EXERCISE - 1 [C]

1. **(5)**

$f(x)=\sqrt{8x+\lambda^2}-\sqrt{14x-x^2-48}$

Let $y=\sqrt{8x-\lambda^2}$

$y^2=8x-x^2$

$x^2+y^2-8x=0$

$x^2-8x+16+y^2=16$

$(x-4)^2+y^2=4^2$

Let $y=\sqrt{14x-x^2-48}$

$y^2+x^2-14x+48=0$

$x^2-14x+49+y^2=1$

$(x-7)^2+y^2=1$

$\therefore f(x)=y_1-y_2$

$f(x)$ is largest along PQ

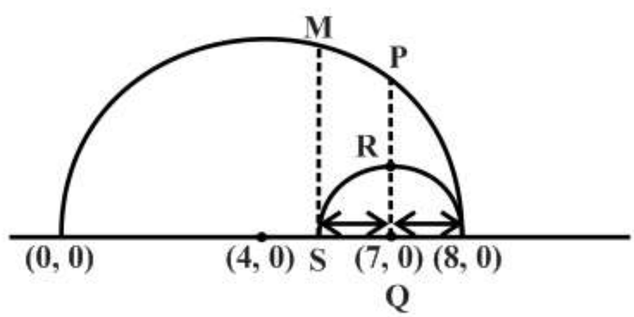
$f(x)_{max}$ = PR or SM

SM is obtained when x = 6

At x = 6, $y=\sqrt{8x-x^2}=\sqrt{48-36}$

$=\sqrt{12}$

$=2\sqrt{3}$

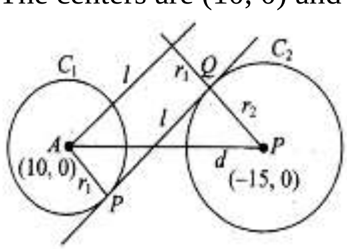


$\therefore$ m = 2, n = 3

m + n = 5

2. **(20)**

The centers are (10, 0) and (-15, 0) and the radii are $r_1=6$ and $r_2=9$. Also, d = 25, $r_1+r_2<d$.



So, the circles are neither intersecting not touching. Therefore,

$PQ=\sqrt{d^2-(r_1+r_2)^2}$

$=\sqrt{625-225}$

= 20

3. **(72)**

The equation of the line y = x in distance form is

$\frac{x}{\cos\theta}=\frac{y}{\sin\theta}=r$, where $\theta=\frac{\pi}{4}$

For point P, $r=6\sqrt{2}$. Therefore, the coordinates of P are given by

$\frac{x}{\cos(\pi/4)}=\frac{y}{\sin(\pi/4)}=6\sqrt{2}$ or $x=6, y=6$

Since P(6, 6) lies on $x^2+y^2+2gx+2fy+c=0$, we have

$72+12(g+f)+c=0$ .....(i)

Since y = x touches the circle, the equation

$2x^2+2x(g+f)+c=0$ has equal roots. Therefore,

$4(g+f)^2=8c$

Or $(g+f)^2=2c$ .....(ii)

From (i), we get

$[12(g+f)]^2=[-(c+72)]^2$

Or $144(g+f)^2=(c+72)^2$

Or $144(2c)=(c+72)^2$

Or $(c-72)^2=0$ or $c=72$

4. **(56)**

The equation of radical axis (i.e., common chord) of the two circles is

$10x+4y-a-b=0$

The center of the first circle is $H(-4,-4)$.

Since the second circle bisects the circumference of the first circle, the center $H(-4,-4)$ of the first circle must lie on the common chord (i). Therefore, a +b = $10\times-4+4\times-4=-40-16=-56$.

5. **(1)**

$xx_1+yy_1-1=0$

$S-S'=0$ ......(1)

$\Rightarrow(\lambda+6)x+(2\lambda-8)y+2=0$ ....(2)

(1) and (2) represent the same line .

$\therefore -2x_1=\lambda+6, -2y=2\lambda-8$

$\therefore -2x_1-6=-y_1+4$

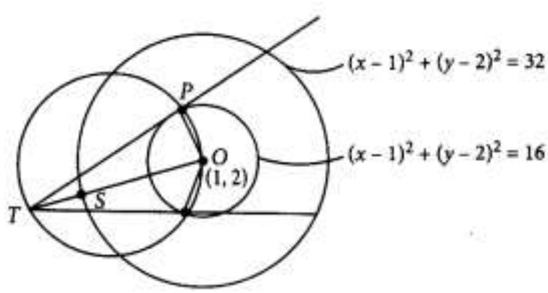
$\Rightarrow 2x-y+10=0$

$\therefore \left.\begin{matrix} p=2 \\ q=-1 \end{matrix}\right\} \Rightarrow p+q=1$

6. **(2)**

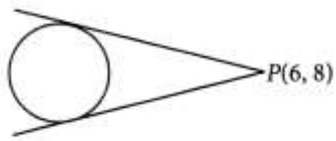
$OS=4\sqrt{2}$

Required distance $TS-OT-SO=12-4\sqrt{2}$



7. **(5)**

Area of the triangle $=\dfrac{r.\left(x_1^{\ 2}+y_1^{\ 2}-r^2\right)^{3/2}}{x_1^{\ 2}+y_1^{\ 2}}$



$$A=\frac{r.\left(100-r^2\right)^{3/2}}{100}$$

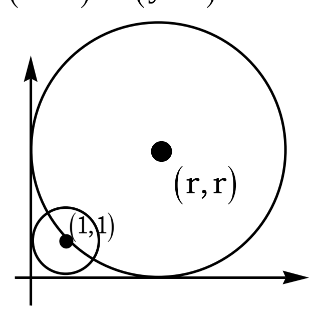
$$\Rightarrow \frac{dA}{dr}=\frac{e.\frac{3}{2}\left(100-r^2\right)^{1/2}(-2r)+\left(100-r^2\right)^{3/2}}{100}$$

$$=\frac{\left(100-r^2\right)^{1/2}}{100=0\Rightarrow r=5}\left(-3r^2+100-r^2\right)=0\Rightarrow r=5$$

$\therefore$ Area is maximum, when $r=5$

8. **(2)**

$(x-r)^2+(y-r)^2=r^2$



(1, 1) lies on it

$\Rightarrow (1-r)^2+(1-r)^2\Rightarrow r^2=\sqrt{2}|1-r|=r\Rightarrow r=2-\sqrt{2},2+\sqrt{2}$

G.E = 2

9. **(2)**

Triangle ABC is right angled at A

P = ortho-centre $\Delta ABC=(1,1)$

$$\frac{PC}{PB}=\frac{AC}{AB}=\frac{\sqrt{8}}{\sqrt{2}}=2$$

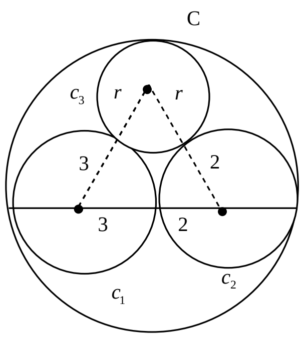
10. **(4)**

The equation $y^2-10y+c=0$ and $x^2-6x+c=0$ must have imaginary roots and also $1+16-6-40+c<0$.

On taking the intersections of all conditions we will get $x\in(25, 29)$

$\Rightarrow$ length of interval $=4$

11. **(8)**



Let O, O_1, O_2, O_3 be the centres and r the radius of C_3

Then, $OO_1=2, OO_2=3, O_1O_3=r+3, OO_3=5-r, O_2O_3=r+2$

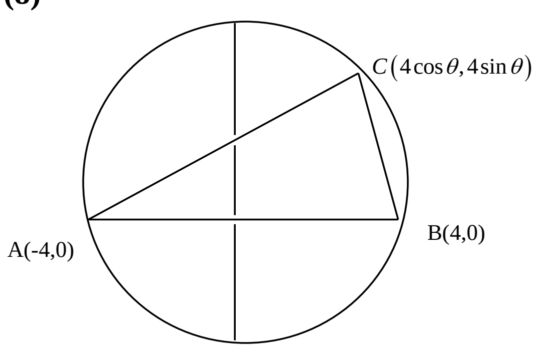
For triangle OO_1O_3 by cosine rule we get

$(r+3)^2=4+(5-r)^2-2.2(5-r)\cos\theta$

$(r+2)^2=9+(5-r)^2+2.3.(5-r)\cos\theta$, where $\theta=\angle O_3OO_1$

Eliminating $\cos\theta$, we find $r=\frac{30}{19}\Rightarrow\frac{30}{19}=\frac{m}{n}, 2n-m=8$

12. **(8)**



Required area

$A=\frac{1}{2}.8.4\sin\theta=|16\sin\theta|$

Now area of integer then the possible values of

$\sin\theta$ are $\frac{1}{16},\frac{2}{16}......\frac{15}{16}$

i.e. 15 points in each quadrant

$\Rightarrow$ 60 + 2 more with $\sin\theta=1$

$\Rightarrow N=62$

13. **(0)**

$S_1 : x^2 + y^2 + 2\lambda x + 4 = 0$

$S_2 : x^2 + y^2 - 4\lambda x + 8 = 0$

Since both represent real circles

$\therefore r_1 \geq 0 \& r_2 \geq 0$

$\therefore \lambda^2 - 4 \geq 0 \quad \therefore \lambda \leq -2 \text{ or } \lambda \geq 2$

$\therefore 4\lambda^2 - 8 \geq 0 \quad \therefore \lambda \leq -\sqrt{2} \text{ or } \lambda \geq \sqrt{2}$

From 1, 2 $\lambda(-\infty, -2] \cup [2, \infty)$

All of these lie within the range

14. **(1)**

Let equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

Let $\left(x, \frac{1}{x}\right)$ be a point on the circle

$\therefore x^4 + 2gx^3 + cx^2 + 2fx + 1 = 0$

$\Rightarrow abcd = \frac{1}{1} = 1$

15. **(3)**

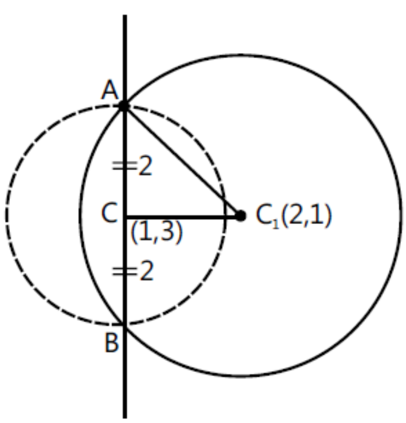
Here radius of smaller circle, $AC - \sqrt{1^2 + 3^2 - 6} = 2$

Clearly, from the figure the radius of bigger circle

$r^2 = 2^2 + \left[(2-1)^2 + (1-3)^2\right]$

$r^2 = 9$

$\Rightarrow r = 3$



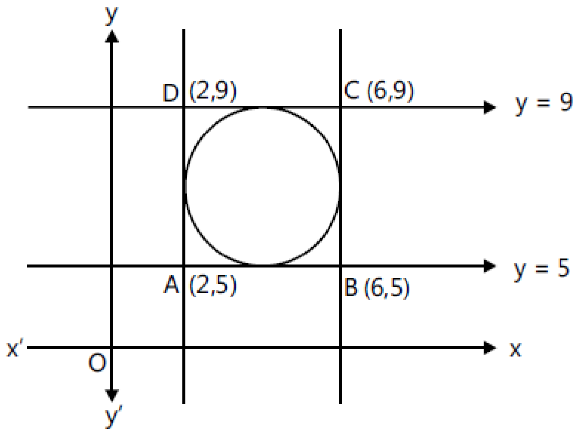
16. **(11)**

Given, circle is inscribed in square formed by the lines

$x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$

$\Rightarrow x = 6$ and $x = 2$, $y = 5$ and $y = 9$

Which could be plotted as



Where ABCD clearly forms a square

$\therefore$ Centre of inscribed circle

= Point of intersection of diagonals

= Mid point of AC or BD

$= \left(\frac{2+6}{2}\right), \left(\frac{5+9}{2}\right) = (4,7)$

$\Rightarrow$ Centre of inscribed circle is (4, 7)

17. **(5)**

The line 5x – 2y+6=0 meets

The y-axis at the point (0, 3) and therefore the tangent as to pass through the point (0, 3) and required length

$= \sqrt{x_1^2 + y_1^2 + 6x_1 + 6y_1 - 2}$

$= \sqrt{0^2 + 3^2 + 6(0) + 6(3) - 2} = \sqrt{25} = 5$

18. **(2)**

Since, the given circles intersect orthogonally.

$2(g_1 g_2 + f_1 f_2) = G + C_2$

$\therefore$ 2(–1) (0) + 2(–k) (–k) = 6 + k

$\Rightarrow 2k^2 - k - 6 = 0 \qquad \Rightarrow k = -\frac{3}{2}, 2$

19. **(1)**

Given, $x^2 + y^2 = 4$

Centre $\equiv C_1 \equiv (0,0)$ and $R_1 = 2$

Again, $x^2 + y^2 - 6x - 8y - 24 = 0$, then $C \equiv (3,4)$and $R_2 = 7$ again, $C_1C_2 = 5 = R_2 - R_1$

Since, the given circles touch internally therefore, they can have just one common tangent at the point of contact.

20. **(1)**

Eq. of circle touching x – a × y at (1, 0) u

$(x-1)^2 + (y-k)^2 = k^2$

Circle passes through (2, 3), then

$(x-1)^2 + (3-k)^2 = k^2$

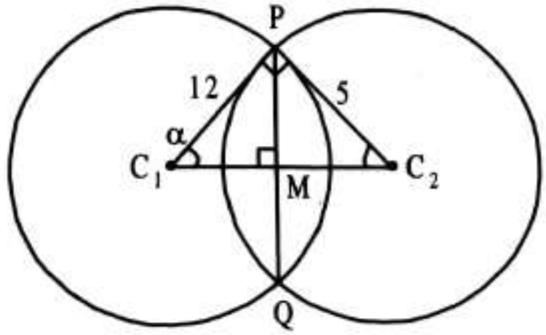
$1 + 9 - 6k + k^2 = k^2$

$\Rightarrow 6k = 10$

$\Rightarrow 2k = \frac{10}{3}$

1. (B)

14. (b)



According to the diagram,

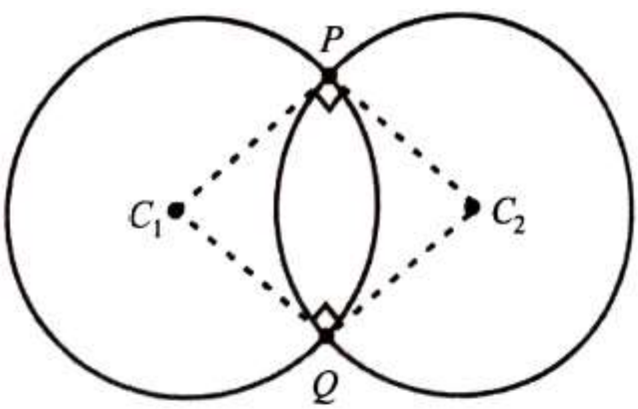
In ΔPC_1C_2, $\tan\alpha = \frac{5}{12} \Rightarrow \sin\alpha = \frac{5}{13}$

In ΔPC_1M, $\sin\alpha = \frac{PM}{12} \Rightarrow \frac{5}{13} = \frac{PM}{12} \Rightarrow PM = \frac{60}{13}$

Hence, length of common chord $(PQ) = \frac{120}{13}$

2. (D)

15. (d)



$2g_1g_2 + 2f_1f_2 = 2(-1)(-3) + 2(-1)(-3) = 12$

$c_1 + c_2 = 14 - 2 = 12$, since, $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

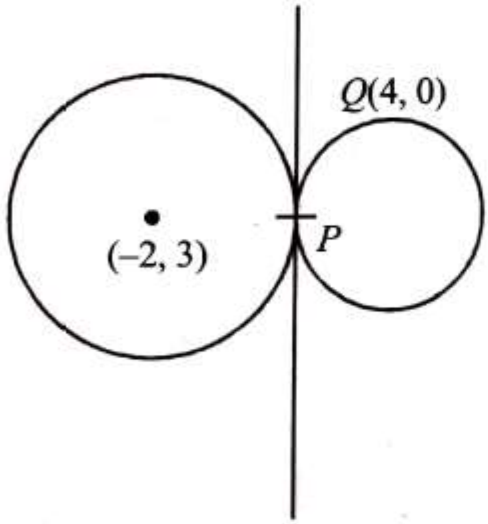
Hence, circles intersect orthogonally

$\therefore$ Area of the quadrilateral PC_1QC_1

$= 2\left(\frac{1}{2}(C_1P)(C_2P)\right) = 2 \times \frac{1}{2} r_1 r_2 = (2)(2) = 4$ sq. units

3. (C)

16. **(c)** The equation of circle $x^2 + y^2 + 4x - 6y = 12$ can be written as $(x+2)^2 + (y-3)^2 = 25$



Let $P = (1, -1)$ & $Q = (4, 0)$

Equation of tangent at $P\,(1, -1)$ to the given circle:

$x(1) + y(-1) + 2(x+1) - 3(y-1) - 12 = 0$

$3x - 4y - 7 = 0$ ...(i)

The required circle is tangent to (i) at $(1, -1)$.

$\therefore \quad (x-1)^2 + (y+1)^2 + \lambda\,(3x - 4y - 7) = 0$ ...(ii)

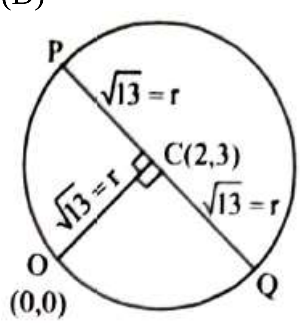
Equation (ii) passes through $Q(4, 0)$

$\Rightarrow \quad 3^2 + 1^2 + \lambda(12 - 7) = 0 \Rightarrow 5\lambda + 10 = 0 \Rightarrow \lambda = -2$

Equation (ii) becomes $x^2 + y^2 - 8x + 10y + 16 = 0$

radius $= \sqrt{(-4)^2 + (5)^2 - 16} = 5$

4. (D)



Slope of $OC = \frac{3}{2}$ $\therefore$ Slope of $PQ = \tan\theta = -\frac{2}{3}$

$\Rightarrow \sin\theta = \frac{2}{\sqrt{13}}$ and $\cos\theta = \frac{-3}{\sqrt{13}}$

Using symmetric from the line

$(P, Q):\left(2 \pm \sqrt{13}\cos\theta, 3 \pm \sqrt{13}\sin\theta\right)$

$\Rightarrow \left(2 \pm \sqrt{13}.\left(-\frac{3}{\sqrt{13}}\right), 3 \pm \sqrt{13}\left(\frac{2}{\sqrt{13}}\right)\right) \Rightarrow (-1, 5) \,\&\, (5, 1)$

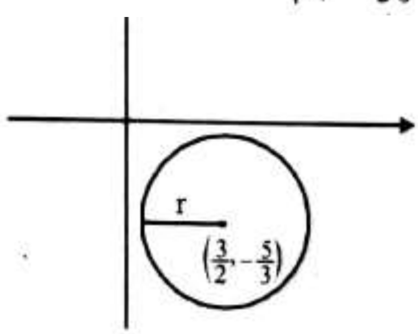
5. (D)

11. **(d)** Since, $S: 36x^2 + 36y^2 - 108x + 120y + C = 0$

$\Rightarrow x^2 + y^2 - 3x + \frac{10}{3}y + \frac{C}{36} = 0$

$\text{Centre} \equiv (-g, -f) \equiv \left(\frac{3}{2}, \frac{-10}{6}\right)$

$\text{radius} = r = \sqrt{\frac{9}{4} + \frac{100}{36} - \frac{C}{36}}$



Since circle neither intersects nor touches the coordinate axes.

$\therefore \quad r < \frac{3}{2} \Rightarrow \frac{9}{4} + \frac{100}{36} - \frac{C}{36} < \frac{9}{4}$

$\Rightarrow C > 100$ ... (i)

Now point of intersection of $x - 2y = 4$ and $2x - y = 5$ is $(2, -1)$, which lies inside the circle S.

$\therefore S(2, -1) < 0$

$\Rightarrow (2)^2 + (-1)^2 - 3(2) + \frac{10}{3}(-1) + \frac{C}{36} < 0$

$\Rightarrow 4 + 1 - 6 - \frac{10}{3} + \frac{C}{36} < 0$

$\Rightarrow C < 156$ ... (ii)

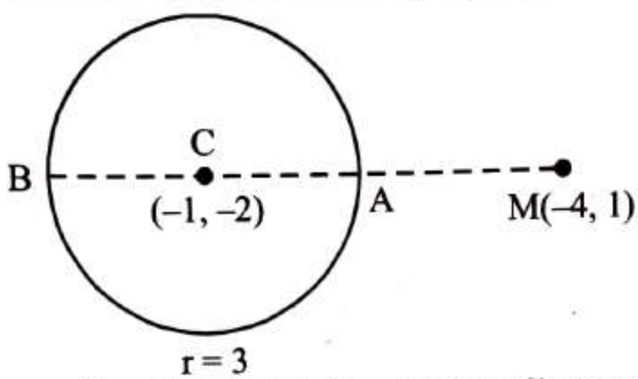
From (i) & (ii), $100 < C < 156$

6. (C)

12. **(c)** Consider the given circle $x^2 + y^2 + 2x + 4y - 4 = 0$

$\Rightarrow (x+1)^2 + (y+2)^2 = 9$

So, the centre of the circle is $(-1, -2)$ and radius = 3 unit.



Centre of smallest circle is A and centre of largest circle is B

$r_2 = |CM - CA| = 3\sqrt{2} - 3$, $r_1 = |CM + CB| = 3\sqrt{2} + 3$

$\frac{r_1}{r_2} = \frac{3\sqrt{2}+3}{3\sqrt{2}-3} = \frac{(3\sqrt{2}+3)^2}{9} = (\sqrt{2}+1)^2 = 3 + 2\sqrt{2}$

Now, comparing with $\frac{r_1}{r_2} = a + b\sqrt{2}$, then $a = 3, b = 2$

Hence, $a + b = 3 + 2 = 5$

7. (B)

1. **(b)** Let $s \equiv \sin t$, $c \equiv \cos t$
Let orthocentre be (h, k)
Since it if an equilateral triangle hence orthocentre coincides with centroid

$$\Rightarrow a + s + c = 3h, b + s - c = 3k$$
$$\Rightarrow (3h - a)^2 + (3k - b^2) = (s + c)^2 + (s - c)^2$$
$$= 2(s^2 + c^2) = 2$$
$$\therefore \left(h - \frac{a}{3}\right)^2 + \left(k - \frac{b}{3}\right)^2 = \frac{2}{9},$$

Now circle centre at $\left(\frac{a}{3}, \frac{b}{3}\right)$

we are given that $\Rightarrow \frac{a}{3} = 1, \frac{b}{3} = \frac{1}{3} \Rightarrow a = 3, b = 1$

$$\therefore a^2 - b^2 = 8$$

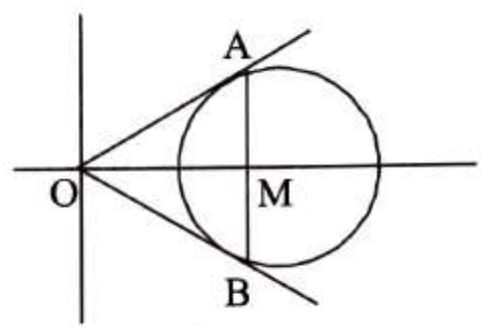
8. (B)

2. **(b)** Equation of given circle is $(x - 2)^2 + y^2 = 1$

Equation of chord AB : $2x = 3$

$$OA = OB = \sqrt{3};\ AM = \frac{\sqrt{3}}{2}$$

$$\Rightarrow AB = 2 \cdot \frac{\sqrt{3}}{2} . 2$$



Area of triangle OAB = $\frac{1}{2}(AB)(OM) = \frac{3\sqrt{3}}{4}$ sq. units

9. (D)

3. **(d)** Given circle $x^2+y^2-2gx+6y-19c=0$

Passes though (6,1)

$12g+19c=43$ ....(i)

Centre (g, –3) lies on given line

So, g +bc = 8 ....(ii)

Solve equations (i) & (ii); c = 1 & g = 2

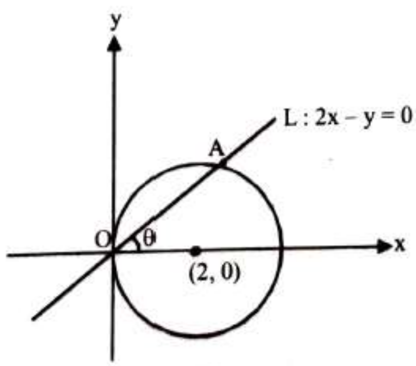
equation of circle $x^2+y^2-4x+6y-19=0$

x-intercept $=2\sqrt{g^2-c}=2\sqrt{23}$

10. (A)

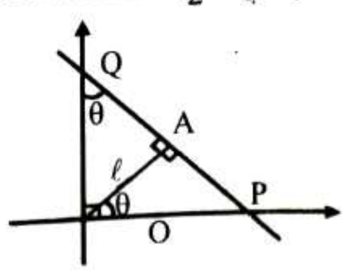
4. **(a)** Given circle is $C_1 : x^2+y^2-4y=0$

$\tan\theta=2$



C_2 is a circle with OA as diameter.

So, tangent at A on C_2 is perpendicular to OR



Therefore, $\frac{QA}{AP}=\frac{l\cot\theta}{l\tan\theta}, \frac{1}{\tan^2\theta}=\frac{1}{4}$

11. (A)

5. **(a)** Let point P(h, k) & Q(p, q).

Equation of circle by using diameter form.

$(x-h)(x-p)+(y-k)(y-q)=0$

(where h, p are the roots of $x^2-4x-6=0$ and k, q are the roots of $y^2+2y-7=0$)

$x^2+y^2-4x+2y-13=0$

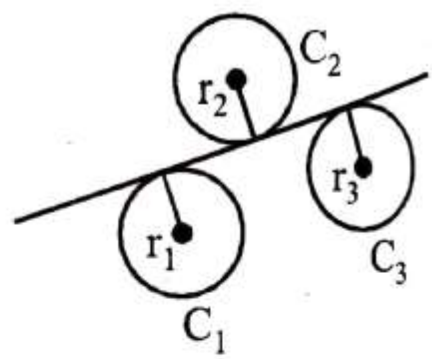
Now, Compare it with the given equation, we get

$a=-2, b=1, c=-13$, Now, $a+b-c=12$

12. (D)

6. **(d)** Given L : $y=mx+c$ be a common tangent

Let r_1 for c_1 is, $r_1=\left|\frac{c}{\sqrt{1+m^2}}\right|$



$r_3=\left|\frac{2m-1+c}{\sqrt{1+m^2}}\right|$

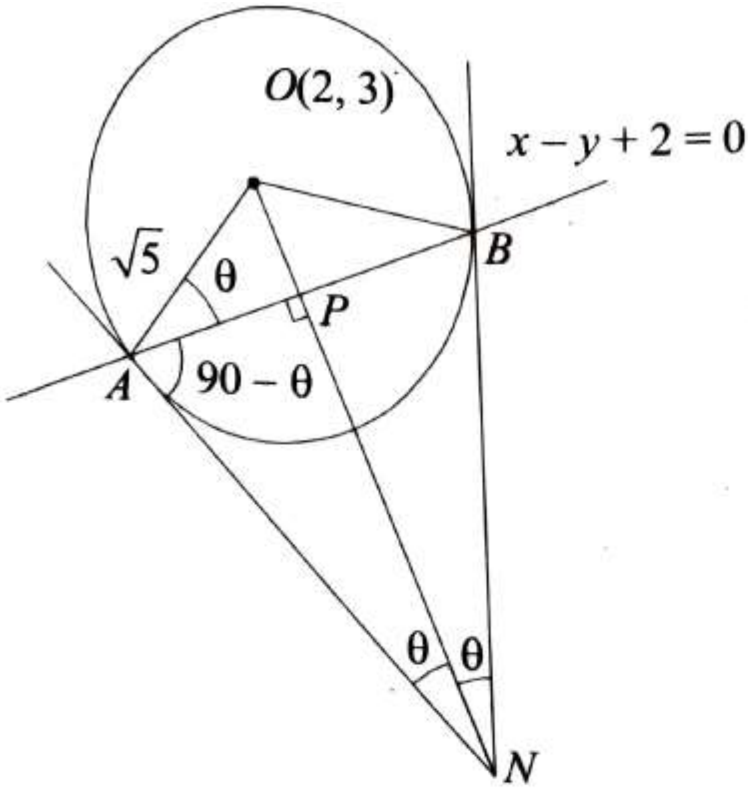
$\because \quad m=\frac{1}{2} \quad [\text{as } r_1=r_3=r_2]$

For $r_2=\left|\frac{m-1+c}{\sqrt{1+m^2}}\right| \Rightarrow 2c=1-m \Rightarrow c=\frac{1}{4}$

Now $20(r^2+c) \Rightarrow 20\left(\frac{1}{20}+\frac{5}{20}\right)=6$

13. (C)

7. **(c)** Here, tangent T to circle C_1, is $x-y+2=0$ will be chord of contact for C_2



$$OP = \left|\frac{2-3+2}{\sqrt{2}}\right| \Rightarrow OP = \frac{3}{\sqrt{2}}$$

now, $AP = \sqrt{OA^2 - OP^2} = \frac{1}{\sqrt{2}}$

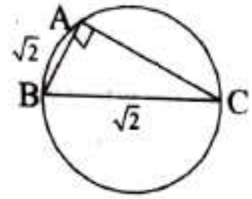
$\tan\theta = 3 \therefore \sin\theta = \frac{3}{\sqrt{10}} = \frac{AP}{AN} \Rightarrow AN = \frac{\sqrt{5}}{3} = BN$

Area of $\Delta ANB = \frac{1}{2}\cdot(AN^2)\sin 2\theta = \frac{1}{6}$

14. (Bonus)

8. **(Bonus)** Radius of given circle is 1.

Centre : $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

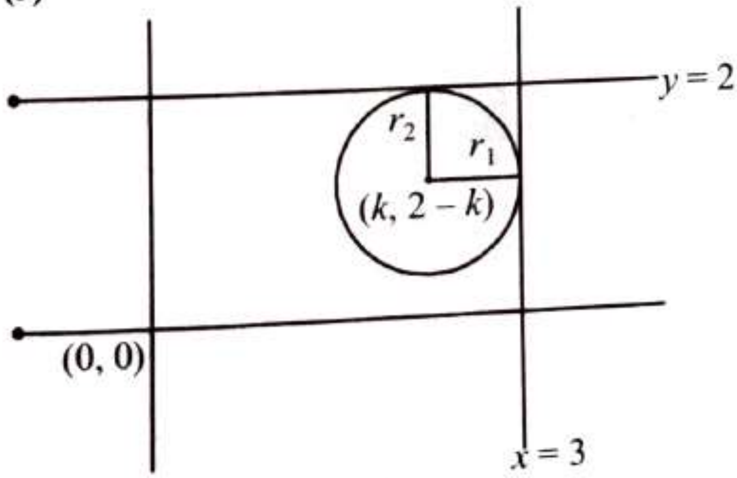


$BC = \text{diameter} = 2, AB = \sqrt{2}$

$AC = \sqrt{BC^2 - AB^2} = \sqrt{2}$, $\Delta ABC = \frac{1}{2} AB.AC = 1$

15. (3)

58. (3)



$\because$ Centre lies on $x + y = 2$

Let $x = k$ $\therefore y = 2 - k$ $\Rightarrow$ Centre $= (k, 2-k)$

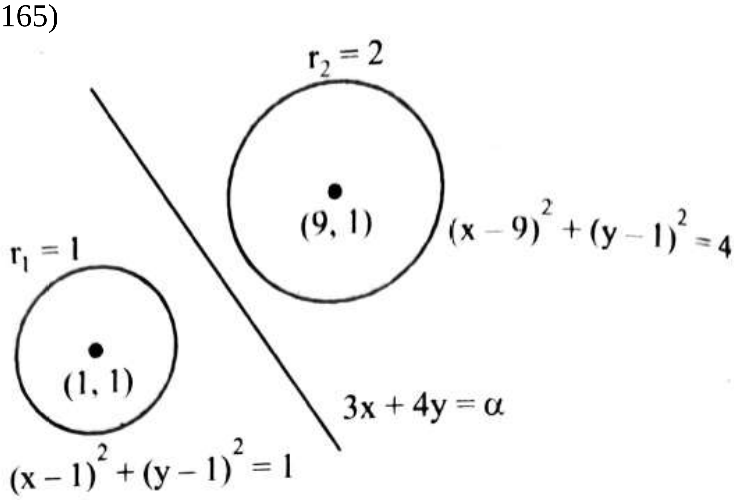
$\Rightarrow$ Radius $(r_1) = 3 - k$

Also, radius $(r_2) = 2 - (2 - k)$

$\therefore 3 - k = 2 - (2 - k) \Rightarrow k = \frac{3}{2}, \quad r = 3 - \frac{3}{2} = \frac{3}{2}$

Hence, diameter = 3.

16. (165)



We can say line lies between the two circles or both centres should lie on either side of the line as well as line can be tangent to circle.

$(3+4-\alpha).(27+4-\alpha)<0$

$(7-\alpha).(31-\alpha)<0 \Rightarrow \alpha \in (7,31)$ ...(i)

d_1 = distance of (1, 1) from line

d_2 = distance of (9, 1) from line

$d_1 \geq r_1 \Rightarrow \frac{|7-\alpha|}{5} \geq 1 \Rightarrow \alpha(-\infty,2] \cup [12,\infty)$ ...(ii)

$d_2 \geq r_2 \Rightarrow \frac{|31-\alpha|}{5} \geq 2 \Rightarrow \alpha \in (-\infty,21] \cup [41,\infty)$ ...(iii)

$(i) \cap (ii) \cap (iii) \Rightarrow \alpha \in [12,21]$

Sum of integers $= 12+13+.....+21 = 165$

17. (1)

56. **(1)** Consider the given equation of first circle

$x^2+y^2-10x-10y+41=0$

Then, centre $C_1 \equiv (5, 5)$ and radius $r_1 = 3$

And the second equation of circle is

$x^2+y^2-24x-10y+160=0$

Then, centre $C_2 \equiv (12, 5)$ and radius = 3

Distance between centres = 7 and sum of radii = 3 + 3 = 6

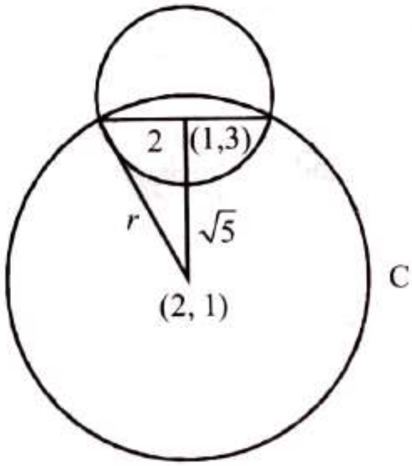
$\therefore$ Distance between centres > Sum of radii

$\Rightarrow$ Circle are separated,

So, the required minimum possible distance $= 7-(3+3) = 1$

18. (3)

57. (3)



Given that $x^2+y^2-2x-6y+6=0$ center $(1, 3)$ and radius $=2$

Distance between $(1, 3)$ and $(2, 1)$ is $\sqrt{5}$

$\therefore\ (\sqrt{5})^2+(2)^2=r^2 \Rightarrow r=3$

19. (12)

47. (12) Image of centre $c_1 \equiv (1, 3)$ in $x-y+1=0$ is given by

$$\frac{x_1-1}{1}=\frac{y_1-3}{-1}=\frac{-2(1-3+1)}{1^2+1^2}$$

$\Rightarrow x_1=2,\ y_1=2$

$\therefore$ Centre of circle $c_2 \equiv (2,2)$

$\therefore$ Equation of c_2 be $x^2+y^2-4x-4y+\frac{38}{5}=0$

Now radius of c_2 is $\sqrt{4+4-\frac{38}{5}}$ $r_2=\sqrt{2/5}$

$(\text{radius of } c_1)^2=(\text{radius of } c_2)^2$

$\Rightarrow 10-\alpha=\frac{2}{5}\Rightarrow \alpha=\frac{48}{5}\ \therefore \alpha+6r^2=\frac{48}{5}+\frac{12}{5}=12$

20. (25)

48. **(25)** Given cricle $x^2 + y^2 + 6x + 8y + 16 = 0$ with centre $(-3, -4)$ and rdius 3 units.

The cricle $x^2 + y^2 + 2\left(3-\sqrt{3}\right)x + 2\left(4-\sqrt{6}\right)y = k + 6\sqrt{3}$ $+8\sqrt{6}, k > 0$ has centre $\left(\sqrt{3}-3, \sqrt{6}-4\right)$ and radius $\sqrt{k+34}$

So, These two circles touch internally hence

$\sqrt{3+6} = \left|\sqrt{k+34} - 3\right|$.

Here, $k = 2$ is only possible $(\because k > 0)$

Equation of common tangent to two circles is

$2\sqrt{3x} + 2\sqrt{6y} + 16 + 6\sqrt{3} + 8\sqrt{6} + k = 0$

Therefore, $k = 2$ then equation is

$x + \sqrt{2}y + 3 + 4\sqrt{2} + 3\sqrt{3} = 0$ ... (i)

So, (α, β) are foot of perpendicular from $(-3, -4)$ To line (i) then

$$\frac{\alpha+3}{1} = \frac{\beta+4}{\sqrt{2}} = \frac{-\left(3-4\sqrt{2}+3+4\sqrt{2}+3\sqrt{3}\right)}{1+2}$$

$$\therefore \alpha + 3 = \frac{\beta+4}{\sqrt{2}} = -\sqrt{3}$$

$$\Rightarrow \left(\alpha+\sqrt{3}\right)^2 = 9 \text{ and} \left(\beta+\sqrt{6}\right)^2 = 16$$

Therefore $\left(\alpha+\sqrt{3}\right)^2 + \left(\beta+\sqrt{6}\right)^2 = 25$

21. (816)

49. **(816)** Normal lines to the circle are

$y+2x=\sqrt{11}+7\sqrt{7},\ 2y+x=2\sqrt{11}+6\sqrt{7}$.

Center of the circle is point of intersection of normals

$\left(\frac{8\sqrt{7}}{3}, \sqrt{11}+\frac{5\sqrt{7}}{3}\right)$

Given equation of tangent is $\sqrt{11}y-3x=\frac{2\sqrt{77}}{3}+11$

Distance of tangent from the centre is shown below.

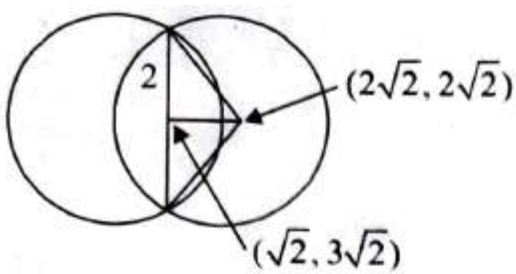
$$r=\left|\frac{8\sqrt{7}-11-\frac{5}{3}\sqrt{77}+\frac{5\sqrt{77}}{3}+11}{\sqrt{20}}\right|$$

$r=\left|\sqrt[4]{\frac{7}{5}}\right|=\sqrt[4]{\frac{7}{5}}$ units.

Now, $(5h-8k)^2+5r^2=816$

22. (10)

50. **(10)**



Now we given that

PQ is diameter of circle

$S: x^2+y^2-2\sqrt{2}x-6\sqrt{2}y+14=0$

Here center of the circle S_1 is $(\sqrt{2}, 3\sqrt{2})$ and radius

$r_1=\sqrt{6}$

Now circle $S_2: (x-2\sqrt{2})^2+(y-2\sqrt{2})^2=r^2$

Here center of the circle S_2 is $(2\sqrt{2}, 2\sqrt{2})$ and radius is r

Now in ΔOCQ

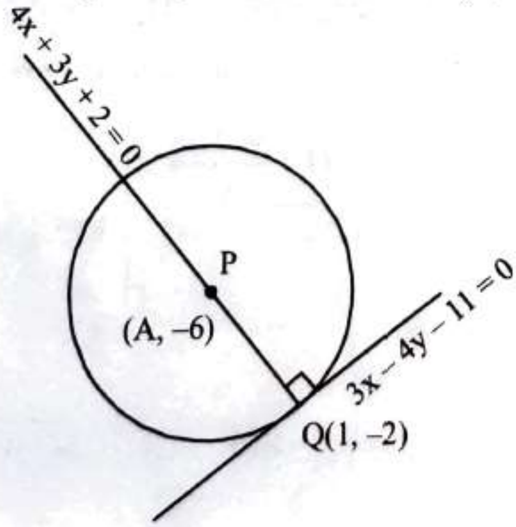
$|OC|^2+|CQ|^2=|OQ|^2$

$\Rightarrow\ 4+6=r^2 \Rightarrow r^2=10$

23. (11)

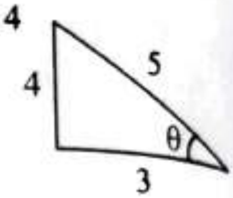
52. **(11)** Given $4x+3y+2=0$ and $3x-4y-11=0$

Intersection point Q of these two lines is $(1, -2)$.



$$\frac{x}{-25}=\frac{y}{50}=\frac{1}{-25};\ \frac{x-1}{\cos\theta}=\frac{y+2}{\sin\theta}=\pm 5$$

$$y=-2+5\left(-\frac{4}{5}\right)=-6,\ x=1+5\left(\frac{3}{5}\right)=4$$



$$\text{Req. distance}=\left|\frac{5(4)-12(-6)+51}{13}\right|$$

$$=\left|\frac{20+72+51}{13}\right|=\frac{143}{13}=11$$

CIRCLES

EXERCISE – 2(A)

Q.1

$4x^2 + 4y^2 - 12x + 4y + 1 = 0$

$x^2 + y^2 – 3x + y + ¼ = 0$

Center = (3/2 , -1/2) ,

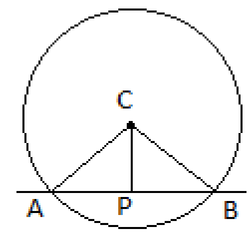
Radius = 3/2

$\angle ACB = 120^0$

⇨ $\angle ACP = 60^0$

AC = 3/2

⇨ CP = $3/2 \cos 60^0$ = ¾



∴ locus of CP is : $(x-3/2)^2 + (y+1/2)^2 = 9/16$

⇨ $X^2 + y^2 – 3x + y + 31/16 = 0$

Ans: C

Q.2

$x^2 + y^2 - 2x – 6y = 0$ has Center : C_1 =(1 , 3) & Radius: $R_1 = 2$

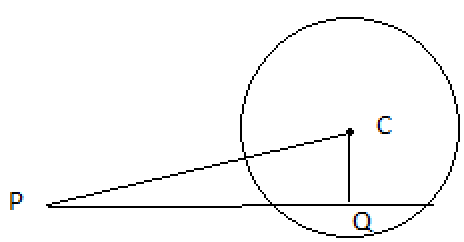
$C_2 = (2 , 1)$

Distance between centers : d= $\sqrt{5}$

∴ $(R_2)^2 = (R_1)^2 + d^2 = 3$

Ans: (C)

Q.3



The locus of Q is circle with PC as diameter

P= (h , k) & C= (0 , 0)

Locus : (x-h)(x-0) + (y-k)(y-0) = 0

Ans :(B)

Q.4

Radical axis of two sides of triangle will pass through the common vertex and will perpendicular to third side.

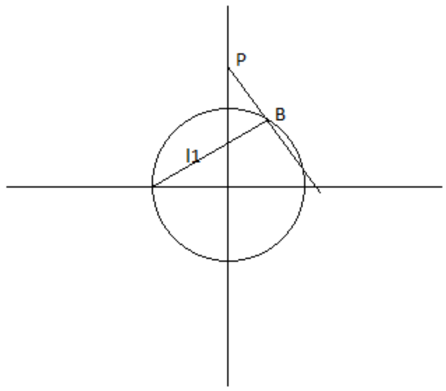
So, intersection of these axis will be orthocenter of triangle ABC

Ans: (D)

Q.5 The point of concurrence will be the pole of the line.

Ans: (D)

Q.6



B is the intersection point of l1 and circle

B = (6, 8), slope of line = ½

Therefore, slope of perpendicular line = -2

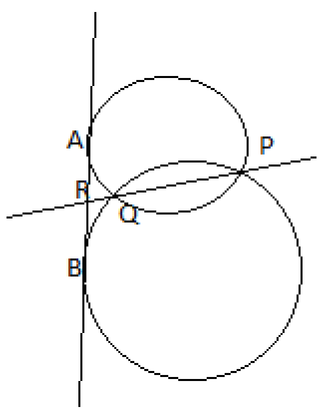
And equation of line will be : (y-8) = -2(x-6)

⇨ 2x + y = 20

So, coordinates of P are (0,20) i.e. t = 20

Ans: (C)

Q.7



P =(1 ,1) & Q = (3,-1)

$AR^2 = (RQ)(RP)$ & $BR^2 = (RQ)(RP)$

⇨ R is the midpoint of AB

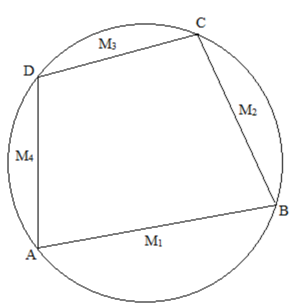
Equation of PQ : (y - 1) = -1(x - 1)

⇨ x + y =2, hence R is (0, 2)

Therefore, $AB^2 = 4(RP)(RQ)$ or $AB = 2\sqrt{6}$

Ans: (B)

Q.8



$M_1 = ½$, $M_2 = -3/4$ & $M_3 = ¼$

$\angle ABC = \Pi - \angle CDA$

⇨ $\frac{M1-M2}{1+M1M2} = -\frac{M3-M4}{1+M3M4}$

⇨ M4 = 9/2

Ans: (D)

Q.9

$d = R_1 + R_2$

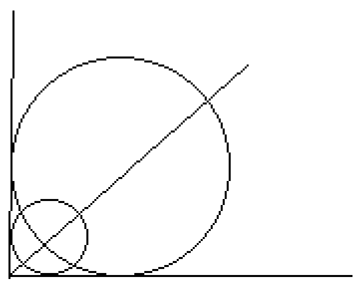
$d^2 - (R_1 - R_2)^2 = 4 * 35$

⇨ $R_1R_2 = 35$

⇨ $(R_1 , R_2) = 35*1$ or $7*5$

Ans: (C)

Q.10



Center lies on $y = x$

One end of the common chord $=(a ,b)$

Other end is the reflection in $y=x$ i.e. $= (b, a)$

⇨ Radical axis : $x + y = a + b$

Q.11

Equation of the family of circle is $x^2 + y^2 - x + ky = 0$

Center $C_1 =(1/2 , -k/2)$ $R_1^2 = 1/4 + k^2/4$

$X^2 + y^2 = 9$

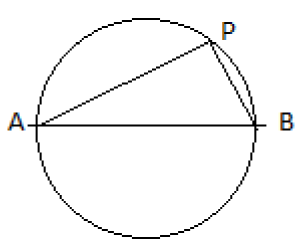
$C_2 = (0,0)$ $R_2= 3$

$\sqrt{(1 + k * k)} = 3$

$k = +- 2\sqrt{2}$

Ans: (B)

Q.12

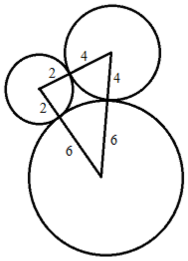


P lies on the circle $x^2 + y^2 = 50$

⇨ $50 = 1^2 + 7^2$ or $7^2 + 1^2$ or $5^2 + 5^2$

Ans: (C)

Q.13



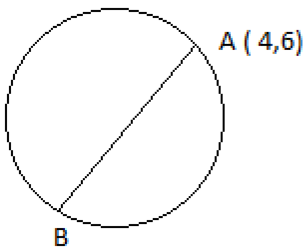
Radical center will be incenter of the triangle formed by the centers

PA = in radius of the triangle formed with sides 8,10,6

i.e = 2

Ans: (A)

Q.14



Let the other end of the chord be B (k, -6)

If B lies on the circle

$K^2 - 4k + 72 = 0$

K is non real

Ans: (A)

Q.15

Let A_k be (x_k, y_k) & let P be (x, y), then

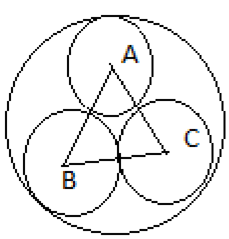
$PA_k{}^2 = (x - x_k)^2 + (y - y_k)^2$.

Now $nx^2 + ny^2 - 2(\sum_{k=1}^{n} x_k)x - 2(\sum_{k=1}^{n} y_k)y + \sum_{k=1}^{n} x_k{}^2 + \sum_{k=1}^{n} y_k{}^2 = \sum_{k=1}^{n} d_k{}^2$

Clearly this is equation of a circle.

Ans: (A)

Q.16



ABC is equilateral triangle

Radius of the required circle is $r + \frac{2r}{2\sin(\frac{\pi}{3})}$

$= r\ (1 + 2/\sqrt{3})$

Ans: (B)

Q.17

Length of common chord, $\ell = \frac{2r_2 r_1 \sin\theta}{PQ}$, where $\cos\theta = \frac{PQ^2 - r_1^2 - r_2^2}{2r_1 r_2}$, P & Q are centers and r_1 & r_2 are the radii.

Ans:(B)

Q.18

OMPN is cyclic quadrilateral

⇨ Diameter of the circle is OP

⇨ Radius = OP/2

⇨ I.e radius = 5/2

Ans:(B)

Q.19

Let the center of w be c(x,y)

Then the radius is the length of the tangent

$R^2 = x_1^2 + y_1^2 - k^2$

⇨ $(x-x_1)^2 + (y-y_1)^2 = x_1^2 + y_1^2 - k^2$

⇨ $X^2+y^2 - 2xx_1 - 2yy_1 + k^2 = 0$

It passes through (a,b)

⇨ $2ax_1 + 2by_1 - (a^2+b^2+k^2) = 0$

Ans: (A)

Q.20

If 3x + 4y=c is a tangent then [c]/5 =5

⇨ C = + - 5

Ans: (C)

Q.21

Let the line be (y-2)=m(x-2)

Perpendicular distance from center = $|6m|/\sqrt{(1+m*m)}$

R=5 gives $R^2 - d^2 = 4$

$25 - 36m^2/(1+m^2) = 16$

$25 + 25m^2 - 36m^2 = 16 + 16m^2$

$27m^2 = 9$ gives $3m^2 = 1$

Ans: (D)

Q.22

$(x-3)^2 + (y-4)^2 = 10$

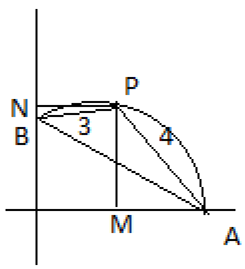
$x-3 = a$

$y-4 = b$

then $a^2 + b^2 < 10$

Ans: (D)

Q.23



$\angle APB = \Pi/2$

PM = y

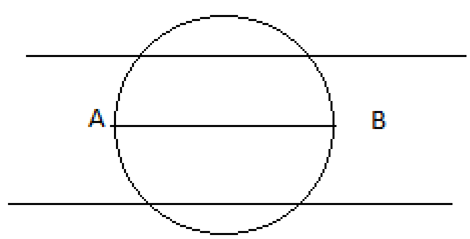
PN = x

Therefore, triangle PNB and Triangle PAM are similar

⇨ $x/3 = y/4$

Ans: (C)

Q.24



AB is the diameter of the circle ½ AB.h = 5

h=2

radius of the circle = 5/2

hence 4 points (C)

Q.25

The equation of the circle is

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$

$X^2 + 2ax - b^2 + y^2 + 2px - q^2 = 0$

Ans: (C)

Q.26

Both the circle pass through (0,0)

If they touch each other then tangent at (0,0) should coincide

⇨ $gx + fy = 0$ and $g_1x + f_1y = 0$ are same

⇨ $g/g_1 = f/f_1$

Ans: (A)

Q.27

Let the tangent be $x\cos\theta + y\sin\theta = r$

⇨ $A = (r/\cos\theta, 0)$

And $B = (0, r/\sin\theta)$

⇨ $C = (r/\cos\theta, r/\sin\theta)$

⇨ $1/x^2 + 1/y^2 = 1/r^2$

Ans:(B)

Q.28

The common chord should be the diameter of $x^2 + y^2 = 16$

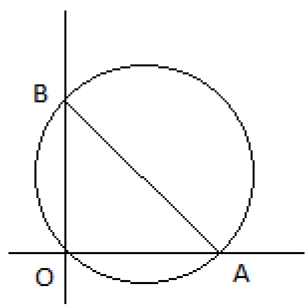
Chord is $3x-4y=0$

$X^2+y^2-16+k(3x-4y)=0$, hence Radius = 5

⇨ $K = \pm 6/5$

Ans:(A)

Q.29



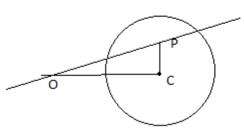
If OA = a , OB = b, hence centroid $G(h,k) = (a/3, b/3)$

$a^2 + b^2 = 36r^2$

$h^2+k^2 = 4r^2$

Ans: (C)

Q.30

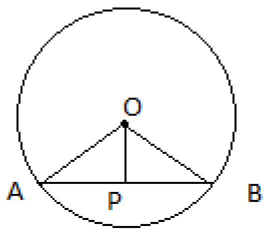


Locus of P is the circle on OC as diameter

⇨ $X^2 + y^2 - ax = 0$

Ans: (C)

Q.31



$\angle AOP = \Pi/4$

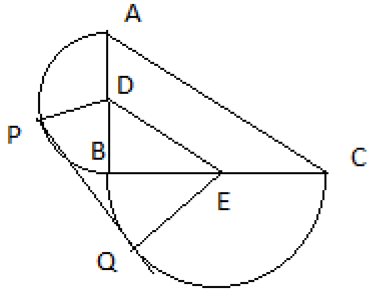
⇨ $OP = \sqrt{2}$

⇨ $OP^2 = 2$

⇨ $X^2 + y^2 = 2$

Ans: (D)

Q.32



D, E are mid points of AB , BC

⇨ $DE = 5/2$

⇨ $\angle DPQ = \angle PQE = \Pi/2$

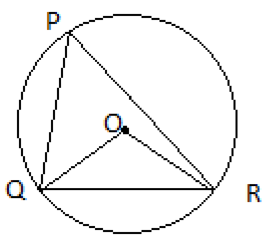
⇨ $PQ^2 = DE^2 - (QE - PD)^2$

$= 25/4 - 1/4 = 6$

⇨ $PQ = \sqrt{6}$

Ans: (B)

Q.33



$OQ = 5$

$OR = 5$

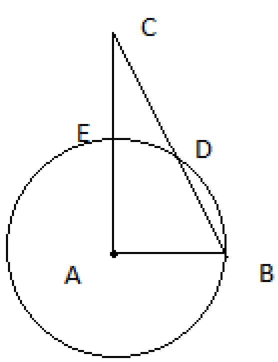
$RQ = 5\sqrt{2}$

$\therefore \angle QOP = \Pi/2$

And $\angle QPR = ½ \angle QOP$ or $\Pi - \angle QOP = \Pi/4$ or $3\Pi/4$.

Ans: (D)

Q.34



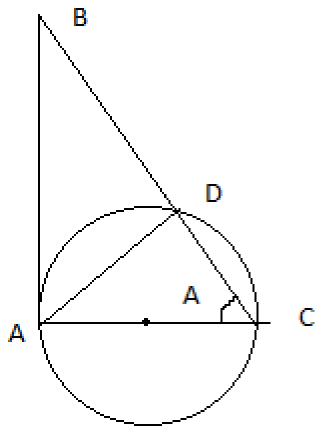
$CD.CB$ = power of the point = $CA^2 - EA^2$

$AC^2 = 16*36 + (BC^2 - AC^2)$

$2AC^2 = 16*36 + 36^2$ gives $AC = 6\sqrt{26}$

Ans: (B)

Q.35



If AC = 2r, then DC = 2rcosθ

AD=2rsinθ, AB=2rtanθ & BC=2rsecθ

⇨ $AC^2 = AB^2 . AD^2/(AB^2 - AD^2)$

Ans: (D)

Q.36

A moves on the circle $x^2 + y^2 = 9$

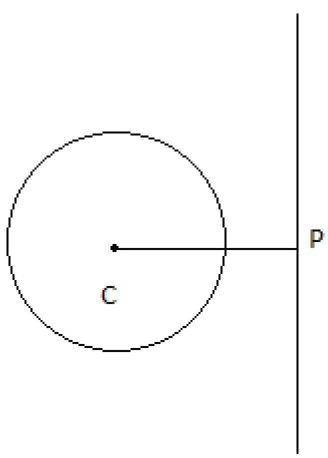
Let A = (3cosθ,3sinθ)

Then the centroid G = (cosθ,sinθ)

⇨ $X^2 + y^2 =1$

Ans: (A)

Q.37



$x^2 + y^2 = 6x -8y$

Center = (3,-4)

Perpendicular distance from center to line

d = CP = |-9 +16-25|/5

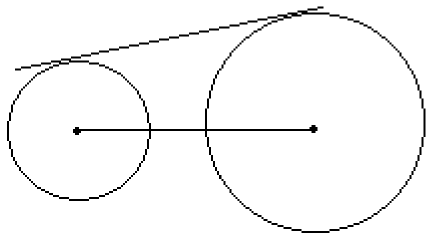
=18/5

Radius = 5

Shortest distance = |18/5 -5| =7/5

Ans: (A)

Q.38



Centers $C_1 = (10,0)$ & $C_2 = (-15,0)$

$R_1=6$ & $R_2=9$

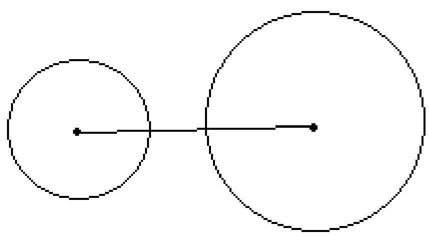
$d = C_1C_2 = 25$

$d > R_1+R_2$

⇨ Circles are non intersecting

Length of the direct common tangent = $(d^2 - (R_1 - R_2)^2)^{1/2} = \sqrt{616}$
PQ = $(d^2 - (R_1 + R_2)^2)^{1/2} = 20$
Ans: (c)

Q.39



$C_1 = (0,1)$ & $C_2 = (4,9)$, $R_1=2$ & $R_2=2$

$C_1C_2 > R_1+R_2$

Center of the smallest circle is mid point of $C_1C_2=(2,5)$

Ans: (D)

Q.40 Equation of the family of circles is
$(x-2)^2 + (y-5)^2 + k(2x-y+1) = 0$
Center = $[-(k-2), (k+10)/2]$
Lies on $x-2y = 4$
$K=-6$
$\therefore$ radius = $3\sqrt{5}$
Ans : (A)

–

23. (AB)

$x^2+y^2-4y+3+\lambda x=0$

$\Rightarrow$ common points are $(0,1),(0,3)$

$\therefore$ Equation of circle $S=0$ is $x^2+(y-1)(y-3)=0$

i.e. $x^2+y^2-4y+3=0$

$\therefore$ Area of $S=0$ is π and radius of director circle $=\sqrt{2}(1)=\sqrt{2}$ units

24. (AB)

$A(\alpha)=(a\cos\alpha, a\sin\alpha)$

$B(\beta)=(a\cos\beta, a\sin\beta)$

$C(\gamma)=(a\cos\gamma, a\sin\gamma)$

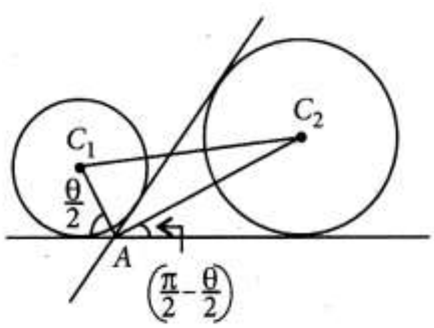
ΔABC is equilateral $S=G$

$(0,0)=\left(\frac{a(\cos\alpha+\cos\beta+\cos\gamma)}{3}\right), a\left(\frac{\sin\alpha+\sin\beta+\sin\gamma}{3}\right)$

$\Rightarrow \cos\alpha+\cos\beta+\cos\gamma=0$, $\sin\alpha+\sin\beta+\sin\gamma=0$

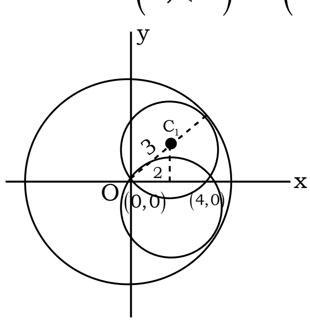
25. (AB)

From the figure it is clear that $\angle C_1AC_2=90^\circ$. Similarly $\angle C_1BC_2=\angle C_1CC_2=\angle C_1DC_2=90^\circ$. Thus ABCD is a cyclic quadrilateral with C_1C_2 as diameter ABCD is clearly not a square



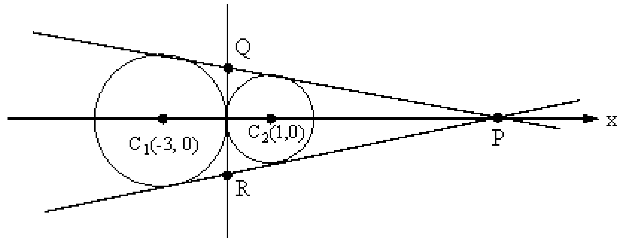
26. (ACD)

centre $=(2,\sqrt{5})$ or $(2,-\sqrt{5})$



$OC_1=\sqrt{9-4}=\sqrt{5}$

27. (ABCD)



$C_1 = (-3, 0)\ \ r_1 = 3$

$C_2 = (1, 0)\ \ r_2 = 1$

ΔPQR is equilateral Δ^{le}

$\therefore S = I = (1, 0)$

In radius of $\Delta PQR = r_1 = 1$

Circum radius of $\Delta PQR = 2$

28. (BC)

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be required circle.
(0, 1) lies on it.
Using conditions for orthogonality
We get g = 7, f = -1, c = +1

29. (AC)

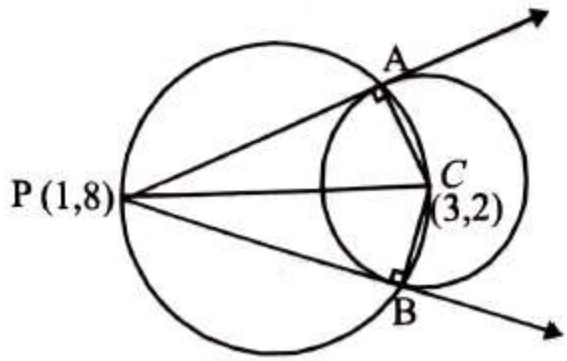
Δ APB must be isosceles for maximum area and P lies on perpendicular bisector of AB

30. (ABC)

Only One Option Correct

1. (b)

19. **(b)** Given that tangents PA and PB are drawn from the point P (1, 3) to circle $x^2 + y^2 - 6x - 4y - 11 = 0$ with centre C (3, 2).



Clearly the circumcircle of ΔPAB will pass through C and as $\angle A = 90°$, PC must be a diameter of the circle.

$\therefore$ Equation of required circle is

$(x-1)(x-3)+(y-8)(y-2)=0$

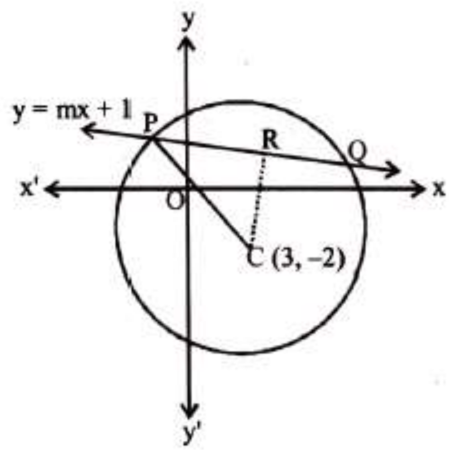
$\Rightarrow \quad x^2 + y^2 - 4x - 10y + 19 = 0$

2. (a)

13. **(a)** Given : Circle $(x-3)^2+(y+2)^2=25$, with centre $C(3,-2)$ and radius 5 is intersected by a line y = mx + 1 at P & Q such that co-ordinates of mid point R of PQ is $-\frac{3}{5}$.

Since x-coordinates of point R is $-\frac{3}{5}$ and point R lies on the line y = mx + 1, therefore y-coordinate of R will be $\frac{3m}{5}+1$.

$\therefore\ R\left(-\frac{3}{5}, \frac{-3m}{5}+1\right)$

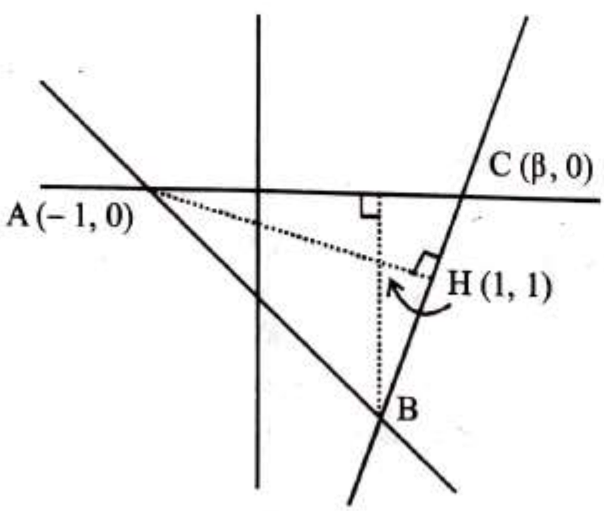


Since R is the mid point of PQ, therefore $CR \perp PQ$

$$\Rightarrow \frac{\frac{-3m}{5}+1+2}{-\frac{3}{5}-3}\times m=-1$$

$$\Rightarrow m^2-5m+6=0 \Rightarrow m=2,3$$

3. (b)



$(1, -2) = (\alpha, -\alpha - 1)$

$\Rightarrow \alpha = 1$

It is clear from question that one of the vertex of triangle is intersection of x-axis and

$x + y + 1 = 0 \Rightarrow A(-1, 0)$

Let vertex B be $(\alpha, -\alpha - 1)$

Line AC $\perp$ BH so, $m_{AC} \cdot m_{BH} = -1$

$$\Rightarrow \; O = -\frac{(1-\alpha)}{\alpha+2} \Rightarrow \alpha = 1 \Rightarrow B(1, -2)$$

Let vertex C be $(\beta, 0)$

Line AH $\perp$ BC

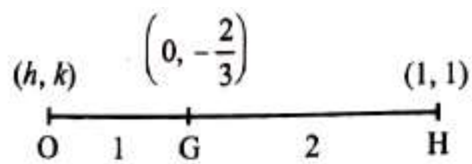
$\therefore \; m_{AH} \cdot m_{BC} = -1$

$$\Rightarrow \; \frac{1}{2} \cdot \frac{2}{\beta - 1} = -1 \Rightarrow \; \beta = 0$$

Centroid of ΔABC is $\left(0, -\frac{2}{3}\right)$

We know that G (centroid) divides line joining circumcentre (O) and orthocentre (H) in the ratio 1 : 2.

$\Rightarrow$



$$2h + 1 = 0 \Rightarrow \frac{2k+1}{3} = -\frac{2}{3}$$

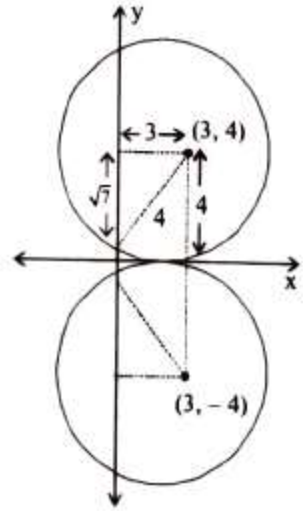
$$\Rightarrow h = -\frac{1}{2} \Rightarrow k = -\frac{3}{2} \Rightarrow \text{Circumcentre is } \left(-\frac{1}{2}, -\frac{3}{2}\right).$$

Equation of circum circle is (passing through C (0, 0)) is $x^2 + y^2 + x + 3y = 0$

One or More than One Option Correct

1. (A, C)

79. **(a, c)** Here, there are two possibilites for the given circle as shown in the figure.



$\therefore$ The equations of circles can be

$$(x-3)^2+(y-4)^2=4^2$$

or $(x-3)^2+(y+4)^2=4^2$

$\Rightarrow$ $x^2+y^2-6x-8y+9=0$

or $x^2+y^2-6x+8y+9=0$

2. (B, C)

78. **(b, c)** Let the equation of circles be

$x^2+y^2+2gx+2fy+c=0$ ...(i)

It passes through (0, 1)

$\therefore 1+2f+c=0$ ...(ii)

Since circle (i) is orthogonal to circle $(x-1)^2+y^2=16$

i.e. $x^2+y^2-2x-15=0$

and $x^2+y^2-1=0$

$\therefore$ $2g\times(-1)+2f\times 0=c-15$

$\Rightarrow$ $2g+c-15=0$ ...(iii)

and $2g\times 0+2f\times 0=c-1$

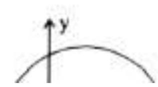
$\Rightarrow$ $c=1$ ...(iv)

Solving (ii), (iii) and (iv), we get

$$c=1, g=7, f=-1$$

$\therefore$ Required circle is $x^2+y^2+14x-2y+1=0$, with centre $(-7, 1)$ and radius = 7

$\therefore$ (b) and (c) are correct options.



3. (A, C)

77. **(a, c)** Given : A circle : $x^2+y^2=1$

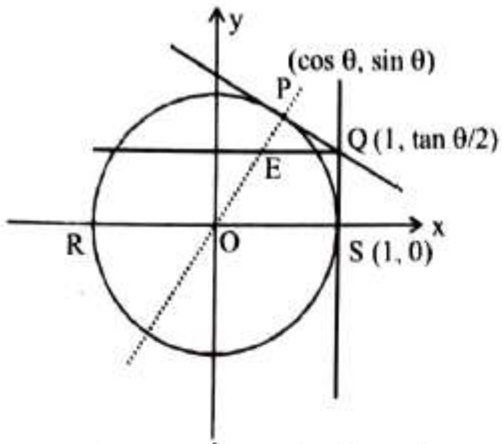
Let coordinates of P = (cos θ, sin θ)

$\therefore$ Equation of tangent at P(cos θ, sin θ) is

$x\cos\theta+y\sin\theta=1$ ...(i)

Equation of normal at P is $y = x\tan\theta$ ...(ii)
Now, equation of tangent at S is $x = 1$ ... (iii)
On solving (i) and (iii), we get the coordinates of Q as

$$\left(1, \frac{1-\cos\theta}{\sin\theta}\right) = \left(1, \tan\frac{\theta}{2}\right)$$



∴ Equation of line through Q and parallel to RS is
$y = \tan\frac{\theta}{2}$ ... (iv)
Intersection point E of normal (ii) and line (iv) can be find out by solving (ii) and (iv).
Now from (ii) and (iv),

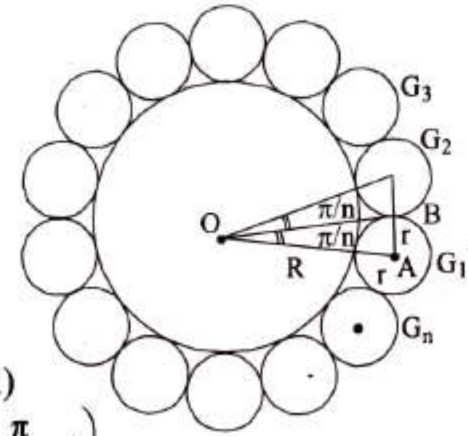
$$\tan\frac{\theta}{2} = x\tan\theta \Rightarrow x = \frac{1-\tan^2\theta/2}{2}$$

∴ Locus of E is $x = \frac{1-y^2}{2} \Rightarrow y^2 = 1-2x$

It is satisfied by the points $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{3}, \frac{-1}{\sqrt{3}}\right)$.

4. (C, D)

40. **(c, d)** Refer to diagram,
In ΔAOB



$$\sin\left(\frac{\pi}{n}\right)=\frac{r}{R+r}$$

$$\Rightarrow \operatorname{cosec}\left(\frac{\pi}{n}\right)=\frac{R}{r}+1$$

$$\Rightarrow R=r\left[\operatorname{cosec}\left(\frac{\pi}{n}\right)-1\right]$$

If $n=4$ then $R=r(\sqrt{2}-1)$

If $n=5$ then $R=r\left(\operatorname{cosec}\frac{\pi}{5}-1\right)$

$$\because \operatorname{cosec}\frac{\pi}{5}<\operatorname{cosec}\frac{\pi}{6}$$

$$\left(\operatorname{cosec}\frac{\pi}{5}-1\right)<2-1=1 \quad \therefore R<r$$

If $n=8$ then $R=r\left(\operatorname{cosec}\frac{\pi}{8}-1\right)$ $\therefore \operatorname{cosec}\frac{\pi}{8}>\operatorname{cosec}\frac{\pi}{4}$

$$\left(\operatorname{cosec}\frac{\pi}{8}-1\right)>\sqrt{2}-1 \Rightarrow R>r\left(\sqrt{2}-1\right)$$

If $n=12$, then $R=r\left(\operatorname{cosec}\frac{\pi}{12}-1\right)$

$R=r(\sqrt{2}(\sqrt{3}+1)-1)$; $R<\sqrt{2}(\sqrt{3}+1)r$

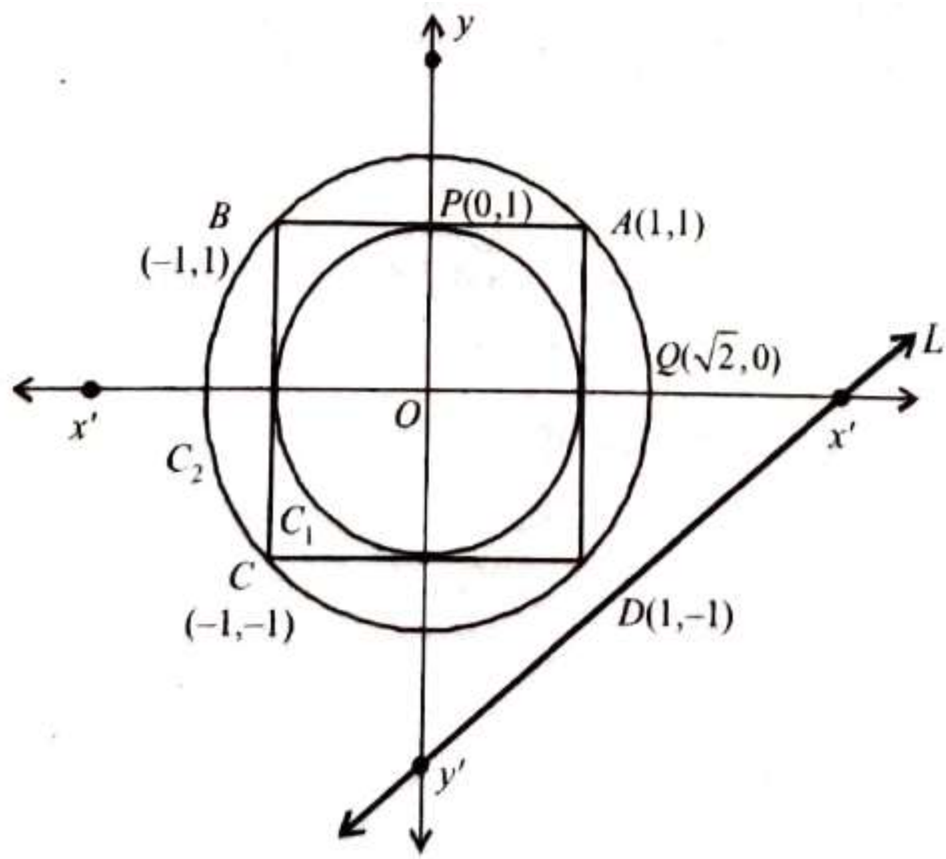
Comprehensions Type Questions

Passage - 1

1. (A)

90. **(a)** According to the given question, we can assume the square $ABCD$ with its vertices $A(1, 1)$, $B(-1, 1)$, $C(-1, -1)$, $D(1, -1)$.

P be the point $(0, 1)$ and Q be the point $(\sqrt{2}, 0)$.

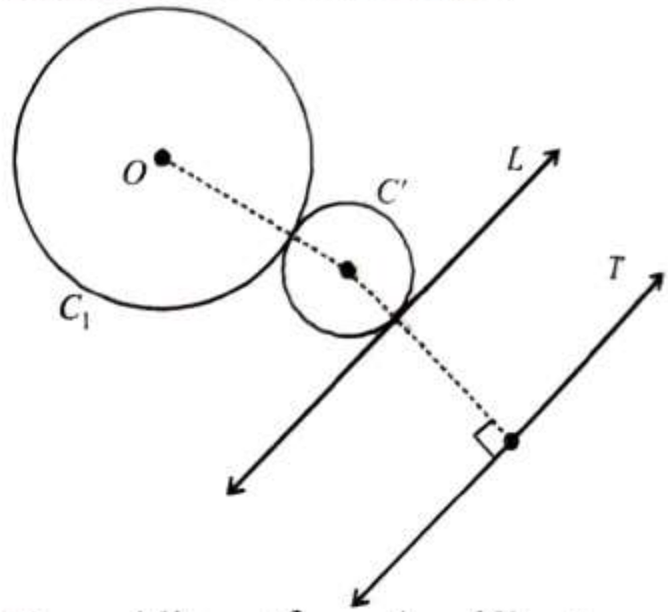


Then, $\frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$

$= \frac{1+1+5+5}{2[(\sqrt{2}-1)^2+1]+2[(\sqrt{2}+1)^2+1]} = \frac{12}{16} = 0.75$

2. (C)

91. **(c)** Let C' be the said circle touching C_1 and L, so that C_1 and C' are on the same side of L. Let us draw a line T parallel to L at a distance equal to the radius of circle C_1, on opposite side of L.
Then the centre of C' is equidistant from the centre of C_1 and from line T.
$\Rightarrow$ Locus of centre of C' is a parabola.



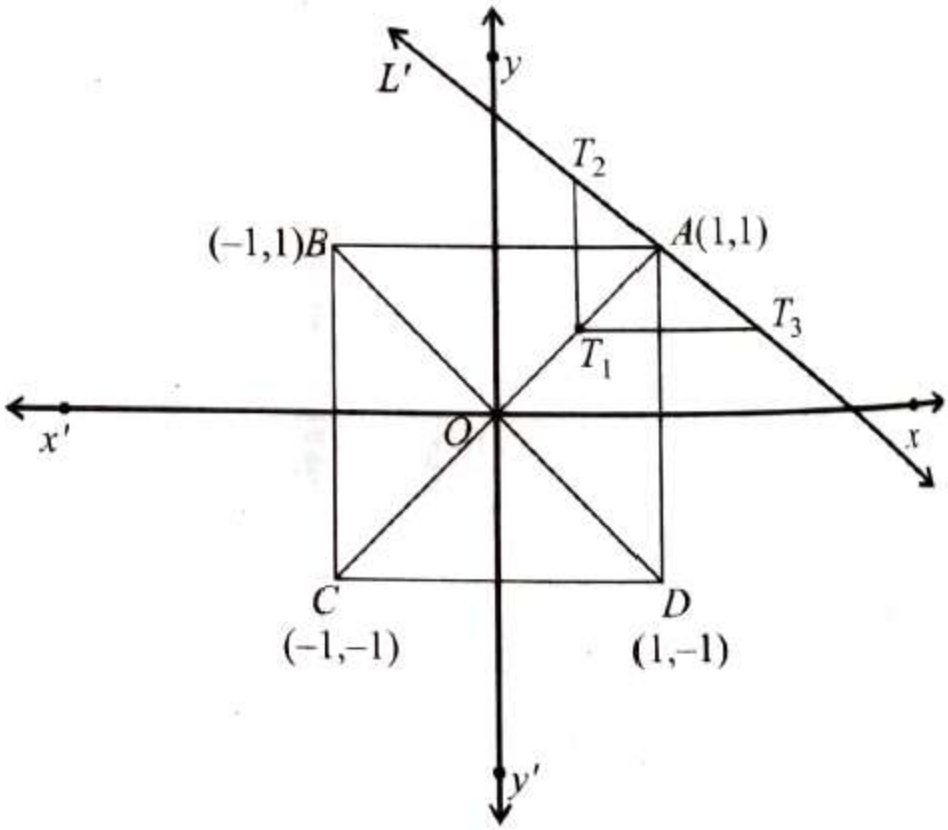
3. (C)

92. **(c)** Since S is equidistant form A and line BD, it traces a parabola. Clearly, AC is the axis, A (1, 1) is the focus and $T_1\left(\frac{1}{2},\frac{1}{2}\right)$ is the vertex of the parabola.

$$AT_1 = \frac{1}{\sqrt{2}}.$$

$T_2 T_3$ = latus rectum of parabola

$$= 4 \times \frac{1}{\sqrt{2}} = 2\sqrt{2}$$

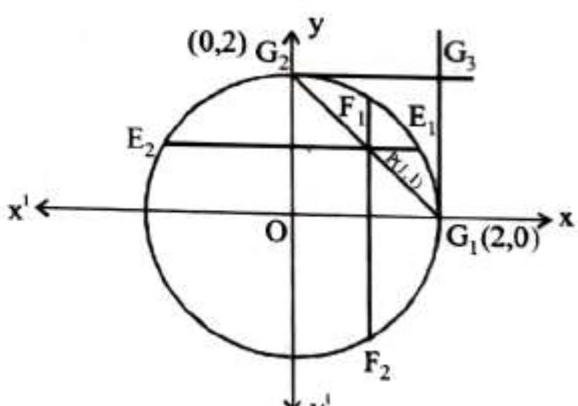


$\therefore \quad \text{Area}(\Delta T_1T_2T_3) = \frac{1}{2} \times \frac{1}{\sqrt{2}} \times 2\sqrt{2} = 1$ sq. units

Passage - 2

4. (A)

86. (a)



Equation of $E_1 E_2$ is $y = 1$
Equation of $F_1 F_2$ is $x = 1$
Equation of $G_1 G_2$ is $x + y = 2$
By symmetry, tangents at E_1 and E_2 will meet on y-axis and tangents at F_1 and F_2 will meet on x-axis

$E_1 \equiv (\sqrt{3}, 1)$ and $F_1 \equiv (1, \sqrt{3})$

Equation of tangent at E_1 is $\sqrt{3}x + y = 4$

Equation of tangent at F_1 is $x + \sqrt{3}y = 4$

$\therefore$ Points $E_3(0, 4)$ and $F_3(4, 0)$

Tangents at G_1 and G_2 are $x = 2$ and $y = 2$ respectively intersecting each other at $G_3(2, 2)$.

Clearly E_3, F_3 and G_3 lie on the curve $x + y = 4$.

5. (D)

87. **(d)** Let point P be $(2\cos\theta, 2\sin\theta)$

Tangent at P is $x\cos\theta + y\sin\theta = 2$

$\therefore \quad M\left(\frac{2}{\cos\theta}, 0\right)$ and $N\left(0, \frac{2}{\sin\theta}\right)$

$\therefore \quad$ Mid point of MN $= \left(\frac{1}{\cos\theta}, \frac{1}{\sin\theta}\right)$

For locus of mid point (x, y) of MN,

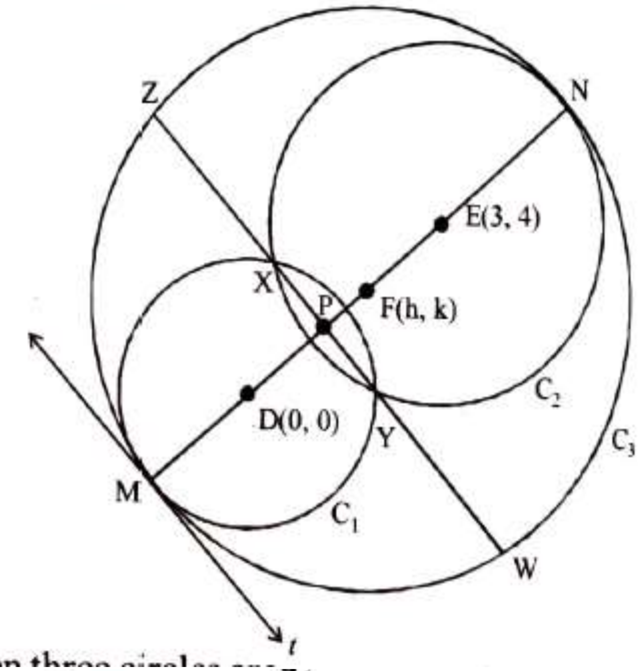
$$x = \frac{1}{\cos\theta}, \quad y = \frac{1}{\sin\theta}$$

$$\Rightarrow \quad \frac{1}{x^2} + \frac{1}{y^2} = 1 \Rightarrow x^2 + y^2 = x^2y^2$$

Passage - 3

6. (D)
7. (A)

For Questions 82 and 83



Given three circles are

$C_1 : x^2 + y^2 = 9$

$C_2 : (x-3)^2 + (y-4)^2 = 16$

$C_3 : (x-h)^2 + (y-k)^2 = r^2$

Centres of circles C_1, C_2, C_3 are D(0, 0), E(3, 4), F(h, k) respectively

and radii of circles $C_1 : C_2 : C_3$ are 3, 4, r respectively.

Equation of DE : $y = \frac{4}{3}x$

Centres of circles C_1, C_2, C_3 are collinear $\Rightarrow$ $F\left(h, \frac{4}{3}h\right)$

$MN = MD + DE + EN = 3 + 5 + 4 = 12 \Rightarrow r = 6$

$\therefore \quad DE = 6 - 3 = 3$

$$\Rightarrow \quad h^2 + \frac{16}{9}h^2 = 9 \Rightarrow h^2 = \frac{81}{25}$$

$\Rightarrow h = \dfrac{9}{5}$ taking h +ve, as lies between D and E

$\therefore \quad F\left(\dfrac{9}{5}, \dfrac{12}{5}\right)$

$\therefore \quad 2h + k = \dfrac{18}{5} + \dfrac{12}{5} = \dfrac{30}{5} = 6$

$\therefore \quad$ (A) – (p)

DE is common chord of circles C_1 and C_2

$\therefore \quad$ Equation of XY : $S_1 - S_2 = 0$

$\Rightarrow 6x + 8y - 18 = 0 \Rightarrow 3x + 4y - 9 = 0$

DE is common chord of circles C_1 and C_2

$\therefore \quad$ Equation of XY : $S_1 - S_2 = 0$

$\Rightarrow 6x + 8y - 18 = 0 \Rightarrow 3x + 4y - 9 = 0$

Length of $\perp$ from D to XY $= \dfrac{9}{5} = DP$

Also $DX = 3, \ \therefore \ PX = \sqrt{9 - \dfrac{81}{25}} = \sqrt{\dfrac{225-81}{25}} = \dfrac{12}{5}$

$\therefore \quad XY = 2PX = \dfrac{24}{5}$

ZW is chord of C_3.

$FP = MF - MP = 6 - \left(3 + \dfrac{9}{5}\right) = 6 - \dfrac{24}{5} = \dfrac{6}{5}$

$\therefore \quad ZP = \sqrt{6^2 - \left(\dfrac{6}{5}\right)^2} = \dfrac{6\sqrt{24}}{5} = \dfrac{12\sqrt{6}}{5} \ \therefore ZW = \dfrac{24\sqrt{6}}{5}$

Hence, $\dfrac{\text{Length of ZW}}{\text{Length of XY}} = \dfrac{24\sqrt{6}/5}{24/5} = \sqrt{6}$

$\therefore$ (B) – (q)

$$\text{Area of } \Delta MZN = \frac{1}{2} MN \times ZP = \frac{1}{2} \times 12 \times \frac{12\sqrt{6}}{5} = \frac{72\sqrt{6}}{5}$$

$$\text{Area of } \Delta ZMW = \frac{1}{2} \times ZW \times MP$$

$$= \frac{1}{2} \times \frac{24\sqrt{6}}{5} \times \frac{24}{5} = \frac{288\sqrt{6}}{25}$$

$$\therefore \quad \frac{\text{Area of } \Delta MZN}{\text{Area of } \Delta ZMW} = \frac{72\sqrt{6}}{5} \times \frac{25}{288\sqrt{6}} = \frac{5}{4}$$

$\therefore$ (C) – (r)

Now common tangent of C_1 and C_3 is $S_1 - S_3 = 0$

$\Rightarrow \quad 2hx + 2ky - h^2 - k^2 = 9 - r^2$

or $\quad \dfrac{18}{5}x + \dfrac{24}{5}y - \dfrac{81}{25} - \dfrac{144}{25} = 9 - 36$

$\Rightarrow \quad 3x + 4y + 15 = 0$

It is tangent to $x^2 = 8\alpha y$

Putting value of y from common tangent in parabola, we get

$$x^2 = -8\alpha\left(\frac{3x+15}{4}\right) \Rightarrow x^2 + 6\alpha x + 30\alpha = 0$$

It should have equal roots

$\therefore \quad 36\alpha^2 - 4 \times 30\alpha = 0 \Rightarrow \alpha = \dfrac{10}{3} \quad \therefore$ (D) – (u)

Thus (B) – (q) is the only correct combination
and (D) – (s) is the only incorrect combination.

Passage - 4

8. (D)

84. **(d)** $\because a_n = \frac{1}{2^{n-1}}$

$$S_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^{n-1}} = 2\left(1 - \frac{1}{2^n}\right) = 2 - \frac{1}{2^{n-1}}$$

For circles C_n to inside M

$$S_{n-1} + a_n < \frac{1025}{513} \Rightarrow 2 - \frac{1}{2^{n-2}} + \frac{1}{2^{n-1}} < \frac{1025}{513}$$

$$\Rightarrow \quad 1 - \frac{1}{2^n} < \frac{1025}{1026} = 1 - \frac{1}{1026}$$

$$\Rightarrow \quad 2^n < 2026 \Rightarrow n \le 10 \Rightarrow k = 10$$

Also $l = 5$

$3k + 2l = 30 + 10 = 40$

9. (B)

85. **(b)** $\because \quad r = \frac{(2^{199} - 1)\sqrt{2}}{2^{198}}$

Now, $\sqrt{2}S_{n-1} + a_n < \frac{2^{199} - 1}{2^{198}}\sqrt{2}$

$$\Rightarrow \quad 2\sqrt{2}\left(1 - \frac{1}{2^{n-1}}\right) + \frac{1}{2^{n-1}} < \frac{2^{199} - 1}{2^{198}}$$

$$\Rightarrow \quad \frac{2\sqrt{2} - 1}{2.2^{n-2}} > \frac{\sqrt{2}}{2^{198}}$$

$$\Rightarrow \quad 2^{n-2} < \left(2 - \frac{1}{\sqrt{2}}\right)2^{197} \therefore n \le 199 \Rightarrow n = 199$$

Numerical Value Answer

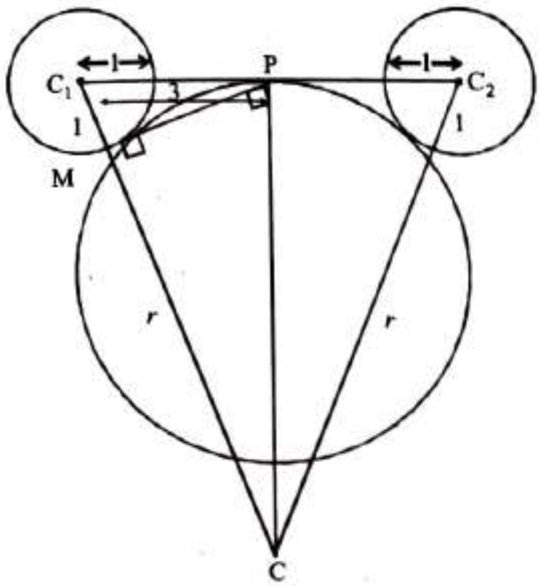
1. (8)

45. **(8)** Let r be the radius of required circle.

Clearly, in ΔC_1CC_2, $C_1C = C_2C = r + 1$

and P is mid point of C_1C_2

$\therefore \quad CP \perp C_1C_2$, Also $PM \perp CC_1$



Now $\Delta PMC_1 \sim \Delta CPC_1$ (By AA similarity)

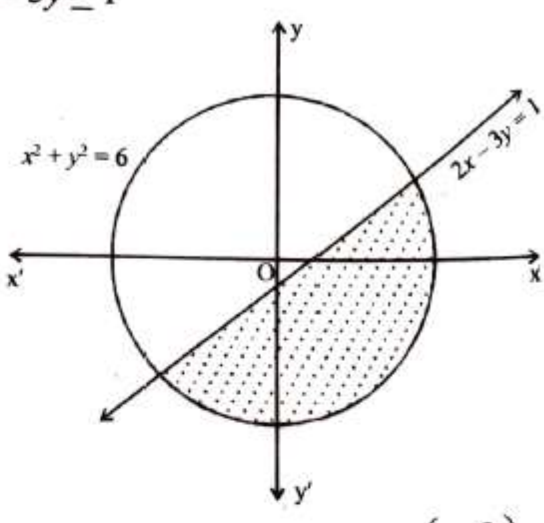
$$\therefore \quad \frac{MC_1}{PC_1} = \frac{PC_1}{CC_1} \Rightarrow \frac{1}{3} = \frac{3}{r+1} \Rightarrow r+1 = 9 \Rightarrow r = 8.$$

2. (2)

44. **(2)** The smaller region of circle is the region given by

$x^2 + y^2 \le 6$ ...(i)

and $2x - 3y \ge 1$ ...(ii)



We observe that only two points $\left(2, \frac{3}{4}\right)$ and $\left(\frac{1}{4}, -\frac{1}{4}\right)$

satisfy both the inequations (i) and (ii)

$\therefore$ 2 points in S lie inside the smaller part.

3. (2)

43. **(2)** Centre of the circle is $(-1, -2)$

Geometrically, circle will have exactly 3 common points with axes in the cases

(i) Passing through origin $\Rightarrow$ $p = 0$

(ii) Touching x-axis and intersecting y-axis at two points

i.e. $f^2 > C$ and $g^2 = C$.

i.e. $4 > -p$ and $1 = -p \Rightarrow p > -4$ and $p = -1$ $\therefore$ $p = -1$

(iii) Touching y-axis and intersecting x-axis at two points

i.e. $f^2 = c$ and $g^2 > C$

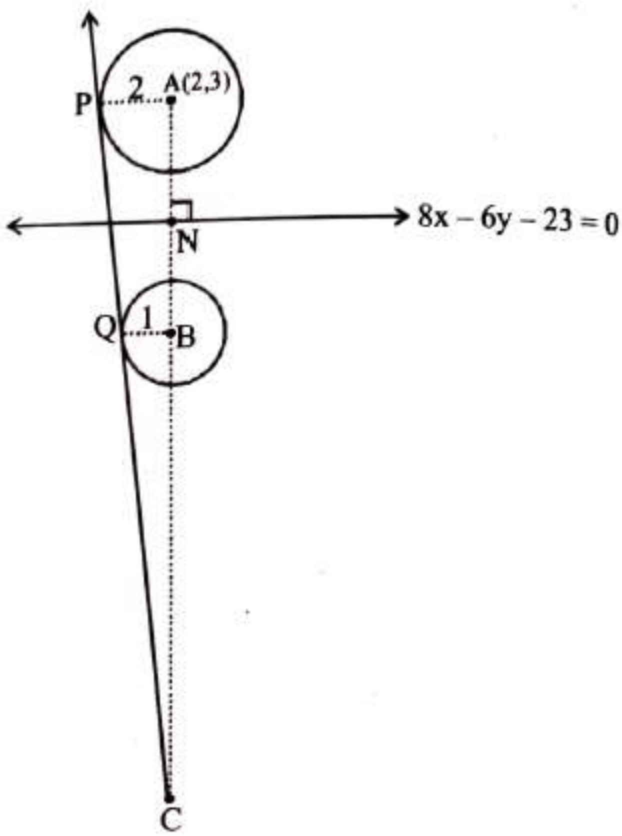
$\Rightarrow$ $4 = -p$ and $1 > -p$

$\Rightarrow$ $p = -4$ and $p > -1$, which is not possible.

$\therefore$ only two values of p are possible.

4. (10)

42. **(10)** $AN = \left|\frac{16-18-23}{\sqrt{64+36}}\right| = \frac{25}{10} = \frac{5}{2} = BN$

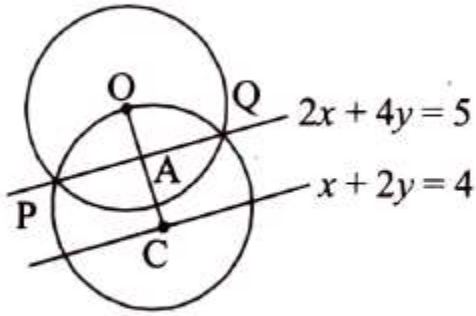


$\because$ $\Delta CPA \sim \Delta CQB$ (By AA similarity)

$\therefore$ $\frac{CA}{CB} = \frac{PA}{QB}$ $\Rightarrow$ $\frac{CA}{CA-5} = \frac{2}{1}$

$\Rightarrow$ $CA = 2CA - 10$ $\Rightarrow$ $CA = 10$

5. (2)

41. (2) 

$\because$ Centre of circle is O (0, 0).

OA = perpendicular distance from point O to line

$2x + 4y = 5 = \left|\frac{0+0-5}{\sqrt{4+16}}\right| = \frac{\sqrt{5}}{2}$

OC = perpendicular distance from point O to line $x + 2y = 4$

$= \left|\frac{0+0-4}{\sqrt{1+4}}\right| = \frac{4}{\sqrt{5}}$

$\therefore CA = OC - OA = \frac{3}{2\sqrt{5}} \;\; \because CQ = OC = \frac{4}{\sqrt{5}}$ (radius)

Now $AQ^2 = CQ^2 - CA^2$ $(\because AC \perp PQ)$ $= \frac{16}{5} - \frac{9}{20} = \frac{11}{4}$

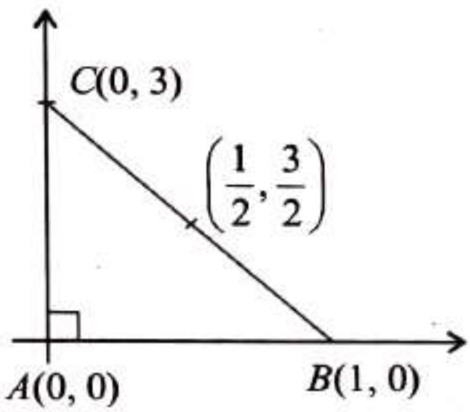
$\therefore OQ = r = \sqrt{OA^2 + AQ^2}$

$\Rightarrow r = \sqrt{\frac{5}{4} + \frac{11}{4}} \Rightarrow r = \sqrt{4} = 2$

6. (0.84)

46. **(0.84)** We have AB = 1, AC = 3 and $\angle BAC = \frac{\pi}{2}$

Let A be the origin B on x-axis, C on y-axis as shown below



$\therefore$ Equation of circumcircle is

$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=\left(\sqrt{(1-0)^2+(0-3)^2}\div 2\right)^2=\frac{5}{2} \quad \text{...(i)}$$

[$\because r$ = Hypotenuse $\div$ 2]

Required circle touches AB and AC, have radius r

$\therefore$ Equation be $(x-r)^2+(y-r)^2=r^2$ ...(ii)

If circle in equation (ii) touches circumcircle internally, we have

$d_{c_1c_2}=|r_1-r_2|$

$$\Rightarrow \left(\frac{1}{2}-r\right)^2+\left(\frac{3}{2}-r\right)^2=\left(\left|\sqrt{\frac{5}{2}}-r\right|\right)^2$$

$$\Rightarrow \frac{1}{4}+r^2-r+\frac{9}{4}+r^2-3r=\left(\sqrt{\frac{5}{2}}-r\right)^2 \text{ or } \left(r-\sqrt{\frac{5}{2}}\right)^2$$

$$\Rightarrow 2r^2-4r+\frac{5}{2}=\frac{5}{2}+r^2-\sqrt{10}\,r$$

$$\Rightarrow r=0 \text{ or } 4-\sqrt{10}$$

$\Rightarrow$ r = 0.837 = 0.84 (on rounding off)

Subjective

1.

110. Given circle:

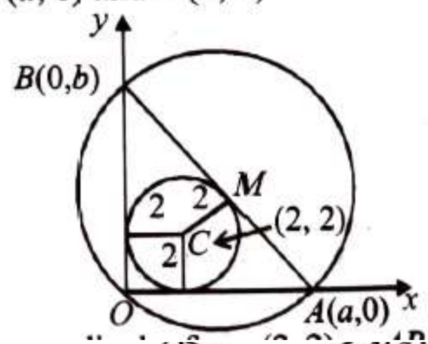
$x^2+y^2-4x-4y+4=0.$

$\Rightarrow \quad (x-2)^2+(y-2)^2=4,$

which has centre $C(2, 2)$ and radius 2.

Let the equation of third side AB of ΔOAB is $\frac{x}{a}+\frac{y}{b}=1$

such that $A(a, 0)$ and $B(0, b)$



Length of perpendicular form (2, 2) on AB = radius = $CM=2$

$$\therefore \quad \frac{\left|\frac{2}{a}+\frac{2}{b}-1\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=2$$

Since (2, 2) and origin lie on same side of AB

$$\therefore \quad \frac{-\left(\frac{2}{a}+\frac{2}{b}-1\right)}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=2 \Rightarrow \frac{2}{a}+\frac{2}{b}-1=-2\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} \quad \ldots(i)$$

$\because \quad \angle AOB=\pi/2.$

Therefore, AB is the diameter of the circle passing through the vertices of the ΔOAB. Hence centre of the circle is the mid-point $\left(\frac{a}{2}, \frac{b}{2}\right)$ of the circle.

Let centre be $(h, k) \equiv \left(\frac{a}{2}, \frac{b}{2}\right)$

then $a=2h$, $b=2k$.
On putting the values of a and b in (i), we get

$$\frac{2}{2h}+\frac{2}{2k}-1=-2\sqrt{\frac{1}{4h^2}+\frac{1}{4k^2}}$$

$$\Rightarrow \quad \frac{1}{h}+\frac{1}{k}-1=-\sqrt{\frac{1}{h^2}+\frac{1}{k^2}}$$

$$\Rightarrow h+k-hk+\sqrt{h^2+k^2}=0$$

$\therefore$ Locus of $M(h, k)$ is,

$$x+y-xy+\sqrt{x^2+y^2}=0 \qquad \ldots\text{(ii)}$$

Comparing it with given equation of locus of circumcentre of the triangle i.e.

$$x+y-xy+k\sqrt{x^2+y^2}=0 \qquad \ldots\text{(iii)}$$

We get, $k=1$

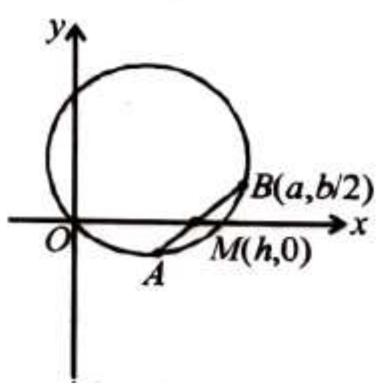
2.

106. Given : A circle

$$2x(x-a)+y(2y-b)=0 \ (a, b\neq 0)$$

$$\Rightarrow \ 2x^2+2y^2-2ax-by=0 \qquad \text{....(i)}$$

Let us consider the chord of this circle which passes through the point $\left(a, \frac{b}{2}\right)$ and whose mid point lies on x-axis.



Let $(h, 0)$ be the mid point of the chord, then equation of chord can be obtained by $T = S_1$

i.e., $2xh + 2y.0 - a(x+h) - \frac{b}{2}(y+0) = 2h^2 - 2ah$

$\Rightarrow \quad (2h-a)x - \frac{b}{2}y + ah - 2h^2 = 0$

This chord passes through $\left(a, \frac{b}{2}\right)$,

$\therefore \quad (2h-a)a - \frac{b}{2}.\frac{b}{2} + ah - 2h^2 = 0$

$\Rightarrow \quad 8h^2 - 12ah + (4a^2 + b^2) = 0$

According to the question, two such chords are there, so we should have two real and distinct values of h from the above quadratic in h.

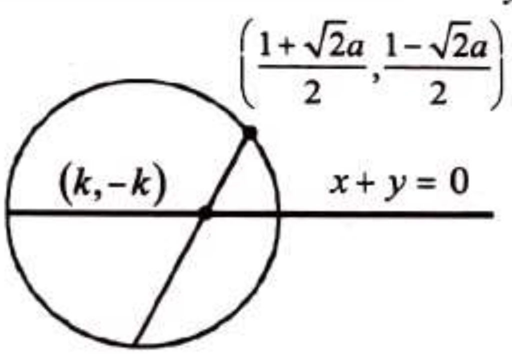
$\therefore \quad D > 0$

$\Rightarrow \quad (12a)^2 - 4 \times 8 \times (4a^2 + b^2) > 0 \Rightarrow a^2 > 2b^2$

3.

104. Let the given point be $(p, \overline{p}) = \left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$, then

the equation of the circle becomes $x^2 + y^2 - px - \overline{p}y = 0$



Since the chord is bisected by the line $x + y = 0$, its mid-point can be chosen as $(k, -k)$. Hence the equation of the chord represented by $T = S_1$ is

$$kx - ky - \frac{p}{2}(x + k) - \frac{\bar{p}}{2}(y - k) = k^2 + k^2 - pk + \bar{p}k$$

Since, it passes through $A(p, \bar{p})$,

$$\therefore \quad kp - k\bar{p} - \frac{p}{2}(p + k) - \frac{\bar{p}}{2}(\bar{p} - k) = 2k^2 - pk + \bar{p}k$$

or $\quad 3k(p - \bar{p}) = 4k^2 + (p^2 + \bar{p}^2) \qquad \ldots(i)$

Put $p - \bar{p} = a\sqrt{2}$ and $p^2 + \bar{p}^2 = 2.\frac{(1+2a^2)}{4} = \frac{1+2a^2}{2}$ $\qquad \ldots(ii)$

Hence, from (i) using (ii), we get

$$4k^2 - 3\sqrt{2}\,ak + \frac{1}{2}(1 + 2a^2) = 0 \qquad \ldots(iii)$$

Since, there are two chords which are bisected by $x + y = 0$, we must have two real values of k from (iii)

$\therefore \quad 18a^2 - 8(1 + 2a^2) > 0$

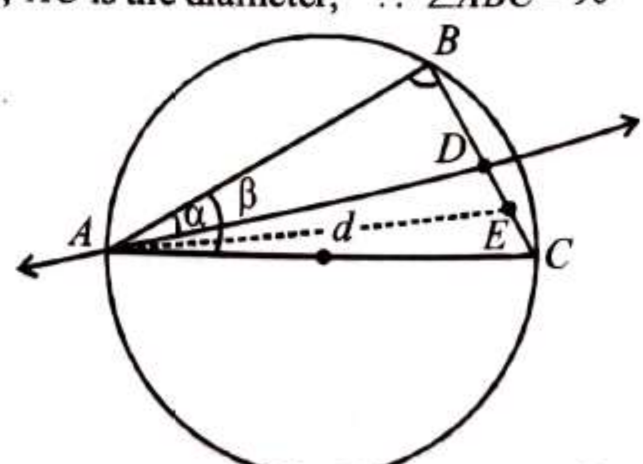
$\Rightarrow \quad a^2 - 4 > 0 \Rightarrow (a + 2)(a - 2) > 0 \Rightarrow a < -2$ or > 2

$\therefore \quad a \in (-\infty, -2) \cup (2, \infty)$

4.

103. Let r be the radius of circle, then $AC = 2r$

Since, AC is the diameter, $\therefore \angle ABC = 90°$



$\therefore \quad$ In ΔABC, $BC = 2r\sin\beta$, $AB = 2r\cos\beta$

In right angled ΔABC,

$BD = AB \tan\alpha = 2r\cos\beta\tan\alpha$

$AD = AB\sec\alpha = 2r\cos\beta\sec\alpha$

$\therefore \quad DC = BC - BD = 2r\sin\beta - 2r\cos\beta\tan\alpha$

Since E is the mid point of DC,

$$\therefore \quad DE = \frac{DC}{2} = \frac{2r\sin\beta - 2r\cos\beta\tan\alpha}{2}$$

$\Rightarrow \quad DE = r\sin\beta - r\cos\beta\tan\alpha$

Now in ΔADC, AE is the median.

$\therefore \quad 2(AE^2 + DE^2) = AD^2 + AC^2$

$\Rightarrow \quad 2[d^2 + r^2(\sin\beta - \cos\beta\tan\alpha)^2]$

$= 4r^2\cos^2\beta\sec^2\alpha + 4r^2$

$$\Rightarrow \quad r^2 = \frac{d^2\cos^2\alpha}{\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta\cos(\beta-\alpha)}$$

$\Rightarrow$ Area of circle $= \pi r^2$

$$= \frac{\pi d^2\cos^2\alpha}{\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta\cos(\beta-\alpha)}$$

5.

102. Given C is the circle with centre at $(0, \sqrt{2})$ and radius r (say), then $C \equiv x^2 + (y-\sqrt{2})^2 = r^2$

$\Rightarrow \quad (y-\sqrt{2})^2 = (r^2 - x^2) \Rightarrow y - \sqrt{2} = \pm\sqrt{r^2 - x^2}$

$\Rightarrow \quad y = \sqrt{2} \pm \sqrt{r^2 - x^2}$ ...(i)

The only rational value which y can have is 0. Suppose the possible value of x for which y is 0 is x_1. Certainly $-x_1$ will also give the value of y as 0 (from (i)). Thus, at the most, there are two rational points which satisfy the equation of C.

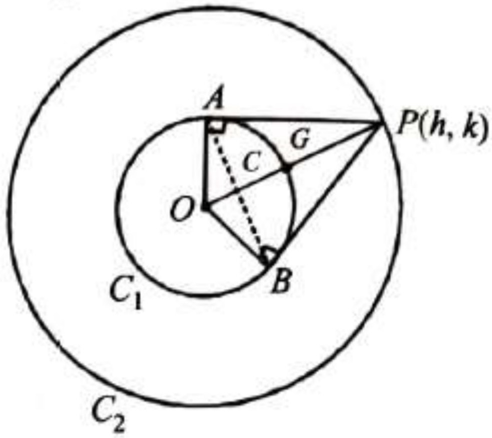
6.

101. Let $P(h, k)$ be on C_2

$\therefore \quad h^2 + k^2 = 4r^2$ ...(i)

Chord of contact of P w.r.t. C_1 is $hx + ky = r^2$

It intersects C_1, $x^2 + y^2 = a^2$ in A and B.



Eliminating y, we get

$$x^2 + \left(\frac{r^2 - hx}{k}\right)^2 = r^2$$

$\Rightarrow \quad x^2(h^2 + k^2) - 2r^2 hx + r^4 - r^2 k^2 = 0$

$\Rightarrow \quad x^2.4r^2 - 2r^2 hx + r^2(r^2 - k^2) = 0$

$$\therefore \quad x_1 + x_2 = \frac{2r^2 h}{4r^2} = \frac{h}{2}, y_1 + y_2 = \frac{k}{2}$$

If (x, y) be the centroid of ΔPAB, then

$$3x = x_1 + x_2 + h = \frac{h}{2} + h = \frac{3h}{2}$$

$\therefore \quad x = \frac{h}{2}$ or $h = 2x$ and similarly $k = 2y$

Putting the value of h and k in (i), we get

$4x^2 + 4y^2 = 4r^2$

$\therefore \quad$ Locus is $x^2 + y^2 = r^2$

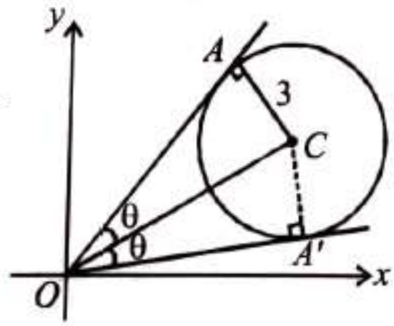
7. ( )

100. The equation $2x^2 - 3xy + y^2 = 0$ represents pair of tangents OA and OA'.

Let angle between these to tangents be 2θ.

Then, $\tan 2\theta = \dfrac{2\sqrt{\left(\frac{-3}{2}\right)^2 - 2\times 1}}{2+1}$

$\left[\because \tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a+b}\right]$



$\dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{1}{3} \Rightarrow \tan^2\theta + 6\tan\theta - 1 = 0$

$\tan\theta = \dfrac{-6 \pm \sqrt{36+4}}{2} = -3 \pm \sqrt{10}$

Since θ is acute, $\therefore$ $\tan\theta = \sqrt{10} - 3$

Now we know that line joining the point through which tangents are drawn to the centre bisects the angle between the tangents,

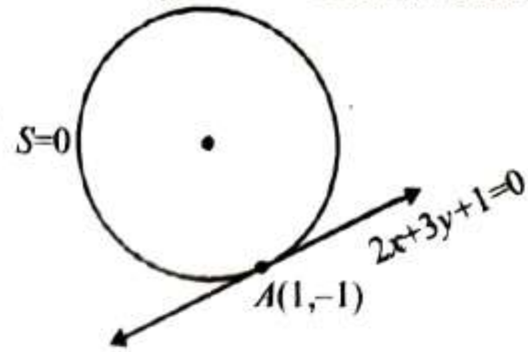
$\therefore$ $\angle AOC = \angle A'OAC = \theta$

In ΔAOC, $\tan\theta = \dfrac{3}{OA}$

$\Rightarrow$ $OA = \dfrac{3}{\sqrt{10}-3} \times \dfrac{\sqrt{10}+3}{\sqrt{10}+3}$, $\therefore$ $OA = 3(3+\sqrt{10})$.

8. ( )

97. Given : A line $2x+3y+1=0$ touches a circle $S=0$ at $(1,-1)$.



$\therefore$ Equation of the circle can be

$(x-1)^2+(y+1)^2+\lambda(2x+3y+1)=0.$

$\Rightarrow \quad x^2+y^2+2x(\lambda-1)+y(3\lambda+2)+(\lambda+2)=0$ ...(i)

But given that this circle is orthogonal to the circle, the extremities of whose diameter are $(0, 3)$ and $(-2,-1)$ i.e.

$x(x+2)+(y-3)(y+1)=0$

$\Rightarrow \quad x^2+y^2+2x-2y-3=0$ ...(ii)

On applying the condition of orthogonality for circles (i) and (ii), $2(\lambda-1).1+2\left(\frac{3\lambda+2}{2}\right).(-1)=\lambda+2+(-3)$

$(\because\ 2g_1g_2+2f_1f_2=c_1+c_2)$

$\Rightarrow \quad 2\lambda-2-3\lambda-2=\lambda-1 \Rightarrow 2\lambda=-3 \Rightarrow \lambda=\frac{-3}{2}$

Substituting this value of λ in equation (i), we get the required circle as

$x^2+y^2-5x-\frac{5}{2}y+\frac{1}{2}=0$

$\Rightarrow \quad 2x^2+2y^2-10x-5y+1=0$