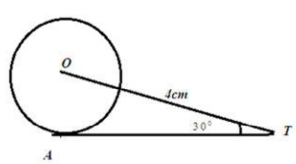
PACE IIT & MEDICAL

Sub. : Maths **Total Marks : 80**

Std. X (CBSE)

Pre. Answer Paper - 02

	SECTION A **Section A consists of 20 questions of 1 mark each.**	
1. **Ans :**	**The polynomial equation x (x + 1) + 8 = (x + 2) (x – 2) is** a) linear equation	1
2. **Ans :**	**There are 312, 260 and 156 students in class X, XI and XII respectively. Buses are to be hired to take these students to a picnic. Find the maximum number of students who can sit in a bus if each bus takes equal number of students** a) 52	1
3. **Ans :**	**Find the value of k if the points A(2, 3), B(4, k) and C(6, –3) are collinear.** c) 0	1
4. **Ans :**	**The (n – 1)th term of an A.P. is given by 7,12,17, 22,… is** d) 5n – 3	1
5. **Ans :**	**If in two triangles ABC and** $\frac{AB}{QR}=\frac{BC}{PR}=\frac{CA}{PQ}$, **then** a) $\Delta PQR \sim \Delta CAB$	1
6. **Ans :**	**If the discriminant of a quadratic polynomial, D > 0, then the polynomial has** b) two real and unequal roots	1
7. **Ans :**	**In the following figure, AT is a tangent to the circle with centre O such that OT = 4 cm and** $\angle OTA = 30^0$. **Then AT is equal to**  c) $2\sqrt{3}$ cm	1
8.	$\frac{2\tan 30^0}{(1+\tan^2 30^0)}=$	1

Ans : a) $\sin 60^0$

9. **If the height of the building and distance from the building foot's to a point is increased by 20%, then the angle of elevation on the top of the building:** 1

Ans : c) Do not change

10. **It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be** 1

Ans : a) 10 m

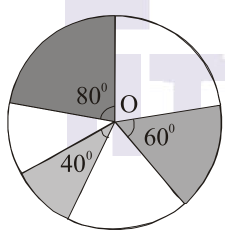
11. **If we change the shape of an object from a sphere to a cylinder, then the volume of cylinder will** 1

Ans : c) Remains unchanged

12. **Mean of 100 items is 49. It was discovered that three items which should have been 60, 70, 80 were wrongly read as 40, 20, 50 respectively. The correct mean is** 1

Ans : c) 50

13. **In the given figure, three sectors of a circle of radius 7 cm, making angles of 60°, 80° and 40° at the centre are shaded. The area of the shaded region (in cm^2) is [Using $\pi = \frac{22}{7}$]** 1



Ans : a) 77

14. **A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is:** 1

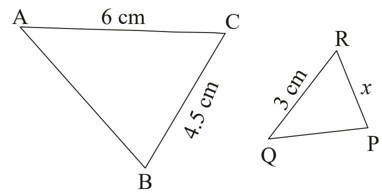
Ans : d) 51

15. **In a hospital, weights of new born babies were recorded, for one month. Data is as shown:** 1

Weight of new born baby (in kg)	1.4 – 1.8	1.8 – 2.2	2.2 – 2.6	2.6 – 3.0
No of babies	3	15	6	1

Then the median weight is:

Ans : c) 2.05 kg

16. **Ans :**	**In the given figure, $\Delta ABC \sim \Delta QPR$. The value of x is.**  a) 2.25 cm	1
17. **Ans :**	**The pair of equations 3x – 5y = 7 and – 6x + 10y = 7 have** c) no solution	1
18. **Ans :**	**The value of cos 0°. cos 1°. cos 2°. cos 3°… cos 89° cos 90° is** c) 0	1
19. **Ans :**	**Assertion(A): If $x \sin^3 \theta + y \cos^3 \theta = \sin\theta \cos\theta$ and $x \sin\theta = y \cos\theta$, then $x^2 + y^2 = 1$.** **Reason(R): For any value of $\theta, \sin^2 \theta + \cos^2 \theta = 1$** a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).	1
20. **Ans :**	**Assertion(A): If one zero of polynomial $p(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of the other, then k = 2.** **Reason(R): If (x – a) is a factor of p(x), then p(a) = 0 i.e., a is a zero of p(x).** b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).	1
	SECTION B **Section B consists of 5 questions of 2 marks each.**	
21. **Ans.**	**Find value of K for which the consecutive terms 3k + 1, 2k + 3, 6k + 2, form an AP.** 3k + 1, 2k + 3 and 6k + 2 form an AP $\therefore\ a_2 - a_1 = a_3 - a_2$ $\therefore\ 2k + 3 - (3k + 1) = 6k + 2 - (2k + 3)$ $2k + 3 - 3k - 1 = 6k + 2 - 2k - 3$ $\therefore\ -k + 2 = 4k - 1$ $\therefore\ 5k = 3$ $\therefore\ k = 3/5$.	2
22. **Ans.**	**The distance between A(1, 3) and B(x, 7) is 5. calculate the possible values of x.** A(1, 3), B(x, 7), and AB = 5 units. Distance AB = 5 units. $\sqrt{(x-1)^2 + (7-3)^2} = 5^2$ ---(distance formula) squaring both sides	2

$\therefore (x-1)^2+4^2=5^2$

$\therefore x^2-2x+1+16=25$

$\therefore x^2-2x+17=25$

$\therefore x^2-2x-8=0$

$\therefore x^2-4x+2x-8=0$

$\therefore x(x-4)+2(x-4)=0$

$\therefore (x-4)(x+2)=0$

$\therefore x-4=0$ or $x+2=0$

$\therefore x=4$ or $x=-2$.

23. **If a circle touches the side BC of $\triangle ABC$ at P and extended sides AB and AC at Q and R respectively, prove that $AQ=\frac{1}{2}(BC+CA+AB)$.** — 2

Ans.

$AQ=AR$ ---(i)

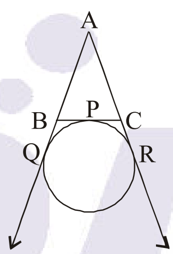
$BP=BQ$ ---(ii)

$CP=CR$ ---(iii)

---(Length of tangents drawn from an external point to a circle are equal.

Perimeter of $\triangle ABC = AB+BC+AC$

$\therefore$ Perimeter of $\triangle ABC=(AQ-BQ)+(BP+PC)+(AR-CR)$



Perimeters of $\triangle ABC=(AQ-BQ)+(BQ+PC)+(AQ-PC)$

$\therefore$ Perimeter of $\triangle ABC=2\,AQ$

$\therefore AQ=\frac{1}{2}$ perimeter of $\triangle ABC$

$\therefore AQ=\frac{1}{2}(BC+CA+AB)$.

24. Find the roots of quadratic equation $x^2-7x+12=0$. — 2

Ans.

$x^2-7x+12=0$ ---(given)

$\therefore x^2-4x-3x+12=0$

$\therefore x(x-4)-3(x-4)=0$

$\therefore (x-4)(x-3)=0$

$\therefore (x-4)=0$ or $(x-3)=0$

$\therefore x=4$ or $x=3$.

OR

Find the value of p so that the quadratic equation $x^2+px+1=0$ has real roots.

Ans.

Given quadratic equation is

$x^2+px+1=0$

$\therefore a=1, b=p$ and $c=1$

For having real roots,

$b^2-4ac=0$

$\therefore p^2-4(1)(1)=0$

$\therefore p^2-4=0$

$\therefore\ p^2 = 4$ $\therefore\ p = \pm 2$

25. **If** $\sqrt{3}\sin\theta - \cos\theta = 0$ **and** $0^0 < \theta < 90^0$, **find the value of** θ. 2

Ans. $\sqrt{3}\sin\theta = \cos\theta$ ---(given)

$\therefore\ \sqrt{3} = \frac{\cos\theta}{\sin\theta}$ $\therefore\ \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$ $\therefore\ \tan\theta = \frac{1}{\sqrt{3}}$

but $\tan 30^0 = \frac{1}{\sqrt{3}}$

$\therefore\ \tan\theta = \tan 30^0$ $\therefore\ \theta = 30^0$

OR

If $x = a\sin\theta$ **and** $y = b\tan\theta$, **then prove that** $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

Ans. $\text{L.H.S.} = \frac{a^2}{x^2} - \frac{b^2}{y^2}$ $= \frac{a^2}{(a\sin\theta)^2} - \frac{b^2}{(b\tan\theta)^2}$

$= \frac{a^2}{a^2\sin^2\theta} - \frac{b^2}{b^2\tan^2\theta}$ $= \frac{1}{\sin^2\theta} - \frac{1}{\tan^2\theta}$ $= \text{cosec}^2\theta - \cot^2\theta = 1$

$\therefore$ L.H.S. = R.H.S.

SECTION C

Section C consists of 6 questions of 3 marks each.

26. **Show that** $\sqrt{5}$ **is an irrational number.** 3

Ans. Let us assume that $\sqrt{5}$ is a rational number.

$\therefore\ \sqrt{5} = \frac{p}{q}$

Where p, q are co-prime integers and $q \neq o$

$\therefore\ 5 = \frac{p^2}{q^2}$ ---(squaring both sides)

$\therefore\ 5q^2 = p^2$ ---(i)

So, 5 divides p

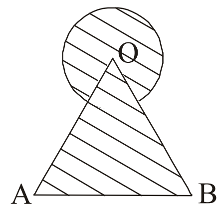
$\therefore$ p is a multiple of 5 $\therefore\ p = 5\text{ m}$ $\therefore\ p^2 = 25\text{ m}^2$ ---(ii)

$\therefore\ 5q^2 = 25\text{ m}^2$ ---[from (i) and (ii)]

$\therefore\ q^2 = 5\text{ m}^2$ $\therefore\ q^2$ is a multiple of 5 $\therefore$ q is a multiple of 5

Hence, p and q have a common factor 5.

This cantradicts our assumption that they are co-primes.

	$\therefore \frac{p}{q}$ is not a rational number $\therefore \sqrt{5}$ is an irrational number.	
27.	**Find the area of the shaded region in the fig., where a circular arc of radius 6cm has been drawn with vertex O of an equilateral triangle OAB of side 12cm as centre.** 	3
Ans.	r = 6 cm, side of an equilateral triangle (a) = 12 cm, $\theta = 60^0$ --- (angle of an equilateral triangle) Area of shaded region = Area of circle + Area of equilateral triangle – A (sector) $= \pi r^2 + \frac{\sqrt{3}}{4}a^2 - \frac{\theta}{360}\pi r^2$ $= \pi(6)^2 + \frac{\sqrt{3}}{4}(12)^2 - \frac{60}{360}\pi(6)^2$ $= 36\pi - \frac{1}{6}36\pi + \frac{\sqrt{3}}{4}\times 144$ $= 36\pi\left(1-\frac{1}{6}\right) + 36\sqrt{3}$ $= 36\pi\left(\frac{5}{6}\right) + 36\sqrt{3}$ $= 36 \times \frac{22}{7} \times \frac{5}{6} + 36\sqrt{3} = \frac{660}{7} + 36\sqrt{3}$ $\therefore$ Area of shaded region $= \left(\frac{660}{7} + 36\sqrt{3}\right)\text{cm}^2$.	
28.	**'Mr. Kasam runs a small business of making earthen pots. He makes certain number of pots on daily basis. Production cost of each pot is ₹ 40 more than 10 times total number of pots, he makes in one day. If production cost of all pots per day is ₹ 600, find production cost of one pot and number of pots he makes per day.**	3
Ans.	Let Mr. Kasam make x number of pots on daily basis. Production cost of each pot = ₹ (10x + 40) According to the given condition, $x(10x + 40) = 600$ $\Rightarrow 10x^2 + 40x = 600$ $\Rightarrow 10x^2 + 40x - 600 = 0$ $\Rightarrow x^2 + 4x - 60 = 0$ …[Dividing both sides by 10] $\Rightarrow x^2 + 10x - 6x - 60 = 0$ $\Rightarrow x(x + 10) - 6(x + 10) = 0$ $\Rightarrow (x + 10)(x - 6) = 0$	

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$= x + 10 = 0$ or $x - 6 = 0$

$= x = -10$ or $x = 6$

But, number of pots cannot be negative.

$= x = 6$

= Production cost of each pot = ₹ (10 x + 40)

= ₹ [(10×6)+ 40]

= ₹ (60 + 40) = ₹ 100

Production cost of one pot is ₹ 100 and the number of pots Mr. Kasam makes per day is 6.

OR

The sum of squares of two consecutive even natural numbers is 244; find the numbers.

Ans. Let the first even natural number be x.

∴ the next consecutive even natural number will be (x + 2).

According to the given condition,

$x^2 + (x + 2)^2 = 244$

$\Rightarrow x^2 + x^2 + 4x + 4 = 244$

$\Rightarrow 2x^2 + 4x + 4 - 244 = 0$

$\Rightarrow 2x^2 + 4x - 240 = 0$

$\Rightarrow x^2 + 2x - 120 = 0$ …[Dividing both sides by 2]

$\Rightarrow x^2 + 12x - 10x - 120 = 0$

$\Rightarrow x(x + 12) - 10(x + 12) = 0$

$\Rightarrow (x + 12)(x - 10) = 0$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$\therefore x + 12 = 0$ or $x - 10 = 0$

$\therefore x = -12$ or $x = 10$

But, natural number cannot be negative.

$\therefore x = 10$ and $x + 2 = 10 + 2 = 12$

∴ The two consecutive even natural numbers are 10 and 12.

29. **If** $5\tan\alpha = 4$, **show that** $\dfrac{5\sin\alpha - 3\cos\alpha}{5\sin\alpha + 2\cos\alpha} = \dfrac{1}{6}$**.** 3

Ans.

$5\tan\alpha = 4$ ---(given)

$\therefore \tan\alpha = \dfrac{4}{5}$

$$\frac{5\sin\alpha - 3\cos\alpha}{5\sin\alpha + 2\cos\alpha}$$

$$= \frac{\dfrac{5\sin\alpha - 3\cos\alpha}{\cos\alpha}}{\dfrac{5\sin\alpha + 2\cos\alpha}{\cos\alpha}}$$ ---(divide Numerator and denominator by $\cos\alpha$)

$$= \frac{\dfrac{5\sin\alpha}{\cos\alpha} - 3\dfrac{\cos\alpha}{\cos\alpha}}{\dfrac{5\sin\alpha}{\cos\alpha} + 2\dfrac{\cos\alpha}{\cos\alpha}} = \frac{5\tan\alpha - 3}{5\tan\alpha + 2} = \frac{5(4/5) - 3}{5(4/5 + 2)} = \frac{4-3}{4+2}$$

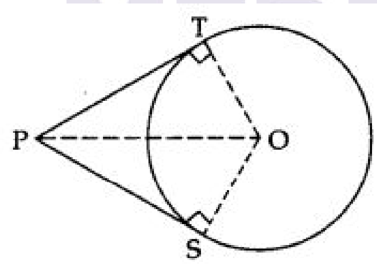
$$\therefore \frac{5\sin\alpha - 3\cos\alpha}{5\sin\alpha + 2\cos\alpha} = \frac{1}{6}.$$

30. **Prove that the lengths of tangents drawn from an external point to a circle are equal** 3

Ans.

Given: PT and PS are tangents from an external point P to the circle with centre O.
To prove: PT = PS

Construction: Join O to P, T and S.



Proof: In Δ OTP and Δ OSP.

OT = OS …[radii of the same circle]

OP = OP …[common]

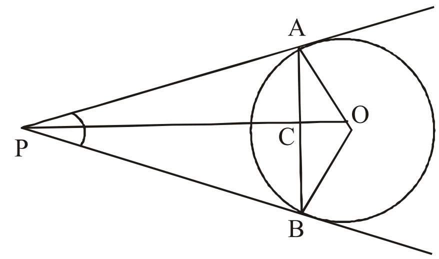
$\angle$OTP = $\angle$OSP …[each 90°]

ΔOTP = ΔOSP …[R.H.S.]

PT = PS …[c.p.c.t.]

OR

If PA and PB are two tangents drawn from a point P to a circle with centre O touching it at A and B. Prove that OP is the perpendicular bisector of AB.



Ans. Let OP intersect AB at a point C, we have to prove that AC = CB and

$\angle ACP = \angle BCP = 90^0$

$\because$ PA, PB are two tangents from a point P to the circle with centre O

$\angle APO = \angle BPO$ [$\because$ O lies on the bisector of $\angle APB$]

In two Δs , ACP and BCP, we have

AP = BP [tangents from P to the circle are equal]

PC = PC Common

$\angle APO = \angle BPO$ Proved

$\therefore \Delta ACP \cong \Delta BCP$ By SAS rule

$\therefore AC = CB$ CPCT

And $\angle ACP = \angle BCP$ [CPCT]

But $\angle ACP + \angle BCP = 180^0$

$\Rightarrow \angle ACP = \angle BCP = 90^0$

Hence, OP is perpendicular bisector of AB.

31. **Find the probability of getting 53 Tuesdays in a leap year.** 3

Ans. In leap year, there are 366 days.

366 days = 52 weeks + 2 days

These 2 days can be

{(Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday), (Sunday, Monday)}

Favourable cases : Getting Tuesday = {Monday, Tuesday), (Tuesday, Wednesday)}

	$\therefore$ Probability that a leap year will contain 53 sundays $= \dfrac{\text{No. of favourable cases}}{\text{Total no. of cases}} = \dfrac{2}{7}$.	
	SECTION D **Section D consists of 4 questions of 5 marks each.**	
32. **Ans.**	**Kargil's temperature was recorded in a week from Monday to Saturday. All readings were in A.P. The sum of temperatures of Monday and Saturday was 5°C more than sum of temperatures of Tuesday and Saturday. If temperature of Wednesday was -30° Celsius then find the temperature on the other five days.** Let the temperatures from Monday to Saturday in A.P. be a, a + d, a + 2d, a + 3d, a + 4d, a + 5d. According to the first condition, $(a) + (a + 5d) = (a + d) + (a + 5d) + 5^0$ $\not{2}a + 5d = 2\not{a} + 6d + 5$ $\therefore\ d = -5^0$ According to the second condition, $a + 2d = -30°$ $= a + 2(-5°) = -30°$ $= a - 10° = -30°$ $a = -30° + 10° = -20°$ $a + d = -20° - 5° = -25°$ $a + 3d = -20° + 3(-5°) = -20° - 15° = -35°$ $a + 4d = -20° + 4(-5°) = -20° - 20° = -40°$ $a + 5d = -20° + 5(-5°) = -20° - 25° = -45°$ = The temperatures on the other five days are – 20°C, -25° C, -35° C, -40° C and -45° C. **OR** **A man borrows ₹ 8000 and agrees to repay with a total interest of ₹ 1360 in 12 monthly instalments. Each instalment being less than the preceding one by ₹ 40. Find the amount of the first and last instalment.**	5
Ans.	i. The instalments are in A.P. Amount repaid in 12 instalments (S_{12}) = Amount borrowed + total interest = 8000 + 1360 $\therefore S_{12} = 9360$ Number of instalments (n) = 12 Each instalment is less than the preceding one by ₹ 40. $\therefore\ d = -40$	

	ii. $S_n = \frac{n}{2}[2a+(n-1)d]$ $\therefore\ S_{12} = \frac{12}{2}[2a+(12-1)(-40)]$ $\therefore\ 9360 = 6[2a+(11)(-40)]$ $\therefore\ 9360 = 6(2a-440)$ $\therefore\ \frac{9360}{6} = 2a-440$ $\therefore\ 1560 = 2a-440$ $\therefore\ 1560+440 = 2a$ $\therefore\ 2000 = 2a$ $\therefore\ a = \frac{2000}{2}$ $\therefore\ a = 1000$ iii. $t_n = a+(n-1)d$ $\therefore\ t_{12} = 1000+(12-1)(-40)$ $= 1000+11(-40)$ $= 1000-440$ $\therefore\ t_{12} = 560$. $\therefore$ Amount of the first instalment is ₹ 1000 and that of the last instalment is ₹ 560.	
33. **Ans.**	**A Solid is in the form of a circular cone mounted on a hemisphere. The radius of the hemisphere is 2.1 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical tub full of water, in such a way that the whole solid is submerged in water. If the radius of the cylinder is 5 cm and its height is 9.8 cm, find the volume of the water left in the cylindrical tub.** Radius of hemisphere and cone (r) = 2.1 cm height of cone (h) = 4 cm Vol. of solid = Vol. of hemisphere + Vol. of cone $= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$ $\quad = \frac{2}{3}\times\pi\times(2.1)^3 + \frac{1}{3}\times\pi\times(2.1)^2\times 4$ $= \frac{1}{3}\pi(2.1)^2[2(2.1)+4]$ $\quad = \frac{1}{3}\times\pi\times 2.1\times 2.1(4.2+4)$ $= \frac{1}{3}\times\frac{22}{7}\times 2.1\times 2.1\times 8.2$ $= 22\times 0.7\times 0.3\times 8.2$ $= 37.884\ cm^3$. Also, Radius of cylindrical tub = 5 cm	5

	Height of cylindrical tub = 9.8 cm Vol. of cylindrical tub $= \frac{22}{7} \times (5)^2 \times 9.8) = 22 \times 25 \times 1.4 = 770 \text{ cm}^3.$ When the solid is submerged in the tub then vol. of water left = Vol. of cylinder tub – Vol. of solid = 770 – 37.884 = 732.116 cm^3 $\therefore$ Vol. of water left = 732.116 cm^3	
34. **Ans.**	**If the length of a rectangle is increased by 2 cm and width by 3 cm, its area is increased by 35 cm^2. If the length and width are decreased by 2cm each, the area is descreased by 18.4 cm^2. Find the dimensions of the rectangle.** Let length of rectangle be x cm & breadth be y cm. $\therefore$ Original area of rectangle = xy $(x + 2)(y + 3) = xy + 35$ --- I^{st} condition $\therefore xy + 3x + 2y + 6 = xy + 35$ $\therefore 3x + 2y + 6 = 35$ $\therefore 3x + 2y = 29$ ---(i) $(x – 2)(y – 2) = xy – 18.4$ --- II^{nd} condition $\therefore xy – 2x – 2y + 4 = xy – 18.4$ $\therefore – 2x – 2y + 4 = – 18.4$ $\therefore –2x – 2y = – 22.4$ ---(ii) adding equation (i) & (ii) x = 6.6 Put x = 6.6 in equation (i) therefore $3(6.6) + 2y = 29$ $\therefore 19.8 + 2y = 29$ $\therefore 2y = 9.2$ $\therefore y = 4.6$ $\therefore$ Length = 6.6 cm and breadth = 4.6 cm.	5
	OR	
    Ans.	**A man travels 300 km partly by train and partly by car. He takes 4 hours if the travels 60 km by train and the rest by car. If he travels 100 km by train and the remaining by car, he takes 10 minutes longer. Find the speeds of the train and the car separately.** Let the speed of the train = x km/hr Let the speed of the car = y km/ hr According to the Question, $\frac{60}{x} + \frac{240}{y} = 4$ ...(i) ...$\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right.$ $\frac{100}{x} + \frac{200}{y} = \frac{25}{6}$ ...(ii) ...$\left[\begin{array}{l}\because 4 \text{ hr} + 10 \text{ min}. \\ = 4 + \frac{10}{60} = \frac{25}{6} \text{ hr}.\end{array}\right.$ Multiplying (i) by 5 and (ii) by 6, we get $\frac{300}{x} + \frac{1200}{y} = 20$	

$$\pm\frac{600}{x} \pm \frac{1200}{y} = -25$$

$$\frac{-300}{x} = -5$$

$5x = 300 \quad \therefore \quad x = 60$

Putting the value of x in (i), we get

$$\frac{60}{60} + \frac{240}{y} = 4 \Rightarrow \frac{240}{y} = 4 - 1$$

$3y = 240 \quad \Rightarrow \quad y = 80$

$\therefore$ Speed of the train = 60 km/hr

and Speed of the car = 80 km/hr

35. **Find the mean marks of students for the following distribution :** 5

Marks	No. of Students
0 and above	80
10 and above	77
20 and above	72
30 and above	65
40 and above	55
50 and above	43
60 and above	28
70 and above	16
80 and above	10
90 and above	8
100 and above	0

Ans.

Marks	Cumulative Frequency	Class Interval	Class Mark (x_i)	Frequency (f_i)	$f_i x_i$
0 and above	80	0 – 10	5	80 – 77 = 3	15
10 and above	77	10 – 20	15	77 – 72 = 5	75
20 and above	72	20 – 30	25	72 – 65 = 7	175
30 and above	65	30 – 40	35	65 – 55 = 10	350
40 and above	55	40 – 50	45	55 – 43 = 12	540
50 and above	43	50 – 60	55	43 – 28 = 15	825
60 and above	28	60 – 70	65	28 – 16 = 12	780
70 and above	16	70 – 80	75	16 – 10 = 6	450
80 and above	10	80 – 90	85	10 – 8 = 2	170
90 and above	8	90 – 100	95	8 – 0 = 8	760
100 and above	0	100 – 110	105	0	0
	Total			**80**	**4140**

Mean $(\overline{x}) = \dfrac{\sum f_i x_i}{\sum f_i}$ $\quad\therefore\ \overline{x} = \dfrac{4140}{80} = 51.75$

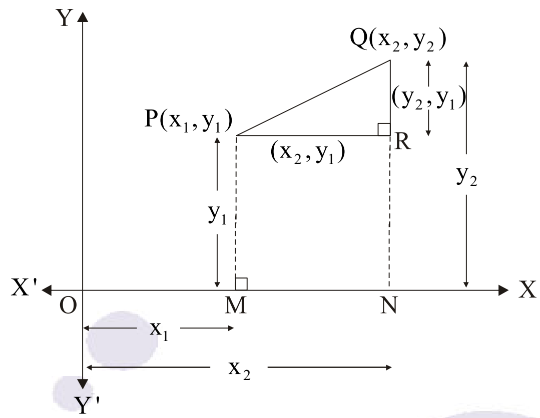
$\therefore$ Mean marks of student is 51.75.

SECTION E

Case study based questions are compulsory.

36. **Case study Based**

Distance Formula



Let P(x_1, y_1) and Q(x_2, y_2) be any two points. PM and QN are perpendiculars drawn from P and Q on the x-axis and $PR \perp QN$.

From the figure, we have

OM = x_1, PM = y_1, ON = x_2, QN = y_2

Thus, PR = MN = ON – OM = $x_2 - x_1$

and QR = QN – RN = $y_2 - y_1$.

Let PQ be d.

In right triangle PQR, right-angled at R,

$PQ^2 = PR^2 + QR^2$

[Using Pythagoras Theorem]

$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\Rightarrow \quad PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

a) **Find the distance between the points A(c, 0) and B(0, –c).** 1

Ans. Distance between the point A(x_1, y_1), B(x_2, y_2) is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(0 - c)^2 + (-c - 0)^2}$$

$$AB = \sqrt{2c^2} = c\sqrt{2} \text{ units.}$$

b) **What is the perpendicular distance of the point P(2, 3) from the x-axis.** 1

Ans. In P(2, 3)

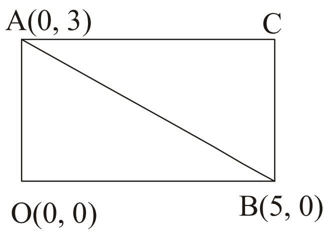
The y coordinate shows the perpendicular distance from x axis

the y coordinate is 3

$\therefore$ The perpendicular distance of p point from the x axis is 3.

c) **AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0) and B(5, 0). Find the length of its diagonal.** 2

Ans.



$A(0, 3) = x_1, y_1$
$B(5, 0) = x_2, y_2$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(5-0)^2 + (0-3)^2}$$

$$= \sqrt{5^2 + (-3)^2}$$

$$= \sqrt{25+9}$$

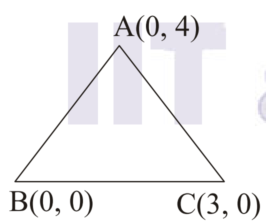
$$= \sqrt{34}$$

$\therefore$ The length of diagonal $= \sqrt{34}$.

OR

Find the perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0).

Ans.



Using distance fromula

$A(0, 4) = x_1, y_1$
$B(0, 0) = x_2, y_2$
$C(3, 0) = x_3. y_3$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0-0)^2 + (0-4)^2}$$

$$= \sqrt{0+16}$$

$$= 4$$

$$BC = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$

$$= \sqrt{(3-0)^2 + (0-0)^2}$$

$$= \sqrt{9+0}$$

$$= \sqrt{9}$$

$$= 3$$

$$AC = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$

$$= \sqrt{(3-0)^2 + (0-4)^2}$$

$$= \sqrt{9+16}$$

$$= \sqrt{25}$$

$$= 5$$

Perimeter of triangle = AB + BC + AC

= 4 + 3 + 5

= 12.

37. **In a marriage ceremony of her daughter, Manita, Rakesh has to make arrangements for the accomodation of 150 persons. For this purpose, he plans to build a conical tent in such a way that each person has 4 sq. m. of the space on the ground and 20 m^3 of the air to breathe.**

a) **Find the volume of conical tent.** 1

Ans. Volume of the conical tent = 150 × 20

= 3000 m^3.

b) **What is the height of conical tent which Rakesh is plannilg to make?** 1

Ans. Total number of people = 150

space required for each person on the ground = $4m^2$

Area of base of the cone = 150 × 4

$$\pi r^2 = 600$$

$$r^2 = \frac{\overset{300}{\cancel{600}} \times 7}{\underset{11}{\cancel{22}}}$$

$$r^2 = \frac{2100}{11}$$

$$\frac{1}{3}\pi r^2 h = 3000$$

$$\frac{1}{\cancel{3}} \times \frac{\overset{2}{\cancel{22}}}{\cancel{7}} \times \frac{\overset{\overset{100}{\cancel{300}}}{\cancel{2100}}}{\cancel{11}} \times h = 3000$$

$$200\ h = 3000$$

$$h = \frac{\overset{15}{\cancel{3000}}}{\cancel{200}}$$

$$h = 15m.$$

c) **The height of cone is 48cm and the radius of its base is 36cm. Find the curved surface area (take $\pi = 3.14$).** 2

Ans.

$h = 48$ cm $\quad r = 36$ cm

$l^2 = h^2 + r^2$

$l^2 = 48^2 + 36^2$

$= 2304 + 1296$

$= 3600$

$l = \sqrt{3600}$

$l = 60$ cm

$\therefore$ The curved surface area $= \pi rl$

$\pi rl = 3.14 \times 36 \times 60 = 6782.4\ cm^2$

OR

If the curved surface area of a right circular cone is 12, 320 cm^2 and its base radius is 56cm then find its height $\left(\pi = \frac{22}{7}\right)$.

Ans. Curved surface area $= \pi rl$

$$12320 = \frac{22}{7} \times 56 \times l$$

$$l = \frac{\overset{\overset{70}{\cancel{1540}}}{\cancel{12320}} \times \overset{1}{\cancel{7}}}{\cancel{22} \times \underset{\underset{1}{\cancel{8}}}{\cancel{56}}}$$

$l = 70$ cm

$l = \sqrt{h^2 + r^2}$ $70 = \sqrt{h^2 + 56^2}$

$4900 = h^2 + 3136$

$h^2 = 1764$

$h = 42$ cm .

38. **Case Study Based**

Basic Proportionality Theorem

(Fig. 1)

A teacher is teaching Basic Proportionality theorem in a class.

He has proved the result $\frac{AD}{BD} = \frac{AE}{EC}$ **for** $\triangle ABC$ **as shown in figure 2.**

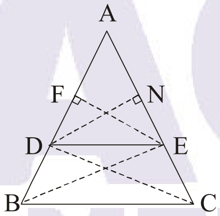


Fig. 2

He proved the theorem using the following steps :

Step 1 : $\frac{ar(\triangle ADE)}{ar(\triangle BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times BD \times EF} = \frac{AD}{BD}$

Step 2 : Similarly, $\frac{ar(\triangle ADE)}{ar(\triangle CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN} = \frac{AE}{EC}$

Step 3 : But $ar(\triangle BDE) = ar(\triangle CDE)$

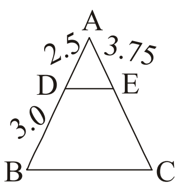
[$\because$ Triangles on the same base DE and between the same parallels DE and BC are equal in area]

$\therefore \quad \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{ar(\Delta ADE)}{ar(\Delta CDE)}$

$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}$

a) **In a ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC. If AD = 2.5cm, BD = 3.0cm and AE = 3.75cm, then find AC.** 1

Ans.



$AD = 2.5$
$BD = 3.0$
$AE = 3.75$
$EC = ?$

$\frac{AD}{BD} = \frac{AE}{EC}$

$\frac{2.5}{3} = \frac{3.75}{EC}$

$EC = \frac{3.75 \times 3}{2.5}$

$= \frac{\overset{2.5}{\cancel{\overset{0.75}{\cancel{3.75}}}} \times 3 \times 10}{\underset{5}{\cancel{25}}}$

$= 0.75 \times 6$

$= 4.50$

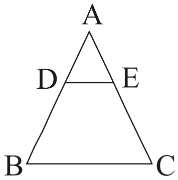
$AC = AE + EC$

$= 3.75 + 4.50$

$= 8.25$ cm.

b) **In a ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC. If AD = 4x –3, AE = 8x – 7, BD = 3x – 1 and CE = 5x – 3, then find value of x.** **1**

Ans.



$AD = 4x - 3$
$AE = 8x - 7$
$BD = 3x - 1$
$CE = 5x - 3$

$x = ?$

$$\frac{AD}{BD} = \frac{AE}{EC}$$

$$\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$$(4x-3)(5x-3) = (8x-7)(3x-1)$$

$$4x(5x-3)-3(5x-3) = 8x(3x-1)-7(3x-1)$$

$$20x^2-12x-15x+9 = 24x^2-8x-21x+7$$

$$20x^2-27x+9 = 24x^2-29x+7$$

$$24x^2-29x+7-20x^2+27x-9 = 0$$

$$4x^2-2x-2 = 0$$

$$2x^2-x-1 = 0$$

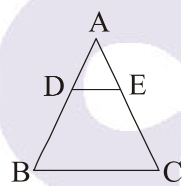
$$2x^2-2x+x-1 = 0$$

$$2x(x-1)+1(x-1) = 0$$

$$(x-1)(2x+1) = 0$$

$$x = 1 \quad x = \frac{1}{2}.$$

c) **In the given Fig., DE || BC and** $\frac{AD}{DB} = \frac{3}{5}$. **If AC = 4.8cm, then find AE.** 2



Ans. **Given :**

$$\frac{AD}{BD} = \frac{AE}{EC}$$

$$AE = x$$

$$AC = AE + EC$$

$$4.8 = x + EC$$

$$EC = 4.8 - x$$

$$\frac{3}{5} = \frac{x}{4.8-x}$$

$$3(4.8-x) = 5x$$

$$14.4 - 3x = 5x$$

$$8x = 14.4$$

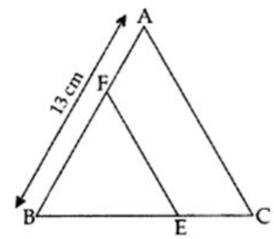
$$x = \frac{\overset{1.8}{\cancel{14.4}}}{\underset{1}{\cancel{8}}}$$

$x = 1.8$

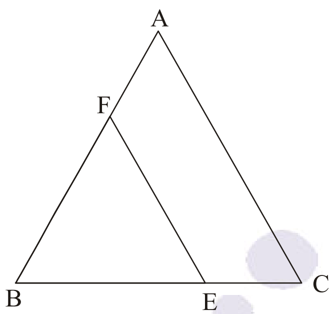
$\therefore$ AE = 1.8cm.

OR

In the figure, EF || AC, BC = 10 cm, AB = 13 cm and EC = 2 cm, find AF.



Ans.



BC = 10 cm

AB = 13 cm

EC = 2 cm

AF = ?

BC = BE + EC©

BE = BC – EC

= 10 – 2

= 8

AB = AF + BF   AF = x

10 = x + BF   BF = 10 – x

$$\frac{BF}{AF} = \frac{BE}{EC}$$

$$\frac{10 - x}{x} = \frac{8}{2}$$

$2(10 - x) = 8x$

$20 - 2x = 8x$

$20 = 8x + 2x$

$20 = 10x$

$$x = \frac{20}{10}$$

$x = 2$

$\therefore$ AF = 2 ·

* * *