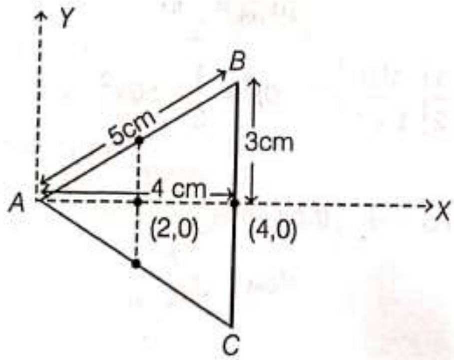
JEE Main Exercise

1. (A)



$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{\lambda(5)(2) + \lambda(5)(2) + (2\lambda)(6)(4)}{\lambda(5) + \lambda(5) + 2\lambda(6)} = \frac{34}{11}\text{ cm}$$

2. (C)

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 - m_2\mathbf{r}_2 - m_3\mathbf{r}_3}{m_1 - m_2 - m_3}$$

$$= \frac{[\sigma\pi(4R)^2)](0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}) - (\sigma\pi R^2)(3R\,\hat{\mathbf{i}}) - (\sigma\pi R^2)(3R\hat{\mathbf{j}})}{\sigma\pi(4R)^2 - \sigma\pi R^2 - \sigma\pi R^2}$$

$$= \frac{-3R}{14}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

3. (D)

$$s_{AC} = 2L \Rightarrow s_A = 2L + s_C$$

$$s_{BC} = -2L \Rightarrow s_B = -2L + s_C$$

$$s_{CM} = \frac{m_As_A + m_Bs_B + m_Cs_C}{m_A + m_B + m_C} = 0$$

$$\Rightarrow \quad \frac{m(2L + s_C) + 2m(-2L + s_C) + 3m\,s_C}{6m} = 0$$

$$\Rightarrow \quad s_C = \frac{L}{3}$$

4. (B)

$s_{AP} = +4 \Rightarrow s_A = 4 + s_P$

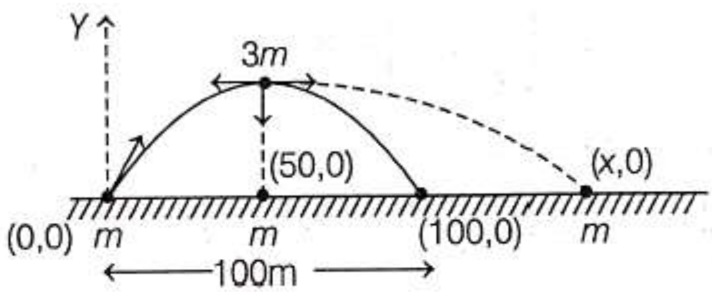
$s_{CP} = -4 \Rightarrow s_C = -4 + s_P$

$$s_{CM} = \frac{m_1 s_1 + m_2 s_2 + m_3 s_3 + m_4 s_4}{m_1 + m_2 + m_3 + m_4}$$

$$\Rightarrow 0 = \frac{40(4 + s_p) + 60(-4 + s_p) + 50 s_p + 90 s_p}{40 + 60 + 50 + 90}$$

$\Rightarrow s_p = \frac{1}{3}$m towards right

5. (C)



$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$\Rightarrow \quad 100 = \frac{m(0) + m(50) + mx}{3m} \Rightarrow x = 250 \text{ m}$$

6. (B)

Friction on wedge will be acting towards right. Due to friction, centre of mass of (wedge + block) system will move rightward and due to gravity centre of mass will move downward.

7. (D)

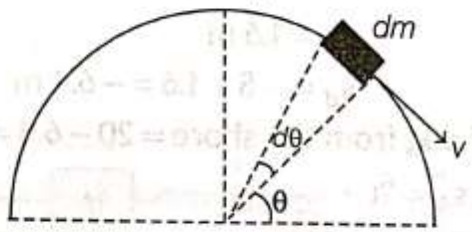
External forces on the (Earth + ball) system is zero. So to keep centre of mass stationary both will move away from each other.

8. (B)

$$dm = \frac{M}{\pi R} R d\theta = \frac{M}{\pi} d\theta$$

$$d\mathbf{p} = d\,mv \sin\theta\,\hat{i} + dmv\cos\theta\,\hat{j}$$

$$\Rightarrow \int d\mathbf{p} = \frac{Mv}{\pi}\int_0^{\pi} \sin\theta\, d\theta\,\hat{i} + \frac{Mv}{\pi}\int_0^{\pi}\cos\theta\, d\theta\,\hat{j}$$



$$\Rightarrow \quad \mathbf{p} = \frac{2Mv}{\pi}\hat{i} \Rightarrow p = \frac{2Mv}{\pi}$$

9. (A)

Applying linear momentum conservation in horizontal

$$0 + 0 = 1v_1 + 2(-v_2)$$

$$\Rightarrow \quad v_1 = 2v_2$$

Applying work-energy theorem,

$$W_{gravity} + W_{internal\ normal} + W_{normal\ from\ ground} = \Delta K$$

$$\Rightarrow \quad 1\times 10(1.1 - 0.1) + 0 + 0$$

$$= \left(\frac{1}{2}\times 1v_1^2 + \frac{1}{2}\times 2v_2^2\right) - (0+0)$$

$$\Rightarrow \quad 10 = \frac{1}{2}(2v_2)^2 + v_2^2$$

$$\Rightarrow \quad v_2 = \sqrt{\frac{10}{3}}\ \text{m/s}$$

10. (A)

m_1 will break off the wall when the spring acquires natural length,

Applying work-energy theorem,

$$W_{spring} + W_{mg} + W_N = \Delta K$$

$$\Rightarrow \quad \frac{1}{2}K(x^2 - 0^2) + 0 + 0 = \frac{1}{2}m_2 v^2 - 0$$

$$\Rightarrow \quad v = \left(\sqrt{\frac{K}{m_2}}\right)x$$

$$v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{m_1(0) + m_2\left(\sqrt{\frac{K}{m_2}}\right)x}{m_1 + m_2} = \frac{(\sqrt{Km_2})x}{m_1 + m_2}$$

11. (C)

Using energy conservation in CM reference frame,

$$K_1 + U_1 = K_2 + U_2$$

$$\Rightarrow \quad \frac{1}{2}\left[\frac{3(6)}{3+6}\right][2-(-1)]^2 + 0 = \frac{1}{2}\left[\frac{3(6)}{3+6}\right](0)^2 + \frac{1}{2}\times 200\, x^2$$

$$\Rightarrow \quad x = 30 \text{ cm}$$

12. (B)

$$F = u\left(\frac{dm}{dt}\right) = 400 \times 0.05 = 20 \text{ N}$$

13. (B)

$$F = \frac{dm}{dt}v$$

$$F_{avg} = \frac{\Delta m}{\Delta t}v = \frac{5}{2.5}\times 4 = 8 \text{ dyne}$$

14. (A)

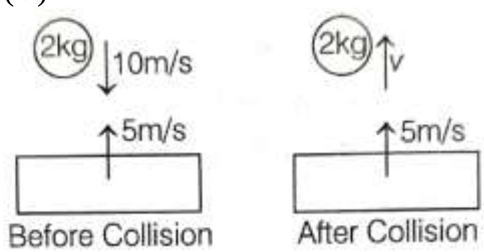
2. (a) $I_1 = m\left[\sqrt{2gh} - \left\{-\sqrt{2g\left(\frac{h}{4}\right)}\right\}\right] = \frac{3m}{2}\sqrt{2gh}$

$$I_2 = m\left[\sqrt{2gh} - \left\{-\sqrt{2g\left(\frac{h}{16}\right)}\right\}\right] = \frac{5}{4}m\sqrt{2gh}$$

$$m\sqrt{2gh} = \frac{2I_1}{3} = \frac{4}{5}I_2$$

$$\Rightarrow \quad 5I_1 = 6I_2$$

15. (D)



$$e = 1 = \frac{v-5}{10-(-5)}$$

$$\Rightarrow \quad v = 20 \text{ m/s}$$

$$I = m(v_2 - v_1) = 2[20-(-10)]$$

$$= 60 \text{ N-s}$$

16. (C)

5. (c) Let time be t when string again gets taut

$$s_1 = s_2 \Rightarrow 4t = 2t + \frac{1}{2}(10)t^2 \Rightarrow t = 0.4 \text{ s}$$

At t = 0.4 s, velocity of lower block

$$v = u + at = 2 + (10)(0.4) = 6 \text{ m/s}$$

Using linear momentum conservation,

$$4(4) + 4(6) = (4+4)v$$

$$v = 5 \text{ m/s}$$

$$I = \int T dt = 4(5-4) = 4 \text{ N-s}$$

17. (A)

7. (a) $mu + m(0) = mv + m(2v)$

$\Rightarrow \quad v = \frac{u}{3}$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{2v - v}{u - 0} = \frac{v}{u} = \frac{1}{3}$$

18. (A)

11. (a) $mu + m(0) = mv_1 + mv_2$

$\Rightarrow \quad v_1 + v_2 = u \quad \text{...(i)}$

$$e = \frac{v_2 - v_1}{u - 0} \Rightarrow v_2 - v_1 = eu \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_1 = \frac{(1-e)u}{2},\ v_2 = \frac{(e+1)u}{2}$$

$$\frac{v_1}{v_2} = \left(\frac{1-e}{1+e}\right)$$

19. (A)

Speed of 2 kg just before collision,

$$v^2 = u^2 + 2as$$

$\Rightarrow \quad v^2 = (1)^2 + 2(-0.2 \times 10)(0.16)$

$\Rightarrow \quad v = 0.6 \text{ m/s}$

Using linear momentum conservation,

$$2(0.6) + 4(0) = 2v_1 + 4v_2$$

$$v_1 + 2v_2 = 0.6 \quad \text{... (i)}$$

$$e = \frac{v_2 - v_1}{0.6 - 0} = 1$$

$\Rightarrow \quad v_2 - v_1 = 0.6 \quad \text{...(ii)}$

Solving Eqs. (i) and (ii), we get

$$v_1 = -0.2 \text{ m/s and } v_2 = 0.4 \text{ m/s}$$

Separation between them when they come to rest,

$$= \frac{v_1^2}{2\mu g} + \frac{v_2^2}{2\mu g} = 0.05 \text{ m} = 5 \text{ cm}$$

20. (A)

13. (a) When A collides with B, they will exchange their velocities.

For collision between B and C

$$mv + 4m(0) = mv_1 + 4mv_2$$

$$\Rightarrow \quad v_1 + 4v_2 = v \quad \text{...(i)}$$

$$e = \frac{v_2 - v_1}{v - 0} = 1 \Rightarrow v_2 - v_1 = v \quad \text{... (ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_1 = -0.6v, \; v_2 = 0.4\,v$$

Now, B will collide with A and they will exchange their velocities. So, finial velocity of A will be 0.6 v, left.

21. (B)

17. (b) Velocity of ball B just after collision $= \sqrt{2gh}$

$$= \sqrt{2 \times 10 \times 5}$$

$$= 10 \text{ m/s}$$

Using linear momentum conservation,

$$m(16) + m(0) = mv_1 + m(10)$$

$$\Rightarrow \quad v_1 = 6 \text{ m/s}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{10 - 6}{16 - 0} = \frac{1}{4}$$

22. (D)

21. (d) $T = \sqrt{\frac{2h}{g}} + \frac{2(e\sqrt{2gh})}{g} + \frac{2(e^2\sqrt{2gh})}{g} + \frac{2(e^3\sqrt{2gh})}{g} + \dots$

$$= \sqrt{\frac{2h}{g}}(1 + 2e + 2e^2 + 2e^3 + \dots)$$

$$= \sqrt{\frac{2h}{g}}[1 + 2e(1 + e + e^2 + \dots)]$$

$$= \sqrt{\frac{2h}{g}}\left(1 + 2e\left[\frac{1}{1-e}\right]\right)$$

$$= \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$$

23. (D)

27. (d) $mu + 0 = mv_1 + nmv_2$

$\Rightarrow \quad v_1 + nv_2 = u \quad$ ...(i)

$$e = \frac{v_2 - v_1}{u - 0} = 1$$

$\Rightarrow \quad v_2 - v_1 = u \quad$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$v_2 = \frac{2u}{(n+1)}$$

$$K_2 = \frac{1}{2}(nm)v_2^2$$

Fraction of incident energy transferred to the heavier ball

$$= \frac{\frac{1}{2}(nm)\left(\frac{2u}{n+1}\right)^2}{\frac{1}{2}mu^2} = \frac{4n}{(1+n)^2}$$

24. (A)

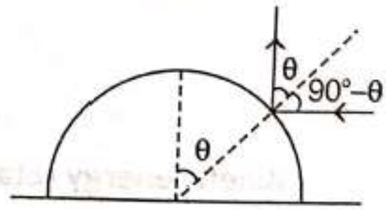
31. (a) $\tan\phi = \frac{\tan\theta}{e} = \frac{\tan 45^\circ}{\left(\frac{1}{\sqrt{2}}\right)}$

$\Rightarrow \quad \phi = \tan^{-1}(\sqrt{2})$

$$v' = \sqrt{(ev\cos\theta)^2 + (v\sin\theta)^2} = \frac{\sqrt{3}}{2}v$$

25. (D)

32. (d) $\tan\phi = \dfrac{\tan\theta}{e}$



$$\Rightarrow \quad \tan\theta = \frac{\tan(90^\circ - \theta)}{(1/3)}$$

$$\Rightarrow \quad \tan^2\theta = 3$$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \quad \theta = 60^\circ$$

26. (A)

$$\mathbf{n} = (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}) = -2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 8\hat{\mathbf{k}}$$

$$e = \frac{v_{\text{sep}}}{v_{\text{app}}} = \frac{|\mathbf{v}\cdot\mathbf{n}|}{|\mathbf{u}\cdot\mathbf{n}|}$$

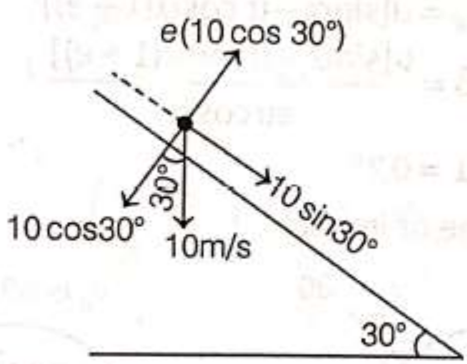
$$= \frac{-4 - 2 + 24}{|-8 - 3 - 40|}$$

$$= \frac{18}{51} = \frac{6}{17}$$

27. (A)

Velocity of ball just before hitting the inclined plane

$$= \sqrt{2gh} = 10\,\text{m/s}$$



$$T = \frac{2u_y}{g\cos 30^\circ} = \frac{2[0.5(10\cos 30^\circ)]}{10\cos 30^\circ} = 1\,\text{s}$$

28. (B)

(b) $F_{ext} + F_{Thrust} = ma$

$\Rightarrow \quad -mg + \frac{dm}{dt}v_{rel} = ma$

$\Rightarrow \quad -5000\times 10 + \frac{dm}{dt}(800) = 5000\times 20$

$\Rightarrow \quad \frac{dm}{dt} = 187.5 \text{ kgs}^{-1}$

29. (C)

(c) $F_{Thrust} = m\frac{dv}{dt}$

$\Rightarrow \quad -\frac{dm}{dt}v_{rel} = m\frac{dv}{dt}$

$\Rightarrow \quad \int_0^v dv = -\int_{m_0}^{m_0/2}\frac{dm}{m}v_{rel}$

$\Rightarrow \quad v = 2\ln 2$

30. (9)

$$x_{CM} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2} = \frac{\sigma\pi(28)^2(0) - \sigma\pi(21)^2(7)}{\sigma\pi(28)^2 - \sigma\pi(21)^2} = -9 \text{ cm}$$

31. (4)

$$x_{CM} = \frac{\int x\,dm}{\int dm} = \frac{\int x\rho\pi y^2 dx}{\int \rho\pi y^2 dx} = \frac{\int_0^6 x\rho\pi\left(\frac{x}{k}\right)dx}{\int_0^6 \rho\pi\left(\frac{x}{k}\right)dx} = 4m$$

$y_{CM} = 0$

So, $\alpha + \beta = 4 + 0 = 4$

32. (3)

$$(s_{CM})_x = \frac{m_1s_1 + m_2s_2}{m_1 + m_2}$$

$\Rightarrow \quad 0 = \frac{2s_m + 1(S_m - 9)}{2+1} \Rightarrow s_m = 3\text{m}$

33. (8)

$$(s_{CM})_x = \frac{m_1 s_1 + m_2 s_2}{m_1 + m_2}$$

$$\Rightarrow \quad 0 = \frac{50(-3 + s_p) + 100 s_p}{50 + 100}$$

$$\Rightarrow \quad s_p = 1 \text{ m}$$

$$s_m = -3 + 1 = -2\text{m}$$

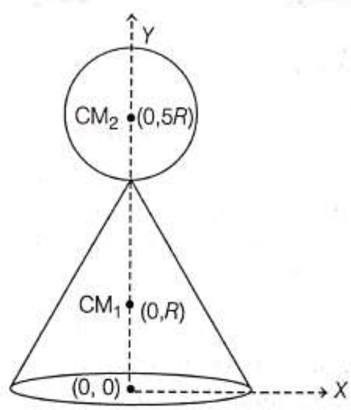
Distance travelled by plank in 5 s $= 2 \times 5 = 10$ m

Distance travelled by man w.r.t. ground $= 10 - 2 = 8$ m

34. ***4R* from *O***

Mass of cone $= \rho\left[\frac{1}{3}\pi(2R)^2(4R)\right] = \frac{16}{3}\rho\pi R^3$

Mass of sphere $= 12\rho\left(\frac{4}{3}\pi R^3\right) = 16\rho\pi R^3$



Centre of mass of solid cone is $\frac{H}{4} = \frac{4R}{4} = R$ above base.

$$y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$= \frac{\frac{16}{3}\rho\pi R^3 (R) + 16\rho\pi R^3 (5R)}{\frac{16}{3}\rho\pi R^3 + 16\rho\pi R^3} = 4R$$

35. $\frac{2r}{3(4-\pi)}$

Let mass per unit area of plate be σ.

m_1 = mass of rectangular plate $= \sigma(2r \times r) = 2\sigma r^2$,

m_2 = mass of semi-circular plate $= \sigma\left(\frac{\pi r^2}{2}\right)$

$$x_{CM} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$$

$$= \frac{(2\sigma r^2)\left(\frac{r}{2}\right) - \sigma\left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)}{2\sigma r^2 - \sigma\left(\frac{\pi r^2}{2}\right)}$$

$$= \frac{2r}{3(4-\pi)}$$

36. **g/4**

37. **1670 N**

$\Sigma F_{ext} = Ma_{CM} = m_1 a_1 + m_2 a_2 + m_3 a_3 + m_4 a_4$

$\Rightarrow \quad T - 40g - 60g - 50g - 20g$

$= 40(-2) + 60(0) + 50(1) + 20(0)$

$\Rightarrow \quad T = 1670$ N

38. ***L*/4**

Let displacement of plank $= s_p$

Displacement of man w.r.t. plank $= s_{mp} = L$

$\Rightarrow \quad s_m - s_p = L \Rightarrow s_m = L + s_p$

Since, external forces on (man + plank) system are zero in horizontal direction and initial velocity of CM of (man + plank) system is zero,

$\Rightarrow \quad (s_{CM})_x = 0 \Rightarrow \frac{m_1 s_p + m_2 s_m}{m_1 + m_2} = 0$

$\Rightarrow \quad \frac{3Ms_p + M(L + s_p)}{4M} = 0$

$\Rightarrow \quad s_p = -\frac{L}{4} = \frac{L}{4}$, left

39. **40/3 cm**

Let displacement of boat $= s_b$

Displacement of Ram w.r.t. boat $= s_{Ab} = +2$

$\Rightarrow \quad s_A = 2 + s_b$

Displacement of Shyam w.r.t. boat $= s_{Bb} = -2$

$\Rightarrow \quad s_B = -2 + s_b$

$$(s_{CM})_x = 0$$

$$\Rightarrow \quad \frac{40s_b + 50(2+s_b) + 60(-2+s_b)}{40+50+60} = 0$$

$$\Rightarrow \quad s_b = \frac{2}{15}\text{ m} = \frac{40}{3}\text{ cm}$$

40. **$h\cot\theta/6$**

Let displacement of wedge $= s_w$

Displacement of block w.r.t. wedge in horizontal

$= s_{bw} = -h\cot\theta$

$\Rightarrow \quad s_b = -h\cot\theta + s_w$

$$(s_{CM})_x = 0 \quad \Rightarrow \quad \frac{m(-h\cot\theta + s_w) + 5m\,s_w}{m+5m} = 0$$

$$\Rightarrow \quad s_w = \frac{h\cot\theta}{6}$$

41. (871.5)

For 8 kg block, $v^2 = u^2 + 2as$

$\Rightarrow \quad 0^2 = u^2 + 2(-0.5 \times 9.8)(0.8)$

$\Rightarrow \quad u = 2.8$ m/s

For 6 kg block, $v^2 = u^2 + 2as$

$\Rightarrow \quad 0^2 = u^2 + 2(-0.5 \times 9.8)(1.25)$

$\Rightarrow \quad u = 3.5$ m/s

Applying conservation of linear momentum,

$0.05\,v = 8 \times 2.8 + 6.05 \times 3.5$

$\Rightarrow \quad v = 871.5$ m/s

42. (32)

Applying linear momentum conservation for throwing of sack,

$$0+0=150\ v_{boat}+50(6)$$
$$v_{boat}=-2\ \text{m/s}$$

Applying linear momentum conservation for landing of sack in the second boat,

$$50\times6+150\times0=200\ v'_{boat}$$
$$v'_{boat}=1.5\ \text{m/s}$$
$$T=\frac{2u_y}{g}\ \Rightarrow\ 0.5=\frac{2u_y}{10}\Rightarrow u_y=2.5\ \text{m/s}$$
$$R=\frac{2u_xu_y}{g}=\frac{2(6)(2.5)}{10}=3\ \text{m}$$

Initial separation between boats = 3m

Distance travelled by first boat while the sack is in the air

$=2\times0.5=1$ m

Distance $=u_{rel}t=[2-(-1.5)](8)=28$ m

Total distance $=3+1+28=32$ m

43. (22)

$$T=\sqrt{\frac{2H}{g}}-\sqrt{\frac{2h}{g}}=\frac{1}{\sqrt{g}}(\sqrt{2\times50}-\sqrt{2\times18})=\frac{4}{\sqrt{g}}$$
$$s_x=u_xt\Rightarrow1=u_x\left(\frac{4}{\sqrt{g}}\right)\Rightarrow u_x=\frac{\sqrt{g}}{4}$$

Applying conservation of linear momentum in horizontal,

$$\Rightarrow\quad 0+0=-2v_{bag}+56\left(\frac{\sqrt{g}}{4}\right)$$
$$\Rightarrow\quad v_{bag}=7\sqrt{g}=7\times\frac{22}{7}=22\ \text{m/s}$$

44. (60°)

According to work-energy theorem

$W=$ Change in kinetic energy

$$Fs\cos\theta=\frac{1}{2}mv^2-\frac{1}{2}mu^2$$

Substituting the given values, we get;

$$20\times 4\times \cos\theta = 40-0 \qquad [\because\ u=0]$$

$$\cos\theta = \frac{40}{80} = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ .$$

45. **15 m**

$$u_{CM} = \frac{m_1u_1 + m_2u_2}{m_1+m_2} = \frac{2(0)+4(15)}{2+4} = 10 \text{ m/s} \uparrow$$

$$a_{CM} = \frac{m_1a_1 + m_2a_2}{m_1+m_2} = \frac{2g+4g}{2+4} = g\downarrow$$

$$H_{max} = \frac{u_{CM}^2}{2a_{CM}} = \frac{(10)^2}{2(10)} = 5 \text{ m}$$

Initial height of CM from ground

$$= \frac{m_1y_1 + m_2y_2}{m_1+m_2}$$

$$= \frac{2(30)+4(0)}{2+4}$$

$$= 10 \text{ m}$$

Maximum height attained by CM from the ground $= 10+5 = 15$ m

46. (a) **288 J** (b) **384 J**

(a) Applying conservation of linear momentum,

$16(0) = 4v_1 + 12(4)$

$\Rightarrow \qquad v_1 = -12$ m/s

Kinetic energy of 4 kg mass $= \frac{1}{2}(4)(12)^2 = 288$ J

(b) Energy released $= K_1 + K_2 = \frac{1}{2}(4)(12)^2 + \frac{1}{2}(12)(4)^2$

$= 384$ J

47. (a) $v/\sqrt{2}$ (b) $(3/2)\,mv^2$

(a) Applying conservation of linear momentum,

$$4m(0)=m(v\hat{i})+m(v\hat{j})+2m\mathbf{v}$$

$$\Rightarrow \quad \mathbf{v}=-\frac{v}{2}\hat{i}-\frac{v}{2}\hat{j}$$

Speed of third fragment $=\sqrt{\left(\frac{v}{2}\right)^2+\left(\frac{v}{2}\right)^2}=\frac{v}{\sqrt{2}}$

(b) Energy released

$$=\frac{1}{2}mv^2+\frac{1}{2}mv^2+\frac{1}{2}(2m)\left(\frac{v}{\sqrt{2}}\right)^2=\frac{3}{2}mv^2$$

48. **$3\sqrt{gl}$**

Applying conservation of linear momentum

$$mv+2m(0)=3mv'$$

$$\Rightarrow \quad v'=\frac{v}{3}$$

Applying work-energy theorem after collision,

$W_{mg}+W_T=\Delta K$

$$\Rightarrow \quad -3mgl(1-\cos 60^\circ)+0=0-\frac{1}{2}(3m)\left(\frac{v}{3}\right)^2$$

$$\Rightarrow \quad v=3\sqrt{gl}$$

49. **220 m/s**

Speed of block just after collision $=\sqrt{2gh}$

$$=\sqrt{2\times 9.8\times 0.1}$$

$$=1.4 \text{ m/s}$$

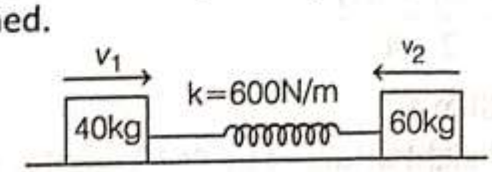
Applying conservation of linear momentum,

$$\left(\frac{10}{1000}\right)(500)+2\times 0=\left(\frac{10}{1000}\right)v+2\times 1.4$$

$$\Rightarrow \quad v=220 \text{ m/s}$$

50. **$v_1=4.5 \text{ m/s},\ v_2=3 \text{ m/s}$**

Let their velocities be v_1 and v_2 when spring becomes unstrectched.



Applying conservation of linear momentum,

$$40(0)+60(0)=40v_1+60(-v_2)$$

$$\Rightarrow \quad v_1=\frac{3v_2}{2} \quad \text{... (i)}$$

Applying conservation of mechanical energy,

$$K_1+U_1=K_2+U_2$$

$$\Rightarrow (0+0)+\frac{1}{2}\times 600\times(1.5)^2=\left(\frac{1}{2}\times 40v_1^2+\frac{1}{2}\times 60v_2^2\right)+0$$

$$\Rightarrow \quad 2v_1^2+3v_2^2=67.5 \quad \text{... (ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_1=4.5 \text{ m/s and } v_2=3 \text{ m/s}$$

51. **10 m/s**

Let velocity of platform $= v_p$

Velocity of man w.r.t. platform $= v_{mp} = 30$

$$\Rightarrow \quad v_m - v_p = 30$$

$$\Rightarrow \quad v_m = (30+v_p)$$

Applying linear momentum conservation for (man + platform) system in horizontal,

$$0+0=100(30+v_p)+200v_p$$

$$\Rightarrow \quad v_p=-10 \text{ m/s}$$

52. $\dfrac{m^2u}{(M+2m)(M+m)}$

Applying conservation of linear momentum when A jumps

$$0+0=(M+m)v_c+m(-u+v_c)$$

$$\Rightarrow \quad v_c=\frac{mu}{M+2m}$$

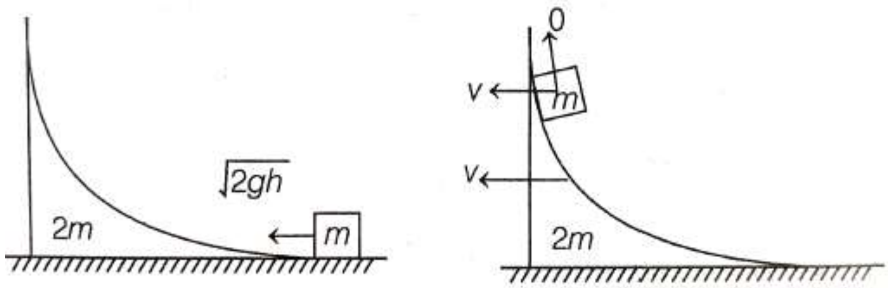
Applying conservation of linear momentum when B jumps

$$(M+m)\left(\frac{mu}{M+2m}\right)=Mv_c'+m(u+v_c')$$

$$\Rightarrow \quad v_c'=-\frac{m^2u}{(M+2m)(M+m)}$$

53. (a) $\sqrt{\frac{2gh}{9}}$ (b) $\frac{2h}{3}$

(a)



When the smaller block reaches maximum height from ground, its velocity w.r.t. the larger block is zero.

Using conservation of linear momentum,

$$2m(0)+m(\sqrt{2gh})=2mv+mv$$

$$\Rightarrow \quad v=\frac{\sqrt{2gh}}{3}$$

(b) Using energy conservation,

$$K_1+U_1=K_2+U_2$$

$$\Rightarrow \left[\frac{1}{2}m(\sqrt{2gh})^2+0\right]+0=\frac{1}{2}(3m)\left(\frac{\sqrt{2gh}}{3}\right)^2+mgh_{max}$$

$$\Rightarrow \quad h_{max}=\frac{2h}{3}$$

54. (3)

3. For ball B, $T-mg=\frac{mv_2^2}{l}$

$$\Rightarrow \quad 40-10=\frac{1v_2^2}{0.3} \Rightarrow v_2=3 \text{ m/s}$$

Using momentum conservation,

$$2v_0+0=2v_1+1(3)$$

$$2v_0=2v_1+3 \quad \text{... (i)}$$

$$e=\frac{1}{2}=\frac{3-v_1}{v_0-0}$$

$$\Rightarrow \quad 6-2v_1=v_0 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_0=3 \text{ m/s}$$

55. (4)

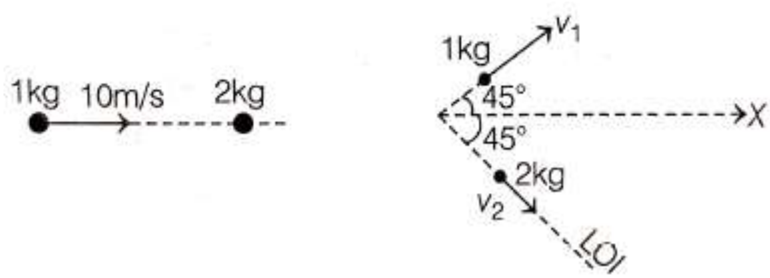
7. Collision doesn't affect the vertical component of velocity. So, total time of flight will be unchanged.

$$T = t_1 + t_2$$

$$\Rightarrow \quad \frac{2u\sin 30^\circ}{g} = \frac{50}{u\cos 30^\circ} + \frac{50}{eu\cos 30^\circ}$$

$$\Rightarrow \quad u = \frac{100}{\sqrt{7}} \text{ m/s}$$

56. **1/2**



Using linear momentum conservation,

In X-direction,

$$1 \times 10 + 2 \times 0 = 1v_1 \cos 45^\circ + 2v_2 \cos 45^\circ$$

$$\Rightarrow \quad v_1 + 2v_2 = 10\sqrt{2} \quad \text{... (i)}$$

In Y-direction, $0 + 0 = 1v_1 \sin 45^\circ - 2v_2 \sin 45^\circ$

$$\Rightarrow \quad v_1 = 2v_2 \quad \text{... (ii)}$$

Solving Eqs. (i) and (ii), we get

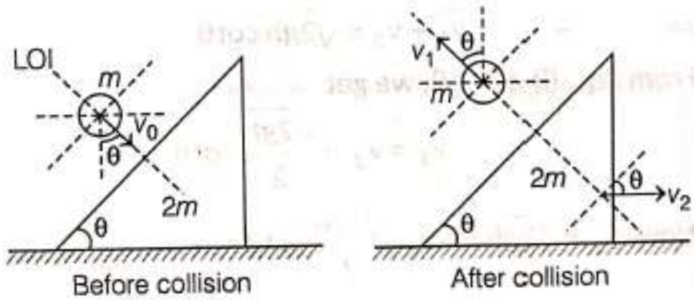
$$v_1 = \frac{10}{\sqrt{2}} \text{ m/s and } v_2 = \frac{5}{\sqrt{2}} \text{ m/s}$$

Line of impact will be along the line of motion of 2 kg after collision.

$$= \frac{v_2 - v_1 \cos 90^\circ}{10\cos 45^\circ - 0} = \frac{5/\sqrt{2}}{10/\sqrt{2}} = \frac{1}{2}$$

57. $\frac{(1+e)v_0 \sin\theta}{2+\sin^2\theta}$

Using linear momentum consevation for (wedge + ball) system is horizontal,



$$mv_0 \sin\theta = -mv_1 \sin\theta + 2mv_2$$

$\Rightarrow$ $$2v_2 - v_1 \sin\theta = v_0 \sin\theta \quad \text{...(i)}$$

$$e = \frac{v_{sep}}{v_{app}} = \frac{v_1 + v_2 \sin\theta}{v_0}$$

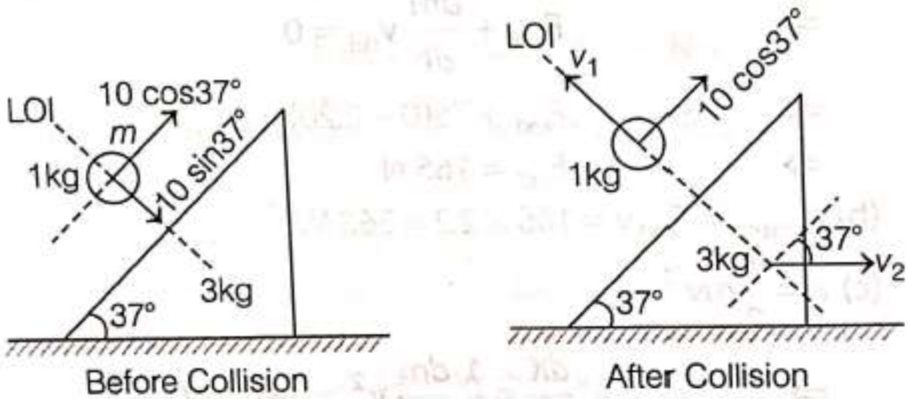
$\Rightarrow$ $$v_1 + v_2 \sin\theta = ev_0 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_2 = \frac{(e+1)v_0 \sin\theta}{2 + \sin^2\theta}$$

58. $v_{\text{Wedge}} = 1.5 \text{ m/s}$

8. Component of velocity of ball perpendicular to the line of impact remains unchanged.



Using conservation of momentum for (wedge + ball) system in horizontal

$1(10)+3(0)=3v_2+1((10\cos 37°)\cos 37°-v_1\sin 37°)$

$\Rightarrow \quad 5v_2-v_1=6 \quad$ ...(i)

$$e=\frac{v_2\sin 37°-(-v_1)}{10\sin 37°}=0.4$$

$\Rightarrow \quad 5v_1+3v_2=12 \quad$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$v_2=1.5\text{ m/s}$$

59. (a) 5.32×10^5 N (b) 3.22×10^5 N (c) $v_0-gt+v_{rel}\ln\frac{M_0}{M}$ (d) 2718 m/s

(a) $F_{Thrust}=\frac{dm}{dt}v_{rel}=190\times2800$

$=5.32\times10^5$ N

(b) $F_{net}=F_{Thrust}-mg$

At blast off, $F_{net}=5.32\times10^5-(21000)(10)$

$=3.22\times10^5$N

(c) $\sum F = m\frac{dv}{dt}$

$$\Rightarrow \quad F_{ext} + F_{Thrust} = m\frac{dv}{dt}$$

$$\Rightarrow \quad -mg + \left(-\frac{dm}{dt}\right)v_{rel} = \frac{m\,dv}{dt}$$

$$\Rightarrow \quad -\int_0^t g\,dt - \int_{M_0}^{M} \frac{dm}{m} v_{rel} = \int_{v_0}^{v} dv$$

$$\Rightarrow \quad -gt + \left[\ln\left(\frac{M_0}{M}\right)\right]v_{rel} = v - v_0$$

$$\Rightarrow \quad v = v_0 - gt + v_{rel}\ln\left(\frac{M_0}{M}\right)$$

(d) Time taken to burn all the fuel $= \frac{15000}{190}\ s = 78.95\ s$

$$v = 0 - 10(78.95) + 2800\ln\left(\frac{21000}{6000}\right)$$

$$= 2718\ m/s$$

60. (a) $\frac{M}{L}\left(gy + v_0^2\right)$ (b) $\frac{Myv_0^2}{2L}$

(a) $F_{Thrust} = \frac{dm}{dt}v_{rel} = \frac{d\left(\frac{M}{L}y\right)}{dt}(0 - v_0) = -\frac{M}{L}v_0^2$

$$F_{ext} + F_{Thrust} = m\frac{dv}{dt}$$

$$\Rightarrow \quad P - \frac{M}{L}yg - \frac{M}{L}v_0^2 = 0$$

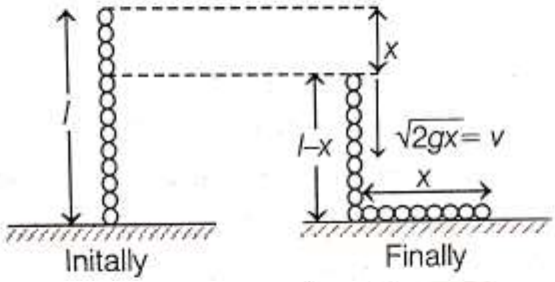
$$\Rightarrow \quad P = \frac{M}{L}(gy + v_0^2)$$

(b) Energy lost during the lifting = Work done by applied force – Increase in mechanical energy of chain

$$= \int_0^y P\,dy - \left(\left(\frac{m}{L}y\right)g\left(\frac{y}{2}\right) + \frac{1}{2}\left(\frac{M}{L}y\right)v_0^2\right)$$

$$= \frac{Myv_0^2}{2L}$$

61. $3mg\,\frac{x}{I}$



$$F_{Thrust} = \frac{dm}{dt} v_{rel} = \frac{d\left(\frac{m}{l}x\right)}{dt}(v-0)$$

$$= \frac{m}{l}\frac{dx}{dt}v = \frac{m}{l}v^2 = \frac{2mgx}{l}$$

$$N = F_{Thrust} + \left(\frac{m}{l}x\right)g$$

$$= \frac{2mgx}{l} + \frac{mgx}{l} = \frac{3mgx}{l}$$

PYQ : JEE Main

1. (b)

Given, $m_1 = 4g, u_1 = 300\,m/s$

$m_2 = 0.8\,kg = 800\,g, u_2 = 0\,m/s$

From law of conservation of momentum,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Let the velocity of combined system $= v\,m/s$ then,

$$4\times300 + 800\times0 = (800+4)\times v \Rightarrow v = \frac{1200}{804} = 1.49\,m/s$$

Now, $\mu = 0.2(\text{given})$

$a = \mu g \Rightarrow a = 0.3\times10 \quad (\text{take } g = 10\,m/s^2)$

$= 3\,m/s^2$

Then, from $v^2 = u^2 + 2as$

$$(1.49)^2 = 0 + 2\times3\times s \Rightarrow s = \frac{(1.49)^2}{6} s = \frac{2.22}{6} = 0.379\,m$$

2. (a)

$$Z_0 = h - \frac{h}{4} = \frac{3h}{4}$$

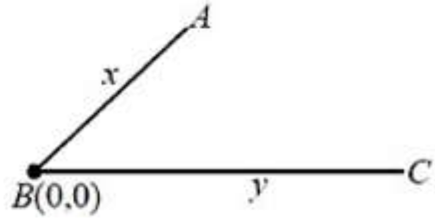
3. (b)

$$E_{initial} = \frac{1}{2}m(2v)^2 + \frac{1}{2}2nm(v)^2 = 3\,mv^2$$

$$E_{final} = \frac{1}{2}3m\left(\frac{4}{9}v^2 + \frac{4}{9}v^2\right) = \frac{4}{3}mv^2$$

$$\therefore \quad \text{Fractional loss} = \frac{3-\frac{4}{3}}{3} = \frac{5}{9} = 56\%$$

4. (c)



$$x_{cm} = \frac{x}{2}\frac{(\rho x)\left(\frac{x}{2}\right)\frac{1}{2} + \rho\, y^{\frac{x}{2}}}{\rho(x+y)}$$

$$\Rightarrow \frac{1}{2}+\frac{y}{x}=\frac{y^2}{x^2}$$

$$\Rightarrow \frac{y}{x}=\frac{1+\sqrt{3}}{2}=1.37$$

5. (b)

From impulse momentum theorem

$$\int_0^1 6t\,dt = mv$$

$\therefore\ v = 3\,m/s$

So, work done by the force $=\Delta K.E.=\frac{1}{2}(1)(3)^2=4.5\text{ J}$

6. (a)

According to law of conservation of linear momentum vertical component,

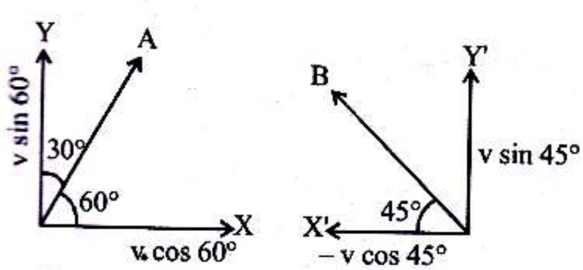
$$2mv'\sin\theta = mv\sin 60^\circ + mv\sin 45^\circ$$

$$2mv'\sin\theta = \frac{mv}{\sqrt{2}}+\frac{mv\sqrt{3}}{2} \quad \ldots\ldots(i)$$

Horizontal component,

$$2mv'\cos\theta = mv\sin 60^\circ - mv\cos 45^\circ$$

$$2mv'\cos\theta = \frac{mv}{2}+\frac{mv}{\sqrt{2}} \quad \ldots\ldots(ii)$$



For particle A   For particle B

Dividing equation (i) by equation(ii),

$$\tan\theta = \frac{\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}}{\frac{1}{2}-\frac{1}{\sqrt{2}}}=\frac{\sqrt{2}+\sqrt{3}}{1-\sqrt{2}}$$

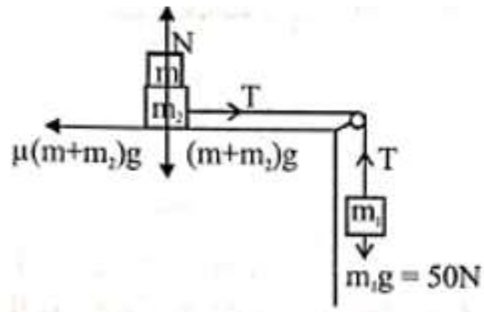
7. (b)

Given : $m_1 = 5$ kg; $m_2 = 10$ κγ; $\mu = 0.15$ m

FBD for m_1, $m_1g - T = m_1a$

$\Rightarrow \quad 50 - T = 5\times a$ and $T - 0.15(m+10)g$

$= (10+m)a$



From rest $a = 0$

Or, $50 = 0.15(m+10)10$

$\Rightarrow \quad 5 = \frac{3}{20}(m+10)$

$\frac{100}{3} = m+10 \quad \therefore \; m = 23.3$ kg ; close to option (b)

8. (b)

$2MV, \cos 30° + mv_2 \cos 45° = 10\,M\cos 30° + 10\cos 45°$

$\Rightarrow \quad v_1\sqrt{3} + \frac{v_2}{\sqrt{2}} = 5\sqrt{3} + 5\sqrt{2} \qquad ...(i)$

$2MV, \sin 30° - mV_2 \sin 45° = -10\,m\sin 30° + 10M\sin 45°$

$V_1 - \frac{V_2}{\sqrt{2}} = -5 + 5\sqrt{2} \qquad ...(ii)$

$V_1 = \frac{5(\sqrt{3}-1)+10\sqrt{2}}{\sqrt{3}+1} = \frac{17.5}{2.7} = 6.5$ m/s

$V_2 = 6.3$ m/s

9. (c)

$k_f = 1.5\,k_i$

$v_1^2 + v_2^2 = 1.5\,v_0^2$

From conservation of momentum

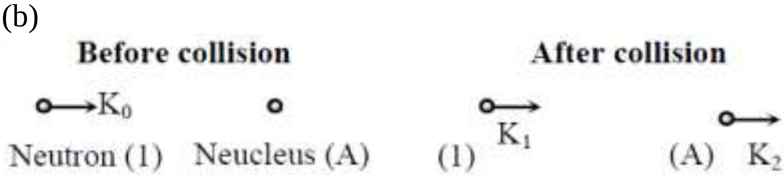
$v_1 + v_2 = v_0$

From (i) and (ii)

$2v_1v_2 = -0.5\,v_0^2$

So, $v_2 - v_1 = \sqrt{v_2^2 + v_1^2 - 2v_1v_2} = \sqrt{1.5v_0^2 + 0.5v_0^2} = \sqrt{2}\,v_0$

10. (b)



$\therefore \quad \sqrt{K_0} = \sqrt{K_1} + \sqrt{AK_2}$ (from conservation of momentum) and $K_0 = K_1 + K_2$ (for elastic collision)

So after solving

$(1+A)\frac{K_1}{K_2} - 2\sqrt{\frac{K_1}{K_0}} = (A-1)$

For Deuterium, $A = 2, 1 - \frac{K_1}{K_0} = 0.89$

For Carbon, $A = 12, A - \frac{K_1}{K_0} = 0.28$

11. (b)

$P = \frac{(2mv\cos 45^\circ)n}{A}$ (P → Pressure, A → Area)

$= \frac{2 \times 3.32 \times 10^{-27} \times 10^{3} \times \frac{1}{\sqrt{2}} \times 10^{23}}{2 \times 10^{-4}}$

$= 2.35 \times 10^{3}\ N/m^2$

12. (b)

$v_{s,m} = v_s - v_m \Rightarrow 0.7 = v_s - v_m$

$P_i = P_f$

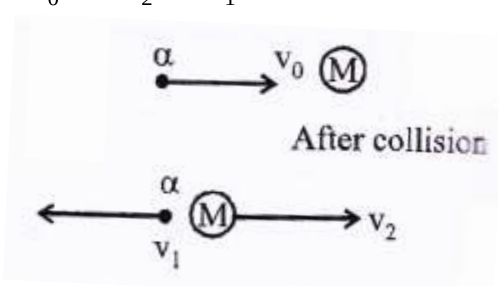
$0 = 20(0.7 - v) - 50v$

Or $v = 0.2 m/s$

13. (d)

Using conservation of momentum

$mv_0 = Mv_2 - mv_1$



$\frac{1}{2}mv_1^2 = 0.36 \times \frac{1}{2}mv_0^2$ $[\therefore M = \text{mass of nucleus}]$

$\Rightarrow v_2 = \sqrt{\frac{m}{M}} \times 0.8v_0$

$mv_0 = \sqrt{mM} \times 0.8v_0 - m \times 0.6v_0$

$\Rightarrow 1.6m = 0.8\sqrt{mM} \Rightarrow 4m^2 = mM$ $\therefore M = 4m$

14. (c)

Kinetic energy of block A

$k_1 = \frac{1}{2}mv_0^2$

$\therefore$ From principle of linear momentum conservation

$$mv_0 = (2m+M)v_f \Rightarrow v_f = \frac{mv_0}{2m+M}$$

According to question, if $\frac{5}{6}$th the initial kinetic energy is lost in while process.

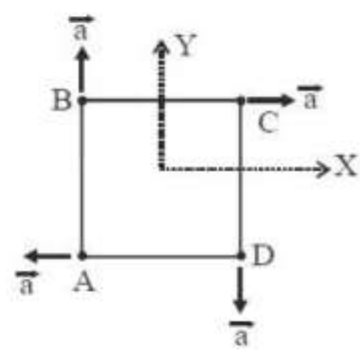
$$\therefore \frac{k_i}{k_f} = 6 \Rightarrow \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}(2m+M)\left(\frac{mv_0}{2m+M}\right)^2} = 6$$

$$\Rightarrow \frac{2m+M}{m} = 6 \qquad \therefore \frac{M}{m} = 4$$

15. (a)

$\vec{a}_A = -a\hat{i}$; $\vec{a}_B = a\hat{j}$

$\vec{a}_c = a\hat{i}$; $\vec{a}_D = -a\hat{j}$



$$\vec{a}_{cm} = \frac{m_a\vec{a}_a + m_b\vec{a}_b + m_c\vec{a}_c + m_d\vec{a}_d}{m_a + m_b + m_c + m_d}$$

$$\vec{a}_{cm} = \frac{-ma\hat{i} + 2m\hat{j} + 3ma\hat{i} - 4ma\hat{j}}{10m}$$

$$= \frac{2ma\hat{i} - 2ma\hat{j}}{10m} = \frac{a}{5}\hat{i} - \frac{a}{5}\hat{j} = \frac{a}{5}(\hat{i} - \hat{j})$$

16. (c)

Applying linear momentum conservation

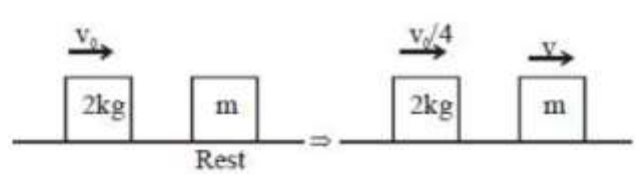
$$m_1v_1\hat{i} + m_2v_2\hat{i} = m_1v_3\hat{i} + m_2v_4\hat{i}$$

$$m_1v_1 + 0.5m_1v_2 = m_1(0.5v_1) + 0.5m_1v_4$$

$$v_1 = v_4 - v_2$$

17. (b)

By conservation of linear momentum: n



$$2v_0 = 2\left(\frac{v_0}{4}\right) + mv \Rightarrow 2v_0 = \frac{v_0}{2} + mv\,2$$

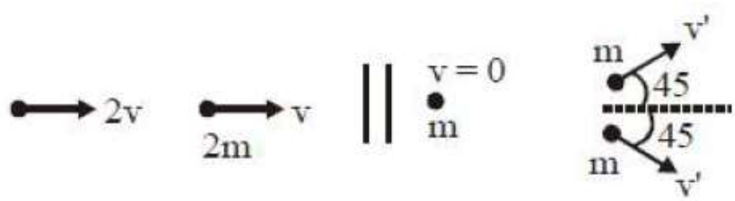
$$\Rightarrow \frac{3v_0}{2} = mv \qquad ...(1)$$

Since collision is elastic $\rightarrow$

$$V_{seperation} = v_{approch}$$

$$\Rightarrow v - \frac{v_0}{4} = v_0 \Rightarrow m = \frac{6}{5} = 1.2\text{ kg}$$

18. (b)



Linear momentum conservation

$$m\,2v + 2m\,v = m \times 0 + m\frac{v'}{\sqrt{2}} \times 2$$

$$v' = 2\sqrt{2}\,v\,.$$

19. (b)

Given,

Mass of block, $m_1 = 1.9\text{kg}$

Mass of bullet, $m_2 = 0.1\text{kg}$

Velocity of bullet, $\nu_2 = 20\,m/s$

Let ν be the velocity of the combined system. It is an inelastic collision.

Using conservation of linear momentum

$$m_1 \times 0 + m_2 \times \nu_2 = (m_1 + m_2)\nu$$

$$\Rightarrow 0.1 \times 20 = (0.1 + 1.9) \times \nu = 1m/s$$

Using work energy theorem

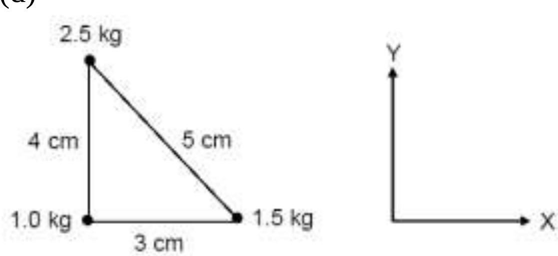
Work done = Change in Kinetic energy

Let K be the Kinetic energy of combined system. $(m_1 + m_2)gh$

$$= K - \frac{1}{2}(m_1 + m_2)v^2$$

$$\Rightarrow 2 \times g \times 1 = K - \frac{1}{2} \times 2 \times 1^2 \Rightarrow K = 21J$$

20. (d)

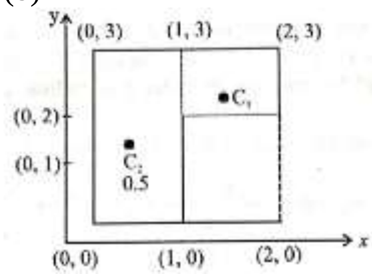


Take 1 kg mass at origin

$$X_{cm} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{5} = 0.9\text{ cm}$$

$$Y_{cm} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{5} = 2\text{ cm}$$

21. (b)



For given Lamina

$$m_1 = 1, C_1 = (1.5, 2.5)$$

$$m_2, C_2 = (0.5, 1.5)$$

$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1m_2} = \frac{1.5+1.5}{4} = 0.75$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{2.5+4.5}{4} = 1.75$$

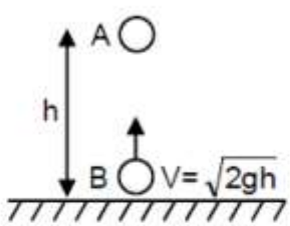
$\therefore$ Coordinate of centre of mass of flag shaped lamina (0.75, 1.75)

22. (d)

Time for collision $t_1 = \frac{h}{\sqrt{2gh}}$

After t_1 $V_A = 0 - gt_1 = -\sqrt{\frac{gh}{2}}$

and $V_B = \sqrt{2gh} - gt_1 = \sqrt{gh}\left[\sqrt{2} - \frac{1}{\sqrt{}}\right]$



At the time f collision

$$\vec{P}_i = \vec{P}_f$$

$$\Rightarrow \quad m\vec{V}_A + m\vec{V}_B = 2m\vec{V}_f$$

$$\Rightarrow \quad -\sqrt{\frac{gh}{2}} + \sqrt{gh}\left[\sqrt{2} - \frac{1}{\sqrt{2}}\right] = 2\vec{V}_f$$

$$V_f = 0$$

and height from ground $= h - \frac{1}{2}gt_1^2 = h - \frac{h}{4} = \frac{3h}{4}$

So time $= \sqrt{2 \times \frac{\left(\frac{3}{4}\right)}{g}} = \sqrt{\frac{3h}{2g}}$

23. (b)

Conserving momentum

$$mv\hat{i} + m\left(\frac{u}{2}\hat{i} + \frac{u}{2}\hat{j}\right) = 2m\left(u_1\hat{i} + u_2\hat{j}\right)$$

On solving

$$u_1 = \frac{3u}{4} \text{ and } u_2 = \frac{u}{4}$$

Change in K.E.

$$\left[\frac{1}{2}mu^2 + \frac{1}{2}m\left(\frac{u}{2}\sqrt{2}\right)^2\right] - \left[\frac{1}{2}(2M)\left(\frac{9u^2}{16} + \frac{u^2}{16}\right)\right]$$

$$= \frac{3mu^2}{4} - \frac{5mu^2}{8} = \frac{mu^2}{8}$$

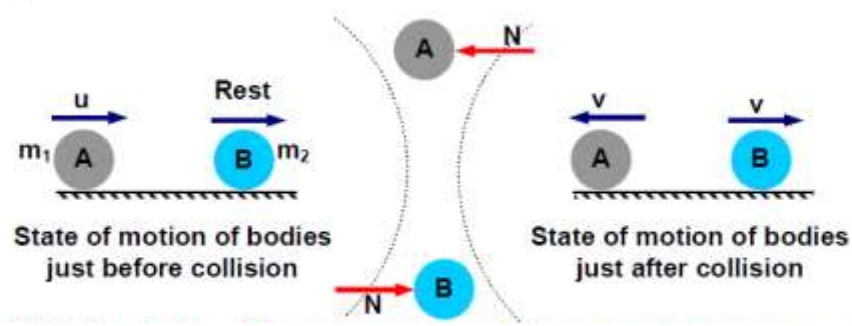
24. (d)

If collision is elastic, C comes to rest after collision. When compression in spring is maximum, velocities of A and B are same, (say v).

Using conversion of Mechanical Energy, we can write

$$\frac{1}{2}mv^2 = 2x\frac{1}{2}mv^2 + \frac{1}{2}kx^2 \Rightarrow x = v\sqrt{\frac{m}{2k}}$$

25. (b)



With the help of Conservation of linear momentum, we can write

$$m_1u = (m_2 - m_1)v \qquad ...(1)$$

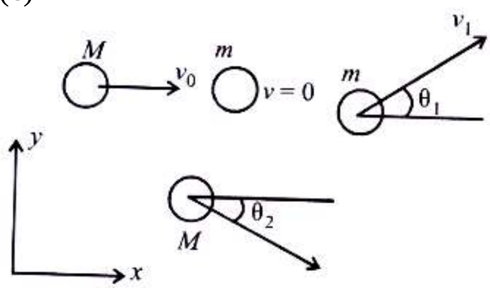
With the help of definition of e, we can write

$$e = \frac{v_s}{v_a} = \frac{2v}{u} \Rightarrow u = \frac{2v}{e} \qquad ...(2)$$

Putting the value of e in equation (1), we have

$$m_1\frac{2v}{e} = (m_2 - m_1)v \Rightarrow 2m_1 = em_2 - em_1 \Rightarrow \frac{m_2}{m_1} = \frac{2+e}{e} = 1 + \frac{2}{e} > 2$$

26. (c)



Let $\theta_1=\theta_2=\theta$. Then

$Mv_0=mv_1\cos\theta+Mv_2\cos\theta$ ..... (i)

And, $0=mv_1\sin\theta-Mv_2\sin\theta$ ..... (ii)

From (i) &(ii), we get

$$Mv_0=mv_1\cos\theta+M\left(\frac{mv_1}{M}\right)\cos\theta$$

$Mv_0=2mv_1\cos\theta$ ..... (iii)

By conservation of K.E.

$$\frac{1}{2}Mv_0^2=\frac{1}{2}Mv_1^2+\frac{1}{2}Mv_2^2$$

$$\Rightarrow\frac{1}{2}M\left(\frac{2mv_1\cos\theta}{M}\right)^2=\frac{1}{2}mv_1^2+\frac{1}{2}M.\frac{m^2v_1^2}{M^2}$$

$$\Rightarrow\frac{1}{2}\left(\frac{4m^2v_1^2\cos^2\theta}{M}\right)^2=\frac{1}{2}mv_1^2+\frac{m^2v_1^2}{2M}$$

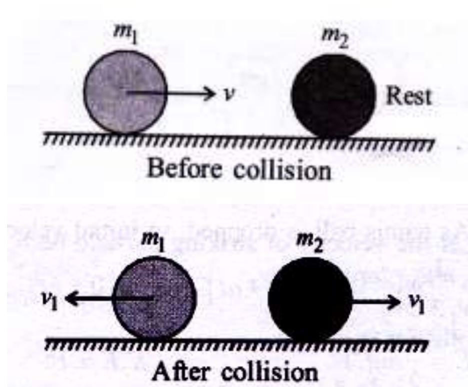
$$\Rightarrow\frac{4m\cos^2\theta}{M}=1+\frac{m}{M}\Rightarrow4\cos^2\theta=\frac{M}{m}+1$$

For largest $\frac{M}{m}$, $\cos\theta=1$ So, $\frac{M}{m}=3$

27. (b)

After the collision the objects move in opposite direction let with velocity v_1 then from law of conservation of momentum $P_i=P_f$

$m_1v=(m_2-m_1)v_1$



$\Rightarrow v_1 = \dfrac{m_1 v}{(m_2 - m_1)}$ ....(i)

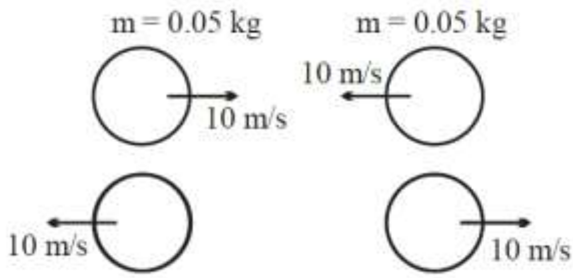
If collision is elastic then, $e = 1 = \dfrac{v_1 - (-v_1)}{v - 0}$

Also, $2v_1 = v \Rightarrow v_1 = \dfrac{v}{2}$ ......(ii)

From equation (i) &(ii)

$\dfrac{1}{2} = \dfrac{m_1}{m_2 - m_1}$ $\therefore \dfrac{m_2}{m_1} = \dfrac{3}{1}$

28. (b)



Change in momentum of any one ball

$|\Delta \vec{P}| = 2 \times 0.05 \times 10$

$|\Delta \vec{P}| = 1$

$|\vec{F}_{av}| = \dfrac{|\Delta \vec{P}|}{\Delta t}$

$F_{av.} = 200$ N

29. (b)

$P_i = P_f$ (no any external force)

$0.2 \times 10 = 10 \times v$

$v = 0.2$ m/sec

Loss in K.E. $= \dfrac{1}{2} \times (0.2) \times 10^2 - \dfrac{1}{2} \times 10 (0.2)^2$

$= \dfrac{1}{2} \times 10 \times (0.2)[10 - 0.2]$

$= 9.8$ J

30. (b)

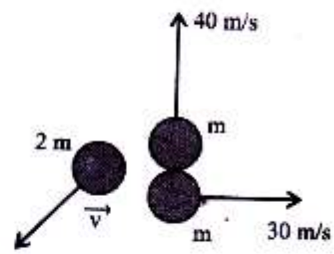
$\vec{P}_i = 0.15 \times 12(\hat{i})$

$\vec{P}_f = 0.15 \times 12(-\hat{i})$

$|\overrightarrow{\Delta P}| = 3.6$ kg – m/s

$3.6 = F\Delta t$
$3.6 = 100\Delta t$
$\Delta t = 0.036\,\text{sec}$

31. (b)



By law of conservation of momentum

$\left|\vec{P}_t\right| = \left|\vec{P}_f\right|$

$\theta = m\left(30\hat{i} + 40\hat{j}\right) + 2m\vec{v}$

$\vec{v} = -15\hat{i} - 20\hat{j}$

So, $\left|\vec{v}\right| = \sqrt{-15^2 + (-20)^2}$

$= \sqrt{625} = 25\text{m/s}$

32. (c)

Let the velocity of striking particle by u_0. Then, $mu_0 = mv_1 + 5mv_2$

$u_0 = v_1 + 5v_2$ ……(i)

As, collision is elastic

So, $e = 1 \Rightarrow \frac{v_2 - v_1}{u_0} = 1$

$\Rightarrow v_2 - v_1 = u_0$ …..(ii)

Adding (i) ad (ii), we get

$2u_0 = 6v_2 \Rightarrow v_2 = \frac{u_0}{3}$

So, $\%\Delta K.E_2 = \frac{\frac{1}{2}(5m)\left(\frac{u_0}{3}\right)^2 - 0}{\frac{1}{2}mu_0^2} \times 100 = \frac{500}{9} \simeq 55.6\%$

33. (d)

$20 \times 10^{-3} \times \frac{180}{60} \times 100 = 10\text{ V}$

$\Rightarrow v = 0.6\text{ m/s}$

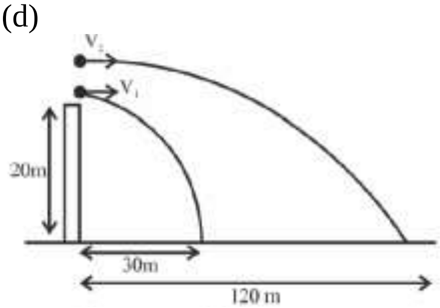
34. (c)

The velocities will be interchanged after collision.

Speed of P just before collision $=\sqrt{2gh}$

$$=\sqrt{2\times10\times0.2}=2\text{ m/s}$$

35. (d)

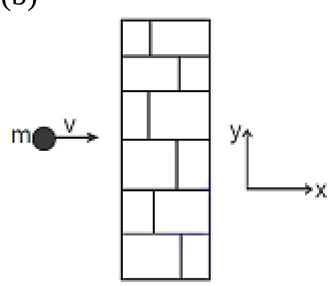


$$v_1=\frac{30}{\sqrt{\frac{2h}{g}}},\ v_2=\frac{120}{\sqrt{\frac{2h}{g}}}$$

$$(0.01)u=(0.2)\frac{30\sqrt{g}}{\sqrt{2h}}+(0.01)\frac{120\sqrt{g}}{\sqrt{2h}}$$

$$u=300+60=360\text{ ms}^{-1}$$

36. (b)



$P_i=Nm\upsilon\hat{i}$ $\vec{P}_f=-Nm\upsilon\hat{i}$

N is Number of balls strikes will wall N = 100

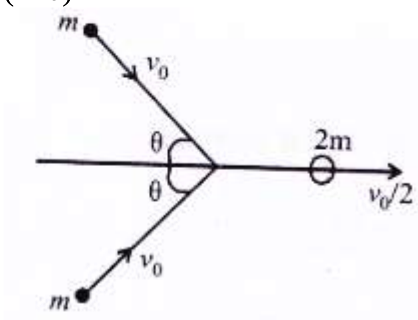
$$\Delta\vec{P}=\vec{P}_f-\vec{P}_i=-2Nm\upsilon\hat{i}$$

$$=-200\text{ }Nm\upsilon\hat{i}$$

$$\vec{F}_{Total}=\frac{\Delta\vec{P}}{\Delta t}=-\frac{200\,mvt}{t}$$

$$|\vec{F}|=\frac{200\,mv}{t}$$

37. (120)



Momentum conservation along x direction,

$$2mv_0 \cos\theta = 2m\frac{v_0}{2} \Rightarrow \cos\theta = \frac{1}{2} \text{ or } \theta = 60^\circ$$

Hence angle between the initial velocities of the two bodies

$$= \theta + \theta = 60^\circ + 60^\circ = 120^\circ.$$

38. (10.00)

From momentum conservation in perpendicular direction of initial motion.

$$mu_1 \sin\theta_1 = 10mv_1 \sin\theta_2 \qquad \ldots\ldots(i)$$

It is given that energy of m reduced by half. If u_1 be velocity of m after collision, then

$$\left(\frac{1}{2}mu^2\right)\frac{1}{2} = \frac{1}{2}mu_1^2 \Rightarrow u_1 = \frac{u}{\sqrt{2}}$$

If v_1 be the velocity of mass 10 m after collision, then $\frac{1}{2}\times 10m \times v_1^2 = \left(\frac{1}{2}\right)m\left(\frac{u^2}{2}\right) \Rightarrow v_1 = \frac{u}{\sqrt{20}}$

From equation (i), we have

$$\sin\theta_1 = \sqrt{10}\sin\theta_2$$

39. (1)

For elastic collision $KE_i = KE_f$

$$\frac{1}{2}m\times 25 + \frac{1}{2}\times m \times 9 = \frac{1}{2}m \times 32 + \frac{1}{2}mv_B^2$$

$$34 = 32 + V_B^2 \Rightarrow V_B = \sqrt{2}$$

$$KE_B = \frac{1}{2}mv_B^2 = \frac{1}{2}\times 0.1 \times 2 = 0.1J = \frac{1}{10}J$$

$$\therefore x = 1$$

40. (30)

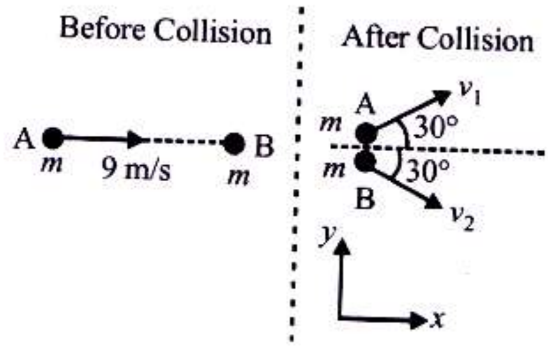
Using Conservation of linear momentum along X-axis, we can write

$$mv_0 = mv_2\cos\theta \Rightarrow \cos\theta = \frac{v_0}{v_2} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$$

41. (4)

$x_{CM} = y_{CM} = \frac{4a}{3\pi}$

42. (1)



From conservation of momentum along y-axis,

$\vec{P}_{iy} = \vec{P}_{fy}$

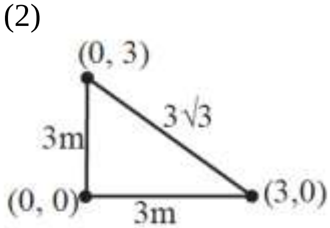
$0 + 0 = mv_1 \sin 30^\circ \hat{j} + mv_2 \sin 30^\circ (-\hat{j})$

$mv_2 \sin 30^\circ = mv_1 \sin 30^\circ$

$v_2 = v_1 \text{ or } \frac{v_1}{v_2} = 1$

43. (12)

44. (2)



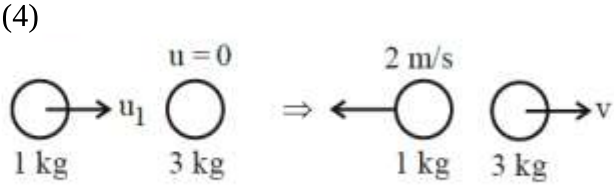
$\vec{r}_{com} = \frac{M(0\hat{i} + 0\hat{j}) + M(3\hat{i}) + M(3\hat{j})}{3M}$

$\vec{r}_{com} = \hat{i} + \hat{j}$

$|\vec{r}_{com}| = \sqrt{2} = \sqrt{x}$

$x = 2$

45. (4)



$1 \times u_1 = -2 + 3v \quad \Rightarrow \quad u_1 = -2 + 3v \quad ...(1)$

$1 = \frac{v + +2}{u_1}$ $\Rightarrow$ $v + 2 = u_1$ ...(2)

Solving (1) and (2)

$u_1 = 4\ \text{m/s}$