1 (A) Solutions

Inequation

1) $\frac{2}{x} < 3 \Rightarrow 3 - \frac{2}{x} > 0 \Rightarrow \frac{3x-2}{x} > 0$

By Wavy curve $x \in (-\infty, 0) \cup (2/3, \infty)$ — C

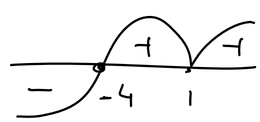
2) $\frac{x+4}{x-3} < 2 \Rightarrow \frac{x+4}{x-3} - 2 < 0 \Rightarrow \frac{10-x}{x-3} < 0$

OR $\frac{x-10}{x-3} > 0 \Rightarrow x \in (-\infty, 3) \cup (10, \infty)$ — A

3) $\frac{2x-3}{3x-5} \geq 3 \Rightarrow \frac{2x-3}{3x-5} - 3 \geq 0 \Rightarrow \frac{-7x+12}{3x-5} \geq 0$

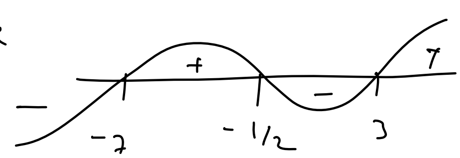
OR $\frac{7x-12}{3x-5} \leq 0 \Rightarrow x \in \left(\frac{5}{3}, \frac{12}{7}\right]$ — B

4) $(x-1)^2(x+4) < 0$ Wavy curve



Hence $x \in (-\infty, -4)$ — B

5) $(2x+1)(x-3)(x+7) < 0$ wavy curve



Sol^n $(-\infty, -7) \cup (-\frac{1}{2}, 3)$ — A

6) $x^2 + 6x - 27 > 0 \Rightarrow (x+9)(x-3) > 0 \Rightarrow x \in (-\infty, -9) \cup (3, \infty)$

Also $x^2 - 3x - 4 < 0 \Rightarrow (x-4)(x+1) < 0 \Rightarrow x \in (-1, 4)$

taking intersection $x \in (3,4)$ — (c)

7) $x^2 - 1 \le 0 \Rightarrow x \in [-1, 1]$

Also $x^2 - x - 2 \geq 0 \Rightarrow (x-2)(x+1) \geq 0$

or $x \in (-\infty, -1] \cup [2, \infty)$

taking intersection $x \in \{-1\}$ — (D)

(8) $\log_{\frac{1}{2}} (x^2 - 5x + 7) > 0$

(i) Domain: $(-\infty, \infty)$ (ii) $x^2 - 5x + 7 < \left(\frac{1}{2}\right)^0$

or $x^2 - 5x + 6 < 0$

or $x \in (2, 3)$

their intersection is $(2,3)$ — (B)

(9) $\log_3 (x^2 - 6x + 11) < 1$

(i) Domain $(-\infty, \infty)$ (ii) $x^2 - 6x + 11 < (3)^1$

$\Rightarrow x^2 - 6x + 8 < 0$

$\Rightarrow x \in (2, 4)$

Intersec^n $(2,4)$ — (B)

(10) $\log_{|x|} (x^2 + (x+1)) \geq 0$

Domain: $x \in R - \{0, \pm 1\}$

Case I $0 < |x| < 1$

$x^2 + x + 1 \le 1$

$\Rightarrow x(x+1) \le 0$

Case II $|x| > 1$

$x^2 + x + 1 \geq 1$

$x(x+1) \geq 0$

$\Rightarrow \quad x(x+1) \le 0$ | $x(x+1) \geq 0$

$x \in [-1, 0]$ | $x \in (-\infty, -1] \cup [0, \infty)$

Intersecⁿ $\Rightarrow S_1 = \boxed{x \in (-1, 0)}$ | Intersecⁿ $\Rightarrow \boxed{S_2 = (-\infty, -1) \cup (1, \infty)}$

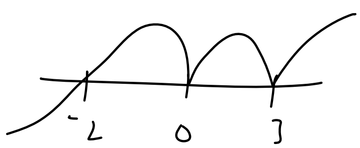
final Ans $S_1 \cup S_2$ or $x \in (-\infty, -1) \cup (-1, 0) \cup (1, \infty)$

— (D)

(11) $x(e^x - 1)(x+2)(x-3)^2 \le 0$

same as $x^2(x+2)(x-3)^2 \le 0$

wavy curve



$\therefore$ Hence $x \in (-\infty, -2] \cup \{0, 3\}$

— (C)

(12) $\left|\dfrac{x^2}{x-1}\right| \le 1 \Rightarrow x^2 \le |x-1|$

Case I $x > 1$

$x^2 - x + 1 \le 0$

$x \in \phi$

Case II $x < 1$

$x^2 + x - 1 \le 0$

$x \in \left[\dfrac{-1-\sqrt{5}}{2}, \dfrac{-1+\sqrt{5}}{2}\right]$

$\therefore$ Solⁿ $x \in \left[\dfrac{-1-\sqrt{5}}{2}, \dfrac{\sqrt{5}-1}{2}\right]$ — (B)

Definition of function

(13) option (c) matches the condition, rest of the options are relation

(14) option (c) as only $x \geq 0$ is allowed but domain is given Real

Even & odd

(15) Conceptual — (D)

(16) Numeration Even, denominator odd

Hence $f(x)$ is odd (A)

(17) $f(x) = \cos\left(\log\left(\sqrt{1+x^2}+x\right)\right)$

$$f(-x) = \cos\left(\log\left(\sqrt{1+x^2}-x\right)\right)$$

$$= \cos\left(\log\left(\frac{1}{\sqrt{1+x^2}+x}\right)\right) \quad \text{Rationalize}$$

$$= \cos\left(-\log\left(x+\sqrt{1+x^2}\right)\right)$$

$$= \cos\left(\log\left(x+\sqrt{1+x^2}\right)\right) = f(x) \Rightarrow \text{even} \quad (A)$$

(18) A & B both odd functions

C is neither even nor odd Hence (D)

(19) (A) $f(x) = x\left(\frac{a^x-1}{a^x+1}\right)$

$$f(-x) = -x\left(\frac{a^{-x}-1}{a^{-x}+1}\right) = -x\left(\frac{1-a^x}{1+a^x}\right) = x\left(\frac{a^x-1}{a^x+1}\right) = f(x)$$

Hence (A)

(20) clearly (B)

(21) As $f(-x) = f(x) \Rightarrow$ Even (C)

Periodic func^n

(22) Since $f(\pi/2 + x) = f(x) \Rightarrow$ fundamental period $\pi/2$ — (B)

(23) Since $f(\pi/2 + x) = f(x) \Rightarrow$ F.P. is $\pi/2$ — (B)

(24) Since $f(x+4) = f(x) \Rightarrow$ F.P is $\pi/4$ — (A)

(25) Since $f(\pi+x) = f(x) \Rightarrow$ F.P is π $\longrightarrow$ (B)

(26) period is $\frac{T}{|k|} = 6\pi$ (A)

(27) (C) Conceptual

Domain, Co-domain & Range

(28) $x+2 > 0 \Rightarrow (-2, \infty)$ — (B)

(29) $f(x) = g(x) \Rightarrow 2x^2 - 1 = 1 - 3x$

$\Rightarrow 2x^2 + 3x - 2 = 0$

$\Rightarrow (2x-1)(x+2) = 0$

$x \in \{-2, 1/2\}$ — (D)

(30) $\frac{3-x}{2} > 0 \Rightarrow x \in (-\infty, 3)$ — (B)

(31) $\cos^{-1}(4x-1) \Rightarrow -1 \le 4x-1 \le 1$

(31) $\cos^{-1}(4x-1) \Rightarrow -1 \le 4x-1 \le 1$

$\Rightarrow 0 \le 4x \le 2$

$\Rightarrow 0 \le x \le \frac{1}{2}$ — (B)

(32) $\log|x^2-9| \Rightarrow |x^2-9| > 0$

Ans $x \in R - \{3,-3\}$ — (C)

(33) $f(x) = \sqrt{x-1} + \sqrt{6-x} \Rightarrow x-1 \geq 0$ & $6-x \geq 0$

Intersecⁿ $x \in [1,6]$ — (B)

(34) $f(x) = \sqrt{2-2x-x^2} \Rightarrow 2-2x-x^2 \geq 0$

$\Rightarrow x^2+2x-2 \le 0$

$x \in [-1-\sqrt{3}, -1+\sqrt{3}]$ — (B)

(35) $f(x) = \sin^{-1} 5x \Rightarrow -\frac{1}{5} \le x \le \frac{1}{5}$ — (B)

(26) Conceptual — (B)

(37) $f(x) = \sin \frac{\pi[x]}{2}$ Basically $\rightarrow \sin \frac{n\pi}{2}$ $n \in$ Integer

$\therefore$ Range $\{-1, 0, 1\}$ — (B)

(38) $f(x) = \begin{cases} 1 & \text{if } x > 3 \\ -1 & \text{if } x < 3 \end{cases}$

Hence (B)

(39) Conceptual — (D)

(40) $-1 \le \sin x \le 1$

$\Rightarrow -7 \le -7\sin x \le 7$

$\Rightarrow 2 \le 9 - 7\sin x \le 16$ Ans (B)

(41) $-1 \le \sin 3x \le 1$

$\Rightarrow 1 \le 2 - \sin 3x \le 3$

$\Rightarrow 1 \ge \frac{1}{2-\sin 3x} \ge \frac{1}{3}$ Ans (A)

(42) Conceptual (C)

(43) put $f(x) = -1 \Rightarrow x^2 = -1 \Rightarrow x \in \phi$ — (C)

(44) $f(x) = \cos 2x - \sin 2x \Rightarrow$ Range $[-\sqrt{2}, \sqrt{2}]$

Its subset is (B)

(45) $f(x) = \frac{+\cancel{x}}{\cancel{x}} = 1$ as $x \in [3,7]$ $\therefore$ option (C)

(46) $f(x) = \frac{1}{\sqrt{x - [x]}}$ $\therefore x - [x] > 0$

We know $x - 1 < [x] \le x \quad \forall x \in R$

i.e. $x \ge [x] \quad \forall x \in R$

Hence for $x > [x]$ $\boxed{x \in R - Z}$ (B)

(47) $f(x) = 2 + x - [x-3]$

$= 5 + (x-3) - [x-3] = 5 + \{x-3\} \Rightarrow f(x) \in [5, 6)$

(B)

(48) $f(x+y) = f(x) f(y) \Rightarrow f(x) = a^x$

Also $f(1) = 2 \Rightarrow a = 2$ $\therefore f(x) = 2^x$

Then $\sum_{k=1}^{n} f(a+k) = f(a+1) + f(a+2) + \dots + f(a+n)$

$= 2^{x+1} + 2^{x+2} + \dots + 2^{x+n}$

$= 2^{x+1}(2^n - 1) \Rightarrow x+1 = 4 \Rightarrow x = 3$ — (C)

(49) Conceptual (C)

(50) $f(x+2) = \frac{(x+1)(x+2)}{2} = \frac{x+2}{x} \cdot \frac{x(x+1)}{2}$

$= \frac{x+2}{x} \cdot f(x+1)$ — (B)

(51) $f(x+ay, x-ay) = axy$ let $x+ay = X$

$x - ay = Y$

$\Rightarrow x = \frac{X+Y}{2}, \quad y = \frac{X-Y}{2a}$

Hence $f(X, Y) = a\left(\frac{X+Y}{2}\right)\left(\frac{X-Y}{2a}\right) = \frac{X^2 - Y^2}{}$ (B)

Hence $f(X, Y) = a\left(\frac{X+Y}{2}\right)\left(\frac{X-Y}{2a}\right) = \frac{X^2-Y^2}{4}$ (B)

(52) $\frac{f(xy) + f(x/y)}{f(x)f(y)} = \frac{\cos(\log x + \log y) + \cos(\log x - \log y)}{\cos(\log x)\cdot\cos(\log y)}$

$= \frac{2\cancel{\cos(\log x)\cos(\log y)}}{\cancel{\cos(\log x)\cos(\log y)}} = 2$ — (D)

(53) $f(x) = |x| + |x-1|$

If $x \in (0,1) \Rightarrow f(x) = x - (x-1)$

$= 1$ — (A)

(54) $f(2x+3y, 2x-7y) = 20x$

let $2x+3y = \alpha$, $2x-7y = \beta \Rightarrow x = \frac{7\alpha+3\beta}{20}$

$f(\alpha, \beta) = \cancel{20}\left(\frac{7\alpha+3\beta}{\cancel{20}}\right)$ — (B)

(55) $f(x) = \log_a x \Rightarrow f(ax) = \log_a(ax)$

$= 1 + \log_a x = 1 + f(x)$ — (B)

(56) $f(y) = \frac{ay-c}{cy-a} = \frac{a\left(\frac{ax-c}{cx-a}\right) - c}{c\left(\frac{ax-c}{cx-a}\right) - a} = x$ — (A)

(57) one-one but not onto as not every integer is attained — (A)

(58) $\left\{ x^2 \right.$, $x \geq 0$

(58) $f(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$ → it is both one-one & onto

— (C)

(59) $f(x) = \dfrac{x^2}{1+x^2}$, $f(-x) = \dfrac{x^2}{1+x^2} = f(x)$

Since Even $\Longrightarrow$ Many-one

& range of $f(x)$ is $[0, 1) \Rightarrow$ Into

(A)

(60) f is a bijection, (draw graph) — (D)

(61) $f(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$ clearly many-one & Into

— (D)

(62) $f : (\pi/2, 3\pi/2) \to R$, By graph '$f$' is bijection — (C)

(63) in $[\pi/2, 3\pi/2]$ Sinx is one-one & since Range = Codomain
it is onto
also — (C)

(6) $f(x) = x - (-1)^x$

$f(x) = \begin{cases} x-1 & \text{when } x \in \text{Even natural} \\ x+1 & \text{when } x \in \text{odd natural} \end{cases}$

which is clearly one-one & onto as range of $f(x)$ is 'N'

— (C)

(65) $f(x) = e^{x} + e^{-x}$

& $f(-x) = e^{-x} + e^{x} = f(x)$ $\Rightarrow$ Even funcⁿ

$\Rightarrow$ Many-one

But By AM/GM $\frac{e^{x}+e^{-x}}{2} \geq 1$ $\Rightarrow$ $f(x) \geq 2$ $\therefore$ Range $\subset$ Codomain

$\Rightarrow$ Into

— (C)

(66) $f: R \to [-1,1]$, in 'R' $\sin x$ is many-one as periodic

& but range is also $[-1,1]$

$\therefore$ onto

— (C)

(67) $f: R \to R$; $\boxed{f(x) = 1}$ $\to$ Many one & Into — (C)

(68) $f: Z \to Z$ $f(x) = x+2$ is a bijection as it is both one-one & onto

— (B)

(69) $f: [-1,1] \to [-1,1]$

$f(x) = \sin\frac{\pi x}{2}$ is bijection as it is one-one & onto — (B)

(70) $f: R \to R^{+}$; $f(x) = e^{-x}$ is onto

as range of e^{-x} is $(0,\infty)$ — (B)

(71) Conceptual — (C)

(72) $f(-1) = 2$
$f(2) = 2$ $\Rightarrow$ clearly many-one

Also $f(x) = x^2 - x$ will not have many integers in its range $\Rightarrow$ Into — (D)

(73) Conceptual. $f(g(x)) = 2g(x) = 2x = g(x) + g(x)$ — (C)

(74) Conceptual (B)

(75) $$f(g(x)) = g^2(x) + 2g(x) - 3$$
$$= (3x-4)^2 + 2(3x-4) - 3$$
$$= 9x^2 - 18x + 5$$ — (B)

(76) As $f(2) = -2 \Rightarrow g(f(2))$ is undefined — (D)

(77) $$g(f(x)) = e^{f(x)} = e^{x^2 + \frac{1}{x^2}} = e^{x^2} \cdot e^{1/x^2}$$ — (D)

(78) $$g(f(x)) = 2x - 1$$
$$= \frac{2}{3}(3x) - 1 = \frac{2}{3}(f(x) - 4) - 1$$

$\therefore g(f(x)) = \frac{2}{3}(f(x)) - 11/3 \quad \therefore g(x) = \frac{2x - 11}{3}$ — (C)

(79) $f(g(x)) = (x+3)^2$ & $g(x) = x+3$

put $x = -6$ $\quad g(-6) = -3$

$f(g(-6)) = 9$

$\Rightarrow f(-3) = 9$ — (C)

(80) If $fog(x) = gof(x) \Rightarrow f$ & g inverse of each other

$f(0) = b \Rightarrow 0 = g(b)$

(80) If $fog(x) = gof(x) \Rightarrow f$ & g inverse of each other

$\Rightarrow f(g(x)) = x \quad \forall x \in$ Domain | $f(0) = b \Rightarrow 0 = g(b)$

Also $g(0) = d \Rightarrow 0 = f(d)$

— (C)

(81) $f(g(x)) = \dfrac{1-g(x)}{1+g(x)} = \dfrac{1-4x+4x^2}{1+4x-4x^2}$ — (A)

(82) Conceptual — (D)

(83) $f(f(x)) = \dfrac{1-\left(\frac{1-x}{1+x}\right)}{1+\left(\frac{1-x}{1+x}\right)} = x$

$\therefore f(f(\sin\theta)) = \sin\theta$ — (A)

(84) $f(f(x)) = \left(a-(f(x))^n\right)^{1/n}$

$= \left(a-(a-x^n)\right)^{1/n} = x$ — (B)

(85) $f(g(x)) = \log\left(\dfrac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{(3x+x^3)}{1+3x^2}}\right) = \log\left(\dfrac{(1+x)^3}{(1-x)^3}\right) = 3f(x)$

— (B)

(86) $f(f(\sqrt{4})) = f(0) = 1$ — (C)

(87) $f(g(y)) = \dfrac{g(y)}{\sqrt{1-g^2(y)}} = \dfrac{\frac{y}{\sqrt{1+y^2}}}{\sqrt{1-\frac{y^2}{1+y^2}}} = y$ — (C)

$\sqrt{1 - \frac{y^2}{1+y^2}}$

(88) $g(f(x)) = \cos(\pi[x]) = \cos(n\pi)$ $\therefore$ Range $\{-1, 1\}$ — (B)

(89) let 'a' be pre-image $\Rightarrow f(a) = 2 \Rightarrow a^2+3 = 2$

$\therefore a = \phi$ — (D)

(90) option (D) is a bijection $\therefore$ it has inverse — (D)

(91) $f(x) = 2^x$ $\therefore$ for Inverse $f(y) = x$

$\Rightarrow 2^y = x$

$\Rightarrow y = \log_2 x$ — (C)

(92) for Inverse $f(y) = x \rightarrow \frac{e^y - e^{-y}}{e^y + e^{-y}} = x - 2$

Apply componendo & dividendo $\Rightarrow \frac{e^y}{-e^{-y}} = \frac{x-1}{x-3}$

$\Rightarrow e^{2y} = \frac{1-x}{x-3} \Rightarrow y = \frac{1}{2}\log\left(\frac{1-x}{x-3}\right)$ — (D)

(93) for Inverse let $f(y) = x \Rightarrow y + \frac{1}{y} = x$

$\Rightarrow y^2 - xy + 1 = 0 \Rightarrow y = \frac{x \pm \sqrt{x^2-4}}{2}$

$f^{-1}(x) = \frac{x + \sqrt{x^2-4}}{2}$ as Range of f^{-1} is $[1, \infty)$ — (A)

(94) let $f(y) = x \Rightarrow x = \ln(y + \sqrt{1+y^2})$

$$y+\sqrt{1+y^2}=e^{x}$$

Rationalize $\Rightarrow$ $\sqrt{1+y^2}-y=e^{-x}$ $\Rightarrow$ $y=\dfrac{e^{x}-e^{-x}}{2}$ — (C)

(95) Range of f is $\{-1, 0, 7, 26\}$

that is domain of f^{-1} — (C)

(96) let $f(y)=x \Rightarrow \left(4-(y-7)^3\right)^{1/5}=x$

$$\Rightarrow 4-(y-7)^3=x^5$$

$$\Rightarrow (4-x^5)^{1/3}+7=y \quad \text{— (C)}$$

(97) $(f\circ g)^{-1}(x) = g^{-1}\circ f^{-1}(x)$

$$= \frac{2+\log_e x}{3} \quad \text{— (B)}$$

$g^{-1}(x)=\dfrac{x+2}{3}$

$f^{-1}(x)=\ln x$

1(B) Solutions

Tuesday, January 30, 2024 2:18 PM

① Conceptual 10^{10} — ⓒ

② $f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right) = \log\left(\frac{(1+x)^2}{(1-x)^2}\right) = 2f(x)$

— ⓒ

③ $[\pi^2] = 9$, $[-\pi^2] = -10$

$\therefore f(x) = \cos 9x + \cos 10x \quad \therefore f(-\pi) = 2 \longrightarrow$ (D)

④ $f(x+y) = f(x) + f(y) \implies f(x) = kx$

As $f(1) = 7 \Rightarrow k = 7$

Hence $\sum_{r=1}^{n} f(r) = \sum_{r=1}^{n} 7r = 7\frac{n(n+1)}{2}$ — (D)

⑤ $f(x) = \frac{1}{\sqrt{x-2+2+2\sqrt{2}\sqrt{x-2}}} + \frac{1}{\sqrt{x-2+2-2\sqrt{2}\sqrt{x-2}}}$

$= \frac{1}{\sqrt{(\sqrt{x-2}+\sqrt{2})^2}} + \frac{1}{\sqrt{(\sqrt{x-2}-\sqrt{2})^2}}$

$= \frac{1}{\sqrt{x-2}+\sqrt{2}} + \frac{1}{|\sqrt{x-2}-\sqrt{2}|}$ put $x = 11$

$= \frac{1}{3+\sqrt{2}} + \frac{1}{3-\sqrt{2}} = \frac{6}{7}$ — ⓒ

⑥ $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ $\quad \therefore x+3 > 0$ & $x^2+3x+2 \neq 0$

$x \in (-3, \infty)$ & $x \neq \{-1, -2\}$

— ⓒ

(7) Domain (i) $x - x^2 \geq 0$ (ii) $4 + x \geq 0$ (iii) $4 - x \geq 0$

$x \in [0,1]$ & $x \in [-4, \infty)$ & $x \in (-\infty, 4]$

$\therefore$ Intersecⁿ $x \in [0,1]$ —— (D)

(8) Domain: $\log(x^2 - 6x + 6) \geq 0$

$\Rightarrow x^2 - 6x + 6 \geq 1$

$\Rightarrow x^2 - 6x + 5 \geq 0$

$\Rightarrow x \in (-\infty, 1] \cup [5, \infty)$ — (C)

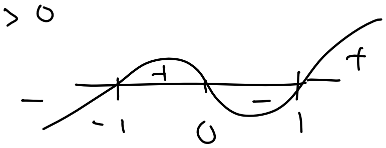
(9) $\cos^{-1}\left(\log_2(x/2)\right)$

$\Rightarrow -1 \leq \log_2(x/2) \leq 1$

or $1/2 \leq \frac{x}{2} \leq 2 \Rightarrow x \in [1, 4]$ — (A)

(10) clearly $4 - x^2 \neq 0$ & $x^5 - x^3 > 0$

$x \neq \{\pm 2\}$ & $x^3(x-1)(x+1) > 0$



& $(-1, 0) \cup (1, \infty)$

Ans $(-1, 0) \cup (1, 2) \cup (2, \infty)$ — (D)

(11) $\log_{3+x}(x^2 - 1)$ $\therefore$ $3 + x > 0$, $3 + x \neq 1$ $\Rightarrow x \in (-3, -2) \cup (-2, \infty)$

& $x^2 - 1 > 0$ $\Rightarrow$ $x \in (-\infty, -1) \cup (1, \infty)$

Their intersection $(-3,-2) \cup (-2,-1) \cup (1,\infty)$

— (C)

(12) Domain $2\sin^{-1}(2x) + \pi/3 \geq 0$ & $x \in [-1/2, 1/2]$

$\sin^{-1}(2x) \geq -\pi/6$

$2x \geq -1/2$

or $x \geq -1/4$ Ans $x \in \left[-\frac{1}{4}, 1/2\right]$ — (A)

(13) $f(x) = \dfrac{1+x^2}{x^2} = 1 + \dfrac{1}{x^2}$

$\therefore$ clearly $f(x) \in (1, \infty)$ — (B)

(14) We know $-1 \leq -\sin 3x \leq 1$

$1 \leq 2 - \sin 3x \leq 3$

$1 \geq \dfrac{1}{2 - \sin 3x} \geq \dfrac{1}{3}$

Range is $\left[\frac{1}{3}, 1\right]$ — (A)

(15) $-|a| \leq a\cos(bx+c) \leq |a|$

$\therefore$ $d-|a| \leq d + a\cos(bx+c) \leq d+|a|$

Assuming 'a' to be positive — (D)

(16) $0 \leq \cos^2 x \leq 1$

$0 \leq \dfrac{\pi}{4}\cos^2 x \leq \pi/4$

$1 \leq \sec\left(\dfrac{\pi}{4}\cos^2 x\right) \leq \sqrt{2}$ — (A)

$1 \le \sec\left(\frac{\pi}{4}\cos^2 x\right) \le \sqrt{2}$ — (A)

(17) let $y = \dfrac{x^2+3x+1}{x^2+x+1} \Rightarrow (y-1)x^2 + (y-3)x + y-1 = 0$

for $x \in R$ $\quad D \geq 0$

$(y-3)^2 - 4(y-1)^2 \geq 0$

$\Rightarrow (y+1)(3y-5) \leq 0$

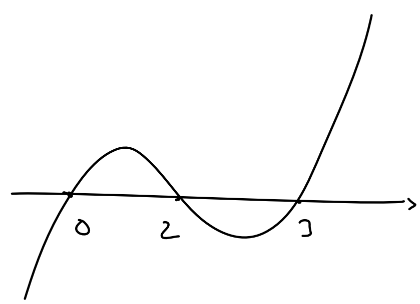
$y \in [-1, 5/3]$ — (D)

(18) Since $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ only $\Rightarrow$ one-one

But Range of $f(x)$ is not all integer $\Rightarrow$ Into

— (B)

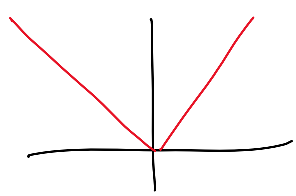
(19) By graph



Many-one & onto (B)

(20) By formula $3^5 - {}^3C_1 2^5 + {}^3C_2$ — (B)

(21) By graph



Many-one & Into

— (D)

(22) put $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ Hence one-one

find range : $y = \dfrac{x-m}{x-n} \Rightarrow x = \dfrac{ny-m}{y-1}$

$\therefore$ Range : $R - \{1\} \neq$ Co-domain

$\therefore$ Range : $R - \{1\} \neq$ Co-domain
$\therefore$ Into — (B)

(23) $f(n) = \begin{cases} \frac{n-1}{2}, & n \text{ odd} \\ -\frac{(n+2)}{2}, & n \text{ Even} \end{cases}$

$\therefore$ when n odd
let $n = 2a+1$ $(a \in W)$
When n Even
$n = 2b$ $(b \in N)$

$f(a,b) = \begin{cases} a, & a \in W \\ -(b+1), & b \in N \end{cases}$ $\Longrightarrow$ $f(x)$ is one-one but not onto as range $\in Z - \{-1\}$ — (A)

(24) (A) $f(x) = x\left(\frac{a^x - 1}{a^x + 1}\right)$

$f(-x) = -x\left(\frac{a^{-x} - 1}{a^{-x} + 1}\right) = -x\left(\frac{1 - a^x}{1 + a^x}\right) = f(x)$ Even
— (A)

(25) Since $f(-x) = f(x)$ $\Longrightarrow$ Symm abt y-axis — (B)

(26) LCM of 4, 6, 4 is 12 $\therefore$ period 12 — (D)

(27) Since odd $\Rightarrow$ $\boxed{f(0) = 0}$ & period 2 $\Rightarrow$ $f(4) = f(0) = 0$
— (A)

(28) Conceptual 'T1' — (B)

(29) $f(x) = 1 + 2x - [2x] = 1 + \{2x\}$ $\therefore$ period $\frac{1}{2}$ — (B)

(30) LCM of $2(n-1)$ & $2n$ is $2n(n-1)$ — (C)

(31) $g(f(x)) = f^2(x) = (2x-1)^2$ — (B)

$\therefore \ g(f(x)) = f(x) = (2x+1)$ —

(32) $g(-3) = 10 \quad \therefore f(g(-3)) = f(10) = 121$ — (A)

(33) Assuming onto, $f(x) = 2^x$ is one-one $\Rightarrow$ invertible — (A)

(34) $gof(x) = 4x^2 - 10x + 4 \quad \& \ g(x) = x^2 + x - 2$

$\therefore g(f(x)) = f^2(x) + f(x) - 2$

clearly $f(x)$ is linear

$\therefore$ let $f(x) = 2x + a$

$g(f(x)) = (2x+a)^2 + (2x+a) - 2$

$\therefore$ upon Comparing $a = -3 \quad \therefore$ (A)

(35) $$f(g(y)) = \frac{\frac{y}{\sqrt{1+y^2}}}{\sqrt{1 - \frac{y^2}{1+y^2}}} = y$$ — (C)

(36) $$f(f(x)) = \frac{2\left(\frac{2x-3}{x-2}\right) - 3}{\frac{2x-3}{x-2} - 2} = \frac{x}{1}$$ — (A)

(37) $f(g(x)) = x + 2\sqrt{x} + 1 + 2$

$= (\sqrt{x}+1)^2 + 2 \quad \Rightarrow f(x) = x^2 + 2$ — (B)

$= g^2(x) + 2$

(38) $$f(f(x)) = \frac{\frac{x}{2x-1}}{\frac{2x}{2x-1} - 1} = x$$ $\rightarrow$ Domain $x \neq \frac{1}{2}$

$\therefore f(f(f(x))) = f(x) = \dfrac{x}{2x-1} \quad \therefore$ Domain $x \neq 1/2$

$\therefore f(f(f(x))) = f(x) = \dfrac{x}{2x-1}$ $\therefore$ Domain $x \neq 1/2$

<u>Ans</u> (B)

(39) $f(x) = \dfrac{2x+1}{3x-2}$ $\therefore f(2) = 5/4$

$\therefore f(f(2)) = \dfrac{5/2+1}{\frac{15}{4}-2} = \dfrac{7/2}{7/4} = 2$ — (D)

(40) $g(f(x)) = [|x|]$; $f(g(x)) = |[x]|$

$\therefore fog(x) \geq g(f(x)) \Rightarrow$ green graph above red $\forall x \in R$ — (D)

(41) let $f(y) = x \Rightarrow 3^{y(y-2)} = x$

$\Rightarrow y^2 - 2y - \log_3 x = 0$

$\Rightarrow y = 1 + \sqrt{1 + \log_3 x}$ [(-)ve rejected as range of f^{-1} is $[1,\infty)$]

—— (B)

(42) $f(x) = \{x\}$ is periodic $\Rightarrow$ Not invertible — (C)

(43) as $\sec^{-1}$ is anyway $[0,\pi]$

so sufficient condition is

$$\frac{2-|x|}{4} \geq 1 \quad \text{or} \quad \frac{2-|x|}{4} \leq -1$$

$\Rightarrow \quad 2-|x| \geq 4$

$|x| \leq -2$

$x \in \phi$

$2-|x| \leq -4$

$|x| \geq 6$

$\Rightarrow \quad x \in (-\infty, -6] \cup [6, \infty)$

— (D)

(44) $y = \dfrac{x-1}{x^2-2x+3} \Rightarrow yx^2 - (2y+1)x + 3y+1 = 0$

for $x \in$ Real, $D \geq 0$

$(2y+1)^2 - 4y(3y+1) \geq 0$

$-8y^2+1 \geq 0 \Rightarrow y \in \left[\dfrac{-1}{2\sqrt{2}}, \dfrac{1}{2\sqrt{2}}\right]$

— (D)

(45) $f(x) = \cos^{-1}\left(\dfrac{1+x^2}{2x}\right) + \sqrt{2-x^2}$

Note: $\dfrac{x^2+1}{2x} \in (-\infty, -1] \cup [1, \infty)$ so for $\cos^{-1}$ to exist x can only be ± 1

$\therefore$ Domain is $x \in \langle -1, 1 \rangle$

$\therefore$ find $f(1)$ & $f(-1)$ $\therefore$ Range $\langle 1, 1+\pi \rangle$ — (D)

(46) $f(x) = \dfrac{\tan(\pi[x^2-x])}{1+\sin(\cos x)}$

Since Numerator is $\tan(n\pi)$

$\therefore$ Numerator is 0 $\forall x \in R$

$\therefore f(x) = 0 \quad \forall x \in R$ — (D)

(47) $f(x) = \dfrac{e^x}{1+[x]}, \quad x \geq 0$

$1+[x]$,

least is when $x = 0$ i.e. 1

max is ∞ when $x \to \infty$

$\therefore$ Range $[1, \infty)$ — D

(48) $f(x) = \dfrac{1}{1-x}$ $(x \neq 1)$

$f(f(x)) = \dfrac{1}{1-\left(\frac{1}{1-x}\right)} = \dfrac{x-1}{x}$ $(x \neq 0, 1)$

$f(f(f(x))) = 1-(1-x) = x$ $\therefore$ $(x \neq 0, 1)$ — C

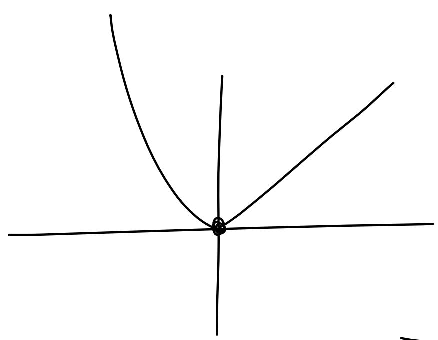
(49) Conceptual (A)

(50) $f(f(x)) = \begin{cases} \cancel{f^2(x), \quad f(x) < 0} & \therefore \text{ rejected} \\ f(x), \quad f(x) \geq 0 \end{cases}$

$f(f(x)) = f(x) \ \forall x \in R$

where

$y = f(x)$



clearly $f(x) \geq 0 \ \forall x \in R$ —— D

(51) LCM of 2π & π is '2π' — A

(52) Conceptual —— D

(53) $f(6\pi+x) = \dfrac{\cos(\sin(6n\pi + nx))}{\tan\left(\frac{6\pi}{n} + \frac{x}{n}\right)} = \dfrac{\cos(\sin nx)}{\tan\left(\frac{6\pi}{n} + \frac{x}{n}\right)} = \dfrac{\cos(\sin nx)}{\tan(x/n)}$

if $n = 1, 2, 3, 6$

Ans — ABCD

54) $f(x) = \sin 3\pi\{x\} + \tan \pi \cancel{[x]}$

periodic & period 1 — A

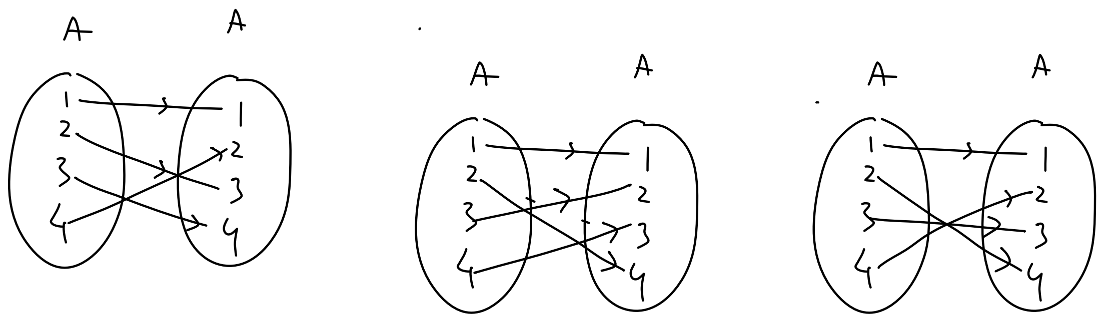
55) odd extension is given by $-f(-x) \ \forall\ x \in (-\infty, 0]$

i.e. $-[\sin(\cos x) + x - \tan x]$

or $-\sin(\cos x) - x + \tan x$ — D

56) C $f(x) = \log(x^2 - x + 1)$

$f(-x) = \log(x^2 + x + 1) \rightarrow$ neither Even nor odd — C

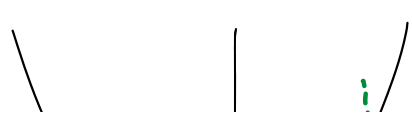
57) 

only 3 functions — D

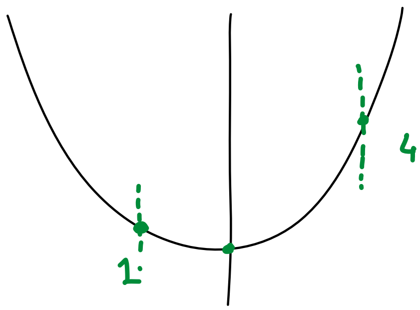
58) $f(x) = \sin(\{x+3\}) = \sin(\{x\})$ ∴ period 1 — C

59) $f(2+t) = f(2-t) \ \forall\ t \in R \Rightarrow$ graph of $f(x)$ is symm abt $x = 2$

∴ vertex of $f(x)$ lies on $x = 2$



clearly $f(4) > f(1) > f(2)$



$x = 2$

clearly $f(4) > f(1) > f(2)$

— (B)

(60) $f(x) = 2\tan 3x + 5\sqrt{2}\,|\sin 3x| \longrightarrow$ period $\pi/3$

$\therefore$ option (A)

(61) $[x]^2 - 5[x] + 6 = 0 \Rightarrow [x] = 2$ or $[x] = 3$

$\Rightarrow x \in [2,3) \cup x \in [3,4)$

$\Rightarrow x \in [2,4)$ — (D)

(62) $\left[\log_2\left(\frac{x}{[x]}\right)\right] \geq 0$ as $[x] \neq 0 \Rightarrow x \notin [0,1)$

$\Rightarrow \log_2\left(\frac{x}{[x]}\right) \geq 0$

$\Rightarrow \frac{x}{[x]} \geq 1$

cases

If $x \in [1,\infty)$

$x \geq [x]$

Always true

$S_1 \equiv [1,\infty)$

or

If $x \in (-\infty, 0)$

$x \leq [x]$

only true at integers

$\therefore$ $x \in$ Negative integers also

Hence — (D)

(63) $2[x] = x + \{x\}$

$\Rightarrow \quad [x] = 2\{x\}$ — (i)

Since LHS integer $\therefore 2\{x\} = 0$ or $2\{x\} = 1$

$\therefore \{x\} = 0$ or $\{x\} = 1/2$

from (i) $[x] = 0$ | from (i) $[x] = 1$

$\therefore x = [x] + \{x\}$ | $x = [x] + \{x\}$

$x = 0$ | $x = 3/2$ — (B)

(64) $[x]^2 = [x] + 2\{x\}$

$\Rightarrow \quad \dfrac{[x]^2 - [x]}{2} = \{x\}$

Since $\{x\} \in [0,1)$ $\therefore [x]$ can only take $0, 1$

If $[x] = 0$ | If $[x] = 1$

$\{x\} = 0$ | $\{x\} = 0$

$\therefore x = 0$ | $\Rightarrow x = 1$

— (A)

(65) $[x^2] + x = a$ Since $a \in N$ $\therefore x$ also must be integer

We can put $x = 1, 2, 3, 4 \rightarrow$ gives 4 values of $a \le 20$

— (C)

(66) $[x + [2x]] < 3$ $\quad \{ [x + I] = [x] + I \}$

$[x] + [2x] < 3$

or $[x] + [2x] \le 2$ — (A)

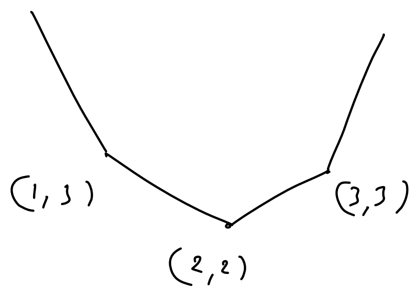
clearly $x \le 0$ is a sol^n —— (1)

If $x > 0$ then (i) $x \in (0, 1/2)$ Ⓐ is true | (ii) $x \in [1/2, 1)$ Ⓐ is true | (iii) $x \ge 1$ ϕ as Ⓐ not true

$\therefore$ final ans $(-\infty, 0] \cup (0, 1/2) \cup [1/2, 1)$

OR $(-\infty, 1)$ —— (d)

(67) plot $f(x)$



$\therefore$ min at $x=2$ is 2

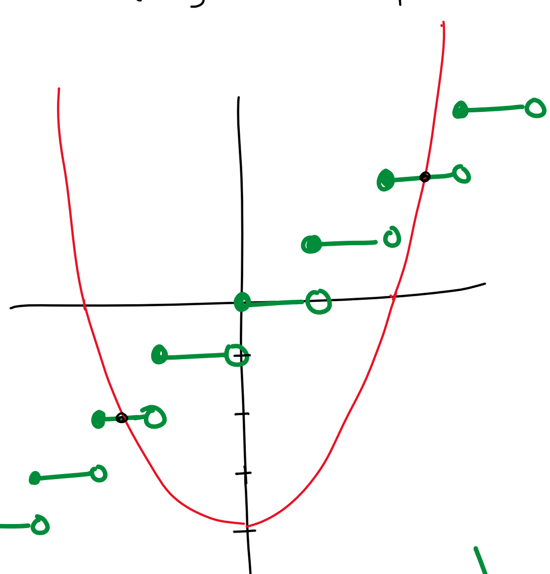
—— (B)

(68) observe that $1 \le |\sin x| + |\cos x| \le \sqrt{2}$ $\forall x \in R$

$\therefore$ $[|\sin x| + |\cos x|] = 1$ $\forall x \in R$ — (C)

(69) $x^2 - 4 = [x]$ let $f(x) = x^2 - 4$, $g(x) = [x]$

plot $f(x)$ & $g(x)$



As 2 intersection pts

$\implies$ 2 sol^n

— (B)

70) plot all 3 $y = x^3$

$y = x^2$

$y = 1/64$

clearly max of all 3 gives

$$f(x) = \begin{cases} \frac{1}{64}, & 0 \le x \le 1/8 \\ x^2, & \frac{1}{8} < x \le 1 \\ x^3, & 1 < x < \infty \end{cases} \quad \text{— (C)}$$

71) $f(-x) = f(x) \quad \forall x \in R$

$$(-ax+\cancel{b})\cos x - (-c\cancel{x}+d)\sin x = (ax+\cancel{b})\cos x + (c\cancel{x}+d)\sin x \quad \forall x \in R$$

Simplify $\quad ax\cos x + d\sin x = 0 \quad \forall x \in R \Rightarrow a, d = 0$ — (c)

72) $f(x) + g(x) = e^x \quad \forall x \in R$

$\therefore f(-x) + g(-x) = e^{-x} \Rightarrow f(x) - g(x) = e^{-x} \quad \forall x \in R$

$\therefore$ solving $\quad f(x) = \dfrac{e^x + e^{-x}}{2}, \quad g(x) = \dfrac{e^x - e^{-x}}{2}$

$\therefore f^2 - g^2 = 1$ — (D)

73) Reflecn of $A(5, k)$ is $B(k, 5)$ & it lies on $f(x)$

$\Rightarrow f(k) = 5 \Rightarrow k = 2$

$\therefore B(2, 5) \therefore$ reflecn about origin is $-2, -5$

— (A)

$\therefore$ B(2,5) $\therefore$ reflecⁿ about origin is $-2,-5$

— A

74) let $f^{-1}(4) = a \Rightarrow 4 = f(a)$

But $f(x) = 2x^3 + 7x - 5$

see $\boxed{f(1) = 4} \Rightarrow a = 1$ — A

75) $f(\pi + x) = f(x) \Rightarrow$ 'π' period

76) Conceptual — A

77) for range to have only integers & 'f' to be continuous it must be a constant funcⁿ — D

78) $g(-1, -3/2) = (-1) - (-3/2) = 1/2$

$g(-4, -1.75) = (-1.75) - (-4) = 2.25$

$\therefore f(1/2, 2.25) = (2.25)^{1/2} = 1.5$ — D

79) it can be seen that expression goes both upto $+\infty$ & $-\infty$ $\therefore$ range R — D

80) Conceptual option D satisfies

1 (C) Solⁿ

① $f(x) = \sqrt{\cos^{-1}(2x) + \pi/6}$ Since $\cos^{-1}(x)$ is always ≥ 0

Domain : $-\frac{1}{2} \leq x \leq \frac{1}{2}$

$\therefore$ $f^{-1}(x)$ range is same as domain of $f(x)$

ie $\left[-\frac{1}{2}, \frac{1}{2}\right]$ $\therefore$ $\boxed{a+b=0}$

② $f : (2,4) \longrightarrow (1,3)$ as $x \in (2,4)$

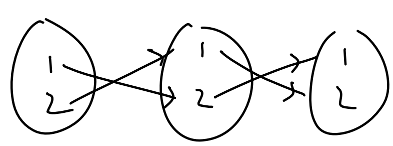
$\frac{x}{2} \in (1,2)$ $\therefore$ $[x/2] = 1$

Hence $f(x) = x - 1$ $\therefore$ $f^{-1}(x) = x+1$ $\therefore$ $\boxed{\alpha = 1}$

③ LCM of 1, 6, 10 is $\boxed{30}$

④ $f(f(i)) = i \quad \forall \ i = 1, 2, 3, \dots 10$

ex



so basically divide 10 no. into 5 groups each containing 2

No. of ways $\dfrac{(10)!}{(2!)^5 \, 5!}$

Every group formation $\Longrightarrow$ 1 unique funcⁿ

$\therefore$ $\boxed{\text{Ans } 945}$

⑤ $f(2x^2-1)$ → then $-1 \leq 2x^2 - 1 \leq 3$

$\Longrightarrow$ $0 \leq 2x^2 \leq 4$

$\Rightarrow \quad x^2 \le 2$

Hence $x \in \{-1, 0, 1\}$ $\quad$ Ans $\boxed{3}$

(6) let $f(y) = x \Rightarrow x = y + \frac{1}{y} \Rightarrow y^2 - xy + 1 = 0$

$\Rightarrow y = \frac{x + \sqrt{x^2 - 4}}{2}$ $\quad$ ((-)ve rejected as range of f^{-1} is $[2, \infty)$)

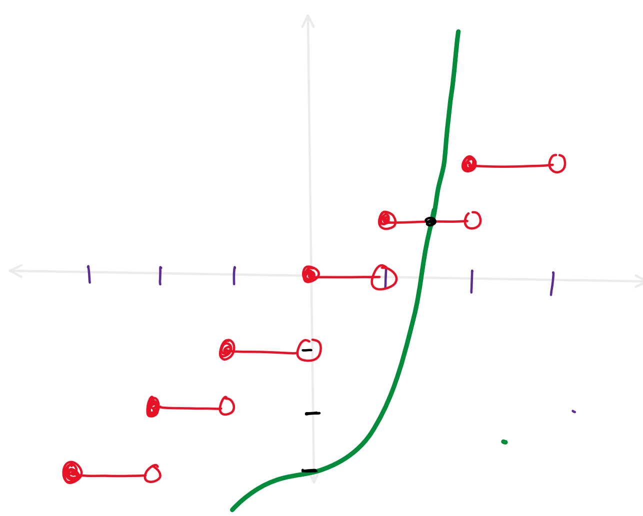
(7) let's find max of $\cos(\sin x)$, which is 1 when $x = 0$

$\therefore \boxed{f(x)_{min} = 1}$

(8) $x^3 - 3 = [x]$

plot $y = x^3 - 3$
& $y = [x]$

clearly only $\boxed{1 \text{ sol}^n}$



(9) Conceptual

(10) let $f(x) = x^2 - 3x + 4$ $\quad$ let $f : [3/2, \infty) \to [7/4, \infty)$

$\therefore f^{-1}(x) = \frac{3}{2} + \sqrt{x - 7/4}$ $\quad$ $f^{-1} : [7/4, \infty) \to [3/2, \infty)$

$\therefore f(x) = f^{-1}(x)$ Same as solving $f(x) = x$ as $f(x)$ Inc

i.e. $x^2 - 4x + 4 = 0$

i.e. $x^2 - 4x + 4 = 0$

$\therefore \boxed{x = 2}$ (which satisfies & in domain)

(11) Conceptual

(12) $f(x) = \begin{cases} \frac{x}{2} + 2, & x \le 2 \\ 5 - x, & 2 < x < 3 \\ 11 - (x-6)^2, & x \geq 3 \end{cases}$

$\therefore f(x) = 2$ possible when $x = 0$, $x = 3, 9$

$f(f(x)) = 2$ let 'α' be a solⁿ $\Rightarrow f(f(\alpha)) = 2$

$\Rightarrow f(\alpha) = 0, 3, 9$

$f(\alpha) = 0 \longrightarrow 2$ values of x

$f(\alpha) = 3 \longrightarrow 3$ " of x

$f(\alpha) = 9 \longrightarrow 2$ values of x

Total values $= 7$

(13) $f\left(\frac{x+1}{x-1}\right) = 2f(x) + \frac{1}{x-1}$ —— (i)

$x \rightarrow \frac{x+1}{x-1} \Rightarrow f(x) = 2f\left(\frac{x+1}{x-1}\right) + \frac{x-1}{2}$ —(ii)

from (i) & (ii)

$f(x) = 4f(x) + \frac{2}{x-1} + \frac{x-1}{2}$

OR $f(x) = -\frac{1}{3}\left(\frac{x-1}{2} + \frac{2}{x-1}\right)$

$\therefore 6f(0) = -2\left[-\frac{1}{2} - 2\right] = 5$

(14) $|2x-1| = 3[x] + 2\{x\} \Rightarrow |2x-1| = 2x + [x]$

Case I $x \geq 1/2$ | Case II $x < 1/2$

$|2x-1| = 3[x] + 2\{x\}$

<u>Case I</u> $\quad x \geq 1/2$

$\cancel{2x} - 1 = \cancel{2x} + [x]$

$[x] = -1$

$\Rightarrow x \in \phi$

<u>Case II</u> $\quad x < 1/2$

$-2x + 1 = 2x + [x]$

$\Rightarrow [x] = 1 - 4x$

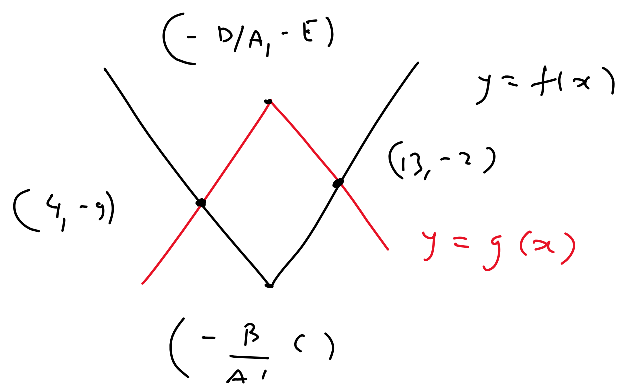
$x = 1/4$

$\therefore \boxed{\text{Ans } 4}$

(15) $\quad f(x) = |Ax + B| + C$

$g(x) = -|Ax + D| - E$

By graph



Since it is a // gm
By equating mid pts $\quad \therefore \quad -\dfrac{(B+D)}{A} = 17 \quad \& \quad C - E = -11$

$$\left| E - C + \frac{B+D}{A} \right| = |11 - 17| = 6$$

1. (A)

Given: $f(x)=2x+\sin x, x\in R$

$\Rightarrow f'(x)=2+\cos x$. Now, $-1\le\cos x\le 1$

$\Rightarrow 1\le 2+\cos x\le 3 \Rightarrow 1\le 2+\cos x\le 3$

$\therefore f'(x)>0, \forall x\in R$

$\Rightarrow f(x)$ is strictly increasing and therefore one-one

Also as $x\to\infty$, $f(x)\to\infty$ and $x\to-\infty$, $f(x)\to-\infty$

$\therefore$ Range of $f(x)=R=$ domain of $f(x)\Rightarrow f(x)$ is onto.

Hence, $f(x)$ is one-one and onto.

2. (B)

Given: $f:[0,\infty)\to[0,\infty)$ and $f(x)=\dfrac{x}{x+1}$

$\therefore f'(x)=\dfrac{1+x-x}{(1+x)^2}=\dfrac{1}{(1+x)^2}>0\,\forall x$

$\therefore f$ is an increasing function $\Rightarrow f$ is one-one.

Now, $D_f=[0,\infty)$

For range let $\dfrac{x}{1+x}=y\Rightarrow x=\dfrac{y}{1-y}$

Now, $x\ge 0 \Rightarrow 0\le y<1$

$\therefore R_f=[0,1)\ne$ Co-domain,

$\therefore f$ is not onto.

3. (D)

$f(x)=x^2+2bx+2c^2 \Rightarrow f(x)=(x+b)^2+2c^2-b^2$

$\Rightarrow f_{\min}=2c^2-b^2$ and $g(x)=-x^2-2cx+b^2$

$g(x)=-(x+c)^2+b^2+c^2 \Rightarrow g_{\max}=b^2+c^2$

For $f_{\min}>g_{\max}\Rightarrow 2c^2-b^2>b^2+c^2$

$\Rightarrow c^2>2b^2\Rightarrow|c|>|b|\sqrt{2}$

4. (D)

$f(x)=e^{x^2}+e^{-x^2} \Rightarrow f'(x)=2x\left(e^{x^2}-e^{-x^2}\right)\ge 0, \forall x\in[0,1]$

$\therefore f(x)$ is in increasing function on [0, 1]

$\therefore f_{\max}=f(1)=e+\dfrac{1}{e}=a;\ g(x)=xe^{x^2}+e^{-x^2}$

$\Rightarrow g'(x)=\left(2x^2+1\right)e^{x^2}-2xe^{-x^2}\ge 0, \forall x\in[0,1]$

$\therefore\ g(x)$ is an increasing function on [0, 1]

$\therefore\ g_{\max} = g(1) = e + \frac{1}{e} = b$

$h(x) = x^2 e^{x^2} + e^{-x^2}$

$\Rightarrow\ h'(x) = 2x\left[e^{x^2}\left(1+x^2\right) - e^{-x^2}\right] \geq 0,\ \forall x \in [0, 1]$

$\therefore\ h(x)$ is an increasing function on [0, 1]

$\therefore\ h_{\max} = h(1) = e + \frac{1}{e} = c$

$\therefore\ a = b = c$.

5. (B)

Given: $f(x) = 2x^3 - 15x^2 + 36x + 1$

$\Rightarrow\ f'(x) = 6x^2 - 30x + 36$

$= 6\left(x^2 - 5x + 6\right) = 6(x-2)(x-3)$

$\because\ f'(x) > 0\ \forall\ x \in [0, 2)$ and $f'(x) < 0\ \forall\ x \in (2, 3)$

$\therefore\ f(x)$ is increasing on [0, 2) and decreasing on (2, 3)

$\therefore\ f(x)$ is many one on [0, 3]

Also $f(0) = 1,\ f(2) = 29,\ f(3) = 28$

$\therefore$ Absolute min = I and Absolute max = 29

$\therefore$ Range of f = [1, 29] = codomain

Hence f is onto.

6. (A)

Number of onto function such that exactly three elements in $x \in A$ such that $f(x) = \frac{1}{2}$ is equal to

$= {}^7C_3, \left\{2^4 - 2\right\} = 14.{}^7C_3$

7. (C)

$$f_1(x) = f_{0+1}(x) = f_0\left(f_0(x)\right) = \frac{1}{1 - \frac{1}{1-x}} = \frac{x-1}{x}$$

$$f_2(x) = f_{1+1}(x) = f_0\left(f_1(x)\right) = \frac{1}{1 - \frac{x-1}{x}} = x$$

$$f_3(x) = f_{2+1}(x) = f_0\left(f_2(x)\right) = f_0(x) = \frac{1}{1-x}$$

$$f_4(x) = f_{3+1}(x) = f_0\left(f_3(x)\right) = \frac{x-1}{x}$$

$$\therefore\quad f_0 = f_3 = f_6 = \ldots\ldots = \frac{1}{1-x}$$

$f_1 = f_4 = f_7 = ... = \frac{x-1}{x}$ f

$f_2 = f_5 = f_8 = ... = x$

$$f_{100}(3) = \frac{3-1}{3} = \frac{2}{3}\ f_1\left(\frac{2}{3}\right) = \frac{\frac{2}{3}-1}{\frac{2}{3}} = -\frac{1}{2}$$

$$f_2\left(\frac{3}{2}\right) = \frac{3}{2}$$

$$\therefore\quad f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right) = \frac{5}{3}$$

8. (D)

$$f(g(x)) = x$$

$$\Rightarrow\quad f\left(3^{10}x-1\right) = 2^{10}\left(3^{10}\cdot x-1\right)+1 = x$$

$$\Rightarrow\quad 2^{10}\left(3^{10}x-1\right)+1 = x$$

$$\Rightarrow\quad x\left(6^{10}-1\right) = 2^{10}-1$$

$$\Rightarrow\quad x = \frac{2^{10}-1}{6^{10}-1} = \frac{1-2^{-10}}{3^{10}-2^{-10}}$$

9. (D)

$$\left.\begin{array}{l} f(1) = 1-5\left[\frac{1}{5}\right] = 1 \\ f(5) = 6-5\left[\frac{6}{5}\right] = 1 \end{array}\right\} \rightarrow \text{Many one}$$

$f(10) = 10-5(2) = 0$ which is not in co-domain.

Neither one-one nor onto.

10. (C)

Domain and codomain = {1, 2, 3, .., 20}.

There are five multiple of4 as 4, 8, 12, 16 and 20 and there are 6 multiple of3 as 3, 6,9, 12, 15, 18.

Since, whenever k is multiple of 4 then $f(k)$ is multiple of 3 then total number of arrangement

$= {}^6C_5 \times 5! = 6!$

Remaining 15 elements can be arranged in 15! ways.

Since, for every input, there is an output

$\Rightarrow$ function $f(k)$ is onto

$\therefore$ Total number of arrangements = 15!.6!

11. (B)

$$\because\quad \phi(x) = ((hof)og)(x)$$

$$\because\quad \phi\left(\frac{\pi}{3}\right) = h\left(f\left(g\left(\frac{\pi}{3}\right)\right)\right) = h\left(f\left(\sqrt{3}\right)\right) = h\left(3^{1/4}\right)$$

$$=\frac{1-\sqrt{3}}{1+\sqrt{3}}=-\frac{1}{2}\left(1+3-2\sqrt{3}\right)=\sqrt{3}-2=-\left(-\sqrt{3}+2\right)$$

$$=-\tan 15^\circ=\tan\left(180^\circ-15^\circ\right)=\tan\left(\pi-\frac{\pi}{12}\right)=\tan\frac{11\pi}{12}$$

12. (B)

$$(gof)(x)=g\left(f(x)\right)=f^2(x)+f(x)-1$$

$$g\left(f\left(\frac{5}{4}\right)\right)=4\left(\frac{5}{4}\right)^2-10\cdot\frac{5}{4}+5=-\frac{5}{4} \qquad [\because\ g\left(f(x)\right)=4x^2-10x+5\ ]$$

$$g\left(f\left(\frac{5}{4}\right)\right)=f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)-1$$

$$-\frac{5}{4}=f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)-1$$

$$f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)+\frac{1}{4}=0$$

$$\left(f\left(\frac{5}{4}\right)+\frac{1}{2}\right)^2=0$$

$$f\left(\frac{5}{4}\right)=-\frac{1}{2}$$

13. (C)

Given that $f:A\to B$ and $g:B\to C$

$\therefore\quad f^{-1}B\to A$ and $g^{-1}:C\to B$

We have $(gof)^{-1}=f^{-1}og^{-1}:C\to A$

$\therefore\quad f$ must be one-one and g will be onto function

14. (B)

For finding inverse of $f(x)$

Let $y=\frac{x-2}{x-3}\ \Rightarrow\ xy-3y=x-2\ \Rightarrow\ x\left(y-1\right)=3y-2$

$\therefore\ f^{-1}(x)=\frac{3x-2}{x-1}$

Similarly for inverse of $g(x)$

$y=2x-3\ \Rightarrow\ x=\frac{y+3}{2}\ \Rightarrow\ g^{-1}(x)=\frac{x+3}{2}$ y

$\therefore\quad f^{-1}(x)+g^{-1}(x)=\frac{13}{2}$

$\therefore\quad \frac{3x-2}{x-1}+\frac{x+3}{2}=\frac{13}{2}$

$\Rightarrow\quad 6x-4+x^2+2x-3=13x-13$

$\Rightarrow\quad x^2-5x+6=0\ \Rightarrow\ \left(x-2\right)\left(x-3\right)=0$

$\Rightarrow\ x = 2$ or 3.

15. (C)

$y = 5^{\log x}$

$\Rightarrow\ \log y = \log x \cdot \log 5$ [taking log on both sides]

$\Rightarrow\ \log x = \dfrac{\log y}{\log 5} = \log_5 y$

$\Rightarrow\ x = e^{\log 5^y}\ \Rightarrow\ x = y^{\log_5 e}\ \Rightarrow\ x = y^{\frac{1}{\log_5}}$

16. (B)

Putting value of K from 1 to 10, we get

$f(1) = f(2) = 2$

$f(3) = f(4) = 4$

$f(5) = f(6) = 6$

$f(7) = f(8) = 8$

$f(9) = f(10) = 10$

Since, $g(f(x)) = f(x)$

$\therefore\ gof(1) = f(1)\ \Rightarrow\ g(2) = f(1) = 2$

$gof(2) = f(2)\ \Rightarrow\ g(2) = f(2) = 2$

$gof(3) = f(3)\ \Rightarrow\ g(4) = f(3) = 4$

$\therefore$ The image of 2, 4, 6, 8, 10 in function $g(x)$ should be 2, 4, 6, 8, 10 respectively. Therefore, image of each of remaining elements can be any of 10 elements.

Hence, number of possible $g(x)$ is 10^5.

17. (D)

$f : N - \{1\} \to N \qquad f(a) = \alpha$

Where α is max of powers of prime P such that p^α divides a. Also $g(a) = a + 1$

$\therefore\ f(2) = 1 \qquad g(2) = 3$

$f(3) = 1 \qquad g(3) = 4$

$f(4) = 2 \qquad g(4) = 5$

$f(5) = 1 \qquad g(5) = 6$

$\Rightarrow\ f(2) + g(2) = 1 + 3 = 4$

$f(3) + g(3) = 1 + 4 = 5$

$f(4) + g(4) = 2 + 5 = 7$

$f(5) + g(5) = 1 + 6 = 7$

$\therefore$ Many one $f(x) + g(x)$ does not contain 1

$\Rightarrow$ into function

18. (B)

Given that f is bijective function and $f(3) \geq f(9) \geq f(15) \geq f(21) \geq ..... \geq f(99)$

So, all elements 3, 9, 15 …. 99 i.e. 17 elements as 1 choice .
Remaining 50 – 17 = 33 elements has taken from 50 elements.
$\therefore$ Number of ways $= {}^{50}P_{33}$

19. (D)

Given function is $f(x) = \begin{cases} 2n; & n = 2, 4, 6, 8, ... \\ (n-1); & n = 3, 7, 11, 15, ... \\ \left(\frac{n+1}{2}\right); & n = 1, 5, 9, 13, ... \end{cases}$

When n = 2, 4, 6, then $2n$ is the multiple of 4,
When n = 3, 7, 11, 15 then $(n – 1)$ is not multiple of 4.
When n = 1, 5, 9, 13, then $\left(\frac{n+1}{2}\right)$ is the odd number.
Every number gives exactly one value.
Thus, f is one-one & onto.

20. (D)

Given, $f(x) = x - 1; \; g(x) = \frac{x^2}{x^2 - 1}$

Now, $f(g(x)) = g(x) - 1$

$= \frac{x^2}{x^2-1} - 1 = \frac{x^2 - x^2 + 1}{x^2 - 1}$

Hence, $f(g(x)) = \frac{1}{x^2 - 1}; \; x \neq \pm 1$

Thus, $f(g(x))$ will be even function

$\Rightarrow$ $f(g(x))$ is may one function.

Let $y = \frac{1}{x^2 - 1}$ or $y.x^2 - y = 1$

$x^2 = \left(\frac{1+y}{y}\right)$

$\left(\frac{1+y}{y}\right) \geq 0$

Range : $y \in (-\infty, -1] \cup (0, \infty)$

Hence, Range $\neq$ Co-domain $\Rightarrow$ $f(g(x))$ is into function.

21. (B)

$f(x)=\frac{x-1}{x+1}$

Given $f^{n+1}(x)=f\left(f^{n}(x)\right)$

$$\Rightarrow \quad f^{2}(x)=f\left(f(x)\right)=\frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}=-\frac{1}{x}$$

$$f^{3}(x)=f\left(f^{2}(x)\right)=f\left(-\frac{1}{x}\right)=\frac{x+1}{1-x}$$

$$\Rightarrow \ f^{4}(x)=f\left(\frac{x+1}{1-x}\right)=x$$

$$\Rightarrow \quad f^{6}(x)=-\frac{1}{x} \Rightarrow \ f^{6}(6)=-\frac{1}{6}$$

$$f^{7}(x)=\left(-\frac{1}{x}\right)=\frac{x+1}{1-x} \Rightarrow \ f^{7}(7)=\frac{8}{-6}=-\frac{4}{3}$$

$$\therefore \ -\frac{1}{6}+\left(-\frac{4}{3}\right)=-\frac{3}{2}$$

22. (19.00)

The desired functions will contain either one element or two elements in its codomain of which '2' always belongs of $f(A)$.

$\therefore$ The set B can be {2}, {1, 2}, {2, 3}, {2, 4}

Total number of functions $=1+\left(2^{3}-2\right)3=19$.

23. (26)

Let $k\ f(k)+2=\lambda(k-2)(k-3)(k-4)(k-5)$ ….. (i)

Put k = 0

We get $\lambda=\frac{1}{60}$

Now, put λ in equation (i)

$$\Rightarrow \ kf(k)+2=\frac{1}{60}(k-2)(k-3)(k-4)(k-5)$$

Put $k=10$

$$\Rightarrow 10f(10)+2=\frac{1}{60}(8)(7)(6)(5)=28 \Rightarrow 10f(10)=26$$

$$\Rightarrow \ 52-10f(10)=52-26=26$$

24. (2)

Given that

$a+\alpha=1$

$b+\beta=2$ and $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}$ ….. (i)

Replace x by $\frac{1}{x}$

$$\Rightarrow \quad af\left(\frac{1}{x}\right)+\alpha f(x)=\frac{b}{x}+\beta x \qquad \text{..... (ii)}$$

Adding (i) and (ii),

$$(a+\alpha)f(x)+(a+\alpha)f\left(\frac{1}{x}\right)=x(b+\beta)+(b+\beta)\frac{1}{x}$$

$$\Rightarrow \quad \frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}=\frac{b+\beta}{a+\alpha}=\frac{2}{1}=2.$$

25. (18)

$f(g(x))=8x^2-2x$

$g(f(x))=4x^2+6x+1$

Let $f(x)=ax^2+bx+c$ and $g(x)=dx+e$

Now, $f(g(x))=a(g(x))^2+b(g(x))+c$

$=a(dx+c)^2+b(dx+e)+c$

$g(f(x))=d(f(x))+e$

$d(ax^2+bx+c)+e$

$\because \quad f(g(x))=8x^2+2x$ and $g(f(x))=4x^2+6x+1$

Now, $ad^2=8,\ 2adc+bd=-2$, $ce^2=be+c=0$ and $ad=4, bd=6, cd+e=1$

On solving, $a=2, b=-1, c=2, d=3, c=1$a

$\Rightarrow \quad f(x)=2x^2+3x+1$

$g(x)=2x$

$\Rightarrow \quad f(2)+g(2)=18$

26. (190)

Given a function $f(n)=\begin{cases} 2n, \text{ if } n=1,2,3,5 \\ 2n-11, \text{ if } n=6,7,\ldots 10 \end{cases}$

Put n =1, 2, 3, 4...10

$f(1)=2,\ f(2)=4,\ f(3)=6,\ f(4)=8, \ldots\ldots f(6)=1,\ f(7)=3,\ f(8)=5\ldots\ldots f(10)=9$.

Take $gof(n)=\begin{cases} (n+1), \text{ if } n \text{ is odd} \\ (n-1), \text{ if } n \text{ is even} \end{cases}$

Put n = 1, 2, 3, …… 10.

$f(g(1))=2,\ f(g(2))=1,\ f(g(3))=4,\ f(g(4))=3,\ f(g(5))=6,\ f(g(10))=9$

As, $f(g(10))=9$, and $f(10)=9$, then $g(10)=10$.

Similarly, $g(1)=1, g(2)=6, g(3)=2, g(4)=7, g(5)=3$

Put the values in the required expression,

$g(10)(g(1)+g(2))+g(3)+g(4)+g(5)$

$\Rightarrow \quad 10(1+6+2+7+3)$

$\Rightarrow \ 10\times(19)=190$.

27. (2)

Given function is $f(x)=\left(2\left(1-\frac{x^{25}}{2}\right)\left(2+x^{25}\right)\right)^{\frac{1}{50}}$

$f(x)=\left[\left(2-x^{25}\right)\left(2+x^{25}\right)\right]^{\frac{1}{50}}=\left(4-x^{50}\right)^{1/50}$

Take, $f(f(x))=\left(4-\left(\left(4-x^{50}\right)^{1/50}\right)^{50}\right)^{1/50}=x$

Now, $g(x)=f(f(f(x)))+f(f(x))$

$=f(x)+x$

Put $x=1$ in above equation

$g(1)=f(1)+1=3^{1/50}+1$

28. (31)

Given expression is $2f(a)-f(b)+3f(c)+f(d)=0$.

$2f(a)+3f(x)=f(b)-d(d)$ ...(i)

As per given range {0, 1, 2, 3, ……. 10}

Let $f(c)=0$ and $f(a)=1,2,3,4$.

Put the values in equation (i),

$2f(a)+3f(c)=f(b)-d(d)$

$2(1)+3(0)=f(b)-f(d)$

$2(1)+3(0)=f(b)-f(d)$

$f(b)-f(d)=2$

So, total number of choices whose difference 2 are 7.

Similarly, for $f(c)=0, 1, 2, 3$.

The total numbers of functions are 31.

2 (A) Solutions

① $f(a+x) = b + \left(1+b^3 - 3b^2 f(x) + 3b f^2(x) - f^3(x)\right)^{1/3} \quad \forall x \in R$

$\Rightarrow \quad f(a+x) = b + \left(1 + (b - f(x))^3\right)^{1/3} \quad \forall x \in R$

$\Rightarrow \quad (f(a+x) - b)^3 = 1 + (b - f(x))^3 \quad \forall x \in R$

$\Rightarrow \quad (f(a+x) - b)^3 + (f(x) - b)^3 = 1 \quad — (i) \quad \forall x \in R$

$x \to a+x$

$(f(2a+x) - b)^3 + (f(a+x) - b)^3 = 1 \quad - (ii)$

(i) − (ii) given

$(f(2a+x) - b)^3 = (f(x) - b)^3 \quad \forall x \in R$

$\Rightarrow \quad f(2a+x) = f(x) \quad \forall x \in R \quad \therefore$ period $2a$ (B)

② $f(x) = \log\left(\dfrac{x^2 - 5x + 6}{x^2 + x + 1}\right) + \sqrt{\dfrac{1}{[x^2 - 1]}}$

(i) $\dfrac{x^2 - 5x + 6}{x^2 + x + 1} > 0$

$\Rightarrow \quad (x-3)(x-2) > 0$

$x \in (-\infty, 2) \cup (3, \infty)$

(ii) $[x^2 - 1] \geq 1$

$\Rightarrow \quad x^2 - 1 \geq 1$

$x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$

taking intersecn

$(-\infty, -\sqrt{2}] \cup [\sqrt{2}, 2) \cup (3, \infty) \quad —$ (D)

③ clearly (i) $-1 \leq \dfrac{1-|x|}{3} \leq 1$ & (ii) $-1 \leq \dfrac{|x| - 3}{5} \leq 1$

$-3 \le |x|-1 \le 3$ & $-5 \le |x|-3 \le 5$

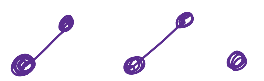
$|x| \le 4$ & $|x| \le 8$

$\Rightarrow x \in [-4,4]$ & $x \in [-8,8]$

Intersection is $[-4,4]$ — (A)

(4) clearly $2\{x\}^2 - 3\{x\} + 1 \geq 0$

$(2\{x\}-1)(\{x\}-1) \geq 0 \Rightarrow \boxed{\{x\} \le 1/2}$



clearly

$x \in [-1, -\frac{1}{2}] \cup [0, \frac{1}{2}] \cup \{1\}$

— (B)

(5) $\cos(\sin x) > 0 \ \forall x \in R$

for domain $\log_x \{x\} \geq 0$

Case I $x \in (0,1)$

$\Rightarrow \{x\} \le 1$

$\boxed{x \in (0,1)}$

— (D)

Case II $x \in (1,\infty)$

$\Rightarrow \{x\} \geq 1$

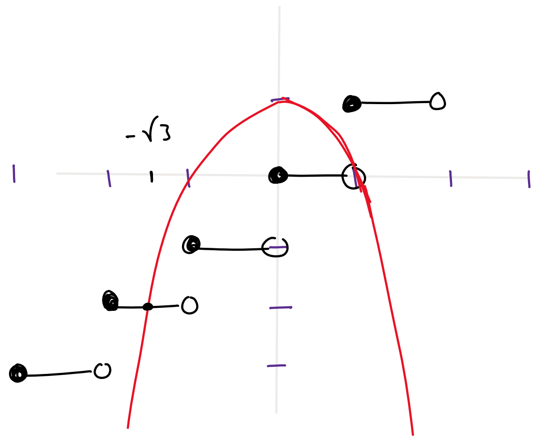
$x \in \phi$

(6) clearly $[x] - 1 + x^2 \geq 0$

OR $[x] \geq 1 - x^2$

plot $y = [x]$

& $y = 1 - x^2$



They intersect when

$[x] = -2$

$\therefore$ sol^n $\quad -2 = 1 - x^2$

$$\boxed{x = -\sqrt{3}}$$

Sol^n $\quad (-\infty, -\sqrt{3}] \cup [1, \infty)$ — D

(7) Since $\sin^{-1}$ $\quad \therefore \quad -1 \le \left[\log_2\left(x^2/2\right)\right] \le 1$

$$\Rightarrow \quad -1 \le \log_2\left(x^2/2\right) < 2$$

$$\Rightarrow \quad \frac{1}{2} \le x^2/2 < 4$$

$$\Rightarrow \quad 1 \le x^2 < 8$$

$\therefore \quad x \in (-2\sqrt{2}, -1] \cup [1, 2\sqrt{2})$ — D

(8) for domain $\quad -1 \le [x] - x^2 + 4 \le 2$

$\Rightarrow \quad [x] \ge x^2 - 5 \quad \& \quad [x] \le x^2 - 2$

rest of solution similar to Q6 — B

(9) $f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$

Domain $\quad x \in [-1, 1]$

But since $\sin^{-1}x + \cos^{-1}x = \pi/2$

$\therefore \quad f(x) = \frac{\pi}{2} + \tan^{-1}x$

as $\quad x \in [-1, 1]$

$-\frac{\pi}{4} \le \tan^{-1}x \le \pi/4$

$\pi/4 \le \frac{\pi}{2} + \tan^{-1}x \le \frac{3\pi}{4}$

Range $\quad [\pi/4, 3\pi/4]$ — D

Range $[\pi/4, 3\pi/4]$ — (D)

(10) $f(x) = \sin^{-1}\left[x^2+\frac{1}{2}\right] + \cos^{-1}\left[x^2-\frac{1}{2}\right]$

Since G.I.F $\Rightarrow$ [ ] can only take $-1, 0$ or 1

Case I $\left[x^2+\frac{1}{2}\right] = -1$

$-1 \le x^2+\frac{1}{2} < 0$

$\Rightarrow -2 \le x^2-\frac{1}{2} < -1$

$\Rightarrow \left[x^2-\frac{1}{2}\right] = -2$

$\therefore$ Second term not-defined

Case II $\left[x^2+\frac{1}{2}\right] = 0$

$\Rightarrow 0 \le x^2+\frac{1}{2} < 1$

$\Rightarrow -1 \le x^2-\frac{1}{2} < 0$

$\Rightarrow \left[x^2-\frac{1}{2}\right] = -1$

$\therefore f(x) = \sin^{-1}0 + \cos^{-1}-1$

$= \pi$

Case III $\left[x^2+\frac{1}{2}\right] = 1$

$1 \le x^2+1/2 < 2$

$0 \le x^2-1/2 < 1$

$\therefore \left[x^2-1/2\right] = 0$

$\therefore f(x) = \sin^{-1}1 + \cos^{-1}0$

$= \pi$

$\therefore$ Range of $f(x) = \{\pi\}$ — (B)

(11) observe that $\frac{3}{4} \le x^2+x+1 < \infty$

$\frac{\sqrt{3}}{2} \le \sqrt{x^2+x+1} < \infty$

or $\frac{\sqrt{3}}{2} \le \sqrt{x^2+x+1} \le 1$ (as $\sin^{-1}$)

$\therefore \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \le \sin^{-1}\sqrt{x^2+x+1} \le \sin^{-1}1$ ($\sin^{-1}$ Increasing function)

$\therefore$ Range $[\pi/3, \pi/2]$ — (C)

(12) observe $0 \le \frac{x^2}{\sqrt{1+x^2}} < \infty$

$\sqrt{1+x^2}$

or $\quad 0 \le \dfrac{x^2}{\sqrt{1+x^2}} \le 1 \quad$ (for $\cos^{-1}$)

$$\cos^{-1} 0 \ge \cos^{-1}\left(\frac{x^2}{\sqrt{1+x^2}}\right) \ge \cos^{-1} 1$$

<u>Ans</u> Range $[0, \pi/2]$ —— (C)

(13) observe $\cos 1 \le \cos(\sin x) \le 1$

$$\ln(\cos 1) \le \ln(\cos(\sin x)) \le 0$$

since this expression is either (-)ve or 0

for f(x) to be defined it can only be '0'

$\therefore$ Range $\{0\}$ —— (D)

(14) $f(f(x)) = \begin{cases} f(x), & f(x) \in Q \\ 1-f(x), & f(x) \in Q^c \end{cases}$

(i) when $x \in Q$
$f(x) \in Q$ & $f(x) = x$,

(ii) when $x \in Q^c$
$f(x) \in Q^c$ & $f(x) = 1-x$

$f \circ f(x) = \begin{cases} x, & x \in Q \\ 1-(1-x), & x \in Q^c \end{cases}$

$\Rightarrow f \circ f(x) = x \quad \forall x \in R$ —— (A)

(15) $f(-x) = -f(x) \quad \forall x \in [-4, 4]$ as 'f' is odd

$$-\cot(\sin x) + \left[\frac{x^2}{|a|}\right] = -\cot(\sin x) - \left[\frac{x^2}{|a|}\right] \quad \forall x \in [-4, 4]$$

$\Rightarrow \quad \left[\frac{x^2}{|a|}\right] = 0 \quad \forall x \in [-4, 4]$

$\Rightarrow$ only possible when $|a| > 16$ we will have $0 \leq \frac{x^2}{|a|} < 1 \quad \forall x \in [-4,4]$

$\therefore$ sol^n $a \in (-\infty, -16) \cup (16, \infty)$ — (B)

(16) let $f^{-1}(x) = y \Rightarrow x = f(y)$

$\Rightarrow x = y(4-y)$

$\Rightarrow y^2 - 4y + x = 0$

$\Rightarrow y = 2 + \sqrt{4-x}$ — (B)

$\left\{\begin{array}{l} (-\text{ive rejected as} \\ \text{range of } f^{-1} \text{ is } [2, \infty) \end{array}\right\}$

(17) let $f^{-1}(x) = y \Rightarrow x = f(y)$

$\Rightarrow x = 2^{y(y-2)}$

$\Rightarrow y(y-2) = \log_2 x$

$\Rightarrow y^2 - 2y - \log_2 x = 0$

$\Rightarrow y = 1 - \sqrt{1 + \log_2 x}$ — (A)

$\left\{\begin{array}{l} (+\text{ive rejected as} \\ \text{range of } f^{-1} \text{ is } (-\infty, 1] \end{array}\right\}$

(18) $f(x) = ax + \cos x$

$f'(x) = a - \sin x$

Case I $f(x)$ is Increasing $\forall x \in R$

$\therefore f'(x) \geq 0 \quad \forall x \in R$

$\Rightarrow a - \sin x \geq \quad \forall x \in R$

$\Rightarrow \boxed{a \geq 1}$

Case II $f(x)$ is decreasing $\forall x \in R$

$\therefore f'(x) \leq 0 \quad \forall x \in R$

$a - \sin x \leq 0 \quad \forall x \in R$

$\Rightarrow \boxed{a \leq -1}$

$\underline{\underline{\text{Sol}^n}}$ $a \in (-\infty, -1] \cup [1, \infty)$ —— (C)

(19) observe $2 \le x^4 - 2x^2 + 3 < \infty$

$$\log_{1/2} 2 \ge \log_{\frac{1}{2}} (x^4 - 2x^2 + 3) > -\infty$$

$$-1 \ge \log_{1/2} (x^4 - 2x^2 + 3) > -\infty$$

$$\Rightarrow \cot^{-1}(-1) \le \cot^{-1}\left(\log_{0.5}(x^4 - 2x^2 + 3)\right) < \cot^{-1}(-\infty)$$

$$\frac{3\pi}{4} \le \cot^{-1}\left(\log_{0.5}(x^4 - 2x^2 + 3)\right) < \pi$$

$\underline{\text{An}}$ (C)

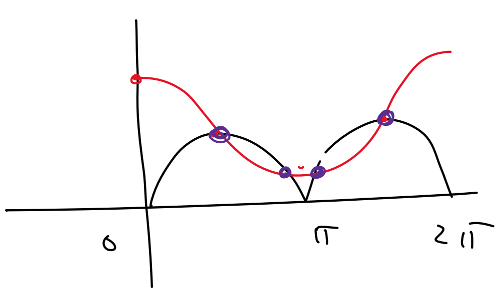
(20) $[x]\{x\} = 1$ — (i) as $\{x\} \ge 0 \Rightarrow [x] > 0$ for (i) to be true

if $[x] = m$ ($m \in N$) $\Rightarrow \{x\} = \frac{1}{m}$

$\therefore$ clearly infinite sol^n $\because x = [x] + \{x\}$

$x = m + \frac{1}{m}$ — (D)

(21) plot graph $y = 2^{\cos x}$, $y = |\sin x|$ in $[0, 2\pi]$ as both periodic



$\therefore$ 4 sol^n in $[0, 2\pi]$

$\therefore$ $12 + 2 = 14$ sol^n in $[-2\pi, 5\pi]$

— (B)

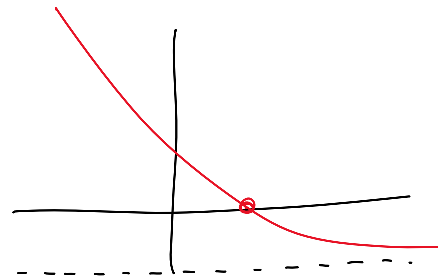
22) let $f(x) = \left(\frac{2}{5}\right)^x + \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$

$$f'(x) = \left(\frac{2}{5}\right)^x \cdot \ln(2/5) + \left(\frac{3}{5}\right)^x \ln(3/5) + \left(\frac{4}{5}\right)^x \ln(4/5)$$

observe $f'(x) < 0 \quad \forall x \in R \Rightarrow f(x)$ decreasing $\forall x \in R$

$\therefore f(x) \to +\infty$ if $x \to -\infty$

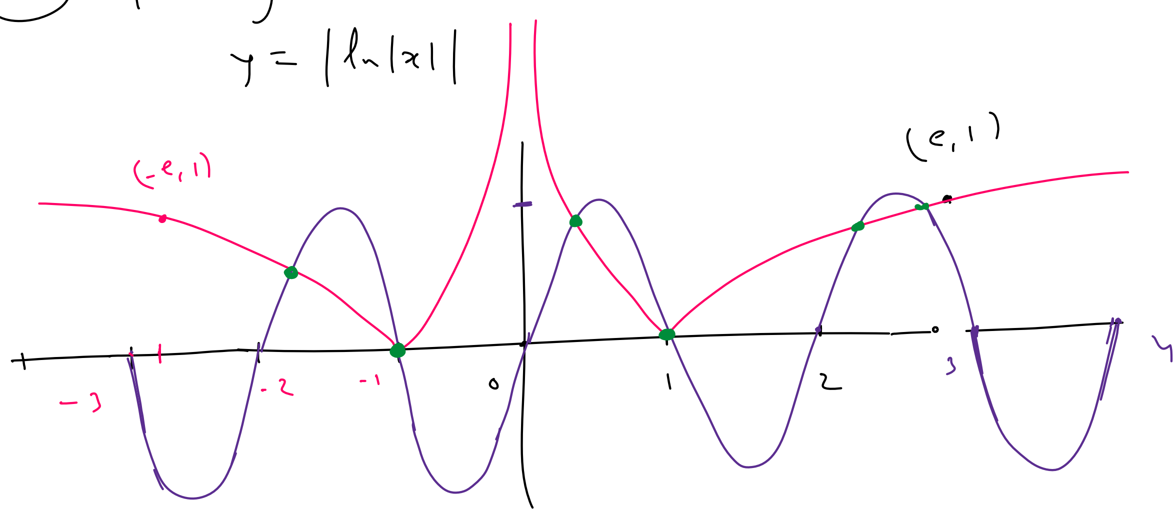
$f(x) \to -1$ if $x \to +\infty$



$\therefore f(x) = 0$ has only 1 soln — (A)

23) plot $y = \sin \pi x$

$y = |\ln|x||$



no. of intersection points = 6 — (D)

24) let $x = I + f$

$\Rightarrow 2I + [f + 1/2] = 2004$

clearly $[f + 1/2] = 0$ or $\cancel{1}$ rejected as RHS is even

Clearly $[f + 1/2] = 0$ or 1

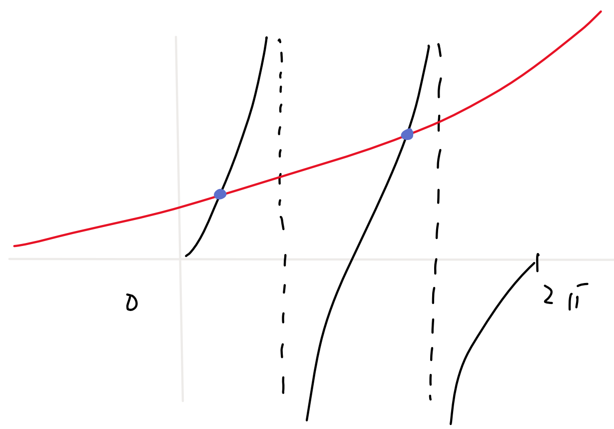
Hence $0 \le f < 1/2$ & $I = 1002$

$\therefore x \in [1002, 1002.5]$ — (D)

(25) plot graph

2 intersec^n pts $\Longrightarrow$ 2 soln

— (B)

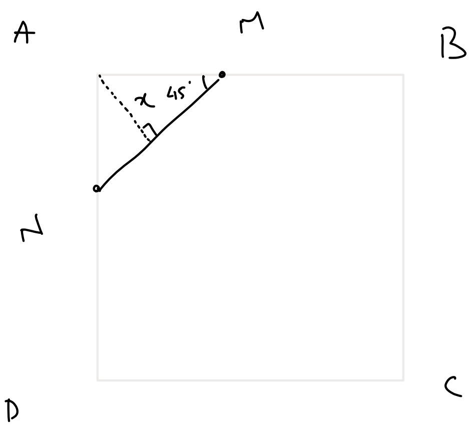


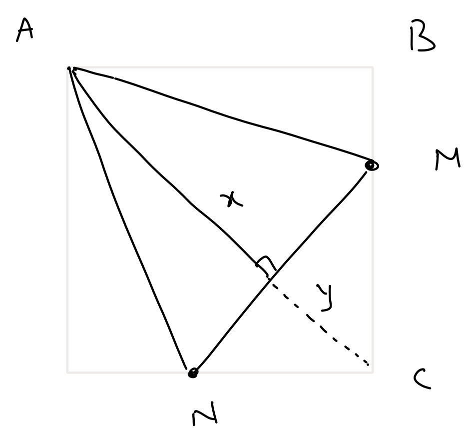
(26) $f(x) = \dfrac{4}{\sqrt{1-x^2}}$ $\therefore f(\sin x) = \dfrac{4}{|\cos x|}$

$f(\cos x) = \dfrac{4}{|\sin x|}$

$\therefore g(x) = |\sin x| + |\cos x|$ $\therefore$ f.p is $\pi/2 \longrightarrow$ (A)

(27)





Case I M & N lie of AB & AD

$\therefore AM = AN = \sqrt{2}x \Rightarrow MN = 2x$

$f(x) = x^2$ $\therefore f(x) \in (0, 2]$

$\forall x \in (0, \sqrt{2}]$

Case II M & N lie on BC & CD

$x + y = 2\sqrt{2}$ & $CM = CN = \sqrt{2}y$

$\therefore MN = 2y$

$f(x) = xy = x(2\sqrt{2} - x)$

$f(x) = 2 - (x - \sqrt{2})^2$

if $x \in (0, \sqrt{2}]$ ↗

$f(x) = xy = x(2\sqrt{2} - x)$

$f(x) = 2 - (x - \sqrt{2})^2$

if $x \in (\sqrt{2}, 2\sqrt{2})$ $\quad f(x) \in [0, 2]$

— (B)

(28)

$$h(x) = \ln(f(x) \cdot g(x))$$

$$= \ln\left(e^{\{e^{|x|}\operatorname{sgn} x\} + [e^{|x|}\operatorname{sgn} x]}\right)$$

$$= \ln\left(e^{(e^{|x|}\operatorname{sgn} x)}\right) = e^{|x|}\operatorname{sgn}(x)$$

which is clearly an odd function — (A)

(29) (A) Even degree ⇒ Many & Range is not R

(B) $f(x) = x^3 + x + 1$

$f'(x) = 3x^2 + 1$ ⇒ injective & surjective both

(C) Range $[1, \infty)$ ⇒ not surjective

(d) odd degree ⇒ Surjective but not Inc $\forall x \in R$

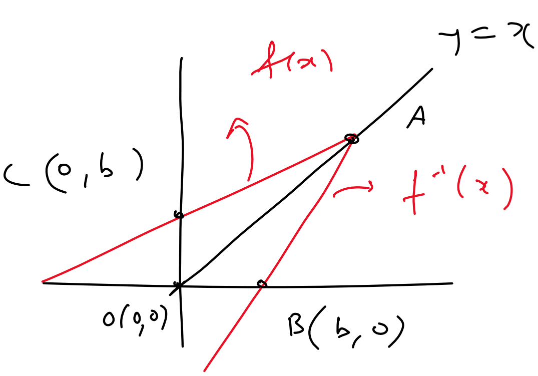
⇒ many-one

— (D)

(30) $f(x) = \dfrac{9x}{25} + b$

Solve with $y = x$

$A\left(\dfrac{25b}{16}, \dfrac{25b}{16}\right)$



∴ Area of ABOC $= 2\left[\dfrac{1}{2} \times b \times \dfrac{25b}{16}\right]$

⇒ $\dfrac{25b^2}{16} = 49$ ⟹ $b = \dfrac{28}{5}$ — (C)

$$\Rightarrow \quad \frac{25b^2}{16} = 49 \quad \Rightarrow \quad b = \frac{28}{5} \quad \text{— (C)}$$

(31) I : not true Ex $f(x) = \sin x$, $g(x) = \cos x$ both f & g one-one in $[0, \pi/2]$ but $f+g$ is not

II : not true $f(x) = \sin x$, $g(x) = \cos x$ both f & g one-one in $[0, \pi/2]$

$\therefore f \cdot g = \frac{\sin 2x}{2} \rightarrow$ is many one

III : not true Ex $f(x) = \sin x$

Ans (D)

(32) $$\frac{1+e^{f(x)}}{1-e^{f(x)}} = \frac{1}{x} \Rightarrow e^{f(x)} = \frac{1-x}{1+x}$$

$$f(x) = \ln\left(\frac{1-x}{1+x}\right)$$

$$f(a)+f(b) = \ln\left(\frac{1-a}{1+a}\right) + \ln\left(\frac{1-b}{1+b}\right)$$

$$= \ln\left(\frac{(1-a)(1-b)}{(1+a)(1+b)}\right) \text{ — (i)}$$

$$f\left(\frac{a+b}{1+ab}\right) = \ln\left(\frac{1-\frac{(a+b)}{1+ab}}{1+\frac{a+b}{1+ab}}\right) = \ln\left(\frac{1-(a+b)+ab}{1+(a+b)+ab}\right)$$

$$= \ln\left(\frac{(1-a)(1-b)}{(1+a)(1+b)}\right) \text{ — (ii)}$$

$\therefore$ Both (i) & (ii) same $\therefore$ it is true $\forall a \in (-1,1)$, $b \in (-1,1)$

— (D)

— (D)

(33) $f(\cos x) = \cos 17x \quad \forall x \in R$

$g(\sin x) = \sin 17x$

$x \to \pi/2 - x \qquad \therefore g(\cos x) = \sin\left(\frac{17\pi}{2} - 17x\right) = \cos 17x$

$\therefore g(\cos x) = f(\cos x) \quad \forall x \in R$ (D)

or $g(x) = f(x) \quad \forall x \in [-1, 1]$

(34) $f(x) = 2\log_a x \qquad$ let $f(y) = x$

$\Rightarrow 2\log_a y = x$

$f^{-1}(x) = y = a^{x/2}$

$\therefore f^{-1}(b+c) = a^{\frac{b+c}{2}} = a^{b/2} \cdot a^{c/2}$

$= f^{-1}(b) \cdot f^{-1}(c)$ — (A)

(35) Conceptual — (D)

FUNCTIONS

EXERCISE – 2 (B)

Q.1 (A, B, C, D)

(A) $f(x)=\log_{x-1}(2-[x]-[x]^2) \Rightarrow 2-[x]-[x]^2 > 0$

$\Rightarrow [x] \in (-2,1)$ So, $[x] = -1, 0 \Rightarrow x \in (-1,1)$

but, $x-1 \neq 0$, $x-1 > 0 \Rightarrow$ x > 1

So f (x) has empty domain.

(B) $g(x)=\cos^{-1}(2-\{x\})$

Now $0 \leq \{x\} < 1 \Rightarrow 1 < 2-\{x\} \leq 2$

but, $\cos^{-1}$x is defined in $[-1, 1]$

So g (x) has empty domain.

(C) $h(x) = \ln\ln(\cos x)$

Now $\ln(\cos x) > 0 \Rightarrow \cos x > 1$

So h (x) has empty domain.

(D) $f(x) = \dfrac{1}{\sec^{-1}(\text{sgn}(e^{-x}))}$

Now $e^{-x} > 0$ for $x \in R$

$\Rightarrow Sgn(e^{-x}) = 1$ for $x \in R$ and thus $\sec^{-1}(\text{sgn}(e^{-x})) = 0$ for $x \in R$.
So h (x) has empty domain.

Q.2 (A, B, D)

A transcendental function is one that cannot be expressed in terms of an algebraic polynomial.

e.g. trigonometric function, exponential, logarithmic function.

So, (A) , (B) , (D) are transcendental function.

But, $f(x)=\sqrt{x^2+2x+1} = \sqrt{(x+1)^2}$

$$= |x+1|$$

$$= x+1 \; ; \; x \geq -1$$

$$= -x-1 \; ; \; x < -1$$

Q.3 (A, B, C)

(A) $$y = \frac{\sin x}{\sqrt{1+\tan^2 x}} + \frac{\cos x}{\sqrt{1+\cot^2 x}}$$

$$= \frac{\sin x}{|\sec x|} + \frac{\cos x}{|\cos ec x|}$$

$$= \sin x|\cos x| + \cos x|\sin x|$$

$$= 0 \;\; \forall \;\; x \in \left[(4n+1)\frac{\pi}{2} \, , \, (2n+1)\pi\right] \cup \left[(4n+3)\frac{\pi}{2}, \, (2n+2)\right]$$

$$= \sin 2x \;\forall\; x \in \left[2n\pi \, , (4n+1)\frac{\pi}{2}\right]$$

$$= -\sin 2x \;\forall\; x \in \left((2n+1)\pi \, , \, (4n+3)\frac{\pi}{2}\right)$$

Hence graph of $y = \frac{\sin x}{\sqrt{1+\tan^2 x}} + \frac{\cos x}{\sqrt{1+\cot^2 x}}$ is dissimilar from $y = \sin 2x$

(B) $$y = \tan x \cdot \cot x = 1 \;\; \forall \;\; x \in (-\infty, \infty) - \frac{x\pi}{2}, \; x \in \mathrm{I}$$

$$y = \sin x \cdot \cos ec x = 1 \;\forall\; x \in (-\infty, \infty) - x\pi \;,\; x \in \mathrm{I}$$

Functions are not identical as domains are not same, hence graphs are dissimilar.

(C) $$y = \frac{|\sec x| + |\cos ec\, x|}{|\sec x||\cos ec\, x|} \Rightarrow y = \frac{1}{|\sec x|} + \frac{1}{|\cos ec x|} \text{ or } y = |\cos x| + |\sin x|, \; x \neq \frac{n\pi}{2}$$

$$y = |\cos x| + |\sin x| \;\; \forall \; x \in (-\infty, \infty)$$

Functions are not identical as domains are not same, hence graphs are dissimilar.

Q.4 (A, B, D)

(A) $\left[x+1+\mathrm{T}\right]=\left[x+1\right]\Rightarrow\left[x+\mathrm{T}\right]=\left[x\right]$

$x+\mathrm{T}-1\le\left[x+\mathrm{T}\right]<x+\mathrm{T}$ & $x-1\le\left[x\right]<x\Rightarrow \mathrm{T}$ is not fixed.

Function is non periodic.

(B) $\sin\left(x+\mathrm{T}\right)^2=\sin x^2\Rightarrow 2\cos\left(\dfrac{\left(x+\mathrm{T}\right)^2+x^2}{2}\right)\sin\left(\dfrac{\left(x+\mathrm{T}\right)^2-x^2}{2}\right)=0.$

$\Rightarrow\dfrac{\left(x+\mathrm{T}\right)^2+x^2}{2}=\dfrac{\left(2n-1\right)\pi}{2}$ or $\dfrac{\left(x+\mathrm{T}\right)^2-x^2}{2}=0$

As value of T is not constant but dependent of x hence $\sin x^2$ is non periodic.

(C) $\sin^2\left(x+\mathrm{T}\right)=\sin^2 x\Rightarrow x+\mathrm{T}=n\pi\pm x\Rightarrow \mathrm{T}=n\pi$

Periodic with period $'\pi'$

$y=\sin^{-1}x\ \rightarrow$ not periodic as $\mathrm{D}=[-1,1]$ & Range $=\left[\dfrac{-\pi}{2},\dfrac{\pi}{2}\right]$

Q.5 (A, C, D)

(A) $f(x)=x+1,\ x\ge-1$ is one – one as linear function are one – one

(B) $f(x)=x+\dfrac{1}{x}\ (x>0)$ has minima at $x=1$

$(g'(x)=0\ \Rightarrow\ x^2-1=0\ \Rightarrow\ x=\pm1)$

So, not one – one in $(0,\infty)$

(C) $h(x)=x^2+4x-5,\ x>0$

$h'(x)=0$ at $x=-2$

So, one – one in $x\in(0,\infty)$

(D) $f(x)=e^{-x}$

$f'(x)<0$ for all $x\in R$

So, one – one in $x \in [0, \infty]$

Q.6 (B, C)

A homogenous function is such that if substitution $y = vx$ is made it should come out to be

$x\ f(v)$.

(A) $x \sin y + y \sin x = v \sin\left(\frac{v}{x}\right) + vx - \sin x$

$$= v\left(\sin\left(\frac{v}{x}\right) + x \sin v\right) \quad \rightarrow \text{ not homogeneous.}$$

(B) $xe^{\frac{y}{x}} + ye^{\frac{x}{y}} = xe^{v} + vx \cdot e^{\frac{1}{v}}$

$$= x\left(e^{v} + ve^{\frac{1}{v}}\right) \quad \rightarrow \text{ homogeneous.}$$

(C) $x^2 - xy = x^2 - vx^2 = x^2(1-v) \quad \rightarrow$ homogeneous.

(D) $\sin^{-1}(xy) = \sin^{-1}(vx^2) \quad \rightarrow$ not homogeneous.

Q.7 (B, C)

Given $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$

Hence, $f(x)$ is a polynomial of degree n.

$$f(x) \cdot f\left(\frac{1}{x}\right) - f(x) - f\left(\frac{1}{x}\right) + 1 = 1$$

$$\Rightarrow (f(x) - 1)\left(f\left(\frac{1}{x}\right) - 1\right) = 1$$

$$\Rightarrow f(x) = 1 + \frac{1}{f\left(\frac{1}{x}\right) - 1}$$

$$\because\ f\left(\frac{1}{x}\right) = \frac{P(x)}{x^n}$$

$$\Rightarrow f(x) = 1 + \frac{x^n}{P(x) - x^n} = 1 + \frac{x^n}{k} \quad ....(I)$$

Hence, $P(x) - x^n = k$ (constant) for $f(x)$ to be polynomial

$$\Rightarrow P(x) = k + x^n$$

$$\Rightarrow f\left(\frac{1}{x}\right) = 1 + \frac{k}{x^n} \Rightarrow f(x) = 1 + k^n \; ....(II)$$

From (I) , (II)

$k = 1$

$\because f(2) = 9 \Rightarrow 2^n + 1 = 9 \Rightarrow n = 3$

Hence, $f(x) = x^3 + 1$

$f(4) = 65$, $f(6) = 216 \Rightarrow 3f(6) \neq 2f(4)$

$f(1) = 2$, $f(3) = 28 \Rightarrow 14 f(1) = f(3)$

$f(3) = 28$, $f(5) = 126 \Rightarrow 9 f(3) = 2 f(5)$

$f(10) = 1001, f(11) = 1332 \Rightarrow f(10) \neq f(11)$

Q.8 (B, D)

$f(x) = x^2$ is may – one in $[-1,1]$

So, can't be inverted

$g(x) = x^3$ is bijective in $[-1,1]$

So, inverse is possible.

$h(x) = \sin 2x$ is may – one in $[-1,1]$

So, not invertible.

$k(x) = \sin\left(\frac{\pi x}{2}\right)$ is one – one in $[-1,1]$

So, invertible.

Q.9 (B, C)

$f(x)=\dfrac{1}{1+x}$ has the range $(-\infty,\infty)-\{0\}$

$f(x)=\dfrac{1}{1+x^2}$ has the range $(0,1)$

$f(x)=\dfrac{1}{1+\sqrt{x}}$ has the range $(0,1)$

$f(x)=\dfrac{1}{\sqrt{3-x}}$ has the range $(0\ ,\ \infty)$

Q.10 (A, B, C)

(A) $f(x)=\cos(2\tan^{-1}x)=\cos\left(\tan^{-1}\dfrac{2x}{1-x^2}\right)$

$=\cos\left(\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)\right)=\dfrac{1-x^2}{1+x^2}$: Domain – R & Range $\in[-1,1]$

$g(x)=\dfrac{1-x^2}{1+x^2}$: Domain – R, Range $\in[-1,1]$

(B) $f(x)=\dfrac{2x}{1+x^2}$: Domain – R, Range $\in[-1,1]$

$g(x)=\sin(2\cot^{-1}x)=\dfrac{2x}{1+x^2}$: Domain – R, Range $\in[-1,1]$

(C) $g\left(x\right)=e^{\ln\left(\operatorname{sgn}\left(\cot^{-1}x\right)\right)}$

$\cot^{-1}x$ must be positive hence domain $\left(0,\infty\right)$.

Now $\cot^{-1}x>0\Rightarrow\operatorname{sgn}\left(\cot^{-1}x\right)=1\Rightarrow e^{\ln\left(\operatorname{sgn}\left(\cot^{-1}x\right)\right)}=1.$

Range : $\{1\}$

$g(x)=e^{\ln[1+\{x\}]}\quad x\in R$

$=[\{x\}]+1=1\ \ \forall\ x\in R$

(D) $f(x) = (a)^{\frac{1}{x}}$, $a > 0$

$f(x) = \sqrt[x]{a}$, $a > 0$

For x being even, there exist 2 value of $g(x) = \pm\sqrt[x]{a}$, $a > 0$

Q.11 (A, B)

$f : R \to R,\ \ f(x) = |x|\ \text{sgn}(x),\ \ x > 0$

$= (-x)(-1);\ x < 0$

$= 0\ ;\ x = 0$

$= (x)(1);\ x > 0.$

$\Rightarrow f(x) = x\ ,\ x \in R\ .$

$g : R \to R,\ \ f(x) = x^{\frac{3}{5}}$ is monotonic.

$h : R \to R,\ \ h(x) = x^4 + 3x^2 + 1$ is many – one

$k : R \to R, k(x) = \dfrac{3x^2 - 7x + 6}{x - x^2 - 2}$

Denominator is always Negative so, Domain – R

Numerator has $D > 0$, $k(x) = 0$ at 2 points thus $k(x)$ is many – one.

Q.12 (A, B)

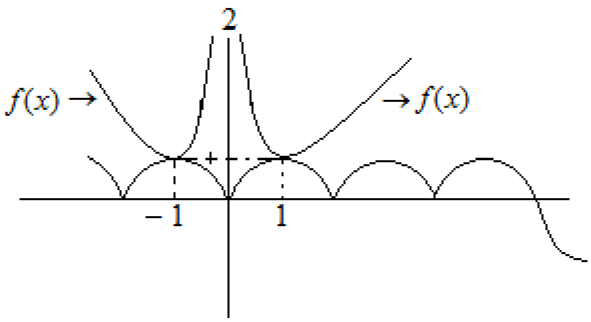
$f(x) = ax + b = y \Rightarrow x = \left(\dfrac{y-b}{a}\right) \Rightarrow f^{-1}(x) = \dfrac{x}{a} - b$.

Now $ax + b = \dfrac{x}{a} - b \Rightarrow a = \dfrac{1}{a}$ & $b = -b$

Hence $(a,b) \to (1,0)$ *or* $(-1,0)$.

Q.13 (B, C)

(A) $x^4 - 2x^2 \sin^2 \dfrac{\pi x}{2} + 1 = 0 \Rightarrow \left(\dfrac{x^4+1}{2x^2}\right) = \sin^2\left(\dfrac{\pi x}{2}\right)$



$$\Rightarrow \frac{1}{2}\left(x^2+\frac{1}{x^2}\right)=\sin^2\frac{\pi x}{2}$$

Let, $f(x)=\frac{1}{2}\left(x^2+\frac{1}{x^2}\right)$, $g(x)=\sin^2\left(\frac{\pi x}{2}\right)$

Has 2 solutions.

(B) $x^2-2x+5+\pi^x=0 \Rightarrow x^2-2x+5=-\pi^x$

$f(x)=x^2-2x+5=(x-1)^2+4>0 \;\forall\; x\in R$

$g(x)=-(\pi^x)<0 \;\forall\; x\in R$

Hence, no solution

(C) $\log_{\frac{3}{2}}(\cot^{-1}x-\text{sgn}(e^x))=2$

As $e^x>0$ thus $\text{sgn}\left(e^x\right)=1.$

$$\Rightarrow \cot^{-1}x-1=\left(\frac{9}{4}\right)$$

$\because \cot^{-1}x\in(0,\pi)$ hence, $\cot^{-1}x-1\in(-1\,,\,\pi-1)$

Hence, no solution.

(D) $\tan\left(x+\frac{\pi}{6}\right)=2\tan x$

$$\Rightarrow \frac{1+\sqrt{3}\tan x}{\sqrt{3}-\tan x}=2\tan x\,.$$

$$\Rightarrow 2\tan^2 x+\sqrt{3}\tan x-2\sqrt{3}=0\,.$$

Hence infinitely many solutions.

Q.14 (A, B, C)

$g(x)$ & $g^{-1}(x)$ is symmetric about line $y=x$

Hence the point P & Q may lie on the line $y = x$ but not necessarily.

(Ex. $g(x) = \dfrac{15 - x^3}{7}$ & $g^{-1}(x) = (15 - 7x)^{1/3}$ intersect in (1, 2) & (2, 1) which do not lie on y = x)

Also there can be more than 1 points of intersection so P & Q need not coincide.

Slope of line joining points of intersections of y = g(x) & y = g^{-1}(x) may be 1 or –1 as either these points will lie on y = x or will be image of each other in y = x.

Q.15 (A, B, C, D)

$$f(2x)\left(1 - f\left(\frac{1}{2x}\right)\right) + f(16x^2y) = f(-2) - f(4xy) \quad x, y \in R - \{0\}$$

$f(4) = -255$, $f(0) = 1$

Put $y = \dfrac{1}{8x^2}$ to get $f(2x)\left(1 - f\left(\dfrac{1}{2x}\right)\right) + f(2x) = f(-2) - f\left(\dfrac{1}{2x}\right)$

$\because$ $f(x)$ is even function $f(2) = f(-2)$

Replacing 2x by t

$$\Rightarrow f(t)\cdot\left(1 - f\left(\frac{1}{t}\right)\right) + f\left(\frac{1}{t}\right) = 0$$

$$\Rightarrow f(t) - f(t)\cdot f\left(\frac{1}{t}\right) + f\left(\frac{1}{t}\right) = 0$$

$$\Rightarrow f(t)\cdot f\left(\frac{1}{t}\right) - f(t) - f\left(\frac{1}{t}\right) + 1 = 1$$

$$\Rightarrow f(t) = 1 + \frac{1}{\left(f\left(\frac{1}{t}\right) - 1\right)}$$

Now, $f(t)$ is a polynomial, So, $f\left(\dfrac{1}{t}\right) = \dfrac{P(t)}{t^n}$

$$\Rightarrow f(t) = 1 + \frac{t^n}{P(t) - t^n}$$

For, $f(t)$ to be polynomial

$P(t)-t^n = k \Rightarrow P(t)=k+t^n$

$\Rightarrow f\left(\frac{1}{t}\right)=\frac{k}{t^n}+1$

$\Rightarrow f(t)=1+k\,t^n$

Hence, $k=\frac{1}{k} \Rightarrow k=\pm 1$

So, $f(x)=\pm x^n+1$

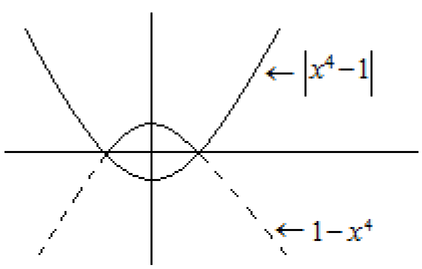
Given $f(4)=-255 \Rightarrow -x^n+1=-255 \Rightarrow n=4$

So, $f(x)=1-x^4$

(A) $f(3)=-80$

(B) $f(x)\cdot f\left(\frac{1}{x}\right)=\frac{(1-x^4)(x^4-1)}{x^4}=\frac{(x^4-1)^2}{x^4}\le 0$

(C) $|f(x)|=k-2$



For 4 different solutions. $k-2 \in (0,1)$

$\Rightarrow k \in (2\ ,\ 3)$

(D) $g(x)=9-2\sqrt{3+f\left(\sqrt{|x|}\right)}$

$f(x)=1-x^4$

$f\left(\sqrt{|x|}\right)=1-\left(\sqrt{|x|}\right)^4=1-x^2$

$$g(x)=9-2\sqrt{3+1-x^2}$$

$$=9-2\sqrt{4-x^2}$$

Hence, $g(x)\in[5,9]$

So, $p^2+4q=25+36=61$

Q.16 (A, C, D)

$$f(x)=\frac{x+2}{x-1}\Rightarrow x=\frac{y+2}{y-1}$$

$$\Rightarrow x=f(y)$$

Range of $f(x)=\mathrm{R}-\{1\}$

Domain of $f(x)=\mathrm{R}-\{1\}$

Q.17 (B, C)

$$f:N\to N,\ f(x)=x+(-1)^{x-1}$$

For, $x\in$ set of even number , $f(x)=x-1,\ x=2m$.

For, $x\in$ set of odd number , $f(x)=x+1,\ x=2m+1$.

Now $y=\begin{cases}x-1, & x=2m\Rightarrow y=2m-1,\ (\text{odd})\\ x+1, & x=2m+1\Rightarrow y=2m+2,\ (\text{even})\end{cases}$

$$\Rightarrow x=\begin{cases}y+1, & y=2m+1\\ y-1, & y=2m+2\end{cases}$$

$$\Rightarrow f^{-1}(x)=\begin{cases}x-1, & x=2m-1\\ x+1, & x=2m\end{cases}$$

Hence $f^{-1}(x)=x-(-1)^x;\ x\in N$

Q.18 (A, B, C)

$$f(x)=\cos[\pi^2]x+\cos[-\pi]x$$

$$=\cos 9x+\cos 4x$$

$f\left(\frac{\pi}{2}\right)=1,\ f(\pi)=0,\ f\left(\frac{-\pi}{2}\right)=1\ \&\ f\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}-1.$

Q.19 (A, B, D)

$f(x)=\sin x+\tan x+\text{sgn}(x^2-6x+10)$

$x^2-6x+10>0$ for all $x\in R$ as $D<0$, hence, $\text{sgn}(x^2-6x+10)=1$

$\Rightarrow f(x)=\sin x+\tan x+1$

Hence f (x) is periodic with fundamental period 2π.

Also $4\pi\ \&\ 8\pi$ can be the periods.

Q.20 (A, C)

$f(x)=\log_2\left(\frac{\sin x-\cos x+3\sqrt{2}}{\sqrt{2}}\right)$

Now $-\sqrt{2}\le\sin x-\cos x\le\sqrt{2}$

$\Rightarrow\frac{2\sqrt{2}}{\sqrt{2}}\le\frac{\sin x-\cos x+3\sqrt{2}}{\sqrt{2}}\le\frac{4\sqrt{2}}{\sqrt{2}}$

$\log_2 2\le\log_2\left(\frac{\sin x-\cos x+3\sqrt{2}}{\sqrt{2}}\right)\le\log_2 4$

Hence, $f(x)\in[1\ ,\ 2]$

Domain $\rightarrow$ R & Range $\rightarrow$ [1 , 2]

PASSAGE – 1

Q.1 (B)

$f(x)=1-e^{\frac{1}{x}-1}$

$f(x)>0\Rightarrow 1-e^{\frac{1}{x}-1}>0$

$\Rightarrow e^{\frac{1}{x}-1}<0$

$\Rightarrow \frac{1}{x} - 1 < 0$

$\Rightarrow \frac{x-1}{x} > 0$

$\Rightarrow x < 0$ or $x > 1$..

Q.2 (A)

$f(x_1) = f(x_2) \Rightarrow 1 - e^{\frac{1}{x_1}-1} = 1 - e^{\frac{1}{x_2}-1}$ *or* $\frac{1}{x_1} = \frac{1}{x_2}$.

Hence *f* (x) is one – one.

$1 - e^{\frac{1}{x}-1} = y \Rightarrow x = \frac{1}{1+\ln(1-y)}$

now for x to be real $1 - y > 0$ & $\ln(1-y) \neq -1$

Hence $y < 1$ & $y \neq 1 - \frac{1}{e}$

Range of $f(x) : (-\infty, 1) - \left\{1 - \frac{1}{e}\right\}$

Hence $f(x)$ is INTO.

Q.3 (B)

Range $= (-\infty, 1) - \left\{1 - \frac{1}{e}\right\}$

PASSAGE – 2

Q.4 (B)

$]x[= \begin{cases} -x, & x > 0 \\ x, & x \leq 0 \end{cases}$

For, $x > 1$, $]x-1[= 2x+3 \Rightarrow 1 - x = 2x + 3$

or $x = -\frac{2}{3}$ (not possible)

For, $x \leq 1$, $]x-1[= 2x+3 \Rightarrow x - 1 = 2x + 3$

or $x=-4$.

Q.5 (A)

$x^2+kx+5=0$

For, $\alpha=-4$

$16-4k+5=0 \Rightarrow k=\frac{21}{4}$

Q.6 (D)

$x^2+kx+5=0$

Product of the roots = 5

one root $=-4$, hence other root $=-\frac{5}{4}$

PASSAGE – 3

(i) $\sqrt{x^2-6x+5} \geq x-4$

Domain : $x^2-6x+5\geq 0 \Rightarrow x\in(-\infty,1]\cup[5,\infty)$

For $x>5$, $\sqrt{x^2-6x+5} \geq x-4 \Rightarrow (x^2-6x+5)\geq(x-4)^2 \Rightarrow x\geq\frac{11}{2}$

For $x<1$, always true as LHS > 0 & RHS < 0.

Hence solution set is $(-\infty,1]\cup\left[\frac{11}{2},\infty\right)$

(ii) $\left(\frac{1}{3}\right)^{x^2-6x-7}>1 \Rightarrow x^2-6x-7<0$

$\Rightarrow (x-7)(x+1)<0$

$x\in(-1,7)$

Q.7 (A)

$[p+q]=\left[1+\frac{11}{2}\right]=6$

Q.8 (B)

Common solution is $(-1, 1] \cup [\frac{11}{2}, 7)$

So, integral values are $0, 1, 6$

Q.9 (D)

$3(p+2q+a+b) = 3(1+11+(-1)+7)$

$=54$

$=2\times 3^3$

No of factor $=2\times4=8$

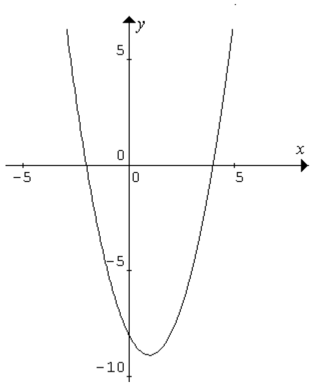
$[x]=8 \Rightarrow x\in[8, 9)$

PASSAGE – 4

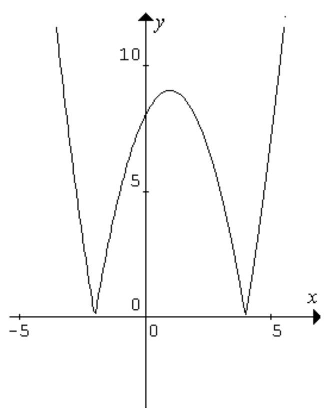
Q.10 (B)

$y=\left|x^2-2x-8\right|$

$f(x)=x^2-2x-8$

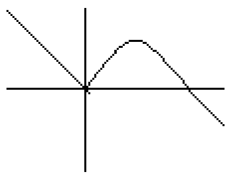


$\rightarrow$

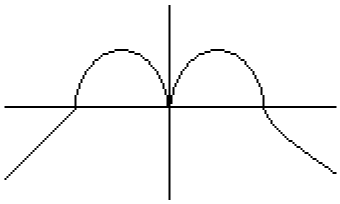


Q.11 (C)

$y=f(x)$
$y=f(|x|)$

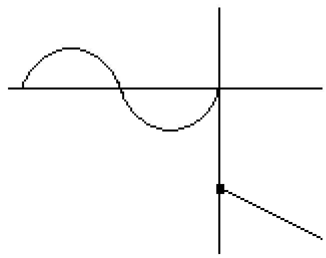


Has the graph same in II & III quad as in I & IV quad.

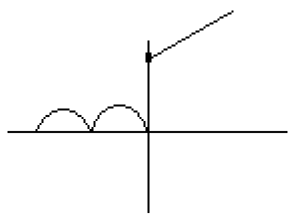


Q.12 (A)

if $y=f(x)$ has graph



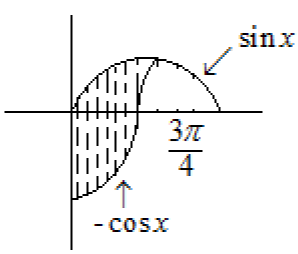
then $y=|f(x)|$ has graph



MATRIX MATCH TYPE

Q.1

(A) for $x\in(0,\pi)$

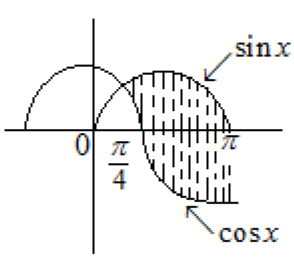


$\sin x+\cos x>0$

$\Rightarrow \sin x>-\cos x$

$x\left(0,\frac{3\pi}{4}\right)$ ....(R)

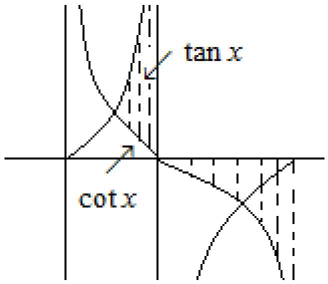
(B)



$\sin x>\cos x$

$x\in\left(\frac{\pi}{4},\pi\right)$ ....(S)

(C)



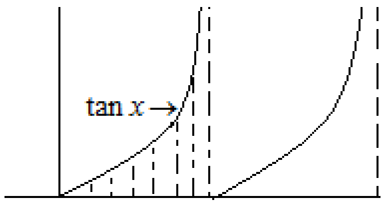
$\tan x - \cot x > 0$

$\tan x > \cot x$

$x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$

$\ldots. (\theta)$

(D)



$\tan x + \cot x > 0$

$\tan x > -\cot x$

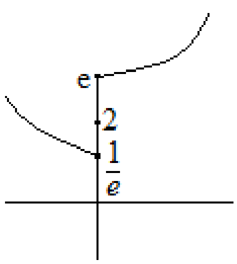
Q.2

(A) $f : R \to R, \ f(x) = e^{\operatorname{sgn} x} + e^{x^2}$

$$= \frac{1}{e} + e^{x^2} \ ; \ x < 0$$

$$f(x) \quad = 2 \ ; \ x = 0$$

$$= e + e^{x^2} \ ; \ x > 0$$



Neither either nor odd

many – one

(R,T)

(B) $f : (-1,1) \to R, \ f(x) = x\,[x^4] + \frac{1}{\sqrt{1-x^2}}$

$$= 0 + \frac{1}{\sqrt{1-x^2}}$$

$\because f(-x) = f(x)$

$\because$ even function So, may – one.

$(Q\ ,\ T)$

(C) $f:R\to\ R,\ f(x)=\dfrac{x(x+1)(x^4+1)+2x^4+x^2+2}{x^2+x+1}$

$$=\frac{(x^4+1)(x(x+1)+2)+x^2}{x^2+x+1}$$

$$=\frac{(x^4+1)(x^2+x+1)+x^4+x^2+1}{x^2+x+1}$$

$$=x^4+1+x^2-x+1$$

$$=x^4+x^2-x+2$$

$f(-x)=x^4+x^2+x+2$

So, neither odd nor even.

$f'(x)$ is a degree equation so at least 1 root.

Hence, not monotonic.

So, (R ,T)

(D) $f:R\to R,\ f(x)=x+3x^3+5x^5+.............101\times 101$

$f'(x)=1+9x^2+25x^4+.............101^2\times 100\ \ >0$ for $x\in R$

Hence, one – one and odd functions.

$\because\ f(-x)=-f(x)$

Q.3

(A) $f:[-1\ ,\ \infty)\to\ (0,\infty)$

$f'(x)=e^{x^2-x}\ \ ;\ x\in[-1,0]$

$$=e^{x^2+x}\ ;\ x>0$$

$f'(x)=0$ at $x=\dfrac{1}{2}$ for $x<0$

$f'(x)=0$ at $x=-\frac{1}{2}$ for $x>0$

(B) $f:(1,\infty)\to[3,\infty)$

$f(x)=\sqrt{10-2x+x^2}$

$\quad=\sqrt{(x-1)^2+9}$

For, $x\geq 1$, $f(x)>3$

Hence, $f(x)$ is never equal to 3 in $(1,\infty)$

So, into, one – one, non – periodic.

(P , Q)

(C) $f:R\to \mathrm{I}$

$f(x)=\tan^5\pi[x^2+2x+3]$

$\quad=\tan^5\pi[(x+1)^2+2]$

For, $x\in R$, $[(x+1)^2+2]\pi$ is a multiple of π

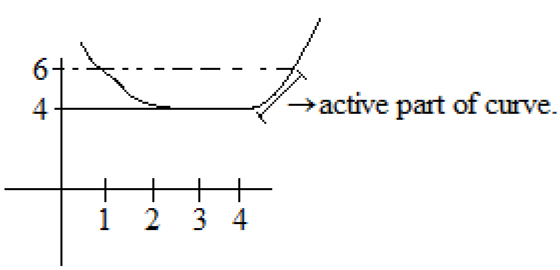
So, $f(x)=0\ \forall\ x\in R$

Hence, periodic, many – one into

(Q, R , T)

(D)

$f:[3,4]\to[4,6]$

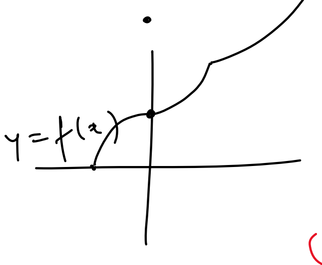


So, one – one, onto.

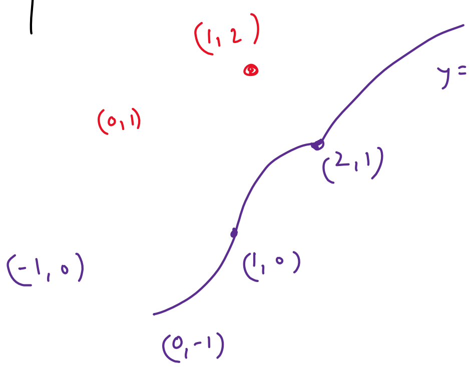
2(B)

Passage-5

$$f(x) = \begin{cases} \sqrt{1-x^2}, & -1 \le x < 0 \\ x^2+1, & 0 \le x < 1 \\ \dfrac{(x-1)^2}{4}+2, & x \ge 1 \end{cases}$$



$y=f(x)$



Range of $f(x) \in [0,\infty)$

If $f \circ g(x) = x$

& $g \circ f(x) = x$ $\forall x \in$ Domain

Then g is inverse of f

$\Rightarrow$ $f : [-1,\infty) \to [0,\infty)$

$g : [0,\infty) \to [-1,\infty)$

(13) $y = f(f(f(g(x)))) = f \circ f(x)$ $\forall x \in [0,\infty)$

as $f(x) \ge 1$

$\therefore f(f(x)) \ge f(1)$ {f is Increasing.}

or $f \circ f \in [2,\infty)$ — (D)

(14) $y = g(g(g(f(x)))) = g \circ g(x)$ $\forall x \in [1,\infty)$

for this composition to exist $x \ge 1$

$\therefore$ Range of $g(g(x)) \in [-1,\infty)$ — (A)

(15) $f(x) = g(x)$ same as $f(x) = x$ (as $f(x)$ Increasing)

$\therefore$ only possible $\dfrac{(x-1)^2}{4} + 2 = x$ $(x \ge 1)$

$\Rightarrow x^2 - 2x + 1 + 8 = 4x$

$\Rightarrow x^2 - 6x + 9 = 0$

$\Rightarrow x = 3$ only solⁿ — (D)

$\Rightarrow \quad x^2 - 6x + 9 = 0$

$\Rightarrow \quad x = 3$ only sol^n — (D)

Matrix Match

(4) (A) $f(x) = \frac{1}{1-x}$ $: x \neq 1$ $\quad f(f(x)) = \frac{1}{1-\left(\frac{1}{1-x}\right)} = 1 - \frac{1}{x}$ $\quad : x \neq 0, 1$

$f^3(x) = 1 - (1-x) = x \quad x \neq 0, 1$

$\therefore A \rightarrow Q$

$\therefore f_{3n}(x)$ is not defined when $x = 0$ or 1

(B) $x^2 f(x) + f(1-x) = 2x - x^4$

$x = 2 \quad \Rightarrow \quad 4f(2) + f(-1) = -12$

$x = -1 \quad f(-1) + f(2) = -3$

$\Rightarrow \quad f(2) = -3 \quad \Rightarrow \quad |f(2)| = 3$ $\quad \boxed{B \rightarrow R}$

(C) $f : [\tfrac{1}{2}, \infty) \rightarrow [3/4, \infty)$

since f is increasing & continuous in $[1/2, \infty)$

$\therefore f(x) = f^{-1}(x)$ same as $f(x) = x$

$x^2 - x + 1 = x$

$x = 1$ $\quad \boxed{C - P}$

(5) $f(x) = [x + 1/2] + [x - 1/2] + 2[-x]$

(A)

$f(0) = -1$

$f(1) = -1$

$\therefore$ not one-one

$f(0) = -1$

$f(1) = -1$ $\therefore$ not one-one

$A \longrightarrow Q, S$

& $f(x)$ range is integer $\therefore$ Into

(B) $f(x) = x^3 + x^2 + 3x + \sin x$

$f'(x) = 3x^2 + 2x + 3 + \cos x$

$= (3x^2 + 2x + 2) + (1 + \cos x)$

$> 0 \ \forall x \in R \qquad \geq 0 \ \forall x \in R$

$\therefore f'(x) > 0 \ \forall x \in R$ $\therefore f(x)$ Increasing $\therefore f(x)$ one-one & onto

$\boxed{B \longrightarrow P, R}$

(C) $f(x) = \dfrac{e^{|x|} - e^{-x}}{e^{x} + e^{-x}}$

$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}, & \text{if } x > 0 \end{cases}$

clearly many one & Into

as $f(x) < 1 \ \forall x \in R$

$\boxed{C \longrightarrow Q, S}$

(d) $f: R \rightarrow R \quad f(x) = e^{\sin(x)} + \sin\left(\dfrac{\pi [x]}{2}\right)$

$f(x)$ is many one & Into

$\boxed{D \longrightarrow Q, S}$

FUNCTIONS

EXERCISE - 2 (C)

Q.1 [03]

$\sin\frac{2x}{3}+\cos 4x+\left|\tan 3x\right|+\mathrm{sgn}(x^2+4x+15)$ has period as LCM of $\left(\frac{2\pi\times 3}{2}\,,\,\frac{2\pi}{4}\,,\,\frac{\pi}{3}\right)$

$\because \mathrm{sgn}(x^2+4x+15)=1$ as $x^2+4x+15>0$ for all $x,$ so period can be any real number.

LCM of $\left(3\pi,\frac{2\pi}{2},\frac{\pi}{3}\right)$ is 3π.

So, k = 3.

Q.2 [05]

$[x]-\{x\}=\frac{x}{3} \Rightarrow 3\big([x]-\{x\}\big)=[x]+\{x\}$

$\Rightarrow \{x\}=\frac{[x]}{2}$

$\because 0 \le \{x\} < 1 \Rightarrow 0\le\frac{[x]}{2}<1$

$\Rightarrow [x]=0,1$ & $\{x\}=0,\frac{1}{2}$

So, $x=\{x\}+[x]$ gives $x=0,\frac{3}{2}$

So, sum of values of x, $\lambda=0+\frac{3}{2}$

Hence, value of $\frac{10\lambda}{3}=\frac{10}{3}\times\frac{3}{2}=5$

Q.3 [02]

$f(x)+f(y)+f(xy)=2+f(x)\cdot f(y)$

at x = 1 , y = 1, $3f(1)=2+f(1)^2$

$\Rightarrow f(1)^2 - 3f(1) + 2 = 0$

$\Rightarrow f(1) = 2$ or $f(1) = 1.$

Now at y = 1, $f(x) + f(1) + f(x) = 2 + f(x) \cdot f(1)$

$\Rightarrow f(x)(2 - f(1)) = 2 - f(1)$

$$\Rightarrow f(x) = \frac{2 - f(1)}{2 - f(1)}$$

Hence if f (1) = 1, then f (x) = 1.

If f (x) = 2, then substitute, y = 1/x to get $f(x) + f\left(\frac{1}{x}\right) + f(1) = 2 + f(x) \cdot f\left(\frac{1}{x}\right)$

$$\Rightarrow f(x) + f\left(\frac{1}{x}\right) = f(x) \cdot f\left(\frac{1}{x}\right)$$

$$\Rightarrow f(x) = 1 + \frac{1}{f\left(\frac{1}{x}\right) - 1}$$

Solution of such polynomial is, $f(x) = 1 \pm x^n$ but, $f(1) = 2 \Rightarrow f(x) = 1 + x^4$

but $f(4) = 17 \Rightarrow 1 + 4^n = 17 \Rightarrow n = 2$

$$f(5) = \frac{5^2 + 1}{13} = \frac{26}{13} = 2.$$

Q.4 [01]

$$\left(\frac{x}{1+x^2}\right)^2 + a\left(\frac{x}{1+x^2}\right) + 3 = 0 \Rightarrow \frac{1}{\left(x + \frac{1}{x}\right)^2} + \frac{a}{\left(x + \frac{1}{x}\right)} + 3 = 0$$

$$\Rightarrow 3\left(x + \frac{1}{x}\right)^2 + a\left(x + \frac{1}{x}\right) + 1 = 0.$$

Let $x + \frac{1}{x} = t$, then $\Rightarrow 3t^2 + at + 1 = 0$.

Now range of $x+\dfrac{1}{x}$ is $(-\infty,-2]\cup[2,\infty)$

Every root of $f(t)=3t^2+at+1=0$ which lies in $(-\infty,-2)\cup(2,\infty)$ gives two values of x and t = 2 or –2 gives one value of x.

Hence exactly two distinct roots are possible when exactly one root lies in (-2, 2) and other root is not equal to -2 or 2.

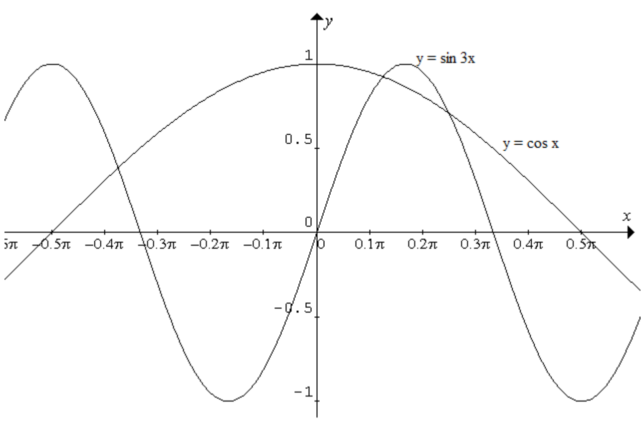
Thus $f(-2)\,f(2)<0$ & $f(\pm 2)\neq 0$

$\Rightarrow (13-2a)(13+2a)<0$

$\Rightarrow a<-\dfrac{13}{2}$ or $a>\dfrac{13}{2}$

Hence $\lambda=\mu=\dfrac{13}{2}\Rightarrow\dfrac{\lambda+\mu}{13}=1$.

Q.5 [03]



Refer the adjoining graph of

$y=\cos x$ & $y=\sin 3x$

Number of points intersection in

$\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$,

k = 3

Q.6 [05]

$f(x)=\sqrt{8x-x^2}-\sqrt{14x-x^2-48}$

$=\sqrt{(8-x)x}-\sqrt{(8-x)(x-6)}$

Domain : $6\le x\le 8$

Now $f(x)=\sqrt{8-x}\left(\sqrt{x}-\sqrt{x-6}\right)$

$$\Rightarrow f'(x) = -\frac{\sqrt{x} - \sqrt{x-6}}{2\sqrt{8-x}} + \sqrt{8-x}\left(\frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x-6}}\right)$$

$$\Rightarrow f'(x) = \left(\sqrt{x-6} - \sqrt{x}\right)\left(\frac{\sqrt{x-6}\sqrt{x} + 8 - x}{2\sqrt{8-x}\sqrt{x-6}\sqrt{x}}\right)$$

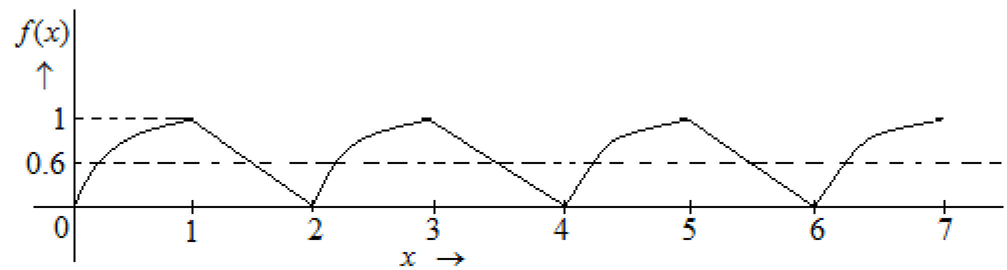
Now $\sqrt{x-6} < \sqrt{x}$ & $\sqrt{x-6}\sqrt{x} > (x-8) \Rightarrow f'(x) < 0$ for $6 \le x \le 8$

Hence $f_{MAX} = f(6) = \sqrt{12}$ & $f_{MIN} = f(8) = 0$.

Thus $m\sqrt{n} = 2\sqrt{3}$.

Q.7 [02]

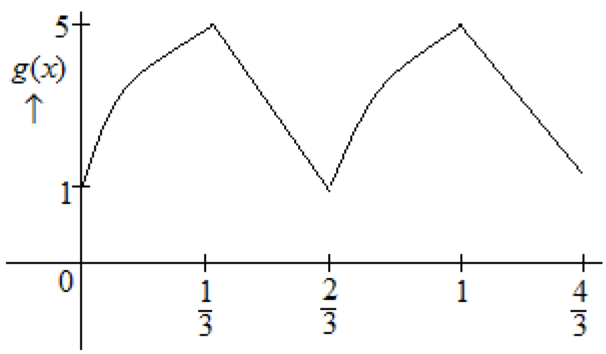
Given $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 2 - x & 1 \le x < 2 \\ f(x+2) & \text{for all } x \end{cases}$



f (x) = 0.6 : $\sqrt{x} = 0.6$ $\Rightarrow x = (0.6)^2 = 0.36$, so sum $= 4 + 6 + 2 \times 0.36 = 10.72$

& $2 - x = 0.6 \Rightarrow x = 0.4$, so sum $= 3 + 0.4 + 5 + 0.4 = 8.8$

A $= 10.72 + 8.8 = 19.52$



Now $g(x) = 4\,f(3x) + 1 \;\forall\; x \in R$

$$\Rightarrow g(x) = \begin{cases} 4\sqrt{3x} + 1 & x \in \left[0, \frac{1}{3}\right) \\ 3 - 4x & x \in \left[\frac{1}{3}, \frac{2}{3}\right) \\ f(3x+2) & x \in \text{all} \end{cases}$$

Fundamental Period $=\left(\frac{2}{3}\right) \Rightarrow$ B $=\frac{2}{3}$.

$g(x)=4f(3x)+1 \Rightarrow g'(x)=12f'(2x)+0$

or $g'\left(\frac{13}{2}\right)=12x\ f'\left(\frac{39}{2}\right)$

g;(6.5) = – 12

So, $|C|=12$

Hence, $\frac{[A]\ B\,|C|}{76}=17\times\frac{2}{3}\times\frac{12}{76}=2$

Q.8 [05]

$x^4-4x^3+6x^2-4x=2008 \Rightarrow (x-1)^4=2009$

$\Rightarrow (x-1)=(2009)^{\frac{1}{4}},-(2009)^{\frac{1}{4}}\ ,\ (2009)^{\frac{1}{4}}i\ ,\ -(2009)^{\frac{1}{4}}i$

So, non-real roots $=1\pm(2009)^{\frac{1}{4}}\cdot i$

product of non-real roots, P $=\left[1+(2009)^{\frac{1}{4}}\cdot i\right]\left[1-(2009)^{\frac{1}{4}}i\right]$

$P=1+(2009)^{\frac{1}{2}}$

So, $[P]=\left[1+(2009)^{\frac{1}{2}}\right]=45$.

Q.9 [03]

Given $f\left(\frac{2x-3}{x-2}\right)=5x-2\ ,\ x\neq 2$

$\Rightarrow$ let , $\frac{2x-3}{x-2}=t$

$\Rightarrow 2x-3=tx-2t$ or $x=\frac{2t-3}{t-2}$

$\Rightarrow f(t) = 5\left(\dfrac{2t-3}{t-2}\right) - 2$

$\Rightarrow f(t) = \dfrac{8t-17}{t-2}$

So, $f(x) = \dfrac{8x-11}{x-2}$

Now let $y = \dfrac{8x-11}{x-2}$

$\Rightarrow x = \left(\dfrac{2y-11}{y-8}\right)$

So, $f^{-1}(x) = \dfrac{2x-11}{x-8}$

$f^{-1'}(13) = \dfrac{26-11}{5} = \dfrac{15}{5} = 3$

Q.10 [04]

$\because P(x)$ has odd degree terms only so $P(-x) = -P(x)$

$P(x)$ divided by $(x-3)$ gives remainder 6 hence $P(3) = 6$

$P(x)$ divided by $(x+3)$ will give remainder $P(-3) = -P(3) = -6$

Now let $P(x) = \left(x^2 - 9\right)Q\left(x\right) + Ax + B$, where g(x) = Ax + B

So, $P(3) = 6 \quad \Rightarrow 3A + B = 6$

& $P(-3) = -6 \Rightarrow -3A + B = -6$

Solving simultaneously gives A = 2, B = 0.

g(2) = 4.

Q.11 [04]

$f : R \to \left(0, \dfrac{2\pi}{3}\right],\ f(x) = \cot^{-1}(x^2 - 4x + \alpha)$

For f (x) to be an ONTO function, $0 \le \cot^{-1}(x^2 - 4x + \alpha) \le \dfrac{2\pi}{3}$ for all real x.

or $x^2 - 4x + \alpha \ge \cot\left(\dfrac{2\pi}{3}\right)$.

$\Rightarrow x^2 - 4x + \alpha \ge -\dfrac{1}{\sqrt{3}}$.

$\Rightarrow x^2 - 4x + \left(\alpha + \dfrac{-1}{\sqrt{3}}\right) \ge 0$ for all real x.

So, $D \le 0 \Rightarrow 16 - 4\left(\alpha + \dfrac{1}{\sqrt{3}}\right) \le 0$.

$\Rightarrow \alpha \ge 4 - \dfrac{4}{\sqrt{3}}$.

So, smallest integral value of α is 4.

Q.12 [04]

$f(x) = \sin^{-1}x + \tan^{-1}x + x^2 + 4x + 1 \Rightarrow f(x) = \sin^{-1}x + \tan^{-1}x + (x+2)^2 - 3$

Now for $x \in [-1, 1]$, all of $\sin^{-1}x, \tan^{-1}x$ & $(x+2)^2$ are increasing functions.

Hence p = f(-1) & q = f(1)

Therefore p + q = 4.

Q.13 [00]

$\log_{\sin x} 2^{\tan x} > 0$

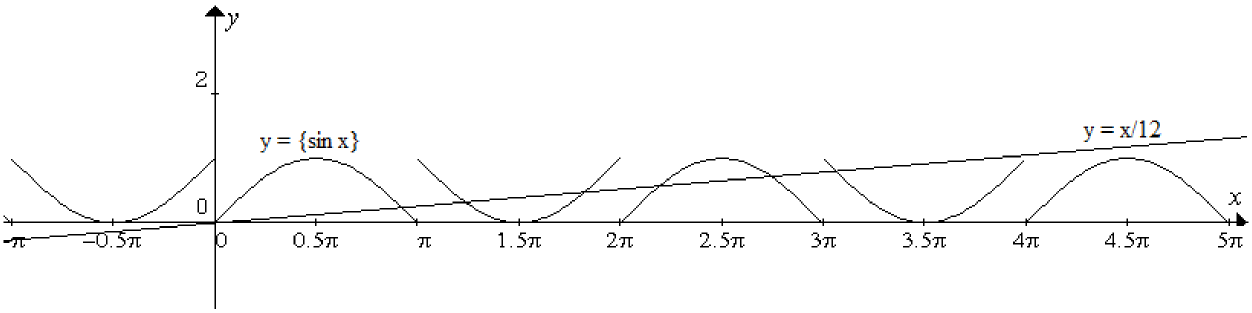
$\Rightarrow (\tan x)\cdot\log_{\sin x} 2 > 0$

$\Rightarrow \dfrac{\tan x}{(\log_2 \sin x)} > 0$

$\tan x > 0$ & $\log_2(\sin x) < 0$ in $\left(0, \dfrac{\pi}{2}\right)$ hence no solution.

{ $\log_a b$ is negative if a > 0 & 0 < a < 1}

Q.14 [07]



$12\{\sin x\} - x = 0$

$\Rightarrow \{\sin x\} = \left(\dfrac{x}{12}\right)$

Refer the adjoining graph.

Q.15 [04]

$[x] + 2\{-x\} = 3x \Rightarrow [x] + 2\{-x\} = 3[x] + 3\{x\}$

Case I : For, $x \in \mathrm{I}$, $\{-x\} = \{x\} = 0$

$\Rightarrow [x] = 3[x]$

$\Rightarrow [x] = 0$

$\Rightarrow x = 0$

Case II : For $x \notin \mathrm{I}$, $[x] + 2(1 - \{x\}) = 3[x] + 3\{x\}$

$\Rightarrow \{x\} = \dfrac{2 - 2[x]}{5}$

Now $0 \le \{x\} < 1$, hence $0 \le \dfrac{2 - 2[x]}{5} < 1$

$\Rightarrow -2 \le -2[x] < 3$

$\Rightarrow -\dfrac{3}{2} < [x] \le -1$

So, $[x] = 1$, $[x] = 0$, $[x] = -1$

$\{x\} = 0, \{x\} = \dfrac{2}{5}$, $\{x\} = \dfrac{4}{5}$

So, $x=1$, $x=\frac{2}{5}, x=-\frac{1}{5}$

Q.16 [02]

$(x)=[x]+1 \ : x \notin \mathrm{I}$

Hence, $[x]^2+([x]+1)^2<4$

$\Rightarrow 2[x]^2+2[x]-3<0$

So, $[x] \in \left(\frac{-1-\sqrt{7}}{2}, \ \frac{-1+\sqrt{7}}{2}\right)$

So, $x \in [-1,1)$

Length of interval = 2

Q.17 [02]

$$g(x)=\left(4\cos^4 x-2\cos 2x-\frac{1}{2}\cos 4x-x^7\right)^{\frac{1}{7}}$$

$$\Rightarrow g(x)=\left[4\cos^4 x-4\cos^2 x+2-\frac{1}{2}(2\cos^2 2x-1)-7\right]^{\frac{1}{7}}$$

$$=\left[4\cos^4 x-4\cos^2 x+2-\cos^2 2x+\frac{1}{2}-x^7\right]^{\frac{1}{7}}$$

$$=\left[4\cos^4 x-4\cos^2 x-(2\cos^2 x-1)^2+\frac{3}{2}-x^7\right]^{\frac{1}{7}}$$

$$=\left[4\cos^4 x-4\cos^2 x-4\cos^4 x+4\cos^2 x-1+\frac{3}{2}-x^7\right]^{\frac{1}{7}}$$

$$=\left(\frac{1}{2}-x^7\right)^{\frac{1}{7}}$$

So, $g(g(x)) = \left[\frac{1}{2} - \left(\frac{1}{2} - x^7\right)^{\frac{1}{7}\times 7}\right]^{\frac{1}{2}}$

$$= \left(\frac{1}{2} - \frac{1}{2} + x^7\right)^{\frac{1}{7}}$$

$$= x$$

So, $\frac{g(g(100))}{50} = \frac{100}{50} = 2$

Q.18 [01]

$f(x) = \frac{3x-2}{x+4} = y \Rightarrow 3x - 2 = xy + 4y$

$\Rightarrow x = \left[\frac{4y+2}{3-y}\right]$

So, $f^{-1}(x) = \frac{4x+2}{3-x} = \frac{x + \frac{1}{2}}{\frac{3}{4} - \frac{x}{4}}$.

Hence $b = \frac{1}{2}, c = -\frac{1}{4} \ \& \ d = \frac{3}{4} \Rightarrow b + c + d = 1.$

Q.19 [02]

$f(x) = \frac{ax^8 + bx^6 + cx^4 + dx^2 + 15x + 1}{x}$

$f(x) = -f(x)$

Hence, $f(-5) = -f(5) = -(-28) = 28$

So, $f\frac{(-5)}{14} = \frac{28}{14} = 2$.

Q.20 [01]

$$\log_2(3-x)+\log_{\frac{1}{2}}\left(\frac{\sin\frac{9\pi}{4}}{5-x}\right) = \cos\frac{11\pi}{3} - \log_{\frac{1}{2}}(x+7)$$

Domain : $x < 3$, $x > -7$

Sol : $\log_2(3-x)+\log_{\frac{1}{2}}\left(\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}} - \log_{\frac{1}{2}}(5-x) = \frac{1}{2} - \log_{\frac{1}{2}}(x+7)$

$\Rightarrow \log_2(3-x)+\log_2(5-x)-\log_2(x+7)=0$

$\Rightarrow \frac{(3-x)(5-x)}{x+7}=1$

$\Rightarrow x^2-8x+15=x+7$

$\Rightarrow x^2-9x+8=0$

$\Rightarrow (x-1)\ (x-8)=0$

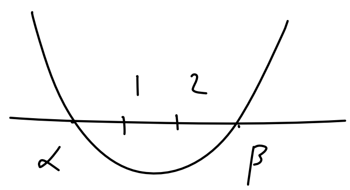
$\Rightarrow x=1$, $x=8$ but, $x\in(-7, 3)$, hence only one integral value of x is possible.

2(C) Solutions

(21) $f(x) = \begin{cases} 4 & , x \in [-4,-2) \\ |x| & , x \in [-2,7) \\ \sqrt{x} & , x \in [7,14) \end{cases}$

$\therefore$ Integers in range are $\{0, 1, 2, 3, \dots 6\}$ $\therefore d = 7$

Also $p(x) = x^2 + mx - 4m + 20$ has roots α & β such that $\alpha < 1,\ \beta > 2$



Conditions are $p(1) < 0$ & $p(2) < 0$

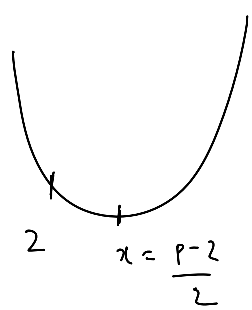
(i) $21 - 3m < 0$ & (ii) $24 - 2m < 0$

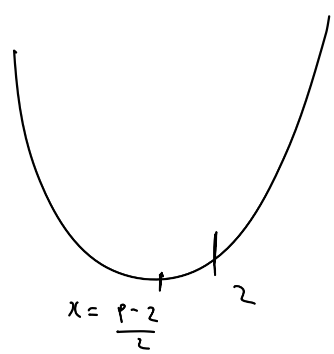
$\boxed{m > 7}$ $\cap$ $\boxed{m > 12}$

$\therefore m \in (12, \infty)$

$\therefore k = 13$ $\therefore$ Ans 8

(22) $f(x) = x^2 - (p-2)x + 3p - 2$ has range $[8, \infty)$ if $x \in [2, \infty)$





Case I $2 \le \frac{p-2}{2}$

Here range is $\left[\frac{-D}{4A}, \infty\right)$

$\therefore \frac{-D}{4A} = 8$

or $(p-2)^2 - 4(3p-2) = -32$

$p^2 - 16p + 46 = 0$

Case II $\frac{p-2}{2} < 2$

Here range is $[f(2), \infty)$.

$\therefore f(2) = 8$

$4 - 2(p-2) + 3p - 2 = 8$

or $\boxed{p = 2}$

or (cut off)

$p^2 - 16p + 46 = 0$

$\boxed{p = 8 + 3\sqrt{2}}$

or $\boxed{p = 2}$

Sum of Values $10 + 3\sqrt{2}$

or $10 + \sqrt{18}$

$\therefore$ Ans (2)

(23) let $-x^{100} = x(x+1)\,q(x) + ax + b$

put $x = 0 \Rightarrow \boxed{b = 0}$

put $x = -1 \Rightarrow -1 = -a \Rightarrow a = 1$

$\therefore \boxed{r(x) = x}$ $\therefore r(10) = 10$

(24) $y = \dfrac{3x^2 + mx + n}{x^2 + 1}$ if $y \neq 3$ only possible if $m = 0$ & $n \neq 3$

$\Rightarrow y = \dfrac{3x^2 + n}{1 + x^2} \Rightarrow y + yx^2 = 3x^2 + n$

or $x^2 = \dfrac{n - y}{y - 3}$

for $x \in R$, $\dfrac{n - y}{y - 3} \geq 0$ or $\dfrac{y - n}{y - 3} \leq 0$

$y \in [n, 3)$

$\therefore \boxed{n = -4}$

$\therefore m^2 + n^2 = 16$

(25) $f(x) \geq 0 \quad \forall x \in R$

$f(1) = a + b + c$

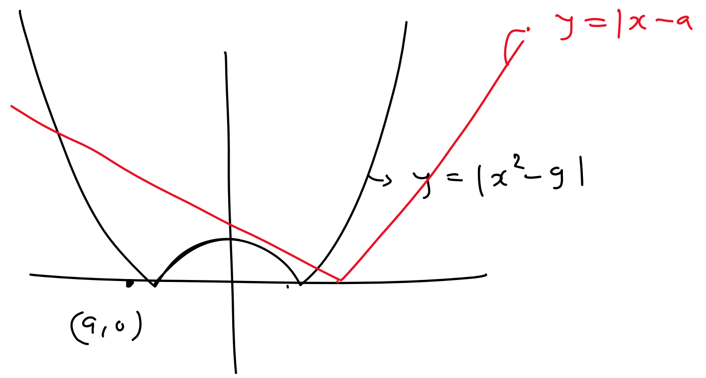
$f(-2) = 4a - 2b + c$

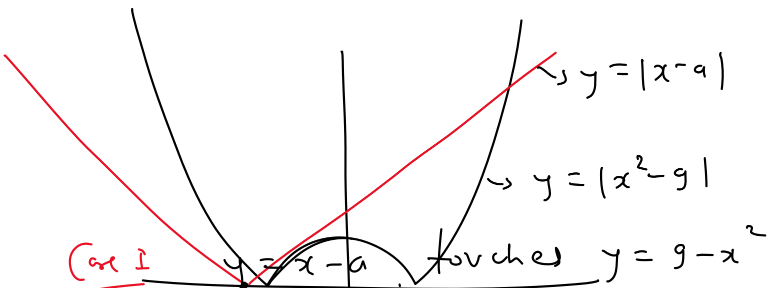
$\therefore f(1) - f(-2) = 3(b - a)$

25) $\dfrac{a+b+c}{b-a} = \dfrac{3+f(1)}{f(1)-f(-2)} = \dfrac{3}{1-\dfrac{f(-2)}{f(1)}}$

$\because$ for min $f(-2)=0$ $\therefore$ min is $\boxed{3}$

26) $|x^2-9| = |x-a|$ has 4 distinct solutions





Case I $y=x-a$ touches $y=9-x^2$

$(9,0)$

Solve $9-x^2=x-a$

$x^2+x-(a+9)=0$

$D=0$

$1+4(a+9)=0 \Rightarrow a=-\dfrac{35}{4}$

Case II $y=a-x$ touches $y=9-x^2$

Solve $a=35/4$ (By symmetry)

If $a \in (-35/4, 35/4)$

then 4 intersecⁿ point

No. of integers. 17

27) $\left[\dfrac{3}{x}\right]+\left[\dfrac{4}{x}\right]=5$ clearly $x>0$ for solⁿ

since eqⁿ is $I_1 + I_2 = 5$ & $I_2 > I_1$

By observation $[3/x]=2$ & $[4/x]=3$

$\therefore$ $2 \le 3/x < 3$ $\cap$ $3 \le \dfrac{4}{x} < 4$

$\Rightarrow$ $1 < x \le 3/2$ $\cap$ $1 < x \le 4/3$

$\therefore$ $\boxed{x \in (1, 4/3]}$ Solⁿ $a=1$.

$\therefore \boxed{x \in (1, 4/3]}$ Sol^n

$a = 1,$
$b = 4$
$c = 3$

$\therefore \boxed{\text{Ans } 20}$

(28) $\left[x - \frac{1}{2}\right]\left[x + \frac{1}{2}\right] = \text{prime}$

Eq^n is $I_1 I_2 = \text{prime}$, for $I_1 I_2$ to prime
$(I_2 > I_1)$

or

$I_2 = 1$
$\Rightarrow [x + 1/2] = 1$
$\Rightarrow [x - 1/2] = 0$
$\therefore I_1 I_2 \neq \text{prime}$
ϕ

or $I_2 = -1$
$[x + 1/2] = -1$
$-1 \leq x + 1/2 < 0$
$-2 \leq x - 1/2 < 0$
$\therefore [x - 1/2] = -2$
$I_1 I_2 = 2$ (prime)
$\therefore S_4 \equiv x \in [-3/2, -1/2)$

$\underline{\underline{\text{Ans}}}$ $x \in [-3/2, -1/2) \cup (3/2, 5/2]$

$\therefore x_1 = -3/2,\ x_2 = -1/2,\ x_3 = 3/2,\ x_4 = 5/2$

$\therefore \underline{\underline{\text{Ans}}}$ $\boxed{\Sigma x_i^2 = 11}$

(29) $\sum_{r=1}^{34} \left[\dfrac{I_1 \ 18 - r}{[x - 1/2]}\right] = 1$

i.e. $r = 1$
$1 \leq x - 1/2 < 2$
$\Rightarrow I_2 = \leq x + 1/2 [\ ] < 3$, $r = 2, 3 \therefore 0 \leq [\ ] \ x + 1/2 < 1$, $r = 4, 5 \therefore [\ ] = 2$
$r = 6, [x + 1/2 [\ ]] = 3 = 2,$
$\vdots$
or $I_2 = 2$
$r = 30, 31 \therefore I_1 I_2 [\ ]_2 = (\text{prime})$

$I_1 = -1$
$[x - 1/2] = -1$
$-1 \leq x - 1/2 < 0$
$\therefore [x + 1/2] = 0$
But $I_1 I_2 = 0$ (not prime)
$\therefore S_2 \equiv \phi$

$r = 32, 33 \therefore [\ ] = 16$, $r = 34 \therefore [\ ] = 17$

Ans $2(1+2+3 \dots 16) + 17 = 289$

30 $f(x) = x^3 + 3x^2 + 4x + b\cos x + c\sin x$

$\therefore f'(x) = 3x^2 + 6x + 4 + c\cos x - b\sin x$

$\therefore$ least of quadratic $\geq -\sqrt{b^2+c^2}$ for $f'(x) \geq 0 \ \forall x \in R$

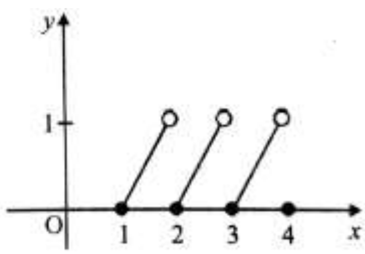
$\Rightarrow -1 \geq -\sqrt{b^2+c^2}$ $\therefore$ max of b^2+c^2 is $\boxed{1}$

Only One Option Correct

1. (A)

$$f(x) = x - [x] = \begin{cases} .... \\ x-1, & 1 \le x < 2 \\ x-2, & 2 \le x < 3 \\ x-3, & 3 \le x < 4 \\ .... \end{cases}$$

$\therefore$ Graph of function $f(x)$ is



Clearly it is a periodic function with period 1.

2. (D)

Given : $2^x + 2^y = 2 \ \forall \ x, y \in R$

But $2^x, 2^y > 0 \ \forall x, y \in R$

$\therefore \quad 2^x = 2 - 2^y < 2 \ \Rightarrow \ 0 < 2^x < 2 \ \Rightarrow \ x < 1$

3. (A)

E = {1, 2, 3, 4} and F = {1, 2}

From E to F we can define, in all, $2 \times 2 \times 2 \times 2 = 16$ functions (2 options for each element of E) out of which 2 are into, when all the elements of E either map to 1 or to 2.

$\therefore$ Number of onto functions = 16 – 2 = 14

4. (D)

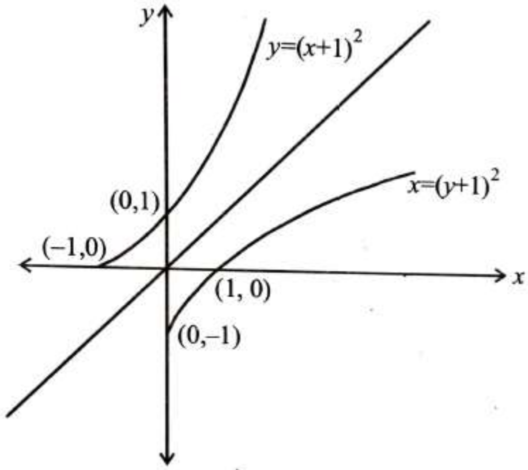
Given : $f(x) = (x+1)^2, x \ge -1$

If $g(x)$ is the reflection of $f(x)$ in the line $y = x$, then it can be obtained by interchanging x and y in $f(x)$

i.e., $y = (x+1)^2$ changes to $x = (y+1)^2$

$\Rightarrow \quad y + 1 = \sqrt{x} \qquad \left[y + 1 \ne -\sqrt{x}, \sin y \ge -1\right]$

$\Rightarrow \quad y = \sqrt{x} - 1 \qquad$ defined $\forall x \ge 0$.



$\therefore \quad g(x) = \sqrt{x} - 1 \ \forall \ x \geq 0$

5. (C)

$f(x)$ is a non-periodic, continuous and odd function

$$f(x) = \begin{cases} -x^2 + x\sin x, \ x < 0 \\ x^2 - x\sin x, \ x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} -2x^2 + \sin x + x\cos x, \ x < 0 \\ 2x - \sin x - x\cos x, \ x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} -(x - \sin x) - x(1 - \cos x), \ x < 0 \\ (x - \sin x) + x(1 - \cos x), \ x > 0 \end{cases}$$

$\because \quad x - \sin x < 0$ if $x < 0$ and $1 - \cos x > 0, \forall \ x \in R$

$\therefore \quad -(x - \sin x) - x(1 - \cos x) > 0$ if $x < 0$ and $(x - \sin x) + x(1 - \cos x) > 0$ if $x > 0$

$\Rightarrow \quad f'(x) > 0 \ \forall \ x \in R \Rightarrow f(x)$ is increasing in R

$\Rightarrow \quad f(x)$ is one-one

$$\therefore \quad \underset{x \to -\infty}{Lt} \left(-x^2\right)\left(1 - \frac{\sin x}{x}\right) = -\infty$$

$$\therefore \quad \underset{x \to \infty}{Lt} \ x^2 \left(1 - \frac{\sin x}{x}\right) = \infty$$

$\Rightarrow$ Range of $f(x) = R \Rightarrow f(x)$ is onto function

One or More than One Correct Answer

1. (B, C)

As (0, 0) and (x, $g(x)$) are two vertices of an equilateral triangle; therefore, length of a side of the triangle

$$= \sqrt{(x-0)^2 + (g(x) - 0)^2} = \sqrt{x^2 + (g(x))^2}$$

$\therefore$ The area of equilateral triangle $= \dfrac{\sqrt{3}}{4}\left(x^2 + (g(x))^2\right)$

But given that area of the equilateral triangle $=\dfrac{\sqrt{3}}{4}$

$\therefore \quad (g(x))^2 = 1-x^2 \Rightarrow g(x)=\pm\sqrt{1-x^2}$

$\therefore$ (B), (C) are the correct options as (A) is not a function.
($\because$ image of x is not unique)

2. (A, C)

$f(x)=\cos\left[\pi^2\right]x+\cos\left[-\pi^2\right]x$

We know that $9<\pi^2<10$ and $-10<-\pi^2<-9$

$\Rightarrow \left[\pi^2\right]=9$ and $\left[-\pi^2\right]=-10$

$\therefore \quad f(x)=\cos 9x+\cos(-10x)$

$f(x)=\cos 9x+\cos 10x$

(A) $f\left(\dfrac{\pi}{2}\right)=\cos\dfrac{9\pi}{2}+\cos 5\pi=-1$ **(true)**

(B) $f(\pi)=\cos 9\pi+\cos 10\pi=-1+1=0$ **(false)**

(C) $f(-\pi)=\cos(-9\pi)+\cos(-10\pi)$

$=\cos 9\pi+\cos 10\pi=-1+1=0$ **(true)**

(D) $f\left(\dfrac{\pi}{4}\right)=\cos\dfrac{9\pi}{4}+\cos\dfrac{5\pi}{2}$

$=\cos\left(2\pi+\dfrac{\pi}{4}\right)+0=\cos\dfrac{\pi}{4}=\dfrac{1}{\sqrt{2}}$ **(false)**

$\therefore$ (A) and (C) are the correct options.

3. (A, B)

Given : $f(x)=\dfrac{b-x}{1-bx}, 0<b<1$

Let $f(x_1)=f(x_2) \Rightarrow \dfrac{b-x_1}{1-bx_1}=\dfrac{b-x_2}{1-bx_2}$

$\Rightarrow b-b^2x_2-x_1+bx_1x_1=b-x_2-b^2x_1+bx_1x_2$

$\Rightarrow x_2\left(1-b^2\right)=x_1\left(1-b^2\right) \Rightarrow x_1=x_2$ as $1-b^2\neq 0$

$\therefore$ f is one-one.

Also, $\dfrac{b-cx}{1-bx}=y \Rightarrow b-x=y-bxy$

$\Rightarrow (by-1)x=;y-b \Rightarrow x=\dfrac{y-b}{by-1}$

For $y=\dfrac{1}{b}$, x is not defined

$\therefore$ f is not onto and hence nor invertible.

Also, $f'(x)=\dfrac{-1(1-bx)-(-b)(b-x)}{(1-bx)^2}=\dfrac{b^2-1}{(1-bx)^2}$

$\therefore \quad f'(b)=\frac{1}{b^2-1}$ and $f'(0)=b^2-1 \Rightarrow f'(b)=\frac{1}{f'(0)}$

$\therefore \quad$ (A) and (B) are the correct options.

4. (A, B)

Given : $f(\cos 4\theta)=\frac{2}{2-\sec^2\theta}=\frac{2\cos^2\theta}{2\cos^2\theta-1}$

$$=\frac{1+\cos 2\theta}{\cos 2\theta}=1+\frac{1}{\cos 2\theta}$$

Let $\cos 4\theta=\frac{1}{3} \Rightarrow 2\cos^2 2\theta-1=\frac{1}{3} \Rightarrow \cos 2\theta=\pm\sqrt{\frac{2}{3}}$

$\therefore \quad f(\cos 4\theta)=1+\frac{1}{\cos 2\theta}=1\pm\sqrt{\frac{3}{2}}$ or $f\left(\frac{1}{3}\right)=1\pm\sqrt{\frac{3}{2}}$

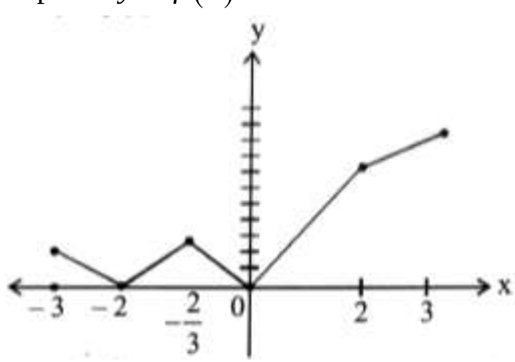
5. (A, B)

Given : $f(x)=2|x|+|x+2|-\big||x+2|-2|x|\big|$

Critical points of the $f(x)$ can be obtained by solving $|x|=0$, $|x+2|=0$ and $\big||x+2|-2|x|\big|=0$, which give $x=0,-2,2,-\frac{2}{3}$

$$\therefore \quad f(x)=\begin{cases} -2x-1, & x\le -2 \\ 2x+4, & -2<x-\frac{2}{3} \\ -4x, & -\frac{2}{3}<x\le 0 \\ 4x, & 0<x\le 2 \\ 2x+4, & x>2 \end{cases}$$

Graph of $y=f(x)$ is as follows :



From graph, $f(x)$ has local minimum at $x=-2$ and $x=0$ and local maximum at $x=-\frac{2}{3}$

6. (B, D)

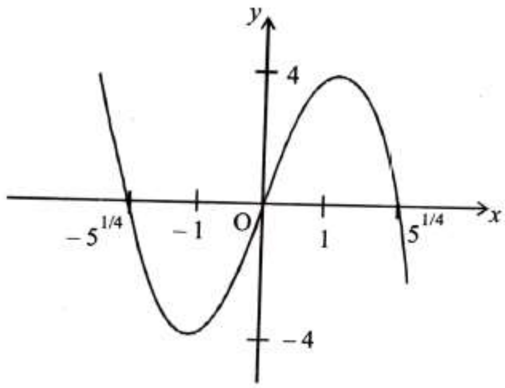
$f(x)=x^5-5x+a$

$f(x)=0 \Rightarrow x^5-5x+a=0 \Rightarrow a=5x-x^2=g(x)$

$\Rightarrow\ g(x)=0$ when $x=0, 5^{1/4}, -5^{1/4}$ and $g'(x)=0 \Rightarrow\ x=1,-1$

Also, $f(-1)=-4$ and $g(1)=4$

Thus graph of $g(x)$ will be as shown below.



From graph, it is clear that if $a \in (-4, 4)$ then $g(x)=a$ or $f(x)=0$ has 3 real roots

If $a>4$ or $a<-4$ then $f(x)=0$ has only one real root.

$\therefore$ Option (B) and (D) are the correct options.

7. (A, B, C)

Given : $f:\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ is given by

$$f(x)=(\log(\sec x+\tan x))^3$$

$$f(-x)=(\log(\sec x-\tan x))^3$$

$$=\left[\log\left(\frac{(\sec x-\tan x)(\sec x+\tan x)}{\sec x+\tan x}\right)\right]^3$$

$$=\left[\log\left(\frac{1}{\sec x+\tan x}\right)\right]^3=\left[-\log(\sec x+\tan x)\right]^3$$

$$=\left[\log\left(\frac{1}{\sec x+\tan x}\right)\right]^3=\left[-\log(\sec x+\tan x)\right]^3$$

$$=-\left[\log(\sec x+\tan x)\right]^3=-f(x)$$

$\therefore$ $f(x)$ is an odd function.

$\therefore$ Option (A) is correct and (D) is not correct.

Now, $f'(x)=3\left[\log(\sec x+\tan x)\right]^2\cdot\frac{\sec x\tan x+\sec^2 x}{\sec x+\tan x}$

$$=3\sec x\left[\log(\sec x+\tan x)\right]^2>0\ \forall\ x\in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$\therefore$ $f(x)$ is increasing on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

We know that strictly increasing function is one-one.

Also $\lim\limits_{x\to\frac{\pi}{2}^-}\left[\log(\sec x+\tan x)\right]^3\to\infty$ and $\lim\limits_{x\to\frac{\pi}{2}^+}\left[\log(\sec x+\tan x)\right]^3\to-\infty$

$\therefore$ Range of $f = (-\infty, \infty) = R = \text{Domain}$

$\therefore$ f is an onto function.

$\therefore$ Option (C) is correct.

8. (A, B, C)

$$f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$$

$$-1 \le \sin x \le 1 \Rightarrow -\frac{\pi}{2} \le \frac{\pi}{2}\sin x \le \frac{\pi}{2}$$

$$\Rightarrow -1 \le \sin\left(\frac{\pi}{2}\sin x\right) \le 1 \Rightarrow \frac{-\pi}{6} \le \frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right) \le \frac{\pi}{6}$$

$$\Rightarrow \frac{-1}{2} \le \sin\left[\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right] \le \frac{1}{2}$$

$\therefore$ Range of $f = \left[\frac{-1}{2}, \frac{1}{2}\right]$

Now, $fog(x) = \sin\left[\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right]$

Range of $fog = \left[\frac{-1}{2}, \frac{1}{2}\right]$

Now, $fog(x)\sin\left[\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right]$

Range of $fog = \left[\frac{-1}{2}, \frac{1}{2}\right]$

Now, $\lim\limits_{x\to 0}\dfrac{f(x)}{g(x)} = \lim\limits_{x\to 0}\dfrac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)}{\frac{\pi}{2}\sin x}$

$$= \lim_{x\to 0}\frac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)}{\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)} \times \frac{\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)}{\frac{\pi}{2}\sin x} = \frac{\pi}{6}$$

$$gof(x) = \frac{\pi}{2}\sin\left(\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right) - \frac{\pi}{2}\sin\left(\frac{1}{2}\right) \le g(f(x)) \le \frac{\pi}{2}\sin\left(\frac{1}{2}\right)$$

Let $\frac{\pi}{2}\sin\left(\frac{1}{2}\right) = p$

Clearly $0 < p < 1$

$\therefore$ $-\frac{\pi}{2}\sin\left(\frac{1}{2}\right) \le g(f(x)) \le \frac{\pi}{2}\sin\left(\frac{1}{2}\right)$

$-p \le g(f(x)) \le p \Rightarrow 0 < p < 1$

$\therefore$ $gof(x) \ne 1$ for any $x \in R$.

Matrix – Match Type :

1. (A) $\to$ Q ; (B) $\to$ R

(A) $f(x)=1+2x,\ D_f=\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

The given function represents a straight line so it is one-one.

But $f_{\min}=1-\pi=f\left(-\frac{\pi}{2}\right),\ f_{\max}=1+\pi=f\left(\frac{\pi}{2}\right)$

$\therefore$ Range of $f=(1-\pi,1+\pi)\in(-\infty,\infty)$

$\therefore$ f is not onto. Hence (A) $\to$ Q.

(B) $f(x)=\tan x$

It is an increasing function on $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and its range = $(-\infty,\infty)=$ co-domain of *f*.

$\therefore$ f is one-one onto. Hence (B) $\to$ R.

2. (A) $\to$ R, S, P ; (B) $\to$ Q, S ; (C) $\to$ Q, S ; (D) $\to$ R, S, P

$$f(x)=\frac{x^2-6x+5}{x^2-5x+6}=\frac{(x-5)(x-1)}{(x-2)(x-3)}$$

(A) If $-1<x<1$ then $f(x)=\frac{(-\text{ve})(-\text{ve})}{(-\text{ve})(-\text{ve})}=+\text{ve}$

$\therefore$ Also $f(x)-1=\frac{-x-1}{x^2-5x+6}=-\frac{(x+1)}{(x-2)(x-3)}$

For $-1<x<1,\ f(x)-1=\frac{-(+\text{ve})}{(-\text{ve})(-\text{ve})}=-\text{ve}$

$\Rightarrow$ $f(x)-1<0\Rightarrow f(x)<1$ **(S)**

$\therefore$ $0<f(x)<1$ (**P**)

(B) If $1<X<2$ then $f(x)=\frac{(-\text{ve})(+\text{ve})}{(-\text{ve})(-\text{ve})}=-\text{ve}$

$\therefore$ $f(x)<0$ **(Q)** and so $f(x)<1$ **(S)**

(C) If $3<x<5$ then $f(x)=\frac{(-\text{ve})(+\text{ve})}{(+\text{ve})(+\text{ve})}=-\text{ve}$

$\therefore$ $f(x)<0$ **(Q)** and so $f(x)<1$ **(S)**

(D) For $x>5$, $f(x)>0$ **(R)**

Also $f(x)-1=\frac{-(x+1)}{(x-2)(x-3)}<0$

For $x>5,\ f(x)<1$ **(S)**

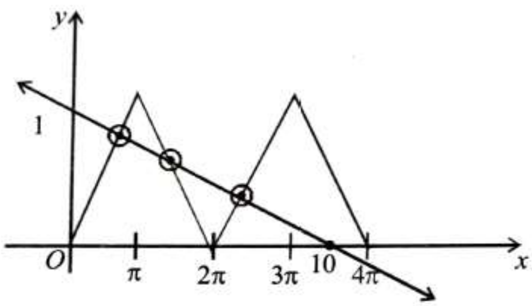
$\therefore$ $0<f(x)<1$ **(P)**

Integer Value Answer/ Non-Negative Integer

1. (3)

Given : $f:[0, 4\pi] \to [0, \pi]$ defined by $f(x) = \cos^{-1}(\cos x)$ and $g(x) = \frac{10-x}{10} = 1 - \frac{x}{10}$

The graph of $y = f(x)$ and $y = g(x)$ are as follows.

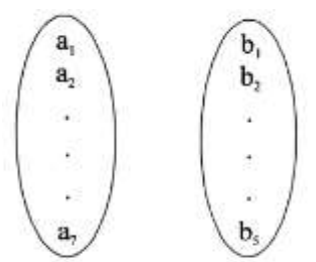


Clearly $f(x) = g(x)$ has 3 solutions.

2. (119)

Here $n(X) = 5$ and $n(Y) = 7$

Number of one-one function $= \alpha = {}^7C_5 \times 5!$ and Number of onto function Y to X = β



1, 1, 1, 1, 3 1, 1, 1, 2, 2

$$= \frac{7!}{3!4!} \times 5! + \frac{7!}{(2!)^2 3!} \times 5! = \left({}^7C_3 + 3 \times {}^7C_3\right) 5!$$

$$= 4 \times {}^7C_3 \times 5!$$

$$\Rightarrow \frac{\beta - \alpha}{5!} = 4 \times {}^7C_3 - {}^7C_5 = 4 \times 35 - 21 = 119$$