PACE-IIT & MEDICAL

MUMBAI / DELHI-NCR / PUNE / NASHIK / AKOLA / GOA / JALGOAN / BOKARO / AMRAVATI / DHULE

IIT – JEE: 2025 **MAJOR TEST - 3** **DATE: 27/01/24**

ANSWER KEY

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	B	**31.**	D	**61.**	A
2.	D	**32.**	C	**62.**	D
3.	A	**33.**	C	**63.**	C
4.	B	**34.**	C	**64.**	B
5.	B	**35.**	D	**65.**	C
6.	D	**36.**	C	**66.**	B
7.	Bonus	**37.**	D	**67.**	B
8.	A	**38.**	A	**68.**	B
9.	B	**39.**	A	**69.**	D
10.	C	**40.**	D	**70.**	C
11.	D	**41.**	C	**71.**	C
12.	D	**42.**	C	**72.**	D
13.	A	**43.**	A	**73.**	C
14.	B	**44.**	C	**74.**	C
15.	B	**45.**	C	**75.**	B
16.	B	**46.**	C	**76.**	A
17.	B	**47.**	A	**77.**	D
18.	D	**48.**	B	**78.**	A
19.	C	**49.**	C	**79.**	A
20.	C	**50.**	B	**80.**	C
21.	9	**51.**	7	**81.**	41
22.	4	**52.**	2.70 to 2.85	**82.**	165
23.	2	**53.**	3	**83.**	8
24.	5	**54.**	1.3	**84.**	10
25.	4	**55.**	500	**85.**	3
26.	1	**56.**	1	**86.**	1
27.	9	**57.**	13	**87.**	0
28.	2	**58.**	3	**88.**	360
29.	4	**59.**	3	**89.**	252
30.	4	**60.**	5	**90.**	3

PART (A) : PHYSICS

1. (B)

$$\frac{40}{100}\left[\frac{I\omega^2}{2}+\frac{1}{2}m(\omega r)^2\right]=\frac{I\omega^2}{2}$$

$$\Rightarrow \frac{3}{5}\frac{I\omega^2}{2}=\frac{1}{5}m\omega^2 r^2$$

$$\Rightarrow I=\frac{2}{3}mr^2$$

2. (D)

Along x axis velocity component of each piece should be $v/2$ after collision according to momentum conservation.

3. (A)

Vertical acceleration $=g-\frac{\mu N}{m}; N=ma=(g-\mu a)m$

Horizontal acceleration $=a$

$$g-ua=a \qquad \Rightarrow a=\frac{g}{1+\mu}$$

4. (B)

For isotropic material, length of any line will increase

$\therefore (d+r_1+r_2)\alpha\Delta T$ is increase in separation between holes.

5. (B)

Obj at $2f \Rightarrow$ image at $2f$, inv

6. (D)

$$(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{F}$$

$$\left(\frac{1.5}{4}\times 3-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{F}$$

$$\frac{0.5}{0.5}\times 4\frac{F'}{F}\Rightarrow F'=4F$$

7. (Bonus)

$$R=\frac{V}{I},\frac{\Delta R}{R}=\frac{\Delta V}{V}+\frac{\Delta I}{I} \quad \Rightarrow R\pm\frac{\Delta R}{R}\times 100=(2\pm 1.125\%)\Omega$$

8. (A)

At the instant parachute opens, velocity is $v_1 = gt = 10\times10 = 100$

If v_2 is velocity on reaching ground,

Then $v_2^2 - v_1^2 = 2\times(-2.5)\times\left(2495 - \frac{1}{2}gt^2\right)$

$\Rightarrow v_2 = 5\,ms^{-1}$

9. (B)

Consider block A

$m_A g - N_{AB} = m_A \times 2$

$\Rightarrow 0.5\times10 - N_{AB} = 0.5\times2$

$\therefore N_{AB} = 4N$

10. (C)

At maximum compression, velocities are same

CLM $\rightarrow m_1v_1 + m_2v_2 = (m_1+m_2)v$

$\Rightarrow v = 4ms^{-1}$

COE $\rightarrow \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}kx^2$

$\Rightarrow x = 0.02m$

11. (D)

Let u_A, u_B & u_C be velocities after 1st impact while v_A, v_B & v_C be after 2nd impact.

Using the formula for velocities after elastic collision as

$$v_1 = \frac{(m_1-m_2)}{m_1+m_2}u_1 + \frac{2m_2u_2}{m_1+m_2} \;\&\; v_2 = \frac{2m_1u_1}{m_1+m_2} + \left(\frac{m_2-m_1}{m_1+m_2}\right)u_2$$

After 1st impact

$u_A = 0, u_B = 3ms^{-1}, u_C = \frac{3}{4}ms^{-1}$

After 2nd impact

$v_A = 0, v_B = 0, v_C = \frac{9}{4}ms^{-1}$

12. (D)

$a_A = \alpha\times2\,, a_B = \alpha\times1$

$\frac{a_A}{a_B} = \frac{2}{1} \Rightarrow a_B = \frac{5}{2}ms^2$

13. (A)

$u_1 = -10cm, f_1 = +10cm, v_1 = \frac{u_1f_1}{u_1+f_1} = \infty ul1$

$u_2 = \infty, f_2 = -20cm, v_2 = -d+10cm$ $\qquad v_2 = \frac{u_2f_2}{u_2+f_2} \Rightarrow d = 10cm$

14. (B)
Number of refracted ray decreases, so the image is less bright

15. (B)
$\frac{p}{v} = pv^{-1} = \text{constant}$
Polytropic process with $k = -1$
$\Delta w = -\frac{nR\Delta T}{k-1} = \frac{R}{2}(T_2 - T_1)$

16. (B)
$\Delta w_{A\to B} = 0$
$\Delta_{B\to C} = nR(T_C - T_B) = 2\times R\times(300-150)$
$= 300RJ$
$\Delta u_{A\to C} = 0$
$\therefore \Delta Q_{A\to C} = \Delta W_{B\to C} = 300R\,J$

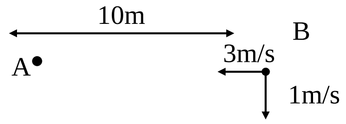
17. (B)
$\because$ t is same, so angular velocities w are same.
$\therefore$ required ratio = $\frac{w^2 r_1}{w^2 r_2} = \frac{r_1}{r_2} =$ 1:5

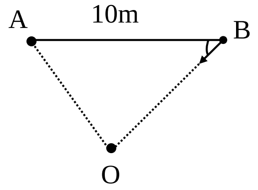
18. (D)
AB process is isothermal process.
Therefore T = constant and $P \propto \frac{1}{V}$
Pressure is decreasing. Therefore volume should increase.
In process BC:
$P \propto T$, therefore V = constant
Further, P and T both are increasing.
CD, process is identical to AB process.

19. (C)
$V = V_0\left(1+\sqrt{\Delta\theta}\right)$
$L^3 = L_0(1+\alpha_1\Delta\theta)\ L_0{}^2(1+\alpha_2\Delta\theta)$
$L_0{}^3 = V_0$ $\quad 1+\sqrt{\Delta\theta} = (1+\alpha_1\Delta\theta)(1+2\alpha_2\Delta\theta)$ $\quad r = \alpha_1 + 2\alpha_2$

20. (C)
Ice will not completely melt, so temperature of mixture is $0^{o}C$.

21. (9)





Maximum distance covered in one jump $= \dfrac{u^2}{g} = 1\text{m}$

So minimum no. of jumps = 9

22. (4)

12 = (3 + 2 +1)a = 6a

$a = \dfrac{12}{6} = 2m/s^2$

Now let F be the net force on 2 kg block in x-direction, then using $\Sigma F_x = \max$ for 3 kg block, we get

$F = (2)(2) = 4N$

23. (2)

Required component

$= \dfrac{m\left(\vec{a}, \vec{v}\right)\vec{v}}{v^2}$

$= \dfrac{\left(2\sqrt{5}\right)(1)\left(2\hat{i} + \hat{j}\right)}{5}$

$\therefore magnitude = \dfrac{2}{\sqrt{5}}.\sqrt{2^2 + 1^2} = 2$

24. (5)

The forces acting on the block are shown in Fig. Since the block is not moving forward for the maximum force F applied, therefore

$F\cos 60° = f = \mu N$ and $F\sin 60° = mg = N$

From equations (i) and (ii)

$F\cos 60° = \mu\left[F\sin 60° + mg\right] \Rightarrow \quad F = \dfrac{\mu mg}{\cos 60° - \mu\sin 60°}$

$$= \dfrac{\dfrac{1}{2\sqrt{3}} \times \sqrt{3} \times 10}{\dfrac{1}{2} - \dfrac{1}{2\sqrt{3}} \times \dfrac{\sqrt{3}}{2}} = \dfrac{5}{\dfrac{1}{4}} = 20\,\text{N}$$

25. (4)

Mass of cut out disc $= m = \frac{M}{\pi R^2}\pi\left(\frac{R}{2}\right)^2 = \frac{M}{4}$

From centre of original uniform disc the distance of centre of mass of final disc

$$= X_{C.M} = \frac{M\times 0 - \frac{M}{4}\left(-\frac{R}{2}\right) + \frac{M}{4}\left(\frac{R}{2}\right)}{M - \frac{M}{4} + \frac{M}{4}} = \frac{R}{4}$$

26. (1)

$$\frac{1}{f_a} = \left({}_a\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

and $\frac{1}{f_a} = \left({}_l\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{f_m} = \left(\frac{{}_a\mu_g}{{}_a\mu_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_m} = \left(\frac{1.5}{1.6} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{f_m}{f_a} = \frac{(1.5 - 1)}{\left(\frac{1.5}{1.6} - 1\right)} = -8$$

or $f_m = -8 \times f_a = -8 \times \frac{-1}{5} = 1.6m$

$$\therefore P_m = \frac{\mu}{f_m} = \frac{1.6}{1.6} = 1D$$

27. (9)

$$S = \left(1 - \frac{1}{\mu}\right)t$$

$$3 = \left(1 - \frac{2}{3}\right)t \Rightarrow t = 9\,cm$$

$$= 6\,\text{cm}$$

28. (2)

Mass of ice melted = 600 – 550 = 50 g

$$M' \times 0.1 \times 350 = (600 \times 0.5 \times 10) + (50 \times 80)$$

$M' = 200\,g$
$M' = M \times 100\,g$
$M = 2$

29. (4)
Change in internal energy for cyclic process (ΔU) = 0.
For process a → b, (P is constant)
$W_{a\to b} = P.\,\Delta V = nR\Delta T = -400R$
For process b → c (T is constant)
$W_{b\to c} = -2R(300)\ln 2$
For process c → d (P is constant)
$W_{c\to d} = +400R$
For process d → a (T is constant)
$W_{d\to a} = +2R(500)\ln 2$
Net work $(\Delta W) = W_{a\to b} + W_{b\to c} + W_{c\to d} + W_{d\to a}$
ΔW = 400 R ln 2
dQ = dU + dW, first laws of thermodynamics.
DQ = 400 R ln 2

30. (4)
If the force is applied to the center of mass of the whole system then the system will perform only transitional motion not rotational motion .
For COM:

$$X_{COM} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$

$$x_{COM} = \frac{0.M + M \times \frac{L}{2}}{M + M}$$

$$X_{COM} = \frac{L}{4}$$

PART (B) : CHEMISTRY

31. (D)

$$H_2 \quad + \quad \frac{1}{2}O_2 \longrightarrow H_2O$$

1 vol. $\quad \frac{1}{2}$ vol.

1 mL H_2 reacts with $O_2 = \frac{1}{2}$ mL

$\therefore$ 30 mL H_2 reacts with $O_2 = \frac{1}{2} \times 30 = 15$ mL

O_2 left at the end of the reaction = 20 – 15 = 5 mL

32. (C)

33. (C)

$$M_{mixture} = \frac{M_1V_1 + M_2V_2}{V_1 + V_2} = \frac{1 \times 2.5 + 0.5 \times 3}{5.5} = \frac{4}{5.5}$$

$$= 0.727\ M \cong 0.73\ M$$

34. (C)

35. (D)

Maximum number of electrons in a shell $= 2n^2$

36. (C)

Excited state of neon

37. (D)

$$\bar{\nu} = \frac{1}{\lambda} = R.Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For Balmer series,

$$\frac{1}{\lambda_{shortest}} = 109678 \times 1\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] = \frac{109678}{4}\ cm^{-1}$$

$$\lambda = \frac{4\ cm}{109678} = \frac{4}{109678 \times 100}\ m$$

$$= \frac{4 \times 10^{10}}{109678 \times 100}\ Å = 3647\ Å$$

38. (A)

After the removal of one electrons, O^+ acquires stable half filled configuration, so it has highest $I.E_2$. F has higher $I.E_2$. than N due to higher nuclear charge and smaller size.

39. (A)

In non-polar molecules magnitude of van der Waal's forces depends upon the number of electrons in it. Out of the given choices, Xe has highest number of electrons, as such it has maximum van der Waal's forces and maximum b.p.

40. (D)
CO_2 has zero dipole moment value due to cancellation of individual bond moments (O = C = O) as it as had linear structure. SO_2 has angular structure so it some dipole moment value.

41. (C)

Molecule / ion	BF_3	NO_2^-	NH_2^-	H_2O
Hybridisation	sp^2	sp^2	sp^3	sp^3

42. (C)
Bond dissociation energy of N_2 (B.O. = 3) > N_2^+ (B.O. = 2.5)

43. (A)
P in $POCl_3$ is sp^3 hybridised i.e.,

$$H = \frac{1}{2}[5+3] = 4$$

P in PCl_3 is also sp^3 hybridised *i.e.*,

$$H = \frac{1}{2}[5+3] = 4$$

44. (C)
(i) $KCl(s) + 20H_2O(l) \longrightarrow KCl(20H_2O)\ ;\ \Delta H = 15.90\ kJ$
(ii) $KCl(s) + 200H_2O(l) \longrightarrow KCl(200H_2O)\ ;\ \Delta H = 18.58\ kJ$
Our aim is $KCl(20H_2O) \longrightarrow KCl(200H_2O);\ \Delta H$ (enthalpy of dilution) = ?
Subtract (i) from (ii), we get

$$KCl(20H_2O) + 180H_2O \longrightarrow KCl(200H_2O);\ \Delta H = 2.68\ kJ$$

45. (C)
$\because\ n_p = n_r$

46. (C)
will behave both as an acid and base (Amphoteric).

47. (A)

$$CH_3COOH \rightleftharpoons CH_3COO^- + H^+$$

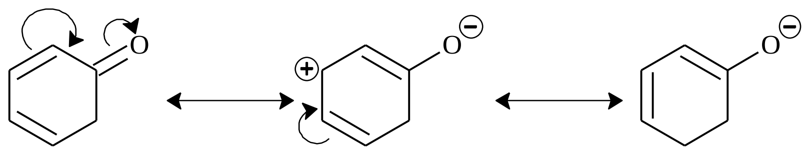
$$K_a = \frac{[CH_3COO^-][H^+]}{[CH_3COOH]}$$

$$[CH_3COOH] = \frac{[CH_3COO^-][H^+]}{K_a}$$

$$= \frac{3.4\times10^{-4}\times3.4\times10^{-4}}{1.7\times10^{-5}} = 6.8\times10^{-3}$$

48. (B)
HCl : Strong acid
NH_4Cl : Salt of weak base + Strong acid $\longrightarrow$ It gives acidic solution.
NaCl : Salt of strong base + Strong base $\longrightarrow$ It gives neutral solution with *p*H = 7.
NaCN : Salt of strong base + weak acid $\longrightarrow$ It gives basic solution with *p*H > 7.
Thus the increasing order of *p*H is HCl < NH_4Cl < NaCl < NaCN

49. (C)
The IUPAC name of aforementioned compound is 1,1-dimethyl-3-cyclohexanol.

50. (B)


Double bond get converted in single bond.

51. (7)
l = 3 means *f*-subshell and it has 7 orbitals.

52. (2.70 to 2.85)
$$\text{E.N.} = \frac{\text{I.E.} + \text{E.A.}}{129} = \frac{275 + 86}{129} \simeq 2.8$$

53. (3)
$$W = -nRT\ln\frac{V_2}{V_1}$$
$$-5229 = -n \times 8.3 \times 300 \times \ln\frac{20}{10}$$
$$5229 = 1743 \times n$$
$$\therefore \quad n = 3$$

54. (1.3)
$$X_2O_4(l) \longrightarrow 2XO_2(g)$$
For the reaction, $\Delta n_g = 2$
$$\Delta H = \Delta U + \Delta n_g RT$$
$$\Delta H = 6100\,\text{cal mol}^{-1} + 2 \times 2\,\text{cal K}^{-1}\,\text{mol}^{-1} \times 300\,\text{K}$$
$$= (6.1 + 1.2)\,\text{k cal mol}^{-1}$$
$$= 7.3\,\text{k cal mol}^{-1}$$
$$\Delta G = \Delta H - T\Delta S$$
$$= 7.3\,\text{k cal mol}^{-1} - 300\,\text{K} \times 0.02\,\text{k cal K}^{-1}\,\text{mol}^{-1}$$
$$= (7.3 - 6)\,\text{k cal mol}^{-1} \qquad = 1.3\,\text{k cal mol}^{-1}.$$

55. (500)

$\Delta S = \sum S^\circ(p) - \sum S^\circ(r)$

$= \left[S^\circ(AB_3)\right] - \left[\frac{1}{2}S^\circ(A_2) + \frac{3}{2}S^\circ(B_2)\right]$

$= 50 - \left(\frac{1}{2}\times 60 + \frac{3}{2}\times 40\right) JK^{-1}\ mol^{-1}$

At equilibrium, $\Delta H = T\Delta S$

$\therefore \quad T = \frac{\Delta H}{\Delta S} = \frac{-20\times 10^3\ J}{\left(50 - \frac{1}{2}\times 60 - \frac{3}{2}\times 40\right)}$

$T = 500\ K$

56. (1)

	$H_2(g)$	+	$I_2(g)$	$\rightleftharpoons$	$2HI(g)$
Initial mol :	4.5		4.5		0
Eqm. Mol :	3.0		3.0		3.0

$K_c = \frac{[HI]^2}{[H_2][I_2]} = \frac{3.0\times 3.0}{3.0\times 3.0} = 1$

Note : Volume of the container is not involved in the calculations.

57. (13)

No. of moles of HCl in 0.365 g $= \frac{0.365}{36.5} = 0.01$ mole

No. of moles of NaOH is 100 cc. of 0.2 M NaOH $= \frac{0.2}{1000}\times 100 = 0.02$ mole

NaOH left unneutralized = 0.02 – 0.01 = 0.01 mole

Volume of solution = 100 cm^3

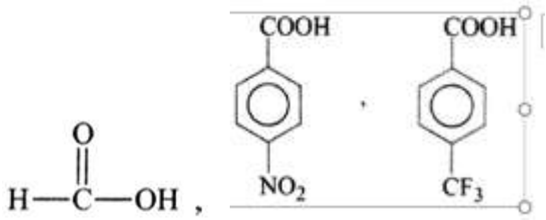
M (NaOH) $= \frac{0.01}{100}\times 1000 = 0.1\ M$

$pOH = -\log(OH^-) = -\log(0.1) = 1$

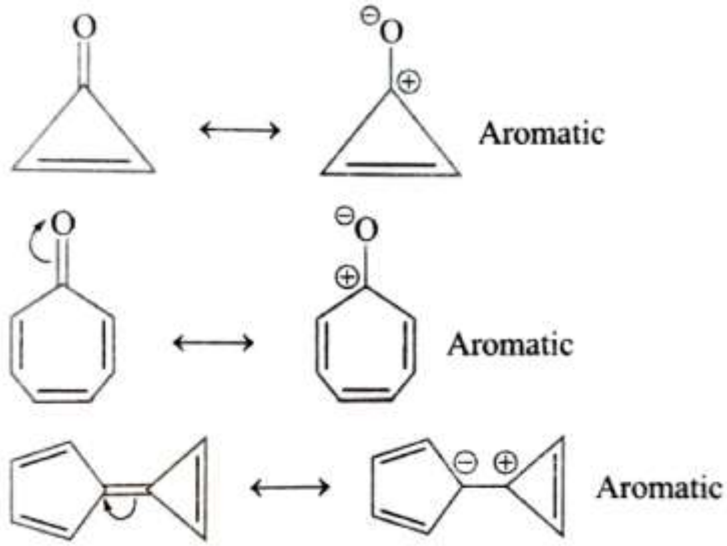
Now, $pH + pOH = 14$

$pH = 14 - pOH = 14 - 1 = 13$

58. (3)



59. (3)



60. (5)

PART (C) : MATHEMATICS

61. (A)

$$f(x)=\frac{1}{\sqrt{|x|-x}}$$

$|x|-x>0$ (We know $|x|\ge x$)

$\Rightarrow \quad x\in(-\infty,0)$

62. (D)

$[x]^2=[x]+2$

$\Rightarrow \quad [x]^2-[x]-2=0$

$\Rightarrow \quad ([x]-2)([x]+1)=0$

$\Rightarrow \quad [x]=-1$ or $[x]=2$

$\Rightarrow \quad x\in[-1,0)$ or $x\in[2,3)$

$\Rightarrow \quad x\in[-1,0)\cup[2,3)$

63. (C)

No. of hands shakes = (selection of 2 from 20) – (hand shakes by wife with own husband) – (hand shake by Indian wife with male)

$={}^{20}C_2-10-5\times9=135$

64. (B)

Required number of rectangles in the figure

= Total possible rectangles – Total number of possible squares

$=\left[{}^{7}C_2\times{}^{5}C_2\right]-\left[(4\times6)\times(3\times5)+(2\times4)+(1\times3)\right]$

$=210-50$

$=160$

65. (C)

$$\sum_{r=0}^{10}{}^{20}C_r={}^{20}C_0+{}^{20}C_1+{}^{20}C_2+\ldots+{}^{20}C_{10}$$

We have ${}^{20}C_0+{}^{20}C_1+{}^{20}C_2+\ldots+{}^{20}C_{20}=2^{20}$

$\Rightarrow\left({}^{20}C_0+\ldots+{}^{20}C_{10}\right)+\left({}^{20}C_{11}+{}^{20}C_{12}+\ldots+{}^{20}C_{20}\right)=2^{20}$

$\Rightarrow\left({}^{20}C_0+\ldots+{}^{20}C_{20}\right)+\left({}^{20}C_9+{}^{20}C_8+\ldots+{}^{20}C_0\right)=2^{20}$

$\Rightarrow\left({}^{20}C_0+\ldots+{}^{20}C_{10}\right)+\left({}^{20}C_0+\ldots+{}^{20}C_{10}-{}^{20}C_{10}\right)=2^{20}$

$\Rightarrow 2\left({}^{20}C_0+\ldots+{}^{20}C_{10}\right)=2^{20}+{}^{20}C_{10}$

$\Rightarrow {}^{20}C_0+\ldots+{}^{20}C_{10}=2^{19}+\frac{1}{2}\,{}^{20}C_{10}$

66. (B)

$e^x + e^{f(x)} = e$

$e^x = e - e^{f(x)}$

$\because e^x > 0 \Rightarrow e - e^{f(x)} > 0 \qquad e^{f(x)} < e$

$\Rightarrow f(x) < 1$

$\Rightarrow f(x) \in (-\infty, 1)$

67. (B)

$$\underset{x\to 1}{\text{Lim}} \frac{(\sqrt{x}-1)(2x-3)}{2x^2+x-3}$$

$$= \underset{x\to 1}{\text{Lim}} \frac{(\sqrt{x}-1)(2x-3)}{(2x+3)(x-1)} = \underset{x\to 1}{\text{Lim}} \frac{(\sqrt{x}-1)(2x-3)}{(2x+3)(\sqrt{x}+1)(\sqrt{x}-1)}$$

68. (B)

$$\lim_{x\to 3}([x-3]+[3-x]-x) \qquad \{\because [x]+[-x]=-1 \text{ when } x \notin I\}$$

$$= \lim_{x\to 3}(-1-x) = -4$$

69. (D)

E_1 : Six occurs

E_2 : Six doesn't occut

A : Man reports that it is a six.

$P(E_1) = \frac{1}{6}, P(E_2) = \frac{5}{6}$

$P\left(\frac{A}{E_1}\right) = \frac{3}{4}, P\left(\frac{A}{E_2}\right) = 1 - \frac{3}{4} = \frac{1}{4}$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{6}\times\frac{3}{4}}{\frac{1}{6}\times\frac{3}{4}+\frac{5}{6}\times\frac{1}{4}}$$

$$= \frac{3}{8}$$

70. (C)

$\cos 2\theta + 2\cos\theta = 2\cos^2\theta + 2\cos\theta - 1$

$= 2\left[\left(\cos\theta + \frac{1}{2}\right)^2 - \frac{1}{4} - \frac{1}{2}\right] \qquad = 2\left(\cos\theta + \frac{1}{2}\right)^2 - \frac{3}{2} \geq -\frac{3}{2}$

71. (C)

For $x \in \mathbf{I},\ f(x) = 0$

For $x \in \mathbf{R} \sim \mathbf{I}$

$(x - [x]) < 1$

$2(x - [x]) < 1 + x - [x]$

Thus $f(x) < \frac{1}{2}$

72. (D)

$f(x) = ||x-1| - 5|$

Case I $x \geq 1$

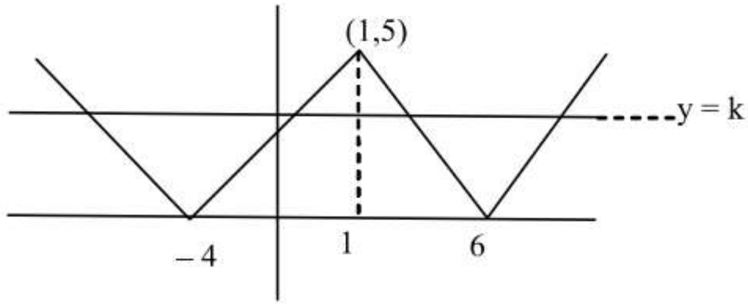
$f(x) = |x - 1 - 5|$

$= |x - 6| = \begin{cases} x - 6 & x \geq 6 \\ -x + 6 & 1 \leq x < 6 \end{cases}$

Case II $x < 1$

$f(x) = |x + 4|$

$= \begin{cases} x + 4 & -4 \leq x < 1 \\ -(x+4) & x < -4 \end{cases}$



73. (C)

$f(x) = x\left\{\left(x + \frac{1}{2}\right)^2 + \frac{11}{4}\right\} + \sin x$

Clearly $x\left\{\left(x + \frac{1}{2}\right)^2 + \frac{11}{4}\right\}$ increases with x.

and its value change over R.

Also, $-1 \leq \sin x \leq 1$.

So, the range of $f = R$

Hence, f is onto and one-one.

74. (C)

$$\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)$$

$$=\lim_{x\to 1}(1-x)\cot\left(\frac{\pi}{2}-\frac{\pi x}{2}\right)$$

$$=\lim_{x\to 1}(1-x).\frac{1}{\tan\left(\frac{\pi}{2}(1-x)\right)}$$

$$=\lim_{x\to 1}\frac{\frac{\pi}{2}(1-x)}{\tan\left(\frac{\pi}{2}(1-x)\right)}.\frac{2}{\pi}$$

$$=\frac{2}{\pi}$$

75. (B)

$\because \alpha,\beta$ are roots of $x^2-ax+b=0$

$\Rightarrow\ x^2-ax+b=(x-\alpha)(x-\beta)$

$$\lim_{x\to\alpha}\frac{e^{x^2-ax+b}-1}{x-\alpha}=\lim_{x\to\alpha}\frac{e^{(x-\alpha)(x-\beta)}-1}{(x-\alpha)(x-\beta)}\times(x-\beta)$$

$$=\alpha-\beta$$

76. (A)

Let E_1 be the event that exactly two players scored more than 50 runs, then

$$P(E_1)=\left(\frac{1}{2}\times\frac{1}{3}\times\frac{3}{4}\times\frac{9}{10}\right)+\left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{4}\times\frac{9}{10}\right)+\left(\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{1}{10}\right)+\left(\frac{1}{2}\times\frac{1}{3}\times\frac{1}{4}\times\frac{9}{10}\right)+\left(\frac{1}{2}\times\frac{1}{3}\times\frac{3}{4}\times\frac{1}{10}\right)+\left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{4}\times\frac{1}{10}\right)$$

$$=\frac{65}{240}$$

Let E_2 be the event that A and B scored more than 50 runs, then

$$P(E_1\cap E_2)=\frac{1}{2}\times\frac{1}{3}\times\frac{3}{4}\times\frac{1}{9}=\frac{27}{240}$$

$$P\left(\frac{E_2}{E_1}\right)=\frac{P(E_1\cap E_2)}{PE_1}$$

$$=\frac{27/240}{65/240}$$

$$=\frac{27}{65}$$

77. (D)

$P(A\cup B)=1-P\left(\overline{A\cup B}\right)=1-\frac{1}{6}=\frac{5}{6}$

We have $P\left(\overline{A}\right)=\frac{1}{4}\Rightarrow P(A)=\frac{3}{4}$

Again, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow \frac{5}{6}=\frac{3}{4}+P(B)-\frac{1}{4}$ $\quad\therefore\ P(B)=\frac{1}{3}$

Now, $P(A\cap B)=P(A)P(B)$ seems to be true.

Thus *A* and *B* are independent.

As $P(A)\neq P(B)$

So, *A* and *B* are not equally likely.

78. (A)

w.r.t. the given data
Let *A* gets *x* no. of objects
Let *B* gets *y* no. of objects
Let *C* gets *z* no. of objects
A.T.Q. $x=y-1, z=y+2$

$x+y+z=16$

$\Rightarrow y-1+y+y+2=16$

$3y=15$

$y=5$

$\therefore\ x=4, z=7$

Required no. of ways $=\frac{16!}{4!5!7!}$

79. (A)

$[x]^2-[x]-6>0$

$\Rightarrow ([x]-3)([x]+2)>0$

$\Rightarrow [x]<-2$ or $[x]>3$

$\Rightarrow [x]\leq -3$ or $[x]\geq 4$

$\Rightarrow x\in(-\infty,-2)\cup[4,\infty)$

80. (C)

M : 1 time
I : 4 times
S : 4 times
P : 2 times
Total no. of required selections

$=(1+1)\cdot(4+1)\cdot(4+1)\cdot(2+1)-1$

$=2\times5\times5\times3-1=149$

81. (41)

$\Rightarrow n(R \cup TV) = 1003 - 63 = 940$

$n(R) = 794$

$n(TV) = 187$

$940 = 794 + 187 - n(R \cap TV)$

$\Rightarrow n(R \cap TV) = 41$

82. (165)

Let $a = 2n_1 + 1, b = 2n_2 + 1, c = 2n_3 + 1, d = 2n_4 + 1$

Now, given $a + b + c + d = 20$

$\Rightarrow 2(n_1 + n_2 + n_3 + n_4) + 4 = 20$

$\Rightarrow n_1 + n_2 + n_3 + n_4 = 8$

Distribution of 8 identical objects in n_1, n_2, n_3, n_4 is ${}^{8+4-1}C_{4-1}$ as $n_1, n_2, n_3, n_4 \geq 0$

$= {}^{11}C_3 = 165$

83. (8)

We know period of $f(x) = |\sin x| + |\cos x|$ is $\frac{\pi}{2}$

$\Rightarrow$ Period of $f(x) = |\sin 4x| + |\cos 4x|$

$= \frac{\pi/2}{4} = \frac{\pi}{8}$

84. (10)

Put $x = 1, -\frac{1}{2}$ is given function respectively, we get

$2f(2) + f\left(\frac{1}{2}\right) = 2$ .....(i)

and $2f\left(\frac{1}{2}\right) + f(2) = -1$ .....(ii)

On solving Equation (i) and (ii), we get $f(2) = \frac{5}{3}$.

85. (3)

$$\lim_{x \to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+9}-3}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{x^2+1}-1\right)\left(\sqrt{x^2+1}+1\right)}{\left(\sqrt{x^2+1}+1\right)\cdot\left(\sqrt{x^2+9}-3\right)} \times \frac{\left(\sqrt{x^2+9}+3\right)}{\left(\sqrt{x^2+9}+3\right)}$$

$$= \lim_{x \to 0} \frac{x^2+1-1}{x^2+9-9} \times \frac{\left(\sqrt{x^2+9}+3\right)}{\sqrt{x^2+1}+1} = \frac{3+3}{1+1} = 3$$

86. (1)

$$\lim_{x\to0}\frac{e^{\tan x}-e^{x}}{\tan x-x}$$

$$=\lim_{x\to0}e^{x}\left[\frac{e^{\tan x-x}-1}{\tan x-x}\right]\Rightarrow\lim_{x\to0}e^{x}\lim_{x\to0}\frac{e^{\tan x-x}-1}{\tan x-x}$$

$=1$

87. (0)

$$\lim_{n\to\infty}\frac{5^{n+1}+3^{n}-2^{2n}}{5^{n}+2^{n}+3^{2n+3}}$$

$$=\lim_{n\to\infty}\frac{5^{n}\left[5+\left(\frac{3}{5}\right)^{n}-\left(\frac{4}{5}\right)^{n}\right]}{9^{n}\left[27+\left(\frac{2}{9}\right)^{n}+\left(\frac{5}{9}\right)^{n}\right]}$$

$$=\lim_{n\to\infty}\left(\frac{5}{9}\right)^{n}\times\left(\frac{5}{27}\right)=0$$

88. (360)

$n(A)=4$ and $n(B)=6$

Let a, b, c, d be the elements of A

Number of way to select image of a from B = 6

Number of way to select image of b from B = 5

Number of way to select image of c from B = 4

Number of way to select image of d from B = 3

Total number of ways $=6\times5\times4\times3=360$

89. (252)

Here total ways will be like $x_i\ge1$ and $x_1+x_2+x_3+x_4+x_5+x_6=11$

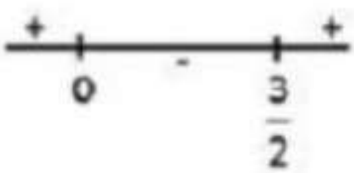
Thus no. of ways $={}^{6+5-1}C_5=252$

90. (3)

$$f(x)=\sqrt{\log_{10}\left(\frac{3-x}{x}\right)}$$

$$\log_{10}\left(\frac{3-x}{x}\right)\ge0$$

$\Rightarrow \frac{3-x}{x}\ge1$ $\Rightarrow \frac{3-2x}{x}\ge0$ $\Rightarrow \frac{2x-3}{x}\le0$



$\Rightarrow x\in\left(0,\frac{3}{2}\right]$