PACE-IIT & MEDICAL
MUMBAI / DELHI-NCR / PUNE / NASHIK / AKOLA / GOA / JALGOAN / BOKARO / AMRAVATI / DHULE

IIT – JEE: 2024 **PART TEST - 2** **DATE: 03/12/23**

ADVANCED

ANSWER KEY

PAPER – 1

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	8.00	**19.**	6	**37.**	3.00
2.	6.00	**20.**	4	**38.**	1.00
3.	4.00	**21.**	5	**39.**	4.00
4.	641.00	**22.**	7	**40.**	1.00
5.	1570.00	**23.**	5	**41.**	9.00
6.	125.6	**24.**	8	**42.**	2.00
7.	1.6	**25.**	6	**43.**	2.00
8.	2.00	**26.**	8	**44.**	2.00
9.	A, B, C	**27.**	A, B, D	**45.**	B, C, D
10.	A, C, D	**28.**	A, B, C	**46.**	B, C
11.	A, D	**29.**	A, B, C	**47.**	A, B, D
12.	C	**30.**	A, B, C, D	**48.**	A, B, C
13.	A, B, C, D	**31.**	B, C, D	**49.**	A, B
14.	B, D	**32.**	B, C, D	**50.**	A, D
15.	C	**33.**	D	**51.**	A
16.	A	**34.**	B	**52.**	C
17.	A	**35.**	C	**53.**	D
18.	A	**36.**	B	**54.**	A

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ANSWER KEY

PAPER – 2

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	6	**19.**	6	**37.**	9
2.	1	**20.**	9	**38.**	4
3.	4	**21.**	6	**39.**	0
4.	8	**22.**	2	**40.**	5
5.	7	**23.**	7	**41.**	2
6.	2	**24.**	5	**42.**	0
7.	2	**25.**	18	**43.**	1
8.	4	**26.**	1	**44.**	1
9.	A, B	**27.**	B, C, D	**45.**	A, B, C, D
10.	A, D	**28.**	A, B, C, D	**46.**	A, B, C, D
11.	A, B, C, D	**29.**	A, B	**47.**	C
12.	A, D	**30.**	A, B, C	**48.**	A, C, D
13.	B, D	**31.**	A, B, C	**49.**	A, B, D
14.	A, C	**32.**	B, C, D	**50.**	A, D
15.	A	**33.**	A	**51.**	B
16.	A	**34.**	B	**52.**	A
17.	C	**35.**	A	**53.**	A
18.	A	**36.**	C	**54.**	A

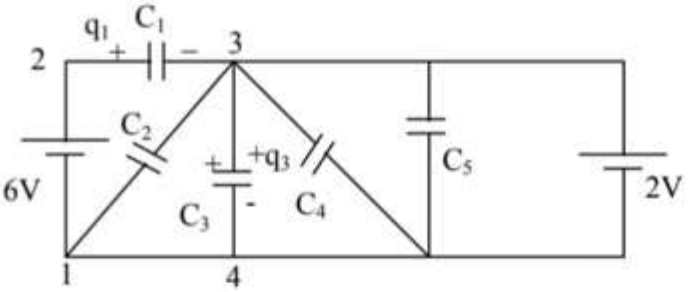
PART (A) : PHYSICS

SOLUTIONS

1. (8.00)

In the loop 1234, $+6-\frac{48}{12}-\frac{q_3}{4}=0,\ q_1=12(6-2)=48\ \mu C$

$q_3=8\ \mu C$



2. (6.00)

$u_1=-\frac{40}{3}$

$f=10$

$\frac{1}{V_1}+\frac{1\times 3}{40}=\frac{1}{10}$

$V_1=40$

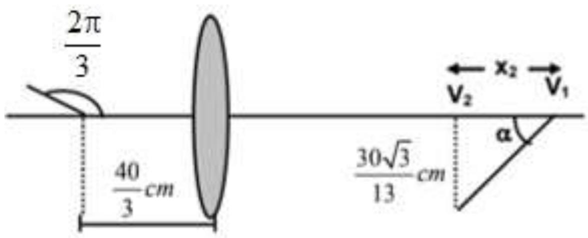
$u_2=-\frac{43}{3}$

$\frac{1}{V_2}=\frac{1}{10}-\frac{3}{43}=\frac{43-30}{430}$

$V_2=\frac{430}{13}$ $\quad\frac{2\pi}{3}$

$x_2=V_1-V_2=40-\frac{430}{13}=\frac{90}{13}$

$\tan\alpha=\frac{30\sqrt{3}}{13\times x}=\frac{30\sqrt{3}\times 13}{13\times 90}=\frac{1}{\sqrt{3}}$



3. (4.00)

$\phi=(B_0+\beta t)A$

$|\varepsilon_{ind}|=\frac{d\phi}{dt}=\beta A$

Applying kVL in the equivalent circuit diagram of the loop.

$\beta A-\frac{Ldi}{dt}-\frac{q}{C}=0$ ...(i)

Also, $i=\frac{dq}{dt}$ ...(ii)

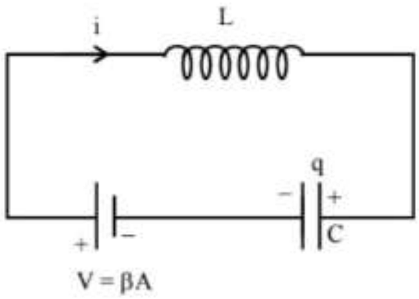
From (i) and (ii),

$L\frac{d^2i}{dt^2} = -\frac{i}{c}$

$i = i_m \sin \omega t \quad \omega = \frac{1}{\sqrt{LC}}$

$\int_0^q dq = i_m \int_0^t \sin(\omega t)\, dt$

$q = \frac{i_m}{\omega}(1 - \cos \omega t)$



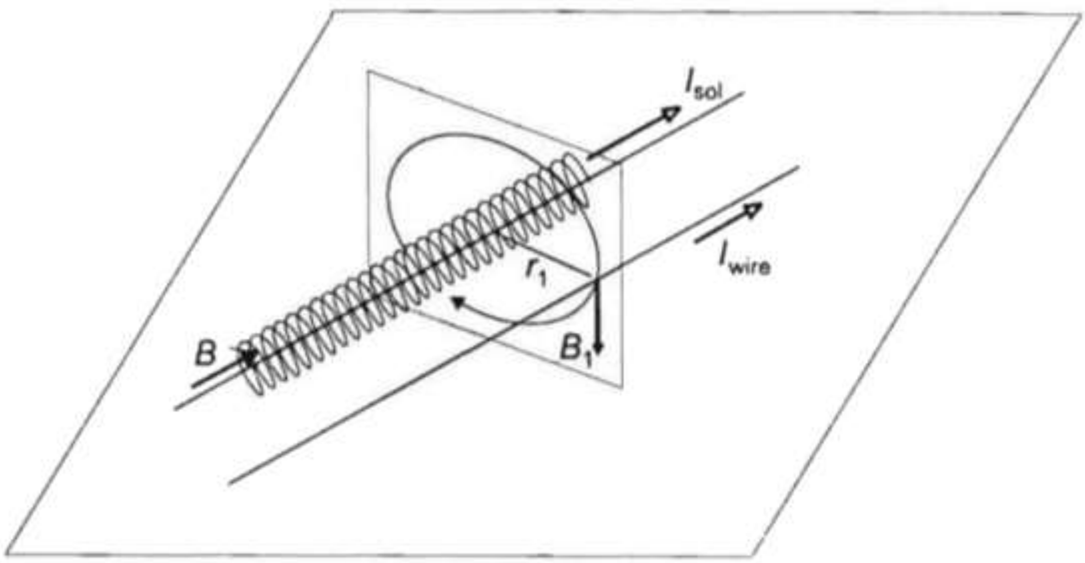
4. (641.00)
For 'n' number of maxima
$d \sin \theta = n\lambda$
$0.32 \times 10^{-3} \sin 30° = n \times 500 \times 10^{-9}$
$n = \left(0.32 \times 10^{-3} / 500 \times 10^{-9}\right) \times \left(\frac{1}{2}\right) = 320$
Hence, total number of maxima's observed in angular range $-30° \le \theta \le 30°$

5. (1570.00)

6. (125.6)

7. (1.6)
Let us denote the given data by : $\frac{N}{l} = 2000\frac{1}{m}$, $r_1 = 5\text{ cm}$, $B = 0.251\text{ T}$, $I = 40\text{ A}$.
First we determine the magnetic field produced by the solenoid at the distance of 5 cm from its axis. Then it is easy to calculate the Lorentz force of this field acting on the wire. The opposite of this force (reaction force) is exerted on the solenoid.
Since, the strength of the homogeneous magnetic field inside the coil is B, the electric current in the solenoid is :

$$I_{sol} = \frac{Bl}{\mu_0 N}$$



Because of the symmetry of the arrangement, the magnetic field produced by the coil outside the solenoid is similar to that of the straight, current-carrying wire, as it is shown in the figure. Let us apply Maxwell's second law to a circle of radius $r_1 = 5\,cm$, whose plane is perpendicular to the axis of the solenoid. Since the surface of the circle is crossed by the current I, the strength of the magnetic field at the distance r_1 from the axis is:

$$B_1 = \mu_0 \frac{I_{sol}}{2r_1\pi}.$$

This magnetic field can be considered homogeneous in the region of the thin wire, so the magnetic Lorentz force exerted on the length *l* of the wire is:

$$F = B_1 I_{wire} l.$$

In details:

$$F = B_1 I_{wire} l = \mu_0 \frac{I_{sol}}{2r_1\pi} I_{wire} l = \mu_0 \frac{Bl}{\mu_0 N}\frac{I_{wire}}{2r_1\pi} l = \frac{Bl^2 I_{wire}}{2r_1\pi N}.$$

It means that the Lorentz force exerted on the length *l* = 1 m of the solenoid is:

$$F = \frac{0.251\frac{Vs}{m^2}\cdot 1m^2 \cdot 40\,A}{2\cdot 0.05\,m\cdot\pi\cdot 2000} = 16\,mN = 1.6\times10^{-2}\,N$$

8. (2.00)

9. (A, B, C)

$$\nu_P = (121-118)\left(\frac{\nu+\nu_D\cos\alpha}{\nu}\right) = 3\left(\frac{\nu+\nu_D\cos\alpha}{\nu}\right)Hz$$

$$\nu_Q = 121-118 = 3\,Hz$$

$$\nu_R = (121-118)\left(\frac{\nu-\nu_D\cos\alpha}{\nu}\right) = 3\left(\frac{\nu-\nu_D\cos\alpha}{\nu}\right)$$

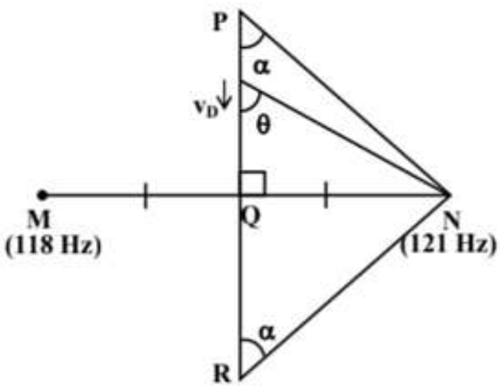
$$\therefore\ \nu_P + \nu_R = 2\nu_Q$$

Now, ν (between P and Q) $= 3\left(\frac{\nu+\nu_D\cos\theta}{\nu}\right)$

$$\frac{d\nu}{d\theta} = -\frac{3\nu_D}{\nu}\sin\theta$$

Now, $\sin\theta = 1$ at Q (maximum)

$\therefore$ rate of change in beat frequency is maximum at Q.



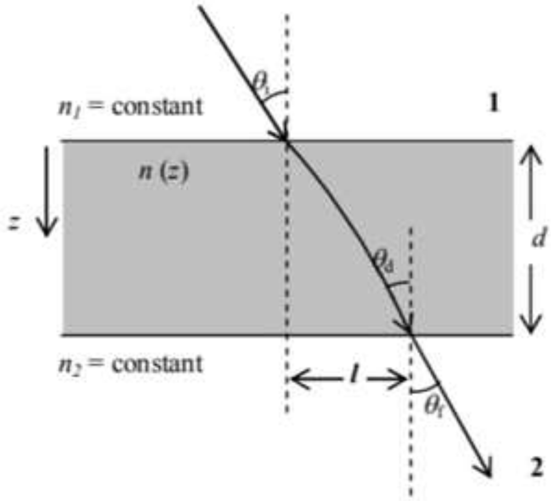
10. (A, C, D)

From Snell's Law

$n_1 \sin\theta_i = n(d)\sin\theta_d = n_2 \sin\theta_f$

The deviation of ray in the slab will depend on $n(z)$

Hence, *l* will depend on $n(z)$ but not on n_2.

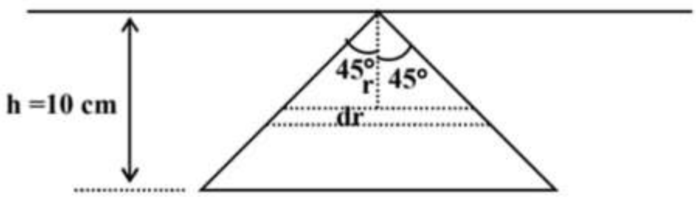


11. (A, D)

$$\phi_{\ell w} = \int_0^h \frac{\mu_0 I}{2\pi r} 2r\,dr = \frac{\mu_0 I h}{\pi}$$

So, mutual inductance $M_{\ell w} = \frac{\mu_0 h}{\pi}$

$$\therefore\ \varepsilon_w = \frac{\mu_0 h}{\pi}\frac{di}{dt} = \frac{\mu_0}{\pi}$$



Due to rotation there is not change in flux through the wire, so there is not extra induced emf in the wire.

From Lenz's Law, current in the wire is rightward so repulsive force acts between the wire and loop.

12. (C)

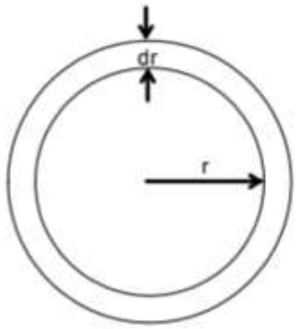
For infinite line,

$$E = \frac{\lambda}{2\pi\varepsilon r}$$

$$\Rightarrow\ dV = \frac{-\lambda}{2\pi\varepsilon r} dr$$

Current through an elemental shell;

$$I = \frac{|dV|}{dR} = \frac{\frac{\lambda}{2\pi\varepsilon r} dr}{\frac{1}{\sigma}\times\frac{dr}{2\pi r\ell}} = \frac{\lambda\sigma\ell}{\varepsilon}$$



This current is radially outwards so;

$$\frac{d}{dt}(\lambda\ell) = \frac{-\lambda\sigma\ell}{\varepsilon} \Rightarrow \frac{d\lambda}{\lambda} = -\left(\frac{\sigma}{\varepsilon}\right)dt$$

$$\Rightarrow\ \lambda = \lambda_0 e^{-(\sigma/\varepsilon)t}$$

$$\text{So, } j = \frac{I}{2\pi r\ell} = \frac{\lambda\sigma}{2\pi\varepsilon r} = \left(\frac{\lambda_0\sigma}{2\pi\varepsilon r}\right)e^{-(\sigma/\varepsilon)t}$$

13. (A, B, C, D)

14. (B, D)

15. (C)
Consider n cap is not an unit vector

16. (A)
$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}$ (concave lens)
$\Rightarrow v=-10$
$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}$ (convex lens)
$\frac{1}{v}=\frac{1}{10}+\frac{1}{-15}$
$\Rightarrow v=+30$

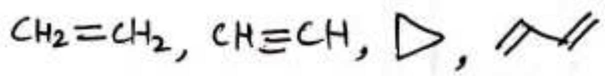
17. (A)

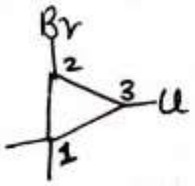
18. (A)

PART (B) : CHEMISTRY

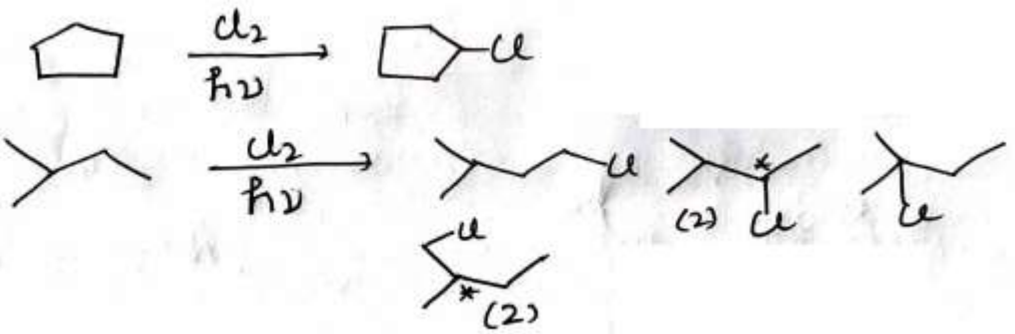
SOLUTIONS

19. (6)
Fructose, Mannose, Maltose, Lactose, Glucose, Altrose

20. (4)


21. (5)


$2+3=5$

22. (7)


$x+y=1+6=7$

23. (5)

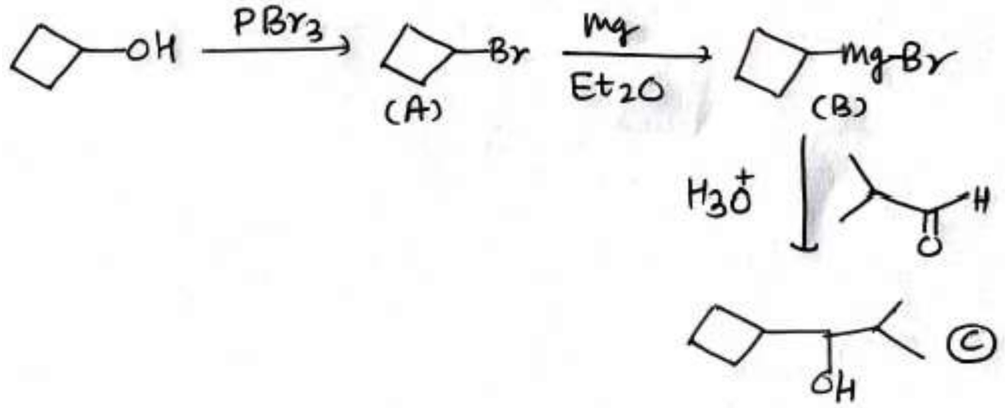
24. (8)

25. (6)

26. (8)

27. (A, B, D)

28. (A, B, C)



29. (A, B, C)

$$C_6H_5-CH_2-COO^-Ag^+ + Br_2 \xrightarrow[\Delta]{CCl_4} AgBr + C_6H_5-CH_2-COOBr$$

$$C_6H_5-CH_2-\overset{O}{\overset{\|}{C}}-O-Br \longrightarrow C_6H_5-CH_2-\overset{O}{\overset{\|}{C}}-\dot{O} + \dot{Br}$$

$$\downarrow$$

$$C_6H_5-\dot{C}H_2 + CO_2 + \dot{Br}$$

$$C_6H_5-\dot{C}H_2 + \dot{Br} \longrightarrow C_6H_5-CH_2-Br$$

30. (A, B, C, D)

$$CH_3-CH_2-OH \xrightarrow[\Delta]{Cu} CH_3-\overset{O}{\overset{\|}{C}}-H$$

$$CH_3-\overset{O}{\overset{\|}{C}}-Cl \xrightarrow[H_2,\ \Delta]{Pd + BaSO_4} CH_3-CHO$$

$$(CH_3-COO)_2Ca + (H\cdot COO)_2Ca \xrightarrow{\Delta} HCHO + CH_3-CHO + CH_3-COCH_3 + CaCO_3$$

$$CH_3-CH=\underset{CH_3}{\underset{|}{C}}-Ph \xrightarrow[Zn,\ H_2O]{O_3} CH_3-CHO + Ph-\overset{O}{\overset{\|}{C}}-CH_3$$

31. (B, C, D)

32. (B, C, D)

33. (D)

34. (B)

35. (C)

36. (B)

PART (C) : MATHEMATICS

SOLUTIONS

37. (3.00)

$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

38. (1.00)

$\int \frac{dy}{1-y} = \int \frac{x\,dx}{1+x^2}$

$\ln(y-1) = -\frac{1}{2}\ln\left(1+x^2\right) + \ln c$

$(y-1)^2 = \frac{c}{1+x^2} \Rightarrow x = 0 \qquad y = \frac{4}{3}$

$x = \frac{1}{9}$

$x = 2\sqrt{2} \quad (y-1)^2 = \frac{1}{9\times 9} = \frac{1}{81} \quad y-1 = \frac{1}{9} \quad y = \frac{10}{9}$

$y\left(\sqrt{8}\right) - \frac{1}{9} = 1$

39. (4.00)

$1-3y^2 = -2y^2 \Rightarrow y = \pm 1$

Req. area = $\int_{-1}^{1}\left(1-3y^2+2y^2\right)dy = 4/3$

$3A = 4$

40. (1.00)

$F(x) = \int \frac{\sec^2 x}{(1+\tan x)^2}\,dx = \frac{-1}{1+\tan x} + C$

$\therefore F(0) = 0 \Rightarrow c = 1$

$\therefore F(x) = \frac{-1}{1+\tan x} + 1$

$\therefore \lim_{x\to\pi/4} F(x) = \frac{-1}{2} + 1 = \frac{1}{2}$

41. (9.00)

$= \int_0^3 \left(x\frac{d}{dx}\left(\int_0^x f(t)\,dt\right) + \int_0^x f(t)\,dt) \right)dx$

Let $\int_0^x f(t)\,dt = \phi(x)$

$$\therefore\ I=\int_0^3\left(x\frac{d}{dx}(\phi(x))+\phi(x)\right)dx=\left[x\phi(x)+c\right]_0^3$$

$$I=\left[x\int_0^x f(t)\,dt+c\right]_0^3=I=3\int_0^3 f(t)\,dt=9$$

42. (2.00)

$$I=\int_0^\pi x^2\frac{\cos x\,dx}{(1+\sin x)^2}=\left(\frac{-x^2}{1+\sin x}\right)_0^\pi-\int_0^\pi\frac{-2x}{1+\sin x}\,dx$$

$$=-\pi^2+2\int_0^\pi\frac{x}{1+\sin x}\,dx$$

But $\int_0^\pi\frac{x}{1+\sin x}\,dx=\pi\int_0^{\pi/2}\frac{1}{1+\sin x}\,dx=\pi\int_0^{\pi/2}\frac{1}{1+\cos x}\,dx$

$$=\int_0^\pi\frac{1}{2\cos^2\frac{x}{2}}\,dx=\frac{\pi}{2}\left[\frac{\tan(x/2)}{(1/2)}\right]_0^{\pi/2}=\pi$$

$I=2\pi-\pi^2$

43. (2.00)

Req. area = $\int_{1-e}^0\log(x+e)\,dx+\int_0^\infty e^{-x}\,dx$

$$=\int_1^e(\log t)\,dt+\int_0^\infty e^{-x}dx$$

Where t = x + e

$$=[t\log t-t]+\left[-e^{-x}\right]_0^\infty$$

$1+1=2$ sq units

44. (2.00)

$f'(x)=-2-3x^2<0\Rightarrow f(x)$ is decreasing

$\therefore f(f(x))<f(-x)\ \Rightarrow\ f(x)>-x$

$\Rightarrow\ 30-x-x^3>0\ \Rightarrow\ x^3+x-30<0$

$\Rightarrow\ (x-3)(x^2+3x+10)<0$

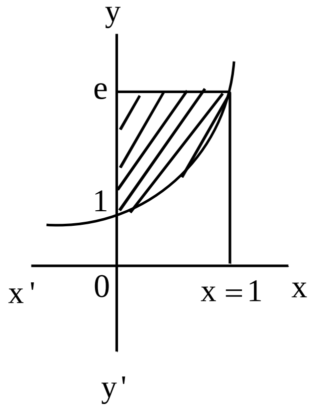
$\Rightarrow\ x<3$

No. of values = 2

45. (BCD)

Req. area $=\int_1^e x\,dy$

$$= \int_1^e \log_e y \, dy = \int_1^e \log_e (1+e-y) \, dy$$



And req. area = $(e \times 1) - \int_0^1 e^x \, dx$

46. (B, C)

$$\text{Let } P(x) = 1 + x + \frac{x^2}{2!} + \ldots.. + \frac{x^{2018}}{2018!}$$

$$P'(x) = 1 + \frac{x}{1!} + \ldots. + \frac{x^{2017}}{2017!}$$

$$\therefore \int (2018)! \frac{(P(x) - P'(x))}{P(x)} dx$$

$$= 2018! \int \left(1 - \frac{P'(x)}{P(x)}\right) dx$$

$$= 2018! (x - \ln P(x)) + C$$

47. (A, B, D)

$$a_n = \prod_{K=0}^{n} \left(1 + \frac{2}{x^{2^K} + x^{-2^K}}\right), |x| > 1$$

$$= \frac{x+1}{x-1} \cdot \frac{x^{2^{n+1}} - 1}{x^{2^{n+1}} + 1}$$

$$\lim_{n \to \infty} a_n = \left(\frac{x+1}{x-1}\right)\left(\frac{1-0}{1+0}\right) = \frac{x+1}{x-1}$$

$$\therefore f(x) = \frac{x+1}{x-1}$$

48. (A, B, C)

$$f(x)\left[f(x)\right)^6+1\right]=x$$

$$f(0)\left[f(0)^6+1\right]=0\Rightarrow f(0)=0$$

$$\&\ 7\left(f(x)\right)^6 f'(x)=1-f'(x)$$

$$f'(x)\left[7\left(f(x)\right)^6+1\right]=1$$

$$\Rightarrow\ f'(x)>0\ \forall\ x\in R$$

$$\Rightarrow\ f(x)\text{ is increasing function }\forall x\in R$$

$$\Rightarrow\ x^7+x=f^{-1}(x)$$

$$\Rightarrow\ \int_0^{\sqrt{2}}(x^7+x)\,dx=3$$

$$\&\ \int_0^a f(x)dx+\int_0^{f(a)} f^{-1}(x)dx=af(a)$$

$$\Rightarrow\ \int_0^{\sqrt{3}} f(x)dx=\frac{f(\sqrt{3})}{8}\left[8\sqrt{3}-\left(f(\sqrt{3})\right)^7-4f(\sqrt{3})\right]$$

49. (A, B)

$$f'(x)\ge f'(1)\ \forall\ x\in[0,1]$$

$$\therefore\ \frac{f'(1)}{f^2(x)+1}\le\frac{f'(x)}{f^2(x)+1}$$

On Integrating

$$f'(1)\int_0^1\frac{dx}{f^2(x)+1}\le\tan^{-1}\left(f(1)\right)-\tan^{-1}\left(f(0)\right)$$

$$\text{i.e. }\int_0^1\frac{dx}{f^2(x)+1}\le\frac{\tan^{-1}f(1)}{f'(1)}\le\frac{f(1)}{f'(1)}\qquad[\because\ \tan^{-1}x\le x\ \forall\ x\ge 0]$$

For equality to hold $\tan^{-1}f(1)=f(1)$

$$\therefore\ f(1)=0\text{, then }\int_0^1\frac{dx}{f^2(x)+1}=0$$

Which is not possible as this is strictly positive function

$$\text{Hence }\int_0^1\frac{dx}{f^2(x)+1}<\frac{f(1)}{f'(1)}$$

50. (A, D)

$\left(f(x)-e^{x}\right)\left(f(x)-e^{x^2}\right)\le 0$

$\Rightarrow e^{x^2}\le f(x)\le e^{x}$ when $x\in(0,1)$

$\Rightarrow e^{x}\le f(x)\le e^{x^2}$ when $x>1$

$\therefore \lim\limits_{x\to1} f(x)=e$.

$$\& \lim_{n\to\infty}\sum_{K=1}^{n}\frac{K^3+6K^2+11K+5}{(K+3)!}$$

$$=\lim_{n\to\infty}\sum_{K=1}^{n}\left(\frac{1}{K!}-\frac{1}{K+3!}\right)$$

$$=\frac{5}{3}$$

51. (A)

(I) req. area $= 2\left[\int_0^1 \frac{2}{1+x^2}\,dx-\int_0^1 x^2\,dx\right]=2(\pi/2-1/3)=\pi-2/3$

(II) $\lim\limits_{n\to\infty} f(n)=\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\left(\frac{n+r}{n^2+r^2}\right)=\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\left(\frac{1}{n}\right)\left(\frac{1+r/n}{1+r^2/n^2}\right)$

$$=\int_0^1\frac{1+x}{1+x^2}\,dx=\left[\tan^{-1}x+\frac{1}{2}\log\left(1+x^2\right)\right]_0^1=\frac{\pi}{4}+\frac{1}{2}\log 2$$

(III) given eq. is $\dfrac{ydx+xdy}{x^2y^2}+\dfrac{1}{y}dy=0$

$$\Rightarrow \frac{d(xy)}{(xy)^2}+\frac{1}{y}dy=0\Rightarrow \frac{-1}{xy}+\log y=C$$

$\therefore y(1)=1\Rightarrow C=-1$

(IV) $I=\int_0^1 \mathrm{Tan}^{-1}\left(\frac{1}{1-x(1-x)}\right)dx=\int_0^1 \mathrm{Tan}^{-1}\left(\frac{1-x+x}{1-x(1-x)}\right)dx$

$$=\int_0^1 \mathrm{Tan}^{-1}(1-x)\,dx+\int_0^1 \mathrm{Tan}^{-1}x\,dx=2\int_0^1 \mathrm{Tan}^{-1}x\,dx$$

52. (C)

(I) $f'(x)=\dfrac{1}{\sqrt{1+(g(x))^3}}\cdot g'(x)$

$g'(x)=\left(1+\sin\left(\cos^2 x\right)\right)\cdot(-\sin x)$

(II) $f(x)=2f(x)f'(x)\qquad f'(x)=\dfrac{1}{2}\qquad f(x)=\dfrac{x}{2}$

(III) $I=2(b-a)+\dfrac{(b-a)(b+a)}{2}-\dfrac{(b-a)\left(b^2+ab+a^2\right)}{3}$

(IV) $\lim\limits_{x\to 0}\dfrac{\sin 2x + bx}{x^3} = -a$

$\lim\limits_{x\to 0}\dfrac{2\cos 2x + b}{x^2} = -3a \qquad b = -2$

$\lim\limits_{x\to 0}\dfrac{-4\sin 2x}{2x} = -3a \qquad a = \dfrac{4}{3}$

53. (D)

(I) $2^x = t \qquad \dfrac{1}{\ln 2}\displaystyle\int\dfrac{dt}{\sqrt{1-t^2}} = \dfrac{1}{\ln 2}\sin^{-1}\left(2^x\right)$

$K = \dfrac{1}{\ln 2}$

(II) $x = t^2 \qquad \displaystyle\int\dfrac{t^5.2t\,dt}{t^7+t^{12}} = 2\int\dfrac{dt}{t^6+t} = 2\int\dfrac{t^{-6}\,dt}{1+t^{-5}}$

$= \dfrac{-2}{5}\ln\left(1+x^{-5/2}\right) \qquad a = +\dfrac{2}{5} \qquad k = \dfrac{5}{2}$

$= \dfrac{2}{5}\ln\left(\dfrac{x^{5/2}}{1+x^{5/2}}\right)$

(III) $\displaystyle\int\dfrac{x^4+2x^2+1}{x\left(x^2+1\right)^2} - \dfrac{2x^2}{x\left(x^2+1\right)^2}$

$= \ln x + \dfrac{1}{x^2+1}$

$k = 1 \quad m = 1$

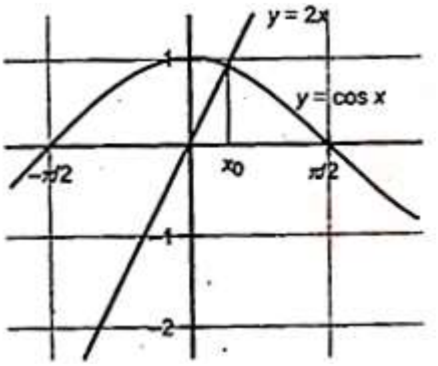
(IV) $\displaystyle\int\dfrac{dx}{5+4\cos x} = \int\dfrac{\sec^2\frac{x}{2}dx}{9+\tan^2\frac{x}{2}} = \dfrac{2}{3}\tan^{-1}\left(\dfrac{\tan\frac{x}{2}}{3}\right)$

$k = \dfrac{2}{3} \qquad m = \dfrac{1}{3}$

54. (A)

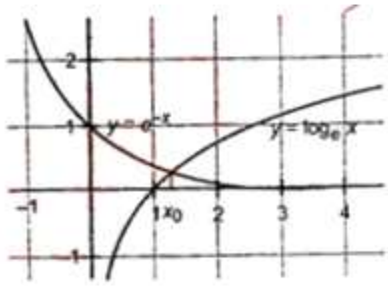
(I) $f(x) = \sin x - x^2 + 1$

$f'(x) = \cos x - 2x$



(II) $f(x) = x\log_e x - x + e^{-x}$

$f'(x) = \log_e x + 1 - 1 - e^{-x} = \log_e x - e^{-x}$



From the graph for $x < x_0, e^{-x} > \log_e x \Rightarrow f'(x) < 0$

For $x > x_0, e^{-x} < \log_e x \Rightarrow f'(x) = 0$

Hence, $x = x_0$ is point of minima.

(III) $f'(x) = -3x^2 + 4x - 3$

Now $D = 16 - 4(-3)(-3) = -20 < 0$

Hence, $f'(x) < 0$, for all real x

$\Rightarrow$ $f(x)$ is always decreasing.

(IV) $f(x) = \cos \pi x + 10x + 3x^2 + x^3$

$\Rightarrow f'(x) = -\pi \sin \pi x + 10 + 6x + 3x^2$

$= \left(x^2 + 2x + \frac{10}{3}\right) - \pi \sin \pi x$

$= 3\left((x+1)^2 + \frac{7}{3}\right) - \pi \sin \pi x$

Now min. value of $3\left((x+1)^2 + \frac{7}{3}\right)$ is 7 but maximum value of $\pi \sin \pi x$ is π.

PART (A) : PHYSICS

SOLUTIONS

1. (6)
Capacitance per unit length $= -\int \vec{E}\cdot d\vec{r} = \Delta V$

$$C = \frac{Q_+}{\delta V} = \frac{\lambda L}{\Delta V}$$

$$\Rightarrow \frac{C}{L} = \frac{\lambda}{-\int \vec{\varepsilon}\cdot d\vec{r}} = \frac{6\pi\varepsilon_0}{\log_e(b/a)}$$

2. (1)

$$\Delta P = \frac{F_{excess}}{A} = 60\times 10^{13}(N/m^2)$$

So, Ans. K = 1

3. (4)
It will oscillate about C.M of system so angular acceleration

$$\alpha = -\omega^2\theta$$

Motion will be S.H.M. of θ is almost 5.1

$$\alpha = -\frac{2qE}{mL}\theta$$

[So, Ans. is n = 4]

4. (8)

5. (7)

$$I_A = \frac{10}{\pi}\times\pi r_A^2 = 1\times 10^{-5}\ W$$

Similarly,

$$I_B = \frac{10}{\pi}\times\pi r_B^2 = 4\times 10^{-5}\ W$$

Path Difference:

$$\delta_{slab} = t\times(\mu-1) = 1000\times 10^{-10}\,m = \frac{\lambda}{6}$$

Therefore Phase Difference $= \frac{2\pi}{\lambda}\times\delta_{slab} = \frac{\pi}{3}$

Now, Resultant Intensity will be as follows:

$$I_R = I_A + I_B + 2\times\sqrt{I_A I_B}\times\cos\left(\frac{\pi}{2}\right) = 7\times 10^{-5}\ W$$

6. (2)

7. (2)

8. (4)

9. (A, B)

10. (A, D)

11. (A, B, C, D)

12. (A, D)

13. (B, D)

14. (A, C)

15. (A)

16. (A)

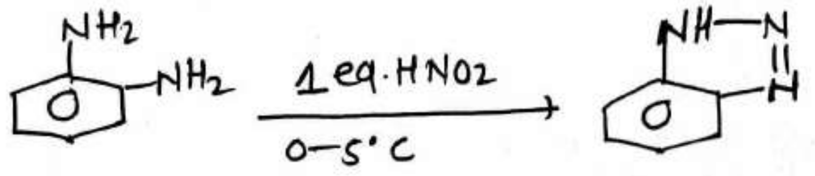
17. (C)
Linear magnification of a simple microscope (m) = 5
Magnification is given by:
$\Rightarrow \quad m = 1 + \frac{D}{f}$
Where D is the near point given by, $D = 25\,\text{cm}$.
On substituting the values we obtain
$\Rightarrow \; 5 = 1 + \frac{25}{f}$
$\Rightarrow \; \frac{25}{f} = 4$
Hence the focal length (f) is given by
$\Rightarrow \; f = \frac{25}{4} = 6.25\,\text{cm}$

18. (A)
For spherical mirror, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \;\Rightarrow\; \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$, $V = \frac{fu}{u - f}$, here f is negative as the mirror is concave and u is also negative as the object is real (lies in front of mirror)
$\therefore\; V = \frac{(-f)(-u)}{(-u)-(-f)} = \frac{fu}{f-u} = \frac{f}{\frac{f}{u} - 1}$
For u < f, it can be observed that V increases as u increases until u will be equal to f at which $V = \infty$.
For u > f, V will be negative, but its numerical value decreases (|V|) as u increases and |V| = f at u $= \infty$

PART (B) : CHEMISTRY

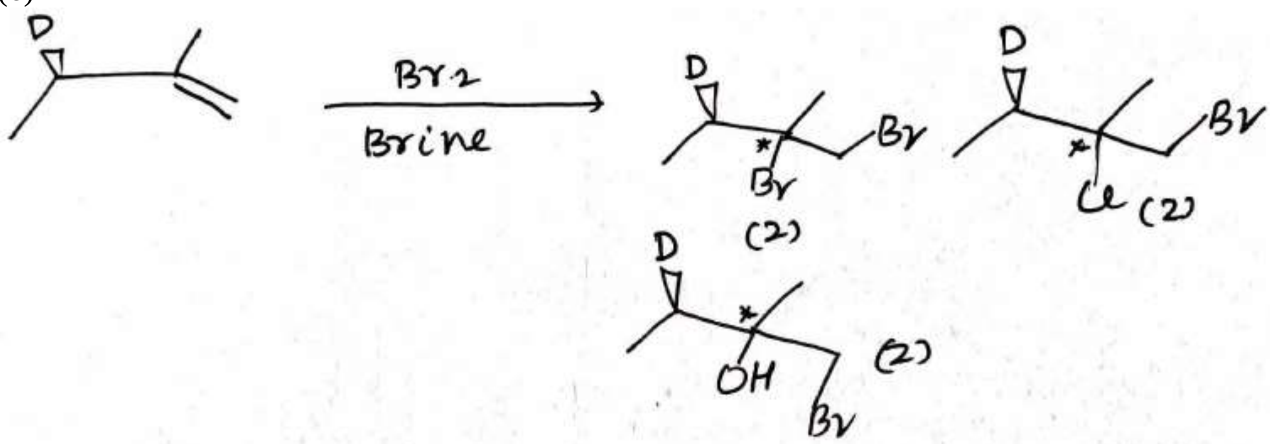
SOLUTIONS

19. (6)



20. (9)

21. (6)



22. (2)

$$CH_3-CH_2-CH_2-OH \xrightarrow[\text{or Cu, 300°C}]{\text{P.C.C, } CH_2Cl_2} CH_3-CH_2-CHO$$

23. (7)

$$R-Cl + Mg \rightarrow R-Mg-Cl \xrightarrow[H^{\oplus}]{CO_2} R-COOH$$

$$R-Cl + KCN \longrightarrow R-CN \xrightarrow{H_3O^+} R-COOH$$

$$R-CHO \xrightarrow[H_2SO_4]{CrO_3} R-COOH$$

$$R-COCl \xrightarrow{H_3O^+} R-COOH$$

$$R-CHO \xrightarrow[\Delta]{[Ag(NH_3)_2]^+NO_3^-} R-COOH$$

$$R-NC \xrightarrow{H_3O^+} R-NH_2 + HCOOH$$

24. (5)

25. (18)

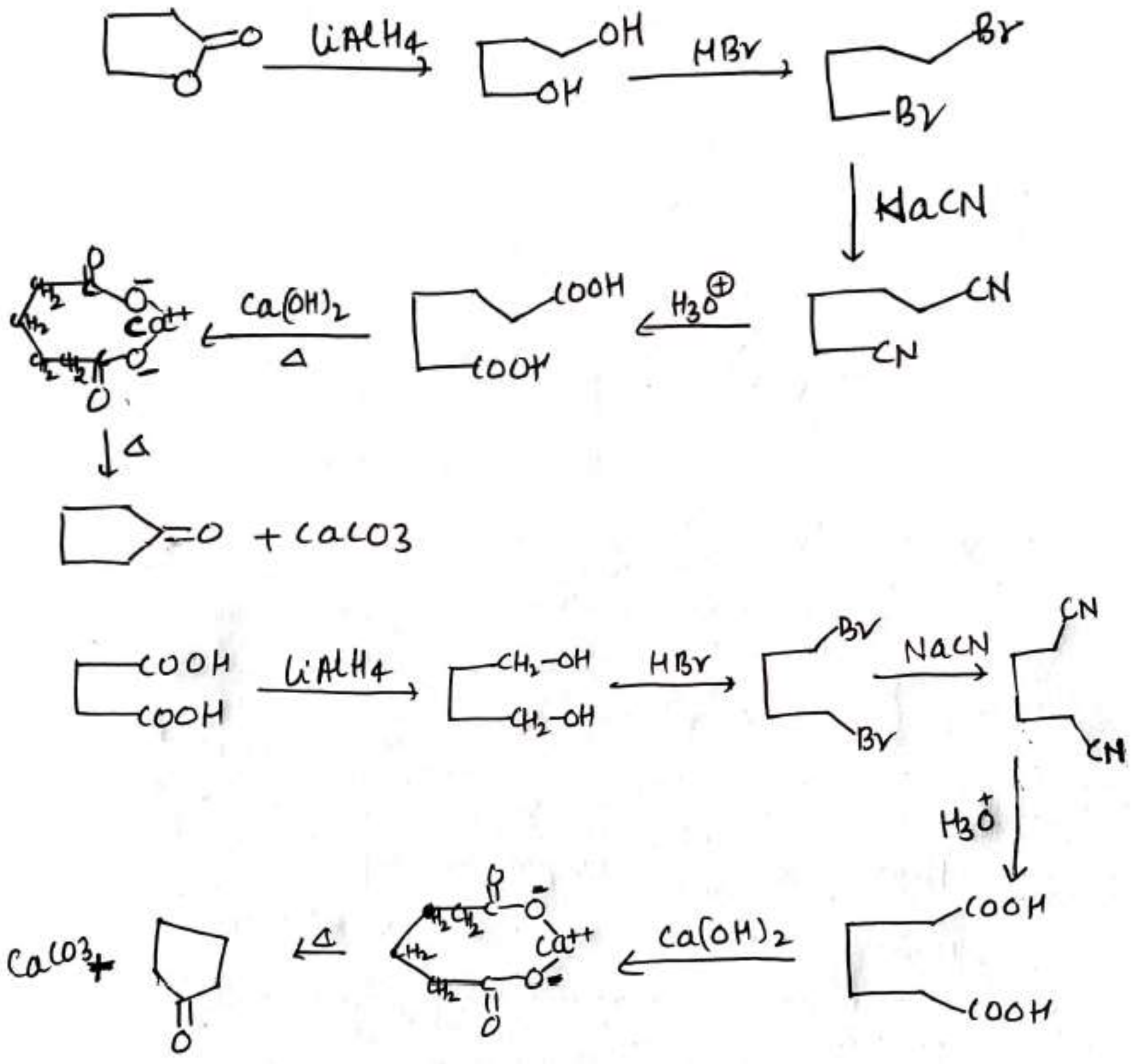
26. (1)

27. (B, C, D)

28. (A, B, C, D)

29. (A, B)

Glucose, Fructose



30. (A, B, C)

31. (A, B, C)

32. (B, C, D)

33. (A)

34. (B)

35. (A)

36. (C)

PART (C) : MATHEMATICS

SOLUTIONS

37. (9)

Using the definition of period , we can get answer.

$[x] = x - \{x\}$.

$$\sin\frac{\pi[x]}{12} = \sin\frac{\pi}{12}(x - \{x\}) \Rightarrow P_1 = \frac{2\pi}{\frac{\pi}{12}} = 24$$

$$\cos\frac{\pi x}{4} \Rightarrow P_2 = \frac{2\pi}{\frac{\pi}{4}} = 8$$

$$\tan\frac{\pi[x]}{3} = \tan\frac{\pi}{3}(x - \{x\}) \Rightarrow P_3 = \frac{\pi}{\frac{\pi}{3}} = 3$$

L.C.M of P_1, P_2, P_3 is λ

L.C.M of 24, 8, 3 is 24

$$\lambda = 24 \Rightarrow \frac{3\lambda}{8} = \frac{3\times 24}{8} = 9$$

38. (4)

$$I = \int_0^{\pi}(\pi - x)\left(\left(\sin^2(\sin x)\right) + \cos^2(\cos x)\right)dx \Rightarrow 2I = 2\pi\int_0^{\frac{\pi}{2}}\left(\sin^2(\sin x) + \cos^2(\cos x)\right)dx$$

$$\Rightarrow I = \pi\int_0^{\frac{\pi}{2}}\left(\sin^2(\sin x)\right) + \cos^2(\cos x)dx = \pi\int_0^{\frac{\pi}{2}}\left(\sin^2(\cos x) + \cos^2(\sin x)\right)dx$$

$$\Rightarrow 2I = \pi\int_0^{\frac{\pi}{2}} 2dx \Rightarrow I = \frac{\pi^2}{2}$$

39. (0)

$f'(x) = f(x)$

$f(x) = Ke^x$

$f(0) = 0$ $\qquad K = 0$

$f(x) = 0$

$f(\ln 5) = 0$

40. (5)

$$\sum_{K=1}^{10}\int_0^1 f(K-1+x)dx=\int_0^1 f(x)dx+\int_0^1 f(x+1)dx.........\int_0^1 f(x+9)$$

$$=\text{putting } x+1=t,(x+2)=t............(x+9)=t$$

$$\Rightarrow \int_0^1 f(x)dx+\int_1^2 f(x)dx+\int_2^3 f(x)dx.........\int_9^{10} f(x)dx$$

$$=\int_0^{10} f(x)dx=5.$$

41. (2)

$$f(x)=\lim_{n\to\infty}\left(\frac{1\times2\times3...\times n}{n\times n\times....\times n}\right)^{\frac{1}{n}}$$

$$\log f(x)=\lim_{n\to\infty}\frac{1}{n}\left(\log\frac{1}{n}+\log\frac{2}{n}+...+\log\frac{n}{n}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\log\frac{r}{n}$$

$$=\int_0^1 \log x\,dx=-1$$

$$f(x)=e^{-1}=\frac{1}{e}$$

Hence $e^2 f(x) = e^2\times\frac{1}{e}=e$

$$\left[e^2 f(x)\right]=[e]=2$$

42. (0)

Put $x\to 1-x$

$$f(1-x)+2f(x)=(1-x)^2+1$$

$$\therefore\ x^2+1+3f(x)=2(1-x)^2+2$$

Put $x=3\Rightarrow\ f(3)=0$

43. **(1)**

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^{\frac{1}{a}}\left\{n^a\,.\,n^{-1/a}+k^a.\,k^{-1/a}\right\}}{n^a\ n}$$

$$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}.\left\{\left(\frac{k}{n}\right)^{1/a}+\left(\frac{k}{n}\right)^{a}\right\}=\int_0^1\left(x^{1/a}+x^a\right)dx$$

$$=\left\{\frac{x^{(1/a)+1}}{\frac{1}{a}+1}+\frac{x^{a+1}}{a+1}\right\}_0^1=\frac{a}{a+1}+\frac{1}{a+1}=1$$

44. **(1)**

$t^2 f(x)-2tf'(x)+f''(x)=0$, has equal roots

Discriminant = $4\left(f'(x)\right)^2-4f(x)\ f''(x)=0$

The above equation can be expressed as: $\frac{f''(x)}{f'(x)}=\frac{f'(x)}{f(x)}$

Integrating both sides, we get : $ln\left(f'(x)\right)=ln\ f(x)-ln\ c$ or $f(x)=cf'(x)$

$\Rightarrow\ f(0)=cf'(0)\ \Rightarrow\ c=\frac{1}{2}$ Hence, $\frac{f'(x)}{f(x)}=2$

$ln\ f(x)=2x+k\xrightarrow{f(0)=1} ln\ f(x)=2x$ or $f(x)=e^{2x}$

$t^2e^{2x}-4te^{2x}+4e^{2x}=0\quad\Rightarrow\quad t^2-4t+4=0$ or $t=2$

Now, $\lim\limits_{x\to 0}\frac{f(x)-1}{x}-\frac{t}{2}=\lim\limits_{x\to 0}\frac{e^{2x}-1}{2x}\times 2-\frac{2}{2}=2-1=1$

45. (A, B, C, D)

$\because\ -1\le f''(x)\le 1$

$\Rightarrow -x\le\int_0^x f''(x)dx\le x\ \forall\ x\in[0,\infty)$ and $x\le\int_x^0 f''(x)\le -x\ \forall\ x\in(-\infty,0)$

$\Rightarrow\ -x\le f'(x)dx\le x,\ x\in[0,\infty)$ and $x\le -f'(x)\le -x,\ \forall x\in(-\infty,0)$

$\Rightarrow\ -\frac{x^2}{2}\le\int_2^x f'(x)dx\le\frac{x^2}{2}$, similarly

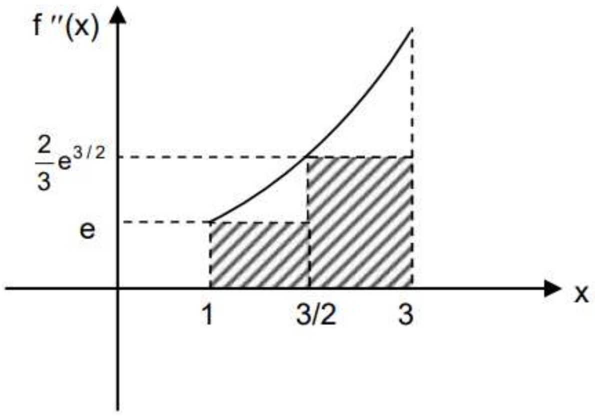
$-\frac{x^2}{2}\le\int_x^0 f'(x)dx\le\frac{x^2}{2}$

$\Rightarrow -\frac{x^2}{2}\le f(x)\le\frac{x^2}{2}$

$\therefore\ -\frac{1}{8}\le f\left(\frac{1}{2}\right)\le\frac{1}{8},-2\le f(2)\le 2,-2\le f(-2)\le 2$

46. (A, B, C, D)

$\because\ f'(x) = \int_1^x \frac{e^t}{t}dt$ and $f''(x) = \frac{e^x}{x} > 0\ \forall x \in [1, \infty)$



Let $g(x) = \frac{e^x}{x}$ $\therefore\ g'(x) = \frac{(x-1)e^x}{x^2} > 0$

$\forall x \in [1, \infty)$

$\Rightarrow$ f ′(x) is increasing and f ′(x) is concave upward function

$\therefore\ f'(2015) = \int_1^{2015} \frac{e^x}{x}dx$

$\because$ g(x) is increasing

$\because\ g(x) > g(1)\ \forall x > 1 \Rightarrow \frac{e^x}{x} > e$

$\therefore\ \int_1^{2015} \frac{e^x}{x}dx > \int_1^{2015} e\,dx$

$\Rightarrow f'(2015) > (2014)e \Rightarrow$ (A) is correct

Also f ′(x) is increasing and concave upward

$\therefore\ \frac{f'(2)+f'(4)}{2} > f'(3)$

$\Rightarrow f'(2) + f'(4) > 2f'(3)$.

Hence (B) is correct

$\because\ f'(3) = \int_1^3 \frac{e^x}{x}dx$

$e \cdot \frac{1}{2} < \int_1^{3/2} \frac{e^x}{x}dx < \int_1^{3/2} e^x\,dx$ ....(i) and

$$\frac{3}{2}\cdot\frac{2}{3}e^{3/2} < \int_{3/2}^{3}\frac{e^x}{x}dx < \int_{3/2}^{3} e^x\,dx \quad ....(ii)$$

(i) + (ii)

$$\frac{e}{2}+e\sqrt{e} < \int_{1}^{3}\frac{e^x}{x}dx < (e^{3/2}-e)+(e^3-e^{3/2})$$

$$\Rightarrow \frac{e}{2}(1+2\sqrt{e}) < \int_{1}^{3}\frac{e^x}{x}dx < (e^2-1)\,e$$

(C) is correct

$\because f'(1)=0$ and $f'(x)$ is decreasing

$\therefore f'(x)=0$ has exactly one real root.

$\therefore f'(x)=0$

47. (C)
Conceptual

48. (A, C, D)

$$\text{(A)L.H.S.} = 2^{-m}\int_{0}^{\pi/2}(\sin 2x)^m dx = \frac{2^{-m}}{2}\int_{0}^{\pi}(\sin t)^m dt$$

$$= 2^{-m}\int_{0}^{\pi/2}(\sin t)^m dt = 2^{-m}\int_{0}^{\pi/2}(\cos t)^m dt = \text{RHS}$$

$$\text{(B)}\,I=\int_{0}^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx \Rightarrow 2I=\pi\int_{0}^{\pi}\frac{\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow 2I = 2\pi\int_{0}^{\pi/2}\frac{\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow I=\pi\int_{0}^{\pi/2}\frac{\sin x}{1+\cos^2 x}dx = \pi\int_{0}^{1}\frac{dt}{1+t^2} \neq \frac{\pi}{2}\int_{0}^{1}\frac{dt}{1+t^2}$$

(C) $\int_0^{\pi/2} \frac{1-\cos 3x}{1+2\cos x}dx = \int_0^{\pi/2} \frac{1+\cos 3x}{2\cos x - 1}dx = 1$

(D) $I = \int_0^{\pi/2} \frac{1+\sin 3x}{1+2\sin x}dx$

Now, using $\int_0^a f(a-x)dx = \int_0^a f(x)dx$

$\int_0^{\pi/2} \frac{1+\cos 3x}{2\cos x - 1}dx = \int_0^{\pi/2} (\text{Cos}2x + \cos x)\, dx = 1$

$\therefore \int_0^{\pi/2} \frac{1+\sin 3x}{1+2\sin x}dx = \int_0^{\pi/2} \frac{1+\cos 3x}{2\cos x - 1}dx$

49. (A, B, D)

$f'(x) = 30x^3(x+1)(x+2)$

$f''(x) = 30x^2(5x^2+12x+6)$

50. (A, D)

$f(x) = \int_0^1 (2-t)\,dt + \int_1^2 t\,dt + \int_2^x t\,dt$

$= \frac{x^2}{2} + 1$

$\therefore f(x) = \begin{cases} \frac{x^2}{2} + 1, if\ \ x > 2 \\ 5x + 1, if\ x \le 2 \end{cases}$

Clearly (1) , (4) are true.

51. (B)

$f'(x) = \begin{cases} 3x^2 + 2x - 10 & -1 < x < 0 \\ \cos x & 0 < x < \pi/2 \\ -\sin x & \pi/2 < x < \pi \end{cases}$

Now $f'\left(\frac{\pi}{2}^-\right)$ = positive , $f'\left(\frac{\pi}{2}^+\right)$ = negative

so at $x = \frac{\pi}{2}$, local maxima

by graph at x = 0, π absolute minima and at x = – 1 absolute maxima

52. (A)

$$g(x) = \underbrace{f\,f\,f....f}_{n\ \text{times}}(x) = \frac{x}{(1+nx^n)^{1/n}}$$

53. (A)

$x = \tan\theta$

54. (A)

Apply integration by parts