1 (A) Solutions

Inequation

1) $\frac{2}{x} < 3 \Rightarrow 3 - \frac{2}{x} > 0 \Rightarrow \frac{3x-2}{x} > 0$

By wavy curve $x \in (-\infty, 0) \cup (2/3, \infty)$ — (C)

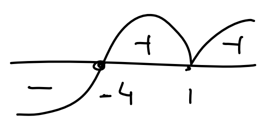
2) $\frac{x+4}{x-3} < 2 \Rightarrow \frac{x+4}{x-3} - 2 < 0 \Rightarrow \frac{10-x}{x-3} < 0$

OR $\frac{x-10}{x-3} > 0 \Rightarrow x \in (-\infty, 3) \cup (10, \infty)$ — (A)

3) $\frac{2x-3}{3x-5} \geq 3 \Rightarrow \frac{2x-3}{3x-5} - 3 \geq 0 \Rightarrow \frac{-7x+12}{3x-5} \geq 0$

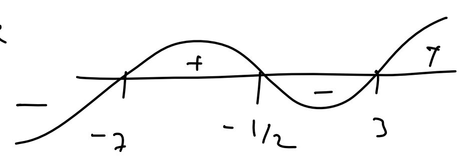
OR $\frac{7x-12}{3x-5} \leq 0 \Rightarrow x \in \left(\frac{5}{3}, \frac{12}{7}\right]$ — (B)

4) $(x-1)^2(x+4) < 0$ Wavy curve



Hence $x \in (-\infty, -4)$ — (B)

5) $(2x+1)(x-3)(x+7) < 0$ wavy curve



Solⁿ $(-\infty, -7) \cup (-\frac{1}{2}, 3)$ — (A)

6) $x^2 + 6x - 27 > 0 \Rightarrow (x+9)(x-3) > 0 \Rightarrow x \in (-\infty, -9) \cup (3, \infty)$

Also $x^2 - 3x - 4 < 0 \Rightarrow (x-4)(x+1) < 0 \Rightarrow x \in (-1, 4)$

taking intersection $x \in (3, 4)$ — (c)

7) $x^2 - 1 \leq 0 \Rightarrow x \in [-1, 1]$

Also $x^2 - x - 2 \geq 0 \Rightarrow (x-2)(x+1) \geq 0$

or $x \in (-\infty, -1] \cup [2, \infty)$

taking intersection $x \in \{-1\}$ — (D)

(8) $\log_{\frac{1}{2}}(x^2 - 5x + 7) > 0$

(i) Domain : $(-\infty, \infty)$ (ii) $x^2 - 5x + 7 < \left(\frac{1}{2}\right)^0$

or $x^2 - 5x + 6 < 0$

or $x \in (2, 3)$

their intersection is $(2, 3)$ — (B)

(9) $\log_3(x^2 - 6x + 11) < 1$

(i) Domain $(-\infty, \infty)$ (ii) $x^2 - 6x + 11 < (3)^1$

$\Rightarrow x^2 - 6x + 8 < 0$

$\Rightarrow x \in (2, 4)$

Intersec^n $(2, 4)$ — (B)

(10) $\log_{|x|}(x^2 + x + 1) \geq 0$

Domain : $x \in R - \{0, \pm 1\}$

Case I $0 < |x| < 1$

$x^2 + x + 1 \leq 1$

$\Rightarrow x(x+1) \leq 0$

Case II $|x| > 1$

$x^2 + x + 1 \geq 1$

$x(x+1) \geq 0$

$\Rightarrow \quad x(x+1) \le 0$

$x \in [-1, 0]$

Intersecⁿ $\Rightarrow S_1 = \boxed{x \in (-1, 0)}$

$x(x+1) \ge 0$

$x \in (-\infty, -1] \cup [0, \infty)$

Intersecⁿ $\Rightarrow \boxed{S_2 = (-\infty, -1) \cup (1, \infty)}$

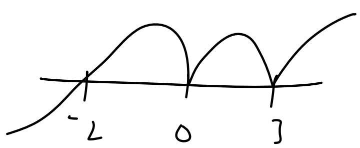
final Ans $S_1 \cup S_2$ or $x \in (-\infty, -1) \cup (-1, 0) \cup (1, \infty)$

— (D)

(11) $x(e^x - 1)(x+2)(x-3)^2 \le 0$

same as $x^2(x+2)(x-3)^2 \le 0$

wavy curve



$\therefore$ Hence $x \in (-\infty, -2] \cup \{0, 3\}$

— (C)

(12) $\left|\frac{x^2}{x-1}\right| \le 1 \Rightarrow x^2 \le |x-1|$

Case I $\quad x > 1$

$x^2 - x + 1 \le 0$

$x \in \phi$

Case II $\quad x < 1$

$x^2 + x - 1 \le 0$

$x \in \left[\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right]$

$\therefore$ Solⁿ $x \in \left[\frac{-1-\sqrt{5}}{2}, \frac{\sqrt{5}-1}{2}\right]$ — (B)

Definition of function

(13) option (c) matches the Condition, rest of the options are relation

(14) option (c) as only $x \geq 0$ is allowed but domain is given Real

Even & odd

(15) Conceptual — (D)

(16) Numeration Even, denominator odd

Hence $f(x)$ is odd (A)

(17) $f(x) = \cos\left(\log\left(\sqrt{1+x^2}+x\right)\right)$

$f(-x) = \cos\left(\log\left(\sqrt{1+x^2}-x\right)\right)$

$= \cos\left(\log\left(\frac{1}{\sqrt{1+x^2}+x}\right)\right)$ Rationalize

$= \cos\left(-\log\left(x+\sqrt{1+x^2}\right)\right)$

$= \cos\left(\log\left(x+\sqrt{1+x^2}\right)\right) = f(x) \Rightarrow$ Even (A)

(18) A & B both odd functions

C is neither even nor odd Hence (D)

(19) (A) $f(x) = x\left(\frac{a^x-1}{a^x+1}\right)$

$f(-x) = -x\left(\frac{a^{-x}-1}{a^{-x}+1}\right) = -x\left(\frac{1-a^x}{1+a^x}\right) = x\left(\frac{a^x-1}{a^x+1}\right) = f(x)$

Hence (A)

(20) clearly (B)

(21) As $f(-x) = f(x) \Rightarrow$ Even (C)

Periodic func^n

(22) Since $f(\pi/2 + x) = f(x) \Rightarrow$ fundamental period $\pi/2$
— (B)

(23) Since $f(\pi/2 + x) = f(x) \Rightarrow$ F.P. is $\pi/2$ — (B)

(24) Since $f(x+4) = f(x) \Rightarrow$ F.P is $\pi/4$ — (A)

(25) Since $f(\pi + x) = f(x) \Rightarrow$ F.P is π $\rightarrow$ (B)

(26) period is $\frac{T}{|k|} = 6\pi$ (A)

(27) (C) Conceptual

Domain, Co-domain & Range

(28) $x+2 > 0 \Rightarrow (-2, \infty)$ — (B)

(29) $f(x) = g(x) \Rightarrow 2x^2 - 1 = 1 - 3x$

$\Rightarrow 2x^2 + 3x - 2 = 0$

$\Rightarrow (2x-1)(x+2) = 0$

$x \in \{-2, 1/2\}$ — (D)

(30) $\frac{3-x}{2} > 0 \Rightarrow x \in (-\infty, 3)$ — (B)

(31) $\cos^{-1}(4x-1) \Rightarrow -1 \leq 4x - 1 \leq 1$

(31) $\cos^{-1}(4x-1) \Rightarrow -1 \le 4x-1 \le 1$

$\Rightarrow 0 \le 4x \le 2$

$\Rightarrow 0 \le x \le \frac{1}{2}$ — (B)

(32) $\log|x^2-9| \Rightarrow |x^2-9| > 0$

Ans $x \in R - \{3,-3\}$ — (C)

(33) $f(x) = \sqrt{x-1} + \sqrt{6-x} \Rightarrow x-1 \geq 0$ & $6-x \geq 0$

Intersecⁿ $x \in [1,6]$ — (B)

(34) $f(x) = \sqrt{2-2x-x^2} \Rightarrow 2-2x-x^2 \geq 0$

$\Rightarrow x^2+2x-2 \le 0$

$x \in [-1-\sqrt{3}, -1+\sqrt{3}]$ — (B)

(35) $f(x) = \sin^{-1} 5x \Rightarrow -\frac{1}{5} \le x \le \frac{1}{5}$ — (B)

(26) Conceptual — (B)

(37) $f(x) = \sin \frac{\pi[x]}{2}$ Basically $\rightarrow \sin \frac{n\pi}{2}$ $n \in$ Integer

$\therefore$ Range $\{-1,0,1\}$ — (B)

(38) $f(x) = \begin{cases} 1 & \text{if } x > 3 \\ -1 & \text{if } x < 3 \end{cases}$

Hence (B)

(39) Conceptual — (D)

(40) $-1 \le \sin x \le 1$

$\Rightarrow -7 \le -7\sin x \le 7$

$\Rightarrow 2 \le 9 - 7\sin x \le 16$ Ans (B)

(41) $-1 \le \sin 3x \le 1$

$\Rightarrow 1 \le 2 - \sin 3x \le 3$

$\Rightarrow 1 \ge \dfrac{1}{2-\sin 3x} \ge \dfrac{1}{3}$ Ans (A)

(42) Conceptual (C)

(43) put $f(x) = -1 \Rightarrow x^2 = -1 \Rightarrow x \in \phi$ — (C)

(44) $f(x) = \cos 2x - \sin 2x \Rightarrow$ Range $[-\sqrt{2}, \sqrt{2}]$

Its subset is (B)

(45) $f(x) = \dfrac{+\cancel{x}}{\cancel{x}} = 1$ as $x \in [3,7]$ $\therefore$ option (C)

(46) $f(x) = \dfrac{1}{\sqrt{x - [x]}}$ $\therefore x - [x] > 0$

We know $x - 1 < [x] \le x \quad \forall x \in R$

i.e. $x \ge [x] \quad \forall x \in R$

Hence for $x > [x]$ $\boxed{x \in R - Z}$ (B)

(47) $f(x) = 2 + x - [x-3]$

$= 5 + (x-3) - [x-3] = 5 + \{x-3\} \Rightarrow f(x) \in [5, 6)$

(B)

(48) $f(x+y) = f(x) f(y) \Rightarrow f(x) = a^x$

Also $f(1) = 2 \Rightarrow a = 2 \quad \therefore f(x) = 2^x$

Then $\sum_{k=1}^{n} f(a+k) = f(a+1) + f(a+2) + \dots + f(a+n)$

$= 2^{x+1} + 2^{x+2} + \dots + 2^{x+n}$

$= 2^{x+1}(2^n - 1) \Rightarrow x+1 = 4 \Rightarrow x = 3$ — (C)

(49) Conceptual (C)

(50) $f(x+2) = \dfrac{(x+1)(x+2)}{2} = \dfrac{x+2}{x} \cdot \dfrac{x(x+1)}{2}$

$= \dfrac{x+2}{x} \cdot f(x+1)$ — (B)

(51) $f(x+ay, x-ay) = axy$ let $x + ay = X$

$x - ay = Y$

$\Rightarrow x = \dfrac{X+Y}{2}, \quad y = \dfrac{X-Y}{2a}$

Hence $f(X, Y) = a\left(\dfrac{X+Y}{2}\right)\left(\dfrac{X-Y}{2a}\right) : \underline{X^2 - Y^2}$ (B)

Hence $f(X, Y) = a\left(\frac{X+Y}{2}\right)\left(\frac{X-Y}{2a}\right) = \frac{X^2-Y^2}{4}$ (B)

(52) $\frac{f(xy) + f(x/y)}{f(x)f(y)} = \frac{\cos(\log x + \log y) + \cos(\log x - \log y)}{\cos(\log x)\cdot\cos(\log y)}$

$= \frac{2\cancel{\cos(\log x)\cos(\log y)}}{\cancel{\cos(\log x)\cos(\log y)}} = 2$ — (D)

(53) $f(x) = |x| + |x-1|$

If $x \in (0,1) \Rightarrow f(x) = x - (x-1)$

$= 1$ — (A)

(54) $f(2x+3y, 2x-7y) = 20x$

let $2x+3y = \alpha$
$2x-7y = \beta$ $\Rightarrow x = \frac{7\alpha+3\beta}{20}$

$f(\alpha, \beta) = \cancel{20}\left(\frac{7\alpha+3\beta}{\cancel{20}}\right)$

— (B)

(55) $f(x) = \log_a x \Rightarrow f(ax) = \log_a(ax)$

$= 1 + \log_a x = 1 + f(x)$ — (B)

(56) $f(y) = \frac{ay-c}{cy-a} = \frac{a\left(\frac{ax-c}{cx-a}\right) - c}{c\left(\frac{ax-c}{cx-a}\right) - a} = x$ — (A)

(57) one-one but not onto as not every integer is attained — (A)

(58) $\left\{ x^2, \; x \geq 0 \right.$

(58) $f(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$ → it is both one-one & onto

— (C)

(59) $f(x) = \dfrac{x^2}{1+x^2}$, $f(-x) = \dfrac{x^2}{1+x^2} = f(x)$

Since Even ⟹ Many-one

& range of $f(x)$ is $[0, 1)$ ⟹ Into

(A)

(60) f is a bijection, (draw graph) — (D)

(61) $f(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$ clearly many-one & Into

— (D)

(62) $f : (\pi/2, 3\pi/2) \to R$, By graph '$f$' is bijection — (C)

(63) in $[\pi/2, 3\pi/2]$ sinx is one-one & since Range = Codomain it is onto also — (C)

(6[illegible]) $f(x) = x - (-1)^x$

$f(x) = \begin{cases} x-1 & \text{when } x \in \text{Even natural} \\ x+1 & \text{when } x \in \text{odd natural} \end{cases}$

which is clearly one-one & onto as range of $f(x)$ is 'N'

— (C)

(65) $f(x) = e^{x} + e^{-x}$

& $f(-x) = e^{-x} + e^{x} = f(x)$ $\Rightarrow$ Even fun^n

$\Rightarrow$ Many-one

But By AM/GM $\frac{e^{x}+e^{-x}}{2} \geq 1$ $\Rightarrow$ $f(x) \geq 2$ $\therefore$ Range $\subset$ Codomain

$\Rightarrow$ Into

— (C)

(66) $f: R \to [-1, 1]$, in R Sin$x$ is many-one as periodic

& but range is also $[-1, 1]$

$\therefore$ onto

— (C)

(67) $f: R \to R$; $\boxed{f(x) = 1}$ $\to$ Many one & Into — (C)

(68) $f: Z \to Z$ $f(x) = x + 2$ is a bijection as it is both one-one & onto

— (B)

(69) $f: [-1, 1] \to [-1, 1]$

$f(x) = \sin \frac{\pi x}{2}$ is bijection as it is one-one & onto — (B)

(70) $f: R \to R^{+}$; $f(x) = e^{-x}$ is onto

as range of e^{-x} is $(0, \infty)$ — (B)

(71) Conceptual — (C)

(72) $f(-1) = 2$
$f(2) = 2$ $\Rightarrow$ clearly many-one

Also $f(x) = x^2 - x$ will not have many integer in its range $\Rightarrow$ Into — (D)

(73) Conceptual. $f(g(x)) = 2g(x) = 2x = g(x) + g(x)$ — (C)

(74) Conceptual (B)

(75) $f(g(x)) = g^2(x) + 2g(x) - 3$

$= (3x-4)^2 + 2(3x-4) - 3$

$= 9x^2 - 18x + 5$ — (B)

(76) As $f(2) = -2 \Rightarrow g(f(2))$ is undefined — (D)

(77) $g(f(x)) = e^{f(x)} = e^{x^2 + \frac{1}{x^2}} = e^{x^2} \cdot e^{1/x^2}$ — (D)

(78)

$g(f(x)) = 2x - 1$

$= \frac{2}{3}(3x) - 1 = \frac{2}{3}(f(x) - 4) - 1$

$\therefore g(f(x)) = \frac{2}{3}(f(x)) - 11/3 \quad \therefore g(x) = \frac{2x - 11}{3}$ — (C)

(79) $f(g(x)) = (x+3)^2$ & $g(x) = x+3$

put $x = -6$ $\qquad g(-6) = -3$

$f(g(-6)) = 9$

$\Rightarrow f(-3) = 9$ — (C)

(80) if $fog(x) = gof(x) \Rightarrow f$ & g inverse of each other

$\mid f(0) = b \Rightarrow 0 = g(b)$

(80) If $f \circ g(x) = g \circ f(x) \Rightarrow f$ & g inverse of each other

$\Rightarrow f(g(x)) = x \quad \forall x \in$ Domain | $f(0) = b \Rightarrow 0 = g(b)$
Also $g(0) = d \Rightarrow 0 = f(d)$

— (C)

(81) $$f(g(x)) = \frac{1-g(x)}{1+g(x)} = \frac{1-4x+4x^2}{1+4x-4x^2}$$ — (A)

(82) Conceptual — (D)

(83) $$f(f(x)) = \frac{1-\left(\frac{1-x}{1+x}\right)}{1+\left(\frac{1-x}{1+x}\right)} = x$$

$\therefore f(f(\sin\theta)) = \sin\theta$ — (A)

(84) $$f(f(x)) = \left(a-(f(x))^n\right)^{1/n}$$

$$= \left(a-(a-x^n)\right)^{1/n} = x$$ — (B)

(85) $$f(g(x)) = \log\left(\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{(3x+x^3)}{1+3x^2}}\right) = \log\left(\frac{(1+x)^3}{(1-x)^3}\right) = 3f(x)$$

— (B)

(86) $f(f(\sqrt{4})) = f(0) = 1$ — (C)

(87) $$f(g(y)) = \frac{g(y)}{\sqrt{1-g^2(y)}} = \frac{\frac{y}{\sqrt{1+y^2}}}{\sqrt{1-\frac{y^2}{1+y^2}}} = y$$ — (C)

$\sqrt{1 - \frac{y^2}{1+y^2}}$

(88) $g(f(x)) = \cos(\pi[x]) = \cos(n\pi)$ $\therefore$ range $\{-1, 1\}$ — (B)

(89) let 'a' be pre-image $\Rightarrow f(a) = 2 \Rightarrow a^2 + 3 = 2$

$\therefore a = \phi$ — (D)

(90) option (D) is a bijection $\therefore$ it has inverse — (D)

(91) $f(x) = 2^x$ $\therefore$ for Inverse $f(y) = x$

$\Rightarrow 2^y = x$

$\Rightarrow y = \log_2 x$ — (C)

(92) for Inverse $f(y) = x \rightarrow \frac{e^y - e^{-y}}{e^y + e^{-y}} = x - 2$

Apply componendo & dividendo $\Rightarrow \frac{e^y}{-e^{-y}} = \frac{x-1}{x-3}$

$\Rightarrow e^{2y} = \frac{1-x}{x-3} \Rightarrow y = \frac{1}{2}\log\left(\frac{1-x}{x-3}\right)$ — (D)

(93) for Inverse let $f(y) = x \Rightarrow y + \frac{1}{y} = x$

$\Rightarrow y^2 - xy + 1 = 0 \Rightarrow y = \frac{x \pm \sqrt{x^2-4}}{2}$

$f^{-1}(x) = \frac{x + \sqrt{x^2-4}}{2}$ as Range of f^{-1} is $[1, \infty)$ — (A)

(94) let $f(y) = x \Rightarrow x = \ln(y + \sqrt{1+y^2})$

$$y+\sqrt{1+y^2}=e^x$$

Rationalize $\Rightarrow$ $\sqrt{1+y^2}-y=e^{-x}$ $\Rightarrow$ $y=\dfrac{e^x-e^{-x}}{2}$ — (C)

(95) Range of f is $\{-1, 0, 7, 26\}$

that is domain of f^{-1} — (C)

(96) let $f(y)=x$ $\Rightarrow$ $\left(4-(y-7)^3\right)^{1/5}=x$

$\Rightarrow$ $4-(y-7)^3=x^5$

$\Rightarrow$ $(4-x^5)^{1/3}+7=y$ — (C)

(97) $(f\circ g)^{-1}(x) = g^{-1}\circ f^{-1}(x)$

$= \dfrac{2+\log_e x}{3}$ — (B)

$g^{-1}(x)=\dfrac{x+2}{3}$

$f^{-1}(x)=\ln x$

1(B) Solutions

(1) Conceptual 10^{10} — (C)

(2) $f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right) = \log\left(\frac{(1+x)^2}{(1-x)^2}\right) = 2f(x)$

— (C)

(3) $[\pi^2] = 9$, $[-\pi^2] = -10$

$\therefore f(x) = \cos 9x + \cos 10x$ $\therefore f(-\pi) = 2 \longrightarrow$ (D)

(4) $f(x+y) = f(x) + f(y) \implies f(x) = kx$

As $f(1) = 7 \Rightarrow k = 7$

Hence $\sum_{r=1}^{n} f(r) = \sum_{r=1}^{n} 7r = \frac{7n(n+1)}{2}$ — (D)

(5) $f(x) = \frac{1}{\sqrt{x-2+2+2\sqrt{2}\sqrt{x-2}}} + \frac{1}{\sqrt{x-2+2-2\sqrt{2}\sqrt{x-2}}}$

$= \frac{1}{\sqrt{\left(\sqrt{x-2}+\sqrt{2}\right)^2}} + \frac{1}{\sqrt{\left(\sqrt{x-2}-\sqrt{2}\right)^2}}$

$= \frac{1}{\sqrt{x-2}+\sqrt{2}} + \frac{1}{\left|\sqrt{x-2}-\sqrt{2}\right|}$ put $x = 11$

$= \frac{1}{3+\sqrt{2}} + \frac{1}{3-\sqrt{2}} = \frac{6}{7}$ — (C)

(6) $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ $\therefore x+3 > 0$ & $x^2+3x+2 \neq 0$

$x \in (-3, \infty)$ & $x \neq \{-1, -2\}$

— (C)

(7) Domain (i) $x - x^2 \geq 0$ (ii) $4 + x \geq 0$ (iii) $4 - x \geq 0$

$x \in [0, 1]$ & $x \in [-4, \infty)$ & $x \in (-\infty, 4]$

$\therefore$ Intersecⁿ $x \in [0, 1]$ ——— (D)

(8) Domain: $\log(x^2 - 6x + 6) \geq 0$

$\Rightarrow \quad x^2 - 6x + 6 \geq 1$

$\Rightarrow \quad x^2 - 6x + 5 \geq 0$

$\Rightarrow \quad x \in (-\infty, 1] \cup [5, \infty)$ — (C)

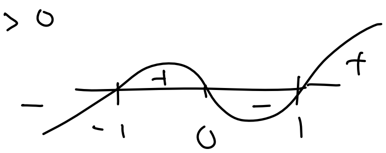
(9) $\cos^{-1}\left(\log_2(x/2)\right)$

$\Rightarrow \quad -1 \leq \log_2(x/2) \leq 1$

or $\quad 1/2 \leq \frac{x}{2} \leq 2 \Rightarrow x \in [1, 4]$ — (A)

(10) clearly $4 - x^2 \neq 0$ & $x^5 - x^3 > 0$

$x \neq \{\pm 2\}$ & $x^3(x-1)(x+1) > 0$



& $(-1, 0) \cup (1, \infty)$

Ans $(-1, 0) \cup (1, 2) \cup (2, \infty)$ — (D)

(11) $\log_{3+x}(x^2 - 1)$ $\therefore$ $3 + x > 0$, $3 + x \neq 1 \Rightarrow x \in (-3, -2) \cup (-2, \infty)$

& $x^2 - 1 > 0 \Rightarrow x \in (-\infty, -1) \cup (1, \infty)$

Their intersection $(-3,-2) \cup (-2,-1) \cup (1,\infty)$

— (C)

(12) Domain $2\sin^{-1}(2x) + \pi/3 \geq 0$ & $x \in [-1/2, 1/2]$

$\sin^{-1}(2x) \geq -\pi/6$

$2x \geq -1/2$

or $x \geq -1/4$ Ans $x \in \left[-\frac{1}{4}, 1/2\right]$ — (A)

(13) $f(x) = \dfrac{1+x^2}{x^2} = 1 + \dfrac{1}{x^2}$

$\therefore$ clearly $f(x) \in (1,\infty)$ — (B)

(14) We know $-1 \leq -\sin 3x \leq 1$

$1 \leq 2 - \sin 3x \leq 3$

$1 \geq \dfrac{1}{2-\sin 3x} \geq \dfrac{1}{3}$

Range is $\left[\frac{1}{3}, 1\right]$ — (A)

(15) $-|a| \leq a\cos(bx+c) \leq |a|$

$\therefore$ $d-|a| \leq d + a\cos(bx+c) \leq d+|a|$

Assuming 'a' to be positive — (D)

(16) $0 \leq \cos^2 x \leq 1$

$0 \leq \frac{\pi}{4}\cos^2 x \leq \pi/4$

$1 \leq \sec\left(\frac{\pi}{4}\cos^2 x\right) \leq \sqrt{2}$ — (A)

$$1 \le \sec\left(\frac{\pi}{4}\cos^2 x\right) \le \sqrt{2} \quad \text{— (A)}$$

(17) let $y = \dfrac{x^2+3x+1}{x^2+x+1} \Rightarrow (y-1)x^2 + (y-3)x + y-1 = 0$

for $x \in R$ $\quad D \geq 0$

$$(y-3)^2 - 4(y-1)^2 \geq 0$$

$$\Rightarrow (y+1)(3y-5) \leq 0$$

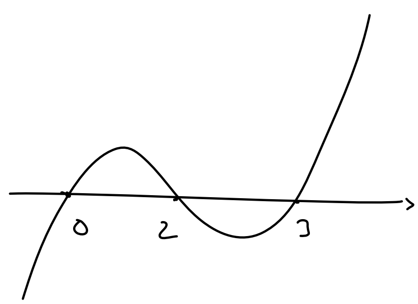
$$y \in [-1, 5/3] \quad \text{— (D)}$$

(18) Since $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ only $\Rightarrow$ one-one

But Range of $f(x)$ is not all integer $\Rightarrow$ Into

— (B)

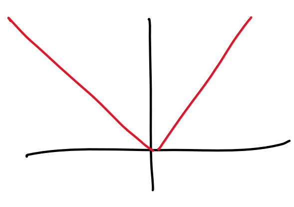
(19) By graph



Many-one & onto (B)

(20) By formula $3^5 - {}^3C_1 2^5 + {}^3C_2$ — (B)

(21) By graph



Many-one & Into

— (D)

(22) put $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ Hence one-one

find range: $y = \dfrac{x-m}{x-n} \Rightarrow x = \dfrac{ny-m}{y-1}$

$\therefore$ Range: $R - \{1\} \neq$ Co-domain

$\therefore$ Range : $R - \{1\} \neq$ Co-domain

$\therefore$ Into — (B)

(23) $f(n) = \begin{cases} \dfrac{n-1}{2}, & n \text{ odd} \\ -\dfrac{(n+2)}{2}, & n \text{ Even} \end{cases}$

$\therefore$ when n odd let $n = 2a+1$ $(a \in W)$

when n Even $n = 2b$ $(b \in N)$

$f(a,b) = \begin{cases} a, & a \in W \\ -(b+1), & b \in N \end{cases} \implies f(x)$ is one-one but not onto as range $\in Z - \{-1\}$ — (A)

(24) (A) $f(x) = x\left(\dfrac{a^x - 1}{a^x + 1}\right)$

$f(-x) = -x\left(\dfrac{a^{-x} - 1}{a^{-x} + 1}\right) = -x\left(\dfrac{1 - a^x}{1 + a^x}\right) = f(x)$ Even

— (A)

(25) Since $f(-x) = f(x) \implies$ Symm abt y-axis — (B)

(26) LCM of 4, 6, 4 is 12 $\therefore$ period 12 — (D)

(27) Since odd $\Rightarrow \boxed{f(0) = 0}$ & period 2 $\Rightarrow f(4) = f(0) = 0$

— (A)

(28) Conceptual 'T' — (B)

(29) $f(x) = 1 + 2x - [2x] = 1 + \{2x\}$ $\therefore$ period $\frac{1}{2}$ — (B)

(30) LCM of $2(n-1)$ & $2n$ is $2n(n-1)$ — (C)

(31) $g(f(x)) = f^2(x) = (2x-1)^2$ — (B)

$\therefore \; g(f(x)) = f(x) = (2x-1) \; — \; ⓘ$

(32) $g(-3) = 10 \quad \therefore f(g(-3)) = f(10) = 121$ — (A)

(33) Assuming onto, $f(x) = 2^x$ is one-one $\Rightarrow$ invertible — (A)

(34) $gof(x) = 4x^2 - 10x + 4$ & $g(x) = x^2 + x - 2$

$\therefore g(f(x)) = f^2(x) + f(x) - 2$

clearly $f(x)$ is linear

$\therefore$ let $f(x) = 2x + a$

$g(f(x)) = (2x+a)^2 + (2x+a) - 2$

$\therefore$ upon comparing $a = -3 \quad \therefore$ (A)

(35) $$f(g(y)) = \frac{\frac{y}{\sqrt{1+y^2}}}{\sqrt{1 - \frac{y^2}{1+y^2}}} = y \quad — \text{(C)}$$

(36) $$f(f(x)) = \frac{2\left(\frac{2x-3}{x-2}\right) - 3}{\frac{2x-3}{x-2} - 2} = \frac{x}{1} \quad — \text{(A)}$$

(37) $f(g(x)) = x + 2\sqrt{x} + 1 + 2$

$= (\sqrt{x}+1)^2 + 2 \quad \Rightarrow f(x) = x^2 + 2$ — (B)

$= g^2(x) + 2$

(38) $$f(f(x)) = \frac{\frac{x}{2x-1}}{\frac{2x}{2x-1} - 1} = x \quad \rightarrow \text{Domain } x \neq \frac{1}{2}$$

$$\therefore f(f(f(x))) = f(x) = \frac{x}{2x-1} \quad \therefore \text{Domain } x \neq 1/2$$

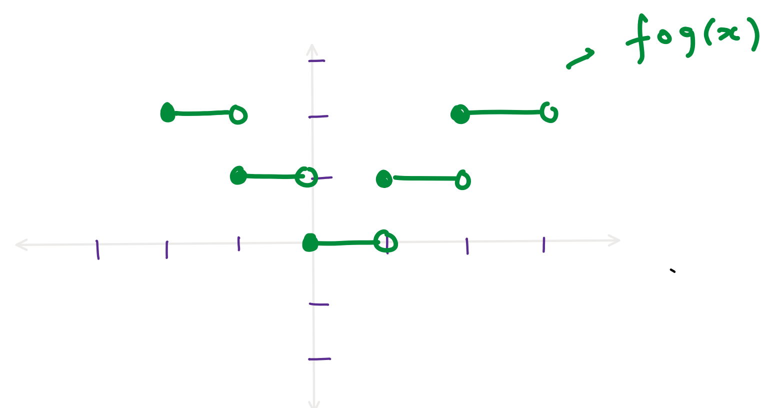
$\therefore f(f(f(x))) = f(x) = \dfrac{x}{2x-1}$ $\therefore$ Domain $x \neq 1/2$

Ans (B)

(39) $f(x) = \dfrac{2x+1}{3x-2}$ $\therefore f(2) = 5/4$

$\therefore f(f(2)) = \dfrac{5/2+1}{\frac{15}{4}-2} = \dfrac{7/2}{7/4} = 2$ — (D)

(40) $g(f(x)) = [|x|]$; $f(g(x)) = |[x]|$



$\therefore fog(x) \geq g(f(x)) \Rightarrow$ green graph above red $\forall x \in R$ — (D)

(41) let $f(y) = x \Rightarrow 3^{y(y-2)} = x$

$\Rightarrow y^2 - 2y - \log_3 x = 0$

$\Rightarrow y = 1 + \sqrt{1+\log_3 x}$ [(-ve rejected as range of f^{-1} is $[1,\infty)$]

(B)

(42) $f(x) = \{x\}$ is periodic $\Rightarrow$ Not invertible — (C)

(43) as $\sec^{-1}$ is anyway $[0,\pi]$

so sufficient condition is

$$\frac{2-|x|}{4} \geq 1 \quad \text{or} \quad \frac{2-|x|}{4} \leq -1$$

$\Rightarrow 2-|x| \geq 4$

$|x| \leq -2$

$x \in \phi$

$2-|x| \leq -4$

$|x| \geq 6$

$\Rightarrow x \in (-\infty, -6] \cup [6, \infty)$

—— (D)

(44) $y = \dfrac{x-1}{x^2-2x+3} \Rightarrow yx^2 - (2y+1)x + 3y+1 = 0$

for $x \in$ Real, $D \geq 0$

$(2y+1)^2 - 4y(3y+1) \geq 0$

$-8y^2+1 \geq 0 \Rightarrow y \in \left[\dfrac{-1}{2\sqrt{2}}, \dfrac{1}{2\sqrt{2}}\right]$

—— (D)

(45) $f(x) = \cos^{-1}\left(\dfrac{1+x^2}{2x}\right) + \sqrt{2-x^2}$

Note: $\dfrac{x^2+1}{2x} \in (-\infty, -1] \cup [1, \infty)$ so for $\cos^{-1}$ to exist x can only be ± 1

$\therefore$ Domain is $x \in \langle -1, 1 \rangle$

$\therefore$ find $f(1)$ & $f(-1)$ $\therefore$ Range $\langle 1, 1+\pi \rangle$ —— (D)

(46) $f(x) = \dfrac{\tan(\pi[x^2-x])}{1+\sin(\cos x)}$

Since Numerator is $\tan(n\pi)$

$\therefore$ Numerator is 0 $\forall x \in R$

$\therefore f(x) = 0 \quad \forall x \in R$ —— (D)

(47) $f(x) = \dfrac{e^x}{1+[x]}, \quad x \geq 0$

$1+[x]$

least is when $x=0$ i.e. 1

max is ∞ when $x \to \infty$ $\therefore$ Range $[1,\infty)$ — (D)

(48) $f(x) = \dfrac{1}{1-x}$ $(x \neq 1)$

$f(f(x)) = \dfrac{1}{1-\left(\frac{1}{1-x}\right)} = \dfrac{x-1}{x}$ $(x \neq 0, 1)$

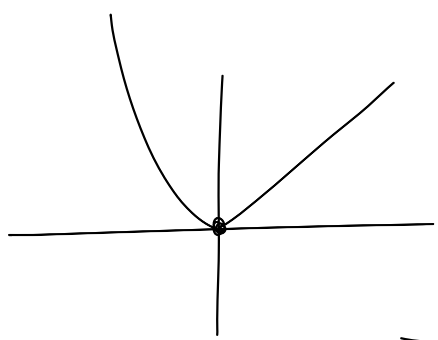
$f(f(f(x))) = 1-(1-x) = x$ $\therefore (x \neq 0, 1)$ — (C)

(49) Conceptual (A)

(50) $f(f(x)) = \begin{cases} \cancel{f^2(x), \quad f(x) < 0} \\ f(x), \quad f(x) \geq 0 \end{cases}$ $\therefore$ rejected

$f(f(x)) = f(x) \ \forall x \in R$

where $y = f(x)$



clearly $f(x) \geq 0 \ \forall x \in R$ — (D)

(51) LCM of 2π & π is '2π' — (A)

(52) Conceptual — (D)

(53) $f(6\pi + x) = \dfrac{\cos(\sin(6n\pi + nx))}{\tan\left(\frac{6\pi}{n} + \frac{x}{n}\right)} = \dfrac{\cos(\sin nx)}{\tan\left(\frac{6\pi}{n} + \frac{x}{n}\right)} = \dfrac{\cos(\sin nx)}{\tan(x/n)}$

If $n = 1, 2, 3, 6$

Ans — A B C D

(54) $f(x) = \sin 3\pi \{x\} + \tan \pi \cancel{[x]}$

periodic & period 1 — (A)

(55) odd extension is given by $-f(-x) \;\forall\, x \in (-\infty, 0]$

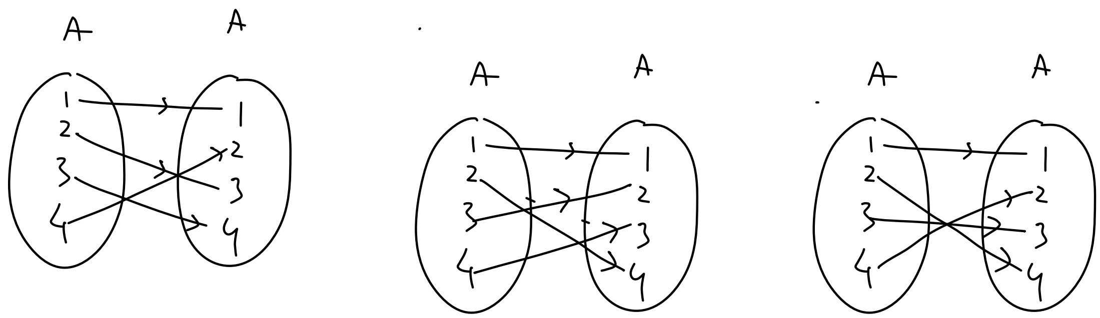
i.e. $-[\sin(\cos x) + x - \tan x]$

or $-\sin(\cos x) - x + \tan x$ — (D)

(56) (C) $f(x) = \log(x^2 - x + 1)$

$f(-x) = \log(x^2 + x + 1) \rightarrow$ neither even nor odd — (C)

(57)

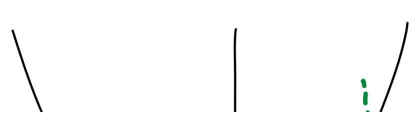


only 3 functions — (D)

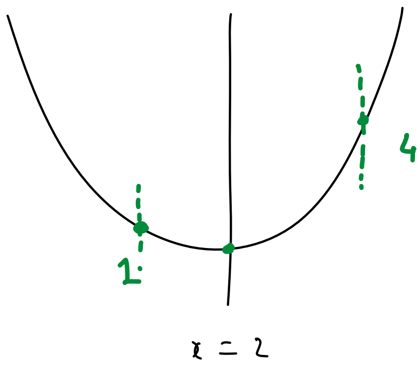
(58) $f(x) = \sin(\{x+3\}) = \sin(\{x\}) \therefore$ period 1 — (C)

(59) $f(2+t) = f(2-t) \;\forall\, t \in R \Rightarrow$ graph of $f(x)$ is symm abt $x = 2$

$\therefore$ vertex of $f(x)$ lies on $x = 2$



clearly $f(4) > f(1) > f(2)$



clearly $f(4) > f(1) > f(2)$

— (B)

(60) $f(x) = 2\tan 3x + 5\sqrt{2}\,|\sin 3x| \longrightarrow$ period $\pi/3$

$\therefore$ option (A)

(61) $[x]^2 - 5[x] + 6 = 0 \Rightarrow [x] = 2$ or $[x] = 3$

$\Rightarrow x \in [2,3) \cup x \in [3,4)$

$\Rightarrow x \in [2,4)$ — (D)

(62) $\left[\log_2\left(\frac{x}{[x]}\right)\right] \geq 0$ as $[x] \neq 0 \Rightarrow x \notin [0,1)$

$\Rightarrow \log_2\left(\frac{x}{[x]}\right) \geq 0$

$\Rightarrow \frac{x}{[x]} \geq 1$

Cases

If $x \in [1,\infty)$

$x \geq [x]$

Always true

$S_1 \equiv [1,\infty)$

or

If $x \in (-\infty, 0)$

$x \leq [x]$

only true at integers

$\therefore$ $x \in$ Negative integers also

Hence — (D)

(63) $2[x] = x + \{x\}$

$\Rightarrow \quad [x] = 2\{x\}$ — (i)

Since LHS integer $\therefore 2\{x\} = 0$ or $2\{x\} = 1$

$\therefore \{x\} = 0$ or $\{x\} = 1/2$

from (i) $[x] = 0$ | from (i) $[x] = 1$

$\therefore x = [x] + \{x\}$ | $x = [x] + \{x\}$

$x = 0$ | $x = 3/2$ — (B)

(64) $[x]^2 = [x] + 2\{x\}$

$\Rightarrow \quad \dfrac{[x]^2 - [x]}{2} = \{x\}$

Since $\{x\} \in [0, 1)$ $\therefore [x]$ can only take $0, 1$

If $[x] = 0$ | If $[x] = 1$

$\{x\} = 0$ | $\{x\} = 0$

$\therefore x = 0$ | $\Rightarrow x = 1$

— (A)

(65) $[x^2] + x = a$ Since $a \in N$ $\therefore x$ also must be integer

We can put $x = 1, 2, 3, 4 \rightarrow$ gives 4 values of $a \leq 20$

— (C)

(66) $[x + [2x]] < 3$ $\{[x + I] = [x] + I\}$

$[x] + [2x] < 3$

or $[x] + [2x] \leq 2$ — (A)

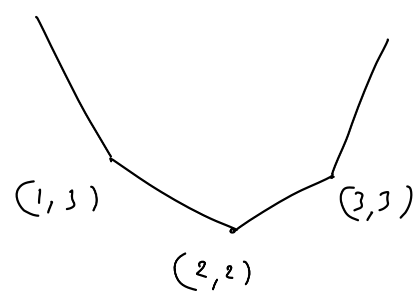
clearly $x \le 0$ is a sol^n — (1)

If $x > 0$ then (i) $x \in (0, 1/2)$ (A) is true | (ii) $x \in [1/2, 1)$ (A) is true | (iii) $x \geq 1$ ϕ as (A) not true

$\therefore$ final ans $(-\infty, 0] \cup (0, 1/2) \cup [1/2, 1)$

OR $(-\infty, 1)$ — (d)

(67) plot $f(x)$



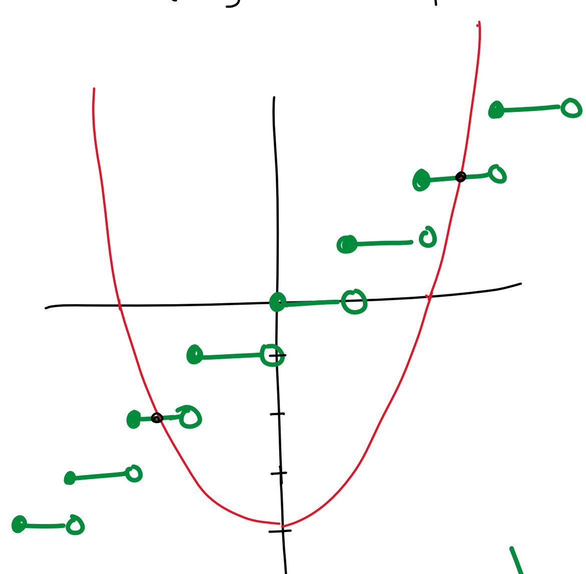
$\therefore$ min at $x = 2$ is 2 — (B)

(68) observe that $1 \le |\sin x| + |\cos x| \le \sqrt{2}$ $\forall x \in R$

$\therefore$ $[|\sin x| + |\cos x|] = 1$ $\forall x \in R$ — (C)

(69) $x^2 - 4 = [x]$ let $f(x) = x^2 - 4$, $g(x) = [x]$

plot $f(x)$ & $g(x)$



As 2 intersection pts $\Rightarrow$ 2 sol^n

— (B)

(70) plot all 3 $y = x^3$

$y = x^2$

$y = 1/64$

clearly max of all 3 gives

$$f(x) = \begin{cases} \frac{1}{64}, & 0 \le x \le 1/8 \\ x^2, & \frac{1}{8} < x \le 1 \\ x^3, & 1 < x < \infty \end{cases}$$ — (C)

(71) $f(-x) = f(x) \quad \forall x \in R$

$(-ax+\cancel{b})\cos x - (-\cancel{cx}+d)\sin x = (ax+\cancel{b})\cos x + (\cancel{cx}+d)\sin x$

$\forall x \in R$

Simplify $ax\cos x + d\sin x = 0 \quad \forall x \in R \Rightarrow a, d = 0$ — (C)

(72) $f(x) + g(x) = e^x \quad \forall x \in R$

$\therefore f(-x) + g(-x) = e^{-x} \Rightarrow f(x) - g(x) = e^{-x} \quad \forall x \in R$

$\therefore$ solving $f(x) = \frac{e^x + e^{-x}}{2}, \quad g(x) = \frac{e^x - e^{-x}}{2}$

$\therefore f^2 - g^2 = 1$ — (D)

(73) Reflecⁿ of $A(5, k)$ is $B(k, 5)$ & it lies on $f(x)$

$\Rightarrow f(k) = 5 \Rightarrow k = 2$

$\therefore B(2,5) \therefore$ reflecⁿ about origin is $-2, -5$

— (A)

$\therefore$ B(2,5) $\therefore$ reflecⁿ about origin is −2,−5

— (A)

(74) let $f^{-1}(4) = a \Rightarrow 4 = f(a)$

But $f(x) = 2x^3 + 7x - 5$

see $\boxed{f(1) = 4} \Rightarrow a = 1$ — (A)

(75) $f(\pi + x) = f(x) \Rightarrow$ 'π' period

(76) Conceptual — (A)

(77) for range to have only integers & 'f' to be continuous it must be a constant funcⁿ — (D)

(78) $g(-1, -3/2) = (-1) - (-3/2) = 1/2$

$g(-4, -1.75) = (-1.75) - (-4) = 2.25$

$\therefore f(1/2, 2.25) = (2.25)^{1/2} = 1.5$ — (D)

(79) it can be seen that expression goes both upto $+\infty$ & $-\infty$ $\therefore$ range R — (D)

(80) Conceptual option (D) satisfies

1 (C) soln

(1) $f(x) = \sqrt{\cos^{-1}(2x) + \pi/6}$ Since $\cos^{-1}(x)$ is always ≥ 0

Domain : $-\frac{1}{2} \leq x \leq \frac{1}{2}$

$\therefore$ $f^{-1}(x)$ range is same as domain of $f(x)$

ie $\left[-\frac{1}{2}, \frac{1}{2}\right]$ $\therefore$ $\boxed{a+b=0}$

(2) $f : (2,4) \longrightarrow (1,3)$ as $x \in (2,4)$

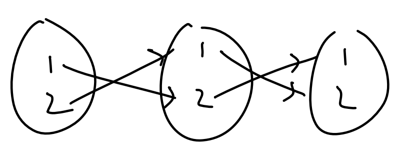
$\frac{x}{2} \in (1,2)$ $\therefore$ $[x/2] = 1$

Hence $f(x) = x - 1$ $\therefore$ $f^{-1}(x) = x + 1$ $\therefore$ $\boxed{\alpha = 1}$

(3) LCM of 1, 6, 10 is $\boxed{30}$

(4) $f(f(i)) = i \quad \forall \ i = 1, 2, 3, \dots 10$

ex



So basically divide 10 no. into 5 groups each containing 2

No. of ways $\frac{(10)!}{(2!)^5 \, 5!}$

Every group formation $\Longrightarrow$ 1 unique func^n

$\therefore$ $\boxed{\text{Ans } 945}$

(5) $f(2x^2 - 1)$ $\rightarrow$ then $-1 \leq 2x^2 - 1 \leq 3$

$\Rightarrow$ $0 \leq 2x^2 \leq 4$

$$\Rightarrow \quad x^2 \le 2$$

Hence $x \in \{-1, 0, 1\}$ Ans $\boxed{3}$

(6) let $f(y) = x \Rightarrow x = y + \frac{1}{y} \Rightarrow y^2 - xy + 1 = 0$

$$\Rightarrow y = \frac{x + \sqrt{x^2 - 4}}{2}$$

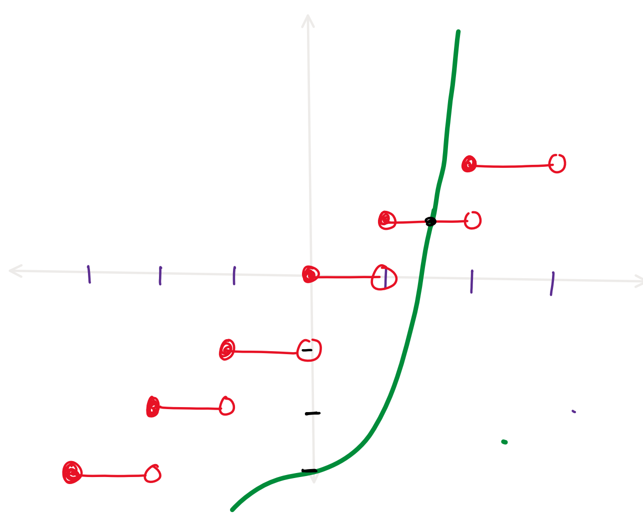
($(-)$ve rejected as range of f^{-1} is $[2, \infty)$)

(7) let's find max of $\cos(\sin x)$, which is 1 when $x = 0$

$\therefore \boxed{f(x)_{min} = 1}$

(8) $x^3 - 3 = [x]$

plot $y = x^3 - 3$
& $y = [x]$



clearly only $\boxed{1 \text{ sol}^n}$

(9) Conceptual

(10) let $f(x) = x^2 - 3x + 4$ let $f : [3/2, \infty) \to [7/4, \infty)$

$\therefore f^{-1}(x) = \frac{3}{2} + \sqrt{x - 7/4}$ $f^{-1} : [7/4, \infty) \to [3/2, \infty)$

$\therefore f(x) = f^{-1}(x)$ Same as solving $f(x) = x$ as $f(x)$ Inc

i.e. $x^2 - 4x + 4 = 0$

ie. $x^2 - 4x + 4 = 0$

$\therefore \boxed{x = 2}$ (which satisfies & in domain)

(11) Conceptual

(12) $f(x) = \begin{cases} \frac{x}{2} + 2 & , x \le 2 \\ 5 - x & , 2 < x < 3 \\ 11 - (x-6)^2 & , x \geq 3 \end{cases}$

$\therefore f(x) = 2$ possible when $x = 0$, $x = 3, 9$

$f(f(x)) = 2$ let 'α' be a solⁿ $\Rightarrow f(f(\alpha)) = 2$

$\Rightarrow f(\alpha) = 0, 3, 9$

$f(\alpha) = 0 \longrightarrow 2$ values of x

$f(\alpha) = 3 \longrightarrow 3$ " of x

$f(\alpha) = 9 \longrightarrow 2$ values of x

Total values $= 7$

(13) $f\left(\frac{x+1}{x-1}\right) = 2f(x) + \frac{1}{x-1}$ —— (i)

$x \to \frac{x+1}{x-1} \Rightarrow f(x) = 2f\left(\frac{x+1}{x-1}\right) + \frac{x-1}{2}$ —(ii)

from (i) & (ii)

$f(x) = 4f(x) + \frac{2}{x-1} + \frac{x-1}{2}$

OR $f(x) = -\frac{1}{3}\left(\frac{x-1}{2} + \frac{2}{x-1}\right)$

$\therefore 6f(0) = -2\left[-\frac{1}{2} - 2\right] = 5$

(14) $|2x-1| = 3[x] + 2\{x\} \Rightarrow |2x-1| = 2x + [x]$

Case I $x \geq 1/2$ | Case II $x < 1/2$

$|2x-1| = 3[x] + 2\{x\}$

Case I $\quad x \geq 1/2$

$\cancel{2x} - 1 = \cancel{2x} + [x]$

$[x] = -1$

$\Rightarrow x \in \phi$

Case II $\quad x < 1/2$

$-2x + 1 = 2x + [x]$

$\Rightarrow [x] = 1 - 4x$

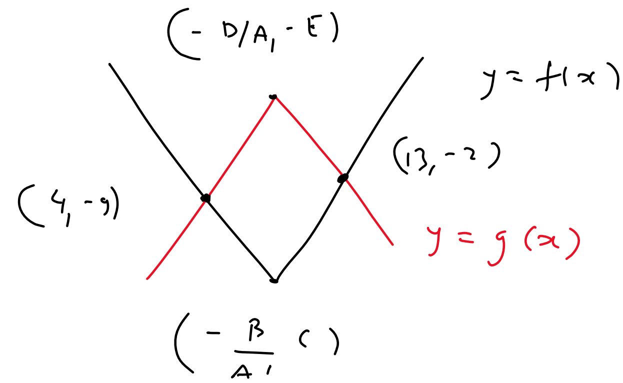
$x = 1/4$

$\therefore \boxed{\text{Ans } 4}$

(15) $\quad f(x) = |Ax + B| + C$

$g(x) = -|Ax + D| - E$

By graph



Since it is a // gm
By equating mid pts

$\therefore \quad \frac{-(B+D)}{A} = 17 \quad \& \quad C - E = -11$

$$\left| E - C + \frac{B+D}{A} \right| = |11 - 17| = 6$$

1. (A)

Given: $f(x)=2x+\sin x,\ x\in R$

$\Rightarrow\ f'(x)=2+\cos x$. Now, $-1\le\cos x\le 1$

$\Rightarrow\ 1\le 2+\cos x\le 3\ \Rightarrow\ 1\le 2+\cos x\le 3$

$\therefore\ f'(x)>0,\ \forall x\in R$

$\Rightarrow\ f(x)$ is strictly increasing and therefore one-one

Also as $x\to\infty,\ f(x)\to\infty$ and $x\to-\infty,\ f(x)\to-\infty$

$\therefore$ Range of $f(x)=R=$ domain of $f(x)\Rightarrow f(x)$ is onto.

Hence, $f(x)$ is one-one and onto.

2. (B)

Given: $f:[0,\infty)\to[0,\infty)$ and $f(x)=\dfrac{x}{x+1}$

$\therefore\ f'(x)=\dfrac{1+x-x}{(1+x)^2}=\dfrac{1}{(1+x)^2}>0\ \forall\, x$

$\therefore\ f$ is an increasing function $\Rightarrow\ f$ is one-one.

Now, $D_f=[0,\infty)$

For range let $\dfrac{x}{1+x}=y\Rightarrow\ x=\dfrac{y}{1-y}$

Now, $x\ge 0\ \Rightarrow\ 0\le y<1$

$\therefore\ R_f=[0,1)\ne$ Co-domain,

$\therefore f$ is not onto.

3. (D)

$f(x)=x^2+2bx+2c^2\ \Rightarrow\ f(x)=(x+b)^2+2c^2-b^2$

$\Rightarrow\ f_{\min}=2c^2-b^2$ and $g(x)=-x^2-2cx+b^2$

$g(x)=-(x+c)^2+b^2+c^2\ \Rightarrow\ g_{\max}=b^2+c^2$

For $f_{\min}>g_{\max}\Rightarrow 2c^2-b^2>b^2+c^2$

$\Rightarrow\ c^2>2b^2\Rightarrow |c|>|b|\sqrt{2}$

4. (D)

$f(x)=e^{x^2}+e^{-x^2}\ \Rightarrow\ f'(x)=2x\left(e^{x^2}-e^{-x^2}\right)\ge 0,\ \forall\, x\in[0,1]$

$\therefore\ f(x)$ is in increasing function on [0, 1]

$\therefore\ f_{\max}=f(1)=e+\dfrac{1}{e}=a;\ g(x)=xe^{x^2}+e^{-x^2}$

$\Rightarrow\ g'(x)=\left(2x^2+1\right)e^{x^2}-2xe^{-x^2}\ge 0,\ \forall\, x\in[0,1]$

$\therefore\ g(x)$ is an increasing function on [0, 1]

$\therefore\ g_{\max} = g(1) = e + \dfrac{1}{e} = b$

$h(x) = x^2 e^{x^2} + e^{-x^2}$

$\Rightarrow\ h'(x) = 2x\left[e^{x^2}\left(1+x^2\right) - e^{-x^2}\right] \geq 0,\ \forall x \in [0, 1]$

$\therefore\ h(x)$ is an increasing function on [0, 1]

$\therefore\ h_{\max} = h(1) = e + \dfrac{1}{e} = c$

$\therefore\ a = b = c$.

5. (B)

Given: $f(x) = 2x^3 - 15x^2 + 36x + 1$

$\Rightarrow\ f'(x) = 6x^2 - 30x + 36$

$= 6\left(x^2 - 5x + 6\right) = 6(x-2)(x-3)$

$\because\ f'(x) > 0\ \forall\ x \in [0, 2)$ and $f'(x) < 0\ \forall\ x \in (2, 3)$

$\therefore\ f(x)$ is increasing on [0, 2) and decreasing on (2, 3)

$\therefore\ f(x)$ is many one on [0, 3]

Also $f(0) = 1,\ f(2) = 29,\ f(3) = 28$

$\therefore$ Absolute min = I and Absolute max = 29

$\therefore$ Range of f = [1, 29] = codomain

Hence f is onto.

6. (A)

Number of onto function such that exactly three elements in $x \in A$ such that $f(x) = \dfrac{1}{2}$ is equal to

$= {}^7C_3, \left\{2^4 - 2\right\} = 14.{}^7C_3$

7. (C)

$$f_1(x) = f_{0+1}(x) = f_0\left(f_0(x)\right) = \frac{1}{1 - \frac{1}{1-x}} = \frac{x-1}{x}$$

$$f_2(x) = f_{1+1}(x) = f_0\left(f_1(x)\right) = \frac{1}{1 - \frac{x-1}{x}} = x$$

$$f_3(x) = f_{2+1}(x) = f_0\left(f_2(x)\right) = f_0(x) = \frac{1}{1-x}$$

$$f_4(x) = f_{3+1}(x) = f_0\left(f_3(x)\right) = \frac{x-1}{x}$$

$$\therefore\ f_0 = f_3 = f_6 = \ldots\ldots = \frac{1}{1-x}$$

$$f_1 = f_4 = f_7 = ... = \frac{x-1}{x} \text{f}$$

$$f_2 = f_5 = f_8 = ... = x$$

$$f_{100}(3) = \frac{3-1}{3} = \frac{2}{3}\ f_1\left(\frac{2}{3}\right) = \frac{\frac{2}{3}-1}{\frac{2}{3}} = -\frac{1}{2}$$

$$f_2\left(\frac{3}{2}\right) = \frac{3}{2}$$

$$\therefore\quad f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right) = \frac{5}{3}$$

8. (D)

$$f(g(x)) = x$$

$$\Rightarrow\quad f\left(3^{10}x-1\right) = 2^{10}\left(3^{10}\cdot x-1\right)+1 = x$$

$$\Rightarrow\quad 2^{10}\left(3^{10}x-1\right)+1 = x$$

$$\Rightarrow\quad x\left(6^{10}-1\right) = 2^{10}-1$$

$$\Rightarrow\quad x = \frac{2^{10}-1}{6^{10}-1} = \frac{1-2^{-10}}{3^{10}-2^{-10}}$$

9. (D)

$$\left.\begin{aligned} f(1) &= 1-5\left[\frac{1}{5}\right] = 1 \\ f(5) &= 6-5\left[\frac{6}{5}\right] = 1 \end{aligned}\right\} \rightarrow \text{Many one}$$

$f(10) = 10-5(2) = 0$ which is not in co-domain.

Neither one-one nor onto.

10. (C)

Domain and codomain = {1, 2, 3, .., 20}.

There are five multiple of4 as 4, 8, 12, 16 and 20 and there are 6 multiple of3 as 3, 6,9, 12, 15, 18.

Since, whenever k is multiple of 4 then $f(k)$ is multiple of 3 then total number of arrangement

$= {}^6C_5 \times 5! = 6!$

Remaining 15 elements can be arranged in 15! ways.

Since, for every input, there is an output

$\Rightarrow$ function $f(k)$ is onto

$\therefore$ Total number of arrangements = 15!.6!

11. (B)

$$\because\quad \phi(x) = ((hof)og)(x)$$

$$\because\quad \phi\left(\frac{\pi}{3}\right) = h\left(f\left(g\left(\frac{\pi}{3}\right)\right)\right) = h\left(f\left(\sqrt{3}\right)\right) = h\left(3^{1/4}\right)$$

$$=\frac{1-\sqrt{3}}{1+\sqrt{3}}=-\frac{1}{2}\left(1+3-2\sqrt{3}\right)=\sqrt{3}-2=-\left(-\sqrt{3}+2\right)$$

$$=-\tan 15^\circ=\tan\left(180^\circ-15^\circ\right)=\tan\left(\pi-\frac{\pi}{12}\right)=\tan\frac{11\pi}{12}$$

12. (B)

$$(gof)(x)=g\left(f(x)\right)=f^2(x)+f(x)-1$$

$$g\left(f\left(\frac{5}{4}\right)\right)=4\left(\frac{5}{4}\right)^2-10\cdot\frac{5}{4}+5=-\frac{5}{4} \qquad [\because\ g\left(f(x)\right)=4x^2-10x+5\ ]$$

$$g\left(f\left(\frac{5}{4}\right)\right)=f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)-1$$

$$-\frac{5}{4}=f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)-1$$

$$f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)+\frac{1}{4}=0$$

$$\left(f\left(\frac{5}{4}\right)+\frac{1}{2}\right)^2=0$$

$$f\left(\frac{5}{4}\right)=-\frac{1}{2}$$

13. (C)

Given that $f:A\to B$ and $g:B\to C$

$\therefore\quad f^{-1}B\to A$ and $g^{-1}:C\to B$

We have $(gof)^{-1}=f^{-1}og^{-1}:C\to A$

$\therefore\quad f$ must be one-one and g will be onto function

14. (B)

For finding inverse of $f(x)$

Let $y=\dfrac{x-2}{x-3}\ \Rightarrow\ xy-3y=x-2\ \Rightarrow\ x(y-1)=3y-2$

$\therefore\ f^{-1}(x)=\dfrac{3x-2}{x-1}$

Similarly for inverse of $g(x)$

$y=2x-3\ \Rightarrow\ x=\dfrac{y+3}{2}\ \Rightarrow\ g^{-1}(x)=\dfrac{x+3}{2}$ y

$\therefore\quad f^{-1}(x)+g^{-1}(x)=\dfrac{13}{2}$

$\therefore\quad \dfrac{3x-2}{x-1}+\dfrac{x+3}{2}=\dfrac{13}{2}$

$\Rightarrow\quad 6x-4+x^2+2x-3=13x-13$

$\Rightarrow\quad x^2-5x+6=0\ \Rightarrow\ (x-2)(x-3)=0$

$\Rightarrow \quad x = 2$ or 3.

15. (C)

$y = 5^{\log x}$

$\Rightarrow \quad \log y = \log x \cdot \log 5$ [taking log on both sides]

$\Rightarrow \quad \log x = \dfrac{\log y}{\log 5} = \log_5 y$

$\Rightarrow \quad x = e^{\log 5^y} \;\Rightarrow\; x = y^{\log_5 e} \;\Rightarrow\; x = y^{\frac{1}{\log_5}}$

16. (B)

Putting value of K from 1 to 10, we get

$f(1) = f(2) = 2$

$f(3) = f(4) = 4$

$f(5) = f(6) = 6$

$f(7) = f(8) = 8$

$f(9) = f(10) = 10$

Since, $g(f(x)) = f(x)$

$\therefore \quad gof(1) = f(1) \Rightarrow g(2) = f(1) = 2$

$gof(2) = f(2) \Rightarrow g(2) = f(2) = 2$

$gof(3) = f(3) \Rightarrow g(4) = f(3) = 4$

$\therefore$ The image of 2, 4, 6, 8, 10 in function $g(x)$ should be 2, 4, 6, 8, 10 respectively. Therefore, image of each of remaining elements can be any of 10 elements.

Hence, number of possible $g(x)$ is 10^5.

17. (D)

$f : N - \{1\} \to N \qquad f(a) = \alpha$

Where α is max of powers of prime P such that p^{α} divides a. Also $g(a) = a + 1$

$\therefore \quad f(2) = 1 \qquad g(2) = 3$

$f(3) = 1 \qquad g(3) = 4$

$f(4) = 2 \qquad g(4) = 5$

$f(5) = 1 \qquad g(5) = 6$

$\Rightarrow \quad f(2) + g(2) = 1 + 3 = 4$

$f(3) + g(3) = 1 + 4 = 5$

$f(4) + g(4) = 2 + 5 = 7$

$f(5) + g(5) = 1 + 6 = 7$

$\therefore$ Many one $f(x) + g(x)$ does not contain 1

$\Rightarrow$ into function

18. (B)

Given that f is bijective function and $f(3) \geq f(9) \geq f(15) \geq f(21) \geq ..... \geq f(99)$

So, all elements 3, 9, 15 …. 99 i.e. 17 elements as 1 choice .
Remaining 50 – 17 = 33 elements has taken from 50 elements.

$\therefore$ Number of ways $= {}^{50}P_{33}$

19. (D)

Given function is $f(x) = \begin{cases} 2n; & n = 2, 4, 6, 8, ... \\ (n-1); & n = 3, 7, 11, 15, ... \\ \left(\dfrac{n+1}{2}\right); & n = 1, 5, 9, 13, ... \end{cases}$

When n = 2, 4, 6, then $2n$ is the multiple of 4,
When n = 3, 7, 11, 15 then $(n – 1)$ is not multiple of 4.
When n = 1, 5, 9, 13, then $\left(\dfrac{n+1}{2}\right)$ is the odd number.

Every number gives exactly one value.
Thus, f is one-one & onto.

20. (D)

Given, $f(x) = x - 1;\ g(x) = \dfrac{x^2}{x^2-1}$

Now, $f(g(x)) = g(x) - 1$

$= \dfrac{x^2}{x^2-1} - 1 = \dfrac{x^2 - x^2 + 1}{x^2-1}$

Hence, $f(g(x)) = \dfrac{1}{x^2-1};\ x \neq \pm 1$

Thus, $f(g(x))$ will be even function

$\Rightarrow$ $f(g(x))$ is may one function.

Let $y = \dfrac{1}{x^2-1}$ or $y.x^2 - y = 1$

$x^2 = \left(\dfrac{1+y}{y}\right)$

$\left(\dfrac{1+y}{y}\right) \geq 0$

Range : $y \in (-\infty, -1] \cup (0, \infty)$

Hence, Range $\neq$ Co-domain $\Rightarrow$ $f(g(x))$ is into function.

21. (B)

$f(x)=\frac{x-1}{x+1}$

Given $f^{n+1}(x)=f\left(f^{n}(x)\right)$

$\Rightarrow \quad f^{2}(x)=f\left(f(x)\right)=\frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}=-\frac{1}{x}$

$f^{3}(x)=f\left(f^{2}(x)\right)=f\left(-\frac{1}{x}\right)=\frac{x+1}{1-x}$

$\Rightarrow \; f^{4}(x)=f\left(\frac{x+1}{1-x}\right)=x$

$\Rightarrow \quad f^{6}(x)=-\frac{1}{x} \Rightarrow f^{6}(6)=-\frac{1}{6}$

$f^{7}(x)=\left(-\frac{1}{x}\right)=\frac{x+1}{1-x} \Rightarrow f^{7}(7)=\frac{8}{-6}=-\frac{4}{3}$

$\therefore \; -\frac{1}{6}+\left(-\frac{4}{3}\right)=-\frac{3}{2}$

22. (19.00)

The desired functions will contain either one element or two elements in its codomain of which '2' always belongs of $f(A)$.

$\therefore$ The set B can be {2}, {1, 2}, {2, 3}, {2, 4}

Total number of functions $=1+\left(2^{3}-2\right)3=19$.

23. (26)

Let $k\;f(k)+2=\lambda(k-2)(k-3)(k-4)(k-5)$ ….. (i)

Put $k = 0$

We get $\lambda=\frac{1}{60}$

Now, put λ in equation (i)

$\Rightarrow \; kf(k)+2=\frac{1}{60}(k-2)(k-3)(k-4)(k-5)$

Put $k=10$

$\Rightarrow 10f(10)+2=\frac{1}{60}(8)(7)(6)(5)=28 \Rightarrow 10f(10)=26$

$\Rightarrow \; 52-10f(10)=52-26=26$

24. (2)

Given that

$a+\alpha=1$

$b+\beta=2$ and $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}$ ….. (i)

Replace x by $\frac{1}{x}$

$$\Rightarrow \quad af\left(\frac{1}{x}\right)+\alpha f(x)=\frac{b}{x}+\beta x \qquad \text{..... (ii)}$$

Adding (i) and (ii),

$$(a+\alpha)f(x)+(a+\alpha)f\left(\frac{1}{x}\right)=x(b+\beta)+(b+\beta)\frac{1}{x}$$

$$\Rightarrow \quad \frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}=\frac{b+\beta}{a+\alpha}=\frac{2}{1}=2.$$

25. (18)

$f(g(x))=8x^2-2x$

$g(f(x))=4x^2+6x+1$

Let $f(x)=ax^2+bx+c$ and $g(x)=dx+e$

Now, $f(g(x))=a(g(x))^2+b(g(x))+c$

$=a(dx+c)^2+b(dx+e)+c$

$g(f(x))=d(f(x))+e$

$d(ax^2+bx+c)+e$

$\because \quad f(g(x))=8x^2+2x$ and $g(f(x))=4x^2+6x+1$

Now, $ad^2=8,\ 2adc+bd=-2$, $ce^2=be+c=0$ and $ad=4, bd=6, cd+e=1$

On solving, $a=2, b=-1, c=2, d=3, c=1$a

$\Rightarrow \quad f(x)=2x^2+3x+1$

$g(x)=2x$

$\Rightarrow \quad f(2)+g(2)=18$

26. (190)

Given a function $f(n)=\begin{cases} 2n, \text{ if } n=1,2,3,5 \\ 2n-11, \text{ if } n=6,7,...10 \end{cases}$

Put n =1, 2, 3, 4...10

$f(1)=2,\ f(2)=4,\ f(3)=6,\ f(4)=8, ...f(6)=1,\ f(7)=3,\ f(8)=5.....f(10)=9$.

Take $gof(n)=\begin{cases} (n+1), \text{ if } n \text{ is odd} \\ (n-1), \text{ if } n \text{ is even} \end{cases}$

Put n = 1, 2, 3, …… 10.

$f(g(1))=2,\ f(g(2))=1,\ f(g(3))=4,\ f(g(4))=3,\ f(g(5))=6,\ f(g(10))=9$

As, $f(g(10))=9$, and $f(10)=9$, then $g(10)=10$.

Similarly, $g(1)=1, g(2)=6, g(3)=2, g(4)=7, g(5)=3$

Put the values in the required expression,

$g(10)(g(1)+g(2))+g(3)+g(4)+g(5)$

$\Rightarrow \quad 10(1+6+2+7+3)$

$\Rightarrow \; 10\times(19)=190$.

27. (2)

Given function is $f(x)=\left(2\left(1-\frac{x^{25}}{2}\right)\left(2+x^{25}\right)\right)^{\frac{1}{50}}$

$f(x)=\left[\left(2-x^{25}\right)\left(2+x^{25}\right)\right]^{\frac{1}{50}}=\left(4-x^{50}\right)^{1/50}$

Take, $f(f(x))=\left(4-\left(\left(4-x^{50}\right)^{1/50}\right)^{50}\right)^{1/50}=x$

Now, $g(x)=f(f(f(x)))+f(f(x))$

$=f(x)+x$

Put $x=1$ in above equation

$g(1)=f(1)+1=3^{1/50}+1$

28. (31)

Given expression is $2f(a)-f(b)+3f(c)+f(d)=0$.

$2f(a)+3f(x)=f(b)-d(d) \qquad ...(\mathrm{i})$

As per given range {0, 1, 2, 3, ……. 10}

Let $f(c)=0$ and $f(a)=1,2,3,4$.

Put the values in equation (i),

$2f(a)+3f(c)=f(b)-d(d)$

$2(1)+3(0)=f(b)-f(d)$

$2(1)+3(0)=f(b)-f(d)$

$f(b)-f(d)=2$

So, total number of choices whose difference 2 are 7.

Similarly, for $f(c)$ = 0, 1, 2, 3.

The total numbers of functions are 31.