PACE
IIT & MEDICAL

Sub. : Maths **Total Marks : 80**
Std. X (CBSE)

Prelim Answer Paper - 04

SECTION A

Section A consists of 20 questions of 1 mark each.

1. **The pair of linear equations x + 2y = 5 and 3x + 12y = 10 has**

Ans : a) unique solution

2. **If the probability that it will rain tomorrow is 0.75, then the probability that it will not rain tomorrow, is**

Ans : d) 0.25

3. **Two tangents, drawn at the end points of diameter of a given circle are always**

Ans : a) parallel

4. **If the distance between the points (x, –1) and (3, 2) is 5 units, then the value of x is**

Ans : d) 7 or – 1

5. **In ΔABC, right anght angled at B, if base line is AB = 12 and BC = 5, then the value of cos C is**

Ans : a) $\frac{5}{13}$

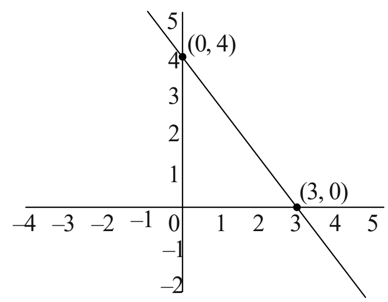
6. **If median = 137 and mean = 137.05, then the value of mode is**

Ans : b) 136.90

7. $\tan^2\theta \sin^2\theta$ **is equal to**

Ans : a) $\tan^2\theta - \sin^2\theta$

8. **The given linear polynomial y = f(x) has**

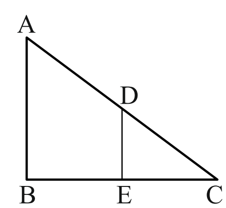


Ans : b) 1 zero and the zero is '3'

9. **Two APs have the same common difference. The first term of one of these is –1 and that of the other is –8. Then, the difference between their 4th terms is**

Ans : c) 7

10. **In ΔABC, DE || AB. AB = a, DE = x, BE = b and EC = c. Express x in terms of a, b and c**



Ans : b) $\frac{ac}{b+c}$

11. **The distance between two parallel tangents to a circle of radius 7 cm, is**

Ans : c) 14 cm

12. **The ratio in which the line 3x + y – 9 = 0 divides the segment joining the points**

(1, 3) and (2, 7) is

Ans : a) 3 : 4

Suppose the line $3x + y - 9 = 0$ divides the line segment joining A(1, 3) and B(2, 7) in the ratio k : 1 at point C.

Then, coordinates of C are $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$.

But C, lies on $3x + y - 9 = 0$.

Therefore, $3\left(\frac{2k+1}{k+1}\right) + \frac{7k+3}{k+1} - 9 = 0$

$\Rightarrow (6k + 3) + (7k + 3) - 9k - 9 = 0$

$\Rightarrow 4k - 3 = 0$

$\therefore \quad k = \frac{3}{4}$

So, the required ratio is 3 : 4 internally.

13. A pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then the Sun's elevation is

Ans : a) 60^0

14. How many terms are there in the arithmectic series 1 + 3 + 5 + ... + 73 + 75?

Ans : d) 38

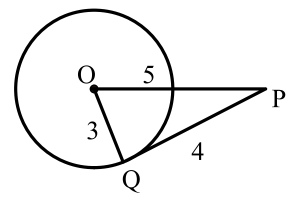
15. The circumference of a circle is equal to the sum of the circumferences of two circles having diameters 34 cm and 28 cm. The radius of the new circle is

Ans : b) 31 cm

16. If radius of circle is 3 cm and tangent drawn from an external point to the circle is 4 cm, then the distance from centre of circle to the external point is

Ans : c) 5 cm

Given, OQ = 3 cm and PQ = 4 cm.



In right angled ΔOPQ, using Pythagoras theorem

$OP = \sqrt{(OQ)^2 + (QP)^2}$

$= \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5\text{cm}$

17. Two cones have their heights in the ratio 1 : 4 and radii in the ratio 4 : 1. The ratio of their volumes is.

Ans : b) 4 : 1

18. The probability of getting 101 marks in out of 100 marks is

Ans : c) 0

Direction : In the question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.

19. Assertion (A) : $5x^2 + 14x + 10 = 0$ has no real roots.

Reason (R) : $ax^2 + bx + c = 0$ has no real roots if $b^2 < 4ac$

Ans : a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

20. Assertion (A) : $\sqrt{2}$ is an irrational number,

Reason (R) : If p be a prime, then $\sqrt{p}$ is an irrational number.

Ans : a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

SECTION B

Section B consists of 5 questions of .

2 marks each.

21. Prove that $\sqrt{2}$ is an irrational number.

Ans : Let us assume that $\sqrt{2}$ is rational.

Then $\sqrt{2}$ can be expressed as $\frac{p}{q}$, where p and q are co-primes and $q \neq 0$.

$\therefore \sqrt{2} = \frac{p}{q}$

On squaring both sides, we get

$2 = \frac{p^2}{q^2} \Rightarrow p^2 = 2q^2$ ...(i)

$\Rightarrow$ p^2 is a multiple of 2

$\Rightarrow$ 2 divides p^2

$\therefore$ 2 divides p.

$\therefore$ p = 2a for some integer a.

Substituting p = 2a in Eq. (i), we get

$(2a)^2 = 2q^2$

$\Rightarrow 4a^2 = 2q^2$

$\Rightarrow q^2 = 2a^2$

$\Rightarrow$ q^2 is a multiple of 2

$\Rightarrow$ 2 divides q^2

$\therefore$ 2 divides q

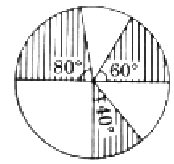
$\Rightarrow$ p and q have atleast 2 as a common factor.

But this contradicts the statement that p and q are co-primes.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.

$\therefore \sqrt{2}$ is irrational.

22. In the given figure, three sectors of a circle of radius 7 cm, making angles of 60^0, 80^0, 40^0 at the centre are shown. Find the area (in cm^2) of the shaded region.



Ans : Given, radius of circle, r = 7 cm

Now, area of shaded region

= Area of three sectors

$$= \frac{\theta_1}{360^0}\pi r^2 + \frac{\theta_2}{360^0}\pi r^2 + \frac{\theta_3}{360^0}\pi r^2$$

$$\left[\because \text{area of sector} = \frac{\theta}{360^0} \times \pi r^2\right]$$

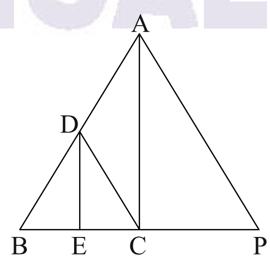
$$= \frac{\pi r^2}{360^0}(\theta_1 + \theta_2 + \theta_3)$$

$$= \frac{22}{7} \times \frac{7}{360^0} \times 7 \times (60^0 + 80^0 + 40^0)$$

$$= 11 \times \frac{1}{180^0} \times 7 \times 180^0$$

$= 77\ cm^2$.

23. In given ΔABC, DE || AC. **IF** DC || AP, **where point P lies on BC produced, then prove that** $\frac{BE}{EC} = \frac{BC}{CP}$.



Ans : Given, in ΔABC,

DE || AC [given]

So, $\frac{BE}{EC} = \frac{BD}{DA}$ ...(i)

[by basic proportionality theorem]

Also in ΔABP, DC || AP

So, $\frac{BC}{CP} = \frac{BD}{DA}$ ...(ii)

[by basic proportionality theorem]

From Eqs. (i) and (ii), we get

$\frac{BE}{EC} = \frac{BC}{CP}$ Hence proved.

OR

In ΔABC and ΔDEF, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3DE$. Then, show that the two triangles are similar but not congruent.

Ans : In ΔABC and ΔDEF, we have

$\angle B = \angle E$

$\angle C = \angle F$

$\Rightarrow \Delta ABC \sim \Delta DEF$

[By AA similarty criterion]

Since, AB and DE are corresponding sides,

But AB = 3DE

We know that two triangles are congruent, if they have the same shape and size.

But there, AB = 3DE i.e. two triangles are not of same size.

$\therefore$ ΔABC is not congruent to ΔDEF.

Hence, the two triangles are similar but not congruent.

24. **If** $\tan(A+B) = \sqrt{3}$ **and** $(A-B) = \frac{1}{\sqrt{3}}$ $0^0 < A+B \leq 90^0$, $A > B$, **then find A and B.**

Ans : We have, $\tan(A+B) = \sqrt{3}$

$\Rightarrow \tan(A+B) = \tan 60^0$

$[\because \tan 60^0 = \sqrt{3}]$

$\therefore A + B = 60^0$ ...(i)

and $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B) = \tan 30^0$

$\left[\because \tan 30^0 = \frac{1}{\sqrt{3}}\right]$

$\therefore A - B = 30^0$ ...(ii)

On adding Eqs. (i) and (ii), we get

$A + B + A - B = 60^0 + 30^0$

$2A = 90^0$

$A = 45^0$

On putting, $A = 45^0$ in Eq. (i), we get

$45^0 + B = 60^0$

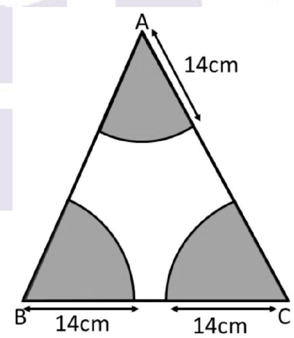
$\therefore$ $B = 60^0 - 45^0 = 15^0$

Hence, $A = 45^0$ and $B = 15^0$.

25. **With vertices A, B and C of ΔABC as centres, arcs are drawn with radii 14 cm and the three portions of the triangle, so**

obtained are removed. Find the total area removed from the triangle.

Ans : According to the given information, the figure is given below.



Area of sector with $\angle A = \frac{\angle A}{360^0} \times \pi \times r^2$

$= \frac{\angle A}{360^0} \times \pi \times (14)^2$

Similarly, other two areas are

$\frac{\angle B}{360^0} \times \pi \times (14)^2$ and

$\frac{\angle C}{360^0} \times \pi \times (14)^2$

Total area removed

$$= \left(\angle A + \angle B + \angle C\right)\frac{\pi \times (14)^2}{360^0}$$

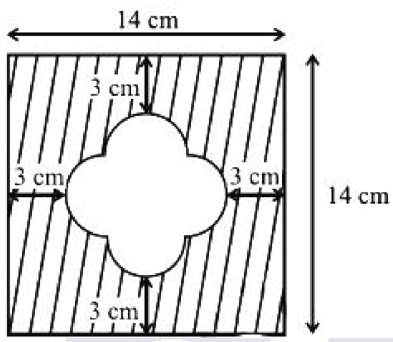
$$= \frac{180^0}{360^0} \times \frac{22}{7} \times 14 \times 14$$

$$[\because \angle A + \angle B + \angle C = 180^0]$$

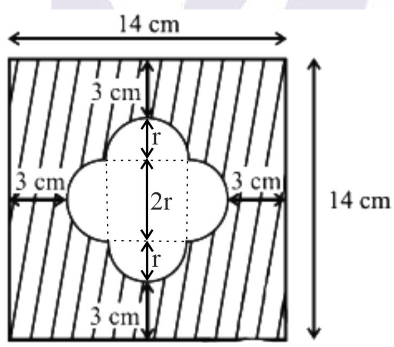
$= 11 \times 2 \times 14 = 308\ cm^2$.

OR

Find the area of the unshaded region shown in the given figure.



Ans : The given figure is



Let the radius of each semi-circle = r

$\therefore$ From figure,

$3 + r + 2r + r + 3 = 14$

$4r + 6 = 14$

$$r = \frac{8}{4} = 2\ cm$$

Area of unshaded region

= 4 × Area of each semi-circle + Area of square with length of each side, 2r

$$= 4 \times \frac{\pi r^2}{2} + (2r)^2$$

$$= 2\pi r^2 + 4r^2 = 2r^2(\pi + 2)\ cm^2.$$

SECTION C

Section C consists of 6 questions of 3 marks each.

26. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and – 3 then find the value of a and b.

Ans : Let $p(x) = x^2 + (a + 1)x + b$

Given that 2 and – 3 are the zeroes of the quadratic polynomial p(x).

$\therefore\ p(2) = 0$ and $p(-3) = 0$

$\Rightarrow 2^2 + (a + 1)(2) + b = 0$

$\Rightarrow 4 + 2a + 2 + b = 0$

$\Rightarrow 2a + b = -6$ ...(i)

and $(-3)^2 + (a + 1)(-3) + b = 0$

$\Rightarrow 9 - 3a - 3 + b = 0$

$\Rightarrow 3a - b = 6$ ...(ii)

On putting the value of a in Eq. (i) we get

$5a = 0$

$\Rightarrow a = 0$

On putting the value of b in Eq. (i), we get

$2 \times 0 + b = -6 \Rightarrow b = -6$

So, the required values are a = 0 and b = – 6

27. Prove the trigonometric identity

$$\sqrt{\frac{\operatorname{cosec} A + 1}{\operatorname{cosec} A - 1}} + \sqrt{\frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}} = 2 \sec A.$$

Ans :
$$\text{LHS} = \sqrt{\frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}} + \sqrt{\frac{\operatorname{cosec} A + 1}{\operatorname{cosec} A - 1}}$$

$$= \frac{\left[\begin{array}{l}(\sqrt{\operatorname{cosec} A - 1})(\sqrt{\operatorname{cosec} A - 1}) + \\ (\sqrt{\operatorname{cosec} A + 1})(\sqrt{\operatorname{cosec} A + 1})\end{array}\right]}{(\sqrt{\operatorname{cosec} A + 1})(\sqrt{\operatorname{cosec} A - 1})}$$

$$= \frac{(\sqrt{\operatorname{cosec} A - 1})^2 + (\sqrt{\operatorname{cosec} A + 1})^2}{(\sqrt{\operatorname{cosec} A + 1})(\sqrt{\operatorname{cosec} A - 1})}$$

$$= \frac{(\operatorname{cosec} A - 1) + (\operatorname{cosec} A + 1)}{\sqrt{\operatorname{cosec}^2 A - 1}}$$

$[\because (\sqrt{a})^2 = a$ and $\sqrt{a+b} \times \sqrt{a-b} = \sqrt{a^2 - b^2}\,]$

$$= \frac{2 \operatorname{cosec} A}{\sqrt{\cot^2 A}}$$

$[\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta]$

$$= \frac{2 \operatorname{cosec} A}{\cot A}$$

$$\left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \frac{2}{\cos A} = 2 \sec A \quad \left[\because \frac{1}{\cos\theta} = \sec\theta\right]$$

= RHS Hence proved.

OR

If $\tan\theta + \sin\theta = m$ **and** $\tan\theta - \sin\theta = n$, **then show that** $(m^2 - n^2)^2 = 16\,mn$ **or** $(m^2 - n^2) = 4\sqrt{mn}$.

Ans : Given, $\tan\theta + \sin\theta = m$ ...(i)

and $\tan\theta + \sin\theta = n$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2\tan\theta = m + n$$

$$\Rightarrow \tan\theta = \frac{m+n}{2}$$

$$\therefore \cot\theta = \frac{1}{\tan\theta} = \frac{2}{m+n} \quad \text{...(iii)}$$

On subtracting Eq. (ii) from Eq. (i), we get

$$2\sin\theta = m - n$$

$$\Rightarrow \sin\theta = \frac{m-n}{2}$$

$$\therefore \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{2}{m-n} \quad \text{...(iv)}$$

We know that $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$\Rightarrow \left(\frac{2}{m-n}\right)^2 - \left(\frac{2}{m+n}\right)^2 = 1$$

[from Eqs. (iii) and (iv)]

$$\Rightarrow \frac{4}{(m-n)^2} - \frac{4}{(m+n)^2} = 1 \quad (1/2)$$

$$\Rightarrow 4\left[\frac{1}{(m-n)^2} - \frac{1}{(m+n)^2}\right] = 1$$

$$\Rightarrow 4\left[\frac{(m+n)^2 - (m-n)^2}{(m-n)^2(m+n)^2}\right] = 1$$

$$\Rightarrow 4\left[\frac{(m^2 + n^2 + 2mn) - (m^2 + n^2 - 2mn)}{(m-n)^2(m+n)^2}\right] = 1$$

$[\because (a \pm b)^2 = a^2 + b^2 \pm 2ab]$

$$\Rightarrow 4\left[\frac{2mn + 2mn}{(m-n)^2(m+n)^2}\right] = 1$$

$$\Rightarrow \frac{16mn}{[(m-n)^2(m+n)]^2} = 1$$

$$\Rightarrow \frac{16mn}{(m^2 - n^2)^2} = 1$$

$$\Rightarrow (m^2 - n^2)^2 = 16mn$$

$$\therefore (m^2 - n^2) = 4\sqrt{mn}$$

28. **Two people are 16 km apart on a straight road. They start walking at the same time.**

If they walk towards each other with different speeds, they will meet in 2 h. Had they walked in the same direction with same speeds as before, they would have met in

8 h. Find walking speeds.

Ans : Let the walking speed of person 1 be x km/h and of person 2 be y km/h.

It is given that if they walk towards each other, they will meet in 2 hr.

$\Rightarrow$ Distance covered by person 1 in 2 hr + Distance covered by person 2 in 2 hr = 16 km

$\Rightarrow 2x + 2y = 16$

[$\because$ distance = speed × time]

$\Rightarrow x + y = 8$ ....(i)

Now, it is also given that if they walk in the same direction, then they will meet in 8 hr.

$\Rightarrow$ Distance covered by person 1 in 8 hr

= Distance covered by person 2 in 8 hr + 16 km

$\Rightarrow 8x - 8y = 16$

$\Rightarrow x - y = 2$ ...(ii)

Adding Eq. (i) and (ii), we get

$$2x = 10 \Rightarrow x = 5$$

From Eq. (ii), $5 - y = 2 \Rightarrow y = 3$

Hence, walking speeds of two persons are 5 km/h and 3 km/h.

29. Find the largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11 and 15, respectively.

Ans : It is given that when dividing 398 by the required number, then there is a remainder of 7.

This means that 398 – 7 = 391 is exactly divisible by the required number.

In other words, required number is a factor of 391.

Similarty, the required positive integer is a factor of 436 – 11 = 425 and 542 - 15 = 527.

Clearly, required number is the HCF of 391, 425 and 527.

Using factor tree, we get the prime factorisations of 391, 425 and 527 as follows :

$391 = 17 \times 23$, $425 = 5^2 \times 17$

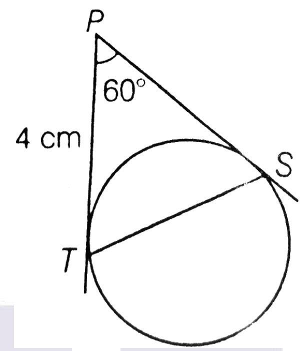
and $527 = 17 \times 31$

$\therefore$ HCF of 391, 425 and 527 = 17

Hence, 17 is the required number.

30. In the given figure, PT and PS are tangents to a circle from a point P such that

PT = 4 cm and $\angle TPS = 60^0$.



Find the length of chord TS. How many lines of same length TS can be drawn in the circle ?

Ans : We know that tangents drawn from external point to the circle are equal in length.

Here, P is an external point.

$\therefore$ PS = PT = 4 cm

So, $\angle PTS = \angle PST$

[$\because$ angles opposite to equal sides are equal]

In ΔPTS, we have

$\angle PTS + \angle PST + \angle TPS = 180^0$

[by angle sum property of triangle]

$\Rightarrow \angle PTS + \angle PTS + 60^0 = 180^0$

[$\because \angle PST = \angle PTS$ and $\angle TPS = 60^0$]

$\Rightarrow 2\angle PTS = 180^0 - 60^0$

$\Rightarrow 2\angle PTS = 120^0$

$\Rightarrow \angle PTS = \dfrac{120^0}{2} = 60^0$

$\therefore\ \Delta PTS$ is an equilateral triangle.

Hence, TS = 4 cm

Here, infinite lines of same length TS can be drawn in a circle.

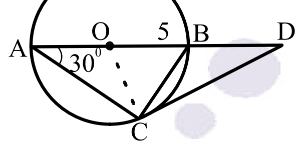
OR

AB is a diameter and AC is a chord of a circle such that $\angle BAC = 30^0$. If the tangent at C intersects AB produced at D, then prove that BC = BD.

Ans : Given AB a diameter of the circle with centre O and DC is the tangent of circle and $\angle BAC = 30^0$.

To prove BC = BD

Construction Join O to C.



Proof : Since, OC $\perp$ CD

[$\because$ the tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\therefore\ \angle OCB + \angle BCD = 90^0$

Now, OC=OA.....(Radii)

$\Rightarrow \angle OCA = \angle OAC$

...[Angles opposite to equal sides are equal]

$\therefore \angle OCA = 30^0$

Now $\angle ACB = 90^0$ [angles in semicircle]

$\therefore \angle OCA + \angle OCB = 90^0$

$\Rightarrow \angle OCB = 60^0$ and $\angle BCD = 30^0$

In ΔACD, $\angle ACD + \angle CAD + \angle ADC = 180^0$

$\Rightarrow 120^0 + 30^0 + \angle ADC = 180^0$

$\Rightarrow \angle ADC = 30^0$

In ΔBCD

$\angle BCD = \angle BDC = 30^0$

$\therefore BC = BD$

31. Find the mean of the following frequency distribution.

Classes	Frequency
25 - 30	14
30 – 35	22
35 – 40	16
40 – 45	6
45 – 50	5
50 – 55	3
55 – 60	4

Ans : Here, class width, h = 30 – 25 = 5

Let the assumed mean, a = 42.5

Now, let us make the table for the given data

Classes	f_i	x_i	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
25 – 30	14	27.5	–15	–3	–42
30 – 35	22	32.5	–10	–2	–44
35 – 40	16	37.5	–5	–1	–16
40 – 45	6	a = 42.5	0	0	0
45 – 50	5	47.5	5	1	5
50 – 55	3	52.5	10	2	6
55 – 60	4	57.5	15	3	12
Total	$\sum f_i = 70$				$\sum f_i u_i = -79$

Here $\sum f_i u_i = -79, \sum f_i = 70$ and h = 5

$$\therefore \text{Mean} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$$

$$= 42.5 + 5\left(\frac{-79}{70}\right)$$

$$= 42.5 - 5.6428 = 36.8571.$$

SECTION D

Section D consists of 4 questions of 5 marks each.

32. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of

6 km/h more than the first speed. If it takes 3 h to complete the journey, what was its first average speed ?

Ans : Let the original average speed of train be x km/h.

Time taken by train to cover 54 km with original speed $= \frac{54}{x}$

Time taken by train to cover 63 km with increased speed $= \frac{63}{x+6}$

According to the question,

$$\frac{54}{x} + \frac{63}{x+6} = 3$$

$\Rightarrow 54(x+6) + 63x = 3x(x+6)$

$\Rightarrow 18(x+6) + 21x = x(x+6)$

$\Rightarrow 18x + 108 + 21x = x^2 + 6x$

$\Rightarrow x^2 - 33x - 108 = 0$

$\Rightarrow x = \frac{33 \pm \sqrt{1089+432}}{2} = \frac{33 \pm 39}{2}$

$\Rightarrow x = \frac{33+39}{2}$ or $x = \frac{33-39}{2}$

$x = 36$ or $x = -3$

Since, speed cannot be negative.

Hence, first average speed is 36 km/h.

OR

Two pipes together can fill a tank in $\frac{15}{8}$h. The pipe with larger diameter takes 2h less than the pipe with smaller diameter to fill the tank separately. Find the time in which each pipe can fill the tank separatelly.

Ans : Let the tap with smaller diameter fills the tank alone in xh.

Let the tap with larger diameter fills the tank alone in (x –2)h.

In 1 hr, the tap with smaller diameter can fill $\frac{1}{x}$ parts of the tank.

in 1 h, the tap will larger diameter can fill $\frac{1}{x-2}$ part of the tank.

The tank is filled up in $\frac{15}{8}$h.

Thus in 1 hr, the tap fill $\frac{8}{15}$ part of the tank.

$$\therefore \frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$$

$\Rightarrow 15(x-2+x) = 8x(x-2)$

$\Rightarrow 15(2x-2) = 8x^2 - 16x$

$\Rightarrow 30x - 30 = 8x^2 - 16x$

$\Rightarrow 8x^2 - 46x + 30 = 0$

$\Rightarrow 4x^2 - 23x + 15 = 0$

$\Rightarrow x = \frac{23 \pm \sqrt{529-240}}{8} = \frac{23 \pm 17}{8}$

$\Rightarrow \therefore x = \frac{40}{8} = 5$ or $= \frac{6}{8} = \frac{3}{4}$

If x = 5, x – 2 = 5 – 2 = 3

If $x = \frac{3}{4}, x - 2 = \frac{3}{4} - 2 = \frac{3-8}{4} = -\frac{5}{4}$,

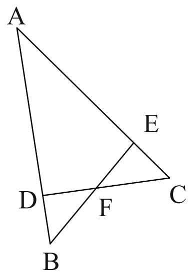
not possible as time can not be negative

Hence, the tap with smaller diameter fills the tank alone in 5 h and the tap with larger diameter fills the tank alone in 3 h.

33. **a) State and prove Basic Proportionality theorem.**

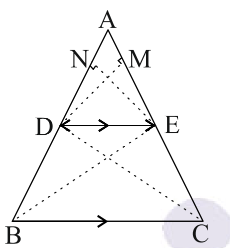
b) In the given figure $\angle CEF = \angle CFE$. F is the mid-point of DC.

Prove that $\frac{AB}{BD} = \frac{AE}{FD}$



Ans : a) **Statement :** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct point, the other two sides are divided in the same ratio,

Proof



Let ABC be a triangle in which a line parallel to side BC intersects the other two sides AB and AC at D and E, respectively.

To prove $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join CD, BE and draw DM $\perp$ AC and EN $\perp$ AB.

Area of $\Delta ADE = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times AD \times NE$$

$\Rightarrow$ $ar(\Delta ADE) = \frac{1}{2} \times AD \times NE$

Area of $\Delta BDE = ar(\Delta BDE)$

$$= \frac{1}{2} \times BD \times NE$$

Also, area of $\Delta ADE = ar(\Delta ADE)$

$$= \frac{1}{2} \times AE \times DM$$

Area of $\Delta CED = ar(\Delta CED)$

$$= \frac{1}{2} \times CE \times DM$$

$$\therefore \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times NE}{\frac{1}{2} \times BD \times NE} = \frac{AD}{BD} \quad ...(i)$$

and $$\frac{ar(\Delta ADE)}{ar(\Delta CED)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times CE \times MD} = \frac{AE}{CE} \quad ..(ii)$$

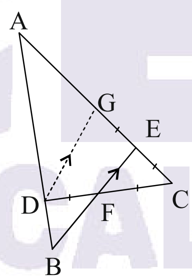
Since, (ΔBDE) and (ΔCED) are on the same base DE and between the same parallel lines DE and BC.

$\therefore$ $ar(\Delta BDE) = ar(\Delta CED)$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$\frac{AD}{BD} = \frac{AE}{CE}$$

b) The given figure is



Draw DG || BE

$\therefore$ In ΔABE,

$$\frac{AD}{BD} = \frac{AG}{GE} \quad ...(i)$$

[using basic proportionality theorem]

Given, F is the mid-point of DC

$\Rightarrow$ DF = CF ...(ii)

Also, $\angle CEF = \angle CFE$

$\Rightarrow$ CE = CF ...(iii)

[sides opposite to equal angles]

In ΔCDG,

$$\frac{CF}{FD} = \frac{CE}{EG}$$

$\Rightarrow CE = EG$ [using Eq. (ii)]

$\Rightarrow GE = CE = CF = FD$ ...(iv)

[using Eqs. (ii) and (iii)]

Adding 1 in Eq. (i), we get

$$\frac{AD}{DB} + 1 = \frac{AG}{GE} + 1$$

$$\Rightarrow \frac{AD + DB}{DB} = \frac{AG + GE}{GE}$$

$$\Rightarrow \frac{AB}{DB} = \frac{AE}{EG} = \frac{AE}{FD} \text{ [using Eq.(iv)]}$$

34. The mean of the following frequency table is 50 but the frequencies f_1 and f_2 in class interval 20-40 and 60-80 are missing. Find the missming frequencies.

Class Interval	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	Total
Frequency	17	f_1	32	f_2	19	120

Ans : Let assumed mean be A = 50

Table for the given data is

Class Interval	Frequency (f_i)	Mid - value (x_i)	$d_i = x_i - A$	$f_i d_i$
0 - 20	17	10	- 40	- 680
20 - 40	f_i	30	- 20	$-20f_i$
40 - 60	32	50	0	0
60 - 80	f_2	70	20	$20f_2$
80 - 100	19	90	40	760
Total	$N = \sum f_i = 68 + f_1 + f_2$			$\sum f_i d_i = 80 - 20f_1 + 20f_2$

Here, $N = \sum f_i = 120$ [given]

$\Rightarrow 68 + f_1 + f_2 = 120$

$\Rightarrow f_1 + f_2 = 52$ ...(i)

But mean = 50 [given]

$$\therefore \quad A + \frac{\sum f_i d_i}{N} = 50$$

$$\Rightarrow 50 + \frac{80 - 20f_1 + 20f_2}{120} = 50$$

$\Rightarrow 80 - 20f_1 + 20f_2 = 0$

$\Rightarrow 4 - f_1 + f_2 = 0$

$\therefore \quad f_1 - f_2 = 4$ ....(ii)

On solving Eqs. (i) and (ii), we get

$f_1 = 28$ and $f_2 = 24$.

35. Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in pond rise by 21 cm?

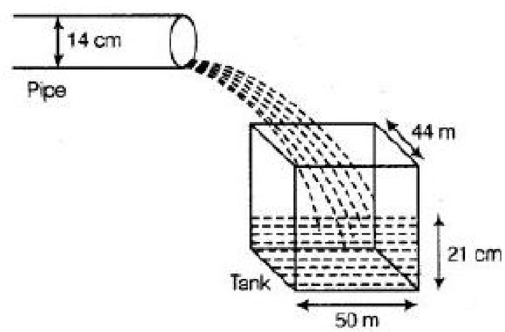
What should be the speed of water, if the rise in water level is to be attained in 1 h ?

Ans : Given, diameter of cylindrical pipe = 14 cm

$\Rightarrow$ Radius of cylindrical pipe (r) = 7 cm

Also, length of the pond (l) = 50m,

Breadth of the pond (b) = 44m,



Height of the water level in the pond (h) = 21 cm

Let t h be the time taken to raise the level of water by 21 cm.

Length of the water column in t h =

$(15 \times t)$cm = 15000 t m^3

Volume of water column that come out from cylindrical pipe in t h

$$= \pi \times \left(\frac{7}{100}\right)^2 \times 15000 \text{ t}$$

$\therefore$ Volume of water column

= Volume of cuboid with height h

$\Rightarrow$

$$\pi \times \left(\frac{7}{100}\right)^2 \times 15000\ t = 50 \times 44 \times \frac{21}{100}$$

$$\Rightarrow \frac{22}{7} \times \frac{7^2}{10} \times 15t = 22 \times 21$$

$\Rightarrow 7 \times 15t = 210 \Rightarrow t = 2h$

Hence, in 2 h, level of water in pond rise by 21 cm.

Let v km/h be the speed of water.

Length of the water column in 1 h = v km

= (v × 1000)m

$\therefore$ Volume of water column

$$= \pi \times \left(\frac{7}{100}\right)^2 \times v \times 1000$$

$$= \frac{22 \times 7 \times v}{10} m^3$$

$$\therefore \frac{22 \times 7 \times v}{10} = 50 \times 44 \times \frac{21}{100}$$

$\Rightarrow v = 30$ km/h

Hence, speed of water is 30 km/h.

OR

A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius of the cylindrical part are 2 m and 14 m, respectively and the total height of the tent is 13.5 m, find the area of the canvas required for making the tent,

keeping a provision of 26 m² **of canvas for stitching and wastage. Also, find the cost of the canvas to be purcased at the rate of ₹ 500 per m².**

Ans : Given, total height of the tent = 13.5 m

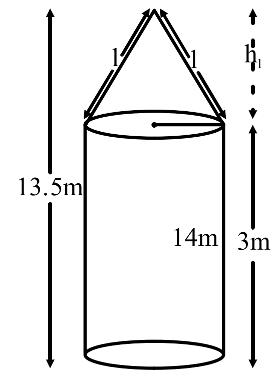
and radius of the base of the tent = 14 m

As shown in figure,

height of the cone, h_1 = Total height

– Height of cylinder

= 13.5 – 3 = 10.5 cm



Radius of cone and radius of cylinder r = 14 m

Slant height l of cone $\sqrt{h_1^2 + r^2}$

$$= \sqrt{(10.5)^2 + (14)^2} = 17.5m$$

Curved surface area of cylinder $= \pi rl$

$$= \pi \times 14 \times 17.5 = 245\pi\ m^2$$

Curved surface area of cylinder $= 2\pi rh_2$,

where h_2 is height of cylinder

$$= 2\pi \times 14 \times 3 = 84\pi\ m^2$$

$\therefore$ Area of canvas required

$= 245\pi + 84\pi + 26$

$$= 245 \times \frac{22}{7} + 84 \times \frac{22}{7} + 26$$

$= 770 + 264 + 26 = 1060\ m^2$

$\therefore$ Cost of canvas = ₹(500×1060)

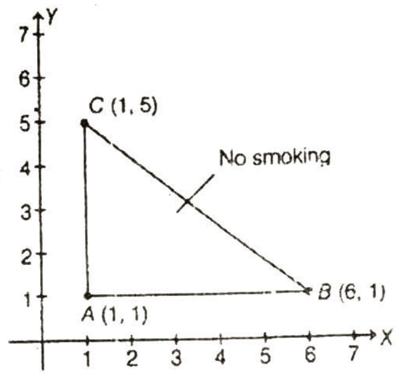
= ₹530000.

SECTION E

Case study based questions are compulsory.

36. No smoking compaign

All of them know that smoking is injurious for health. So, college students decide to make a campaign.



To raise social awareness about hazards of smoking, a school decided to start "No SMOKING" campaign.

10 students are asked to prepare campaign banners in the shape of triangle (as shown in the figure)

On the basis of above information, answer the following questions.

i) If cost of per cm^2 **of banner is ₹ 2, then find the overall cost incurred on such campaign.** **2**

Or

If we want to draw a circumscribed circle of given, then find the coordinates of the centre of circle.

ii) If we draw the image of figure about the line BC, then find the total area. 1

iii) Find the centroid of the given triangle. 1

Ans : i) Here, from the figure,

coordinates of A = (1, 1),

coordinates of B = (6, 1)

and coordinates of C =(1, 5)

$\therefore$ Area of banner = Area of ΔABC

Now, area of one banner

$$= \frac{1}{2} \times AB \times AC$$

$$= \frac{1}{2} \times 5 \times 4$$

= 10 sq units

Then, area of 10 banners

= 10 × Area of one banner

$\therefore$ Cost of 10 banners at the rate of ₹ 2 per cm^2

= 2 × Area of 10 banners

= 2 × 10 × 10

= ₹ 200

OR

A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius of the cylindrical part are 2 m and 14 m, respectively and the total height of the tent is 13.5 m, find the area of the canvas required for making the tent,

keeping a provision of 26 m^2 of canvas for stitching and wastage. Also, find the cost of the canvas to be purcased at the rate of ₹ 500 per m^2.

Ans : The centre of circumscribed circle of given triangle is the mid-point of hypotenuse.

Centre of circle = Mid-point of BC

$$= \left(\frac{1+6}{2}, \frac{5+1}{2}\right)$$

$$= \left(\frac{7}{2}, \frac{6}{2}\right)$$

= (3.5, 3)

ii) Total area of the required figure

= 2 × Area of ΔABC

= 2 × 10 = 20 sq units

iii) Centroid of the triangle (x, y)

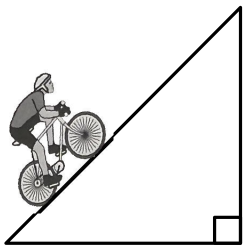
$$= \left(\frac{1+1+6}{3}, \frac{5+1+1}{3}\right)$$

$$= \left(\frac{8}{3}, \frac{7}{3}\right) = (2.6, 2.3)$$

37. **Case Study :**

A cyclist is climbing through a 20 m long rope which is highly stretched and tied from the top of a vertical pole to the ground as

shown below

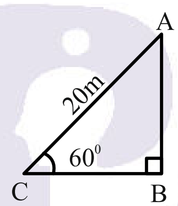


Based on the above information, answer the following questions

i) Find the height of the pole, if angle made by rope with the gound level is 60^0. **1**

ii) If the angle made by the rope with the ground level is 45^0, then find the height of the pole. **2**

Ans : i) Let in ΔABC, AC will be rope and AB be a vertical pole.



Then, AC = 20m, $\angle C = 60^0$ $\angle B = 90^0$

In ΔABC, $\sin 60^0 = \frac{AB}{AC}$

$$\frac{\sqrt{3}}{2} = \frac{AB}{20}$$

$\Rightarrow AB = 10\sqrt{3}m$

ii) In ΔABC, $\angle C = 45^0$

$\sin 45^0 = \frac{AB}{AC}$ $[\angle C = 45^0]$

$\frac{1}{\sqrt{2}} = \frac{AB}{20} \Rightarrow \frac{20}{\sqrt{2}} = AB$

$\Rightarrow AB = \frac{20}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ [by rationalising]

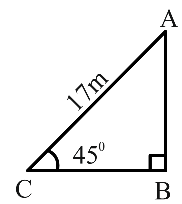
$= 10\sqrt{2}m.$

OR

If the angle made by the rope with the ground level is 45^0 and 3 m rope is broken, then what will be the height of the pole.

iii) If the angle made by the rope with the ground level is 60^0 then calculate the distance between artist and pole at ground level. **1**

Ans : Length of rope = 20 – 3 = 17 m



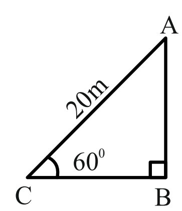
$\sin 45^0 = \frac{AB}{AC}$ $[\angle C = 45^0]$

$\sin 45^0 = \frac{AB}{AC}$

$\frac{1}{\sqrt{2}} = \frac{AB}{17}$

$\Rightarrow AB = \frac{17}{\sqrt{2}} m$

iii) In ΔABC, $\angle C = 60^0$



$\cos 60^0 = \frac{BC}{AC}$

$\Rightarrow \frac{1}{2} = \frac{BC}{20}$

$BC = 10m$

38. Case Study :

Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of ₹ 118000 by paying every month starting with the first installment of ₹ 1000. If he increases the installment by ₹ 100 every month.

On the basis of above information, answer the following questions

i) What amount does he still have to pay after 30th installment ? 1

ii) Find the amount paid by him in 30th installment. 2

Ans : i) After 30th installment, he still have to pay

= 118000 – 735000 = ₹ 44500

ii) Since, he pays first installment of ₹ 1000 and next consecutive months he pay the installment are 1100, 1200, 1300, ...

Thus, we get the AP sequence,

1000, 1100, 1200,.....

Here, a = 1000 and d = 1100 – 1000 = 100

Now, $a_{30} = a + (30 - 1)d$

$[\because\ T_n = a + (n-1)d]$

= 1000 + 29 × 100

= 1000 + 2900 = 3900

Hence, the amount paid by him in 30th installment is ₹ 3900.

OR

Find the amount paid by him in the 30 installments.

iii) If total installments are 40, then what amount paid in the last installment ? 1

Ans : $\therefore \quad S_{30} = \frac{30}{2}[2a + (30-1)d]$

$\left[\because S_n = \frac{n}{2}\{2a + (n-1)d\}\right]$

= 15(2 × 100 + 29 × 100)

= 15(2000 + 2900)

= ₹ 73500

iii) The amount in last 40th installment is

$a_{40} = a + (40 - 1)d$

= 1000 + 39 × 100

= 1000 + 3900

= ₹ 4900.

* * *