

23. (A, B)



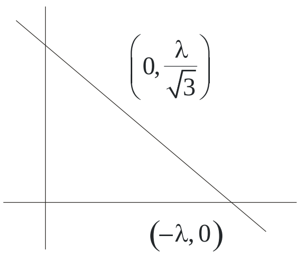
If $P(\sin\theta, \cos\theta)$ inside the Δ

$$\sin\theta > 0 \ \& \cos\theta > 0 \quad \& \quad \sin\theta + \cos\theta < \frac{\sqrt{3}}{\sqrt{2}}$$

$\Rightarrow \quad q \in 1^{st}$ quadrant & $\sin\left(\theta + \frac{\pi}{4}\right) < \sin\frac{\pi}{3}$

24. (B,D)

Let the line be $x - \sqrt{3y} + \lambda = 0$



$\therefore$ length of intercept

$$\Rightarrow \quad \lambda^2 + \frac{\lambda^2}{3} = 100$$

$$\lambda = y5\sqrt{3}$$

25. (C, D)

$$\begin{vmatrix} 1 & 1 & -1 \\ m-1 & m^2-7 & -5 \\ m-2 & 2m-5 & 0 \end{vmatrix} = 0$$

By property

0

$$\begin{vmatrix} 0 & 0 & -1 \\ -m^2+m+6 & m^2-12 & -5 \\ -m+3 & 2m-5 & 0 \end{vmatrix} = 0$$

$\Rightarrow \quad (2m-5)(-m^2+m+6)+(m-3)(m^2-12)=0$

$\Rightarrow \quad (m-3)\left[-(2m-5)(m12)+(m^2-12)\right]=0$

m = 3 or $m^2-m+2=0$ discard

fro m = 3 lives are parallel

26. (B, C)

Let live have slope 'm'

$= \frac{1}{2} = \left|\frac{m+2}{1-2m}\right|$

$\frac{m+2}{1-2m} = \frac{1}{2}$ or $\frac{-1}{2}$

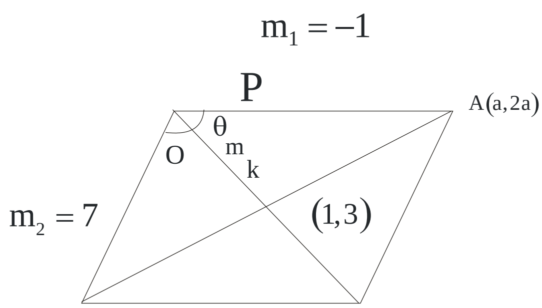
$2m+4=1-2m$ or $2m+2=-1+2m$

4m = - 3 $\Rightarrow m = \infty$

$m = \frac{-3}{4}$

$\therefore$ lives G.M. $\frac{y-3}{x-2} = \frac{-3}{4}$ or ∞

27.



Now diagonal bisects the angle

$\therefore$ equality $\tan\theta$

$\frac{m+1}{1-m} = \frac{7-m}{1+7m}$

$\Rightarrow \quad m = \frac{1}{3}$ or 3

Diagonal are

$\frac{y-3}{x-1}=\frac{1}{3}$ or -3

Put (a, 2a) on there live

$A=\frac{8}{5}$ or $\frac{6}{5}$

28. (A,B,C,D)

Given $\frac{m_1}{m_2}=\frac{9}{2}$

& $\frac{m_1-m_2}{1+m_1m_2}=\pm\frac{7}{9}$

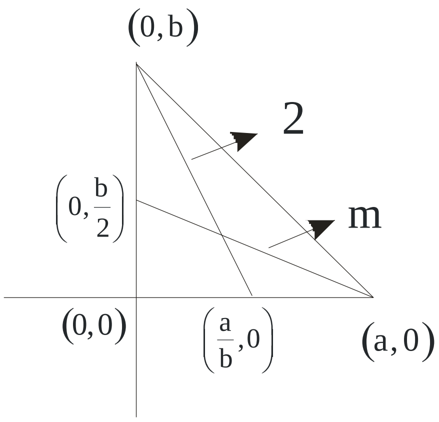
$\frac{7m_2}{2+gm_2}=\frac{7}{9}$ or $\frac{-7}{9}$

$9m_2=2+9m_2{}^2$ or $-9m_2=2+9m_2^2$

$9m_2^2-9m_2^2+2=0$ $\quad$ $9m_2^2+9m_2+2=0$

$m_2=\frac{2}{3}\text{or}\frac{1}{3}$ or $m_2=\frac{-2}{3}\text{or}\frac{-1}{3}$

29. Consider (0,b)



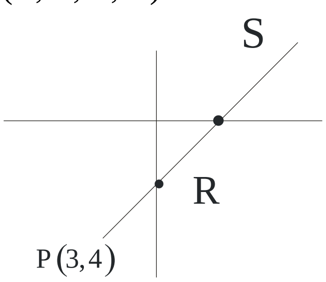
Here $\frac{-2b}{a}=2$ $\quad$ Or $\frac{-b}{2a}=2$

$\Rightarrow\frac{b}{a}=-1$ $\quad$ $\Rightarrow\frac{-b}{a}=4$

$m=\frac{-b}{2a}=\frac{1}{2}$ $\quad$ & $m=\frac{-2b}{a}$

= + 8

30. (A, B, C, D)



Let PR = r_1

PS = r_2

$\therefore$ $R\left(3+r,\cos\theta,\ 4+r,\sin\theta\right)$

$\Rightarrow \quad 3 + r, \cos\theta = 6$

$r_1 = 3\sec\theta$

The $S(3 + r_2, \cos\theta,\ 4 + r_2, \sin\theta)$

$\Rightarrow \quad 4 + r_2 \sin\theta = 8$

$R = 4\operatorname{cosec}\theta$

31. (A, B, & C)

Put $\frac{y}{x} = m$

$1 + m - m^2 = m^3$

Or $m^3 + m^2 = m + 1$

$\cancel{(m+1)}m^2 = \cancel{(m+1)}$

$m = -1 \qquad$ or $\qquad m^2 = 1$

m = -1 $\qquad m = \pm 1$

32. $x^2 + mxy - 2y^2 + 3y - 1 = 0$

A = 1, B = - 2, G = 0, $f = \frac{3}{2}$ C = -1 $H = \frac{M}{2}$

$$\Delta = 0 \quad \begin{vmatrix} 1 & \frac{m}{2} & 0 \\ \frac{m}{2} & -2 & \frac{3}{2} \\ 0 & \frac{3}{2} & -1 \end{vmatrix} = 0$$

$1\left(2 - \frac{9}{4}\right)\frac{-m}{2}\left[\frac{-m}{2}\right] = 0$

$\frac{m^2}{4} - \frac{1}{4} = 0 \Rightarrow m = \pm 1$

Find intersection Ph

33. (A, B, D)

$\Rightarrow$ Angle between lives must be $180^o - 2\alpha$

$\Rightarrow |\tan 2\alpha| = \left|\frac{2\sqrt{h^2 - 1}}{2}\right|$

$\tan^2 2\alpha = h^2 - 1$

$h = |8a^2\alpha|$

34. (A, B, C, D)

Use condition of both roots common

$3\left(\frac{y}{x}\right)^2 + p\left(\frac{y}{x}\right) + 2 = 0 \qquad$ has roots m & m_1

$-3\left(\frac{y}{x}\right)^2 + 9\left(\frac{y}{n}\right) + 2 = 0 \qquad$ has roots m & m_2

$m + m_2 = \frac{-p}{3}, mm_1 = \frac{2}{3} \quad \& \quad m + m_2 = \frac{9}{3}$

Also $m_1 m_2 = 1$ $\quad$ $mm_2 = \frac{-2}{3}$

From here $m^2 = \frac{4}{9}$

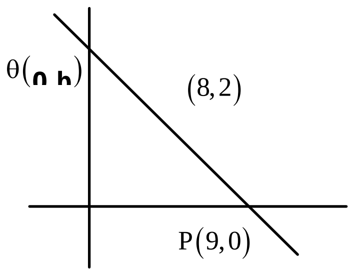
$7m = \frac{2}{3}$ $\quad$ OR $\quad$ $7m = \frac{-2}{3}$

$7m = \frac{2}{3}$ $\quad$ $m_1 = -1$ & $m_2 = 1$

$m_1 = 1$ & $m_2 = -1$ $\quad$ $p = 5, \quad q = +1$

$P = -5$ $\quad$ $q = -1$

23.



a, b (+) ve

also $\frac{9}{a}+\frac{2}{b}=1$

Weighted A.M $\geq$ Weighted H.M

$$\frac{\left(2\sqrt{2}\right)\left(\frac{2\sqrt{2}}{a}\right)+\left(\sqrt{2}\right)\left(\frac{\sqrt{2}}{b}\right)}{2\sqrt{2}+\sqrt{2}}$$

$$\frac{1}{3\sqrt{2}} \geq \frac{3\sqrt{2}}{a+b}$$

$a+b \geq 18$

M, n 18

24. From Q. 23

Use A.M $\geq$ G.M

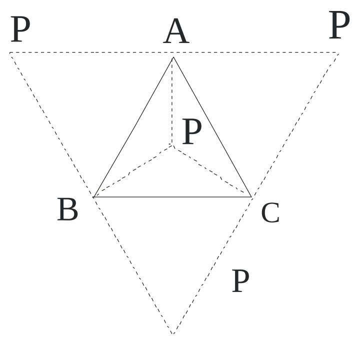
$$\frac{\frac{8}{a}+\frac{1}{b}}{2} \geq \sqrt{\frac{16}{ab}}$$

$$\frac{1}{4} \geq \frac{16}{4b}$$

$ab \geq 64$

$A \geq 32$

25. 4 position possible



Here P such that P,A, B, & C form is any order vertices taken & 4H position is centriod

26. given ax $x^2 - (y-2)^2 = 0$

$X + y - 2 = 0$ or $x - y + 2 = 0$

Bisection Are

$$\frac{x+y-2}{\cancel{\sqrt{2}}} = 1\frac{(x-y+2)}{\cancel{\sqrt{2}}}$$

i.e. $\cancel{x}+y-2=\cancel{x}-y+2$ $\qquad$ $x+\cancel{y}-2=-x+\cancel{y}-2$

$y = 2$ $\qquad$ $x = 0$

3 lives x + y = 3, y = 2 x = 0

3 vertices (0,2) (0,3)(1,2) $A = \frac{1}{2}$

27. Given $2x + 3y = 6$

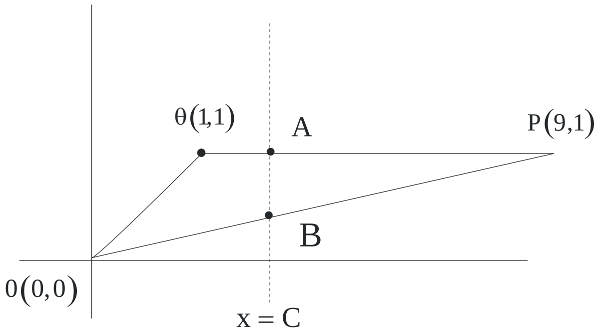
Use equality – Inequality

$\left(2x + 3y\right) \le \sqrt{2^2 + 3^2}\sqrt{x^2 + y^2}$

$\Rightarrow \quad \frac{6}{\sqrt{13}} \le \sqrt{x^2 + y^2}$

$m = \frac{6}{\sqrt{13}}$

28.



A is (C , 1)

B is $\left(C_1, \frac{C}{3}\right)$

$\therefore$ Are of Δ ABP should be 2

Area of ΔOPQ

$\therefore \frac{1}{2}\left|\left(\frac{C-9}{3}\right)^2\right| = 2$

Discard C = 15 or C = 3

Take C = 3

29. $a\left(2x + y - 3\right) + b\left(x + 3y + 1\right) = 0$

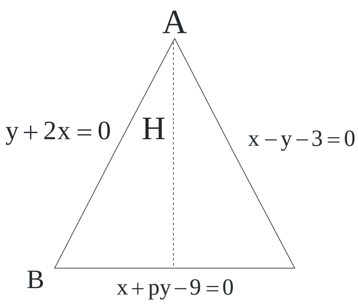
This live passes through intersection of $\begin{matrix} 2x + y - 3 = 0 \\ x + 3y + 1 = 0 \end{matrix}$ i.e. $\left(2, -1\right)$

It also satisfies live mx + 2y + 6 = 0

2m – 2 + 6 = 0

m = - 2

30.



Altitudes from A is $\left(y + 2n\right) + \lambda\left(x - y - 3\right) = 0$

It slope should be P Also H satisfies it

$$-\frac{(\lambda+2)}{1-\lambda}=p$$

$$7+\lambda(-7)$$

$$\lambda=\frac{7}{4}$$

$\Rightarrow$ P is 5

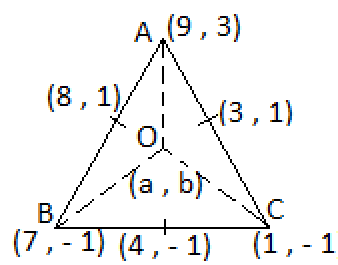
Similarly get q = 45

$$\frac{p+q}{10}=5$$

<u>COMPREHENSION TYPE</u>

Passage – 1

Q.1 (D)



Area of ΔDEF $= \frac{1}{2}\begin{vmatrix} 8 & 1 & 1 \\ 4 & -1 & 1 \\ 5 & 1 & 1 \end{vmatrix}$

$= \frac{1}{2}\left|8(-2)-1(-1)+1(9)\right| = \frac{1}{2}\left|-16+10\right| = \frac{1}{2}\left|-6\right| = 3$

Q.2 (B)

$OA = OB \Rightarrow (a-4)^2 + (b-3)^2 = (a-7)^2 + (b+1)^2$

$4a + 8b = 40 \Rightarrow 2a + 4b = 20$ ....(1)

$OA = OC \quad \Rightarrow \quad (a-a)^2 + (b-3)^2 \; = (a-1)^2 + (b+1)^2$

2a + b = 11 ....(2)

From (1) and (2) we get,

a = 4 b = 3

$\therefore R = \sqrt{(4-a)+(3-3)} \; = \; \sqrt{(5)^2} \; = 5$

a + b + R = 3 + 4 + 5 = 12

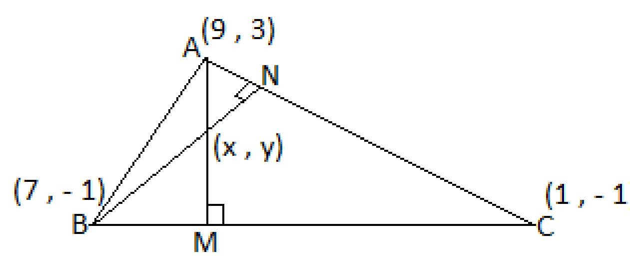
Q.3 (D)

slope of BC $= \frac{-1+1}{1-7} = 0$

slope of AM $= \frac{1}{0}$

equation of AM $\Rightarrow x = 9$ ....(1)

slope of AC $= \frac{4}{8} = \frac{1}{2}$



Slope of BN = – 2.

Equation of BN $\Rightarrow y+1 = -2(x-7)$ ....(2)

Solving (1) and (2)

x = 9 y = – 5

Passage – 2

4 ~~Q.5~~ **(A)**

$$4(a^2+b^2)=(c_1-c_2) \quad \Rightarrow \quad \frac{|c_1-c_2|}{\sqrt{a^2+b^2}}=2$$

L_1 : ax + by +c_1 = 0

L_2 : ax + by +c_2 = 0 distance between them = 2.

$\therefore t \in \{0 , 2\}$

5 ~~Q.6~~ **(A)(C)**

Equation of line passing through P and parallel L_1

Let L_p : ax + by + λ

L_1 : ax + by + c_1

$$\text{diastance} = \frac{c_1-\lambda}{\sqrt{a^2+b^2}} = t$$

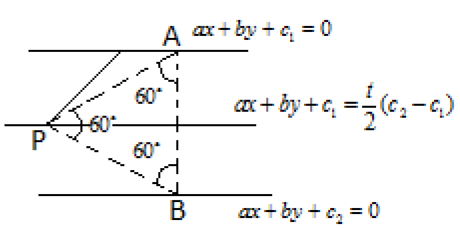
$$\Rightarrow \lambda = c_1 \pm \frac{t}{2}(c_1-c_2)$$

$$\therefore LP : ax+by+c_1=\frac{t}{2}(c_1-c_2)$$ ………………..<u>(A)</u>

$$ax+by+c_1=\frac{t}{2}(c_2-c_1)$$ ………………..<u>(C)</u>

6 ~~Q.7~~ **(B)**

$$\text{Area of } \Delta ABP = \frac{1}{\sqrt{3}}\left(t^2-2t+4\right)$$



7 **Q.8 (A)**

Area of equilateral triangle $\frac{\sqrt{3}}{4}a^2 = \frac{1}{\sqrt{3}}(t^2 - 2t + 4)$

$$a^2 = \frac{4}{\sqrt{3 \times \sqrt{3}}}(t^2 - 2t + 4) \quad \Rightarrow \quad a = \frac{2}{\sqrt{3}}\sqrt{t^2 - 2t + 4}$$

8 **Q.9 (A)(B)(C)(D)**

$t \in \{0, 2\}$

$\Rightarrow \quad AB = \frac{2}{\sqrt{3}}\sqrt{t^2 - 2t + 4}$

$\Rightarrow \quad AB \le \frac{4}{\sqrt{3}}$

$\therefore 2 \le AB \le \frac{4}{\sqrt{3}}$

Area $= \frac{1}{\sqrt{3}}(t^2 - 2t + 4)$ $\qquad \sqrt{3} \le$ Area $\le \frac{4\sqrt{3}}{3}$

When AB = 0, P can obtain four position.

When AB = 2, P can obtain 2 position.

[redacted]

Area $= \frac{1}{\sqrt{3}}(t^2 - 2t + 4)$

Max when t = 0 (A)

Min when t = 2 (B)

$\sqrt{3} <$ Area $< \frac{4\sqrt{3}}{3}$ (C)

Passage – 3

9 **Q.11** **(A)**

$\cdot\ A_1 = \text{AreaBCQD}$

$\quad = \text{Area}\,CQD + \text{Area}\,BCD$

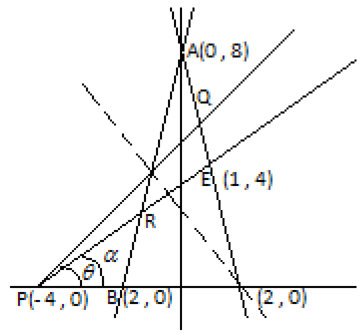
$AC: y = -4x + 8$

$PQ:\ y = \tan\theta x + \tan\theta + 4$

solvong them we get,

$\theta \equiv \left(\dfrac{4-\tan\theta}{4+\tan\theta}, \dfrac{16+12\tan\theta}{4+\tan\theta}\right)$

$\perp$ distance from θt BC is



BC : 4x + 3y – 8 = 0

$$\perp \text{distance} = \frac{32 + 24\tan\theta}{5(4 + \tan\theta)}$$

$$\text{Area CQD} = \frac{1}{2} \times 5 \times \frac{(32 + 24\tan\theta)}{5(4 + \tan\theta)}$$

$$= \frac{32 + 24\tan\theta}{2(4 + \tan\theta)}$$

Area of BCD = 8

$$\therefore A_1 = \frac{48 + 20\tan}{4 + \tan\theta}$$

10 **Q.12** **(A)**

AB : y = 4x – 8

$$\text{PE}\ : y = \tan\alpha\, x + 4 - \tan\alpha$$

$$R \equiv \left(\frac{\tan\alpha - 12}{\tan\alpha - 4}, \frac{4\tan\alpha + 16}{\tan\alpha - 4}\right)$$

BE : 4x – 3y + 8 = 0

$$\perp \text{Distance from R to BE} \equiv \frac{24\tan\alpha - 32}{\tan\alpha - 4}$$

$\therefore$ Area BER $= \dfrac{12\tan\alpha - 16}{\tan\alpha - 4}$

Area BCE = 8

$\therefore A_2 = \dfrac{48 - 20\tan\alpha}{4 - \tan\alpha}$

11 **Q.13** **(A)**

when $\tan\theta = 0$ min $A_1 = \dfrac{48}{4} = 12$

when $\tan\theta = 4$ max $A_1 = \dfrac{128}{8} = 16$

(12 , 16)

12 **Q.14** **(A)**

$\tan\alpha = \dfrac{4}{3}$ min $A_2 = 8$

$\tan\alpha = 0$ max $A_2 = 12$

A_2 : (8 , 12)

13 **Q.15** **(A)**

$$\frac{48 + 20\tan\theta}{4 + \tan\theta} \times \frac{4 - \tan\alpha}{48 - 20\tan\alpha}$$

$$\frac{A_1}{A_2} \equiv (1, 2)$$

<u>Passage – 4</u>

15 **Q.17** **(A)**

$M(\beta, \beta + 1)$ satisfies y = x + 1.

Q.18 **(C)**

BC : 8y = x + 2

$(24 - 2)(8(\beta - 1) - \beta - 2) > 0$

$\Rightarrow \beta > -2/3.$

16 **Q.19** **(C)**

AC : 3y + x = 9

$(3(\beta+1)+\beta-9)\ (-2-9)>0$

$4\beta - 6 < 0 \qquad \beta < \frac{3}{2}$

AB = 2y = 3x + 6 = 0

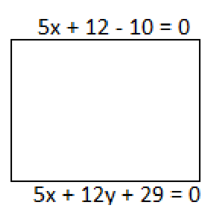
$(3\beta - 2\beta - 2 + 6)\ (18 - 2 + 6) > 0$

$-\frac{6}{7} < \beta < \frac{3}{2}$

Passage – 5

18 **Q.20** **(B)**

M = (- 3, 5)
So, one line will pass through (- 3 , 5) and other have two options.
So, Two square are possible



19 **Q.21** **(A)**

$$\text{distance between lines } = \frac{-10-29}{\sqrt{5^2+12^2}} = \frac{39}{13} = 3$$

Area = 9

20 **Q.22** **(B)**

$12x - 5y + \lambda = 0$

Will pass through (- 3 , 5)

$\Rightarrow (-36) - 25 + \lambda = 0$

$\lambda = 61$

$12x - 5y + \lambda = 0$

Other line parallel to $12x - 5y + \lambda = 0$ will be $12x - 5y + c = 0$

distance $3 = \dfrac{|\lambda - c|}{13} \Rightarrow c = 22$

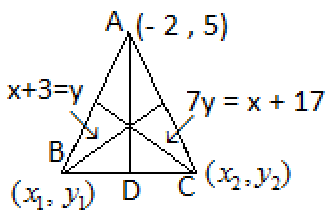
Passage – 6

21 **Q.23 (A)**

$x + 3 = y$

$x + 17 = y$

$x = -\dfrac{2}{3} \quad y = \dfrac{7}{3}$



22 **Q.24 (A)**

Centroid divide median in 2 : 1 ratio.

D (0 , 1)

23 **Q.25 (A)**

(x_1 , y_1) will pass through x + 3 = y

i.e. $x_1 + 3 = y_1$ and $7y_2 = x_2 + 17$

(0 , 1) is the mid – point of (x_1 , y_1) and (x_2 , y_2)

$\dfrac{x_1 + x_2}{2} = 0 \qquad \Rightarrow x_1 + x_2 = 0$

$\dfrac{y_1 + y_2}{2} = 2 \qquad \Rightarrow y_1 + y_2 = 0$

Solving all four equations we get

$(x_1 , y_1) \equiv (-4, -1)$

$(x_2 , y_2) \equiv (4, 3)$

24 C **Q.26 (B)(C)**

slope of AB $= \frac{6}{2} = 3$

slope of BC = $\frac{4}{8} = \frac{1}{2}$

$$\tan\theta = \frac{\left|3-\frac{1}{2}\right|}{\left|1+\frac{1}{2}\times 3\right|} = \frac{\frac{5}{2}}{\frac{5}{2}} = 1$$

$\theta = 45^\circ$

slope of AC $= -\frac{1}{3}$

$\therefore AB \perp AC$

$\therefore$ ABC is right angle and isosceles

25 C Q.27. Area ABC $= \frac{1}{2}[-2(-1,-3)+(-4)(3-5)+4(5+1)] \;=\; 2\,0$

Match the Column

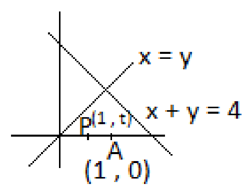
Q.1 $A \to r,\ B \to p,\ C \to u,\ D \to u$

(A) P divides A and B in t : 1 – t ratio hence 0 < t < 1

(B) x + 2y = 1 & $x + 2y = \frac{1}{2}$

$$1 \le 1 + \frac{t}{\sqrt{2}} + 2\left(2 + \frac{t}{\sqrt{2}}\right) \le \frac{15}{2}$$
$$\Rightarrow -\frac{4\sqrt{2}}{3} \le t \le \frac{5\sqrt{2}}{6}$$

(C) Point P and A are same on side of x + y = 4



$\Rightarrow$ (1 + t - 4) (1 + 0 - 4) > 0

$t < 3$

$\Rightarrow$ And P and A are also same side of x = y

$\Rightarrow (1-t)(1-0) > 0 \Rightarrow t < 1$

(D) $\begin{vmatrix} 1 & -t & -m \\ 0 & 1 & 2 \\ m & -1 & 0 \end{vmatrix} = 0 \quad \Rightarrow \quad t = \frac{m^2+2}{2}$

Q.2 $A \to r,\ B \to q,\ C \to p, q, s\ \ D \to p, q, s$

(A) $k = 0 \quad \Rightarrow \quad mx - y = 0 \qquad mx - y = 0$

$x - my = 0 \quad x - my = 0$

(B) mx – y = 0

mx – y = 2k slope = + m pependiculer line.

x – my = k

x – my = – k slope $= \frac{1}{m}$ pependiculer line.

$\therefore$ are sides of rhombus

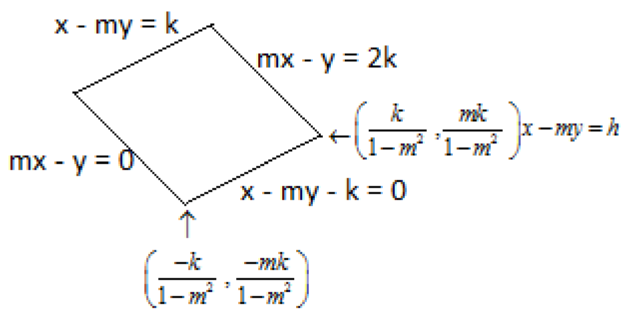
side $= \frac{2k\sqrt{m^2+1}}{m^2-1}$

and diagonal are parallel with

+ 1 , - 1

(C) PQ , S same.

(D) PQ , S same.



Q.3 $\mathrm{A} \to \mathrm{s}, \mathrm{B} \to \mathrm{r}, \mathrm{C} \to \mathrm{q}, \mathrm{D} \to \mathrm{p}$

(A) 2x + y = 0

2x + y = 8

2x – y = 4 Rhombus

2x – y = 4

(B)

$\left.\begin{aligned} 3x - 4y + 11 = 0 \\ 3x - 4y - 9 = 0 \end{aligned}\right\}$ disatnce $= \frac{20}{5}$

$\left.\begin{aligned} 4x + 3y + 17 = 0 \\ 4x + 3y - 3 = 0 \end{aligned}\right\}$ distance $= \frac{20}{5}$

(C) 3x + 2y = 0

3x + 2y = 10

2x – 3y = 0

2x – 3y = 35

(D) 2x + y = 0

$2x + y = 0$

$x - y = 8$

$x - y = 14$

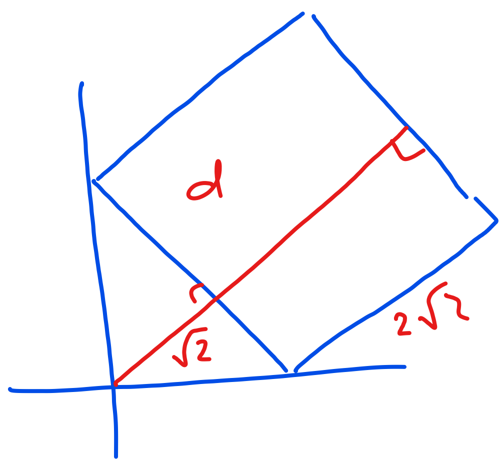
4 A Given $\frac{1}{a} + \frac{4}{b} = 1$ $\therefore a+b$ min

$$\frac{1\left(\frac{1}{a}\right) + 2\left(\frac{2}{b}\right)}{1+2} \geq \frac{1+2}{a + 2(b/2)}$$

$\Rightarrow$ $a+b \geq 9$

B Q must be $(-1, 4)$ $\therefore PQ = 5\sqrt{2}$

C



$d_{max} = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$

d eliminate 'λ'

$\sqrt{2}(x-4) = \frac{y+1}{\sqrt{2}}$ or $y+1 = 2(x-4)$

put $y = 0$

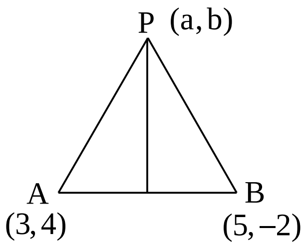
$x = 9/2$

STRAIGHT LINES

EXERCISE - 2(C)

Q.1

A (3, 4) B (5, – 2)



$\Rightarrow PA = PB = \sqrt{(a-3)^2 + (b-4)^2} = \sqrt{(a-5)^2 + (b+2)^2}$

$\Rightarrow 1 - 6a + 9 - 8b + 16 = -10a + 25 + 4b + 4$

$\Rightarrow 4a - 12b = 4$

$\Rightarrow a - 3b = 1$ …………(1)

$$\Rightarrow \frac{1}{2}\begin{vmatrix} a & b & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix} = 10$$

$\Rightarrow a\,(6) - b\,(-2) + 1\,(-26) = 10$

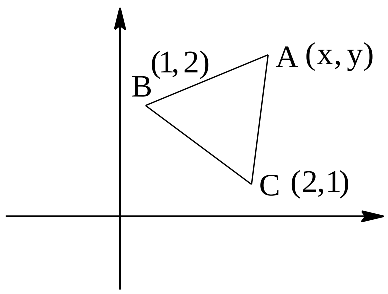
$\Rightarrow 6a + 2b = 46$ ………...(2)

solve (1) & (2)

$\Rightarrow a = 7,\ b = 2$

So $a + b = 9$

Q.2



$$\Rightarrow \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 6$$

$$\Rightarrow \frac{1}{2}\left(x(2-1) - y(-1) + 1(-3)\right) = 6$$

$\Rightarrow x + y = 15$

Q.3

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ x & 2 & 1 \\ 1 & y & 1 \end{vmatrix} = \pm 6$$

$$\Rightarrow (2-y) - 2(x-1) + (xy-2) = \pm 12$$

$\Rightarrow -2x - y + xy = 10$ or -14

Now $-2x - y + xy = 10 \Rightarrow (x-1)(y-2) = 12$

We can write 12 as a product of two integers in 12 ways.

Further $-2x - y + xy = -14 \Rightarrow (x-1)(y-2) = -12$

We can write -12 as a product of two integers in 12 ways.

Hence total 24 pairs of (x, y) are possible.

Q.4

By the property of parabola.

Ratio of area of triangle by three points of parabola to the area of triangle by intersection points of tangent are those point $=\frac{1}{2}$

So required answer = $4\times 2=8$

Q.5

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 2a & a^3 & 1 \\ 2b & b^3 & 1 \\ 2c & c^3 & 1 \end{vmatrix}=A_1$$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 2(a-c) & a^3-c^3 & 0 \\ 2(b-c) & b^3-c^3 & 0 \\ 2c & c^3 & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}\times 2\times(a-c)(b-c)\begin{vmatrix} 1 & a^2+c^2+ac & 0 \\ 1 & b^2+c^2+bc & 0 \\ c & c^3 & 1 \end{vmatrix}=A_1$$

$$\Rightarrow A_1=(a-c)(b-c)\left(b^2+c^2+bc-a^2-c^2-ac\right)$$

$$\Rightarrow A_1=(a-c)(b-c)(b-a)(b+a+c)$$

$$\Rightarrow A_1=(a-b)(b-c)(c-a)(a+b+c)$$

$$\Rightarrow A_2=\frac{1}{2}\begin{vmatrix} a & bc & 1 \\ b & ca & 1 \\ c & ab & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} a-c & b(c-a) & 0 \\ b-c & a(c-b) & 0 \\ c & ab & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}(a-c)(b-c)\begin{vmatrix} 1 & -b & 0 \\ 1 & -a & 0 \\ c & ab & 1 \end{vmatrix}$$

$\Rightarrow \frac{1}{2}(a-c)(b-c)(b-a)$

$\Rightarrow A_2 = \frac{1}{2}(a-b)(b-c)(c-a)$

$\Rightarrow \therefore A_1 : A_2 = 2(a+b+c) = 8$

Q.6

$\Rightarrow 5y = x \Rightarrow \tan\alpha = \frac{1}{5}$

$\Rightarrow 5y = 5x \Rightarrow \tan\beta = \frac{5}{k}$

Given that $\beta = 2\alpha$

$\Rightarrow \tan\beta = \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

$\Rightarrow \frac{5}{k} = \frac{2\left(\frac{1}{5}\right)}{1-\frac{1}{25}} = \frac{10}{24}$

$\Rightarrow k = 12$

Q.7

$\Rightarrow 2ax - 3y - a = 0 \Rightarrow$ slope $m_1 = \frac{2a}{3}$

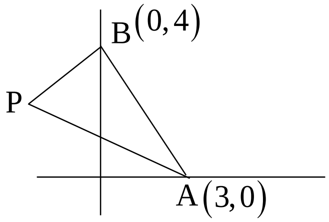
$\Rightarrow 3x + 4y + 1 = 0 \Rightarrow$ slope $m_2 = \frac{-3}{4}$

$\Rightarrow \therefore m_1 m_2 = -1$

$\Rightarrow a = 2$

Q.8

$\Rightarrow \left.\begin{array}{ll} 3x+4y=9 & \text{........(1)} \\ 4x-3y=-12 & \text{........(2)} \end{array}\right\}$ Intersection point $P\left(\frac{-21}{25}, \frac{72}{25}\right)$



$\because$ Both the lines are perpendicular to each other so they intersect at point P at 90^o.

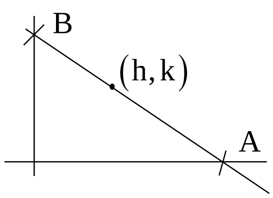
$\Rightarrow \therefore |AB| = \text{diameter circumcircle of } \Delta PAB$

$\Rightarrow \quad = 5$

Q.9

Equation of variable straight lines is $(x+2y-1)+\lambda(2x-y-1)=0$

$\Rightarrow A\left(\frac{\lambda+1}{1+2\lambda},0\right),\ B\left(,\frac{\lambda+1}{2-\lambda}\right)$



$\Rightarrow \therefore\ 2h = \frac{\lambda+1}{1+2\lambda},\ 2k = \frac{\lambda+1}{2-\lambda}$

By elimination of λ we get

$\Rightarrow$ x + 3y = 10 xy

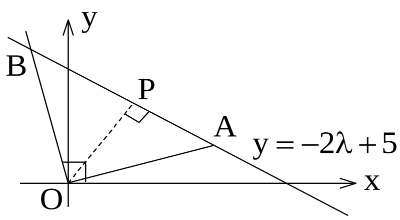
So k = 10

Q.10

$\Rightarrow \because\ \angle AOB = 90^o$

So OP is angle bisector of $\angle AOB$ because Δ AOB is iscoceles triangle.

$\Rightarrow \therefore\ OP = \left|\frac{5}{\sqrt{1+4}}\right| = \sqrt{5}$



$\Rightarrow \therefore$ Area of triangle $\Delta AOB = 2(\text{Area of } AOP)$

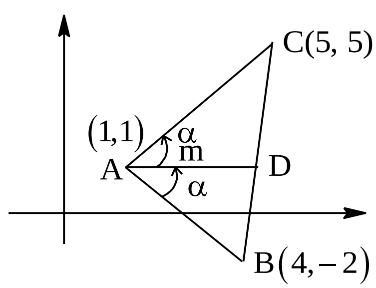
$= 2\left(\frac{1}{2}\times\sqrt{5}\times\sqrt{5}\right) = 5$

Q.11

AD is angle bisector

$\Rightarrow m_{AB} = \frac{-3}{3} = -1$

$\Rightarrow m_{AC} = 1$



So clearly m = 0

So equation of perpendicular AD form C is

$\Rightarrow y-5=\infty(x-5)$

$\Rightarrow$x = 5; so k = 5

Q.12

$\Rightarrow y^2 = 4x, y = 2x+3$

Let suppose a point P (t2, 2t) on parabola. & Image of point P about given line is Point Q.

So locus of point Q is given by

$$\left(4x+3y+6\right)^2 + K\left(3x-4y+12\right) = 0$$

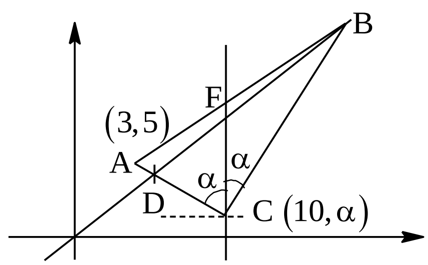
Coordinate of Point Q are given by $\dfrac{x-t^2}{2} = \dfrac{y-2t}{-1} = \dfrac{-(2t^2-2t+3)}{5}$

Eliminate “t” to find the locus, which is given by

$$\left(4x+3y+6\right)^2 + 20\left(3x-4y+12\right) = 0$$

So K=20.

Q.13



Equation of BC is 6y – x + k = 0

By figure its clear that

$$\Rightarrow m_{BC} = -m_{CA}$$

$$\Rightarrow \left(\frac{1}{6}\right) = -\left(\frac{a-5}{10-3}\right)$$

$\Rightarrow$7 = – 6a + 30

$\Rightarrow$6a = 23

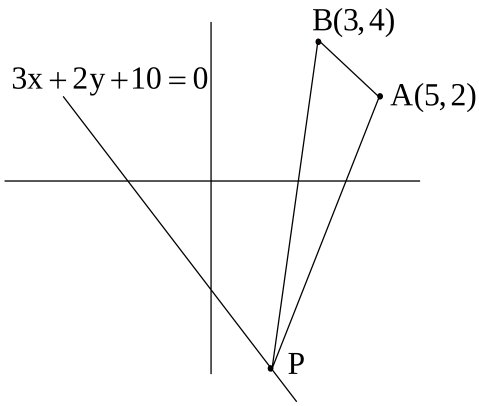
So, $C\left(10, \dfrac{23}{6}\right)$ will satisfy equation of BC.

$\Rightarrow$6(a) – (10) + k = 0

$\Rightarrow$23 – 10 + k = 0

$\Rightarrow$k = – 13

Q.14



$\Rightarrow |PA - PB| \leq AB$

$\Rightarrow |PA - PB|_{max} = AB$ and that will occur only if points P,A & B are collinear.

$\Rightarrow 3h + 2k + 10 = 0$ ....(1)

$\Rightarrow \left(\frac{k-2}{h-5}\right) = \frac{4-2}{3-5} = -1$

$\Rightarrow k - 2 = -h + 5$

$\Rightarrow (h + k) = 7$

Q.15

Line AB is $y - 1 = m(x - 1)$

$\Rightarrow A\left(1-\frac{1}{m}, 0\right), B(0, 1-m)$

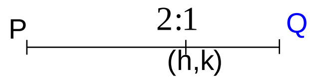
$\Rightarrow C\left(1-\frac{1}{m}, 1-m\right)$

Line PQ is $y-(1-m) = m\left(x-\left(1-\frac{1}{m}\right)\right)$

$\Rightarrow y = (1 - m) + mx - m + 1$

$\Rightarrow y = mx + 2(1 - m)$

$$\Rightarrow P\left(-2\frac{(1-m)}{m},0\right),\ Q(0,2(1-m))$$



So $\dfrac{\dfrac{-2(1-m)}{m}}{3}=h$ ……..(2), $\dfrac{4(1-m)}{3}=k$ …….(1)

$$\Rightarrow \frac{-1}{2(m)}=\frac{h}{k}\Rightarrow m=-\frac{k}{2h}\ \text{ put it in (1)}$$

$$\Rightarrow \left(1+\frac{k}{2h}\right)=\frac{3k}{4}$$

$\Rightarrow$4h + k = 3kh

$\Rightarrow$3xy = 2 (2x + y)

So, k = 2

Q.16

$\Rightarrow$ – 3x + 2y + 1 = 0

$\Rightarrow$2x – 3y + 1 = 0

$$\Rightarrow a_1a_2+b_1b_2=-6-6<0$$

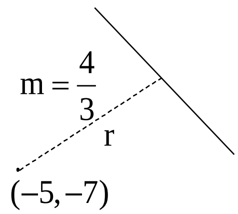
Equation of angle bisectors are

$$\Rightarrow \frac{-3x+2y+1}{\sqrt{13}}=\pm\left(\frac{2x-3y+1}{\sqrt{13}}\right)$$

Negative sign will produce obtuse angle bisector

$\Rightarrow$(– 3x + 2y + 1) = – (2x – 3y + 1)

$\Rightarrow$x + y – 2= 0 …………(1)



Let's take point is at distance r

$$\Rightarrow \tan\alpha = \frac{4}{3}$$

So $\left[(-5 + r\cos\alpha), (-7 + r\sin\alpha)\right]$ is on (1)

$$\Rightarrow -5 + r\left(\frac{3}{5}\right) + \left(-7 + r \times \frac{4}{5}\right) - 2 = 0$$

$$\Rightarrow r\left(\frac{7}{5}\right) = 14$$

$\Rightarrow$ r = 10

Q.17

For concurrent lines

$$\Rightarrow \begin{vmatrix} 4 & -1 & 6 \\ 3 & -4 & -6 \\ 1 & 6 & k \end{vmatrix} = 0$$

$\Rightarrow$ k = 20

Q.18

Point A $\equiv$ (intersection of AB & AD)

$$\equiv \left(y = 2x + 1 \ \ \& \ \ y = \frac{2x+7}{4}\right)$$

$$\equiv \left(\frac{1}{2}, 2\right)$$

Point C $\equiv$(intersection of 3y = x + 1 & y = 4x – 7)

So equation of AC $\equiv y-1=\dfrac{2-1}{\frac{1}{2}-2}(x-2)$

$\Rightarrow y-1=\dfrac{-2}{3}(x-2)$

$\Rightarrow 2x + 3y - 5 = 0$

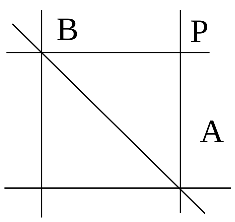
$\Rightarrow a = 2, b = 3, c = 5$

$\Rightarrow |a+b+c|=10$

Q.19

Variable line through intersection point is

$\Rightarrow (2x+3y-1)+\lambda(3x+2y-1)=0$



$\Rightarrow A\left(\dfrac{\lambda+1}{2+3\lambda},0\right), B\left(0,\dfrac{\lambda+1}{3+2\lambda}\right), P\left(\dfrac{\lambda+1}{2+3\lambda},\dfrac{\lambda+1}{3+2\lambda}\right)$

$\Rightarrow h=\dfrac{\lambda+1}{2+3\lambda}$ …….(1)

$\Rightarrow k=\dfrac{\lambda+1}{3+2\lambda}$ …….(2)

By elimination of λ by two equation we get

$\Rightarrow \dfrac{1}{h}+\dfrac{1}{k}=5$ or $\dfrac{1}{x}+\dfrac{1}{y}=5$

So k = 5

Q.20

Variable point on the Line QPR is

$\Rightarrow \left\{\left(\sqrt{2}\sec\theta + r\cos\alpha\right), \left(\sqrt{3}\tan\theta + r\sin\alpha\right)\right\}$ …..Slope of line QPR= $\tan\alpha = 2$

$\Rightarrow \therefore$ equation of pair of straight lines is

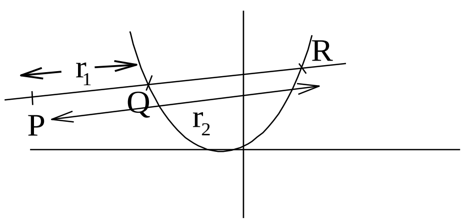
$\Rightarrow 3x^2 - 2y^2 = 0$

$\Rightarrow \frac{x^2}{2} - \frac{y^2}{3} = 0$

$\Rightarrow 3\left(2\sec^2\theta + r^2\cos^2\alpha + 2\sqrt{2}\,r\sec\theta\cot\alpha\right) - 2\left(3\tan^2\theta + r^2\sin^2\alpha + 2\sqrt{3}r\sin\alpha\tan\theta\right) = 0$

$\Rightarrow r^2\left(3\cos^2\alpha - 2\sin^2\alpha\right) + 2\sqrt{2}\left(3\sec\theta\cos\alpha - \sqrt{6}\sin\alpha\tan\theta\right)r + 6\sec^2\theta - 6\tan^2\theta = 0 \Rightarrow$

$\therefore r_1r_2 = \frac{6\sec^2\theta - 6\tan^2\theta}{3\cos^2\alpha - 2\sin^2\alpha} = \frac{6\left(\sec^2\alpha\right)}{3 - 2\tan^2\alpha}$



$\Rightarrow r_1r_2 = \frac{6(1+4)}{3-2(4)} = -6$

$\Rightarrow \left|r_1r_2\right| = 6$

Q.21

Pair of lines passing through the points in which the lines $6x^2 + xy - y^2 - 6x + 7y - 12 = 0$ meet the coordinate axes will be given by

$6x^2 + xy - y^2 - 6x + 7y - 12 + \lambda xy = 0$ or $6x^2 + (1+\lambda)xy - y^2 - 6x + 7y - 12 = 0$

As its equation of a pair of lines hence

$$6(-1)(-12) + 2\left(\frac{7}{2}\right)(-3)\left(\frac{1+\lambda}{2}\right) - 6\left(\frac{7}{2}\right)^2 - (-1)(-3)^2 - (-12)\left(\frac{1+\lambda}{2}\right)^2 = 0$$

$\Rightarrow 2(1+\lambda)^2 - 7(1+\lambda) + 5 = 0 \Rightarrow \lambda = \frac{3}{2}$

Hence the required pair of lines is $12x^2 + 5xy - 2y^2 - 12x + 14y - 24 = 0$.

Value of 'a' is 12.

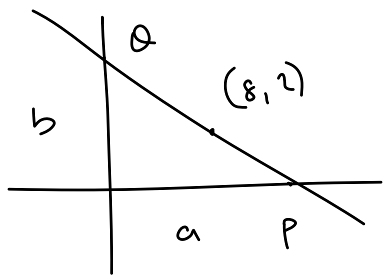
Q.22 Pair of angle bisectors of the lines $x^2 - 2pxy - y^2 = 0$ will be

$$\frac{x^2 - y^2}{2} = \frac{xy}{-p} \text{ or } px^2 + 2xy - py^2 = 0$$

Which must be same as $x^2 + 2qxy - y^2 = 0$.

Comparing the two equations gives pq = 1.

(23) Given $\frac{8}{a}+\frac{2}{b}=1$ & $a+b=?$



By weighted AM/GM

$$\frac{2\sqrt{2}\left(\frac{2\sqrt{2}}{a}\right)+\sqrt{2}\left(\frac{\sqrt{2}}{b}\right)}{2\sqrt{2}+\sqrt{2}} \geq \frac{2\sqrt{2}+\sqrt{2}}{2\sqrt{2}\left(\frac{a}{\sqrt{2}}\right)+2\sqrt{2}\left(\frac{b}{\sqrt{2}}\right)}$$

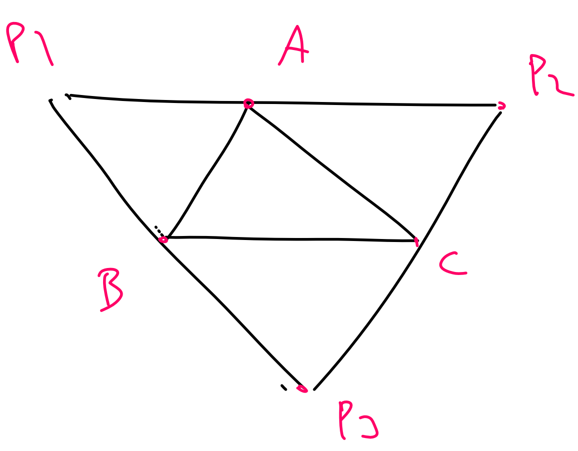
$$\Rightarrow \quad a+b \geq 18$$

(24) Given $\frac{8}{a}+\frac{2}{b}=1$ By AM/GM $\frac{8}{a}+\frac{2}{b} \geq 2\sqrt{\frac{16}{ab}}$

$$\Rightarrow \quad \frac{1}{4} \geq \frac{16}{ab}$$

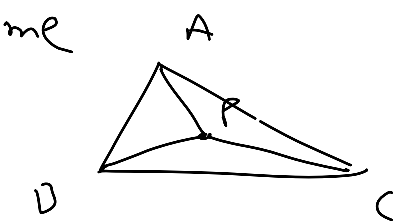
$$ab \geq 64$$

(25) P, A, B, C form a //gm if all have same area



for all 3 positions of P the areas are Equal

Also if P centroid then also areas same



$\Rightarrow$ 4 points P

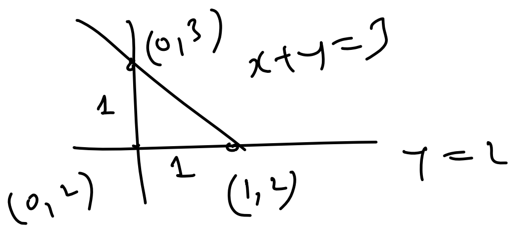
(26) $x^2-y^2+4y-4=0$

(26) $x^2 - y^2 + 4y - 4 = 0$

$\Rightarrow \quad x^2 - (y-2)^2 = 0$

$\Rightarrow \quad x - y + 2 = 0$ or $x + y - 2 = 0$

its bisectors are $x = 0,\ y = 2$



$\therefore$ Area $= \frac{1}{2}$

(27) $\sqrt{x^2+y^2}$ is distance of x, y from $(0,0)$

$\therefore$ least distance is $\perp^r$ distance from $(0,0) = \frac{6}{\sqrt{13}}$

(28) Conceptual

(29) $(2a+b)x + (a+3b)y + b - 3a = 0$

$a(2x + y - 3) + b(x + 3y + 1) = 0$

$\Rightarrow$

$2x + y - 3 = 0$

$x + 3y + 1 = 0$ are concurrent

$mx + 2y + 6 = 0$

$$\Rightarrow \begin{vmatrix} m & 2 & 6 \\ 2 & 1 & -3 \\ 1 & 3 & 1 \end{vmatrix} = 0$$

(30) $\perp^r$ dist $= \left| 2\sqrt{\frac{g^2 - ac}{a(a+b)}} \right|$

$$= 2\sqrt{\frac{\cancel{g}k^2}{\cancel{g}\times 25}} = 4 \Rightarrow |k| = 10$$

Only One Option Correct

1. (D)

2. (A)

3. (D)

4. (D)

5. (C)

6. (B)

7. (D)

Numerical Stem Questions

1. (9)

Let locus point $P(x, y)$

$\therefore$ According to question

$$\left|\frac{\sqrt{2}x + y - 1}{\sqrt{3}}\right|\left|\frac{\sqrt{2}x - y + 1}{\sqrt{3}}\right| = \lambda^2$$

$$\Rightarrow \left|\frac{2x^2 - (y-1)^2}{3}\right| = \lambda^2$$

So, $C : \left|2x^2 - (y-1)^2\right| = 3\lambda^2$

Let the line $y = 2x + 1$ meets C at two points $R(x_1, y_1)$ and $S(x_2, y_2)$

$\Rightarrow y_1 = 2x_1 + 1$ and $y_2 = 2x_2 + 1$ ....(i)

$\Rightarrow (y_1 - y_2) = 2(x_1 - x_2)$

$\therefore RS = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

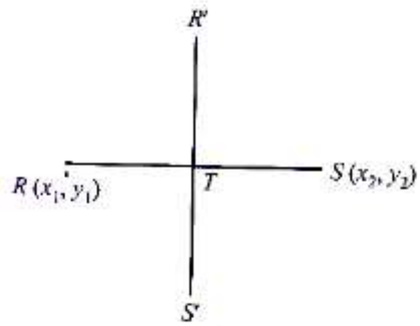
$RS = \sqrt{5(x_1 - x_2)^2} = \sqrt{5}|x_1 - x_2|$

On solving equation curve C and line $y = 2x + 1$, we get

$\left|2x^2 - (2x)^2\right| = 3\lambda^2 \qquad \Rightarrow x^2 = \frac{3\lambda^2}{2}$

$\therefore RS = \sqrt{5}\left|\frac{2\sqrt{3}\lambda}{\sqrt{2}}\right| = \sqrt{30}\lambda = \sqrt{270} \Rightarrow 30\lambda^2 = 270 \Rightarrow \lambda^2 = 9$

2. (77.14)



Perpendicular bisector of RS

$T \equiv (0,1)$ [From (i)]

Equation of R'S'

$(y-1) = -\frac{1}{2}(x-0) \Rightarrow x+2y=2$

Let $R'(a_1, b_1)$ and $S'(a_2, b_2)$

$\therefore D = (a_1 - a_2)^2 = (b_1 - b_2)^2 = 5(b_1 - b_2)^2$

On solving $x+2y=2$ and $\left|2x^2 - (y-1)^2\right| = 3\lambda^2$, we get

$\Rightarrow \left|8(y-1)^2 - (y-1)^2\right| = 3\lambda^2$

$\Rightarrow (y-1)^2 = \left(\frac{\sqrt{3}\lambda}{\sqrt{7}}\right)^2$

$y - 1 = \pm\frac{\sqrt{3}\lambda}{\sqrt{7}}$

$\Rightarrow b_1 = 1 + \frac{\sqrt{3}\lambda}{\sqrt{7}}, b_2 = 1 - \frac{\sqrt{3}\lambda}{\sqrt{7}}$

$D = 5\left(\frac{2\sqrt{3}\lambda}{\sqrt{7}}\right)^2 = \frac{5\times4\times3\lambda^2}{7} = \frac{5\times4\times27}{7} = 77.14$

Numerical Value Answer

1. (6)

Let the point P be (x, y)

The, $d_1(P) = \left|\frac{x-y}{\sqrt{2}}\right|$ and $d_2(P) = \left|\frac{x+y}{\sqrt{2}}\right|$

For P lying in first quadrant $x > 0, y > 0$

Now $2 \le d_1(P) + d_2(P) \le 4$

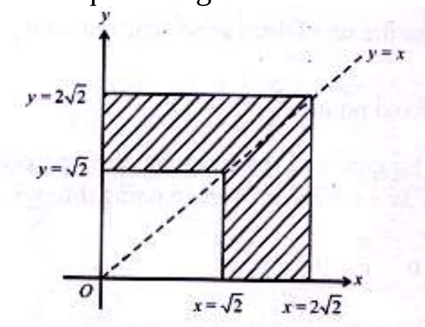
$\Rightarrow 2 \le \left|\frac{x-y}{\sqrt{2}}\right| + \left|\frac{x+y}{\sqrt{2}}\right| \le 4$

If $x > y$, then $2 \le \frac{x-y+x+y}{\sqrt{2}} \le 4 \Rightarrow \sqrt{2} \le x \le 2\sqrt{2}$

If $x < y$, then

$$2 \le \frac{y-x+x+y}{\sqrt{2}} \le 4 \text{ or } \sqrt{2} \le y \le 2\sqrt{2}$$

The required region is the shaded region in the figure given below.



$\therefore$ Required area $= \left(2\sqrt{2}\right)^2 - \left(\sqrt{2}\right)^2 = 8 - 2 = 6$ sq.units

Subjective Problems

3. (18)