PART (A) : PHYSICS

SOLUITONS

1. (B)

If $|\vec{A}| = |\vec{B}| = x$, then $|\vec{C}| = \sqrt{2}x$

Now, $\vec{A}+\vec{B} = -\vec{C}$

Or $(\vec{A}+\vec{B}).(\vec{A}+\vec{B}) = (-\vec{C}).(-\vec{C})$

$\Rightarrow A^2 + B^2 + 2AB\cos\theta = C^2$

$\Rightarrow x^2 + x^2 + 2x^2\cos\theta = 2x^2$

Or $\cos\theta = 0$ or $\theta = 90^\circ$

Again, $\vec{A}+\vec{C} = -\vec{B}$

$\Rightarrow (\vec{A}+\vec{C}).(\vec{A}+\vec{C}) = -\vec{B}.\vec{B} \Rightarrow (A+C).(A+C) = -B.B$

Or $A.A + C.C + 2A.C = B^2$

Or $x^2 + 2x^2 + 2x^2\sqrt{2}\cos\theta = x^2$

Or $\cos\theta = -\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 135^\circ$

Again, $\vec{B}+\vec{C} = -\vec{A}$

Or $(\vec{B}+\vec{C}).(\vec{B}+\vec{C}) = (-\vec{A}).(-\vec{A})$

Or $x^2 + 2x^2 + 2x^2\sqrt{2}\cos\theta = x^2$

Or $\cos\theta = \frac{-2x^2}{2x^2\sqrt{2}\cos\theta} = -\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 135^\circ$

2. (D)

For vector $\vec{A}_1 + \vec{A}_2$, we have

$|\vec{A}_1 + \vec{A}_2|^2 = (\vec{A}_1 + \vec{A}_2).(\vec{A}_1 + \vec{A}_2) \qquad [\because x \cdot x = |x|^2]$

$\Rightarrow |\vec{A}_1 + \vec{A}_2|^2 = |\vec{A}_1|^2 + |\vec{A}_2|^2 + 2\vec{A}_1.\vec{A}_2$

Given, $|\vec{A}_1| = 3, |\vec{A}_2| = 5$ and $|\vec{A}_1 + \vec{A}_2| = 5$

So, we have, $(5)^2 = 9 + 25 + 2A_1.A_2$

$A_1.A_2 = -\frac{9}{2}$

Now, $(2A_1 + 3A_2).(3A_1 - 2A_2)$

$= 6|A_1|^2 - 4A_1.A_2 + 9A_1.A_2 - 6|A_2|^2$

$= 6|A_1|^2 - 6|A_2|^2 + 5A_1.A_2$

Substituting values, we have

$(2A_1 + 3A_2).(3A_1 - 2A_2)$

$= 6(9) - 6(25) + 5\left(-\frac{9}{2}\right) = -118.5$

3. (D)

As, $\frac{dv}{dt} = -kv^3$

$\Rightarrow \int_{v_0}^{v} \frac{dv}{v^3} = -k\int_0^t dt$

$\Rightarrow -\frac{1}{k}\int_{v_0}^{v} v^{-3}dv = t$

Or $-\frac{1}{k}\left|\frac{v^{-3+1}}{-3+1}\right| = t$

Or $\frac{1}{2k}\left[\frac{1}{v^2} - \frac{1}{v_0^2}\right] = t$

Or $\frac{1}{v^2} - \frac{1}{v_0^2} = 2kt$

Or $\frac{1}{v^2} = \frac{1}{v_0^2} + 2kt$

Or $\frac{1}{v^2} = \frac{1 + 2v_0^2kt}{v_0^2}$

Or $v = \frac{v_0}{\sqrt{2v_0^2kt + 1}}$

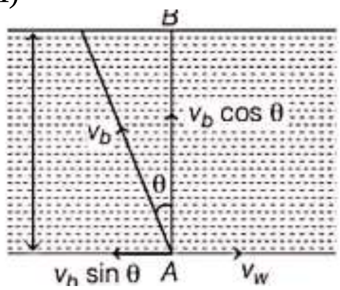
4. (C)

As, $\Delta x = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2$

$= \frac{1}{2}g\left[t^2 - (t-1)^2\right] = \frac{1}{2}g(2t-1)$

$= \frac{1}{2} \times 9.8 \times 5\,m = 24.5\,m$

5. (A)



$v_b \sin\theta = v_\omega$

$\sin\theta = \frac{v_\omega}{v_b} = \frac{1}{2}$

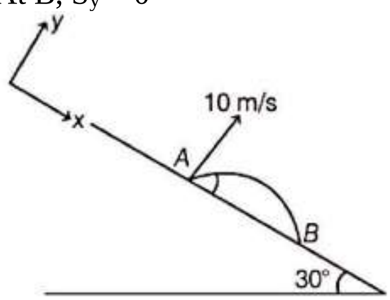
$\Rightarrow \quad \theta = 30^\circ$

Time taken to cross the river,

$t = \frac{D}{v_b \cos\theta} = \frac{D}{v_b \cos 30^\circ} = \frac{2D}{v_b \sqrt{3}}$

6. (B)

At B, $S_y = 0$



$\therefore \quad u_y t + \frac{1}{2} a_y t^2 = 0$

Or $t = -\frac{2u_y}{a_y} = \frac{-2(10)}{-10 \times \sqrt{3}/2} = \frac{4}{\sqrt{3}}$ s

Now, AB = R = $\frac{1}{2} a_x t^2$

$= \frac{1}{2}\left(10 \times \frac{1}{2}\right)\left(\frac{16}{3}\right) = 13.33\,\text{m}$

7. (D)

$\int_0^{\pi/4} \sin x dx - \int_0^{\pi/4} \cos x dx + \int_0^{\pi/4} \sec^2 x dx$

$= [-\cos x]_0^{\pi/4} - [\sin x]_0^{\pi/4} + [\tan x]_0^{\pi/4}$

$= \frac{-1}{\sqrt{2}} + 1 - \frac{1}{\sqrt{2}} + 1 = 2 - \frac{2}{\sqrt{2}}$

$= 2 - \sqrt{2}$

8. (A)

$y = \ln(x)^{3/4} = \frac{3}{4}\ln x$

$\Rightarrow \frac{dy}{dx} = \left(\frac{3}{4}\right)\left(\frac{1}{x}\right) = \frac{3}{4} \times \frac{3}{4}$

$\frac{dy}{dx} = \frac{9}{16}$

9. (D)

Given, $u_x = u\cos\theta_0 = 30 \times \frac{4}{5} = 24 m/s$

And $u_y \sin\theta = 30 \times \frac{3}{5} = 18 m/s$

$v_y = u_y + a_y t$

$v_y = 18 - 10(1) = 8$

After 1 s, u_x will remain as it is and u_y will decrease by

$\therefore \tan\theta = \frac{u_y}{u_x} = \frac{8}{24} = \frac{1}{3}$

10. (B)

As, $\frac{V^2 \sin 2\theta}{g} = \frac{\sqrt{3}v^2}{2g}$

Or $\sin 2\theta = \frac{\sqrt{3}}{2}$

Or $2\theta = 60^\circ$

Or $\theta = 30^0$

Let us cross check with the help of data for vertical range.

$\frac{v^2 \sin^2\theta}{2g} = \frac{v^2}{8g}$ or $\sin^2\theta = \frac{1}{4}$

Or $\sin\theta = \frac{1}{2}$

Or $\theta = 30^\circ$

11. (A, B, C, D)

If $R = x\hat{i} + y\hat{j} + z\hat{k}$

If $R = 0, x = 0, y = 0$ and $z = 0$

12. (A, B)
$\because$ Particle is in equilibrium
$\sum F_x = 0$ and $\sum F_y = 0$
$\therefore$ Then sum of x-component is
$\sum F_x = 10\cos 30^\circ + F_2 \sin 30^\circ - 15\sin 53^\circ = 0$
Or $5\sqrt{3} + \frac{F_2}{2} - 12 = 0$
$\therefore F_2 = 24 - 10\sqrt{3} = 24 - 17 = 7$ N
The sum of y-component of force is
$\sum F_y = F_2 \cos 30^\circ + 10\sin 30^\circ + 15\cos 53^\circ - F_1 = 0$
Or $\frac{\sqrt{3}F_2}{2} + 5 + 15 \times \frac{3}{5} - F_1 = 0$
$\Rightarrow F_1 = 19.95$ N

13. (A, B, C)
(A) The x-component of F_1 is
$F_1 \cos 30^\circ = 6\cos 30^\circ = + \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$N
(B) The x-component of F_2 is
$-F_2 \sin 30^\circ = -4 \times \frac{1}{2} = -2$N
Here, sum of x-component of forces
$F_x = 6\cos 30^\circ - 4\sin 30^\circ = \left(3\sqrt{3} - 2\right)$N
And the sum of y-component of forces is
$F_y = 6\sin 30^\circ + 4\cos 30^\circ = \left(3 + 2\sqrt{3}\right)$N
The magnitude of resultant is
$F = \sqrt{F_x^2 + F_y^2} = \sqrt{52}$N

14. (A, B, D)
The magnitude of unit vector is 1
$\therefore |\vec{a}| = 1$ or $\sqrt{a_1^2 + a_2^2 + a_3^2} = 1$
The equation is satisfied by options (A), (B) and (D). Hence options (A), (B) and (D) are correct

15. (A, C)
$\because \alpha = \beta = \gamma$
But $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$
Or $3\cos^2 \alpha = 1$ or $\cos \alpha = \pm \frac{1}{\sqrt{3}}$

$\therefore F = F\cos\alpha\hat{i} + F\cos\beta\hat{j} + F\cos\gamma\hat{k}$

$F = \pm\left(\hat{i} + \hat{j} + \hat{k}\right)N$

16. (A, C, D)

$F_z = 50\cos 45^\circ = 25\sqrt{2}N$

$F_{xy} = 50\sin 45^\circ = 25\sqrt{2}N$

$\therefore F_x = F_{xy}\cos 45^\circ = 25\sqrt{2} \times \frac{1}{\sqrt{2}} = 25N$

And $F_y = F_{xy}\sin 45^\circ = 25\sqrt{2} \times \frac{12}{\sqrt{2}} = 25N$

Hence, options (A), (C) and (D) are correct

17. (B, C, D)

If C and r are perpendicular to each other

Then, $C.r = 0$

This condition is satisfied by options (B), (C) and (D)

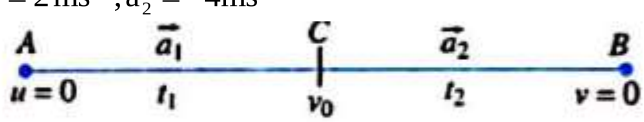
18. (B, C, D)

A body having a constant speed can have a varying velocity due to change in the direction of velocity. Thus a body having constant speed can have an acceleration

If velocity and acceleration are in the same direction, then distance is equal to displacement, because then there is not change in direction of motion. The body will continuously travel in one direction only.

19. (A, C)

$a_1 = 2\,ms^{-2}, a_2 = -4ms^{-2}$



$\nu_0 = a_1t_1 = 2t_1, \nu_0 = a_2t_2 = 4t_2$

$t_1 + t_2 = 6 \Rightarrow \frac{\nu_0}{2} + \frac{\nu_0}{4} = 6 \Rightarrow \nu_0 = 8\,ms^{-1}$

Total distance travelled

$S = AC + CB$

$= \frac{1}{2}a_1t_1^2 + \frac{1}{2}a_2t_2^2 = 24$

20. (A, B, C)

Let they meet at height h after time t

$h = 100t - \frac{1}{2}gt^2 \rightarrow$ for first arrow

$=100(t-5)-\frac{1}{2}g(t-5)^2 \rightarrow$ for second arrow

$\Rightarrow t=-12.5s$ (after solving). So (A) is correct

Time of flight of first arrow: $T=\frac{2u}{g}=\frac{2\times100}{10}=20s$

Second arrow will reach after 5s of reaching first. So (B) is correct

$\nu_1=100-10\times20=-100\,ms^{-1}$

$\nu_2=100-10\times15=-50\,ms^{-1}$

Ratio: $\frac{\nu_1}{\nu_2}=2:1$. So (C) is correct

Maximum height attained

$H=\frac{u^2}{2g}=\frac{(100)^2}{2\times10}=500m$

Hence (D) is incorrect

PART (B) : CHEMISTRY

SOLUTIONS

21. (D)

In one atom of ${}^{14}_{6}C$, neutrons

$=14-6=8$

$1.4\,mg = 1.4\times10^{-3}\,g\,C$

$= \dfrac{1.4\times10^{-3}}{14}\,mol$

$=1\times10^{-4}\,mol$

Number of ${}^{14}_{6}C$ atoms

$=6.02\times10^{19}\,atoms$

And neutrons $=6.02\times10^{19}\times8$ neutrons

$=4.816\times10^{20}$ neutrons

22. (D)

$m = \dfrac{100M}{(1000d - Mm_2)}$

$d = \dfrac{1}{1000}\left(\dfrac{1000M}{m} + Mm_2\right)$

$= \dfrac{1}{1000}\left[\dfrac{1000\times9}{10} + 9\times50\right]$

$=1.35\,g\,mL^{-1}$

$d = \dfrac{1}{1000}\left[\dfrac{1000M}{m} + Mm_2\right]$

m = molality

M = molarity

d = density $(g\,mL^{-1})$

m_2 = molar mass of solute

23. (A)

$1\,mol\,H_2O$

$=18\,g\,H_2O = 18g\times1\,g\,mL^{-1}$

$=18\,mL$

Also, $1\,mol\,H_2O = N$ molecules

$=6.0\times10^{23}$ molecules

Volume of 6.0×10^{23} molecules $=18\,mL$

$\therefore$ Volume of 1.0×10^{6} molecules

$= \dfrac{18\times1.0\times10^{6}}{6.0\times10^{23}} = 3.0\times10^{-17}\,mL$

24. (B)

$10^{6}\,g\,(or\,mL)$ solution has $Fe^{2+} = 1g$

$\therefore 250\,mL$ solution has Fe^{2+}

$= 250\times10^{-6}\,g$

$= \dfrac{250\times10^{6}}{56}\,mol$

$= 4.464\times10^{-6}\,mol$

$\therefore FeSO_4(NH_4)_2SO_4.6H_2O$

$= 4.464\times10^{-6}\times392g$

$= 1.75\times10^{3}\,g$

Each mol of salt has one mol Fe^{2+}

25. (D)

Case I

$\left(\dfrac{4E}{3} - E\right) = h\nu = \dfrac{hc}{3\lambda}$

$\dfrac{E}{3} = \dfrac{hc}{3\lambda}$

$\therefore E = \dfrac{hc}{\lambda}$

Case II

$(2E - E) = h\nu = \dfrac{hc}{\lambda'}$

$E = \dfrac{hc}{\lambda'}$

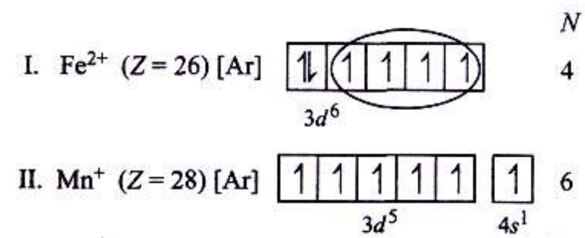
$\therefore \dfrac{hc}{\lambda} = \dfrac{hc}{\lambda'}$

$\therefore \lambda' = \lambda$

26. (A)

Magnetic moment $= \sqrt{N(N+2)}$

Where, N is number of unpaired electrons. Thus, larger the value of N larger will be the magnetic moment.



III. $Cr(\text{as in II})[Ar]3d^5 4s^1$ 6

IV. $Co^{3+}(\text{as in I})[Ar]3d^6$ 4

Thus, $I = IV < II = III$

27. (B)

Angular nodes $= l$

Radial nodes $= (n - l - 1)$

Orbital	n	l	Radial nodes
4s	4	0	3(x)
3p	3	1	1(y)
3d	3	2	0(z)

28. (C)

Electronic configuration	l	m	$(l + m)$	Electrons
$1s^2$	0	0	0	2
$2s^2$	0	0	0	2
$2p^6$	1	$\left.\begin{matrix}-1\\0\\+1\end{matrix}\right]$	0	2
$3s^2$	0	0	0	2
$3p^6$	1	$\left.\begin{matrix}-1\\0\\+1\end{matrix}\right]$	0	2
$4s^2$	0	0	0	2

In $2p^6$ and $3p^6$ only electrons with $l = 1$ and $m = -1$ were taken.

29. (C)

Any sub orbit is represented as nl such the n is the principal quantum number (in the form of values) and l is the azimuthal quantum number (its name). Value of $l < n$

$l = 0 \quad 1 \quad 2 \quad 3 \quad 4$

s p d f g

Value of m: $-l$ to $+l$

Value of s: $+\frac{1}{2}$ or $-\frac{1}{2}$

Thus, for $4f : n = 4, l = 3, m =$ any value between -3 to $+3$ and $s =$ may be $\pm\frac{1}{2}$

30. (B)

$Na \longrightarrow Na^{+} + e^{-}$ First IE

$Na^{+} + e^{-} \longrightarrow Na$

Electron gain enthalpy of Na^{+} is reverse of (IE) Because reaction is reverse so

$\Delta H(eg) = -5.1 eV$

31. (C, D)

1g atom of oxygen atom means

$= 16g$ oxygen atom $= \frac{16}{32}$

$= 0.5 mol$ of O_2

$= 11.2 L\, O_2$ gas at 1 atm and 273 K(at STP)

32. (A, B, D)

	Electron	Proton
H	1	1
O	8	8
N	7	7
C	6	6
F	9	9

1mol H_2O has electron $= 10 N_0$ and protons $= 10 N_0$

Thus, protons in 1 mol $CH_4 = 10 N_0$

Proton in 1 mol $NH_3 = 10 N_0$

Protons in $\frac{1}{3}$ mol $C_4H_8 = \frac{32}{3} N_0$

Protons in 1 mol $HF = 10 N_0$

33. (A, B)

$C \quad + \quad O_2 \quad \longrightarrow \quad CO_2$

1 mol   1 mol   1 mol

$\frac{60}{12} = 5 mol$ $\frac{100}{32} = 3.125 mol$?

$C + \frac{1}{2}O_2 \longrightarrow CO$

1mol   0.5 mol   1 mol

5 mol   3.125 mol   ?

There is excess of O_2 for the formation of CO but insufficient for the formation of CO_2. CO formed by 5 mol of $C = 5$ mol

O_2 consumed by 5 mol of $C = 2.5\,\text{mol}$

Thus, O_2 left unreacted $= 3.125 - 2.50$

$= 0.625\,\text{mol}\,O_2$

This unreacted O_2 can oxidise CO to CO_2

$$CO + \frac{1}{2}O_2 \longrightarrow CO_2$$

1 mol   0.5 mol   1 mol

0.625mol

CO oxidised to CO_2 by $0.625\,\text{mol}\,O_2$

$$= \frac{0.625}{0.5} = 1.25$$

Thus, CO_2 formed $= 1.25\,\text{mol}$, CO left

$= 5.00 - 1.25$

$= 3.75\,\text{mol}$

34. (B, C)

Fe^{2+} : (Ar) [↑↓] [↑] [↑] [↑] [↑]

N(unpaired electrons) =4

$\therefore \mu = \sqrt{N(N+2)}$ B.M

$= \sqrt{24}$ B.M. $= 4.90$ B.M.

Mn^{+} : (Ar) [↑] [↑] [↑] [↑] [↑] $3d^5$ [↑] $4s^1$

$N = 6$

$\mu = \sqrt{48} = 6.93$ B.M

Cr : (Ar) [↑] [↑] [↑] [↑] [↑]   [↑]

$N = 6$

$\therefore \mu = 6.93$ BM

Co^{3+} : (Ar) [↑↓] [↑] [↑] [↑] [↑]

$N = 4$

$\therefore \mu = \sqrt{24} = 4.90$ B.M.

35. (A, B, D)

36. (B, C, D)

37. (A, B, C, D)

$$\bar{\nu} = \frac{1}{\lambda} = \bar{R}_H Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For $\alpha -$ line is Balmer series, $n_1 = 2, n_2 = 3$

$$\therefore xm^{-1} = \bar{R}_H (2)^2 \left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= \bar{R}_H (4)\left(\frac{5}{36}\right)$$

$$\therefore \bar{R}_H = \frac{9}{5}x$$

(A) $\frac{1}{\lambda_a} = \left(\bar{R}_H\right)(4)^2 \left[\frac{1}{2^2} - \frac{1}{3^2}\right]$

$$= \left(\frac{9}{5}x\right)(16)\left(\frac{5}{36}\right) = 4x$$

$$\therefore \lambda_\alpha = \frac{1}{4x} m$$

(B) For $\beta -$ line in Balmer series

$n_1 = 2, n_2 = 4$

$$\therefore \bar{\nu}_\beta = \bar{R}_H (4)^2 \left[\frac{1}{2^2} - \frac{1}{4^2}\right]$$

$$= \left(\frac{9}{5}x\right)(16)\frac{12}{64}$$

$$= \frac{27}{5}x = 5.4\, x\, m^{-1}$$

(C) $\frac{1}{\lambda_{max}} = \bar{R}_H Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

λ_{max} is when $n_2 = (n_1 + 1), n_1 = 2$ for Balmer series.

$$\frac{1}{\lambda_{max}} = \frac{9}{5}x(3)^2 \left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

Thus, it is $\alpha -$ line

$$\frac{1}{\lambda_{max}} = \frac{81x}{5}\left(\frac{5}{36}\right)$$

$$\frac{1}{\lambda_{max}} = \frac{9}{4}x$$

$$\therefore \lambda_{max} = \frac{4}{9x} m$$

(D) $\frac{1}{\lambda_{min}}$ is when $n_2 \to \infty$

$$\therefore \frac{1}{\lambda_{min}} = \frac{\overline{R}_H Z^2}{n_1^2}$$

$$= \frac{9}{5} x \frac{(4)^2}{(3)^2} = \frac{16x}{5}$$

$$\therefore \lambda_{min} = \frac{5}{16x} m$$

38. (C, D)
(A) is incorrect because NO is neutral oxide
(B) is incorrect because CrO is basic oxide

39. (A, D)
Both (A) and (D) are correct. The three electrons in the 2p-orbitals must have same spin, no matter up sin or down spin.

40. (B, D)
Isotones have same number of neutrons $_{32}Ge^{76}, _{33}As^{77}$ and $_{34}Se^{78}$ have same number (44) of neutrons, hence they are isotones.

PART (C) : MATHEMATICS

SOLUTIONS

41. (C)

$|\alpha-\beta|=|\alpha_1-\beta_1|$

$\sqrt{\frac{\Delta}{a^2}}=\sqrt{\frac{\Delta_1}{p^2}} \Rightarrow \frac{b^2-4ac}{q^2-4pr}=\frac{a^2}{p^2}$

42. (A)

$\sqrt{4\sin^2\theta}+2+2\sin\theta=-2\sin\theta+2+2\sin\theta=2$

43. (A)

$x^2-6x+5=(x-3)^2-4\geq -4$

$\min=\min\{f(2), f(3), f(4)\}$

$\max=\max\{f(2), f(3), f(4)\}$

44. (D)

$x-2$ is a common factor

$\Rightarrow \alpha=2$ is common root

$\therefore\ 4+2a+b=0$ and $4+2c+d=0$

$\Rightarrow 2(a-c)+(b-d)=0$

$\Rightarrow \frac{b-d}{c-a}=2$

45. (C)

Both the roots are less than 5

$\Rightarrow \frac{2k\pm\sqrt{4k^2-4(k^2+k-5)}}{2}<5$

$\Rightarrow k\pm\sqrt{5-k}<5 \Rightarrow \pm 1<5-k$

$\Rightarrow 1<5-k$

$\Rightarrow k<4 \Rightarrow k\in(-\infty, 4)$

46. (A)

$\tan A=\frac{5}{12}$; $\quad \cos B=\frac{-3}{5}$

$A+C=180°$ $\quad B+D=180°$

$\cos C=-\cos A=-\frac{12}{13}$ $\quad \tan D=\frac{4}{3}$

47. (D)

Given: Quadratic equation: $x^2 + px + q = 0$ and its roots $= \alpha$ and β.

We know that the standard quadratic equation is: $ax^2 + bx + c = 0$.
Comparing the given equation with the standard equation, we get $a = 1, b = p$ and $c = q.$

We also know that sum of the roots $(\alpha + \beta) = -\frac{b}{a} = -\frac{p}{1} = -p.$

And product of the roots $(\alpha\beta) = \frac{c}{a} = \frac{q}{1} = q.$

We also know that $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ ….. (i)

Substituting the values of $(\alpha + \beta)$ and $\alpha\beta$ in equation (i), we get

$\alpha^2 + \beta^2 = (-p)^2 - 2q = p^2 - 2q.$

We also know that $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$

$= (p^2 - 2q)^2 - 2(q)^2 = p^4 + 4q^2 - 4p^2q - 2q^2 = p^4 + 2q^2 - 4p^2q$

48. (B)

Given: First quadratic equation: $x^2 - 5x + 16 = 0$ and its roots $= \alpha$ and β

Second quadratic equation: $x^2 + px + q = 0$ and its roots $= (\alpha^2 + \beta^2)$ and $\frac{\alpha\beta}{2}$.

We know that the standard quadratic equation is: $ax^2 + bx + c = 0.$
Comparing the first equation with the standard equation, we get $a = 1$, $b = -5$ and $c = 16$.

We also know that sum of the roots $(\alpha + \beta) = -\frac{b}{a} = -\frac{(-5)}{1} = 5$

And product of the roots $(\alpha\beta) = \frac{c}{a} = \frac{16}{1} = 16$

We also know that $\alpha^2 + \beta^2 = (\alpha + \beta) - 2\alpha\beta = -7$

Comparing second equation with the standard equation Since $(\alpha^2 + \beta^2)$ and $\frac{\alpha\beta}{2}$ are roots of equation

$x^2 + px + q = 0$

$(\alpha^2 + \beta^2) + \frac{\alpha\beta}{2} = -p \Rightarrow p = -1$

$(\alpha^2 + \beta^2)\left(\frac{\alpha\beta}{2}\right) = q \quad \Rightarrow q = -56.$

49. (C)

$p(x) = 0$ has real roots, if $b^2 - 4ac \geq 0$ and $Q(x) = 0$ has real roots, if $d^2 + 4ac \geq 0$

Now, $ac \neq 0$. If $ac < 0, b^2 - 4ac \geq 0$ and hence $p(x) = 0$ has real roots.

If $ac > 0$, then $d^2 + 4ac \geq 0$ & hence $Q(x) = 0$ has real roots.

Hence, atleast two roots of $P(x).\,Q(x)=0$ are real.

50. (B)

For equal roots $b^2=4ac$

$k=2$ and $k=-\frac{10}{9}$

51. (B, C)

As the coefficients are not rational, irrational roots need not appear in conjugate pair.

Here, $\alpha+\beta=-3\sqrt{2},\ \alpha\beta=\frac{1}{2}$. Let $\alpha=p+\sqrt{q}$.

Then prove that $\beta=-p+\sqrt{q}$.

52. (B, C)

Clearly $x = m$ is a root of the equation. Therefore, the other root must be $-m$.

$\Rightarrow \frac{1}{-m}+\frac{1}{-m+b}=\frac{1}{m}+\frac{1}{m+b}$

$\Rightarrow \frac{1}{b-m}-\frac{1}{b+m}=\frac{2}{m}$

$\Rightarrow \frac{b+m-b+m}{b^2-m^2}=\frac{2}{m}$

$\Rightarrow 2m^2=2b^2-2m^2$

$\Rightarrow 2m^2=b^2$

53. (A, B, C)

Let $f(x)=(k-2)x^2+8x+k+4$

$f(x)>0 \Rightarrow a>0$ and $D<0$

$k-2>0$ and $64-4(k-2)(k+4)<0$

$k>2$ and $k^2+2k-24>0$

$k>2$ and $(k<-6\, or\, k>4)$

$\therefore$ k can take values greater than 4, $k>4 \Rightarrow$ Least integral value of $k = 5$

54. (A, B)

Let $\alpha+i\beta$ and $\alpha-i\beta$ be two roots of $ax^2+bx+c=0$. Then

$(\alpha+i\beta)(\alpha-i\beta)=\frac{c}{a}$

$\Rightarrow \alpha^2+\beta^2=\frac{c}{a}$

$\Rightarrow \frac{c}{a}>0$

$\Rightarrow$ a and c are of the same sign.

But, $a+c>0$. Therefore, a and c are both positive.
Let $f(x)=ax^2+bx+c$.
Since, $f(x)=0$ has no real roots. Therefore, $f(x)>0$ for all x or $f(x)<0$ for all x
But, $f(0)=c>0$. Therefore, $f(x)>0$ for all x
$f(-1)>0$ and $f(1)>0 \Rightarrow a-b+c>0$ and $a+b+c=0$

55. (A., B, C)

$$\frac{(x-2)^2(x-3)^3(x-1)(x-4)^2}{x+1}\geq 0$$

$$x\in(-1,1]\cup[3,\infty)\cup\{2\}$$

56. (B, C)

$$y=\frac{x^2-3x+4}{x^2+3x+4}$$

$$\Rightarrow x^2(y-1)+3x(y+1)+4(y-1)=0$$

Since x is real, so $D\geq 0$

$$9(y+1)^2-16(y-1)^2\geq 0$$

$$\Rightarrow 7y^2-50y+7\leq 0$$

$$y\in\left[\frac{1}{7},7\right]$$

57. (A, C)

$$\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2=\left(\frac{\sin\theta+2\sin^2\theta/2}{\sin\theta+2\cos^2\theta/2}\right)^2=\tan^2\theta/2 \text{ or } \frac{1-\cos\theta}{1+\cos\theta}$$

58. (B, C, D)

59. (B, D)

60. (A, C)

PART (A) : PHYSICS

SOLUTIONS

1. (C)

Here, $\left(A = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\right)$

Let, $B = \left(\hat{i} - \hat{j}\right)$

Then, $\hat{B} = \frac{B}{B} = \frac{\hat{i} - \hat{j}}{\sqrt{(1)^2 + (-1)^2}} = \frac{\hat{i} - \hat{j}}{\sqrt{2}}$

Component of A along the direction of B is

$$A.\hat{B} = \left(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\right).\frac{\left(\hat{i} - \hat{j}\right)}{\sqrt{2}} = \frac{\left(a_x - a_y\right)}{\sqrt{2}}$$

2. (A)

As, $A = 2\hat{i} + 4\hat{j}$ and $B = 5\hat{i} - p\hat{j}$

$\therefore \quad A = \sqrt{2^2 + 4^2} = \sqrt{20}$

And q $B = \sqrt{5^2 + p^2}$

Now, $A.B = 10 - 4p$

If $A \parallel B$, then

$A.B = AB\cos 0^\circ = AB$

$10 - 4p = \sqrt{20}\sqrt{25 + p^2}$

Squaring, $100 + 16p^2 - 80p$

$= 20\left(25 + p^2\right) = 500 + 20p^2$

Or $20p^2 - 16p^2 + 80p + 400 = 0$

Or $p^2 + 20p + 100 = 0$

Or $\left(p - 10\right)^2 = 0$

$\therefore \quad p = -10$

$\therefore \quad B = 5\hat{i} + 10\hat{j}$

$B = \sqrt{5^2 + (10)^2} = \sqrt{125} = 5\sqrt{5}$

3. (D)

Given, A + B + C = 0, then A, B and C are in one plane and are represented by the three sides of a triangle taken in one order.

(a) $\therefore \quad B \times \left(A + B + C\right) = B \times 0 = 0$

Or $\quad B \times A + B \times B + B \times C = 0$

Or $\quad B \times A + 0 + B \times C = 0$

Or $A\times B = B\times C$ .....(i)

$\therefore\ (A\times B)\times C = (B\times A)\times C$;

It cannot be zero.

If $B \parallel C$, then $B\times C = 0,$, then $(B\times C)\times C = 0,$

Thus, option (a) is correct.

(b) $(A\times B).C = (B\times C).C = 0$

If $B \parallel C$, then $B\times C = 0,$, then $(B\times C)\times C = 0,$

Thus, option (b)

(c) $(A\times B) = \vec{D} = AB\sin\theta\,\hat{D}$. The direction of D is perpendicular to the plane containing A and B.

$(A\times B)\times C = D\times C$. Its direction is in the plane of A, B and C.

Thus, option (c) is correct.

(d) If $C^2 = A^2 + B^2$, then the angle between A and B is 90°

$\therefore\ (A\times B).C = (AB\sin 90^\circ).C = AB(D.C)$

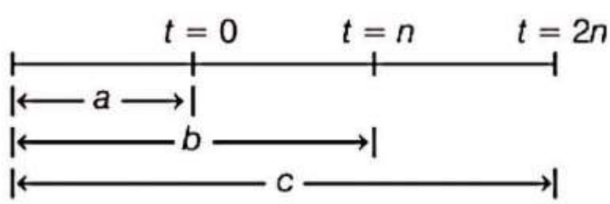
$= ABC\cos 90^\circ = 0$

Thus, option (d) is false.

4. (A)

As, b – a = un + $\frac{1}{2}An^2$

$\therefore\ 2b - 2a = 2un + An^2$



Again, $c - a = u(2n) + \frac{1}{2}A(2n)^2$

Subtracting Eq.(i) from Eq. (ii), we get

$c - a - 2b + 2a = An^2$

$A = \frac{c - 2b + a}{n^2}$

5. (C)

Acceleration of body along AB is $g\cos\theta$

Distance travelled in time t sec = AB = $\frac{1}{2}(g\cos\theta)t^2$

From ΔABC, $AB = 2R\cos\theta$

$2R\cos\theta = \frac{1}{2}g\cos\theta t^2$

$$t^2 = \frac{4R}{g}$$

$$t = 2\sqrt{\frac{R}{g}}$$

6. (B)
Initially velocity keeps on decreasing at a constant rate, then it increases in negative direction with same rate.

7. (D)
Slope of line = $-\frac{2}{3}$

Equation of line is (v - 20) = $-\frac{2}{3}$ (s - 0)

$\Rightarrow \quad v = 20 - \frac{2}{3}s \qquad$ ....(i)

Velocity at s = 15 m i.e.,

$$v = \left.\frac{ds}{dt}\right|_{s=15m} = 20 - \frac{2}{3}(15) = 10 ms^{-1}$$

Differentiate Eq.(i) with respect to time,

Acceleration = $\frac{dv}{dt} = -\frac{2}{3}\frac{ds}{dt}$

$\therefore \quad a = \left.\frac{dv}{dt}\right|_{s=15m} = -\frac{20}{3} ms^{-2}$

8. (C)

$$\frac{R/2}{H} = \frac{\sqrt{3}H}{H} = \sqrt{3}$$

Or $\frac{\left(v_0^2 \sin\theta\cos\theta\right)/g}{\left(v_0^2 \sin^2\theta\right)/2g} = \sqrt{3}$

$$2\cot\theta = \sqrt{3}$$

$$\tan\theta = \frac{2}{\sqrt{3}}$$

Or $\theta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

9. (B)

$$y = \sqrt{x}\ \ln x$$

$$\frac{dy}{dx} = \frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x} = \frac{\ln x + 2}{2\sqrt{x}}$$

10. (C)

$\int \tan x dx - \int \sec^2 x dx - \int \operatorname{cosec}^2 x dx$

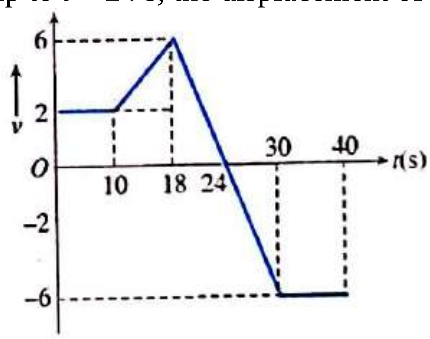
$\int \frac{\sin x}{\cos x} dx - \int \sec^2 x dx - \int \operatorname{cosec}^2 x dx$

$= -\ln \cos x - \tan x + \cot x + c$

$= \ln|\sec x| - \tan x + \cot x + c$

11. (A, C, D)

Maximum value of position coordinates = Initial coordinate + Area under the graph up to $t = 24 s$ (As up to t = 24 s, the displacement of the particle will be positive (figure).



Maximum value of position coordinates

$$= -16 + \left[(2\times10) + \left(\frac{2+6}{2}\right)\times(18-10) + \frac{1\times6}{2}\times(24-18)\right]$$

$$= -16 + [20 + 32 + 18] = 54 \text{ m}$$

At time $t = 18 s$

Position = –16 + Area of graph up to t = 18 s

= –16 + [20 + 32 + 18] = 54 m

At time $t = 30 s$

Position = –16 + Area of graph up to $t = 30 s$

$$= -16 + \left[70 - \frac{1}{2}\times6\times6\right] = 36 \text{ m}$$

12. (A, B)

$\vec{v}_{AW} = -20\hat{j}$

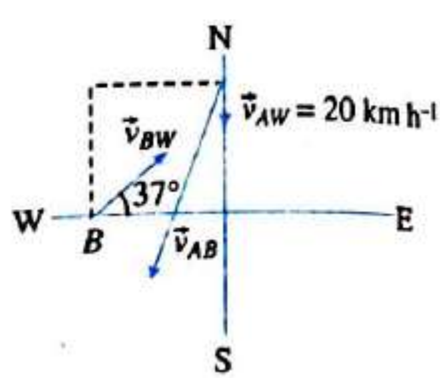
$\vec{v}_{BW} = +32\hat{i} + 24\hat{j}$

$\vec{v}_{AB} = \vec{v}_{AW} - \vec{v}_{BW}$

$= -32\hat{i} - 44\hat{j}$

$\frac{d\vec{r}_{AB}}{dt} = \vec{v}_{AB}$

$= -32\hat{i} - 44\hat{j}$



$$\int_{3\hat{i}+4\hat{j}}^{\vec{r}_{AB}} d\left(\vec{r}_{AB}\right) = -\int_0^t 32dt\,\hat{i} - \int_0^t 44\,dt\,\hat{j}$$

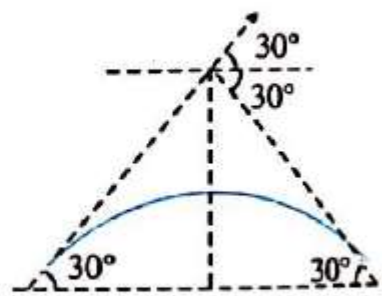
$$\vec{r}_{AB} = \left(3-32t\right)\hat{i} + \left(4-44t\right)\hat{j}$$

At $t = \frac{1}{11}h$, $\hat{j}$ component of $\vec{r}_{AB}$ is zero. At this time, its $\hat{i}$ component is $3-32t = 3-\frac{32}{11} = \frac{1}{11}$ km

It means at $t = \frac{1}{11}h$, A will be east of B. So at no time, A will be west of B.

13. (B, C)

$$T = \frac{2u\sin\theta}{g} = \frac{2\times 10\times\frac{1}{2}}{10} = 1\text{s}$$



14. (A, C)

$\frac{dx}{dt} = t^2$ ….. (i)

$y = \frac{1}{2}\frac{t^3}{3} \Rightarrow \frac{dy}{dt} = \frac{t^2}{2}$ ….. (ii)

$t = 1, v_x = 1, v_y = \frac{1}{2} \Rightarrow \vec{v} = \hat{i} + \frac{1}{2}\hat{j}$

$\frac{d^2x}{dt^2} = 2t$ ….. (iii)

$\frac{d^2y}{dt^2} = t$ ….. (iv)

At $t = 1\text{s}, a_x = 2$ and $a_y = 1$

$\Rightarrow \vec{a} = 2\hat{i} + \hat{j}$

15. (A, D)

So, velocity of first particle

$$= 3\cos 30°\hat{i} + 3\sin 30°\hat{j} = \frac{12}{5}\hat{i} + \frac{9}{5}\hat{j}$$

Velocity of second particle

$$= 4\cos 53°\hat{i} + 4\sin 53°\hat{j} = \frac{12}{5}\hat{i} + \frac{16}{5}\hat{j}$$

So, relative horizontal velocity is zero. So their relative velocity is vertical only. Since both particles are moving under gravity, so that relative acceleration is zero.

Their relative velocity $= \frac{16}{5} - \frac{9}{5} = \frac{7}{5} = 1.4\ \text{m/s}$

16. (4)

If **a** and **b** are perpendicular to each other.

$$R = \sqrt{a^2 + b^2}$$

If a and b are opposite to each other,

$$\frac{R}{\sqrt{2}} = a - b$$

$$\Rightarrow \frac{R^2}{2} = a^2 + b^2 - 2ab$$

$$\Rightarrow a^2 + b^2 = 2a^2 + 2b^2 - 4ab$$

$$\Rightarrow a^2 + b^2 - 4ab = 0$$

$$\Rightarrow \frac{a^2 + b^2}{ab} = 4$$

or $\frac{a}{b} + \frac{b}{a} = 4$

17. (9)

Given, sum of **P** and **Q** is **R**. Let angle between **P** and **Q** is β, then resultant of **P** and **Q**,

$$|\mathbf{R}| = \sqrt{|\mathbf{P}|^2 + |\mathbf{Q}|^2 + 2|\mathbf{P}||\mathbf{Q}|\cos\beta}$$

As, $|\mathbf{R}| = |\mathbf{P}|$ (given)

So, $|\mathbf{P}|^2 = |\mathbf{P}|^2 + |\mathbf{Q}|^2 + 2|\mathbf{P}||\mathbf{Q}|\cos\beta$

or $|\mathbf{P}|\cos\beta = -\frac{Q}{2}$

If resultant of 2**P** and **Q** makes θ with **Q**, then angle θ is given by

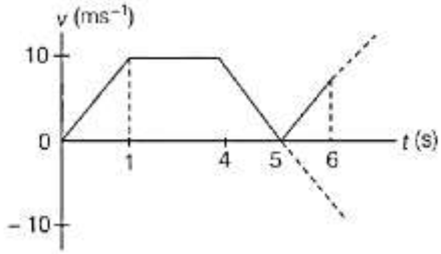
$$\tan\theta = \frac{|2\mathbf{P}|\sin\beta}{|\mathbf{Q}| + |2\mathbf{P}|\cos\beta}$$

Substituting the value of $|\mathbf{P}|\cos\beta$ from Eq. (i) in above equation, we get

$$\tan\theta = \infty \Rightarrow \theta = \frac{\pi}{2} = 90°$$

18. (2)

The corresponding velocity time graph is



Distance travelled = area of velocity time graph

$$= \frac{1}{2} \times 10(3+5) + \frac{1}{2} \times 10 \times 1$$

$$= 40 + 5 = 45\,\text{m}$$

$$\therefore \quad <\upsilon> = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{45}{6} = \frac{15}{2}\,\text{ms}^{-1}$$

$\therefore \quad n = 2$

19. (3)

As horizontal range of the two stones is same. So the sum of angles of projection of two stones must be 90°,

$$30° + \theta = 90° \text{ or } \theta = 90°$$

According to question, $y = \frac{u^2 \sin^2 30°}{2g} = \frac{u^2 (1/2)^2}{2g}$

And $y' = \frac{u^2 \sin^2 60°}{2g} = \frac{u^2 \left(\sqrt{3}/2\right)^2}{2g}$

$\therefore \quad \frac{y'}{y} = 3$ or $y' = 3y$

20. (3)

If range is R then, $\frac{u^2 \sin 2\times 75°}{g} = R - 10$ … (i)

and $\frac{u^2 \sin 2\times 45°}{g} = R + 10$

or $\frac{u^2}{g} = R + 10$

From Eq. (i), $(R+10)\sin 150° = R - 10$

or $(R+10)\frac{1}{2} = R - 10$

or $R = 30\,\text{m}$

PART (B) : CHEMISTRY

SOLUTIONS

21. (C)

$1\,mol\,(NH_4)_3PO_4 = 12\,mol\,H$

$= 4\,mol\,O$

If 12 mol H then O is $= 4\,mol$

Where, $3.18\,mol\,H = \frac{4}{12} \times 3.18 = 1.060\,mol\,O - atom$

22. (D)

If $(V_1 + V_2) = 75mL$

If $V_1 = xL$

Then $V_2 = (75 - x)mL$

$\therefore \frac{M_1V_1 + M_2V_2 + M_3V_3}{V_1 + V_2 + V_3} = M(Mixture)$

$\frac{0.07x + 0.12(75 - x) + 0.15 \times 25}{100} = 0.50$

$0.07x + 9 - 0.12x + 3.75 = 10$

$x = 55mL$

$\therefore V_1 = 55mL$

$V_2 = 20mL$

$V_3 = 25mL$

23. (A)

Time period $\propto \frac{n^3}{Z^2}$

$\therefore \frac{T(H)}{T(He^+)} = \left(\frac{n^3}{Z^2}\right)_H \left(\frac{Z^2}{n^3}\right)_{He^+}$

$\frac{(2)^3}{(1)^2}\frac{(2)^2}{(4)^3} = \frac{32}{64} = \frac{1}{2}$

24. (B)

The radius of nth Bohr orbit $= \frac{n^2 a_0}{Z}$

For $Li^{2+}, Z = 3$

And for second orbit $n = 2$,

$r_2 = \frac{4a_0}{3}$

25. (B)

Electronic configuration of $Cr(Z = 24)$

	n	l	Electrons
$1s^2$	1	0	2
$2s^2$	2	0	2
$2p^6$	2	1	6
$3s^2$	3	0	2
$3p^6$	3	1	6
$3d^5$	3	2	5
$4s^1$	4	0	1

Thus, electrons with $l = 1$, are 12 and with $l = 2$, are 5

26. (D)

Among isolectronic species, the relation in size is

cation < neutral < anion

Hence, Na^+ has smallest size.

27. (C)

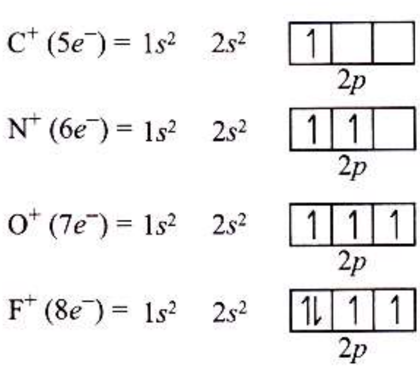
Electronegativity increases from left to right in a period and decreases from top to bottom in a group. Variation is more rapid in ground than in a period, hence the desired order is Electronegativity: $Si < P < C < N$.

28. (B)

Hydration energy depends on charge of ions an ionic radius. Higher the charge, greater the hydration energy. On the other hand, smaller the size, greater the hydration energy. Charge is considered first for comparison. Hence , Mg^{2+} has higher hydration energy than Na^+.

29. (C)

For second ionisation potential, electron will have to be removed from valence shell of the following ions:



In general, ionisation energy increases from left to right in a period. However, exception occur between adjacent atoms in a period, greater amount energy is required for removal of electron from completely half-filled or completely filled orbital than the same for adjacent atom with either less than completely half-filled or less than completely filled orbital. Therefore, ionisation potential of O^+ is greater than that of F^+. Also ionisation potential of N^+ is greater than C^+ but less than both O^+ and F^+ (periodic trend). Hence, overall order is $2nd\ IP : O > F > N > C$.

30. (C)
Ionisation energy increases along a period from left to right and decreases down a group. The position of given elements in the periodic table is as

Group No.2	16	18
Ca	S	Ar
Ba	Se	

Thus, the order of increasing ΔH is
$Ba < Ca < Se < S < Ar$

31. (B, D)

32. (A, B, C)

33. (A, C)
Info based

34. (A, B, C, D)
$$E_n = \frac{-13.6Z^2}{n^2}$$
$$E_4\left(Be^{3+}\right) = -13.6\left(\frac{4}{4}\right)^2 = -13.6\,eV$$
(A) $E_1(H) = -13.6eV$

(B) $E_2\left(He^+\right) = -13.6\left(\frac{2}{2}\right)^2 = -13.6eV$

(C) $E_3\left(Li^{2+}\right) = -13.6\left(\frac{3}{3}\right)^2 = -13.6eV$

(D) $E_{11} = \left(Na^{10+}\right) = -13.6\left(\frac{3}{3}\right)^2 = -13.6eV$
$= -13.6eV$

35. (A, B, C)
(A) and (B) are infact the same statements and both are correct N has slightly greater ionisation energy than oxygen which is against periodic trend. This exception is due to completely half-filled $\left(2p^3\right)$ orbital in nitrogen that make ionisation slightly difficult than oxygen.
(C) Also correct: Although N has greater first ionisation potential than oxygen, two values of

ionisation potentials are comparable since they are adjacent in a period, i.e. electrons are removed from same orbit during ionisation

(D) Incorrect-opposite to (C) of the bonded atoms which in turn has periodic trend in long form of periodic table.

36. (6)

Molar mass of alkene

$$= \frac{22.4\,\text{L mol}^{-1}}{0.8\,\text{L g}^{-1}} = 28\,\text{g mol}^{-1}$$

$$C_nH_{2n} = 12n + 2n = 28$$

$$n = 2$$

Thus, alkene is C_2H_4

Total number of C and H atoms in one formula unit $= 6$

37. (8)

$$XBr_2 \;+\; Cl_2 \longrightarrow XCl_2 \;+\; Br_2$$

$(X+160)$ $\qquad\qquad$ $(X+71)$

3.36 $\qquad\qquad$ 1.58

$$\left(\frac{X+160}{3.36}\right) = \frac{X+71}{1.58}$$

$$X = 8$$

38. (6)

$$h\nu = h\nu_0 + \frac{1}{2}mv^2$$

$$\therefore KE = \frac{1}{2}mv^2 = (h\nu - h\nu_0)$$

$$= \left(\frac{hc}{\lambda} - \text{work function}\right)$$

$$= \left(\frac{6.626\times10^{-34}\times3\times10^{8}}{1850\times10^{-10}}\right)$$

$$-(2.96\times1.6\times10^{-19})$$

$$= (10.74\times10^{-19} - 4.74\times10^{-19})\,J$$

$$= 6.0\times10^{-19}\,J$$

$$1eV = 1.6\times10^{-19}\,J$$

$$hc = 2.0\times10^{-25}\,J - m$$

39. (1)

Bohr's radius of H-atom in ground state $= a_0$

$\frac{n^2 a_0}{Z} = a_o$

$n^2 = Z$

$n^2 = 4\left(Be^{3+}\right)$

$n = 2$

But it is $(n = 2)$ is n^{th} excited state. Thus, first excited state.

40. (3)

$E = Nhv = N\frac{hc}{\lambda}$

$\therefore N = \frac{E\lambda}{hc} = \frac{101 \times 590 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^{8}}$

$= 3.0 \times 10^{20} = y \times 10^{20}$

$hc = 2.0 \times 10^{-25}\,J\,m$

PART (C) : MATHEMATICS

SOLUTIONS

41. (B)

Given: cubic equation: $2x^3+mx^2-13x+n=0$ and its roots = 2 and 2.
Substituting x = 2 in the given equation, we get
$2\times(2)^3+m\times(2)^2-13\times(2)+n=0 \Rightarrow 4m+n=10$ …. (i)
Again substituting x = 3 in the given equation, we get
$2\times(3)^3+m\times(3)^2-13\times3+n=0 \quad 9m+n=-15$ …. (ii)
Subtracting equation (i) from (ii), we get $5m=-25$
$\Rightarrow \quad m=-5$ and $n=30$.

42. (C)

$$\Rightarrow 3\left(\sin^2 x+\cos^2 x-\sin 2x\right)^2+6\left(\sin^2 x+\cos^2 x+\sin 2x\right)+$$
$$4\left(\sin^2 x+\cos^2 x\right)\left(\sin^4 x+\cos^4 x-\sin^2 x\cos^2 x\right)$$
$$\Rightarrow 3(1-\sin 2x)^2+6(1+\sin 2x)+4\left(1-3\sin^2 x\cos^2 x\right)$$
$$\Rightarrow 3+3\sin^2 2x-6\sin 2x+6+6\sin 2x+4-3\sin^2 2x$$
= 13

43. (C)

$$\sqrt{3}\sin x+\cos x=2\left(\sin x\frac{\sqrt{3}}{2}+\frac{1}{2}\cos x\right)$$
$$=2\sin\left(x+\frac{\pi}{6}\right) \text{ max. at } x=\frac{\pi}{3} \text{ i.e. } 60^o$$

44. (B)

Based on sign of trigonometric ratios

45. (A)

Since, α,β are the roots of $x^2-px-(p+c)=0$
$\alpha+\beta=p$ and $\alpha\beta=-(p+c)$
Now, $(\alpha+1)(\beta+1)=1-c$
$$\therefore \frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+c}+\frac{\beta^2+2\beta+1}{\beta^2+2\beta+c}$$
$$\frac{(\alpha+1)^2}{(\alpha+1)^2-(1-c)}+\frac{(\beta+1)^2}{(\beta+1)^2-(1-c)}$$

$\therefore \dfrac{(\alpha+1)^2}{(\alpha+1)^2-(\alpha+1)(\beta+1)}+\dfrac{(\beta+1)^2}{(\beta+1)^2-(\alpha+1)(\beta+1)}$

$\therefore \dfrac{(\alpha+1)^2}{(\alpha+1)(\alpha-\beta)}+\dfrac{(\beta+1)^2}{(\beta+1)(\beta-\alpha)}$

$\Rightarrow \dfrac{\alpha+1}{\alpha-\beta}-\dfrac{\beta+1}{\alpha-\beta}$

$\Rightarrow \dfrac{\alpha-\beta}{\alpha-\beta}=1$

46. (A)

Use $2\sin A\sin B = cso(A-B)-\cos(A+B)$

47. (A)

$\cos 2\theta+\cos\theta = 2\cos^2\theta+\cos\theta-1$

Min. value $\Rightarrow \dfrac{-(1-4.2(-1)}{4.2}=\dfrac{-9}{8}$

48. (B)

Use the formule $\sin\alpha+\sin(\alpha+\beta)+\sin(\alpha+2\beta)+.........+\sin(\alpha+(n-1)\beta)$

$=\dfrac{\sin\left(\alpha+\left(\dfrac{n-1}{2}\right)\beta\right)}{\sin\dfrac{\beta}{2}}.\sin\dfrac{n\beta}{2}$

49. (B)

If $\tan x\tan y = a$ and $x+y=\dfrac{\pi}{6}$, $\tan\dfrac{\pi}{6}=\tan(x+y)=\dfrac{\tan x+\tan y}{1-a} \Rightarrow \dfrac{1-a}{\sqrt{3}}=\tan x+\tan y$

So, x & y satisfies $\sqrt{3}x^2-(1-a)x+a\sqrt{3}=0$

50. (A)

Taking $\cos^2 25-\sin^2 25=\cos(25+25)\cdot\cos(25-25)=\cos 50$

So, $\cos 50=(\cos 25-\sin 25)(\cos 25+\sin 25)=k\sqrt{2-k^2}$

51. (A, B)

Simplify $\tan 15° = \tan(45-30°)$

$=\dfrac{\tan 45-\tan 30°}{1+\tan 45\tan 30°}$

52. (B, C)
θ is obtuse angle

53. (A, D)
$3\cos 2\theta + 4\sin 2\theta = 5$ has α & β as its solution $3 - 3\tan^2\theta + 8\tan\theta = 5 + 5\tan^2\theta$
$\Rightarrow 8\tan^2\theta - 8\tan\theta + 2 = 0$
$\Rightarrow \tan^2\theta - \tan\theta + \frac{1}{4} = 0$, Let $\tan\alpha, \tan\beta$ are the roots.
$\Rightarrow \tan\alpha + \tan\beta = 1$
$\Rightarrow \tan\alpha\tan\beta = \frac{1}{4}$

54. (A, C)
$\sin^2\alpha + 3\cos^2\alpha = \frac{3p}{4}$
$3\sin^2\alpha\cos^2\alpha = \frac{k}{4}$

55. (ACD)

56. (2)
$\alpha, 4\beta$ are roots of $x^2 - 6x + a = 0$ and $\alpha, 3\beta$ are roots of $x^2 - cx + 6 = 0$
$\alpha + 4\beta = 6$ and $4\alpha\beta = a$
$\alpha + 3\beta = c$ and $3\alpha\beta = 6$
$\Rightarrow a = 8 \Rightarrow \alpha = 2$

57. (3)
Let the number be P.
So, P – 129 is divisible by 189.
Let Q be the quotient then, $\frac{P-129}{189} = Q$
$\Rightarrow P = 189Q + 129$
$\Rightarrow \frac{P}{27} = \frac{189Q + 129}{27}$
189 is divisible by 27,
$\therefore$ When 129 divided by 27, leaves a remainder of 21.

58. (4)
$x^2 - 2ax + a^2 - 1 = 0 \Rightarrow (x-a)^2 = 1$
Roots are $a-1$ and $a+1$.
Both roots lie in (–3, 4).
$\therefore a - 1 > -3, a + 1 < 4$

$-2 < a < 3 \Rightarrow a = -1, 0, 1, 2$

59. (8)

$\alpha + \beta = 15, \alpha\beta = 1$

$\left(\frac{1}{\alpha} - 15\right)^{-2} + \left(\frac{1}{\beta} - 15\right)^{-2}, \frac{1}{\alpha} = \beta, \frac{1}{\beta} = \alpha$

$= \frac{1}{\alpha^2} + \frac{1}{\beta^2} = (\alpha + \beta)^2 - 2$

$= 225 - 2 = 223$

60. (2)

The value of (1 – cot 23°) (1-cot 22°)

$\Rightarrow \frac{(\sin 23° - \cos 23°)(\sin 22° - \cos 22°)}{\sin 23° \sin 22°}$

$\Rightarrow \frac{\sqrt{2}\sin(23° - 45°).\sqrt{2}\sin(22° - 45°)}{\sin 23° \sin 22°}$

$\Rightarrow 2$