Sub. : Maths

Std. X (CBSE)

Prelim Answer Paper - 03

Total Marks : 80

SECTION A

Section A consists of 20 questions of 1 mark each.

1. **If HCF (306, 657) = 9, what will be the LCM (306, 657) ?**

Ans : b) 22338

2. **For what value of k, –4 is a zero of the polynomial $x^2 - x - (2k + 2)$?**

Ans : c) 9

3. **If the pair of linear equations 3x + y = 3 and 6x + ky = 8 does not have a solution, then the value of k is**

Ans : a) 2

4. **The non-zero value of k for which the quadratic equation $3x^2 - kx + k = 0$ has equal roots, is**

Ans : c) 12

5. **Which term of the AP 2, –1, –4, –7, ... is – 40?**

Ans : c) 15^{th}

6. **If the point p(k, 0) divides the line segment joining the points A(2, –2) and B(–7, 4) in the ratio 1 : 2, then the value of k is**

Ans : d) -1

7. **The probability of passing a certain test is $\frac{x}{24}$. If the probability of not passing is $\frac{7}{8}$, then x is equal to**

Ans : b) 3

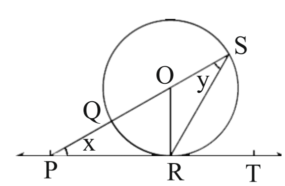
8. **If in two triangles $\triangle FED$ and $\triangle PQR$, $\angle D = \angle Q$ and $\angle R = \angle E$, then which of the following is not true ?**

Ans : b) $\frac{DE}{QR} = \frac{EF}{QP}$

9. **If radius of circle is 3 cm and tangent drawn from an external point to the circle is 4 cm, then the distance from centre of circle to the external point is**

Ans : c) 5 cm

10. **In the given figure, PT is a tangent to a circle with centre O, at point R. If diameter SQ is produced, it meets with PT at point P with $\angle SPR = x$ and $\angle QSR = y$, then the value of $\angle x + 2\angle y$ is**



Ans : d) 90^0

11. **$\sin^6\theta + \cos^6\theta$ is equal to**

Ans : d) $1 - 3\sin^2\theta\cos^2\theta$

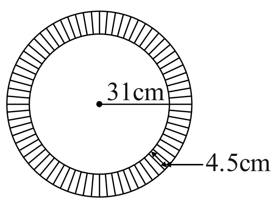
12. **If the perimeter and the area of a circle are numerically equal, then the radius of the circle is**

Ans : a) 2 units

13. **If 8 tan θ = 15, then the value of sin θ – cos θ is**

Ans : a) $\frac{7}{17}$

14. **The area of the shaded portion is**



Ans : a) 940.5 cm^2

15. **If $\tan^2 45^0 - \cos^2 30^0 = x \sin 45^0 \cos 45^0$, then the value of x is**

Ans : a) $\frac{1}{2}$

16. **Which of the following numbers cannot be the probability of happening of an event?**

Ans : b) $\frac{7}{0.01}$

17. **C is mid-point of PQ is P is (4, x), C is (y, –1) and Q is (–2, 4), then x and y respectively are**

Ans : a) -6 and 1

18. **If every term of the statistical data consisting of n terms is decreased by 2, then the mean of the data is**

Ans : a) decreases by 2

Direction : In the question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.

19. **Assertion(A): The common difference of an AP, whose nth term is $a_n = (3n + 7)$, is 3.**

Reason(R): The nth term of an AP is $a_n = a + (n – 1)d$.

Ans : a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

20. **Assertion (A) : Total surface area of the top is the sum of the curved surface area of the hemisphere and the curved surface area of the cone.**

Reason (R) : Top is obtained by fixing the plane surfaces of the hemisphere and cone together.

Ans : a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

SECTION B

Section B consists of 5 questions of 2 marks each.

21. **Using prime factorisation, find HCF and LCM of 96 and 120.**

Ans: We have,

$$\begin{array}{r|l} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

The prime factorisation of

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

and prime factorisation of

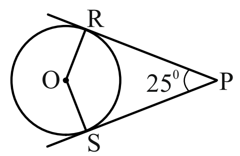
$120 = 2 \times 2 \times 2 \times 3 \times 5$

$= 2^3 \times 3 \times 5$

Now, HCF (96, 120) = $2^3 \times 3 = 24$

and LCM (96, 120) = $2^5 \times 3 \times 5 = 480$.

22. **In the given figure, if $\angle RPS = 25^0$, then find the value of $\angle ROS$**



Ans : Since OR $\perp$ PR and OS $\perp$ SP

$\therefore \quad \angle ORP = \angle OSP = 90^0$.

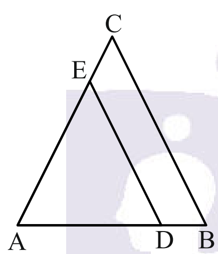
In quadrilateral ORPS.

$\angle ROS + \angle ORP + \angle RPS + \angle OSP = 360^0$

$\Rightarrow \quad \angle ROS + 90^0 + 25^0 + 90^0 = 360^0$

$\Rightarrow \quad \angle ROS = 360^0 - 205^0 = 155^0$

23. In given figure, $DE \parallel BC$. If AD = x. DB = x – 2, AE = x + 2 and EC = x - 1, Find the value of x.



Ans : In $\triangle ABC$, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

[by using basic proportionality theorem]

$$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$$

$\Rightarrow x(x - 1) = (x - 2)(x + 2)$

$\Rightarrow x^2 - x = x^2 - 4$

$\Rightarrow x = 4$

24. Prove that

(cosec A - sin A) (sec A - cos A)

$$= \frac{1}{\tan A + \cot A}$$

Ans : LHS = (cosec A - sin A) (sec A - cos A)

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$$

$$= \frac{\cos^2 A}{\sin A} . \frac{\sin^2 A}{\cos A} = \sin A \cos A . \quad ..(i)$$

$$RHS = \frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{\sin A \cos A}{1} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$= \sin A \cos A$ ... (ii)

From Eqs. (i) and (ii), we have

LHS = RHS

Hence, (cosec A – sin A) (sec A – cos A)

$$= \frac{1}{\tan A + \cot A} \quad \text{Hence proved}$$

OR

If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

Ans : Let $\cos\theta - \sin\theta = x$ ...(i)

Given $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$ ...(ii)

On squaring Eqs. (i) and (ii) and adding. we get

$\cos^2\theta + \sin^2\theta - 2\cos\theta . \sin\theta + \cos^2\theta + \sin^2\theta = 2\cos\theta\sin\theta = x^2 + 2\cos^2\theta$

$\Rightarrow 1 + 1 = x^2 + 2\cos^2\theta$

$\Rightarrow 2 = x^2 + 2(1 - \sin^2\theta)$

$\Rightarrow 2 = x^2 + 2 - 2\sin^2\theta$

$\Rightarrow x = \sqrt{2}\sin\theta$

Hence, $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

25. The short and long hands of a clock are 6 cm and 8 cm long. respectively. Then, find the sum of the distance travelled by their tips in 1 day. [take π = 22/7]

Ans : In 1 day i.e. 24h, short (hour) hand of the clock make 2 revolutions and long (minute) hand make 24 revolutions.

In 1 revolution, distance travelled by tip of hour hand

= Circumference of circle of radius 6 cm

$$= 2 \times \frac{22}{7} \times 6$$

In 1 revolution distance travelled by tip of minute hand

=Circumference of circle of radius 8 cm

$$= 2 \times \frac{22}{7} \times 8$$

$\therefore$ Sum of distances travelled by tips of both hand in 1 day

$$= 2\times 2\times\frac{22}{7}\times 6 + 24\times 2\times\frac{22}{7}\times 8$$

$$= 2\times\frac{22}{7}\times(12+192) = 2\times\frac{22}{7}\times 204$$

= 1282.29 cm (approx).

OR

The radius of the wheel of a bus is 25 cm. If the speed of the bus is 33 km/h, then how many revolutions will the wheel make in 1 min ?

Ans : In 1 h, distance covered by wheel = 33km

In 1 min, distance covered by wheel

$$= \frac{33\times 1000}{60}\,\text{m}$$

= 550 m

Now, number of revolutions made in 1 min

$$= \frac{\text{Distance covered by wheel}}{\text{Circumference of the wheel}}$$

$$= \frac{550}{2\times\frac{22}{7}\times\frac{25}{100}}$$

$$\left[\because 25\text{ cm} = \frac{25}{100}\text{m}\right]$$

$$= \frac{550\times 7\times 100}{2\times 22\times 25} = 350.$$

SECTION C

Section C consists of 6 questions of 3 marks each.

26. Prove that $5\sqrt{2}$ is irrational.

Ans : Let us assume that $5\sqrt{2}$ is a rational number .

Then, there exist coprime positive integers a and b such that

$$5\sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \quad \sqrt{2} = \frac{a}{5b}$$

$\because$ 5, a and b are integers, so $\frac{a}{5b}$ is a rational number.

$\Rightarrow$ $\sqrt{2}$ is a rational number.

But this contradicts the fact that $\sqrt{2}$ is irrational So, our assumption is not correct.

Hence, $5\sqrt{2}$ is an irrational number.

27. If α and β are the zeroes of the quadratic polynomial f(x) = $3x^2 - 5x - 2$, then find the value of $\alpha^3 + \beta^3$.

Ans : Given, α and β are the zeroes of quadratic polynomial

$f(x) = 3x^2 - 5x - 2$

On comparing with $f(x) = ax^2 + bx + c$, we get

$a = 3$, $b = -5$ and $c = -2$

We know that

$$\alpha+\beta=-\frac{b}{a}=\frac{-(-5)}{3}=\frac{5}{3} \text{ and } \alpha.\beta=\frac{c}{a}=-\frac{2}{3}$$

$$\therefore\ \alpha^3+\beta^3 = (\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha.\beta\right]$$

$$=\frac{5}{3}\left[\left(\frac{5}{3}\right)^2-3\times\left(\frac{-2}{3}\right)\right]$$

$$=\frac{125}{27}+2\times\frac{5}{3}$$

$$=\frac{125+90}{27}=\frac{215}{27}.$$

28. Three chairs and two tables cost Rs. 1850 Five chairs and three tables cost Rs 2850. Find the cost of seven chairs and three tables.

Ans : Let the cost of one chair and one table are Rs x and Rs y. respectively then

According to the question,

$3x + 2y = 1850$ ...(i)

$5x + 3y = 2850$ ... (ii)

On multiplying Eq, (i) by 3 and Eq (ii) by 2 and then subtracting Eq. (ii) from Eq. (i) we get

$3(3x+2y)-2(5x+3y)=1850\times3-2850\times2$

$\Rightarrow 9x - 10x = 5550 - 5700$

$\Rightarrow -x = -150$

$\Rightarrow x = 150$

On putting $x = 150$ in Eq (i), we get

$3 \times 150 + 2y = 1850$

$\Rightarrow 450 + 2y = 1850$

$\Rightarrow 2y = 1850 - 450 = 1400$

$$\Rightarrow y = \frac{1400}{2} = 700$$

$\therefore$ Cost of one chair is Rs 150 and cost of one table is Rs 700.

Hence, the cost of seven chairs and three tables

$= 7x + 3y = 7 \times 150 + 3 \times 700$

$= 1050 + 2100$

$=$ Rs 3150

OR

The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digit of the number differ by 2. Find the number. How many such numbers are there?

Ans : Let the ten's and the unit's digit be x and y respectively

Then, the number $= 10x + y$

After reversing the order of the digits, the number is $10y + x$

According to the question.

$(10x + y) + (10y + x) = 66$

$\Rightarrow 11x + 11y = 66$

$\Rightarrow x + y = 6$ ...(i)

Also, it is given that the digits differ by 2, therefore either

$x - y = 2$ ....(ii)

or $y - x = 2$ ...(iii)

If $x - y = 2$, then adding Eqs. (i) and (ii) we get

$2x = 8 \Rightarrow x = 4$

From Eq (i).

$4 + y = 6 \Rightarrow y = 2$

Thus, the number $= 10 \times 4 + 2 = 42$

If $y - x = 2$, then adding Eq (i) and (iii) we get

$2y = 8 \Rightarrow y = 4$

From Eq. (i)

$x + 4 = 6 \Rightarrow x = 2$

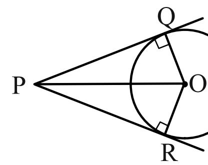
Thus, the number $= 10 \times 2 + 4 = 24$

Hence, there are two such numbers 42 and 24.

29. Prove that the angle between the two

tangents drawn from external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Ans : Let PQ and PR be two tangents drawn from an external point p to a circle with centre O



To prove : $\angle QOR = 180^0 - \angle QPR$

or $\angle QOR + \angle QPR = 180^0$

Proof : In $\triangle OQP$ and $\triangle ORP$

PQ = PR [$\because$ tangents drawn from an external point are equal in length]

OQ = OR [radii of circle]

OP = OP [common sides]

$\therefore$ $\triangle OQP \cong \triangle ORP$

[by SSS conguence rule]

Then, $\angle QPO = \angle RPO$ [by CPCT]

and $\angle POQ = \angle POR$ [by CPCT]

$$\Rightarrow \left.\begin{aligned} \angle QPR &= 2\angle OPQ \\ \text{and } \angle QOR &= 2\angle POQ \end{aligned}\right\}$$

Now, in right angled ΔOQP,

$\angle QPO + \angle QOP = 90^0$

$\Rightarrow 2\angle QOP = 180^0 - 2\angle QPO$

[multiplying both sides by 2]

$\Rightarrow \angle QOR = 180^0 - \angle QPR$

[from Eq.(i)]

$\Rightarrow \angle QOR + \angle QPR = 180^0$

Hence proved.

OR

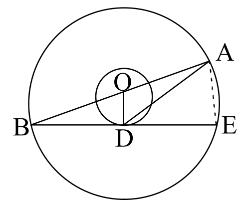
The radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle. BD is a tangent to the smaller circle touching it at D. Find the length AD.

Ans : Let the line BD intersects the bigger circle at E.

Now, Join AE.

Let O be the centre of the bigger circle, then O is the mid-point of AB.

[$\because$ AB is a diameter of thebigger circle]



BD is a tangent to the smaller circle and OD is a radius through the point of contact D. Then,

$OD \perp BD \Rightarrow OD \perp BE$

Since, OD is perpendicular to the chord BE of bigger circle.

$\therefore$ BD = DE

[$\because$ perpendicular drawn from the centre to a chord bisects the chord]

D is the mid-point of BE

$\therefore$ In ΔBAE, O is the mid-point AB and D is the mid-kpoint of BE.

$\therefore$ $OD = \dfrac{1}{2} AE$

[$\because$ line segment joining the mid-points of any two sides of a triangle is half of the third side]

$\Rightarrow AE = 2(OD) = 2 \times 8 = 16$ cm

In right angled ΔOBD, using Pythagoras theorem,

$OD^2 + BD^2 = OB^2$

$\Rightarrow BD = \sqrt{OB^2 - OD^2} = \sqrt{13^2 - 8^2}$

[$\because OB = 13$ cm]

$= \sqrt{169 - 64} = \sqrt{105}$

$\therefore \quad DE = BD = \sqrt{105}$

In right angled ΔAED, using Pythagoras theorem, we have

$$AD = \sqrt{(AE)^2 + (DE)^2}$$

$$= \sqrt{16^2 + \sqrt{105}^2} = \sqrt{256 + 105}$$

$$= \sqrt{361} = 19 \text{ cm.}$$

30. If tan A = *n* tan B and sin A = m sin B, then prove that $\cos^2 A = \dfrac{m^2-1}{n^2-1}$.

Ans : We have to find $\cos^2 A$ in terms of m and n. This means $\angle B$ is to be eliminated from the given relations.

Now, $\tan A = n \tan B \Rightarrow \tan B = \dfrac{1}{n} \tan A$

$$\Rightarrow \quad \cot B = \frac{n}{\tan A} \quad \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

and $\sin A = m \sin B \Rightarrow \sin B = \dfrac{1}{m} \sin A$

$$\Rightarrow \quad \operatorname{cosec} B = \frac{m}{\sin A}$$

We know that, $\operatorname{cosec}^2 B - \cot^2 B = 1$

Now, substituting the value of cot B and cosec B. We get

$$\left(\frac{m}{\sin A}\right)^2 - \left(\frac{n}{\tan A}\right)^2 = 1$$

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$$

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\dfrac{\sin^2 A}{\cos^2 A}} = 1 \quad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2 \cos^2 A}{\sin^2 A} = 1$$

$$\Rightarrow \frac{m^2 - n^2 \cos^2 A}{\sin^2 A} = 1$$

$$\Rightarrow m^2 - n^2 \cos^2 A = \sin^2 A$$

$$\Rightarrow m^2 - n^2 \cos^2 A = 1 - \cos^2 A$$

$$[\because \sin^2\theta = 1 - \cos^2\theta]$$

$$\Rightarrow m^2 - 1 = n^2 \cos^2 A - \cos^2 A$$

$$\Rightarrow m^2 - 1 = \cos^2 A (n^2 - 1)$$

$\therefore \quad \cos^2 A = \dfrac{m^2-1}{n^2-1}$ Hence proved.

31. A bag contains 18 balls out of which, x balls are red.

i) If one ball is drawn at random from the bag. then what is the probability that it is red ball?

ii) If 2 more red balls are put in the bag, then the probability of drawing a red ball will be 9/8 times that of probability of red ball coming in part (i). Find the value of x.

Ans : i) $\because$ Total number of balls in the bag = 18

Total number of red balls in the bag = x

$\therefore$ P (drawing a red ball) = $\dfrac{x}{18}$

ii) $\because$ Number of red balls added to the bag = 2

$\therefore$ Total number of red balls in the bag

= 18 + 2 = 20

and total number of red balls in the bag

= x + 2

Now, P (drawing red ball) = $\dfrac{x+2}{20}$

According to the question

$$\frac{x+2}{20} = \frac{9}{8}\left(\frac{x}{18}\right)$$

$$\Rightarrow \frac{x+2}{20}=\frac{x}{16}$$

$\Rightarrow 16x + 32 = 20x$

$\Rightarrow 4x = 32$

$\Rightarrow x = 8$

SECTION D

Section D consists of 4 questions of 5 marks each.

32. A motor boat whose speed is 18 km/h in still water takes 1 h more to go 24 km upstream than to return downstream to the same spot. Find the speed of stream.

Ans : Let the speed of the stream be x km/h

Then, speed of the boat downstream

$= (18 + x)$ km/h

and speed of the boat upstream

$= (18 - x)$km/h

Then, according to the question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$\Rightarrow 24(18 + x) - 24(18 - x)$

$= (18 - x)(18 + x)$

$\Rightarrow 24x + 24x = 324 - x^2$

$\Rightarrow x^2 + 48x - 324 = 0$

$\Rightarrow x^2 + 54x - 6x - 324 = 0$

$\Rightarrow x(x + 54) - 6(x + 54) = 0$

$\therefore$ x = 6 [$\because$ x cannot be negative]

$\therefore$ Speed of stream = x = 6 km/h

OR

Two water taps together can fill a tank in $9\frac{3}{8}$ h. The tap of larger diameter takes 10 h less than the smaller one to fill the tank separately.Find the time in which each tap can seperately fill the tank.

Ans : Let time taken by tap of smaller diameter be t

Time taken by tap of larger diameter $= (t - 10)$h.

Given : Time taken by both the taps to fill the tank

$$= 9\frac{3}{8} = \frac{75}{8}\text{ h}$$

Portion of the tank filled by tap of smaller diameter in $1\text{h} = \frac{1}{t}$

Portion of the tank filled by tap of larger diameter in $1\text{h} = \frac{1}{t-10}$

Portion of the tank filled by both the taps in $1\text{h} = \frac{1}{\frac{75}{8}} = \frac{8}{75}$

$$\therefore \frac{1}{t} + \frac{1}{t-10} = \frac{8}{75}$$

$$\Rightarrow \frac{(t-10)+t}{t(t-10)} = \frac{8}{75}$$

$$\Rightarrow \frac{2t-10}{t^2-10t} = \frac{8}{75}$$

$\Rightarrow 150t - 750 = 8t^2 - 80t$

$\Rightarrow 8t^2 - 230t + 750 = 0$

$\Rightarrow 4t^2 - 115t + 375 = 0$

$\Rightarrow 4t^2 - 100t - 15t + 375 = 0$

$\Rightarrow 4t(t - 25) - 15(t - 25) = 0$

$\Rightarrow (t - 25)(4t - 15) = 0$

$\Rightarrow t = 25$ or $t = \frac{15}{4}$

If t = 25 then time taken by tap of larger diameter = 25 - 10 = 15 h

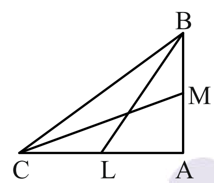
If $t = \frac{15}{4}$, then time taken by tap of larger diameter

$= \frac{15}{4} - 10 = \frac{25}{4}$ which is not possible

$\therefore$ Time take by taps of smaller and larger diameter are 25 h and 15h, respectively.

33. BL and CM are medians of $\triangle ABC$ right angled at A. Prove that $4\left(BL^2 + CM^2\right) = 5BC^2$

Ans : In figure $\angle A = 90^0$ Since BL and CM are medians, so the lines BL and CM divide CM divide AC and AB respectively, into two equal parts.



$\therefore \quad CL = AL = \frac{AC}{2}$ ...(i)

and $AM = BM = \frac{AB}{2}$ ...(ii)

In $\Delta BAC, \quad \angle A = 90^0$

Using Pythagoras theorem, we get

$BC^2 = AB^2 + AC^2$

In $\Delta BAL, \quad \angle A = 90^0$

Using Pythagoras theorem, we get

$BL^2 = AB^2 + AL^2$

$\Rightarrow BL^2 = AB^2 + \left(\frac{AC}{2}\right)^2$ [using Eq.(i)]

$\Rightarrow BL^2 = AB^2 + \frac{AC^2}{4}$

$\Rightarrow 4BL^2 = 4AB^2 + AC^2$ ...(iv)

In ΔMAC, $\angle A = 90^0$

Using Pythagoras theorem, we get

$CM^2 = AC^2 + AM^2$

$\Rightarrow CM^2 = AC^2 + \left(\frac{AB}{2}\right)^2$

$\Rightarrow 4CM^2 = 4AC^2 + AB^2$ ....(v)

On adding Eqs. (iv) and (v), we get

$\Rightarrow 4(BL^2 + CM^2) = 5BC^2$

[Using Eq.(iii)]

Hence proved.

34. From a solid cylinder whose height is 12 cm and diameter is 10 cm, a conical cavity of same height and same diameter is hollowed out. Find the volume and total surface area of the remaining solid.

Ans : Given, diameter of the cylinder, d = 10 cm

$\Rightarrow r = 5$ cm

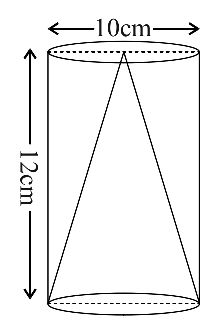
and height of the cylinder, h = 12 cm

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 5 \times 5 \times 12 = \frac{6600}{7} \text{cm}^3$

and volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12 = \frac{2200}{7} \text{cm}^3$



Now, volume of remaining solid

= Volume of the cylinder - Volume of the cone

$= \frac{6600}{7} - \frac{2200}{7}$

$= \frac{4400}{7} = 628.57 \text{ cm}^3$

Since, slant height of the cone.

$l = \sqrt{r^2 + h^2} = \sqrt{(5)^2 + (12)^2}$

$= \sqrt{169} = 13\text{cm}$

$\therefore$ Curved surface area of the cone $= \pi r l$

$= \frac{22}{7} \times 5 \times 13 = \frac{1430}{7} \text{cm}^2$

Clearly, curved surface are of the cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 5 \times 12 = \frac{2640}{7} \text{cm}^2$

and area of upper base of the cylinder $= \pi r^2$

$= \frac{22}{7} \times 5 \times 5 = \frac{550}{7} \text{cm}^2$

Now, total surface area of the remaining solid

= Curved surface area of the cylinder + Curved surface area of the cone + Area of upper base of the cylinder

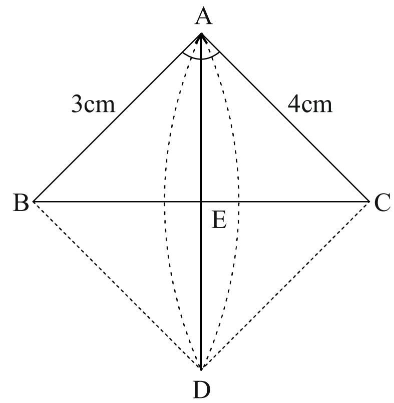
$= \frac{2640}{7} + \frac{1430}{7} + \frac{550}{7}$

$= \frac{4620}{7} = 660\text{cm}^2$.

OR

A right angled triangle whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone, so formed. [choose the value of π as found appropriate]

Ans : Let ABC be a right angled triangle, right angled at A and BC is the hypotenuse.



Also, let AB = 3 cm

and AC = 4 cm.

Then, $BC = \sqrt{3^2 + 4^2}$

[by Pythagoras theorem]

$= \sqrt{9+16} = \sqrt{25} = 5\text{cm}$

As, ΔABC revolves about the hypotenuse BC. It forms two cones ABD and ACD.

In ΔAEB and ΔCAB, $\angle AEB = \angle CAB$ [each 90^0]

$\angle ABE = \angle ABC$ [common]

$\therefore \Delta AEB \sim \Delta CAB$ [by AA similarity criterion]

$\therefore \frac{AE}{CA} = \frac{AB}{BC}$ [$\because$ in similar triangles corresponding sides are proportional]

$\Rightarrow \frac{AE}{4} = \frac{3}{5} \Rightarrow AE = \frac{12}{5} = 2.4$

So, radius of the base of each cone,

AE = 2.4 cm

Now, in right angled ΔAEB,

$BE = \sqrt{AB^2 - AE^2}$

[by pythagoras theorem]

$= \sqrt{(3)^2 - (2.4)^2}$

$= \sqrt{9 - 5.76} = \sqrt{3.24} = 1.8$

So, height of the cone ABD = BE = 1.8 cm

$\therefore$ Height of the cone,

ACD = CE = BC – BE

= 5 – 1.8 = 3.2 cm

Now, Volume of the cone ABD

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.4)^2 \times 1.8$$

$$= \frac{22}{21} \times 10.368 = 10.86 cm^3$$

and volume of the cone ACD

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.4)^2 \times 3.2$$

[$\because$ radius will be same as AD is common]

$$= \frac{405.504}{21} = 19.31\ cm^3$$

$\therefore$ Required volume of double cone = 10.86 + 19.31

= 30.17 cm^3

Now, surface area of cone ABD $= \pi r l$

$$= \frac{22}{7} \times 2.4 \times 3 = \frac{158.4}{7}$$

= 22.63 cm^2

and surface are of cone of cone ACD $= \pi r l$

$$= \frac{22}{7} \times 2.4 \times 4 = \frac{211.2}{7} = 30.17\ cm^2$$

$\therefore$ Required surface area of double cone

= 22.63 + 30.17 = 52.8 cm^2.

35. If the median of the distribution given below is 30, then find the values of x and y.

Class interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	Total
Number of Student	5	x	20	15	y	5	60

Ans : Cumulative frequency table

Class interval	Frequency (f_i)	Cumulative frequency (cf)
0 – 10	5	5
10 – 20	x	5 + x
20 – 30	20	25 + x
30 – 40	15	40 + x
40 – 50	y	40 + x + y
50 – 60	5	45 + x + y
Total	$\sum f_{i0} = 60$	

$\because$ Median = 30

So, it lies in the interval 30-40.

Thus, 30-40 is the median class.

$\therefore$ l = 30, h = 10, f = 15, cf = 25 + x and N = 60

where, l is the lower limit of median class,

N is the total number of observations,

cf is the cumulative frequency of class preceding the median class,

f is the frequency of the median class

and h is the class size.

$$\because \text{ Median } = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$\therefore \quad 30 = 30 + \frac{\frac{60}{2} - (25 + x)}{15} \times 10$$

$$\Rightarrow 30 - (25 + x) = 0 \Rightarrow x = 5$$

Also, we have

$$60 = 45 + x + y$$

$$\Rightarrow 15 = x + y \Rightarrow 15 = 5 + y \Rightarrow y = 10.$$

SECTION E

Case study based questions are compulsory.

36. Your friend Veer wants to participate in a 200 m race. He can currently run that

distance in 51 sec and with each day of

practice it takes him 2 sec less. He wants to do in 31 sec.

i) **If n^{th} term of an AP is given by $a_n = 2n + 3$, then find the common difference of an AP.** **1**

ii) **Find the term s of AP for the given situation and determine the 10th terms from the end.** **2**

Ans : i) Given, $a_n = 2n + 3$

$\therefore$ Common difference $= a_{n+1} - a_n$

$= 2(n + 1) + 3 - (2n + 3)$

$= 2n + 2 + 3 - 2n - 3$

$= 2$

ii) In first day, Veer takes 51 sec to complete the 200 m race. But in every next day he takes 2 sec lesser than the previous day.

Thus, AP series will formed

51, 49, 47,...., 31

Here, $l = 31$ and $d = 49 - 51 = -2$

$\therefore$ 10th term from the end $= l - (10 - 1)d$

$= l - 9d$

$= 31 - 9(-2)$

$= 31 + 18 = 49$

OR

What is the minimum number of days he needs to practice till his goal is achieved?

iii) **Find the value of x, for which 2x, x + 10, 3x + 2 are three consecutive terms of an AP.** **1**

Since, Veer wants to achieve the goal in 31 sec.

Let Veer takes n days to achieve the target.

$\therefore$ $a_n = a + (n - 1)d$

Here, $a = 51$, $d = 49 - 51 = -2$

$\therefore$ $31 = 51 + (n - 1)(-2)$

$\Rightarrow (n - 1)2 = 20$

$\Rightarrow (n - 1) = 10 \Rightarrow n = 11$

Hence, he needs minimum 11 days to achieve the goal.

iii) Given, terms 2x, x + 10, 3x + 2 are in AP.

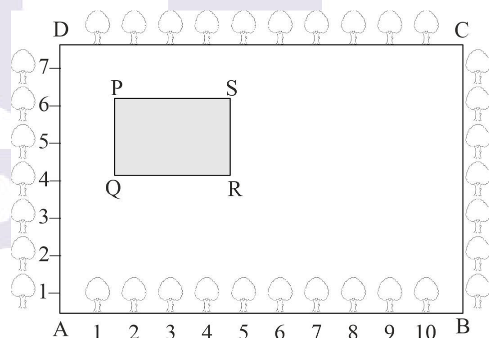
$$\therefore \quad x + 10 = \frac{2x + (3x + 2)}{2}$$

$\Rightarrow 2x + 20 = 5x + 2$

$\Rightarrow 3x = 18 \Rightarrow x = 6.$

37. Tree Plantation to control Pollution

The class X students of a secondary school in Krishnagar have been alloted a rectangular plot of land for this gardening activity.



Sapling of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a rectangular gracy lawn in the plot as shown in above figure.

The students sowing seeds of flowering plants on the remaining area of the plot.

i) **Find the coordinates of point Q and S.** **1**

ii) **If the point m divides the line QS in the ratio 3 : 2, then find the coordinates of m.** **2**

Ans : i) The coordinates of points Q and S are (2, 3) and (6, 6)

ii) By using internal division formula,

Q (2, 3) —3— M —2— S (6, 6)

Coordinates of

$$M = \left(\frac{3\times6+2\times2}{3+2}, \frac{3\times6+2\times3}{3+2}\right)$$

$$= \left(\frac{18+4}{5}, \frac{18+6}{5}\right)$$

$$= \left(\frac{22}{5}, \frac{24}{5}\right)$$

OR

If the point G divides the line QR in the ratio 1 : 2, then find the coordinates of G.

iii) Find the distance between the vertices of diagonal Q and S. **1**

Ans : By using internal division formula,

Q (2, 3) —1— M —2— S (6, 3)

Coordinates of

$$G = \left(\frac{1\times6+2\times2}{1+2}, \frac{1\times3+2\times3}{1+2}\right)$$

$$= \left(\frac{6+4}{3}, \frac{3+6}{3}\right)$$

$$= \left(\frac{10}{3}, \frac{9}{3}\right) = \left(\frac{10}{3}, 3\right)$$

iii) Distance between the vertices of diagonal Q and S

$$= \sqrt{(6-2)^2+(6-3)^3}$$

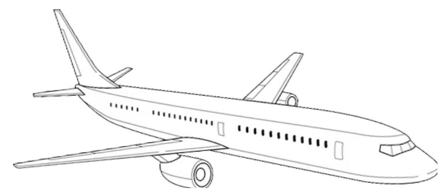
$$= \sqrt{4^2+3^2}$$

$$= \sqrt{16+9}$$

$$= \sqrt{25} = 5 \text{ units.}$$

38. **Flight Features : The aviation technology has evolved many upgradations in the last few years. It has taken in account, speed, direction, and distance as well as other**

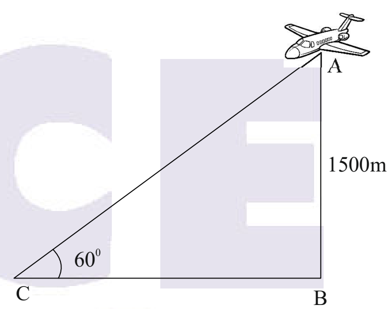
features of the flight. Even the wind plays a vital role, when a plane travels.



Angle of elevation : The angle of

elevation of an object viewed is the angle formed by the line of sight with the

horizontal when it is above the horizontal level.



Based on the above information, answer the following questions

i) If the point C moves towards the point B, then how does the angle of elevation vary? **1**

ii) Find the distance of point C from the object. **2**

Ans : i) If the point C moves towards the point B, then angle of elevation increases.

ii) In right angled ΔACB,

$$\sin 60^0 = \frac{AB}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{1500}{AC}$$

$$\Rightarrow AC = 1500 \times \frac{2}{\sqrt{3}} = \frac{3000}{3} \times \sqrt{3}$$

$= 1000\sqrt{3}\text{m}$

OR

What is the value of distance BC ?

iii) If angle of elevation changes from 60^0 to 45^0, then what will be the new distance of BC? **1**

Ans : In $\Delta ABC, \tan 60^0 = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{1500}{BC}$

$$\Rightarrow BC = \frac{1500}{\sqrt{3}} = 500\sqrt{3}\text{m}$$

iii) If angle of elevation changes from 60^0 to 45^0 then in ΔABC.

$$\tan 45^0 = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{1500}{BC}$$

$$\Rightarrow BC = 1500 \text{ m}.$$

* * *