EXERCISE - 1 [A]

1. (A)
Since $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
And $\sin^{-1}\left(\frac{1}{x}\right) + \cos^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$
So $\sin^{-1} x + \cos^{-1} x + \sin^{-1}\left(\frac{1}{x}\right) + \cos^{-1}\left(\frac{1}{x}\right) = \pi$

2. (D)
Since domain of $\sin^{-1} x$ & $\cos^{-1} x$ is [-1, 1] but since x > 0
so 2π + x > 1 so the given terms is not defined

3. (D)
Given $\cos^{-1}(\frac{\pi}{3} + \sec^{-1}(-2)) = \cos^{-1}(\frac{\pi}{3} + \cos^{-1}\left(\frac{1}{-2}\right)) = \cos^{-1}(\frac{\pi}{3} + \frac{2\pi}{3}) = \cos^{-1}(\pi) = -1$

4. (A)
Given, $\sin^{-1}\left(\sin\left(\frac{23\pi}{7}\right)\right) + \cos^{-1}\left(\cos\left(\frac{39\pi}{7}\right)\right)$
Given $\sin^{-1}\left(\sin\left(\frac{23\pi}{7}\right)\right) = \sin^{-1}\left(\sin\left(2\pi + \frac{9\pi}{7}\right)\right) = \sin^{-1}\left(\sin\left(\frac{9\pi}{7}\right)\right) = -\frac{9\pi}{7} + \pi$
So $\cos^{-1}\left(\cos\left(\frac{39\pi}{7}\right)\right) = \cos^{-1}\left(\cos\left(4\pi + \frac{11\pi}{7}\right)\right) = \cos^{-1}\left(\cos\left(\frac{11\pi}{7}\right)\right) = -\frac{11\pi}{7} + 2\pi$
So $\sin^{-1}\left(\sin\left(\frac{23\pi}{7}\right)\right) + \cos^{-1}\left(\cos\left(\frac{39\pi}{7}\right)\right) = -\frac{9\pi}{7} + \pi - \frac{11\pi}{7} + 2\pi = \frac{\pi}{7}$

5. (D)
$\cos^{-1}\left(\cos\left(-\frac{17}{15}\pi\right)\right) = \cos^{-1}\left(\cos\left(\frac{17}{15}\pi\right)\right) = -\frac{17}{15}\pi + 2\pi = \frac{13\pi}{15}$

6. (C)
$sin\left(\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = sin\left(\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right) = \frac{1}{2}$

7. (A)
$\sin\left(\frac{\pi}{6} + sin^{-1}\left(-\frac{1}{2}\right)\right) = \sin\left(\frac{\pi}{6} - \frac{\pi}{6}\right) = 0$

8. (C)
$\tan\left(90^o - \cot^{-1}\left(\frac{1}{3}\right)\right) = \cot\left(\cot^{-1}\left(\frac{1}{3}\right)\right) = \frac{1}{3}$

9. (A)

$\sin\left(\cos^{-1}\frac{12}{13}\right)$ let $\cos^{-1}\left(\frac{12}{13}\right) = \theta, \cos\theta = \frac{12}{13}, so \sin\theta = \frac{5}{13}$

10. (D)

$\text{Sin}^{-1}\left(\cos\frac{33\pi}{5}\right) = \sin^{-1}\left(\cos\left(6\pi + \frac{3\pi}{5}\right)\right) = \sin^{-1}\left(\cos\frac{3\pi}{5}\right) = \sin^{-1}\left(\cos(\frac{\pi}{2} + \frac{\pi}{10}\right)$

$= \sin^{-1}\left(\sin\left(\frac{\pi}{10}\right)\right) = \frac{\pi}{10}$

11. (B)

Given $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$

So $\left(\frac{\pi}{2} - \cos^{-1} x\right) + \left(\frac{\pi}{2} - \cos^{-1} y\right) = \frac{2\pi}{3}$

Or $\cos^{-1} x + \cos^{-1} y = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

12. (B)

13. (C)

Here θ = 10 rad doesn't lie between $-\pi^2$ and π^2

But, 3π − θ lies between −π2 andπ2

Also, sin (3π − 10) = sin10

$\Longrightarrow \sin^{-1}(\sin 10) = \sin^{-1}(\sin(3\pi - 10)) = 3\pi - 10$

14. (B)

Let $y = \cos^{-1}\left(\sqrt{\frac{1+\cos x}{2}}\right) = \cos^{-1}\left(\sqrt{\frac{2\cos^2\left(\frac{x}{2}\right)}{2}}\right) = \cos^{-1}\left[\cos\left(\frac{x}{2}\right)\right] = \frac{x}{2}$

15. (A)

Let $\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3} = \theta \Rightarrow \cos 2\theta = \frac{\sqrt{5}}{3}$

But $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} \Rightarrow \frac{\sqrt{5}}{3} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\Rightarrow \sqrt{5} + \sqrt{5}\tan^2\theta = 3 - 3\tan^2\theta$

$\Rightarrow \left(\sqrt{5} + 3\right)\tan^2\theta = 3 - \sqrt{5} \Rightarrow \tan^2\theta = \frac{3 - \sqrt{5}}{3 + \sqrt{5}}$

$\Rightarrow \tan^2\theta = \frac{\left(3 - \sqrt{5}\right)^2}{4} \Rightarrow \tan\theta = \frac{3 - \sqrt{5}}{2}$

On rationalizing $\Rightarrow \tan\theta = \frac{3-\sqrt{5}}{2} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{2}{3+\sqrt{5}}$

16. (B)

Given expression is: $\tan^{-1}\left(\frac{a-b}{1+ab}\right)+\tan^{-1}\left(\frac{b-c}{1+bc}\right)$

Let, $y=\tan^{-1}\left(\frac{a-b}{1+ab}\right)+\tan^{-1}\left(\frac{b-c}{1+bc}\right)$ ...

We know that: $\tan^{-1}a-\tan^{-1}b=\tan^{-1}\left(\frac{a-b}{1+ab}\right)$ ...

So, applying equation (2) in equation (1) we get:

$$\begin{aligned}y&=(\tan^{-1}a-\tan^{-1}b)+(\tan^{-1}b-\tan^{-1}c)\\&=\tan^{-1}a-\tan^{-1}b+\tan^{-1}b-\tan^{-1}c\\&=\tan^{-1}a-\tan^{-1}c\end{aligned}$$

Therefore, the expression reduces to $\tan^{-1}a-\tan^{-1}\mathbf{c}$.

17. (C)

Given that, $\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{2}{3}=\sin^{-1}x$

Now using the identity, $\sin^{-1}a+\sin^{-1}b=\sin^{-1}\left[a\sqrt{(1-b^2)}+b\sqrt{((-a^2)}\right]$.

Here, $a=\frac{1}{3}, b=\frac{2}{3}$

Substituting values we get:

$$\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{2}{3}=\sin^{-1}\left[\frac{1}{3}\sqrt{\left(1-\frac{4}{9}\right)}+\frac{2}{3}\sqrt{\left(1-\frac{1}{9}\right)}\right]$$

$$=\sin^{-1}\left[\frac{1}{3}\sqrt{\frac{5}{9}}+\frac{2}{3}\sqrt{\frac{8}{9}}\right]=\sin^{-1}\left[\frac{1}{9}\sqrt{5}+\frac{4}{9}\sqrt{2}\right]=\sin^{-1}\left[\frac{\sqrt{5}+4\sqrt{2}}{9}\right]$$

18. (B)

Given $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\frac{2x+3x}{1-2x\times3x}=\frac{\pi}{4}$

$\Rightarrow \frac{5x}{1-6x^2}=1$

$\Rightarrow 6x^2+5x-1=0$

$\Rightarrow (6x-1)(x+1)=0$

$\Rightarrow x=\frac{1}{6},-1$

19. (A)

L.H.S

$= \cot^{-1}\left(\frac{xy+1}{x-y}\right) + \cot^{-1}\left(\frac{yz+1}{y-z}\right) + \cot^{-1}\left(\frac{zx+1}{z-x}\right)$

$= \tan^{-1}\left(\frac{x-y}{xy+1}\right) + \tan^{-1}\left(\frac{y-z}{yz+1}\right) + \tan^{-1}\left(\frac{z-x}{zx+1}\right)$

$= [\tan^{-1}x - \tan^{-1}y] + [\tan^{-1}y - \tan^{-1}z] + [\tan^{-1}z - \tan^{-1}x]$

(since $0 < xy, yz, zx < 1$)

$= 0$

$=$ RHS

20. (B)

$y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$

$y = \frac{\pi}{2} - \tan^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$

$y = \frac{\pi}{2} - 2\tan^{-1}(\sqrt{\cos x})$

$\frac{\pi}{2} - y = 2\tan^{-1}(\sqrt{\cos x})$

$\Rightarrow \cos\left(\frac{\pi}{2} - y\right) = \cos\left(2\tan^{-1}(\sqrt{\cos x})\right)$

Now apply, $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$ on R.H.S.

$$\sin y = \frac{1 - \tan^2\left[\tan^{-1}(\sqrt{\cos x})\right]}{1 + \tan^2\left[\tan^{-1}(\sqrt{\cos x})\right]}$$

$$\sin y = \frac{1 - \cos x}{1 + \cos x}$$

$$\sin y = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$\Rightarrow \sin y = \tan^2\frac{x}{2}$$

1. (C)
The above expression is true for
$\alpha = 1, \beta = 1$ and $\gamma = 1$
Since $\frac{-\pi}{2} \le \sin^{-1}(x) \le \frac{\pi}{2}$
Hence
$\alpha\beta + \beta\gamma + \gamma\alpha$
$= (1) + (1) + (1)$
$= 3$

2. (B)
$\frac{-2\pi}{5}$
$= -\sin^{-1}\left(\sin\left(\frac{2\pi}{5}\right)\right)$
$= \sin^{-1}\left(-\sin\left(\frac{2\pi}{5}\right)\right)$
$= \sin^{-1}\left(\sin\left(\pi + \frac{2\pi}{5}\right)\right)$
$= \sin^{-1}\left(\sin\left(\frac{7\pi}{5}\right)\right)$

3. (C)
$cos^{-1}\left(-sin\frac{7\pi}{6}\right) = cos^{-1}\left(\cos\frac{\pi}{3}\right) = \frac{\pi}{3}$

4. (C)
$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$
$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$
$= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$

5. (A)
Since $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$
$\tan^{-1}x + \cot^{-1}x + \sin^{-1}x = \frac{\pi}{2} + \sin^{-1}x$
As $-\frac{\pi}{2} \le \sin^{-1}\text{x} \le \frac{\pi}{2} \Rightarrow 0 \le \frac{\pi}{2} + \sin^{-1}\text{x} \le \pi$
$\Rightarrow 0 \le \tan^{-1}x + \cot^{-1}x + \sin^{-1}x \le \pi$
$\therefore \text{a} = \text{o}, \text{b} = \pi$

6. (A)
If $sin^{-1}x + tan^{-1}x = y(-1 < x < 1)$ then $y = \frac{3\pi}{2}$

7. (B)

We have

$\sin^{-1}x - \cos^{-1}x = \pi/6$

$\Rightarrow \sin^{-1}x + \cos^{-1}x - 2\cdot\cos^{-1}x = \pi/6$

$\Rightarrow \pi/2 - 2\cdot\cos^{-1}x = \pi/6$

$\Rightarrow -2\cdot\cos^{-1}x = \pi/6 - \pi/2$

$\Rightarrow -2\cdot\cos^{-1}x = \dfrac{\pi - 3\pi}{6}$

$\Rightarrow -2\cdot\cos^{-1}x = \dfrac{\pi}{6} - \dfrac{3\pi}{6}$

$\Rightarrow 2\cdot\cos^{-1}x = 2\pi/6$

$\Rightarrow \cos^{-1}x = \dfrac{2\pi}{6.2}$

$\Rightarrow \cos^{-1}x = \pi/6$

$\Rightarrow x = \cos\pi/6$

8. (C)

The formula

$\tan^{-1}[\tan(a)] = a$

Works for $a \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

In our case $\frac{5\pi}{7} > \frac{\pi}{2}$ but we can use the periodicity of tan:

$\tan\left(\dfrac{5\pi}{7}\right) = \tan\left(\dfrac{5\pi}{7} - \pi\right) = \tan\left(-\dfrac{2\pi}{7}\right)$

$\tan^{-1}\left[\tan\left(\dfrac{5\pi}{7}\right)\right] = \tan^{-1}\left[\tan\left(-\dfrac{2\pi}{7}\right)\right] = -\dfrac{2\pi}{7}$

9. (D)

The number of positive integral solutions of $cos^{-1}\left(4x^2 - 8x + \frac{7}{2}\right) = \frac{\pi}{3}$ is None of the above

10. (D)

Given $a\sin^{-1}\mathrm{x} - \mathrm{b}\cos^{-1}\mathrm{x} = \mathrm{c}$

$\Rightarrow a\sin^{-1}x - b\left(\dfrac{\pi}{2} - \sin^{-1}x\right) = c$

$\Rightarrow (a+b)\sin^{-1}x = c + \dfrac{b\pi}{2}$

$\Rightarrow \sin^{-1}x = \dfrac{2c + b\pi}{2(a+b)}$

$a\sin^{-1}x + b\cos^{-1}x = a\sin^{-1}x + b\left(\dfrac{\pi}{2} - \sin^{-1}x\right)$

$= (a-b)\sin^{-1}x + b\dfrac{\pi}{2}$

$= \dfrac{(a-b)(2c+b\pi)}{2(a+b)} + \dfrac{b\pi}{2}$

$= \dfrac{2c(a-b) + b\pi(a-b+a+b)}{2(a+b)}$

$= \dfrac{c(a-b) + ab\pi}{(a+b)}$

11. (B)

$$\cos^{-1}\left(\frac{1+x^2}{2x}\right)=\frac{\pi}{2}+\left(\sin^{-1}x+\cos^{-1}x\right)$$

$$\Rightarrow \quad \cos^{-1}\left(\frac{1+x^2}{2x}\right)=\pi$$

$$\Rightarrow \quad \left(\frac{1+x^2}{2x}\right)=\cos\pi=-1$$

$$\Rightarrow \quad x^2+1+2x=0$$

$$\Rightarrow \quad x=-1$$

12. (C)

We have $\cos^{-1}x+\cos^{-1}(2x)=-\pi$, which is not possible as $\cos^{-1}x$ and $\cos^{-1}2x$ never take negative values

13. (B)

The given equation is $ax^2+\sin^{-1}\left((x-1)^2+1\right)+\cos^{-1}\left((x-1)^2+1\right)=0$.

Now, $-1\le(x-1)^2+1\le 1 \Rightarrow x=1$

So, we have $a+\frac{\pi}{2}=0 \Rightarrow a=-\frac{\pi}{2}$

14. (C)

Put $\sin^{-1}\frac{5}{x}=A\Rightarrow\frac{5}{x}=\sin A$

$\sin^{-1}\frac{12}{x}=B\Rightarrow\frac{12}{x}=\sin B \qquad \Rightarrow A+B=\frac{\pi}{2}$

$$\Rightarrow \quad \sin A=\sin\left(\frac{\pi}{2}-B\right)=\cos B=\sqrt{1-\sin^2 B}$$

$$\Rightarrow \quad \frac{5}{x}=\sqrt{1-\frac{144}{x^2}} \quad \Rightarrow \frac{169}{x^2}=1$$

$\Rightarrow \quad x^2=169 \qquad \Rightarrow \quad x=13 \qquad [\because\ x=-13$ does not satisfy the given equation]

15. (D)

$$\sin\left(2\sin^{-1}(0.8)\right)=\sin\left(\sin^{-1}\left(2\times 0.8\sqrt{1-(0.8)^2}\right)\right)=\sin\left(\sin^{-1}0.96\right)=0.96$$

16. (B)

Let $x=\sin\theta$ where $-\frac{1}{2}\le x\le 1 \Rightarrow -\frac{\pi}{6}\le\theta\le\frac{\pi}{2}$

Then $f(x)=\sin^{-1}\left(\frac{\sqrt{3}}{2}x-\frac{1}{2}\sqrt{1-x^2}\right)$

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right)$$

$$= \sin^{-1}\left(\sin\left(\theta - \frac{\pi}{6}\right)\right)$$

$$= \theta - \frac{\pi}{6} = \sin^{-1}x - \frac{\pi}{6} \qquad \left[\because\ \theta - \frac{\pi}{6} \in \left(\frac{-\pi}{3}, \frac{\pi}{3}\right)\right]$$

17. (C)

$\sin^{-1}x = 2\sin^{-1}a$

Now $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$

$\Rightarrow \quad -\frac{\pi}{2} \le 2\sin^{-1}a \le \frac{\pi}{2}$

$\Rightarrow \quad -\frac{\pi}{4} \le \sin^{-1}a \le \frac{\pi}{4}$

$\Rightarrow \quad \frac{1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$

$\Rightarrow \quad |a| \le \frac{1}{\sqrt{2}}$

18. (C)

$$\sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right) = \tan^{-1}\left(\frac{\sqrt{r}-\sqrt{r+1}}{1+\sqrt{r(r-1)}}\right)$$

$$\Rightarrow \quad \sum_{r=1}^{n}\sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right) = \sum_{r=1}^{n}\left(\tan^{-1}\sqrt{r} - \tan^{-1}\sqrt{r-1}\right) = \tan^{-1}\sqrt{n}$$

19. (B)

$x = \sin(\theta+\beta)$ and $y = \sin(\theta-\beta)$

$\Rightarrow\ 1 + xy = 1 + \sin(\theta+\beta)\sin(\theta-\beta) = 1 + \sin^2\theta - \sin^2\beta = \sin^2\theta + \cos^2\beta$

20. (B)

$$\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right]$$

Let $\frac{1}{2}\cos^{-1}\frac{a}{b} = \theta \ \Rightarrow\ \cos 2\theta = \frac{a}{b}$

$$\tan\left[\frac{\pi}{4}+\theta\right] + \tan\left[\frac{\pi}{4}-\theta\right] = \frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta}$$

$$= \frac{1+\tan^2\theta + 2\tan\theta + 1 + \tan^2\theta - 2\tan\theta}{\left(1-\tan^2\theta\right)} = \frac{2}{\cos 2\theta} = \frac{2b}{a}$$

21. (D)

$$\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{3\sin 2x}{5+3\cos 2x}\right)=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{\frac{6\tan x}{1+\tan^2 x}}{5+\frac{3\left(1-\tan^2 x\right)}{1+\tan^2 x}}\right)$$

$$=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{6\tan x}{8+2\tan^2 x}\right)$$

$$=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{3\tan x}{4+\tan^2 x}\right)$$

$$=\tan^{-1}\left(\frac{\frac{\tan x}{4}+\frac{3\tan x}{4+\tan^2 x}}{1-\frac{3\tan^2 x}{4\left(4+\tan^2 x\right)}}\right)$$

$$=\tan^{-1}\left(\frac{16\tan x+\tan^3 x}{16+\tan^2 x}\right)$$

$$=\tan^{-1}\left(\tan x\right)=x$$

1. (0)

Let $y=\sin^{-1}\left(\frac{-1}{2}\right)$

We know that

$\sin^{-1}(-x)=-\sin^{-1}x$

$y=-\sin^{-1}\left(\frac{1}{2}\right)$ Since $\sin\frac{\pi}{6}=\frac{1}{2}$

$y=-\frac{\pi}{6}$ $\frac{\pi}{6}=\sin^{-1}\left(\frac{1}{2}\right)$

$\sin\left(\frac{\pi}{6}-\frac{\pi}{6}\right)=0$

2. (9)

$9\,cot\left(cot^{-1}\frac{1}{3}\right)$ = 9x 1/3 = 3

3. (18)

$sin\left(cos^{-1}\frac{12}{13}\right)=\ sin\left(sin^{-1}\frac{5}{13}\right)=\frac{5}{13}$

5 + 13 = 18

4. (0)

Given, $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$

This will happen only when $\sin^{-1}x=\sin^{-1}y=\sin^{-1}z=\frac{\pi}{2}$

Since $\sin^{-1}\text{x}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

then $=\sin\frac{\pi}{2}=1\Rightarrow x=y=z=1$

Hence desired value is $=1+1+1+-\frac{9}{1+1+1}=3-\frac{9}{3}=0$

5. (15)

$\sec^2(\tan^{-1}2)+\csc^2(\cot^{-1}3)$

$=1+\tan^2(\tan^{-1}2)+1+\cot^2(\cot^{-1}3)$

$=1+[\tan(\tan^{-1}2)]^2+1+[\cot(\cot^{-1}3)]^2=1+2^2+1+3^2=15$

6. (9)

$\sin^{-1}\sin 15+\cos^{-1}\cos 20+\tan^{-1}\tan 25=30-9\pi$

So k = 9

7. (1)

$\cos^{-1}\left(\frac{3}{5}\right)-\sin^{-1}\left(\frac{4}{5}\right)=\cos^{-1}\text{x}$

or, $\frac{\pi}{2}-\sin^{-1}\left(\frac{3}{5}\right)-\sin^{-1}\left(\frac{4}{5}\right)=\cos^{-1}\text{x}$

or, $\frac{\pi}{2}-\sin^{-1}\left(\frac{3}{5}\cdot\frac{3}{5}+\frac{4}{5}\cdot\frac{4}{5}\right)=\cos^{-1}\text{x}$

or, $\frac{\pi}{2}-\sin^{-1}1=\cos^{-1}\text{x}$

or, $\cos^{-1}\text{x}=0$

or, x = 1.

8. (0)

$\theta=\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}}+\tan^{-1}\sqrt{\frac{b(a+b+c)}{ac}}+\tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$

$a=\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}}, \beta=\tan^{-1}\sqrt{\frac{b(a+b+c)}{ac}}, Y=\tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$

$\tan\alpha=\sqrt{\frac{a(a+b+c)}{bc}}, \tan\beta=\sqrt{\frac{b(a+b+a)}{ac}},\ \tan\gamma=\sqrt{\frac{c(a+b+c)}{ab}}$

$\tan\alpha+\tan\beta+\tan\gamma=\sqrt{\frac{a(a+b+c)}{bc}}+\sqrt{\frac{b(a+b+c)}{ac}}+\sqrt{\frac{c(a+b+c)}{ab}}$

$=\frac{(a+b-c)^{\frac{1}{2}}}{\sqrt{3bc}}$

$=\tan\alpha\tan\beta\tan\gamma$

$=\tan\theta=\left[\frac{(\tan\alpha+\tan\beta+\tan\gamma)-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\beta\tan\gamma-\tan\gamma\tan\alpha}\right]$

$=\tan\theta=0$

9. (2)

$3\sqrt{5}\,tan\left\{\left(cos^{-1}\left(-\frac{2}{7}\right)-\frac{\pi}{2}\right)\right\}=\ -3\sqrt{5}\,tan\left\{\left(sin^{-1}\left(\frac{2}{7}\right)\right)\right\}=\ -3\sqrt{5}\,tan\left\{\left(tan^{-1}\left(\frac{2}{3\sqrt{5}}\right)\right)\right\}=$

= 2

10. (7)

$sin^{-1}\left(-\frac{1}{2}\right)=\ -\frac{\pi}{6}$

$tan^{-1}(1)=\frac{\pi}{4}$

$cos^{-1}\left(\cos(-\frac{\pi}{2})\right)=\frac{\pi}{2}$

$sin^{-1}\left(-\frac{1}{2}\right)+tan^{-1}(1)+cos^{-1}\left(\cos(-\frac{\pi}{2})\right)=\ \frac{7\pi}{12}$

k = 7

11. (2)

$sin\left[cot^{-1}\left(cot\frac{17\pi}{3}\right)\right]=\ \frac{\sqrt{3}}{2}\ ,k=2$

12. (20)

$sin^{-1}x_i=\frac{\pi}{2}\ so\ x_i=1$

So $\sum_{i=1}^{20}x_i=20$

13. (2)

Here, $\tan^{-1}x+\cos^{-1}\frac{y}{\sqrt{1-y^2}}=\sin^{-1}\frac{3}{\sqrt{10}}$

or $\tan^{-1}x+\tan^{-1}\left(\frac{1}{y}\right)=\tan^{-1}(3)$

or $\tan^{-1}\left(\frac{1}{y}\right)=\tan^{-1}3-\tan^{-1}(x)$

or $\tan^{-1}\left(\frac{1}{y}\right)=\tan^{-1}\left(\frac{3-x}{1+3x}\right)$

or $y=\frac{1+3x}{3-x}$

14. (0)

We have, $\sin^{-1}\left\{\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}\frac{\sqrt{12}}{4}+\sec^{-1}\sqrt{2}\right)\right\}$

$=\sin^{-1}\left\{\cot\left(\sin^{-1}\frac{\sqrt{3}-1}{2\sqrt{2}}+\cos^{-1}\frac{\sqrt{3}}{2}+\sec^{-1}\sqrt{2}\right)\right\}$

$=\sin^{-1}\left\{\cot\left(\frac{\pi}{12}+\frac{\pi}{6}+\frac{\pi}{4}\right)\right\}$

$=\sin^{-1}\left(\cot\frac{\pi}{2}\right)$

$=\sin^{-1}0=0$

15. (2)

We know that $\cos^{-1}x\in[0,\pi]$

$\therefore\cos^{-1}(a)+\cos^{-1}(b)+\cos^{-1}(c)=3\pi$ is possible iff $a=b=c=-1$

Now, $f(1)=2$ and $f(x+y)=f(x)\cdot f(y)$

Put $x=y=1$, we get

$f(2)=f(1)\cdot f(1)=4$

Put $x=2,y=1$, we get

$f(3)=f(2)\cdot f(1)=4\times 2=8$

$\therefore a^{2f(1)}+b^{2f(2)}+c^{2f(3)}+\frac{a+b+c}{a^{2f(1)}+b^{2f(2)}+c^{2f(3)}}$

$=1+1+1-\frac{3}{1+1+1}$

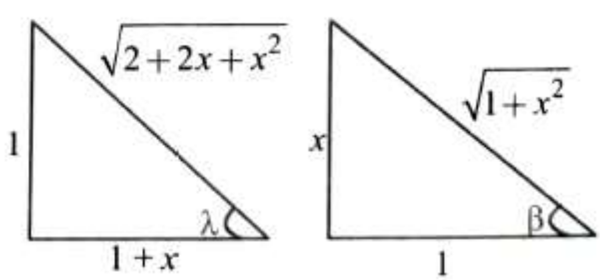
$=2$

1. (A)
Since, x, y, z are in A.P.
Also, we have
$2\tan^{-1} y = \tan^{-1} x + \tan^{-1}(z)$
$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$
$\Rightarrow \frac{x+z}{1-y^2} = \frac{x+z}{1-xz} \quad (\because 2y = x+z)$
$\Rightarrow y^2 = xz$ or $x+z=0$
$\Rightarrow x = y = z = 0$

2. (C)
Consider, $\tan^{-1}\left[\cot\frac{43\pi}{4}\right] = \tan^{-1}\left[\cot\left(10\pi + \frac{3\pi}{4}\right)\right]$
$= \tan^{-1}\left[\cot\frac{3\pi}{4}\right] \quad [\because \cot(2n\pi+\theta) = \cot\theta]$
$= \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)\right] = \frac{\pi}{2} - \frac{3\pi}{4}$
$= \frac{2\pi - 3\pi}{4} = \frac{-\pi}{4}$

3. (C)
Given that, $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left[\frac{2x}{1-x^2}\right]$
$= \tan^{-1} x + 2\tan^{-1} x = 3\tan^{-1} x$
$\tan^{-1} y = \tan^{-1}\left[\frac{3x - x^3}{1-3x^2}\right] \Rightarrow y = \frac{3x-x^3}{1-3x^2}$

4. (A)
$\sin\left[\cot^{-1}(1+x)\right] = \cos\left(\tan^{-1} x\right)$



Let $\cot\lambda = 1+x, \tan\beta = x$
$\Rightarrow \sin\lambda = \cos\beta$

$$\Rightarrow \frac{1}{\sqrt{x^2+2x+2}}=\frac{1}{\sqrt{1+x^2}}$$

$$\Rightarrow x^2+2x+2=x^2+1 \Rightarrow x=-\frac{1}{2}$$

5. (A)

Let $x^2=\cos 2\theta \Rightarrow \theta=\frac{1}{2}\cos^{-1}x^2$

$$\Rightarrow \tan^{-1}\left[\frac{\sqrt{1+x^2}}{\sqrt{1+x^2}}-\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}\right]=\left[\frac{\sqrt{1+\cos 2\theta}}{\sqrt{1+\cos 2\theta}}-\frac{\sqrt{1-\cos 2\theta}}{\sqrt{1-\cos 2\theta}}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{1+\tan\theta}{1-\tan\theta}\right]=\tan^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right]$$

$$=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$$

6. (D)

$\because \cos\alpha=\frac{3}{5}$, then $\sin\alpha=\frac{4}{5}$

$\Rightarrow \tan\alpha=\frac{4}{3}$ and $\tan\beta=\frac{1}{3}$

$$\because \tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\cdot\tan\beta}$$

$$=\frac{\frac{4}{3}-\frac{1}{3}}{1+\frac{4}{9}}=\frac{1}{\frac{13}{9}}=\frac{9}{13}$$

$$\therefore \alpha-\beta=\tan^{-1}\left(\frac{9}{13}\right)=\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)=\cos^{-1}\left(\frac{13}{5\sqrt{10}}\right)$$

7. (D)

$$x=\sin^{-1}(\sin 10)$$

$$\Rightarrow x=3\pi-10 \qquad \begin{cases} 3\pi-\frac{\pi}{2}<10<3\pi+\frac{\pi}{2} \\ \Rightarrow 3\pi-x=10 \end{cases}$$

$$\text{and } y=\cos^{-1}(\cos 10) \qquad \begin{cases} 3\pi<10<4\pi \\ \Rightarrow 4\pi-x=10 \end{cases}$$

$$\Rightarrow y=4\pi-10$$

$$\therefore y-x=(4\pi-10)-(3\pi-10)=\pi$$

8. (C)

$\therefore\ f(x)=\sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$

$\therefore\ -1\le\frac{|x|+5}{x^2+1}\le 1$

$\Rightarrow |x|+5\le x^2+1 \qquad [\because\ x^2+1\ne 0]$

$\Rightarrow x^2-|x|-4\ge 0$

$\Rightarrow \left(|x|-\frac{1-\sqrt{17}}{2}\right)\left(|x|-\frac{1+\sqrt{17}}{2}\right)\ge 0$

$\Rightarrow x\in\left(-\infty,-\frac{1+\sqrt{17}}{2}\right]\cup\left[\frac{1+\sqrt{17}}{2},\infty\right)$

$\therefore\ a=\frac{1+\sqrt{17}}{2}$

9. (C)

$-1\le\frac{x-1}{x+1}\le 1 \Rightarrow \frac{x-1}{x+1}\le 1$

$\Rightarrow \frac{2}{x+1}\ge 0 \Rightarrow x\in(-1,\infty)$ ….. (i)

and $\frac{x-1}{x+1}\ge -1 \Rightarrow \frac{2x}{x+1}\ge 0 \Rightarrow x\in(-\infty,-1)\cup[0,\infty)$ ….. (ii)

from (i) and (ii), $x\in[0,\infty)$ ….. (iii)

Now, $-1\le\frac{3x^2+x-1}{(x-1)^2}\le 1 \Rightarrow \frac{3x^2+x-1}{(x-1)^2}\le 1$

$\Rightarrow \frac{2x^2+3x-2}{(x-1)^2}\le 0$

$\Rightarrow \frac{(2x-1)(x+2)}{(x-1)^2}\le 0 \Rightarrow x\in\left[-2,\frac{1}{2}\right]$ ….. (iv)

and $\frac{3x^2+x-1}{(x-1)^2}\ge -1 \Rightarrow \frac{x(4x-1)}{(x-1)^2}\ge 0$

$\Rightarrow x\in(-\infty,0]\cup\left[\frac{1}{4},\infty\right)$ ….. (v)

From (iv) and (v); $x\in[-2,0]\cup\left[\frac{1}{4},\frac{1}{2}\right]$ ….. (vi)

From (iii) and (vi); $x\in\{0\}\cup\left[\frac{1}{4},\frac{1}{2}\right]$

10. (A)

Let $\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}=\theta \Rightarrow \sin 4\theta=\frac{\sqrt{63}}{8}$

or $\cos 4\theta=\frac{1}{8}$ $\left[\because \cos\theta=\sqrt{1-\sin^2\theta}\right]$

$\Rightarrow 2\cos^2 2\theta-1=\frac{1}{8}$ $\left[\because \cos 2\theta=2\cos^2\theta-1\right]$

$\Rightarrow \cos^2 2\theta=\frac{9}{16} \Rightarrow \cos 2\theta=\frac{3}{4}$

$\Rightarrow 2\cos^2\theta-1=\frac{3}{4} \Rightarrow \cos^2\theta=\frac{7}{8} \Rightarrow \cos\theta=\frac{\sqrt{7}}{2\sqrt{2}}$

$\therefore \tan\theta=\frac{1}{\sqrt{7}}$ $[\because \sin\theta=\sqrt{\sec^2\theta}-1]$

11. (A)

$0\le x^2-x+1\le 1$

$\Rightarrow x^2-x\le 0 \Rightarrow x\in[0,1]$

Also, $0<\sin^{-1}\left(\frac{2x-1}{2}\right)\le\frac{\pi}{2}$

$\Rightarrow 0<\frac{2x-1}{2}\le 1 \Rightarrow 0<2x-1\le 2$

$1<2x\le 3 \Rightarrow \frac{1}{2}<x\le\frac{3}{2}$

From (i) and (ii), we get

$x\in\left(\frac{1}{2},1\right] \Rightarrow \alpha=\frac{1}{2},\beta=1$

So, $\alpha+\beta=\frac{3}{2}$

12. (C)

$f(x)=\sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$

Now, take domain of $\sin^{-1}$

$\frac{x^2-3x+2}{x^2+2x+7}\ge -1$ and $\frac{x^2-3x+2}{x^2+2x+7}\le 1$

$2x^2-x+9\ge 0$ and $5x\ge -5 \Rightarrow x\ge -1$

$x\in R$

Hence, Domain $x\in[-1,\infty)$.

13. (D)

$$-1 \le \frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi} \le 1$$

$$-\frac{\pi}{2} \le \sin^{-1}\frac{1}{4x^2-1} \le \frac{\pi}{2}$$

On solving inequalities we get

Always $-1 \le \frac{1}{4x^2-1} \le 1$

$$x \in \left(-\infty, \frac{-1}{\sqrt{2}}\right) \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$$

14. (B)

$$\left|\frac{x^2-4x+2}{x^2+3}\right| \le 1$$

$$\Rightarrow \left(x^2-4x+2\right)^2 \le \left(x^2+x^3\right)^2$$

$$\Rightarrow \left(x^2-4x+2\right)^2 - \left(x^2+3\right)^2 \le 0$$

$$\Rightarrow \left(2x^2-4x+5\right)\left(-4x-1\right) \le 0$$

$$-4x-1 \le 0 \Rightarrow x \ge \frac{-1}{4}$$

15. (A)

We are given that

$$\cos^{-1}x - 2\sin^{-1}x = \cos^{-1}2x \qquad \text{...A}$$

$$\Rightarrow \cos^{-1}x - 2\left(\frac{\pi}{2} - \cos^{-1}x\right) = \cos^{-1}2x$$

$$\Rightarrow \cos^{-1}x - \pi + 2\cos^{-1}x = \cos^{-1}2x$$

$$\Rightarrow 3\cos^{-1}x = \pi + \cos^{-1}2x \qquad \text{... (i)}$$

$$\Rightarrow \cos\left(3\cos^{-1}x\right) = \cos\left(\pi + \cos^{-1}2x\right) \qquad \left[\because 3\cos^{-1}x = \cos^{-1}\left(4x^3-3x\right)\right]$$

$$\Rightarrow 4x^3 - 3x = -2x$$

$$\Rightarrow 4x^3 = x \Rightarrow x = 0, \pm\frac{1}{2}$$

Here all values of x satisfy the eqn. (A)

$\therefore$ Sum of all the solutions of the eqn. $= -\frac{1}{2} + \frac{1}{2} + 0 = 0$

16. (C)

Given functions is

$$f(x) = \sin^{-1}\left[2x^3-3\right] + \log_2\left(\log_{\frac{1}{2}}\left(x^2-5x+5\right)\right)$$

Take angle of $\sin^{-1}$ as T_1. Which lies between –1 and 1

$T_1 : -1 \le \left[2x^2 - 3\right] < 1$

$\Rightarrow\ -1 \le 2x^2 - 3 < 2\ \Rightarrow\ 2 < 2x^2 < 5$

$\Rightarrow\ 1 < x^2 < \frac{5}{2}\ \Rightarrow\ T_1 : x \in \left(-\frac{5}{2}, -1\right) \cup \left(1, \frac{5}{2}\right)$

Similarly,

$T_2 : x^2 - 5x + 5 > 0$

$\Rightarrow\ \left(x - \left(\frac{5-\sqrt{5}}{2}\right)\right)\left(x - \left(\frac{5+\sqrt{5}}{2}\right)\right) > 0$

$T_3 : \log_{\frac{1}{2}}\left(x^2 - 5x + 5\right) > 0$

$\Rightarrow\ x^2 - 5x - 5 < 1$

$\Rightarrow\ x^2 - 5x + 4 < 0$

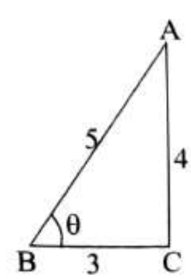
$\Rightarrow\ T_3 : x \in (1, 4)$

Now, take intersection of T_1, T_2, &T_3,

$T_1 \cap T_2 \cap T_3 = \left(1, \frac{5-\sqrt{5}}{2}\right)$

17. (C)

Let $\tan^{-1}\frac{4}{3} = \theta\ \Rightarrow\ \tan\theta = \frac{4}{3}$



$\Rightarrow\ \cos^{-1}\left(\frac{3}{10}\cos\theta + \frac{2}{5}\sin\theta\right)$

$= \cos^{-1}\left(\frac{3}{10} \times \frac{3}{5} + \frac{2}{5} \cdot \frac{4}{5}\right)$

$= \cos^{-1}\left(\frac{9}{50} + \frac{8}{25}\right) = \cos^{-1}\left(\frac{25}{50}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

18. (D)

Given function domain is [–1, 1].

$-1 \le \frac{x^2 - 5x + 6}{x^2 - 9} \le 1$

Take maximum value and subtract l both sides.

$\frac{x^2 - 5x + 6}{x^2 - 9} - 1 \le 0,\ \frac{1}{x+3} \ge 0$

$x \in (-3, \infty)$ ….. (i)

Take minimum value and add 1 both sids.

$\frac{x^2 - 5x + 6}{x^2 - 9} + 1 \ge 0,\ \frac{2x+1}{x+3} \ge 0$

$x \in (-\infty, -3) \cup \left[-\frac{1}{2}, \infty\right)$ ….. (ii)

Now, take the intersection of two equations (i) and (ii)

$x \in \left[-\frac{1}{2}, \infty\right)$

Now, take $x^2 - 3x + 2 > 0,\ x \in (-\infty, 1) \cup (2, \infty)$

$x^2 - 3x + 2 \neq 1,\ x \neq \frac{3 \pm \sqrt{5}}{2}$

Take intersection of all the solutions.

$\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

19. (B)

Given line is $\frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta} = k$

$\sin^{-1} x = k\alpha \Rightarrow \cos^{-1} x = k\beta$

$k = \frac{\pi}{2(\alpha+\beta)}$

$\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right) = \sin\left(4\sin^{-1} x\right)$

$2\sin\left(2\sin^{-1} x\right)\cos\left(2\sin^{-1} x\right)$

$= 4x\sqrt{1-x^2}\left(1-2x^2\right)$

20. (A)

We have $f(x) = \sin^{-1} 2x + \sin 2x + \cos^{-1} 2x + \cos 2x$

$= \sin(2x) + \cos(2x) + \frac{\pi}{2}$

Now, $f(0) = 1 + \frac{\pi}{2}(m)$ and $f\left(\frac{\pi}{8}\right) = \sqrt{2} + \frac{\pi}{2}(M)$

Now, $m + M = 1 + \sqrt{2} + \pi$

21. (A)

Given equation is

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = 2\pi - \frac{7\pi}{3} = \frac{5\pi}{6}$

$tab^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}(-1) = \tan^{-1}\left(\tan\left(-\frac{\pi}{4}\right)\right) = -\frac{\pi}{4}$

$\Rightarrow \sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$$=\frac{\pi}{3}+\frac{5\pi}{6}-\frac{\pi}{4}=\frac{4\pi+10\pi-3\pi}{12}=\frac{11\pi}{12}$$

22. (A)

Take function $\tan^{-1}\frac{1}{1+n+n^2}=\tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$

$=\tan^{-1}(n+1)-\tan^{-1}n$

So, $\sum_{n=1}^{50}\left(\tan^{-1}(n+1)-\tan^{-1}n\right)$

$=\tan^{-1}51-\tan^{-1}1$

Take cot both sides,

$$\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)=\cot\left(\tan^{-1}51+\tan^{-1}1\right)$$

$$=\frac{1}{\tan\left(\tan^{-1}51-\tan^{-1}1\right)}=\frac{1+51\times1}{51-1}=\frac{52}{50}=\frac{26}{25}$$

23. (D)

$$\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sin\left(\cos^{-1}x\right)-x}{1-\tan\left(\cos^{-1}x\right)}$$

Let $x=\cos y$

$\Rightarrow \quad \cos^{-1}(x)=\sin^{-1}\sqrt{1-x^2}$

$$\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sin\left(\sin^{-1}\sqrt{1-x^2}\right)-x}{1-\tan\left(\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sqrt{1-x^2}-x}{1-\left(\frac{\sqrt{1-x^2}}{x}\right)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}}(-x)=-\frac{1}{\sqrt{2}}$$

24. (B)

Given equation is $\tan^{-1}\left[\dfrac{\cos\left(\dfrac{15\pi}{4}\right)-1}{\sin\dfrac{\pi}{4}}\right]$

$$\tan^{-1}\left[\frac{\cos\left(4\pi-\frac{\pi}{4}\right)-1}{\sin\frac{\pi}{4}}\right] \quad \Rightarrow \quad \tan^{-1}\left(\frac{\cos\frac{\pi}{4}-1}{\sin\frac{\pi}{4}}\right)$$

$$\tan^{-1}\left(\frac{1-\sqrt{2}}{1}\right)=-\frac{\pi}{8}$$

25. (130)

Given curves are

$$x=\sin\left(2\tan^{-1}\alpha\right)=\frac{2\alpha}{1+\alpha^2} \text{ and } y=\sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)=\sin\left(\sin^{-1}\frac{1}{\sqrt{5}}\right)=\frac{1}{\sqrt{5}}$$

We have set with relation $y^2=1-x$

$\dfrac{1}{5}=1-\dfrac{2\alpha}{1+\alpha^2}\ldots$ {from value of x and y)

$$\Rightarrow 1+\alpha^2=5+5\alpha^2-10\alpha \Rightarrow 2\alpha^2-5\alpha+2=0$$

So, $\alpha=2\cdot\dfrac{1}{2}$

Take, $\sum\limits_{a\in S}16\alpha^3=16\times 2^3+16\times\dfrac{1}{2^3}=130$

26. (29)

$$50\tan\left(3\tan^{-1}\left(\frac{1}{2}\right)+2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}\left(2\sqrt{2}\right)\right)$$

$$=50\tan\left(\tan^{-1}\left(\frac{1}{2}\right)+2\left(\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}(2)\right)\right)+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}\left(2\sqrt{2}\right)\right)$$

$$=50\tan\left(\tan^{-1}\left(\frac{1}{2}\right)+2\cdot\frac{\pi}{2}\right)+4\sqrt{2}\times\frac{1}{\sqrt{2}}$$

$$=50\left(\tan\left(\tan^{-1}\left(\frac{1}{2}\right)\right)\right)+4=25+4=29$$