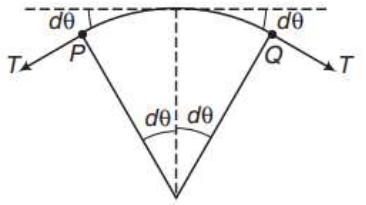
PART (A) : PHYSICS

1. (D)

$\cos(d\theta)$ components of T are cancelled and $\sin(d\theta)$ components towards centre provide the necessary centripetal force to small portion PQ



$\therefore 2T\sin(d\theta) = (m_{PQ})(R)\omega^2$

For small angle, $\sin d\theta \approx d\theta$

$\therefore 2T\,d\theta = \left(\frac{m}{2\pi}\right)(2\theta)(R)(2n\pi)^2$

$\therefore T = 2\pi mn^2R$

Substituting the value we get,

$T = (2\pi)(2\pi)(300/60)^2(0.25)$

$\approx 250N$

2. (C)

$\frac{mv^2}{R} = \mu mg$

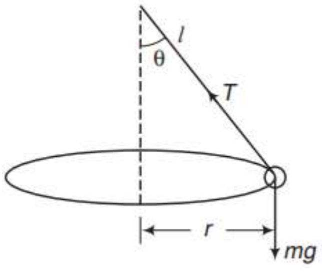
$\therefore v = \sqrt{\mu Rg} = \sqrt{0.3\times300\times10}$

$= 30\,m/s = 108\,km/h$

3. (D)

$T\cos\theta = mg$ ....(i)

$T\sin\theta = mr\omega^2 = m(l\sin\theta)\omega^2$ ......(ii)



Solving these two equations we get,

$\cos\theta = \frac{g}{l\omega^2} = \frac{g}{l(2\pi n)^2}$

$= \frac{10}{\left[2\pi\times\frac{2}{\pi}\right]^2}$ $(l = 1m)$

$\therefore \theta = \cos^{-1}(5/8)$

4. (B)

Given, $R = H$

$$\frac{u^2 \sin 2\alpha}{g} = \frac{u^2 \sin^2 \alpha}{2g}$$

or $2\sin\alpha\cos\alpha = \frac{\sin^2\alpha}{2}$

or $\frac{\sin\alpha}{\cos\alpha} = 4$

or $\tan\alpha = 4$

$\therefore$ $\alpha = \tan^{-1}(4)$

5. (A)

$v = \frac{dS}{dt} = (4t)\hat{i}$

$P = F.v = 12t^2$

$\therefore W = \int_0^2 P dt = \int_0^2 (12)t^2 dt$

$= 24J$

6. (A)

Decrease in potential energy = Work done against friction

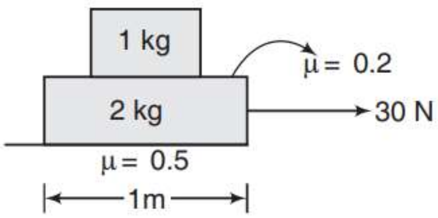
$\therefore mg(h+d) = F.d$

Here F = average resistance

$\Rightarrow F = mg\left(1 + \frac{h}{d}\right)$

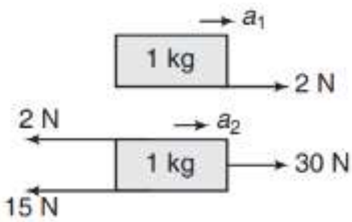
7. (A)

$(f_1)_{max}$ = between 1 kg and 2 kg



$= 0.2 \times 1 \times 10 = 2N$

$(f_2)_{max}$ = between 2 kg and ground



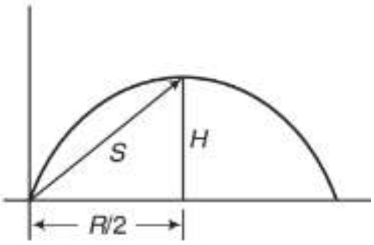
$= 0.5 \times 3 \times 10 = 15N$

$a_1 = \frac{2}{1} = 2m/s^2$

$$a_2 = \frac{30-15-2}{2}$$
$$= 6.5\,m/s^2$$
$$a_r = a_2 - a_1$$
$$= 4.5\,m/s^2$$
$$t = \sqrt{\frac{2S_r}{a_r}}$$
$$= \sqrt{\frac{2\times1}{4.5}} = \frac{2}{3}s$$

8. (B)

$$S = \sqrt{H^2 + R^2/4}$$



Average velocity $= \frac{S}{t} = \frac{S}{T/2} = \frac{2S}{T}$

9. (C)

$$a = \frac{F}{t}$$
$$b = \frac{F}{t^2}$$

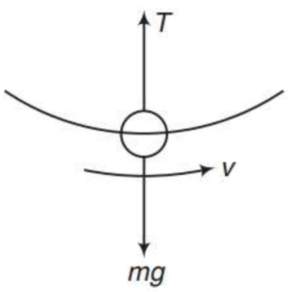
10. (A)

$$[F]^a [L]^b [T]^c = [M]$$
$$\therefore [MLT^{-2}]^a [L]^b [T]^c = [M]$$

Equating the powers we get,
$a = 1, b = -1$
And $c = 2$

11. (D)

$$v^2 = 2gh$$



$T - mg = \frac{mv^2}{l}$

Or $T = mg + \frac{m(2gh)}{l}$

$= mg\left(1 + \frac{2h}{l}\right)$

12. (B)

Maximum range is obtained at 45°

$E = \frac{1}{2}mu^2$ .....(i)

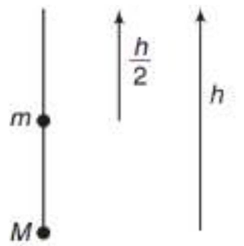
At highest point, $v = u\cos 45^\circ = \frac{u}{\sqrt{2}}$

$\therefore E' = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2$

$= \frac{\left(\frac{1}{2}mu^2\right)}{2} = \frac{E}{2}$

13. (A)

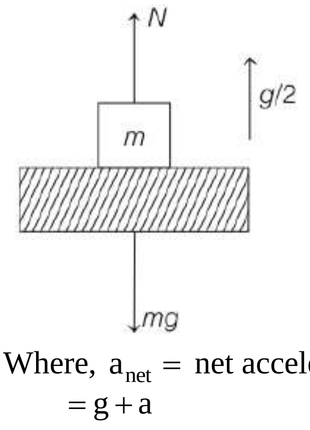
$W = MgH + mg\frac{h}{2} = \left(M + \frac{m}{2}\right)gh$



14. (B)

Normal reaction force on the block,

$N = ma_{net}$



Where, $a_{net} =$ net acceleration of block.

$= g + a$

$= g + \frac{g}{2} = \frac{3g}{2}$

$\Rightarrow \quad N = m\left(g+\frac{g}{2}\right)$

$= \frac{3mg}{2}$

Now, the time t, block moves by a displacement s given by

$s = 0+\frac{1}{2}at^2 = \frac{1}{2}\left(\frac{g}{2}\right)t^2 \quad (\because u = 0)$

Here, $a = \frac{g}{2}$ (given)

$\therefore$ Work done = Force $\times$ Displacement

$\Rightarrow \quad W = \frac{3mg}{2}\times\frac{gt^2}{4}$

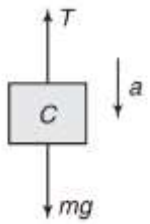
$= \frac{3mg^2t^2}{8}$

15. (B)

$a = \frac{\text{Net pulling force}}{\text{Total mass}}$

$= \frac{(2+2-2)g}{2+2+2} = \frac{g}{3}$

FBD of C



$mg - T = ma = \frac{mg}{3}$

$\therefore T = \frac{2}{3}mg$

$= \frac{2}{3}(20) = 13.3N$

16. (A)

Only two forces are acting, mg and net contact force (resultant of friction and normal reaction) from the inclined plane. Since the body is at rest. Therefore these two forces should be equal and opposite.
$\therefore$ Net contact force = mg (upwards)

17. (B)

$t = \sqrt{\frac{2S}{a}} \propto \frac{1}{\sqrt{a}}$

$\therefore \frac{t_1}{t_2} = \sqrt{\frac{a_2}{a_1}}$

$$\frac{2}{1}=\sqrt{\frac{g\sin\theta}{g\sin\theta-\mu_k g\cos\theta}}=\sqrt{\frac{1}{1-\mu_K}}$$

As, $\sin\theta=\cos\theta$ at 45°

On solving the above equation, we get

$$\mu_K=\frac{3}{4}$$

18. (B)

19. (C)

$$V=\frac{4}{3}\pi R^3$$

$$\therefore(\%\text{error in V})=3(\%\text{ error in R})$$

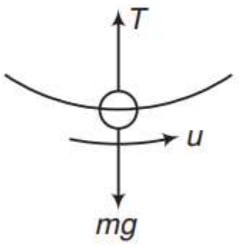
$$=3(1\%)$$

$$=3\%$$

20. (D)

$$P\perp Q=P.Q=0$$

21. (6)



$$T-mg=\frac{m\left(\sqrt{5}gR\right)^2}{R}$$

$$\therefore T=6mg$$

22. (2)

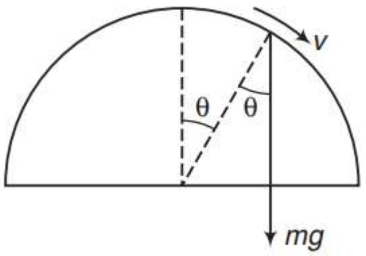
$$\frac{v^2}{R}=a_t=a \qquad \text{(Here, } a_t=a \text{ say)}$$

Or $\frac{(at)^2}{R}=a$

$$\therefore t=\sqrt{\frac{R}{a}}=\sqrt{\frac{20}{5}}=2s$$

23. (6)

At the time of leaving contact



$N = 0$

$\therefore mg\cos\theta = \frac{mv^2}{R} = \frac{m(2gh)}{R}$

$\therefore \cos\theta = \frac{2h}{R} = \frac{2\left[\frac{R}{4} + R(1-\cos\theta)\right]}{R}$

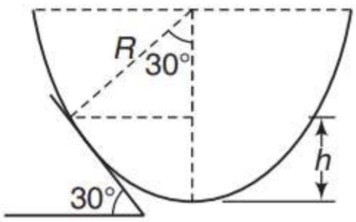
On solving this equation, we get

$\cos\theta = 5/6$ or $\theta = \cos^{-1}\left(\frac{5}{6}\right)$

24. (2)

Angle of repose $\theta = \tan^{-1}(\mu)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$



So, particle may be placed maximum upto 30°, as shown in figure

$h = R - R\cos 30^\circ = \left(1 - \frac{\sqrt{3}}{2}\right)R$

25. (7)

Using $s = ut + \frac{1}{2}at^2$ in vertical direction

$\therefore -70 = (50\sin 30^\circ)t + \frac{1}{2}(-10)t^2$

On solving this equation, we get

$t = 7s$

26. (3)

$h = \frac{u^2}{2g}$ or $u \propto \sqrt{h}$

27. (2)

Decrease in gravitational potential energy of block

= Increaser in spring potential energy

$\therefore mg\left(x_m \sin\theta\right) = \frac{1}{2}K x_m^2$

$\therefore x_m = \frac{2mg\sin\theta}{K}$

28. (4)

Let retarding force is F

Then, $Fx = \frac{1}{2}mv^2$ ....(i)

And $F\left(x'\right) = \frac{1}{2}m\left(2v\right)^2$ ......(ii)

Solving these two equations, we get

$x' = 4x$

29. (2)

$T_1 = \frac{2u\sin\theta}{g}, T_2 = \frac{2u\cos\theta}{g}$

$R = \frac{2\left(u\sin\theta\right)\left(u\cos\theta\right)}{g}$

$= \frac{2\left(\frac{gT_1}{2}\right)\left(\frac{gT_2}{2}\right)}{g}$

$= \frac{1}{2}gT_1T_2$

30. (7)

We know, $\theta = \left(\frac{\omega_1 + \omega_2}{2}\right)t$

Let number of revolutions be N.

$\therefore \quad 2\pi N = 2\pi\left(\frac{900+2460}{60\times 2}\right)\times 26$

$N = 728$

PART (B) : CHEMISTRY

31. (C)

$x = 2\ell + 1 \Rightarrow \ell = \frac{x-1}{2}$

32. (B)

Magnetic moment $= \sqrt{n(n+2)}$

The value of $\mu = 2.83\,BM$ corresponds to the presence of two unpaired electrons.

So the ion is $Ni^{2+}(3d^8)$.

33. (B)

$B.E. \Rightarrow Cl_2 > Br_2 > F_2 > I_2$

34. (C)

Apply Fajan's rules (a large positive charge, and a small cation favour covalency)

35. (A)

Water molecule can form maximum four Hydrogen bond.

36. (A)

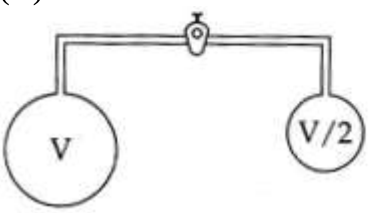
$PCl_3 \rightarrow$ Pyramidal

$SO_3, CO_3^{2-}, NO_3^- \rightarrow$ Trigonal planar

37. (A)

$(NC)_2C = C(CN)_2$

38. (A)



Let the total number of molecules of the gas be n, of which n_1 are in the larger sphere and n_2 in the smaller sphere after the stopcock is opened

$n = n_1 + n_2$ and $pV = nRT$

$$\frac{pV}{RT_1} = \frac{p'V}{RT_1} + \frac{p'V}{2T_2R} \Rightarrow \frac{P}{T_1} = P'\left(\frac{2T_2 + T_1}{2T_1T_2}\right)$$

$$p' = \frac{2pT_2}{2T_2 + T_1}$$

39. (D)

$V_{N_2O_4} = 100 \times \frac{2}{3}$ and $V_{NO_2} = 100 \times \frac{1}{3}$

By the law of mixtures,

Mass of N_2O_4 + mass of NO_2 = mass of mixture

The vapour density of NO_2 is 23 and that of N_2O_4 is 46

$\frac{200}{3} \times 46 + \frac{100}{3} \times 23 = 100 \times d_{mix'}$

Where d_{mix} is the vapour density of the mixture

$\therefore$ vapour density $d = 38.3$

40. (D)

$2NaHCO_3 \xrightarrow{150^\circ C} Na_2CO_3 + CO_2 + H_2O$

$\frac{n_{NaHCO_3}}{n_{CO_2}} = \frac{2}{1}$

$n_{NaHCO_3} = 2n_{CO_2} = 2 \times \frac{112}{22400} = 0.01 \text{mole}$

$W_{NaHCO_3} = 0.01 \times 84 = 0.84g$

$W_{Na_2CO_3} = 1.00 - 0.84 = 0.16g$

$\% Na_2CO_3 = 16$

41. (B)

$PV = nRT = \frac{w}{m} RT$

$P = \frac{dRT}{m}$

$d \propto \frac{P}{T}$

$\frac{d_{(top)}}{d_{(bottom)}} = \frac{710}{273} \times \frac{303}{760} = 1.04 : 1$

42. (C)

$W = 2.303\, nRT \log \frac{P_2}{P_1}$

43. (D)

Enthalpy is extensive property as it's value changes on division of system.

44. (C)

B_2H_6 have 2 electron 3 centre bond.

45. (C)

$\mu_{cal} = e \times l = \left(4.802 \times 10^{-10} \text{esu}\right)\left(1.275 \times 10^{-8} \text{cm}\right)$

$= 6.12D \left(1 \text{Debye} = 10^{-18} \text{esu cm}\right)$

Percentage of ionic character $= \frac{\mu_{obs}}{\mu_{cal}} \times 100 = \frac{1.03}{6.12} \times 100 = 17$

46. (A)
EA order $\Rightarrow$ N < Na < F < Cl

47. (A)
PbI_4 disproportionate easily: $PbI_4 \rightarrow PbI_2 + I_2$. (Inert pair Effect)

48. (A)

49. (A)
$$^5C_2 = \frac{5!}{3!2!} = \frac{120}{6\times 2} = 10$$

50. (D)
Let $P_1 = 3\text{atm}, T_1 = 220 + 273 = 493\text{K}, V_1 = 1.65\,\text{L}$
$P_2 = 0.7\text{atm}, T_2 = 110 + 273 = 383\text{K}, V_2 = 1\text{L}$
Using the gas $PV = nRT$ equation, we get
$$n_1 = \frac{P_1V_1}{RT_1} \text{ and } n_2 = \frac{P_2V_2}{RT_2}$$
Thus, fraction remaining is
$$\frac{n_1}{n_2} = \frac{P_1V_2}{RT_2} \times \frac{RT_1}{P_1V_1} = \frac{0.7\times 1}{R\times 383} \times \frac{R\times 493}{3\times 1.65} = 0.182$$
Fraction escaped $= 1 - 0.182 = 0.818$
Percentage escaped $= 0.818 \times 100 = 81.8\%$
So the correct choice is (D)

51. (1)
Moles of $I_2 = \frac{25.4}{254} = 0.1$
Moles of $Cl_2 = \frac{14.2}{71} = 0.2$
So, moles of ICl and ICl_3 formed are 0.1 and 0.1.

52. (3)
Lyman first line, $\frac{1}{\lambda_1} = \frac{3RZ^2}{4}$
$$\lambda_1 = \frac{4}{3RZ^2}$$
Balmer first line, $\frac{1}{\lambda_2} = RZ^2\left(\frac{5}{36}\right)$
$$\lambda_2 = \frac{36}{5RZ^2}$$
$\lambda_2 - \lambda_1 = 59.3\,\text{nm}$
$$\frac{36}{5RZ^2} - \frac{4}{3RZ^2} = 59.3\,\text{nm}$$

$Z^2 = \frac{1}{59.3R}\left(\frac{88}{15}\right)$
$Z^2 = 9$
$Z = 3$

53. (6)
$CH_2Cl_2, NH_3, PCl_2F_3, SF_4, XeO_2F_2, NO_2^-$

54. (8)
$NH_3, SF_4, IF_5, XeO_3, H_2SO_3, HNO_2, O_3, HClO_3$

55. (4)
N, Ar, Be, Mg

56. (10)
Use $M = \frac{\% \text{ by weight} \times 10 \times d}{M_{w_2}}$
$M_1V_1 = M_2V_2$
$\frac{90 \times 10 \times 0.8}{46} = \frac{10 \times 10 \times 0.9}{46} \times 80$
$V = \frac{10 \times 0.9 \times 80}{90 \times 0.8} = 10\text{mL}$

57. (32)
Diameter of hydrogen atom $= 16.92\,\mathring{A}$
Radius of an atom $= \frac{16.92}{2} = 8.46\,\mathring{A}$
We know:
$r_n = \frac{0.53n^2}{Z}\mathring{A}$
$8.46 = \frac{0.53n^2}{1}$
Or $n = 4$
Therefore, the maximum number of electron in the fourth orbit are $= 2n^2 = 2 \times (4)^2 = 32$

58. (15)
$13.22\text{eV} = 13.6\left[\frac{1}{1^2} - \frac{1}{n^2}\right] = 13.6 - \frac{13.6}{n^2}$
$\frac{13.6}{n^2} = 13.6 - 13.22 = 0.38$
$n^2 = \frac{13.8}{0.38} = 36$
$n = 6$
The electron of H atom is excited from $n_1 \to 1$ to $n_2 \to 6$

Number of spectral line $=\frac{6(6-1)}{2}=5$

The number of photons emitted is equal to the number of spectral line $=15$

59. (3)

Extensive properties which depends on mass are $(e),(g),(h)$.

60. (8)

$$\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$$

$n=1$

$$\left(P+\frac{a}{v^2}\right)(v-B)=RT$$

If b is negligible

$$P=\frac{RT}{V}-\frac{a}{V^2}$$

The equation is quadratic in V thus

$$V=\frac{+RT\pm\sqrt{R^2T^2-4aP}}{2P}$$

Since V has one value at given P and T, thus numerical value of discriminant $=0$

$R^2T^2=4aP$

$$P=\frac{R^2T^2}{4a}=\frac{(0.0821)^2(300)^2}{4\times3.592}$$

$$\therefore\frac{P}{5.227}=8$$

PART (C) : MATHEMATICS

61. (A)

$\Rightarrow 10^{\log_p(\log_q(\log_r x))} = 1$

$\Rightarrow \log_q(\log_r x) = 1$

$\therefore \log_r x = q$

$\Rightarrow x = r^q \qquad ...(1)$

$\log_q(\log_r(\log_p x)) = 0$

$\Rightarrow \log_r(\log_p x) = 1$

$\Rightarrow \log_p x = r$

$\Rightarrow x = p^r \qquad ...(2)$

From (1) and (2), $r^q = p^r$

$\Rightarrow p = r^{q/r}$

62. (C)

$\tan^2\theta = \sin\theta\cos\theta$

$\Rightarrow \sin\theta = \cos^3\theta$

Also the given expression is

$\sin^6\theta - 3\sin^4\theta + 3\sin^2\theta - 1 + 1 + \sin^2\theta$

$= (\sin^2\theta - 1)^3 + 1 + \sin^2\theta$

$= -(1 - \sin^2\theta)^3 + 1 + \sin^2\theta$

$= -\cos^6\theta + 1 + \sin^2\theta$

$= -\sin^2\theta + 1 + \sin^2\theta = 1$

63. (D)

$5\cos A + 3 = 0$

$\Rightarrow \cos A = -3/5$

$\Rightarrow \sin A = 4/5$

$\Rightarrow \tan A = -4/3$

Now equation whose roots are $4/5$ and $-4/3$ is

$(x - 4/5)(x + 4/3) = 0$

Or $15x^2 + 8x - 16 = 0$

64. (A)

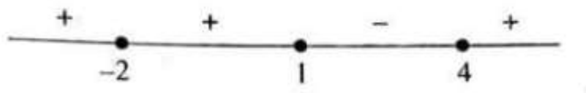
$|x^2 - 2x - 8| + |x^2 + x - 2| = 3|x + 2|$

Now $(x^2 + x - 2) - (x^2 - 2x - 8) = 3x + 6 = 3(x + 2)$

$\therefore (x^2 - 2x - 8)(x^2 + x - 2) \le 0$

$(x - 4)(x + 2)^2(x - 1) \le 0$

The sign scheme is shown below



From the sign scheme, $x \in [1,4] \cup \{-2\}$

65. (Bonus)

$$\frac{|x+2|-|x|}{\sqrt{8-x^3}} \geq 0$$

We must have $8 - x^3 > 0$

$\Rightarrow x^3 < 8 \Rightarrow x < 2$

Also $|x+2| - |x| \geq 0$

$\Rightarrow |x+2| \geq |x| \Rightarrow (x+2)^2 - x^2 \geq 0 \Rightarrow 2(x+1) \geq 0$

$\Rightarrow x \geq -1$

So $x \in [-1, 2)$

66. (C)

We know that,

$$\cos\alpha \cdot \cos(2\alpha)\cos(2^2\alpha)...(2^{n-1}\alpha) = \frac{\sin(2^n\alpha)}{2^n \sin\alpha}$$

$$\therefore \cos\frac{\pi}{2^2} \cdot \cos\frac{\pi}{2^3} .... \cos\frac{\pi}{2^{10}} \cdot \sin\frac{\pi}{2^{10}}$$

$$= \left\{\frac{\sin\left(\frac{\pi}{2^{10}} 2^9\right)}{2^9 \sin\left(\frac{\pi}{2^{10}}\right)}\right\} \sin\frac{\pi}{2^{10}} \qquad [\because \text{ here, } \alpha = \frac{\pi}{2^{10}} \text{ and } n = 9]$$

$$= \frac{1}{2^9}\sin\left(\frac{\pi}{2}\right) = \frac{1}{2^9} = \frac{1}{512}$$

67. (B & C)

Since $2a, b, 2c$ are in A.P., therefore

$2a + 2c = 2b$

$\Rightarrow a - b + c = 0$

$\therefore f(1) = 0$

Also $f(0) = c > 0$

Therefore, product of roots is positive

Therefore, other root is also positive.

68. (A)

As $1, a_1, a_2, a_3, ..., a_{87}, a_{88}, a_{89}, 89$ are in A.P.,

So $a_1 + a_{89} = a_2 + a_{88} = ... = 1 + 89 = 90$

$$\therefore \sum_{r=1}^{89} \log\left(\tan a_r^{\circ}\right)$$

$=\log\left(\tan a_1^\circ.\tan a_2^\circ.....\tan a_{88}^\circ.\tan a_{89}^\circ\right)$
$=\log 1=0$

69. (B)
Let $x=e, y=e^2, z=e^3$
$\therefore$ Given terms are $\log_{e^2} e, \log_{e^3} e, \log_{e^4} e$
Or $\frac{1}{2},\frac{1}{3},\frac{1}{4}$ which are in H.P.

70. (B)
$S=1+4x+7x^2+10x^3+........$
$x.S=x+4x^2+7x^3+......$
Subtracting
$S(1-x)=1+3x+3x^2+3x^3+......$
$S(1-x)=1+3x\left(\frac{1}{1-x}\right)\ |x|<1$
$\therefore S=\frac{1+2x}{(1-x)^2}$
Given $\frac{1+2x}{(1-x)^2}=\frac{35}{16}$
$\Rightarrow 16+32x=35+35x^2-70x$
$\Rightarrow (5x-1)(7x-19)=0$
But $|x|<1$ $\therefore x=\frac{1}{5}$

71. (A)
$T_n=\frac{1}{3}\left[\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\frac{1}{21}+......\right]$
$=\frac{2}{3}\left[\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+\frac{1}{5.6}+......\right]$
Hence $T_n=\frac{2}{3}\frac{1}{(n+1)(n+2)}$
$=\frac{2}{3}\left[\frac{1}{n+1}-\frac{1}{n+2}\right]$
$\therefore S_\infty=\frac{2}{3}.\frac{1}{2}=\frac{1}{3}$

72. (B)
Let T_n be the n^{th} term of the series
$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+......$

Then $T_n = \frac{n}{1+n^2+n^4} = \frac{n}{\left(1+n^2\right)^2 - n^2}$

$= \frac{n}{\left(n^2+n+1\right)\left(n^2-n+1\right)}$

$= \frac{1}{2}\left[\frac{1}{n^2-n+1} - \frac{1}{1+n\left(n+1\right)}\right]$

Now $\sum_{r=1}^{n} T_r = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{1+1.2}\right] + \frac{1}{2}\left[\frac{1}{1+1.2} - \frac{1}{1+2.3}\right]$

$+ \frac{1}{2}\left[\frac{1}{1+2.3} - \frac{1}{1+3.4}\right] + ... + \frac{1}{2}\left[\frac{1}{1+\left(n-1\right)n} - \frac{1}{1+n\left(n+1\right)}\right]$

$= \frac{1}{2}\left[1 - \frac{1}{1+n\left(n+1\right)}\right] = \frac{n\left(n+1\right)}{2\left(n^2+n+1\right)}$

73. (A)
We know that
$2^{n-1} = {}^nC_0 + {}^nC_2 + {}^nC_4 + ... = {}^nC_1 + {}^nC_3 + {}^nC_5 + ...$
So, ${}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + ... + {}^{10}C_9 = 2^{10-1} = 2^9$

74. (B)
$T_{r+1} = {}^{256}C_r \left(\sqrt{3}\right)^{256-r} \left(\sqrt[8]{5}\right)^r = {}^{256}C_r \left(3\right)^{\frac{256-r}{2}} \left(5\right)^{r/8}$
Term would be integral if $\frac{256-r}{2}$ and $\frac{r}{8}$ both the positive integer
As $0 \le r \le 256$, $\therefore r = 0, 8, 16, 24, ..., 256$
For above values of $r, \left(\frac{256-r}{2}\right)$ is also an integer.
$\therefore$ Total number of values of $r = 33$.

75. (C)
Obviously $T_n = \frac{1^3+2^3+3^3+......+n^3}{1+3+5+.....\text{up to n terms}}$

$= \frac{\sum n^3}{\frac{n}{2}\left[2+\left(n-1\right)2\right]} = \frac{1}{4}\frac{n^2\left(n+1\right)^2}{n^2} = \frac{1}{4}\left(n^2+2n+1\right)$

76. (A)
$a\alpha^2 + b\alpha + c = a\alpha^2 + p\alpha + q = 0$
$\Rightarrow \alpha = \frac{q-c}{b-p}$ (i)
And $b^2 - 4ac = p^2 - 4aq$
$\Rightarrow b^2 - p^2 = 4a\left(c-q\right)$

$\Rightarrow b+p=\dfrac{4a(c-q)}{b-p}=-4a\alpha$ (from (i))

$\therefore \alpha=\dfrac{-(b+p)}{4a}=\dfrac{\frac{-b}{a}-\frac{p}{a}}{4}$

Which is AM of all roots of $f(x)=0$ and $g(x)=0$

77. (B)

$f(x)=\cos^2 x+\dfrac{\sec x}{4}$

$=\cos^2 x+\dfrac{1}{4\cos x}$

For $x\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right), \cos x>0$

Now $f(x)=\cos^2 x+\dfrac{1}{8\cos x}+\dfrac{1}{8\cos x}$

Using A.M. $\geq$ G.M.

$\Rightarrow \dfrac{\cos^2 x+\dfrac{1}{8\cos x}+\dfrac{1}{8\cos x}}{3}\geq\left(\dfrac{1}{8^2}\right)^{\frac{1}{3}}=\left[\left(\dfrac{1}{4}\right)^3\right]^{\frac{1}{3}}$

$\Rightarrow \cos^2 x+\dfrac{1}{4\cos x}\geq\dfrac{3}{4}$

78. (A)

$(1+t^2)^{12}(1+t^{12})(1+t^{24})$

$=(1+{}^{12}C_1t^2+{}^{12}C_2t^4+.......+{}^{12}C_4t^8+.....+{}^{12}C_{10}t^{20}+......)(1+t^{12}+t^{24}+t^{36})$

$\therefore$ Coefficient of $t^{24}={}^{12}C_6+2$

79. (A)

$(1+x)^n=C_0+C_1x+C_2x^2+.....=C_rx^r+.....$ .....(i)

$\left(1+\dfrac{1}{x}\right)=C_0+C_1\dfrac{1}{x}+C_2\dfrac{1}{x^2}+......+C_r\dfrac{1}{x^r}+........$ ......(ii)

Multiplying both sides and equating coefficient of x^r in $\dfrac{1}{x^n}(1+x)^{2n}$ or the coefficient of x^{n+r} in $(1+x)^{2n}$ we get the value of required expression

$={}^{2n}C_{n+r}=\dfrac{(2n)!}{(n-r)!(n+r)!}$

80. (B)

$\dfrac{(n+1)(n+2)}{2}=45$ ir $n^2+3n-88=0\Rightarrow n=8$

81. (729)

$\log_3(\log_2 x) + \log_{1/3}(\log_{1/2} y) = 1$

$\Rightarrow \log_3(\log_2 x) - \log_3(-\log_2 y) = 1$

$\Rightarrow \log_3\left(-\dfrac{\log_2 x}{\log_2 y}\right) = 1$

$\Rightarrow -\dfrac{\log_2 x}{\log_2 y} = 3$

$\Rightarrow xy^3 = 1$

Also, $xy^2 = 9$

$\Rightarrow y = \dfrac{1}{9}$

$\therefore x = 729$

82. (9)

$18\sin^2\theta + 2\operatorname{cosec}^2\theta - 3$

$= \left(\sqrt{18}\sin\theta - \sqrt{2}\operatorname{cosec}\theta\right)^2 + 9$

83. (2)

Given expression $= \dfrac{\sin 50^\circ + \sin 140^\circ + \sin 170^\circ}{2\sin 25^\circ \sin 70^\circ \sin 85^\circ}$

$= \dfrac{4}{2}\left(\text{Using } \sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C, \text{where } A + B + C = 180^\circ\right)$

= 2

84. (4)

$3^{\sin 2x} + 2\cos^2 x + 3^{1+2\left(1-\cos^2 x\right)-\sin 2x} = 28$

$\Rightarrow 3^{\sin 2x + 2\cos^2 x} + \dfrac{27}{3^{\sin 2x + 2\cos^2 x}} = 28$

$\Rightarrow \left(3^t\right)^2 - 28\left(3^t\right) + 27 = 0 \Rightarrow 3^t = 27 \text{ or } 1$

$\Rightarrow \sin 2x + 2\cos^2 x = 0 \text{ or } 3(\text{not possible})$

$\Rightarrow \sin 2x + 2\cos^2 x = 0$

$\Rightarrow \sin 2x + \cos 2x = -1$

$\Rightarrow \sin 4x = 0(\text{by squaring})$

$\Rightarrow 4x = n\pi, n \in Z$

$\Rightarrow x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}, \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

85. (4)

$\sqrt[4]{|x-3|^{x+1}} = \sqrt[3]{|x-3|^{x-2}}$

Taking log on both the sides

$\dfrac{x+1}{4}\log|x-3| = \dfrac{x-2}{3}\log|x-3|$

$\Rightarrow \log|x-3|\left[\frac{x+1}{4}-\frac{x-2}{3}\right]=0$

$\Rightarrow \log|x-3|=0 \text{ or } \left[\left(\frac{x+1}{4}\right)-\left(\frac{x-2}{3}\right)\right]=0$

$\Rightarrow x=4 \text{ or } x=11$

86. (25)

Given A.P. is $a_1, a_2, a_3, \ldots\ldots, a_{15}$

$a_1+a_{15}=a_2+a_{14}=\ldots\ldots=2a_8$

$a_1+a_{15}+a_8=\frac{3}{2}(a_1+a_{15})=15$

$\Rightarrow a_1+a_{15}=10$

$a_2+a_3+a_8+a_{13}+a_{14}=2(a_1+a_{15})+a_8$

$=2(10)+5=25$

87. (42)

Let r is the common ratio.

$\Rightarrow \frac{a+ar}{2}=6$ and $\frac{ar^2+ar^3}{2}=54$

$\Rightarrow r^2=9 \Rightarrow r=\pm 3 \Rightarrow r=3 (r\neq -3)$

When $r=3$, $a=3$ AM of a and d $=\frac{a+ar^3}{2}=42$

88. (32)

$a+b=12$

$ab+\frac{6ab}{a+b}=48$

$\therefore ab+\frac{ab}{2}=48$

$\therefore ab=32$

89. (14)

$T_r={}^{14}C_{r-1}x^{r-1}; T_{r+1}={}^{14}C_r x^r; T_{r+2}={}^{14}C_{r+1}x^{r+1}$

By the given condition $2.{}^{14}C_r={}^{14}C_{r-1}+{}^{14}C_{r+1}$ …….(i)

$\Rightarrow 2.\frac{14!}{r!(14-r)!}=\frac{14!}{(r-1)!(15-r)!}+\frac{14!}{(r+1)!(13-r)!}$

$\Rightarrow \frac{2}{r.(r-1)!(14-r).(13-r)!}$

$=\frac{1}{(r-1)!(15-r).(14-r).(13-r)!}+\frac{1}{(r+1)r(r-1)!(13-r)!}$

$\Rightarrow \frac{2}{r(14-r)}=\frac{1}{(15-r)(14-r)}+\frac{1}{(r+1)r}$

$$\Rightarrow \frac{1}{r(14-r)}-\frac{1}{(15-r)(14-r)}=\frac{1}{(r+1)r}-\frac{1}{r(14-r)}$$
$$\Rightarrow \frac{(15-r)-r}{r(15-r)(14-r)}=\frac{(14-r)-(r+1)}{(r+1)r(14-r)}$$
$$\Rightarrow \frac{15-2r}{15-r}=\frac{13-2r}{r+1}$$
$$\Rightarrow 15r+15-2r^2-2r=195-30r-13r+2r^2$$
$$\Rightarrow 4r^2-56r+180=0 \Rightarrow r^2-14r+45=0$$
$$\Rightarrow (r-5)(r-9)=0 \Rightarrow r=5,9$$

90. (7920)

We have $(x)^{12-r}\left(\frac{1}{x^2}\right)^r = x^0 \Rightarrow x^{12-3r}=x^0 \Rightarrow r=4$

Hence the required term is ${}^{12}C_4 2^8\left(-\frac{1}{2}\right)^4 = 7920$