IIT – JEE: 2025 **TW TEST (ADV)** **DATE: 26/11/23**

ANSWER KEY

CENTRE OF MASS

1.	(C)	2.	(D)	3.	(C)	4.	(A)	5.	(C)
6.	(AB)	7.	(BD)	8.	(AD)	9.	(BCD)	10.	(BC)
11.	(ABD)	12.	(BD)	13.	(BD)	14.	(AC)	15.	(C)
16.	(2)	17.	(4)	18.	(5)	19.	(7)	20.	(4)

ENERGETICS

21.	(B)	22.	(B)	23.	(A)	24.	(A)	25.	(A)
26.	(ABCD)	27.	(AB)	28.	(C)	29.	(ABC)	30.	(ACD)
31.	(ABC)	32.	(AC)	33.	(BCD)	34.	(AC)	35.	(C)
36.	(6.00)	37.	(2.00)	38.	(5.00 to 5.05)	39.	(6.00 to 6.20)	40.	(425.00)

BINOMIAL THEOREM

41.	(D)	42.	(B)	43.	(A)	44.	(D)	45.	(B)
46.	(BD)	47.	(BD)	48.	(ABC)	49.	(ABC)	50.	(AC)
51.	(ABCD)	52.	(BD)	53.	(BC)	54.	(BCD)	55.	(ABCD)
56.	(337.00)	57.	(21.00)	58.	(10.00)	59.	(21.00)	60.	(1.00)

TOPIC: CENTRE OF MASS

SOLUTIONS

1. (C)
Initially particle will remain stationary till time t_1,
Here $t_1 = \frac{L}{V}$
Centre of mass velocity $V_{CM} = \frac{MV}{m+M}$
After this time particle will fall down vertically and reaches ground in time t_2
$t_2 = \sqrt{\frac{2h}{g}}$
$x_{CM} = V_{CM}(t_1 + t_2)$

2. (D)

3. (C)

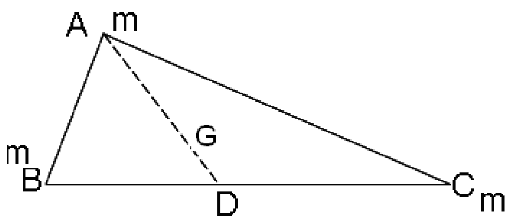
4. (A)
There are no external horizontal forces acting on the man plus boat system. (The forces exerted by the man and the boat on each other are internal forces for the system.) Therefore, the centre of mas of the system, which is initially at rest, will always be at rest.

5. (C)
The centre of mass of the block plus wedge must move with speed $\frac{mu}{m+\eta m} = \frac{u}{1+\eta} = v_{CM}$.
$\therefore \frac{1}{2}mu^2 - mgh = \frac{1}{2}(m+\eta m)v_{CM}^2$.

6. (A, B)

7. (BD)
Conceptual

8. (AD)
Mass at B and C can be replaced by 2m at D. Then m at A and 2m at D can be replaced by 3m at G
$\therefore$ A is correct



Statement in (b) is correct for equilateral triangle not for all triangles
For any quadrilateral the statement is not correct centre of mass of body can exist out side the body e.g. semi-circular ring, hemispherical shell etc.

9. (BCD)
Centre of mass always lies b/w centre of mass of segments and always lies on axis of symmetry.

10. (B, C)

11. (ABD)
F_{ext} on the system $(m_1 + m_2 + M)$ in horizontal direction is zero.
$\therefore$ centre of mass of the system remains stationary.
Again $\vec{v}_{m_1g} = \vec{v}_{m_1t} + \vec{v}_{tg} = (-u_{rel} + v)\hat{i}$

$$\vec{v}_{(m_2+M)g} = v\hat{i}$$

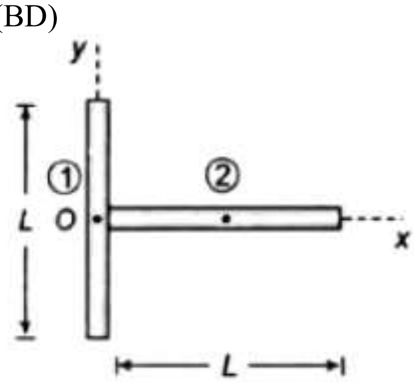
Conserver linear momentum in horizontal direction

$$m_1(-u_{rel} + v)\hat{i} + (m_2 + M)v\hat{i} = 0$$

$$\therefore \quad v = \left(\frac{m_1 u_{rel}}{m_1 + m_2 + M}\right)\hat{i}$$

12. (BD)
$F \neq 0$ implies that $a_{cm} \neq 0$

13. (BD)



This system is symmetrical about the x-axis. Hence we need to find $x_{c.m.}$.
Here we will take coordinates of c.m. of rods.

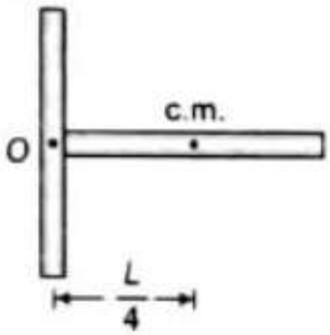
	x-coordinates
Rod (1) $m_1 = m$	0

Rod (2) $m_2 = m$ $\qquad \frac{L}{2}$

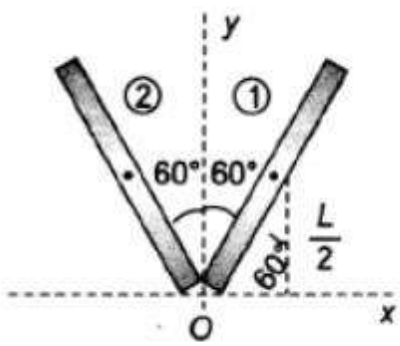
$$x_{\text{c.m.}} = \frac{m \times 0 + m \times \frac{L}{2}}{m+m} \frac{m\frac{L}{2}}{2m} = \frac{L}{4}$$

$$y_{\text{c.m.}} = 0$$

$$\text{c.m.} = \left(\frac{L}{4}, 0\right)$$



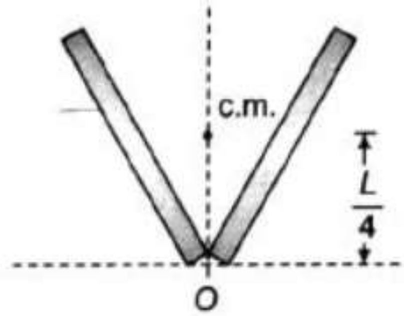
(c)



Symmetry about the y-axis:

		y-coordinates
Rod (1)	$m_1 = m$	$\frac{L}{2}\cos 60° = \frac{L}{4}$
Rod (2)	$m_2 = m$	$\frac{L}{2}\cos 60° = \frac{L}{4}$

$$y_{\text{c.m.}} = \frac{m \times \frac{L}{4} + m \times \frac{L}{4}}{m+m} = \frac{L}{4}$$

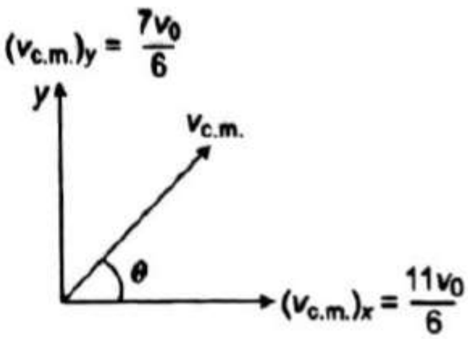


14. (AC)

$$(v_{c.m.})_x = \frac{2m(5v_0\cos 37^\circ)+3mv_0}{m+2m+3m=6m} = \frac{11v_0}{6}$$

$$(v_{c.m.})_y = \frac{mv_0+2m(5v_0\sin 37^\circ)}{m+2m+3m} = \frac{7v_0}{6}$$

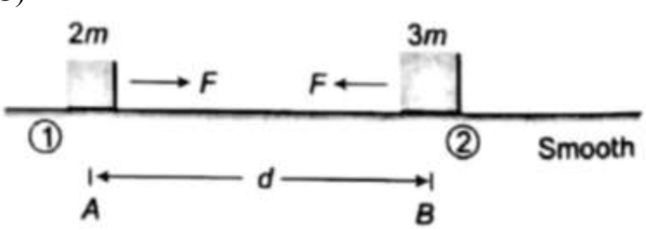
$$\vec{v}_{c.m.} = (v_{c.m.})_x\hat{i} + (v_{c.m.})_y\hat{j} = \frac{11v_0}{6}\hat{i} + \frac{7v_0}{6}\hat{j}$$



$$v_{c.m.} = \sqrt{\left(\frac{11v_0}{6}\right)^2 + \left(\frac{7v_0}{6}\right)^2} = \frac{v_0}{6}\sqrt{170}$$

$$\tan\theta = \frac{7v_0/6}{11v_0/6} = \frac{7}{11} \Rightarrow \theta = \tan^{-1}\left(\frac{7}{11}\right)$$

15. (C)

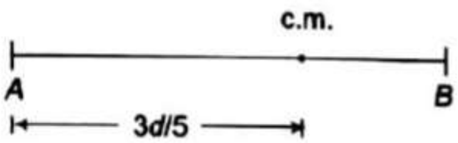


(a) Since $\vec{F}_{ext} = 0, \vec{a}_{c.m.} = 0$.

(b) Initially the particle are at rest $v_1 v_2 = 0$, therefore, $v_{c.m.} = 0$. Since $\vec{F}_{ext} = 0$, the velocity of c.m. is constant and hence $v_{c.m.}$ is always zero whenever the separation becomes $\frac{d}{2}, \frac{d}{3}$, etc.

(c) First. Locate the c.m.

$$x_{c.m.} = \frac{2m \times 0 + 3m \times d}{2m + 3m} = \frac{3d}{5}$$



Since the c.m. is at rest, its position is fixed, hence particles will meet at cm. i.e. at distance $\frac{3d}{5}$ from A.

16. (2)

$$dm = \left[\frac{M_0}{L} x\right] dx$$

For centre of mass $x_{C.M} = \frac{\int_0^L dmx}{\int_0^L dm} = \frac{2L}{3}$

17. (4)

Mass of cut out disc $= m = \frac{M}{\pi R^2} \pi \left(\frac{R}{2}\right)^2 = \frac{M}{4}$

From centre of original uniform disc the distance of centre of mass of final disc

$$= X_{C.M} = \frac{M \times 0 - \frac{M}{4}\left(-\frac{R}{2}\right) + \frac{M}{4}\left(\frac{R}{2}\right)}{M - \frac{M}{4} + \frac{M}{4}} = \frac{R}{4}$$

18. (5)

$$(75+25)4 = 75(V_p + 2) + 25 \times V_p$$

$$400 = 100V_p + 150$$

$$V_p = 2.5 m/s$$

$$d = 2.5 \times \frac{4}{2} = 5m$$

19. (7)

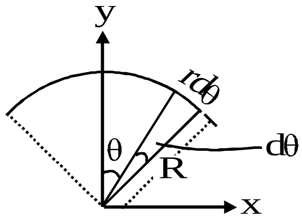
$$\frac{L}{7}\left[1 + \frac{6}{5} + \frac{7}{5} + \frac{4}{5}\right] = \frac{22L}{35}$$

20. (4)

$$dm = \frac{m}{\frac{\pi R}{2}}(d\ell)$$

$$= \frac{2M}{\pi R}(Rd\theta)$$

$$= \frac{2Md\theta}{\pi}$$



$$y_{cm} = \frac{\int y\,dm}{M} = \frac{\frac{2MR}{\pi}\int_{-\pi/4}^{\pi/4}\cos\theta d\theta}{M}$$

$$= \frac{2R}{\pi}[\sin\theta]_{-\pi/4}^{\pi/4} = \frac{2R}{\pi}\times\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right]$$

$$\frac{2\sqrt{2}R}{\pi} = \frac{2\sqrt{2}\times\sqrt{2}\pi}{\pi} = 4$$

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TOPIC: ENERGETICS

SOLUTIONS

21. **(B)**

$\Delta H = q_p$ and C_p is heat evolved or absorbed per mole for 1^o fall or rise in temperature. Here, fall in temperature $= 320 - 293 = 27K$.

Molar mass of $CCl_2F_2 = 12 + 2\times35.5 + 2\times19$

$= 121 g\ mol^{-1}$

$\therefore$ Heat evolved from 1.25 g of the sample on being cooled from 320 K to 293 K at constant pressure $= \frac{80.7}{121}\times1.25\times27\,J = 22.51J$

i.e., $\Delta H = -22.51J$.

At const. P,

$\Delta U = q_P - P.\Delta V$

$= -22.851 - 1.013\times10^5(248-274)\times10^{-6}$

$= -19.88J$

22. **(B)**

Evolution of 680 kcal is accompanied by CO_2

$= 6\times44 = 264g$

Evolution of 170 kcal will be accompanied by

$CO_2 \quad = \frac{264}{680}\times170 = 66g$.

23. **(A)**

1 mole of butane, (C_4H_{10}), i.e., 58 g liberate energy = 2600 kJ

$\therefore 11.6kg,\ i.e., 11600g$

Energy consumed in one day $= 2.0\times10^4 J$

$= 20000J$

$\therefore$ No. of days for which the cylinder will last

$= \frac{2600\times11600}{58}\times\frac{1}{20000} = 26$

24. **(A)**

The given diagram represents that the process is carried out in infinite steps. Hence, it is isothermal reversible expansion of the ideal gas from pressure 2.0 atm to 1.0 atm at 298 K.

$W = -2.303nRT\log\frac{p_1}{p_2}$

$W = -2.303 \times 1 \times 8.314 \times 298 \times 0.3010 J$
$W = -1717.46 J$

25. **(A)**
In (a), for expansion $A \rightarrow B, P = \text{constant}$.
Hence, $\frac{V}{T} = \text{const}$. As V increases, T also increases. Expansion B to C is accompanied by fall of temperature, this means that it is adiabatic expansion represented by adiabatic expansion curve BC.

26. **(ABCD)**

27. **(AB)**

28. **(C)**
$C + O_2 \longrightarrow CO_2(g); \Delta_r H = x kJ\, mol^{-1}$
$C_{(graphite)} + \frac{1}{2} O_2(g) \longrightarrow CO(g); \Delta_r H = y kJ\, mol^{-1}$
$CO(g) + \frac{1}{2} O_2(g) \rightarrow CO_2(g); \Delta_r H = z kJ\, mol^{-1}$
$(i) = (ii) + (iii) \Rightarrow \quad x = y + z$

29. **(ABC)**

30. **(ACD)**

31. **(ABC)**

32. **(AC)**

33. **(BCD)**

34. (A, C)
$$\left(\frac{\partial H}{\partial T}\right)_P - \left(\frac{\partial U}{\partial T}\right)_V = R, \left(\frac{dV}{dT}\right)_P = \frac{nR}{P}$$

35. **(C)**

36. **(6.00)**
Volume, Heat capacity, Internal energy, Enthalpy, Entropy and free energy are extensive. The remaining are intensive.

37. **(2.00)**
Heat and work are not state function. All other given properties are state function.

38. **(5.00 to 5.05)**
200 mL of $0.22 M\, H_2SO_4$
$= 200 \times 0.22$ millimole
$= 44$ millimole = 0.044 mole
$= 0.088$ mole of H^+ ions

400 mL of 0.50 M NaOH

$= 400 \times 0.50$ millimole

$= 200$ millimole

$= 0.200$ mole OH^- ions

Thus, 0.88 mole of H^+ ions will neutralize 0.088 mole of OH^- ions

1 mole of H^+ ions on neutralization produce

Heat $= 57.1 kJ$

$\therefore$ 0.88 mole of H^+ ions will produce heat

$= 57.1 \times 0.088 kJ$

$= 5.0 kJ$

39. **(6.00 to 6.20)**

$$\Delta_{fus}S = \frac{\Delta_f H}{T_f} = \frac{6025 J mol^{-1}}{273 K}$$

$= 22.07 JK^{-1}mol^{-1}$

Thus, for melting of 18 g ice at 0^oC, entropy

Change $= 22.07 JK^{-1}$

$\therefore$ For melting of 5 g ice at 0^oC, enthalpy change

$$= \frac{22.07}{18} \times 5 = 6.1 JK^{-1} = 6 JK^{-1}$$

40. **(425.00)**

$H_2 + Cl_2 \rightarrow 2HCl$

$\Delta H_{reaction} = \Delta H_{H-H} + \Delta H_{Cl-Cl} - 2\Delta H_{H-Cl}$

Or $-180 = 430 + 240 - 2\Delta H_{H-Cl}$

$\therefore \Delta H_{H-Cl} = \frac{430+240-(-180)}{2} = \frac{850}{2} = 425\ kJ\ mol^{-1}$

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TOPIC: BINOMIAL THEOREM

Solutions

41. **(D)**

$$\frac{10!}{r!s!(10-r-s)!} \quad 2^{r/2}\, 3^{s/3} \quad 5^{\frac{10-r-s}{6}}$$

(r, s) ≡ (4,0), (10,0) (4,6) for rational terms.

42. **(B)**

$$a_r + a_{r+1} = {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$$

$$\sum_{r=0}^{n-1}\left({}^{n+1}C_{r+1}\right)^2 = {}^{2n+2}C_{n+1} - {}^{n+1}C_0 - {}^{n+1}C_{n+1}$$

$$= {}^{2n+2}C_{n+1} - 2$$

43. **(A)**

$$(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_rx^r + ... \qquad …..(i)$$

$$\left(1+\frac{1}{x}\right)^n = C_0 + C_1\frac{1}{x} + C_2\frac{1}{x^2} + ... + C_r\frac{1}{x^r} + ... \qquad …..(ii)$$

Multiplying both sides and equating coefficient of x^r in $\frac{1}{x^n}(1+x)^{2n}$ or the coefficient of x^{n+r} in $(1+x)^{2n}$ we get the value of required expression

$$= {}^{2n}C_{n+r} = \frac{(2n)!}{(n-r)!(n+r)!}$$

44. **(D)**

$(1 + n)^{m+1} - n^{m+1} = 1 + {}^{m+1}C_1.n + {}^{m+1}C_2.\ n^2 + ...+....+ {}^{m+1}C_m\ .\ n^m$

put n = 1, 2, 3,..... n and add

45. **(B)**

Let y + z = a

$(x + a)^{2007} + (x - a)^{2007}$

$= 2[x^{2007} + {}^{2007}C_2\, x^{2005}a^2 + {}^{2007}C_4x^{2003}.\ a^4 + ...]$

No. of terms

$= 1 + 3 + 5 + ...+2007 = (1004)^2$

46. **(BD)**

$2^{4n} - 2^n(7n + 1) = 2^n\ .\ 8^n - 2^n(7n + 1) = 2^n[(1 + 7)^n - 7n - 1] = 2^n[{}^nC_2\ .\ 7^2 + {}^nC_3\ .\ 7^3 + ...]$

Clearly multiple of 49.

47. **(BD)**

$$S_n = \text{coeff of } x^n \text{ in } (1+x)^n + \frac{(1+x)^{n+1}}{2} + \frac{(1+x)^{n+2}}{2} + \ldots\ldots + \frac{(1+x)^{2n}}{2^n}$$

$$S_n = \text{coeff of } x^n \text{ in } (1+x)^n \times \left[\left(\frac{1+x}{2}\right)^{n+1} - 1\right] \div \left[\frac{1+x}{2} - 1\right]$$

$$= \text{coeff of } x^n \text{ in } \frac{1}{2^n} \times \left[\frac{(1+x)^{2n+1} - 2^{n+1}(1+x)^n}{x-1}\right]$$

$$= \text{coeff of } x^n \text{ in } \frac{1}{2^n} \times \left[2^{n+1}(1+x)^n - (1+x)^{2n+1}\right]\left[1 + x + x^2 + \ldots.. + \infty\right]$$

$$= \frac{1}{2^n} \times \left[2^{n+1}\left({}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n\right) - \left({}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots.. + {}^{2n+1}C_n\right)\right]$$

$$= \frac{1}{2^n} \times \left[2^{n+1} . 2^n - \frac{2^{2n+1}}{2}\right] = 2^{n+1} - 2^n = 2^n$$

$\Rightarrow S_n = 2^n$

48. **(ABC)**

$(13^n - 3^n) + 7^n = 10k + 7^n$

Last digit will be 3 if n = 4k + 3; $k \in W$

49. **(ABC)**

$f(x) = (1 + x^2 - x^3)^{1000} = \sum A_r . x^r$

Coeff. of x^{20} in f(x) and f(-x) are same $\Rightarrow$ a = d

$g(x) = (1 - x^2 + x^3)^{1000} = \sum B_r . x^r$

Coeff. of x^{20} in g(x) and g(-x) are same $\Rightarrow$ b = c

Coeff. of x^{20} in $(1 + x^2 + x^3)^{1000}$ largest compared to b, c since it involves sums of all positive terms while b and c are made by combination +ve and –ve terms.

50. **(AC)**

Let $f' = \left(5\sqrt{3} - 8\right)^{2n+1}$

[x] + {x} - f′ = even

$\Rightarrow$ {x} - f′ = 0 , [x] even

$\Rightarrow$x . {x} = x. f′ = $(11)^{2n+1}$

51. **(ABCD)**

$(1 + i)^n = P_n + i Q_n$; $(1 - i)^n = P_n - i Q_n$

by multiplying $(1 + 1)^n = P_n^2 + Q_n^2 \Rightarrow P_n^2 + Q_n^2 = 2^n$

$(1 + i)^8 = ((1 + i)^2)^4 = (2i)^4 = 16 = P_8 + iQ_8$

$P_8 = 16,\ Q_8 = 0$

$(1 + i)^{10} = (2i)^5 = 32i = P_{10} + iQ_{10}$

$Q_{10} = 32,\ P_{10} = 0$

52. **(BD)**

Since $\left(1 - x^3\right) = \left(1 - x\right)\left(1 + x + x^2\right)$

So $\left(1+x+x^2\right)^{50}(1-x)^{53}=\left(1-x^3\right)^{50}(1-x)^3$

Coefficient of $x^{10}=3\left({}^{50}C_3\right)$

53. **(BC)**

54. **(BCD)**

(A) ${}^{n}C_3\left(\frac{x}{a}\right)^{n-3}\left(\frac{1}{x}\right)^3=\frac{5}{2}\Rightarrow n=6$

$a^3=8\Rightarrow a=2$

(B) $\sum_{p=1}^{p=4}{}^{4}C_r\ .\ {}^{r}C_p=\sum_{p=1}^{p=4}\sum_{r=p}^{r=4}{}^{4}C_p\ .\ {}^{4-p}C_{r-p}$

$=\sum_{p=1}^{p=4}{}^{4}C_p\ .\ 2^{4-p}=3^4-2^4=65$

(C) $(1-x)^5\ \frac{(1-x^4)^4}{(1-x)^4}=(1-x)(1-x^4)^4\Rightarrow$ coeff of x^{13} = $-(-{}^{4}C_3)=4$

(D) $\sum_{r=0}^{4}{}^{4}C_r\,(r-2)^2=4+4+4+4=16$

55. **(ABCD)**

(A) $\left(1+1+1+1\right)^{10}=\sum\frac{10!}{a!b!c!d}$

(B) $\left(2-3+5+7\right)^{10}=\sum\frac{2^a(-3)^b5^c7^d}{a!b!c!d}\times 10!$

(C) $\left(2+3+4+5\right)^{10}+(2+3+4-5)^{10}=2\sum\frac{10!\ 2^a3^b5^d4^c}{a!b!c!d}$ where d is even

(D) $\left(2+3+4+5\right)^{10}-(3+4+5)^{10}=\sum\frac{2^a3^b4^c5^d}{a!b!c!d}\times 10!$ where a is natural

56. **(337.00)**

x = $\left(\sqrt{3}+1\right)^{2018}$, $N=[x]+1$

Let y = $\left(\sqrt{3}-1\right)^{2018}$

$x+y=[x]+f+y=\left(\sqrt{3}+1\right)^{2018}+\left(\sqrt{3}-1\right)^{2018}=2^{1009}\left[\left(2+\sqrt{3}\right)^{1009}+\left(2-\sqrt{3}\right)^{1009}\right]$

$[x]+1=2^{1009}.2\ [{}^{1009}C_0\ .\ 2^{1009}+{}^{1009}C_2\ .\ 2^{1007}\ .\ 3+......+......+{}^{1009}C_{1008}\ .\ 2^1\ 3^{504}]$

$[x]+1=2^{1009}\ .2^1.\ 2^1\ [{}^{1009}C_0.2^{1008}+{}^{1009}C_2.2^{1006}\ .3+\ ....+....+{}^{1009}C_{1006}\ .\ 2^2\ .\ 3^{503}+\ {}^{1009}C_{1008}.\ 3^{504}]$

$N=2^{1011}$. ODD

$N=(2^3)^{337}$. ODD

P = 337

57. **(21.00)**

$(7-1)^{2007}+(7+1)^{2007}=49K+2007\times 7\times 2=49\lambda+21$

58. **(10.00)**

Coeff. of x^{10} in $^{10}C_0(1 + x)^{20} - {}^{10}C_1(1 + x)^{18} + {}^{10}C_2(1 + x)^{16} - {}^{10}C_3(1 + x)^{14} + ...+{}^{10}C_{10}(1 + x)^0$

= coeff. of x^{10} in $((1 + x)^2 - 1)^{10}$

= coeff. of x^{10} in $(2 + x)^{10} .x^{10} = 2^{10}$

59. **(21.00)**

$7 - 2r = 3 \Rightarrow r = 2$

$\therefore$ The coefficient is $^7C_2 = 21$.

60. **(1.00)**

$$\frac{(18+7)^3}{(3+2)^6} = \frac{25^3}{5^6} = 1$$