PART (A) : PHYSICS

SOLUTIONS

1. (D)

$x = ae^{-\alpha t} + be^{\beta t}$

$v = \frac{dx}{dt} = -a \propto e^{-\alpha t} + b\beta e^{\beta t}$

$v = \frac{-a\alpha}{e^{\alpha t}} + b\beta e^{\beta t}$

As t increasing with time.

2. (B)

$x \propto t^3$

$v = \frac{dx}{dt} \propto t^2$

$v = \frac{dv}{dt} \propto t$

3. (B)

$x = at + bt^2 - ct^3$

$v = \frac{dx}{dt} = a + 2bt - 3ct^2$

$a = \frac{dv}{dt} = 2b - 6ct$

$a = 0 \Rightarrow t = \frac{b}{3c}$

$\therefore v = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2$

$= a + \frac{2b^2}{3c} - \frac{b^2}{3c}$

$= a + \frac{b^2}{3c}$

4. (B)

$V_{\alpha v} = \frac{V_1 + V_2 + V_3}{3} = \frac{3+4+5}{3} = 4 \text{ m/s}$

5. (B)

$V = \frac{1}{2} \times 10 \times 11 = 55 \text{ m/s}$

6. (D)

$$t = \sqrt{\frac{2h}{g}}$$

7. (B)

$$t = \frac{d}{\sqrt{v^2 - u^2}} \Rightarrow \frac{1}{4} = \frac{1}{\sqrt{5^2 - u^2}} \Rightarrow u = 3\,\text{km/h}$$

8. (D)

$$AB\sin\theta = \sqrt{3}AB\cos\theta$$

$$\tan\theta = \sqrt{3} \qquad \theta = 60^\circ$$

$$|A + B| = \left(A^2 + B^2 + 2AB\cos\theta\right)^{1/2}$$

$$= \left(A^2 + B^2 + AB\right)^{1/2}$$

9. (A)

$$\vec{\tau} = \vec{r} \times \vec{F} = 14\hat{i} - 38\hat{j} + 16\hat{k}$$

10. (B)

$$AB\sin\theta = AB\cos\theta$$

$$\tan\theta = 1 \Rightarrow \theta = 45^\circ$$

11. (A)

$$\vec{P}\cdot\vec{Q} = 0 \Rightarrow a^2 - 2a - 3 = 0$$

$$\Rightarrow a^2 - 3a + a - 3 = 0 \Rightarrow (a+1)(a-3) = 0$$

$$a = -1 \text{ or } a = 3$$

12. (A)

$$\vec{r} = \vec{a} + \vec{b} + \vec{c} = \hat{i} + \hat{j} - \hat{k}$$

$$\hat{r} = \frac{\hat{i} + \hat{j} - k}{\sqrt{3}}$$

13. (B)

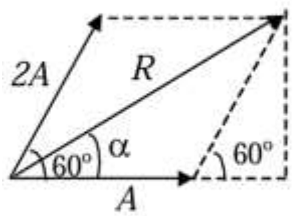
$$S_y = u_y t - \frac{1}{2}gt^2$$

$$-40 = 10t - 5t^2 \Rightarrow t^2 - 2t - 8 = 0$$

$$t = 4\,s$$

$$\therefore R = u_\eta t = 10\sqrt{3} \times 4 = 40\sqrt{3}\text{ m}$$

14. (C)



From the figure,

$$\tan\alpha = \frac{2A\sin 60^{\circ}}{A+2A\cos 60^{\circ}} = \frac{\sqrt{3}}{2}$$

15. (A)

$$\vec{v} = \vec{\omega}\times\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 0 & 4 & -3 \end{vmatrix} = -2\hat{i}+3\hat{j}+4\hat{k}$$

Now $\overleftarrow{v}.\vec{v} = \left(-2\hat{i}+3\hat{j}+4\hat{k}\right).\left(-2\hat{i}+3\hat{j}+4\hat{k}\right)$

Or $v^1 = 4+9+16 = 29$

$v = \sqrt{29}$ units

16. (D)

The resultant is

$\overrightarrow{BA}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DA}$

$=\overrightarrow{BA}+\left(\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DA}\right)$

$=\overrightarrow{BA}+\overrightarrow{BA}$

$=2\overrightarrow{BA}$

Hence (D) is correct.

17. (C)

18. (A)

$$V = \frac{ds}{dt} \Rightarrow \int ds = \int V.dt$$

$$S = \int_0^{\pi/4} \sec^2 t\cdot dt = \left[\tan t\right]_0^{\pi/4}$$

$$= \tan\frac{\pi}{4} - \tan\theta$$

$= 1$ m

19. (C)

$y = \ln x^2$

$$\frac{dy}{dx} = \frac{1}{x^2}\times 2x = \frac{2}{x} = \frac{2}{e}$$

20. (D)

$v = bs^2$

$$a = \frac{vdv}{ds} = bs^2(2bs)$$
$$= 2b^2s^3$$

21. (3)

$$\int_1^2 y.dx = \left(\frac{ax^3}{3} + 2x^2 - 3x\right)_1^2 = 10$$
$$\Rightarrow a = 3$$

22. (2)

23. (4)

$$d = V_m(60)$$
$$d = V_e(40)$$
$$t = \frac{d}{V_m + V_e}$$
$$\Rightarrow t = \frac{d}{\frac{d}{60} + \frac{d}{40}} = 24\,s$$

24. (5)

$$|S_{5^{th}}| = |S_{6^{th}}|$$
$$\Rightarrow u - \frac{g}{2}(2(5)-1) = -\left(u - \frac{g}{2}(2(6)-1)\right)$$
$$u - \frac{9g}{2} = -u + \frac{11g}{2}$$
$$2u = \frac{20}{2}g$$
$$\Rightarrow 10x = \frac{10g}{2} \Rightarrow x = 5$$

25. (9)

$$x = u_x\sqrt{\frac{2h}{g}} \Rightarrow 9 = u\sqrt{\frac{2\times 5}{10}}$$
$$u = 9\ m/s$$

26. (2)

$$P^2 + Q^2 + 2PQ\cos\alpha = R^2$$
$$(P = R)$$
$$2P\cos\alpha = -Q$$
$$\therefore \tan\theta = \frac{2P\sin\alpha}{Q + 2P\cos\alpha}$$

$$\tan\theta = \frac{2P\sin\alpha}{Q-Q} = \frac{2P\sin\alpha}{0}$$

$$\theta = \frac{\pi}{2}$$

27. (5)

$\frac{2v}{g} = 6s$ (motion from B to B)

$v = 30\,m/s$

$30^2 = u^2 - 2\times10\times80$ (motion from A to B)

$\therefore u = 50\,m/s$

28. (4)

Slope is maximum from $t = 30$ to 40 s

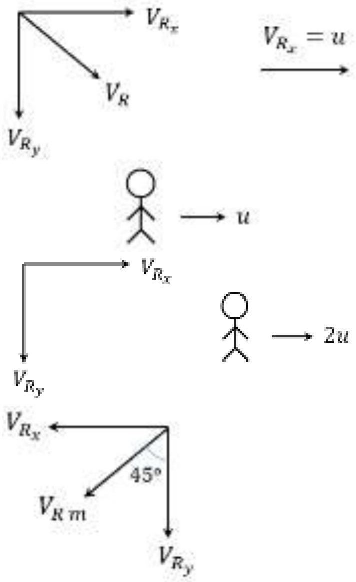
Hence $a_{max} = \frac{60-20}{40-30} = \frac{40}{10} = 4\,m/s^2$

29. (8)

$$\ell^2 = x^2 + y^2 \Rightarrow \frac{2\ell\,d\ell}{dt} = \frac{2x\,dx}{dt} + \frac{2y\,dy}{dt}$$

$$0 = 4\times6 + \frac{3x\,dy}{dt} \Rightarrow \frac{dy}{dt} = -8\,cm/s$$

30. (2)



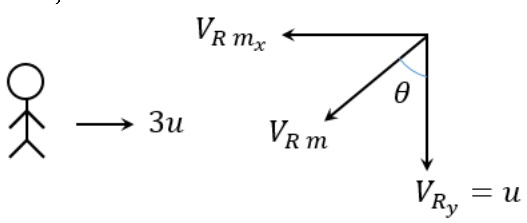
then $V_{R_m} = V_{R_y}$

$V_{R\,m_x} = V_{R_x} - 2u$
$= u - 2u$
$= -u$

$V_{R\,m_x} = V_{R_x} - 2u$
$= u - 2u$
$= -u$

$\therefore V_{R_y} = u$

Now,



$V_{R\,m_x} = u - 3u$
$= -2u$

$\tan\theta = \frac{2u}{u}$

$\theta = \tan^{-1}(2)$

PART (B) : CHEMISTRY

SOLUTIONS

31. (B)
Isoelectronic species

32. (B)
$Li^+ < Li$

33. (A)

34. (B)
Larger are the ionic radii, smaller is the value of lattice energy. Ionic radii vary in the order:
$K^+ > Na^+ > Li^+;\ Br^- > Cl^- > F^-$.

35. (A)
$Ca(OH)_2 + H_2SO_4 \rightarrow CaSO_4 + 2H_2O$

1 : 1 : 1
Given 0.2 0.5
$Ca(OH)_2$ is limiting reagent
$\therefore$ Moles of $CaSO_4$ formed = 0.2

36. (C)
Un for 1
hex for 6
So, Ununhexium will be 116.

37. (A)
The formula of compound A_4O_6 has A : O = 2 : 3
10 g A_4O_6 has A = 5.72 g and O = 4.28 g
Hence amount of A which will combine with 16×6 g oxygen
$$= \frac{5.72}{4.28} \times 96 = 128.3\,g$$
Or at. Mass of $A = \frac{128.3}{4} = 32\,u$
$(4a + 96)\,g\,A_4O_6$ has $4a\,g\,A$ *or*
$$\therefore\ 10g\ A_4O_6\ \text{has}\ \left[\frac{4a \times 10}{4a + 96}\right] gA$$
$$\therefore\ \frac{4a \times 10}{4a + 96} = 5.72$$
$a = 32$

38. (D)
Ce is 1st element of 4f series.

39. (B)
Only NaOH will react with H_2SO_4
$2NaOH + H_2SO_4 \rightarrow Na_2SO_4 + 2H_2O$
Applying molarity equation
$\frac{M_{NaOH} \times 100}{2} = 0.5 \times 10$
Or $M_{NaOH} = \frac{0.5 \times 100 \times 2}{100} = 0.1$
Amount of NaOH per litre = $0.1 \times 40 = 4.0\,g$
Hence NaOH in 100 mL = 0.4 g

40. (A)
Amt. of Fe in g = $\frac{0.334}{100} \times 67200$
Fe atoms in one molecule
$= \frac{224.448}{56} = 4$

41. (A)
According to reaction stoichiometry, oxygen is L.R.
$\therefore$ 1 mole of oxygen will react with ammonia
$= \frac{4}{5} = 0.8$
Ammonia left unconsumed
$= 1.0 - 0.8 = 0.2$ moles

42. (D)
$n_{NH_3} = \frac{4.25}{17} = 0.25$ {given}
1 mole NH_3 has atoms = $4N_A$
$\therefore$ 0.25 mole will have = $0.25 \times 4 \times 6.02 \times 10^{23}$
$= 6.02 \times 10^{23}$

43. (C)

Element	%	Mote Ratio	Simple Ratio
C	18.50	$\frac{18.5}{12}=1.54$	$\frac{1.55}{1.54}=1$
H	1.55	$\frac{1.55}{1}=1.55$	$\frac{1.55}{1.54}=1$
Cl	55.14	$\frac{55.14}{35.5}=1.55$	$\frac{1.55}{1.54}=1$
O	34.81	$\frac{24.81}{16}=1.55$	$\frac{1.55}{1.54}=1$

E. Formula = CHClO

44. (B)

$$\bar{v}_H = R.Z^2\left[\frac{1}{(2)^2}-\frac{1}{(3)^2}\right]$$

$\because$ $n_1 = 2$ and $n_2 = 3$ {first line in Balmar series}

Or $\bar{v}_H = R\times\frac{5}{36}$

Now to have same wave number for He^+ ion

$$\bar{v}_{He^+} = R.Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

$$\bar{v}_{He^+} = R.(2)^2\left(\frac{1}{(n_1)^2}-\frac{1}{(n_2)^2}\right)$$

Or $\bar{v}_{He^+} = R(2)^2\left[\frac{1}{(n_1\times 2)^2}-\frac{1}{(n_2\times 2)^2}\right]$

Hence $n_1 = n_1\times 2 = 2\times 2 = 4$

And $n_2 = n_2\times 2 = 3\times 2 = 6$

45. (C)

In absence of magnetic field p-orbitals have same energy level and are degenerate orbiltals.

46. (A)

Magnetic moment $\sqrt{n(n+2)}$ B.M.

$\therefore$ $n(n+2)-24$, $\therefore n = 4$

Thus Fe^{2+} has four unpaired electron, i.e., it is Fe^{2+} or $[Ar]3d^6$

47. (A)

Radial node = n – *l* – 1 = 2 – 0 – 1 = 1

48. (A)

$\frac{1}{\lambda_A} = R_H\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right]$ for shortest λ of Lyman series of H

$\frac{1}{\lambda_B} = Z^2R_H\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$ for longest λ of Balmar series of He^+

$\therefore \quad \frac{\lambda_B}{\lambda_A} = \frac{1\times 36}{5\times 4}$

$\therefore \quad \lambda_B = \frac{9}{5}.\lambda_A = \frac{9}{5}x$

49. (D)

$\bar{v} = \frac{1}{\lambda} = R_H\left[\frac{1}{2^2} - \frac{1}{3^2}\right];\ n_1 = 2$ for Balmar series

$n_2 = 3$ for H_a line.

50. (C)

From Bohr's concept $\frac{mu^2}{r} = \frac{e^2}{r^2}$

Or $\frac{mr^2.mu^2}{r} = \frac{e^2mr^2}{r^2}$

Or (angular momentum)^2 = e^2mr

Alternative:
Angular momentum in n^{th} orbit,

$L = \frac{nh}{2\pi}$

Also, $r_n = \frac{n^2}{Z}r_0 = r$

So, $n = \sqrt{\frac{Z}{r_0}r}$

So, angular momentum, $L = \sqrt{\frac{Z}{r_0}r} \times \frac{h}{2\pi}$

$L = \sqrt{r} \times \text{constant}$

So, $L \propto \sqrt{r}$

51. (6)

$10\times a + 40\times 1 = 2\times 50$

$10a + 40 = 100$

$a = 6$

52. (3)

Ti	+	Cl_2	$\rightarrow$	$TiCl_x$
1		2.21875 g		3.21875 g

$\frac{1}{\frac{48}{x}} = \frac{2.21875}{35.5}$

$x = 3$

53. (3)

$$\frac{\frac{0.98}{98}}{x} = 30 \times 0.5 \times 2 \times 10^{-3}$$

$$0.01x = 30 \times 10^{-3}$$

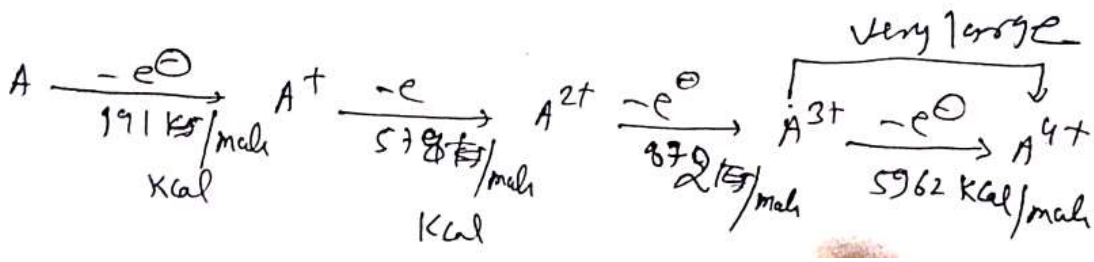
$$x = 3$$

54. (5)

Number of moles $= \frac{8}{16} = 0.5$ moles

Number of moles of electrons $= 0.5 \times 10 = 5$

55. (3)



So values Cl electrons = 3.

56. (4)
Li, B, P, H

57. (2)

$$V_n = V_0\left(\frac{Z}{n}\right)$$

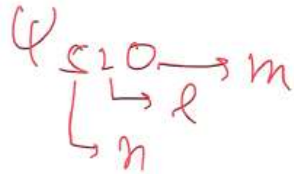
$$\frac{V_H}{V_{He^+}} = \frac{\frac{1}{1}}{\frac{2}{4}} = 2$$

58. (6)

Number of spectrum line $= \frac{(n_2 - n_1)(n_2 - n_1 + 1)}{2}$

$$= \frac{(6-3)(6-3+1)}{2} = 6$$

59. (5)



$\ell = 2 \quad d\text{-subshell}$

$m = \underbrace{-2, -1, 0, +1, +2}_{5\ \text{d-orbital}}$

60. (2)

$$\frac{\left(\frac{e}{m}\right)_{H^+}}{\left(\frac{e}{m}\right)_{He^{2+}}} = \frac{\frac{1}{1}}{\frac{2}{4}} = 2$$

PART (C) : MATHEMATICS

SOLUTIONS

61. (B)

One of the factors of the expression $\left(x^4 - px^2 + q\right) = x^2 - 3x + 2.$

Dividing $x^4 - px^2 + q$ by $x^2 - 3x + 2$, we get remainder $(15-3p)x + (2p+q-14)$

Since $x^2 - 3x + 2$ is one of the factors of $x^4 - px^2 + q$, therefore after dividing $x^4 - px^2 + q$ by $x^2 - 3x + 2$, we get remainder equal to 0.

$\Rightarrow (15-3p)x + (2p+q-14) = 0.$

Comparing both sides, we get $15 - 3p = 0 \Rightarrow p = 5.$

And $2p + q - 14 = 0 \Rightarrow q = 4.$

62. (B)

Apply componendo and dividend

63. (A)

Sum of roots $= 2\sqrt{2}$, and product of roots = 1

64. (B)

$\Delta > 0$ and $ab > 0, ac > 0$

65. (C)

$y = -2x^2 - 6x + 9$ and $a < 0$

$y_{\max} = \frac{-D}{4a} = 13.5$ at $x = \frac{-b}{2a} = -1.5$

66. (D)

67. (C)

Roots are equal if $D = 0$

So, $1 - 4ab = 0$

$ab = \frac{1}{4}$

68. (A)

$\cos^2 B - \cos^2 A = \sin(A+B).\sin(A-B)$

69. (D)

Put $A = B = C = 60°$ and verify

70. (C)
Put $A = 30°$ and $B = 45°$ and verify

71. (C)
$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

72. (D)
Maximum of $-2x^2 + 5x + 4 = 0 = \frac{-D}{4a} = \frac{57}{8}$

Minimum of $x^2 + x + 1 = 0 = \frac{-D}{4a} = \frac{3}{4}$

Difference $= \frac{51}{8}$

73. (C)
Let each term = k, then $\frac{k}{x} = \cos\theta$, $\frac{k}{y} = \cos\left(\theta + \frac{2\pi}{3}\right)$, $\frac{k}{z} = \cos\left(\theta + \frac{4\pi}{3}\right)$ and add.

74. (B)
$a = \sin 170° + \cos 170°$
$= \sin 10° - \cos 10° < 0$

75. (D)
$(a+b)^2 = a^2 + b^2 + 2ab$
$6^2 = 10 + 2ab$
$36 = 10 + 2ab$
$ab = 13$

76. (B)
Multiply and divide by 2 and $2\cos A.\cos B = \cos(A+B) + \cos(A-B)$

77. (B)
$x^2 - 2x + 2 = x^2 y + 3xy + 9y$
$x^2(y-1) + x(2+3y) + (9y-2) = 0$
x is real; $\Delta \geq 0$
$(3y+2)^2 - 4(y-1)(9y-2) \geq 0$
$\Rightarrow\ 27y^2 - 56y + 4 \leq 0$
2 and $\frac{2}{27}$

78. (A)

Put $x^2+x=y$, so that Eq. (1) becomes

$$(y-2)(y-3)=12$$

or $y^2-5y-6=0$

or $(y-6)(y+1)=0$ or $y=6,-1$

When $y=6$, we get

$$x^2+x-6=0$$

$\Rightarrow$ $(x+3)(x-2)=0$ or $x=-3,2$

When $y=-1$, we get

$$x^2+x+1=0$$

Which has non-real roots and sum of roots is –1.

79. (A)

The quadratic equation is

$$x^2-7(\alpha+\beta)x+\left(12(\alpha+\beta)^2+\alpha\,\beta\right)=0$$

$$x^2+\frac{21}{2}x+(27-2)=0$$

$$2x^2+21x+50=0$$

80. (D)

Here, $x=0$ is not a root. Divide both the numerator and denominator by x and put $x+\frac{3}{x}=y$ to obtain

$$\frac{4}{y+1}+\frac{5}{y-5}=-\frac{3}{2}\ \Rightarrow\ y=-5,3$$

$x+\frac{3}{x}=-5$ has two irrational roots and $x+\frac{3}{x}=3$ has imaginary roots.

81. (1)

82. (8)

Apply formula

$$\cos\theta\cos(60^\circ+\theta)\cos(60^\circ-\theta)=\frac{1}{4}\cos 3\theta$$

83. (2)

Sum of roots = 0 $\Rightarrow a=0$

Product of roots = $a+2=2$

84. (2)

Substitute $x^2=t$. Hence, $t^2-8t-9=0\Rightarrow(t-9)(t+1)=0\ \Rightarrow\ t=9$ and $t=-1$

$t=9 \Rightarrow x^2=9 \Rightarrow x=\pm 3$

$t=-1 \Rightarrow x^2=-1 \Rightarrow x=\pm i$

85. (4)

$(\tan 9^\circ+\cot 9^\circ)-(\tan 27^\circ+\cot 27^\circ)$

$=\dfrac{2}{\sin 18^o}-\dfrac{2}{\cos 36^o}=4$

86. (7)

$x=\sqrt{42+x}$

87. (4)

$\tan A+\tan B=2; \tan A\tan B=2$

$\tan(A+B)=\dfrac{2}{1-2}=-2$

$\sin^2(A+B)=\dfrac{4}{5}$

88. (4)

$(x^2+x+2)(x^2+x+3)=0 \Rightarrow x^2+x+2=0$ or $x^2+x+3=0$

Sum of roots of $x^2+x+2=0$ is $\dfrac{-b}{a}=-1(S_1)$

Sum of roots of $x^2+x+3=0$ is $\dfrac{-b}{a}=-1(S_2)$

Answer $=S_1+S_2=-2$

89. (1)

Substitute the corresponding values.

90. (5)

$\left|\alpha^2-\beta^2\right|=24$

$\dfrac{|b|}{a^2}\sqrt{b^2-4ac}=24$

$36-4q=16$

$q=5$