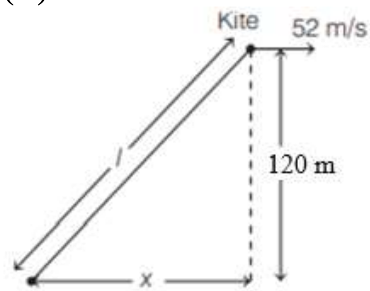
PART (A) : PHYSICS

1. (A)



$$l^2 = (120)^2 + x^2$$

$$2l\frac{dl}{dt} = 2x\frac{dx}{dt}$$

$$\frac{dl}{dt} = \frac{x}{l}\frac{dx}{dt}$$

$$= \frac{50}{130}(52) = 20\,m/s$$

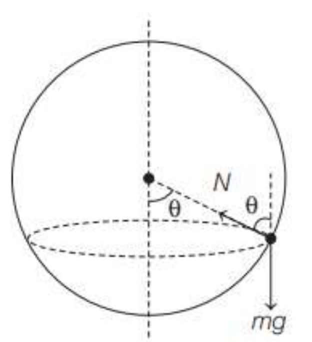
2. (B)

$$[a] = [t^2] = [T^2] \text{ and } \left[\frac{t^2}{bx}\right] = [p]$$

$$\Rightarrow \quad [b] = \left[\frac{t^2}{px}\right] \Rightarrow [b] = \left[\frac{T^2}{ML^{-1}T^{-2}L}\right] = [M^{-1}T^4]$$

$$\text{So, } \left[\frac{a}{b}\right] = \left[\frac{T^2}{M^{-1}T^4}\right] = [MT^{-2}]$$

3. (B)



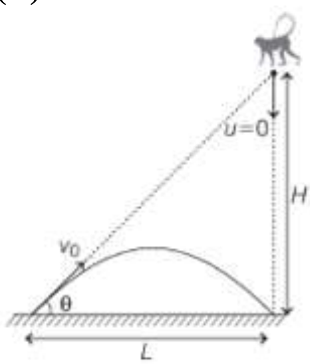
$$\Sigma F_y = 0 \Rightarrow N\cos\theta = mg \quad \text{...(i)}$$

$$\Sigma F_x = ma_x \Rightarrow N\sin\theta = m\omega^2\left(\frac{a}{2}\right) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\omega = \left(\frac{2g}{\sqrt{3}a}\right)^{1/2}$$

4. (A)



For minimum speed,arrow will hit the monkey just at ground.

$$T = \frac{2v_0 \sin\theta}{g} = \sqrt{\frac{2H}{g}}$$

$$\Rightarrow \quad v_0 \sin\theta = \sqrt{\frac{gH}{2}} \quad \text{...(i)}$$

$$R = L = (v_0 \cos\theta)\sqrt{\frac{2H}{g}}$$

$$v_0 \cos\theta = \sqrt{\frac{gL^2}{2H}} \quad \text{...(ii)}$$

Adding and squaring Eqs. (i) and (ii), we get

$$v_0 = \sqrt{\frac{g(H^2+L^2)}{2H}}$$

5. (A)

Lets take maximum elongation in the spring = x and displacement of 2 M as y.

From string constraint

$$2y = x$$

Applying work-energy theorem for system

$$W_{mg} + W_N + W_T + W_{spring} = \Delta K_{system}$$

$$\Rightarrow \quad +2Mg\left(\frac{x}{2}\right) + 0 + 0 + \frac{1}{2}k(0^2 - x^2)$$

$$= (0+0) - (0+0)$$

$$\Rightarrow \quad x = \frac{2Mg}{k}$$

6. (B)

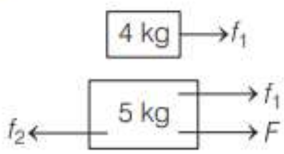
$v = \omega r$ and $a = \omega^2 r$

If radius becomes half, then both v and a will become half of the original values.

7. (C, D)

$f_{1L} = 0.2(40) = 8$ N

$f_{2L} = 0.1(90) = 9$ N



For 4 kg,

$$\Sigma F_x = ma$$

$\Rightarrow$ $f_1 = 4a$

and $f_1 \leq f_{1L}$

$4a \leq 8$

$\Rightarrow$ $a \leq 2\ \text{m/s}^2$

$\Rightarrow$ $a_{max} = 2\ \text{m/s}^2$

For (4 kg + 5 kg) system,

$$\Sigma F_x = ma_x$$

$\Rightarrow$ $F - 9 = 9a$

$\Rightarrow$ $F = 9a + 9$

8. (A,B,D)

(A) With respect to plank, tension is always perpendicular to velocity. So, work done by tension is zero.

(B) In time $\frac{\pi l}{2v_0}$, it will complete quarter circle.

$$W_T = \Delta K = \frac{1}{2}m(2v_0)^2 - \frac{1}{2}m(\sqrt{2}v_0)^2 = mv_0^2$$

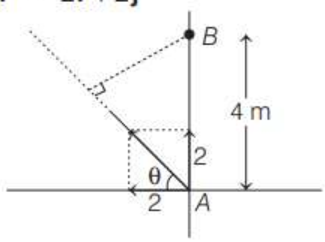
(C) In one revolution, $W_T = 0 \because \Delta K = 0$

(D) In half revolution, initial and final speeds are $\sqrt{2}v_0$.

So, $\Delta K = 0 \Rightarrow W_T = 0$

9. (B,C,D)

$\mathbf{v}_{AB} = 2\hat{\mathbf{j}} - 2\hat{\mathbf{i}} = -2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$



$\tan\theta = 1$

$r_{min} = 4\cos\theta = 2\sqrt{2}$ m

$t = \frac{4\sin\theta}{2\sqrt{2}} = 1$ s

10. (B, C)

$U = 2 + 20x - 5x^2$

At $x = 3; U = 2 + 20(-3) - 5(-3)^2 = -103$ J

$K = 0$

$E = U + K = -103$ J

For maximum value of x, $v = 0$

$\Rightarrow K = 0$

$\Rightarrow U = -103$ J

$\Rightarrow 2 + 20x - 5x^2 = -103$

$\Rightarrow 5x^2 - 20x - 105 = 0$

$\Rightarrow x^2 - 4x - 21 = 0$

$\Rightarrow (x+3)(x-7) = 0$

$\Rightarrow x = -3$ and $x = 7$

11. (B,C,D)

$R = \frac{u^2 \sin 2\theta}{g} \Rightarrow R = \frac{(20)^2 \sin 30^\circ}{10} = 20$ m

$100 \times \frac{\Delta R}{R} = 2\frac{\Delta u}{u} \times 100$

$\Rightarrow$ % error in $R = 2(3\%) = 6\%$

So, $R = (20 \pm 6\%) = (20 \pm 1.2)$ m

12. (A, C)

13. (3.00)

Using work-energy theorem between A and B,

$$W_{mg} + W_N = \Delta K$$

$$\Rightarrow \quad +mg(2R) + 0 = \frac{1}{2}mv_2^2 - 0$$

$$\Rightarrow \quad v_2 = \sqrt{4gR}$$

At point 2,

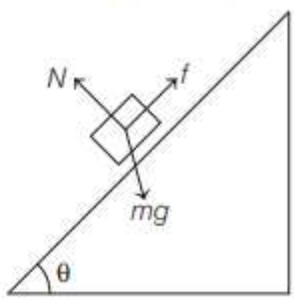
$$N + mg = \frac{mv_2^2}{R}$$

$$\Rightarrow \quad N + mg = 4mg$$

$$\Rightarrow \quad N = 3mg$$

14 (30.00)

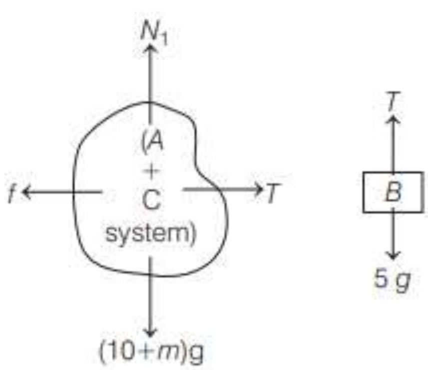
Since, the block is at rest, $N = mg\cos\theta$



$$f = mg\sin\theta$$

$$\text{Contact force} = \sqrt{N^2 + f^2} = mg = 30\text{N}$$

15. (15.00)



For B, $\quad \Sigma F_y = 0 \Rightarrow T = 5g$

For $(A + C)$ system,

$$\Sigma F_y = 0$$

$\Rightarrow$ $N_1 = (10+m)g$

$$f_L = \mu N_1 = 0.2(10+m)g$$

$$\Sigma F_x = 0 \Rightarrow T = f$$

$\Rightarrow$ $f = 5g$

$$f \le f_L$$

$\Rightarrow$ $5g \le 0.2(10+m)g$

$\Rightarrow$ $10 + m \ge 25$

$\Rightarrow$ $m \ge 15$ kg

$\Rightarrow$ $m_{min} = 15$ kg

16. (2.00)

$$s_x = u_x t + \frac{1}{2}a_x t^2$$

$$(\sqrt{3}+1) = (v_1 \cos 45^\circ)t \quad \text{...(i)}$$

$$(\sqrt{3}+1) = (v_2 \cos 60^\circ)t \quad \text{...(ii)}$$

$$s_y = u_y t + \frac{1}{2}a_y t^2$$

$$-y = (v_1 \sin 45^\circ)t - \frac{1}{2}gt^2 \quad \text{...(iii)}$$

$$h - y = (v_2 \sin 60^\circ)t - \frac{1}{2}gt^2 \quad \text{...(iv)}$$

From Eq. (i), we get $v_1 t = (\sqrt{3}+1)\sqrt{2}$

From Eq. (ii), we get $v_2 t = (\sqrt{3}+1)2$

Subtracting Eq. (iii) from Eq. (iv), we get

$$h = (v_2 t \sin 60^\circ - v_1 t \sin 45^\circ)$$

$\Rightarrow$ $h = (\sqrt{3}+1)2\left(\frac{\sqrt{3}}{2}\right) - (\sqrt{3}+1)\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$

$\Rightarrow$ $h = (\sqrt{3}+1)(\sqrt{3}-1) = 2$ m

17. (0.16)

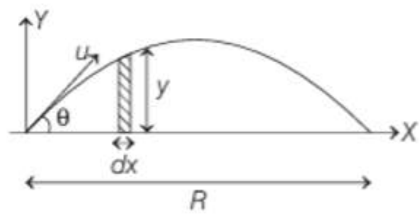
$$\mathbf{v} = 2\hat{\mathbf{i}} + 4t\hat{\mathbf{j}}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 4\hat{\mathbf{j}}$$

$$P = \mathbf{F}\cdot\mathbf{v} = (m\mathbf{a})\cdot\mathbf{v} = 2(16t)$$

At $t = 5$, $\quad P = 160\text{ W}$

18. (3.00)



Equation of trajectory,

$$y = x\tan\theta\left(1 - \frac{x}{R}\right)$$

$$A = \text{Area under trajectory} = \int_0^R y\,dx$$

$$A = \int_0^R \left(x\tan\theta - \frac{x^2\tan\theta}{R}\right)dx$$

$$A = \left[\frac{x^2\tan\theta}{2} - \frac{x^3\tan\theta}{3R}\right]_0^R$$

$$A = \frac{R^2\tan\theta}{2} - \frac{R^3\tan\theta}{3R}$$

$$A = \frac{R^2\tan\theta}{6}$$

$$A = \frac{\left(\dfrac{u^2\,2\sin\theta\cos\theta}{g}\right)^2\tan\theta}{6}$$

$$A = \frac{2}{3}\frac{u^4}{g^2}\sin^3\theta\cos\theta$$

To maximise area,

$$\frac{dA}{d\theta} = 0$$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \quad \theta = \frac{\pi}{3}$$

PART (B) : CHEMISTRY

19. (B)

$C_7H_8 + 3HNO_3 \rightarrow C_7H_5(NO_2)_3 + 3H_2O$

1 mole C_7H_8 needed $3HNO_3$

$\frac{276\text{ g}}{92} = 3\text{ moles}$

Requires = 9 moles HNO_3

So, mass $= 9 \times 63 = 567\text{ g}$.

20. (C)

21. (B)

22. (B)

23. (B)

Boiling point $\propto a$

24. (A)

$q = +600\text{ J} \qquad w = -300\text{ J}$

$\Delta U = q + w$

$\Delta U = 600 - 300$

$\Delta U = 300\text{ J}$

25. (A, B, D)

26. (A, B, C)

At constant T and mols, PV = constant

$\therefore$ PV vs anything is a straight line

Also, as $PV = c \Rightarrow P = \frac{c}{v} \Rightarrow P$ vs $\frac{1}{v}$ is a straight line passing through the origin.

27. (A, B, C)

System surroundings are always separated by a boundary. This boundary is real in case of closed systems and imaginary in case of open systems. A perfectly isolated system is impossible to create. During a reversible process system and surrounding are always in equilibrium. Hence, it is called quasi static.

28. (B, D)

29. (A, B, C)

According to Fajan's rule, larger the anion and greater the charge density on cation more is the covalent character.

30. (A, C)

31. (6.00)

$$C_{process} = C_V + \frac{R}{1-(-3)}$$

$$\frac{\Delta U}{Q} = \frac{nC_V\Delta T}{nC_{Process}\Delta T} = \frac{1.5R}{1.5R + 0.25R} = \frac{6}{7}$$

32. (0.00)
Extensive $\rightarrow$ (ii), (iii), (v), (vi), (ix), (xii)
Intensive $\rightarrow$ (i), (iv), (vii), (viii), (x), (xi)

33. (25.00)

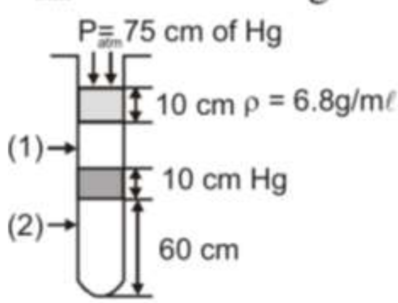
$$\lambda_1 = \left(\frac{150}{V_1}\right)^{1/2} \Rightarrow V_1 = \frac{150}{\lambda_1^2}$$

$$\Rightarrow V_1 = \frac{150}{(1.41)^2} = 75\text{ V} \text{ and } V_2 = \frac{150}{(1.73)^2} = 50\text{ V}$$

Hence, potential should be dropped by 25 V.

34. (90.00)

$P_{atm} = 75$ cm of Hg

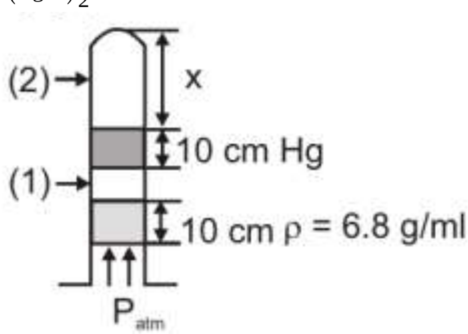


Converting the pressure due to the first liquid

$10 \times 6.8 = 13.6 \times h_1$

$\Rightarrow h_1 = 5 \text{ cm of Hg}$

$(P_{gas})_2 = 75 + 5 + 10 = 90 \text{ cm of Hg}$



$\left(P'_{gas}\right)_2 = 75 - 5 - 10 = 60 \text{ cm of Hg}$

$\Rightarrow P_1V_1 = P_2V_2$

$60x = 90 \times 60$

$\Rightarrow x = 90 \text{ cm}$

35. (39.00)

$CaCO_3(s) \xrightarrow{\Delta} CaO(s) + CO_2(s)$

Moles of $CaCO_3$ formed $= \frac{25}{100} =$ moles of CO_2 formed

Mass of CaO formed $= \frac{25}{100} \times 56g = 14 \text{ g}$

Volume occupied by CaO $= \frac{14}{3.5} \text{cc} = 4 \text{ mL}$

$\therefore$ Volume available for

$CO_2(g) = 504 - 4 \text{ mL} = 0.5 \text{ L}$

Now applying the Vander Waal's equation of state

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

$$\left[P + \frac{4 \times (0.25)^2}{(0.5)^2}\right][0.5 - 0.25 \times 0.04]$$

$= 0.25 \times 0.08 \times 980$

$$\Rightarrow P = 40 - \frac{4 \times (0.25)^2}{(0.5)^2} = 39 \text{ atm}$$

36. (5.14)

Let the moles of $CO_2 = x$, then moles of $SO_2 = \frac{x}{2}$ moles of carbon in sample = x and moles of sulphur in a sample $= \frac{x}{2}$

$12x + \frac{x}{2} \times 32 = 12 \quad \Rightarrow \quad x = \frac{12}{28} = \frac{3}{7}$

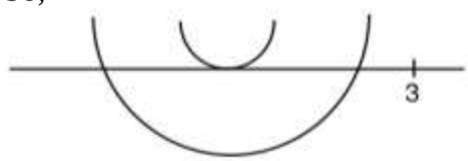
Mass of carbon in sample $= \frac{3}{7} \times 12 = \frac{36}{7} = 5.14 \text{ gm}$

PART (C) : MATHEMATICS

37. (A)

$x^2 - 2ax + (a^2 + a - 3) = 0$ both roots are less than 3

So,



$D = 4a^2 - 4(a^2 + a - 3) \geq 0$

$a - 3 \leq 0$

$a \leq 3$

$\frac{-b}{2a} = \frac{2a}{2} < 3$

$\Rightarrow a < 3$

$f(3) = 9 - 6a + a^2 + a - 3 > 0$

$\Rightarrow a^2 - 5a + 6 > 0$

$(a-3)(a-2) > 0$

$a \in (-\infty, 2) \cup (3, \infty)$

So, $a < 2$

38. (A)

$1 + \tan^2 3x < 2$

$\tan^2 3x < 1$

$-1 < \tan 3x < 1$

$3x \in \left(n\pi - \frac{\pi}{4}, n\pi + \frac{\pi}{4}\right)$

$x \in \left(\frac{n\pi}{3} - \frac{\pi}{12}, \frac{n\pi}{3} + \frac{\pi}{12}\right)$

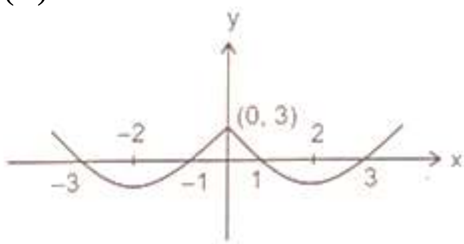
39. (A)

$T_6 = {}^8C_5 \left(\frac{1}{x^{8/3}}\right)^3 (x^2 \log_{10} x)^5 = 5600$

$\Rightarrow x^2 (\log_{10} x)^5 = 100$

$\Rightarrow x = 10$

40. (A)



41. (C)

$-\sqrt{a^2+b^2} \le 20p+35 \le \sqrt{a^2+b^2}$

$-\sqrt{99^2+(-20)^2} \le 20p+35 \le \sqrt{99^2+(-20)^2}$

$-101 \le 20p+35 \le 101$

$-136 \le 20p \le 66$

$-6.8 \le p \le 3.3$

So, total number of integers = 10

42. (B)

$\left(3^{1/7}+5^{1/2}\right)^{14}$

General term $= {}^{14}C_r\left(3^{1/7}\right)^{14-r}\cdot\left(5^{1/2}\right)^r$

$= {}^{14}C_r 3^{\frac{14-r}{7}} 5^{r/2}$

For rational term

$\frac{r}{2}$ should be rational $\Rightarrow r=0, r=14$

Coefficient $= {}^{14}C_0.3^2.5^0 + {}^{14}C_{14}3^0 5^7$

$= 3^2+5^7$

43. (B, D)

$\therefore \quad \cos^4\theta-\sin^4\theta=\cos 2\theta$

$\Rightarrow \quad \cos^4\theta-\sin^4\theta=\cos^2\theta-\sin^2\theta$

$\Rightarrow \quad \left(\cos^4\theta+p\right)-\left(\sin^4\theta+p\right)=\left(\cos^2\theta+q\right)-\left(\sin^2\theta+q\right)$

$\Rightarrow \quad \frac{\sqrt{4a^2-4a}}{1}=\frac{\sqrt{16-8}}{1} \quad \left[\because\ \alpha-\beta=\frac{\sqrt{D}}{a}\right]$

$\Rightarrow \quad 4a^2-4a=8$ or $a^2-a-2=0$

or $\quad (a-2)(a+1)=0$ or $a=2,-1$

44. (A)

$$\frac{n(n+1)}{2}-(2k+1)=\frac{105}{4}(n-2)$$

$$2n^2+2n-8k-4=105n-210$$

$$2n^2-103n-8k+206=0$$

$\therefore$ n must be even

$\because$ $k\in[1, n-1]$

For $k=1,\ 2n^2-103n+198=0 \Rightarrow n=2, \frac{99}{2}$

For $k=n-1,\ 2n^2-111n+214=0 \Rightarrow n=2, \frac{107}{2}$

$\therefore$ $n\in\left(\frac{99}{2}, \frac{107}{2}\right) \Rightarrow n=50$ or 52

For $n=50 \Rightarrow k=\frac{5000-5150+206}{8}=7$

For $n=52 \Rightarrow k=\frac{5408-5356+206}{8}=\frac{258}{8}$

45. (A, B)

$$21-4a-a^2\geq 0$$

$$a^2+4a-21\leq 0$$

$$(a+7)(a-3)\leq 0$$

$$a\in[-7, 3]$$

Case-I $a>-1$

On squaring

$$a^2+3a-10\geq 0$$

$$a\in(-\infty, -5]\cup[2, \infty)$$

$a\in[2, \infty)$ ...(i)

Case-II $a<-1$

L.H.S. is negative ≤ 1

$a\in(-\infty, -1)$

i.e. $a\in[-7, -1)\cup[2, 3]$ ...(ii)

46. (A,C, D)

$$RS=\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\ldots\ldots\cdot\frac{224}{225}=\frac{1}{225}$$

$R<S \Rightarrow R^2<RS \Rightarrow R<\frac{1}{15}$ and $S>\frac{1}{15}>\frac{1}{17}$

47. (A, B, C, D)

$$N=\frac{\log_3 135}{\log_{15} 3}-\frac{\log_3 5}{\log_{405} 3}=(\log_3 27+\log_3 5)-(\log_3 15)\log_3 5.\log_3 405$$
$$=(3+\log_3 5)(1+\log_3 5)-\log_3 5\log_3(81\times 5)=(3+\log_3 5)(1+\log_3 5)-\log_3 5(4+\log_3 5)=3$$

48. (A, B)

$$5\sin x\cos y=1 \qquad \Rightarrow \sin x\cos y=\frac{1}{5} \qquad .....(\text{i})$$
and $\frac{4\sin x}{\cos x}=\frac{\sin y}{\cos y}$
$$\Rightarrow \quad 4\sin x\cos y=\cos x\sin y$$
$$\Rightarrow \quad \cos x\sin y=\frac{4}{5} \qquad .....(\text{ii})$$
$$(\text{i})+(\text{ii});\ \sin(x+y)=1 \Rightarrow x+y=2n\pi+\frac{\pi}{2} \qquad ...(\text{i})$$
$$(\text{i})-(\text{ii});\ \sin(x-y)=\frac{-3}{5} \qquad \Rightarrow x-y=m\pi+(-1)^m\sin^{-1}\left(\frac{-3}{5}\right) \qquad .....(\text{ii})$$
$(1)+(2)$
$$2x=2n\pi+m\pi+\frac{\pi}{2}+(-1)^m\sin^{-1}\left(\frac{-3}{5}\right)$$
$$\Rightarrow \quad x=(m+2n)\frac{\pi}{2}+\frac{\pi}{4}+(-1)^m\frac{1}{2}\sin^{-1}\left(\frac{-3}{5}\right) \text{ and } y=2n\pi+\frac{\pi}{2}-x$$
$$\Rightarrow \quad y=(2n-m)\frac{\pi}{2}+\frac{\pi}{4}+(-1)^{m+1}\frac{1}{2}\sin^{-1}\left(\frac{-3}{5}\right)$$

49. (0.50)

$$f(x)=\frac{1-\sin 2x+\cos 2x}{2\cos 2x}=\frac{1}{2}\left[\frac{(\cos x-\sin x)^2}{\cos^2 x-\sin^2 x}+1\right]=\frac{1}{2}[\tan(45°-x)+1]$$
$$f(16°)=\frac{1}{2}[\tan 29°+1] \qquad \Rightarrow \qquad f(29°)=\frac{1}{2}[\tan 16°+1]$$
$$\Rightarrow \quad f(16°)\,f(29°)=\frac{1}{4}[1+\tan 16°][1+\tan 29°]=\frac{1}{4}(2)=\frac{1}{2}$$

50. (8.00)

$$\sum_{m=97}^{100}\frac{100!}{m!(100-m)!}\cdot\frac{m!}{97!(m-97)!}$$
$$=\sum_{m=97}^{100}\frac{100!}{97!3!}\cdot\frac{3!}{(100-m)!(m-97)!}={}^{100}C_{97}\cdot\sum_{m=97}^{100}{}^{3}C_{m-97}$$
$$={}^{100}C_{97}\cdot 2^3$$

51. (1.33)

$$\sum_{r=0}^{n}\frac{2r+3}{r+1}\cdot {}^{n}C_r=\sum_{r=0}^{n}2\cdot {}^{n}C_r+\sum_{r=0}^{n}\frac{1}{r+1}\cdot {}^{n}C_r=2\cdot 2^n+\frac{1}{n+1}\cdot\sum_{r=0}^{n}\cdot {}^{n+1}C_{r+1}$$

$$=2^{n+1}+\frac{1}{n+1}\cdot\left(2^{n+1}-1\right)=\frac{(n+2)\cdot 2^{n+1}-1}{n+1}$$

$\therefore\ 3k=4$

52. (0.1)

$(\log x)^2-\log x-2\geq 0$

$x>0$ .....(i)

$(\log x-2)(\log x+1)\geq 0$

$\Rightarrow\ \log x\leq -1$ or $\log x\geq 2$

$\Rightarrow\ x\leq\frac{1}{10}$ or $x\geq 100$ .....(ii)

$(\text{i})\cap(\text{ii})\ \Rightarrow\ x\in\left(0,\frac{1}{10}\right]\cup[100,\infty)$

53. (2.20)

$\because\ xy+x+y=71\ \Rightarrow\ xy+(x+y)=71$

and $x^2y+xy^2=880\ \Rightarrow\ xy(x+y)=880$

$\Rightarrow$ xy and $(x+y)$ are the roots of the quadratic equation.

$t^2-71t+880=0$

$\Rightarrow\ (t-55)(t-16)=0$

$\Rightarrow\ x+y=16$ and $xy=55$

$\therefore\ x=11,\ y=5$

$\therefore\ \frac{x}{y}=\frac{11}{5}=2.20$

54. (11.00)

2, 5, 8, ……

$a=2$, $d=3$

$\Rightarrow\ S_{2n}=n\left(4+(2n-1)3\right]=n(6n+1)$

$\Rightarrow$ 57, 95, 61, ...

$S_n=\frac{n}{2}\left[2\times 57+(n-1)2\right]\quad = n[57+n-1]=n(56+n)$

$n(6n+1)=n(56+n)\quad\Rightarrow\ 5n=55\ \Rightarrow\ n=11.$

PART (A) : PHYSICS

1. (1)

$\mathbf{F}_1 = 3\hat{\mathbf{k}}$

$\mathbf{F}_2 = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

$\mathbf{F}_3 = -4\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$

$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 = 0$

$\Rightarrow \quad -\hat{\mathbf{i}} + 3\hat{\mathbf{k}} + \mathbf{F}_4 = 0$

$\Rightarrow \quad \mathbf{F}_4 = \hat{\mathbf{i}} - 3\hat{\mathbf{k}}$

$\Rightarrow \quad F_4 = \sqrt{10}\,\text{N}$

2. (3)

$$H_{max} = \frac{u^2}{2g}$$

$$15 = \frac{\left(\frac{\sqrt{u^2 + 2g(15)}}{2}\right)^2}{2g}$$

$$300 = \frac{u^2 + 300}{4}$$

$$u^2 = 900$$

$$u = 30\,\text{m/s}$$

3. (4)

$v_{MR} = 5\,\text{m/s}$

$v_R = 4\,\text{m/s}$

$$t = \frac{d}{\sqrt{v_{MR}^2 - v_R^2}} = \frac{480}{3} = 160\,\text{s}$$

4. (4)

$$a_1 = \frac{40}{9}\,\text{m/s}^2$$

$$a_2 = \frac{50 - 40}{9} = \frac{10}{9}\,\text{m/s}^2$$

5. (1)

Applying work-energy theorem,

$$W_{mg} + W_N + W_{\text{friction}} + W_{\text{spring}} = \Delta K$$

$$0 + 0 - (\mu mg)x + \frac{1}{2}K(0^2 - x^2) = 0 - \frac{1}{2}mu^2$$

$\Rightarrow$ $-50x - 50x^2 = -\frac{1}{2} \times 50 \times 2^2$

$\Rightarrow$ $x^2 + x - 2 = 0$

$\Rightarrow$ $x = 1\,\text{m}$

6. (6)

Area =36 m

7. (A, B, D)

Lets take magnitude of each of them to be x.

$\mathbf{A} = x\hat{\mathbf{j}}, \mathbf{B} = x\cos 45^\circ\,\hat{\mathbf{i}} + x\sin 45^\circ\,\hat{\mathbf{j}},$

$\mathbf{C} = x\hat{\mathbf{i}}, \mathbf{D} = x\cos 45^\circ\,\hat{\mathbf{i}} - x\sin 45^\circ\,\hat{\mathbf{j}}$

$\mathbf{A} - \mathbf{C} = x\hat{\mathbf{j}} - x\hat{\mathbf{i}} = -\sqrt{2}\mathbf{D}$

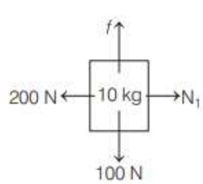
$\mathbf{B} + \mathbf{D} - \sqrt{2}\mathbf{C} = \left(\frac{x}{\sqrt{2}}\hat{\mathbf{i}} + \frac{x}{\sqrt{2}}\hat{\mathbf{j}}\right) + \left(\frac{x}{\sqrt{2}}\hat{\mathbf{i}} - \frac{x}{\sqrt{2}}\hat{\mathbf{j}}\right) - \sqrt{2}x\,\hat{\mathbf{i}} = 0$

$$\frac{\mathbf{A} + \mathbf{C}}{\sqrt{2}} = \frac{x\hat{\mathbf{j}} + x\hat{\mathbf{i}}}{\sqrt{2}} = \mathbf{B}$$

8. (C, D)

9. (A, B, C, D)

w.r.t car



$\Sigma F_x = 0$

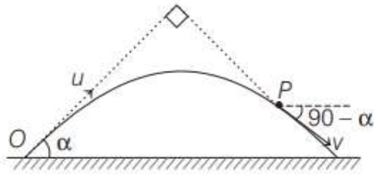
$\Rightarrow$ $N_1 - 200 = 0 \Rightarrow N_1 = 200\,\text{N}$

So, $f_L = \mu_s N_1 = 0.6 \times 200 = 120\,\text{N}$

Since, mg is less than $f_L = 120\,\text{N}$, friction between the box and car will be 100 N (static).

So, $f = 100\,\text{N}$ and the box will not slide down the vertical wall.

Contact force $= \sqrt{N^2 + f^2} = \sqrt{(200)^2 + (100)^2} = 100\sqrt{5}\,\text{N}$



(d) $v\cos(90-\alpha) = u\cos\alpha$

$$v = u\cot\alpha$$

(c) $\tan(-(90-\alpha)) = \dfrac{u\sin\alpha - gt}{u\cos\alpha}$

$$\Rightarrow \quad t = \frac{u}{g\sin\alpha}$$

(b) $\mathbf{r} = (u\cos\alpha)\left(\dfrac{u}{g\sin\alpha}\right)\hat{\mathbf{i}} + \left(u\sin\alpha\left(\dfrac{u}{g\sin\alpha}\right)\right) - \dfrac{1}{2}g\left(\dfrac{u}{g\sin\alpha}\right)^2\hat{\mathbf{j}}$

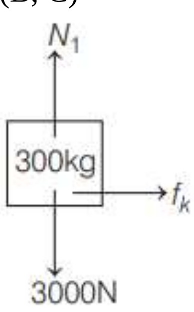
$$\mathbf{r} = \frac{u^2}{g}\cot\alpha\,\hat{\mathbf{i}} + \left(\frac{u^2}{g} - \frac{u^2}{2g}\operatorname{cosec}^2\alpha\right)\hat{\mathbf{j}}$$

$$\mathbf{r} = \frac{u^2}{g}\cot\alpha\,\hat{\mathbf{i}} + \frac{u^2}{g}\left(1 - \frac{\operatorname{cosec}^2\alpha}{2}\right)\hat{\mathbf{j}}$$

$$OP = \sqrt{\left(\frac{u^2}{g}\cot\alpha\right)^2 + \left(\frac{u^2}{g}\left(1 - \frac{\operatorname{cosec}^2\alpha}{2}\right)\right)^2}$$

(a) $\tan\theta = \dfrac{\dfrac{u^2}{g}\left(1 - \dfrac{\operatorname{cosec}^2\alpha}{2}\right)}{\dfrac{u^2}{2}\cot\alpha}$

10. (B, C)



For crate,

$$\Sigma F_y = 0$$

$\Rightarrow$ $+N_1 - 3000 = 0$

$\Rightarrow$ $N_1 = 3000\,\text{N}$

$$f_k = \mu_k N = 0.4\,(3000) = 1200\,\text{N}$$

$$\Sigma F_x = ma$$

$\Rightarrow$ $1200 = 300a$

$\Rightarrow$ $a = 4\,\text{m/s}^2$

$$v = u + at$$

$\Rightarrow$ $4 = 0 + 4t_0$

$\Rightarrow$ $t_0 = 1\,\text{s}$

$$s_{\text{belt}} = ut + \frac{1}{2}at^2 = 4 \times 1 = 4\,\text{m}$$

$$s_{\text{block}} = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}(4)(1)^2$$

$\Rightarrow$ $s_{\text{block}} = 2\,\text{m}$

Work done friction on the crate $= (1200)\,(2) = 2.4\,\text{kJ}$

Work done by friction on the belt $= -\,(1200)\,(4)$

$$= -\,4.8\,\text{kJ}$$

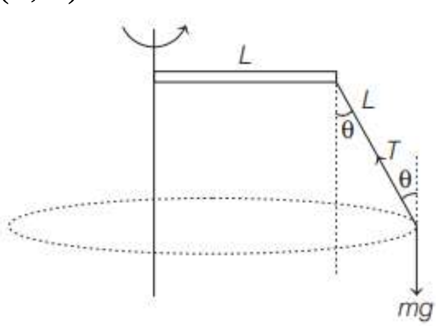
Using work-energy theorem for belt,

$$W_{\text{motor}} + W_{\text{friction}} = \Delta K$$

$\Rightarrow$ $W_{\text{motor}} - 4.8\,\text{kJ} = 0 - 0$

$\Rightarrow$ $W_{\text{motor}} = 4.8\,\text{kJ}$

11. (B, C)



$$\Sigma F_y = 0$$

$\Rightarrow$ $T\cos\theta = mg$ ...(i)

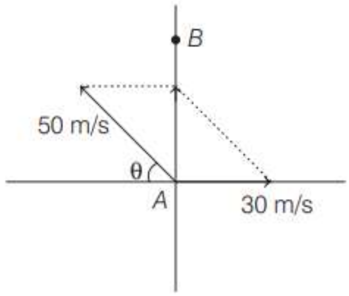
$$\Sigma F_x = ma_x$$

$\Rightarrow$ $T\sin\theta = m\omega^2 L(1 + \sin\theta)$ ...(ii)

From Eqs. (i) and (ii), we get

$$\tan\theta = \frac{\omega^2 L(1 + \sin\theta)}{g}$$

12. (A, B)



$50\cos\theta = 30$

$\theta = 53°$ North of West

$$\text{Time taken} = \frac{720}{50\sin\theta \times \frac{18}{5}} = 5\text{ h}$$

13. (2.00)

$$a = v\frac{dv}{dx}$$

$$a = 2\tan(120°) = -2\sqrt{3}\text{ m / s}^2$$

14. (0.26 – 0.27)

$$y = \frac{t^2 + t^3}{8}$$

$$\frac{dy}{dt} = \frac{2t + 3t^2}{8}$$

$$dy = \left(\frac{2t + 3t^2}{8}\right)dt$$

$$100 \times \frac{dy}{y} = \frac{\left(\frac{2t + 3t^2}{8}\right)t}{y}\left(\frac{dt}{t}\right) \times 100$$

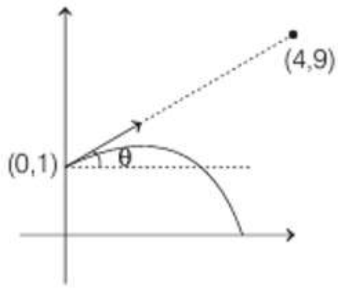
$$\%\text{ error in } y = \left(\frac{2t^2 + 3t^3}{8y}\right)\left(\frac{dt}{t} \times 100\right)$$

$$= \frac{32}{8\left(\frac{12}{8}\right)} \times (0.1\%) = 0.267\%$$

15. (2.00)

$$\sin\theta = \frac{8}{\sqrt{8^2 + 4^2}}$$

$$\Rightarrow \sin\theta = \frac{8}{\sqrt{80}} = \frac{2}{\sqrt{5}}$$



$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad -1 = \left(u \frac{2}{\sqrt{5}}\right)(1) + \frac{1}{2}(-10)(1)^2$$

$$\Rightarrow \quad u = 2\sqrt{5}\ \text{m/s}$$

$$x = (u\cos\theta)\,t = (2\sqrt{5})\frac{(4)}{\sqrt{80}}(1) = 2\ \text{m}$$

16. (6.00)

Using work-energy theorem between A and B

$$W_{mg} + W_N + W_{spring} = \Delta K$$

$$\Rightarrow +mg(2R) + 0 + \frac{1}{2}\frac{mg}{R}(R^2 - (0)^2) = \frac{1}{2}mv^2 - ($$

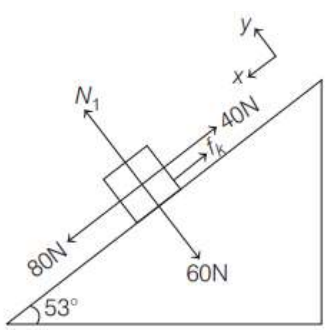
$$\Rightarrow \quad v = \sqrt{5gR}$$

At B,

$$N + k(0) - mg = \frac{m(\sqrt{5gR})^2}{R}$$

$$\Rightarrow \quad N = 6mg$$

17. (6.00)



$$\Sigma F_y = 0$$

$$\Rightarrow \quad N_1 = 60\ \text{N}$$

$$f_K = \mu N_1$$

$$f_K = \frac{1}{3}(60) = 20\ \text{N}$$

$$\Sigma F_x = ma_x$$

$\Rightarrow \quad 80 - 40 - 20 = 10a$

$\Rightarrow \quad a = 2\,\text{m/s}^2$

$s = ut + \frac{1}{2}at^2$

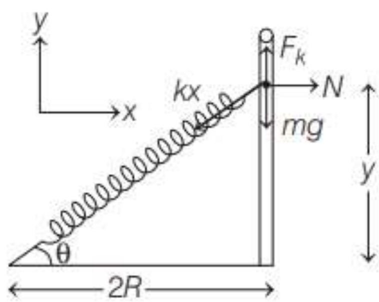
$\Rightarrow \quad s = 0 + \frac{1}{2}(2)(10)^2$

$\Rightarrow \quad s = 100\text{ m}$

$W_F = Fs\cos\theta = 40(100)\cos(180°)$

$= -4000\text{ J}$

18. (8.00)



$\Sigma F_x = 0 \Rightarrow N = kx\cos\theta$

$f_k = \mu N = \mu kx\cos\theta$

$W_{\text{friction}} = \int (\mu kx\cos\theta)\,dy$

$= \int \mu k(2R\sec\theta - 2R)\cos\theta\,dy$

$\begin{bmatrix} y = 2R\tan\theta \\ dy = 2R\sec^2\theta\,d\theta \end{bmatrix}$

$= 4\mu kR^2 \int (1 - \cos\theta)\sec^2\theta\,d\theta$

$= 4\mu kR^2 [\tan\theta - \ln(\sec\theta + \tan\theta)]_{37°}^{0°}$

$= -16\mu mgR\left(\frac{3}{4} - \ln 2\right)$

Using work-energy theorem for ring,

$$W_{\text{gravity}} + W_N + W_{\text{spring}} + W_{\text{friction}} = \Delta K$$

$$+mg\left(\frac{3R}{2}\right) + 0 + \frac{1}{2}k\,[(0.5R)^2 - 0^2]$$

$$-16\,\mu mgR\left(\frac{3}{4} - \ln 2\right) = \frac{1}{2}m\,(\sqrt{3gR})^2 - 0$$

$$\Rightarrow \qquad \mu = \frac{1}{8(3 - 4\ln 2)}$$

PART (B) : CHEMISTRY

19. (1)
$N(SiH_3)_3$ only

20. (7)
$SBr_6, PH_5, XeF_3^-, PH_4F, PH_3F_2, OF_4, XeS_2$

21. (6)
At constant temperature, decreases in molecular masses causes flattening of the graph. For same molecular mass of gas, increase in temperature causes flattening of the graph. From the above graph,

$$(V_{mp})_{T_1} < (V_{mp})_{T_2}$$

$\therefore \quad \frac{T_1}{M_A} < \frac{T_2}{M_B}$

$\therefore \quad \frac{T_2}{T_1} < \frac{M_B}{M_A}$

But, $\frac{M_B}{M_A}$ may be less than or greater than 1.

$\therefore$ Statements (1), (2) & (3) are wrong.
$\therefore$ Reported answer = 1 + 2 + 3 = 6.

22. (5)
$$b = 4N_A\left(\frac{4}{3}\pi r^3\right)$$
$$(4\pi\times10^{-4})\times10^3 = 4\times6.0\times10^{23}\left(\frac{4}{3}\pi r^3\right)$$
$$r = \frac{10^{-8}}{2}\,m$$
$\therefore \quad r = 5\times10^{-9}$ cm
$\therefore \quad z = 5$

23. (3)
$D = b^3, C = b^2$
$$\frac{D}{C} = \frac{b^3}{b} = b = \frac{V_C}{3} = \frac{V_C}{x}$$
$\therefore \ x = 3$

24. (5)
State function are (i), (iii), (iv), (v), (vi)

25. (A, B, C)
$\Delta T = 0, \Delta E = 0, \Delta H = 0$

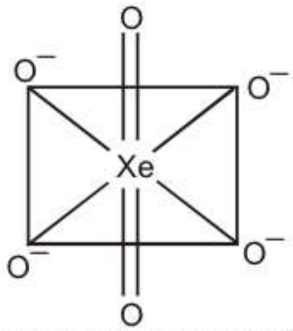
26. (C, D)
(C) from the first graph.
(D) from the second graph.

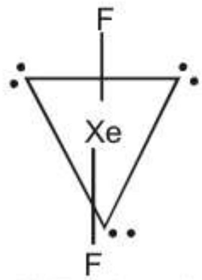
27. (A, B, D)
$P_T = (1+3x) = 1+3\times 0.1 = 1.3\,\text{atm}$
$\Delta P = 0.3\,\text{atm}$ or; $76\times 0.3\,\text{cm of Hg} = 228\,\text{Mn of Hg}$
$760\times 0.3\,\text{mm of Hg}$

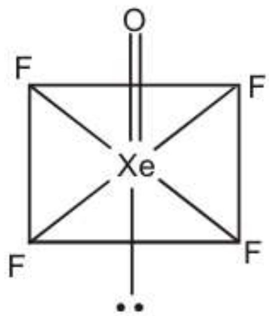
28. (A, C, D)
(A) Fact
(B) $p = mv = \frac{M}{N_A}\sqrt{\frac{3RT}{M}} = \frac{\sqrt{3MRT}}{N_A}$
(C) Max well distribution
(D) Fact

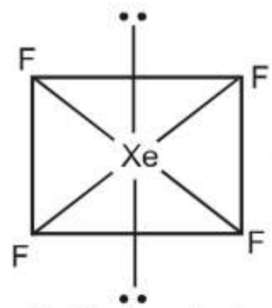
29. (D)
(D) $\underset{sp^2}{H_2C} = \underset{sp^2}{CH} - \underset{sp}{C} \equiv \underset{sp}{CH}$
(C) $\underset{sp^2}{CH_2} = \underset{sp^2}{CH} - \underset{sp}{C} \equiv \underset{sp}{N}$

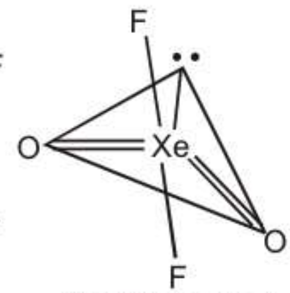
30. (A, B)

(a)

Perxenate ion $[XeO_6]^{4-}$

(b)

XeF_2(Linear)

(c)

$XeOF_4$(one l.p.)

(d)

$XeOF_4$ (two l.p.)


$XeOF_2$(one l.p.)

31. (0.75)

$$M_1 = \frac{10\times1.18\times49}{98}$$

$$M_1V_1 = M_2V_2$$

$$\frac{1.18\times49\times10}{98}\times75 = M_2\times590$$

$$\therefore\ M_2 = 0.75$$

32. (3.75)

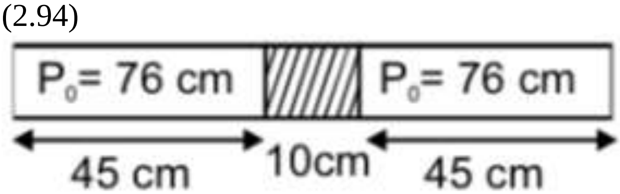
$$\Delta E = \frac{12400}{6200} = 2\text{ eV}$$

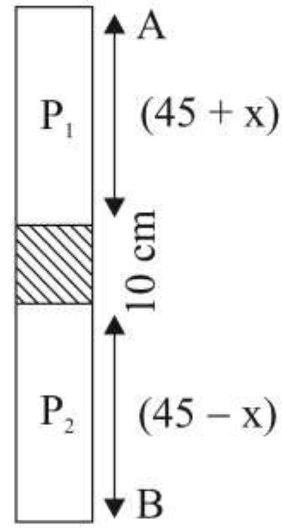
$$2\times1.6\times10^{-19}\times n = 40\times60\times0.5$$

$$n = 3.75\times10^{21}$$

33. (8.00)

34. (2.94)





Initially: $(A)\ 76\times45\times A$

$= 76\times45\times A(B)$

A = Area of cross section
When tube is made vertical,
Let Hg column gets displaced by x cm towards A.

For A side: $P_1\times(45+x)\times A = 76\times45\times A$

For B side: $P_2\times(45-x)\times A = 76\times45\times A$

Also $P_2 = P_1 + 10$

$$\frac{76\times45}{(45-x)} = \frac{76\times45}{(45+x)} + 10 \Rightarrow \frac{10}{76\times45} = \frac{2x}{45^2 - x^2}$$

$$\Rightarrow\ 76\times45\left[\frac{1}{(45-x)} - \frac{1}{(45+x)}\right] = 10$$

$$76\times45\left[\frac{45+x-45+x}{(45-x)(45+x)}\right] = 10 \Rightarrow x = 2.94\text{ cm}$$

35. (4.00)

B.L.: $CO > CO^+, N_2^- > N_2, NO > NO^+, H_2^+ < H_2^-$

36. (4.32)

$$\frac{\mathbf{r}_{mixture}}{\mathbf{r}_{O_2}} = \frac{1/311}{2/20\times60} = \sqrt{\frac{32}{M_{mix}}}$$

$\Rightarrow M_{mix} = 8.59$

$\Rightarrow V.D._{mix} = 4.32$

PART (C) : MATHEMATICS

37. (3)

$\because \quad n^2 \sin^2 x - 2\sin x - (2n+1) = 0$

$\Rightarrow \quad \sin x = \dfrac{2 \pm \sqrt{4 + 4n^2(2n+1)}}{2n^2}$ [By Shridharacharya method]

$= \dfrac{1 \pm \sqrt{(2n^3 + n^2 + 1)}}{n^2}$

$\because \quad 0 \le \sin x \le 1 \quad \left[\because\ x \in \left[0, \dfrac{\pi}{2}\right]\right]$

$\Rightarrow \quad 0 \le \dfrac{1 + \sqrt{(2n^3 + n^2 + 1)}}{n^2} \le 1$

$\Rightarrow \quad \sqrt{(2n^3 + n^2 + 1)} \le (n^2 - 1) \quad [\because\ n \ge 1]$

On squaring both sides, we get

$2n^3 + n^2 + 1 \le n^4 - 2n^2 + 1$

$\Rightarrow \quad n^4 - 2n^3 - 3n^2 \ge 0$

$\Rightarrow \quad n^2 - 2n - 3 \ge 0$

$\Rightarrow \quad (n-3)(n+1) \ge 0$

$\Rightarrow \quad n \ge 3$

$\therefore \quad n = 3, 4, 5, ...$

Hence, the minimum positive integer value of n is 3.

38. (3)

$S = \dfrac{2}{3}\left[\dfrac{1}{2\cdot 3} + \dfrac{1}{3\cdot 4} + \dfrac{1}{4\cdot 5} + \dfrac{1}{5\cdot 6} + ...\right]$

$= \dfrac{2}{3}\left[\dfrac{1}{2} - \dfrac{1}{\infty}\right] = \dfrac{1}{3}$

$\therefore \quad S^{-1} = 3$

39. (2)

$n = 42, 43, 44, ..., 55$

$\therefore$ Total 14 values of n are possible

$\therefore \quad 7m = 14 \Rightarrow m = 2$

40. (1)

$$\sum_{n=3}^{10} \frac{{}^nC_2}{{}^nC_3 \cdot {}^{n+1}C_3} = \sum_{n=3}^{10} \frac{6\times 3}{n(n-1)(n-2)(n+1)}$$

$$=6\sum_{n=3}^{10}\left(\frac{1}{n(n-1)(n-2)}-\frac{1}{(n+1)(n)(n-1)}\right)$$

$$=6\left[\frac{1}{1\cdot2\cdot3}-\frac{1}{11\cdot10\cdot9}\right]$$

$$=1-\frac{1}{11\times5\times3}=1-\frac{1}{165}=\frac{164}{165}$$

$\therefore \quad |p-q|=1$

41. (1)

$a_{n+1}=1-\dfrac{1}{a_1}$

$a_1=2,\ a_2=\dfrac{1}{2},\ a_3=-1$

$a_4=2,\ a_5=\dfrac{1}{2},\ a_6=-1$

$\vdots$

$a_{2019}=2,\ a_{2021}=\dfrac{1}{2},\ a_{2022}=-1$

$\therefore \ P_{2022}=1$

42. (6)

$$\begin{array}{r} ab \\ -ba \\ \hline 36 \end{array}$$

$\therefore\ 10a+b-10b-a=36$

$9(a-b)=36$

$a-b=4$

$\therefore\ a=4,5,6,7,8,9$

Total 6 cases.

43. (A, B)

$\left|\sin^2 x+17-x^2\right|=\left|16-x^2\right|+\left|\sin^2 x+1\right|$

$|x+y|=|x|+|y|$

$xy\ge0$

$\left(16-x^2\right)\left(\sin^2 x+1\right)\ge0$

$\Rightarrow\ x\in[-4,4]$

44. (A, B, C, D)

45. (A, B, D)

$$\log_{10}\left\{\frac{(x^2+y^2)^2}{5}\right\}=\log_{10}\left\{2(x^2+y^2)+75\right\} \Rightarrow \frac{(x^2+y^2)^2}{5}=2(x^2+y^2)+75$$

Put $x^2+y^2=t \Rightarrow \frac{t^2}{5}=2t+75$

$t=25=x^2+y^2 \quad \Rightarrow \quad xy=12$

Also, $x>0$

$y>0$

(4, 3) & (3, 4)

46. (A, B)

Here,

$$r\le\frac{22}{1+\left|\frac{x}{12}\right|}=m$$

$\therefore$ m should be integer & x should be –ve.

$\therefore$ $x=-10,-21$

47. (A, D)

Let r be the common ratio of the G.P., then $\beta=\alpha r, \gamma=\alpha r^2$ and $\delta=\alpha r^3$

$\therefore \quad \alpha+\beta=1 \Rightarrow \alpha+\alpha r=1$

or $\alpha(1+r)=1$ … (i)

and $\alpha\beta=p \Rightarrow \alpha(\alpha r)=p$

or $\alpha^2 r=p$ … (ii)

and $\gamma+\delta=4 \Rightarrow \alpha r^2+\alpha r^3=4$

or $\alpha r^2(1+r)=4$ … (iii)

and $\gamma\delta=q$

$\Rightarrow (\alpha r^2)(\alpha r^3)=q$

or $\alpha^2 r^5=q$ … (iv)

On dividing Eq. (iii) by Eq. (i), we get

$r^2=4 \Rightarrow r=-2, 2$

If we take $r=2$, then α is not integer, so we take $r=-2$

On substituting $r=-2$ in Eq. (i), we get $\alpha=-1$

Now, from Eqs. (ii) and (iv), we get

$$p=\alpha^2 r=(-1)^2(-2)=-2$$

and $q=\alpha^2 r^5=(-1)^2(-2)^5=-32$

Hence, $(p,q)=(-2,-32)$

48. (C, D)

AM ≥ GM

$$\frac{\tan A+\tan B+\tan C}{3} \geq \left(\tan A\cdot\tan B\cdot\tan C\right)^{1/3}$$

$$\therefore \quad \left(\tan A\cdot\tan B\cdot\tan C\right)^{2/3} \geq 3$$

$$\cot A\cdot\cot B\cdot\cot C \leq \frac{1}{3\sqrt{3}} < 3\sqrt{3}$$

49. (0.20)

$$\sum_{r=1}^{\infty}\frac{(4r+5)5^{-r}}{r(5r+5)} = \lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{(5r+5)-r}{r(5r+5)}\right)\cdot\frac{1}{5^r}$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{5r+5}\right)\frac{1}{5^r}$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{r\cdot 5^{-r}}-\frac{1}{(r+1)5^{r+1}}\right)$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{5}-\frac{1}{(n+1)5^{n+1}}\right) = \frac{1}{5}-0=\frac{1}{5}=0.20$$

50. (7.85 or 7.86)

$$\sin\frac{6x}{5}=0;\ \cos\frac{x}{5}=0$$

$$\frac{6x}{5}=n\pi;\ \frac{x}{5}=(2n+1)\frac{\pi}{2}$$

$$x=\frac{5n\pi}{6};\ \ x=\frac{5\pi}{2};\frac{15\pi}{2}$$

$$\frac{15\pi}{2}-\frac{5\pi}{2}=5\pi$$

$$\frac{5\pi}{2}+5n\pi$$

$$=5\pi\left(n+\frac{1}{2}\right),\ x=\frac{5\pi}{2}=7.85$$

51. (1.25)

$$y=\frac{\sin 3x\cdot 2\cos 5x\cos x}{\sin x(2\cos 5x\cos 3x)}$$

$$y=\frac{3-4\sin^2 x}{4\cos^2 x-3}=1+\frac{2}{1-4\sin^2 x}$$

$$y=\left(-\infty,\frac{1}{3}\right)\cup(3,\infty)\ \ a,\sin^2 x\in[0,1]$$

$a=\frac{1}{3}, b=3$

$\therefore \frac{b+a}{b-a}=\frac{3+\frac{1}{3}}{3-\frac{1}{3}}=\frac{10}{8}=1.25$

52. (0.25)

$\log_{10}^2 x+\log_{10} x^2=\log_{10}^2 2-1 \quad ; \quad \log_{10}^2 x+2\log_{10} x+1=\log_{10}^2 2$

$\Rightarrow (\log_{10} x+1)^2=\log_{10}^2 2$

$\log_{10} x+1=\pm\log_{10} 2$

$x=\frac{1}{20}$ and $\frac{1}{5}$

53. (4.00)

$S=$ coefficient of x^4 in

$=\left[{}^4C_0\cdot\left((1+x)^{101}\right)^4-{}^4C_1\left((1+x)^{101}\right)^3+{}^4C_2\left((1+x)^{101}\right)^2-{}^4C_3\left((1+x)^{101}\right)^1+{}^4C_4\right]-{}^4C_4$

$=\left((1+x)^{101}-1\right)^4-1$

$=\left(1+{}^{101}C_1+{}^{101}C_2x^2+{}^{101}C_3x^3+{}^{101}C_4x^4+......-1\right)^4-1$

$=x^4\left({}^{101}C_1+{}^{101}C_2x+{}^{101}C_3x^2+.....\right)^4-1$

$=x^4\left(101+\left({}^{101}C_2x++{}^{101}C_3x^2+....\right)\right)^4-1$

$=101^4$

$k=4$

54. (1.00)

$\tan\frac{\pi}{6}=\frac{3x-x^3}{1-3x^2}$

Squaring both sides

$3x^6-27x^4+33x^2=1$