KINEMATICS

SOLUTIONS

LEVEL – 1

DISTANCE AND DISPLACEMENT, SPEED AND VELOCITY, AVG. SPEED AND AVG. VELOCITY

1. A particle moves in the direction of east for 2 s with velocity of 15 ms^{-1}. Then it moves towards north for 8s with a velocity of 5 ms^{-1}. The average velocity of the particle is
(a) 1 ms^{-1} (b) 5 ms^{-1} (c) 7 ms^{-1} (d) 10 ms^{-1}

2. An aeroplane moves 400 m towards north, 300 m towards west and then 1200 m vertically upwards. Then its displacement from the initial position is
(a) 1300 m (b) 1400 m (c) 1500 m (d) 1600 m

OP is the displacement

displacement in horizontal plane is

$h = \sqrt{(400)^2 + (300)^2} = 500m$

Now net displacement is

$s = \sqrt{(500)^2 + (1200)^2} = 1300m$

3. An athelete completes one round of a circular track of radius R in 40 sec. What will be his displacement at the end of 2 min. 20 sec.
(a) zero (b) 2 R (c) 2 π R (d) 7 π R

Total time duration in consideration = 2 min 20 sec = 140 sec

He completed 1 round in 40 sec

In 140 sec, he will cover $= \frac{140}{40} = 3.5$ rounds

After 3.5 rounds, he would be on the opposite side of the starting point

So, the displacement will be the diameter or 2R.

4. A bus traveling the first one-third distance at a speed of 10 km/h, the next one-fourth at 20 km/h and the remaining at 40 km/h. The average speed of the bus is nearly
(a) 9 km/h (b) 16 km/h (c) 18 km/h (d) 48 km/h

Total distance = x

As s = vt

$$t_1 = \frac{\frac{x}{3}}{10} = \frac{x}{30}$$

$$t_2 = \frac{\frac{x}{3}}{20} = \frac{x}{60}$$

$$t_3 = \frac{\frac{x}{3}}{60} = \frac{x}{180}$$

Average velocity $= \dfrac{x}{\text{total time taken}}$

total time taken $= \dfrac{x+3x+6x}{180} = \dfrac{10x}{180}$

Thus Average velocity $= \dfrac{x \times 180}{10 \quad x} = 18$ km/h

5. An insect crawls a distance of 4m along north in 10 seconds and then a distance of 3m along east in 5 seconds. The average velocity of the insect is:
(a) 7/15 m/s (b) 1/5 m/s (c) 5/15 m/s (d) 12/15 m/s

6. A projectile is thrown with an initial velocity of $\left[a\hat{i}+b\hat{j}\right]$ m/s. If the range of projectile is twice the maximum height reached by it, then
(a) b = a/2 (b) b = a (c) b = 2a (d) b = 4a

Angle of projection, $\theta = \tan^{-1}\dfrac{v_y}{v_x} = \tan^{-1}\dfrac{b}{a}$

$\therefore \tan\theta = \dfrac{b}{a}$.............. (i)

From formula R = 4H cot θ = 2H

$\Rightarrow \cot\theta = \dfrac{1}{2}$ ∴ tan θ = 2 [As R = 2H given]..................(ii)

From Eqs (i) and (ii), b = 2a

7. A hall has the dimensions 10m × 10m × 10m. A fly starting at one corner ends up at a diagonally opposite corner. The magnitude of its displacement is nearly :
(a) $5\sqrt{2}$m (b) $10\sqrt{3}$m (c) $20\sqrt{3}$m (d) $30\sqrt{3}$m

Initial coordinates of fly (0,0,0) assume starting end of room to be origin.
Final coordinates of fly (10,10,10), hence it undergoes a displacement of 10 along each dimension.

Magnitude of displacement = $\sqrt{(x^2+y^2+z^2)} = 10\sqrt{3}$m

8. A particle starts from the origin, goes along y-axis to the point (0, 30m) and then returns along the same line to the point (0, –30m). The distance and displacement of the particle during the trip is :
(a) 0 , 0 (b) 90m, –30m (c) –30m, 90m (d) 0, –30m
Displacement=–30–0= –30m
Distance=30+[30–(–30)]
=90m

9 A train moves with a speed of 30 km/h in the 1st 15 minutes, with another speed of 40 km/h in the next 15 minutes and then with a speed of 60 km/h in the last 30 minutes. The average speed of the train for this journey is :
(a) 60 km/h (b) 47.5 km/h (c) 45 km/h (d) 50 km/h

d1 = $30 \times 15 = 450$m
d2 = $40 \times 15 = 600$m
d3 = $60 \times 30 = 1800$m
Total dorten = $1800 \times 600 + 450$
=2850m
Total level = $15+15+30$
=60

Avg sp = $\frac{2850}{60} = \frac{285}{6} = 47.5\text{kmh}$

AVG. ACCELERATION AND APPLICATION OF CALCULAS

10. The position of a particle moving along x-axis is given by $x = 10t - 2t^2$. Then the time (*t*) at which it will
momently come to rest is

(a) 0 (b) 2.5 s (c) 5 s (d) 10 s

Given that,

$x = 10t - 2t^2$

so,

$v = \frac{dx}{dt} = 10 - 4t$

The particle will come to rest if v = 0

$10 - 4t = 0$

$t = 2.5$ s

11. At any instant, the velocity and acceleration of a particle moving along a straight line are *v* and *a*. The speed of the particle is increasing if

(a) v > 0, a > 0 (b) v < 0, a > 0 (c) v > 0, a < 0 (d) v > 0, a = 0

acceleration of fan must be incensing in order to gain speed. if acceleration is increasing then velocity also increases.

12. If *v* is the velocity of a body moving along *x*-axis, then acceleration of body is

(a) $\frac{dv}{dx}$ (b) $v\frac{dv}{dx}$ (c) $x\frac{du}{dx}$ (d) $v\frac{dx}{dv}$

We know that ⇒ $a = \frac{du}{dt}$

Now, velocity is for a body moving along x-axis

So, V. is x- dependent

So, we use partial differentiation.

$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt}$

$\frac{dx}{dt} = v$

So, we get $a = \frac{dv}{dx} \times v$

13. If a body is moving with constant speed, then its acceleration

(a) Must be zero (b) May be variable (c) May be uniform (d) Both (b) & (c)

14. The initial velocity of a particle is u (at t = 0) and the acceleration a is given by $\alpha t^{3/2}$. Which of the following relations is valid?

(a) $v = u + \alpha t^{3/2}$ (b) $v = u + \frac{3\alpha t^3}{2}$ (c) $v = u + \frac{2}{5}\alpha t^{5/2}$ (d) $v = u + \alpha t^{5/2}$

As f = at $\Rightarrow \frac{dv}{dt} = at$

$\Rightarrow \int_u^v dv = \int_0^t at\ dt \Rightarrow v - u = \frac{at^2}{2}$

$\Rightarrow v = u + \frac{1}{2}at^2$

15. The position x of particle moving along x-axis varies with time t as x = Asin (ωt) where A and ω are positive constants. The acceleration a of particle varies with its position (x) as

(a) $a = Ax$ (b) $a = -\omega^2 x$ (c) $a = A\,\omega\, x$ (d) $a = \omega^2 x A$

Given, $x = A\sin(\omega t)$

$\Rightarrow v = A\omega\cos(\omega t)$

$\Rightarrow a = -A\omega^2\sin(\omega t) \Rightarrow a = -\omega^2 x$

16. A body is moving with variable acceleration (a) along a straight line. The average acceleration of body in time interval t_1 to t_2 is

(a) $\frac{a[t_2 + t_1]}{2}$ (b) $\frac{a[t_2 - t_1]}{2}$ (c) $\frac{\int_{t_1}^{t_2} a\,dt}{t_2 + t_1}$ (d) $\frac{\int_{t_1}^{t_2} a\,dt}{t_2 - t_1}$

Since acceleration is variable, average acceleration = $\frac{\int adt}{\int dt}$

$a_{AV} = \frac{\int adt}{t_2 - t_1}$

17. The position of a particle moving along x-axis given by x = $(-2t^3 + 3t^2 + 5)$m. The acceleration of particle at the instant its velocity becomes zero is

(a) 12 m/s^2 (b) -12 m/s^2 (c) -6 m/s^2 (d) Zero

The position of a particle moving along x-axis is given by

$x = (-2t^3 + 3t^2 + 5)$m

Velocity, $v = \frac{dx}{dt} = 0$

$-6t^2 + 6t = 0.......(1)$

$6t(-t + 1) = 0$

$-t + 1 = 0$

$t = 1$sec

Acceleration, $a = \frac{dv}{dt}$

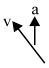
$a = -12t + 6$

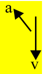
$a|_{t=1sec} = -12 \times 1 + 6$

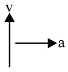
$a|_{t=1sec} = -6 m/s^2$

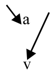
18. A body is projected vertically upward direction from the surface of earth. If upward direction is taken as positive, then acceleration of body during its upward and downward journey are respectively
(a) Positive, negative (b) Negative, negative
(c) Positive, positive (d) Negative, positive

CONSTANT ACCELERATION AND FREE FALL

19. Shown here are the velocity and acceleration vectors for an object in several different types of motion. In which case is the object slowing down and turning to the left?
(a) 

(b) 

(c) 

(d) 

As the man is slowing down therefore, the acceleration component should be opposite to the velocity. so, the angle between velocity vector and acceleration vector should be obtuse.
As the man is turning to the right therefore, the acceleration vector component should be towards the left side.

20. A body is projected vertically upwards with a velocity v. It returns to the point from which it was projected after some time. The average velocity and speed for the total time of flight are respectively.
(a) $\frac{v}{2}$, v (b) 0, $\frac{v}{2}$ (c) 0, 0 (d) $\frac{v}{2}$,0.

Average velocity = 0 because net displacement of the body is zero.

Average speed = Total distance covered / Time of flight $= \frac{2H_{max}}{2u/g}$

$\Rightarrow v_{av} = \frac{2u^2/2g}{2u/g} \Rightarrow v_{av} = u/2$

Velocity of projection = v (given)

$\therefore$ v = v/2

21. A body is released from the top of a tower of height h metre, it takes t second to reach the ground. Where is the body at the time $\frac{t}{2}$?
(a) at h/2 meters from the ground
(b) at h/4 m from the ground
(c) depends on the mass and velocity of body
(d) at 3h/4 m from the ground

$H = \frac{1}{2}gt^2$

$At \frac{t}{2}, s = \frac{1}{2}g(t/2)^2 = \frac{1}{4}\left(\frac{1}{2}gt^2\right)$

$s = \frac{1}{4}H$

Therefore, the height from ground is given by,

$x = H - s = H - \frac{1}{4}H = \frac{3}{4}H$

22. A particle is projected vertically upwards and it reaches the maximum height H in time T seconds. The height of the particle at any time t will be

(a) $g(t - T)^2$ (b) $H - \frac{1}{2}g(t - T)^2$ (c) $\frac{1}{2}g(t - T)^2$ (d) $H - g(t - T)$

At the maximum height, final velocity is zero.

$v = 0 m/s$

From 1st equation of motion,

$v = u + at$

$0 = u - gT$

$u = gT$

From 2nd equation of motion,

$H = ut - \frac{gt^2}{2}$

$H = gT^2 - \frac{gT^2}{2}$

$H = 0.5gT^2$

At anytime, t,

$h = ut - gt^2$

$h = gTt - 0.5gt^2$

Add and subtract $0.5gT^2$, we get

$h = (gtT - 0.5gt^2 - 0.5gT^2) + 0.5gT^2$

Put $H = gT^2$

$h = -0.5g(T - t)^2 + H$

$h = H - \frac{1}{2}g(T - t)^2$

23. A body starts from rest with uniform acceleration. If its velocity after n seconds is v, then its displacement in the last two seconds is

(a) $\frac{2v(n-1)}{n}$ (b) $\frac{v(n-1)}{n}$ (c) $\frac{v(n+1)}{n}$ (d) $\frac{2v(2n+1)}{n}$

Initial velocity of body $u = 0$

Let the acceleration be a.

Using $v = u + at$

We get $v = 0 + an$

$\Rightarrow a = \frac{v}{n}$

For displacement in last two seconds, $t = 2$

Using $S = vt - \frac{at^2}{2}$

Or $S = v(2) - \frac{v}{2n}(2)^2 = \frac{2v(n-1)}{n}$

24. A particle start moving from rest state along a straight line under the action of a constant force and travel distance x in first 5 seconds. The distance travelled by it in next five seconds will be

(a) x (b) 2 x (c) 3 x (d) 4 x

we know distance covered in straight line is given as $S=ut+at^2/2$

for starting from rest take initial velocity u=0 we get $S=at^2/2$

It is given that for t=5second S=x so $x=a5^2/2=25a/2$

Now distance in the duration of 6 to 10 second will be distance in 10 second - distance in 5 seconds $=a10^2/2-a5^2/2=75a/2=3\times25a/2=3x$.

Answer is 3x

25. A body is projected vertically upward with speed 40 m/s. The distance travelled by body in the last second of upward journey is [take g = 9.8 m/s2 and neglect effect of air resistance]

(a) 4.9 m (b) 9.8 m (c) 12.4 m (d) 19.6 m

Let the horizontal distance be H and

let the distance traveled in the last second be d

$$H = \frac{u^2}{2g}$$

$$H = \frac{1600}{19.6}$$

$$H = 81.63m$$

Now time taken by the body to reach a height of 81.36 m= $\sqrt{\frac{2H}{g}} = 4.08s$

Now from equation of motion, we have

$$s = ut + \frac{1}{2}gt^2$$

$$H - d = (40)(t-1) - \frac{1}{2}g(t-1)^2$$

$$81.63 - d = (40)(4.08-1) - \frac{1}{2}(9.8)(4.08-1)^2$$

$$d = 4.85m$$

So, the correct answer is '4.9m'.

26. A ball is dropped from a bridge of 122.5 metre above a river. After the ball has been falling for two seconds, a second ball is thrown straight down after it. Initial velocity of second ball so that both hit the water at the same time is

(a) 49 m/s (b) 55.5 m/s (c) 26.1 m/s (d) 9.8 m/s

For the first ball: Let it takes 't' sec to river.

$$\Rightarrow 122.5 = \frac{1}{2} \times g \times t^2 \text{------------------------(1)\$}$$

for second ball: time taken = t − 2sec.

$$122.5 = u(t-2) + \frac{1}{2}g(t-2)^2 \text{--------------(2)}$$

$$\Rightarrow \frac{gt^2}{2} = ut - 2u + \frac{gt^2}{2} + 2g - 2gt$$

$$\Rightarrow (2g-u)t = 2g - 2u$$

$$\Rightarrow t = \frac{2g-2u}{2g-u}$$

from(1)

$$t^2 = \frac{2\times122.5}{g}$$

$$t = \sqrt{\frac{2\times122.5}{98}} = \frac{35}{7} = 5 \text{ sec}$$

$$\Rightarrow 10g - 5u = 2g - 2u$$

$$\Rightarrow 8g = 3u$$

$$u = \frac{8\times9.8}{3} = 26.1 \text{ m/s}$$

27. A balloon starts rising from ground from rest with an upward acceleration 2 m/s^2. Just after 1 s, a stone is dropped from it. The time taken by stone to strike the ground is nearly

(a) 0.3 s (b) 0.7 s (c) 1 s (d) 1.4 s

When the stone is dropped from the balloon, its initial velocity is the same as the velocity of the balloon at that instant.

The upward acceleration of the balloon is a= 2m/s^2

The balloon starts from rest, so u= 0 m/s

The balloon rises for 1 s before the stone is dropped, hence we have

v =u + at

v=0+=2 m/s in the vertically upw ard direction.

This is the initial velocity of the stone after being dropped.

The distance moved up by the balloon in 1 second is

$v^2 = u^2 + 2as$

$v^2 = 2as$

$\therefore S = \frac{v^2}{2a} = \frac{4}{4} = 1m$

Hence, the stone falls by 1 m before hitting the ground

Now, the acceleration on the stone after being dropped is g = 9.8m/s^2

Hence, for the stone,

we have

$u = -2m/s;\ s = 1m;\ a = 9.8m/s^2;\ t = 1s$

u is negative as it is in the upward direction

Hence, using third equation of motion,

we get time as

$s = ut + \frac{1}{2}at^2$

$1 = -2t + \frac{1}{2} \times 9.8t^2$

$4.9t^2 - 2t - 1 = 0$

$\therefore = t = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 4.9 \times (-1)}}{2 \times 9.8} = \frac{2 \pm \sqrt{4 + 19.6}}{2 \times 9.8} = \frac{2 \pm \sqrt{23.6}}{9.8}$

Since, time cannot be negative, we have

$t = \frac{2 + \sqrt{23.6}}{9.8} = 0.7s$

Hence, the stone will strike the ground 0.7 s after being dropped from the balloon.

28. A boy throws balls into air at regular interval of 2 second. The next ball is thrown when the velocity of first ball is zero. How high do the ball rise above his hand? [Take g = 9.8 m/s^2]

(a) 4.9 m (b) 9.8 m (c) 19.6 m (d) 29.4 m

Velocity of a ball becomes zero when the ball is at maximum height.

The interval is 2 s, which means that after the 1 st ball reaches max height at that time 2nd ball is thrown

It means time of ascent of ball is 2 s

So, time of ascent= u/g = 2
So u= 19.6 m/s
Then ,the max height= u^2/2g =19.6*19.6/2*9.8
Max height = 19.6 m

29. A ball projected from ground vertically upward is at same height at time t_1 and t_2. The speed of projection of ball is [Neglect the effect of air resistance]

(a) $g[t_2 - t_1]$ (b) $\frac{g[t_1 + t_2]}{2}$ (c) $\frac{g[t_2 - t_1]}{2}$ (d) $g[t_1 + t_2]$

Let us say, the ball take Δt
time to reach at to most point 3 from point 1
The ball will take time Δt again to reach from portion 3 to portion 2

$\Rightarrow t_1 \Delta t + \Delta t = t_2$

$\Rightarrow t_1 + 2\Delta t = t_2$

$\Rightarrow \Delta t = \frac{t_2 - t_1}{2}$

Let the projected speed of ball be it at position 3 the speed of ball = 0
Applying I equation of motion

$o = v - g(t_1 + \Delta t)$

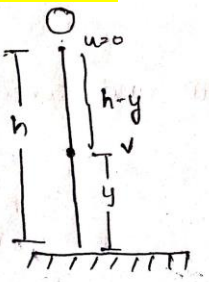
$\Rightarrow v = g(t_1 + \Delta t)$

$v = g\left(t_1 + \frac{t_2 - t_1}{2}\right)$

$\Rightarrow v = g\frac{(t_1 + t_2)}{2}$

30. A ball is dropped from a height *h* above ground. Neglect the air resistance, its velocity (*v*) varies with its height above the ground as

(a) $\sqrt{2g(h-y)}$ (b) $\sqrt{2gh}$ (c) $\sqrt{2gy}$ (d) $\sqrt{2g(h+y)}$



using,

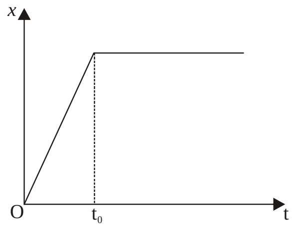
$v^2 - u^2 = 2as$

$v^2 = 2g(h-y)$

$v = \sqrt{2g(h-y)}$

GRAPHICAL ANALYSIS

31. Figure shows the displacement (x)-time (t) graph of a particle moving on the X-axis.



(a) The particle is at rest
(b) The particle is continuously going along X-direction
(c) The velocity of particle increases upto time t_0 and then becomes constant
(d) The particle moves at a constant velocity up to a time t_0, and then stops.

The displacement-time graph is a straight line inclined to time axis upto time t0, indicates a uniform velocity. After time t0, the displacement-time graph is a straight line parallel to time axis indicates the particle is at rest.

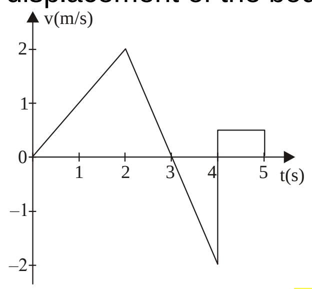
32. The displacement-time graph for two particles A and B are straight lines inclined at angles of 30º and 60º with the time axis. The ratio of velocities of $V_A : V_B$ is

(a) 1 : 2 (b) 1 : √3 (c) √3 : 1 (d) 1 : 3

As velocity $v = \frac{dx}{dt}$ = slope of x – t graph.

so $V_A : V_B = \text{slope}_A : \text{slope}_B = \tan 30^0 : \tan 60^0 = \frac{1}{\sqrt{3}} : \sqrt{3} = 1 : 3$

33. The velocity versus time graph of a body in a straight line is as shown in Figure. The displacement of the body in five seconds is



(a) 2 m (b) 2.5 m (c) 4 m (d) 5 m

"**Area under the velocity-Time graph gives the Displacement**."

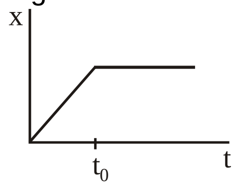
we divide the area into 3 parts

$A = A_{0-3} + A_{3-4} + A_{4-5}$

$A = \frac{1}{2} \times 3 \times 2 - \frac{1}{2} \times 1 \times 2 + 1 \times 1$

$A = 3m$

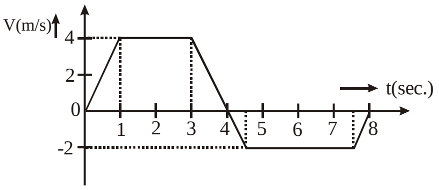
34. Figure shows the displacement-time graph of a particle moving on the X-axis.



(a) the particle is continuously going in positive x direction
(b) the particle is at rest
(c) the velocity increases up to a time t_0, and then becomes constant
(d) the particle moves at a constant velocity up to a time t_0, and then stops.

When an object travels equal distances in equal intervals of time, it moves with uniform speed. The distance traveled by the object is directly proportional to the time taken. The particle moves at a constant velocity up to a time t_0.
After time t_0, the displacement is constant because the graph is parallel to the time axis, which shows the particle has stopped.

35. The velocity - time graph of a linear motion is shown in figure. The displacement from the origin after 8 sec. is :



(a) 5 m (b) 16 m (c) 8 m (d) 6 m.

Distance is the area under the curve of v − t graph

Then,

Distance travelled = $A_1 + A_2 + A_3 + A_4 + A_5 + A_6$

$= \left(\frac{1}{2}\right)(1)4 + 2(4) + \frac{1}{2}(1)(4) + \frac{1}{2}\left(\frac{1}{2}\right)(2) + (2)(3)$

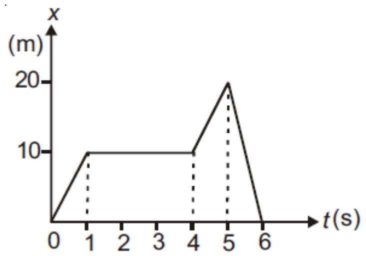
$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(2) = 19\text{ m}$

Displacement fill t = 8 s = $A_1 + A_2 + A_3 - A_4 - A_5 - A_6$

$= \frac{1}{2}(1)(4) + 2(4) + \frac{1}{2}(1)(4) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)2$

$-(2)(3)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(2) = 12 - 7 = 5\text{ m}$

36. Figure shows the graph of *x*-coordinate of a particle moving along *x*-axis as a function of time. Average velocity during *t* = 0 to 6 s and instantaneous velocity at *t* = 3 s respectively, will be



(a) 10 m/s, 0 (b) 60 m/s, 0 (c) 0, 0 (d) 0, 10 m/s

Let the total displacement 0 s to 6 s is D and the time is t.

The average velocity from 0 s to 6 s is given as,

$v_{avg} = \frac{D}{t}$

$= \frac{10 + 0 + 10 - 20}{6}$

$= 0\text{ m/s}$

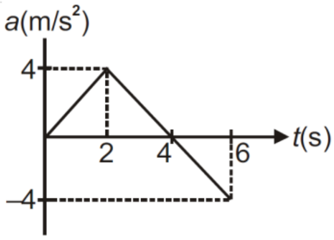
Since, the instantaneous velocity at a time is equal to the slope of velocity time graph at that time.

The instantaneous velocity at 3 s is given as,

$v_i = 0$

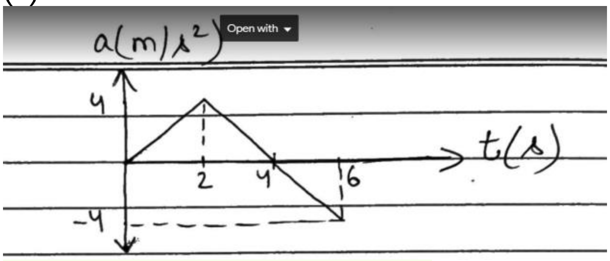
Thus, the average velocity from 0 s to 6 s is 0 m/s and the instantaneous velocity at 3 s is 0 m/s.

37. For the acceleration-time (a-t) graph shown in figure, the change in velocity of particle from t = 0 to t = 6 s is



(a) 10 m/s (b) 4 m/s (c) 12 m/s (d) 8 m/s

(2)



Area under acceleration (a) – time (t)

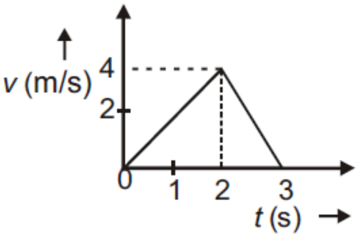
curve gives change in velocity.

Area $= \left(\frac{1}{2} \times 4 \times 4\right) + \left(\frac{1}{2} \times 2 \times (-4)\right)$

Area $= 8 - 4 = 4$

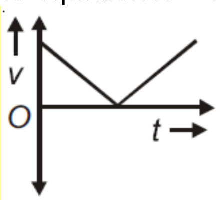
So change in velocity $= 4$ m/s

38. The velocity versus time graph of a body moving in a straight line is as shown in the figure below

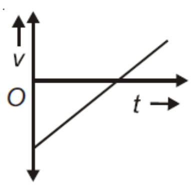


(a) The distance covered by the body in 0 to 2 s is 8 m
(b) The acceleration of the body in 0 to 2 s is 4 ms^{-2}
(c) The acceleration of the body in 2 to 3 s is 4 ms^{-2}
(d) The distance moved by the body during 0 to 3 s is 6 m

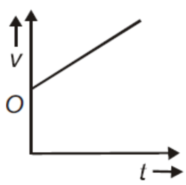
39. A particle moves along x-axis in such a way that its x-co-ordinate varies with time according to the equation $x = 4 - 2t + t^2$. The speed of the particle will vary with time as



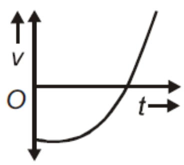
(a)



(b)

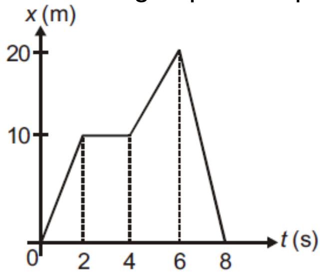


(c)



(d)

40. The position (x)-time (t) graph for a particle moving along a straight line is shown in figure. The average speed of particle in time interval $t = 0$ to $t = 8$ s is



(a) Zero (b) 5 m/s (c) 7.5 m/s (d) 9.7 m/s

(b)

average speed is $v = \frac{\text{total distance}}{\text{total time}}$

total distance is 40m and total time is 8sec

$v = \frac{40}{8} = 5ms^{-1}$

41. If R and H are the horizontal range and maximum height attained by a projectile, than its speed of projection is

(a) $\sqrt{2gR + \frac{4R^2}{gH}}$ (b) $\sqrt{2gH + \frac{R^2 g}{8H}}$ (c) $\sqrt{2gH + \frac{8H}{Rg}}$ (d) $\sqrt{2gH + \frac{R^2}{H}}$

(b)

Let the velocity of the projection is 'V' and angle of projection "θ" with horizontal

At maximum height H, $v_y = 0$

Using Newton's third equation of motion

$v^2 - u^2 = 2as$

$u = 0, u = v\sin\theta; a = -g; s = H$

$0 - v^2 \sin^2\theta = 2 \times (-g)H$

$v\sin\theta = \sqrt{2gH} \longrightarrow 1$

Also the half time of flight-from A to B and from B to C

Using Newton's second equation of motion from B to C

$u = o; t = \frac{T}{2}; S = -H, \quad a = -g$ (Here T is total time of flight)

$S = ut + \frac{1}{2}at^2$

$-H = 0 + \frac{1}{2}(-g)\left(\frac{T}{2}\right)^2$

$T = \sqrt{\frac{8H}{g}}$

Range (R)= $v\cos\theta \times T$

$$R = v\cos\theta \times \sqrt{\frac{8H}{g}} \longrightarrow 2$$

$$v\cos\theta = R\sqrt{\frac{g}{8H}} \longrightarrow 3$$

Squaring and adding 1 and 2

$$v^2 = 2gH + \frac{R^2 g}{8H}$$

$$v = \sqrt{2gH + \frac{R^2 g}{8H}}$$

GROUND TO GROUND PROJECTION

42. Two projectiles A and B are projected with angle of projection 15º for the projectile A and 45º for the projectile B. If R_A and R_B be the horizontal range for the two projectiles, then :

(a) $R_A < R_B$

(b) $R_A = R_B$

(c) $R_A > R_B$

(d) the information is insufficient to decide the relation of R_A with R_B.

We know that, Range is given by, $R = \frac{u^2 \sin(2\alpha)}{g}$

Information is insufficient to decide the relation of R_A with R_B.

Here, Information of the initial velocity is not given in the question.

Hence, option (D) is correct

43. A projectile is thrown with an initial velocity of $\left[a\hat{i} + b\hat{j}\right]$ m/s. If the range of projectile is twice the maximum height reached by it, then

(a) b = a/2 (b) b = a (c) b = 2a (d) b = 4a

$$h = \frac{u^2 \sin\theta}{2g}$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\frac{u^2 \sin\theta\cos\theta}{g} = \frac{2 \times u^2 \sin^2\theta}{2g}$$

$$(u\sin\theta)(u\cos\theta) = \frac{(u\sin\theta)^2}{2}$$

$$a.b = \frac{b}{2}$$

$$b = 2a$$

44. The angle of projection, at which the horizontal range and maximum height of projectile are equal, is

(a) 45º (b) $\theta = \tan^{-1}(0.25)$ (c) $\theta = \tan^{-1} 4$ (d) 60º.

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$$

$$2 \sin\theta \cos\theta = \frac{\sin^2\theta}{2}$$

$$u \cos\theta = \sin\theta$$

$$\tan\theta = 4$$

$$\theta = \tan^{-1} 4$$

45. A javelin thrown into air at an angle with the horizontal has a range of 200 m. If the time of flight is 5 second, then the horizontal component of velocity of the projectile at the highest point of the trajectory is

(a) 40 m/s (b) 20 m/s (c) 9.8 m/s (d) 5 m/s

Range: $S = u\cos\theta \times T$

$$\therefore u\cos\theta = \frac{200}{5} = 40 \text{ m/s}$$

The horizontal component of velocity $u\cos\theta$ remains same throughout journey.

46. The position coordinates of a projectile projected from ground on a certain planet (with no atmosphere) are given by $y = (4t - 2t^2)$m and $x = (3t)$ metre, where t is in second and point of projection is taken as origin. The angle of projection of projectile with vertical is

(a) 30° (b) 37° (c) 45° (d) 60°

→ Given

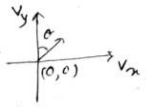
$y = 4t - 2t^2$

$x = 3t$

To, Find the angle of projection

$\frac{dx}{dt} = 3$

$\frac{dy}{dt} = 4 - 4t$



at $t = 0$

At $t = 0$

$V_x = 3$

$V_y = 4 - 4 \times 0 = 4$

$\tan\theta = 3/4$

$\therefore \theta = 37°$

Hence, option 'C' is Correct

47. A particle is projected from ground with speed 80 m/s at an angle 30° with horizontal from ground. The magnitude of average velocity of particle in time interval t = 2 s to t = 6 s is [Take $g = 10$ m/s^2]

(a) $40\sqrt{2}\text{m/s}$ (b) 40m/s (c) Zero (d) $40\sqrt{3}\text{m/s}$

Time of flight $= \frac{2u\sin\theta}{g}$

$= \frac{2\times 80\times \sin 30^\circ}{10} = 8$

As particle during journey at t = 2sec and at t = 6sec will be at same height and hence vertical displacement is zero.

Horizontal displacement $=(u\cos\theta)\times(6-2)$

$=80\times\frac{\sqrt{3}}{2}\times 4$

$=160\sqrt{3}\text{m}$

$\therefore$ Average velocity $= \frac{160\sqrt{3}}{4} = 40\sqrt{3}$ m/sec

48. Two objects are thrown up at angles of 45° and 60° respectively, with the horizontal. If both objects attain same vertical height, then the ratio of magnitude of velocities with which these are projected is

(a) $\sqrt{\frac{5}{3}}$ (b) $\sqrt{\frac{3}{5}}$ (c) $\sqrt{\frac{2}{3}}$ (d) $\sqrt{\frac{3}{2}}$

Given,

Angle $45^0, 60^0$

So,

$$\left(\frac{V_1}{V_2}\right)^2 = \frac{\frac{\sin^2\alpha_1}{2g}}{\frac{\sin^2\alpha_2}{2g}}$$

$$\alpha_1 = 45^\circ, \alpha_2 = 60^\circ$$

Where $\left(\frac{V_1}{V_2}\right)^2 = \left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}}\right)^2$

$$= \frac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{2}$$

So,

$$\frac{V_1}{V_2} = \sqrt{\frac{3}{2}}$$

Thus, the ratio of magnitude of velocities with which these are projected is

$$\frac{V_1}{V_2} = \sqrt{\frac{3}{2}}$$

49. For an object projected from ground with speed u horizontal range is two times the maximum height attained by it. The horizontal range of object is

(a) $\frac{2u^2}{3g}$ (b) $\frac{3u^2}{4g}$ (c) $\frac{3u^2}{2g}$ (d) $\frac{4u^2}{5g}$

Given: Velocity of Projection = u.
Range = 2 Hmax.

To find: Horizontal Range = ?

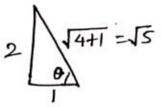
Sol^n: $R = \frac{u^2 \sin 2\theta}{g}$

$H_{max} = \frac{u^2 \sin^2\theta}{2g}$

According to question.

$\frac{u^2 \sin 2\theta}{g} = 2 \times \frac{u^2 \sin^2\theta}{2g}$

$2 \sin\theta \cos\theta = \sin^2\theta \Rightarrow \boxed{\tan\theta = 2}$



$\sin\theta = \frac{2}{\sqrt{5}}$ $\cos\theta = \frac{1}{\sqrt{5}}$

$R = \frac{u^2 \times 2 \sin\theta \cos\theta}{g} = \frac{u^2 \times 2 \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}}}{g}$

$\boxed{R = \frac{4u^2}{5g}}$

50. The velocity at the maximum height of a projectile is $\frac{\sqrt{3}}{2}$ times its initial velocity of projection (u). Its range on the horizontal plane is

(a) $\frac{\sqrt{3}u^2}{2g}$ (b) $\frac{3u^2}{2g}$ (c) $\frac{3u^2}{g}$ (d) $\frac{u^2}{2g}$

speed at the maximum height is only going to be horizontal component which is ucosθ

$$u \cos\theta = \frac{\sqrt{3}u}{2}$$

$$\theta = 30^o$$

Therefore range R = $\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 60^o}{g} = \frac{\sqrt{3}u^2}{2g}$

51. A projectile is thrown into space so as to have a maximum possible horizontal range of 400 metres. Taking the point of projection as the origin, the co-ordinates of the point where the velocity of the projectile is minimum are

(a) (400, 100) (b) (200, 100) (c) (400, 200) (d) (200, 200)

Maximum possible range $R_{max} = 400$ m

Maximum height attained $H = \frac{R_{max}}{4} = 100$ m

The particle has minimum velocity at the top most point of its projectile motion whose co-ordinates are $(R_{max}/2, H)$

So, x co-ordinate $x = \frac{400}{2} = 200$ m

Y-co-ordinate $y = H = 100$ m

52. If the time of flight of a bullet over a horizontal range *R* is *T*, then the angle of projection with horizontal is

(a) $\tan^{-1}\left(\frac{gT^2}{2R}\right)$ (b) $\tan^{-1}\left(\frac{2R^2}{gT}\right)$ (c) $\tan^{-1}\left(\frac{2R}{g^2T}\right)$ (d) $\tan^{-1}\left(\frac{2R}{gT}\right)$

53. When a particle is projected at some angle to the horizontal, it has a range *R* and time of flight t_1. If the same particle is projected with the same speed at some other angle to have the same range, its time of flight is t_2, then

(a) $t_1 + t_2 = \frac{2R}{g}$ (b) $t_1 - t_2 = \frac{R}{g}$ (c) $t_1 t_2 = \frac{2R}{g}$ (d) $t_1 t_2 = \frac{R}{g}$

Range R is same for θ and 90° − θ

Now, $t_1 = \frac{2u\sin\theta}{g}$

$t_2 = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$

Hence, $t_1 t_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g}$

$= \frac{2u^2\, 2\sin\theta\cos\theta}{g^2}$

$t_1 t_2 = \frac{2u^2\sin 2\theta}{g}$ $\left[R = \frac{u^2\sin 2\theta}{g}\right]$

$t_1 t_2 = \frac{2R}{g}$

54. A projectile is thrown with velocity v at an angle θ with horizontal. When the projectile is at a height equal to half of the maximum height, the vertical component of the velocity of projectile is

(a) $v\sin\theta \times 3$ (b) $\frac{v\sin\theta}{3}$ (c) $\frac{v\sin\theta}{\sqrt{2}}$ (d) $\frac{v\sin\theta}{\sqrt{3}}$

A projectile is thrown with velocity v at an angle θ with horizontal.

then, $v_x = v\cos\theta, v_y = v\cos\theta$

we have to find vertical component of velocity of the projectile at height equal to half of the maximum height.

we know, $H_{max} = \frac{u^2\sin^2\theta}{2g}$

here u is intial velocity of projectile. but here given initial velocity is v

so, $H_{max} = \frac{v^2\sin^2\theta}{2g}$

now, time taken to reach half of maximum height, t

$$Y = v_y t + \frac{1}{2}a_y t^2$$

$$\frac{v^2\sin^2\theta}{2g} = v\sin\theta.t - 5t^2$$

$$200t^2 - 40v\sin\theta.t + v^2\sin^2\theta = 0$$

$$t = \frac{(\sqrt{2} \pm 1)\, v\sin\theta}{10\sqrt{2}}$$

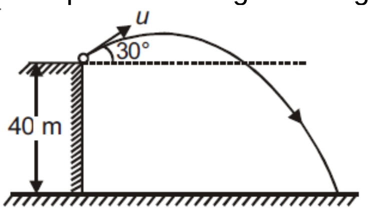
$$v_y' = v_y + a_y t$$

$$= v\sin\theta - \frac{(\sqrt{2}-1)\, v\sin\theta}{\sqrt{2}}$$

$$= \frac{v\sin\theta}{\sqrt{2}}$$

PROJECTILE FROM A HEIGHT

55. Figure shows a projectile thrown with speed *u* = 20 m/s at an angle 30° with horizontal from the top of a building 40 m high. Then the horizontal range of projectile is



(a) $20\sqrt{3}$m (b) $40\sqrt{3}$m (c) 40 m (d) 20 m

Given that,

Speed u = 20 m/s

Angle $\theta = 30^0$

Height h = 40 m

Now, along y axis

$u_y = u \sin 30^0$

$u_y = 20 \times \frac{1}{2}$

$u_y = 10\ m/s$

Now, we know that

$a = -10\ m/s^2$

$s = -40\ m$

Now, from equation of motion

$s = u_y t - \frac{1}{2} g t^2$

$-40 = 10t - 5t^2$

$5t^2 - 10t - 40 = 0$

$t^2 - 2t - 8 = 0$

$t^2 - (4-2)t - 8 = 0$

$t(t-4) + 2(t-4) = 0$

$(t+2)(t-4)$

Now, neglect of negative value

So, t = 4 s

Now, the range is

$R = u \cos 30^0 \times t$

$R = 20 \times \frac{\sqrt{3}}{2} \times 4$

$R = 40\sqrt{3}\ m$

Hence, the range is $40\sqrt{3}$ m

56. Two paper screens *A* and *B* are separated by distance 100 m. A bullet penetrates *A* and *B*, at points *P* and *Q* respectively, where Q is 10 cm below *P*. If bullet is travelling horizontally at the time of hitting *A*, the velocity of bullet at *A* is nearly

(a) 100 m/s (b) 200 m/s (c) 600 m/s (d) 700 m/s

Solution:

Let Velocity at A be V

⇒ Time taken to reach $B = \frac{100}{V} = t$ ——(i)

Also in this same time interval t, height of bullet will decrease by 10 cm = 0.1 m

$h = \frac{1}{2}gt^2 \Rightarrow 0.1 = \frac{1}{2} \times 10 t^2$

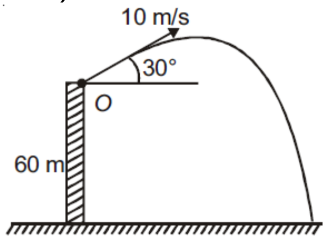
$\Rightarrow t^2 = \frac{2}{200} \Rightarrow t = \sqrt{\frac{2}{200}}$

$\Rightarrow t = 1/\sqrt{50}$

Putting in equation (i), we get $V \approx 700$ m/s

Hence, Option D is correct.

57. A ball is projected from a point O as shown in figure. It will strike the ground after (g = 10 m/s^2)



(a) 4 s (b) 3 s (c) 2 s (d) 5 s

Time period of projectile motion:

$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 10 \times \sin 30^\circ}{10} = 2 \times \frac{1}{2} = 1\text{s}$$

Then for vertically downward motion:

initial velocity $(u_y) = 10\sin 30^\circ = 5$ m/s

$a = g$ and $h = 60$m

So applying equation $h = ut + \frac{1}{2}at^2$

$\Rightarrow 60 = 5t + \frac{1}{2}(10)t^2$

$\Rightarrow 60 = 5t + 5t^2$ or $t^2 + t - 12 = 0$

$(t-3)(t+4) = 0$

$\Rightarrow t = 3$s ($t = -4$s is not possible)

Therefore total time of flight = T + t = 4s.

RELATIVE MOTION IN ONE DIMENSION

58. A body A of mass 4 kg is dropped from a height of 100 m. Another body B of mass 2 kg is dropped from a height of 50 m at the same time. Then :

(a) Both the bodies reach the ground simultaneously.

(b) A takes nearly 0.7th of time required by B

(c) B takes nearly 0.7th of time required by A

(d) A takes double the time required by B.

$h_1 = 100\text{ m}$

$h_2 = 50\text{ m}$

$h_1 = 2h_2$

$h_1 = u_1 t + \frac{1}{2} g t_1^2$

$h_1 = 0 + \frac{1}{2} g t_1^2$ —(i)

Simillarly

$h_2 = \frac{1}{2} g t_2^2$ —(ii)

(i) ÷ (ii)

$\frac{h_1}{h_2} = \frac{t_1^2}{t_2^2}$

$2 = \frac{t_1^2}{t_2^2}$

$t_2^2 = \frac{t_1^2}{2}$

$t_2 = \frac{t_1}{\sqrt{2}} = 0.7\, t_1$

59. Two particles are moving with velocities v_1 and v_2. Their relative velocity is the maximum when the angle between their velocities is :

(a) zero (b) $\pi/4$ (c) $\pi/2$ (d) π

The relative velocity of one body w.r.t. other body is maximum if they move in opposite direction. Thus the angle between their velocities must be π so that their relative velocity is maximum.

60. A body falls freely from a height of 50 m. Simultaneously, another body is thrown from the surface of earth with a certain initial velocity. The two bodies meet at a height of 10 m. What is the initial velocity of the second body ?

(a) 4.9 m/s (b) 9.8 m/s (c) 17.5 m/s (d) 19.6 m/s

let ball A dropped from top

ball B thrown up with velocity u

time taken by A to travel $s_A = 50 - 10 = 40$

$$\frac{gt^2}{2} = 40$$

$$t = 2.85\text{s}$$

$S_B = 10, t = 2.85$

$$s_B = ut - \frac{gt^2}{2}$$

$$10 = u \times 2.85 - 40$$

$$u = \frac{50}{2.85}$$

$$u = 17.5\text{m/s}$$

61. A body is projected vertically upward with speed 10 m/s and other at same time with same speed in downward direction from the top of a tower. The magnitude of acceleration of first body w.r.t. second is {take g = 10 m/s^2}

(a) Zero (b) 10 m/s^2 (c) 5 m/s^2 (d) 20 m/s^2

when body is projected vertically upward
$u = 10$ m/s
$V = 0$
$a = a_1$
$t = 1$ sec

$\Rightarrow V = u + a_1 t$
$0 = 10 + a(1) \Rightarrow a_1 = -10 m/s^2$

now 2nd body at same time with same speed in downward
$u = 0$
$V = 10$ m/s
$a = a_2$
$t = 1$ sec

$\Rightarrow V = u + a_2 t \Rightarrow 10 = a_2(1) \Rightarrow a_2 = 10 m/s^2$

hence magnitude of second body w.r.t first body $= 10 - 10 = 0$

62. Two balls are projected upward simultaneously with speeds 40 m/s and 60 m/s. Relative position (x) of second ball w.r.t. first ball at time t = 5 s is [Neglect air resistance].

(a) 20 m (b) 80 m (c) 100 m (d) 120 m

speed of first ball=40m/s
second ball=60 m/s
time=5s
Distance covered by first ball
S=ut+1/2at^2
S=40x5-1/2x10 x 5 x5
s=200-125=75m
Distance travelled by second ball:
S=60x5-1/2x10x5x5
=300-125=175m
Relative position of second ball with respect to first=175-75=100m

RELATIVE MOTION IN TWO DIMENSION

63. A swimmer wishes to reach directly opposite bank of a river, flowing with velocity 8 m/s. The swimmer can swim 10 m/s in still water. The width of the river is 480 m. Time taken by him to do so:

(a) 60 sec (b) 48 sec (c) 80 sec (d) none of these.

so to reach directly opposite bank of river he has to swim at an angle where component of his velocity cancel out the flow of water so that he can go directly opposite bank of the river.
so if we assume he is swimming at an angle θ with the flow, we get
$10\cos\theta = 8$
and also $10\sin\theta$ will be component which will help him cross the river.
so if $\cos\theta = 0.8$ then $\sin\theta = 0.6$
velocity helping in crossing the river $= 10 \times 0.6 = 6$m/s
also distance is given width = 480m
time taken will be $t = \frac{480}{6} = 80$

64. A man who can swim at the rate of 2 km/h crosses a river to an exactly opposite point on the other bank by swimming in a direction of 120º to the flow of the water in the river. The velocity of the water current in km/h is :

(a) 1 (b) 2 (c) 1/2 (d) 3/2

V_m= Velocity of man,

V_r= Velocity of rain,

$V_{mr} = V_m - V_r$

Taking components,

$V_{mr} \cos 120° = V_m \cos 90° - V_r \cos 0°$

$2 \times \left(-\frac{1}{2}\right) = -V_r$

So, $V_r = 1\text{km/hr}$

65. A boat covers certain distance between two spots in a river taking t_1 hrs going downstream and t_2 hrs going upstream. What time will be taken by boat to cover same distance in still water?

(a) $\frac{t_1+t_2}{2}$ (b) $2(t_2-t_1)$ (c) $\frac{2t_1t_2}{t_1+t_2}$ (d) $\sqrt{t_1t_2}$

Let the velocity of a boat in still water is u and velocity of the river is v and distance is d

down the stream speed will add total velocity is u+v it takes time t_{1}

in upstream speed will be subtracted total velocity is u-v it take time t_{2}

$u + v = \frac{d}{t_1}$ eq(1)..

$u - v = \frac{d}{t_2}$ eq(2)..

$2U = \frac{d}{t_1} + \frac{d}{t_2}$

$u = \frac{d(t_1+t_2)}{2t_1t_2}$

time take by the still boat is $t = \frac{d}{u} = \frac{2t_1t_2}{(t_1+t_2)}$

66. A train of 150 m length is going towards North at a speed of 10 m/s. A bird is flying at 5 m/s parallel to the track towards South. The time taken by the bird to cross the train is

(a) 10 s (b) 15 s (c) 30 s (d) 12 s

Relative velocity of the parrot w.r.t. the train $[10-(-5)] = (10-(-5))\text{m/s} = 15\text{m/s}$

Time taken by the parrot to cross the train

$= \frac{150}{15} = 10\text{s}.$

67. Two cars are moving in the same direction with a speed of 30 km/h. They are separated from each other by 5 km. Third car moving in the opposite direction meets the two cars after an interval of 4 minutes. The speed of the third car is

(a) 30 km/h (b) 25 km/h (c) 40 km/h (d) 45 km/h

Let the speed of third car be x km/hr

t = 4min = 1/15hr

D = 5km

Relative speed = 30 + x = D/t = 75

Thus x = 75 − 30 = 45 km/hr

68. Four persons *P, Q, R* and *S* are initially at the four corners of a square of side *d*. Each person now moves with a constant speed *v* in such a way that *P* always moves directly towards *Q, Q* towards *R, R* towards *S*, and *S* towards *P*. The four persons will meet after time

(a) $\frac{d}{2v}$ (b) $\frac{d}{v}$ (c) $\frac{3d}{2v}$ (d) They will never meet

Here, velocity components will be $v\cos 45 = \frac{v}{\sqrt{2}}$

And, displacement will be $\frac{d}{\sqrt{2}}$

So time taken will be

$$t = \frac{d}{v}$$

$$= \frac{\frac{d}{\sqrt{2}}}{\frac{v}{\sqrt{2}}} = \frac{d}{v}$$

69. Two trains each of length 100 m moving parallel towards each other at speed 72 km/h and 36 km/h respectively. In how much time will they cross each other?

(a) 4.5 s (b) 6.67 s (c) 3.5 s (d) 7.25 s

Total lengh to be covered (l) = 200m

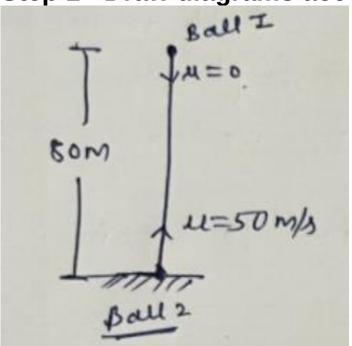
Total relative speed $(v) = (\vec{v}_1 - \vec{v}_2) = 72 - (-36) = 108\text{km/h} = 108 \times \frac{5}{18} = 30\text{m/s}$

So, Time taken to cross each other $= \frac{l}{v} = \frac{200}{30} = 6.67\text{s}$

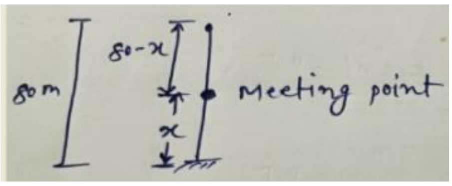
70. A ball is dropped from the top of a building of height 80 m. At same instant another ball is thrown upwards with speed 50 m/s from the bottom of the building. The time at which balls will meet is

(a) 1.6 s (b) 5 s (c) 8 s (d) 10 s

Step 1 - Draw diagrams according to given data Refer Figure



Speed of ball 2 at which it is thrown, v=50 m/s
initial velocity of ball 1 is zero
Let both the balls meet when distance covered by ball 2 is x
So distance covered by ball 1 in this time is 80−x
Let t is the time at which both the balls meet.



Step 2 - Calculation of time at which balls will meet

By 2nd equation of motion $\left(S = ut + \frac{1}{2}at^2\right)$

$$x = 50t - \frac{1}{2}gt^2 \quad \text{....(1)}$$

$$80 - x = 0 + \frac{1}{2}gt^2 \quad \text{....(2)}$$

From equation (1) and equation (2)

$$80 - 50t + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$$

$$80 = 50t$$

$$t = \frac{8}{5} = 1.6 \text{ s}$$

71. A shell is fired vertically upwards with a velocity v_1 from a trolley moving horizontally with velocity v_2. A person on the ground observes the motion of the shell as a parabola, whose horizontal range is

(a) $\frac{2v_1^2 v_2}{g}$ (b) $\frac{2v_1^2}{g}$ (c) $\frac{2v_2^2}{g}$ (d) $\frac{2v_1 v_2}{g}$

Vertical velocity of shell $u_y = v_1$

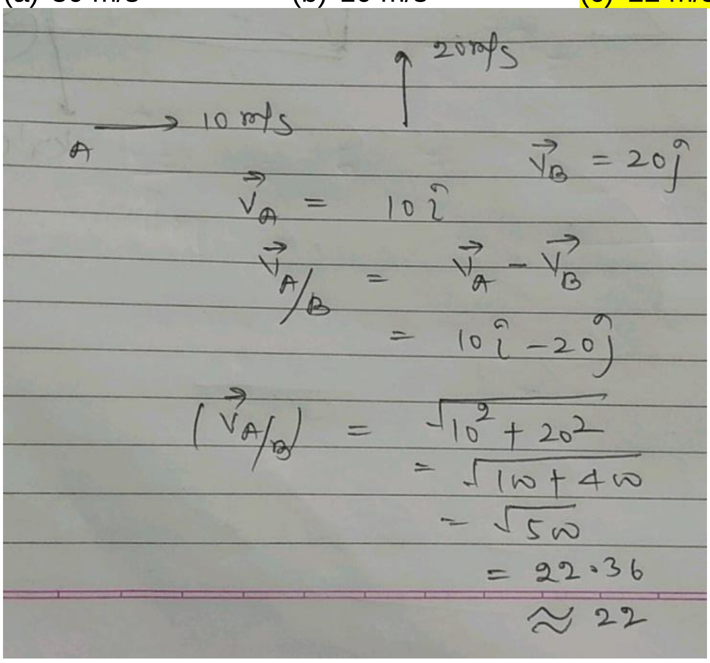
Horizontal velocity of shell $u_x = v_2$

Horizontal range of projectile $R = \frac{2u_x u_y}{g}$

$$\Rightarrow R = \frac{2v_1 v_2}{g}$$

72. Out of the two cars *A* and *B*, car *A* is moving towards east with a velocity of 10 m/s whereas *B* is moving towards north with a velocity 20 m/s, then velocity of *A* w.r.t. *B* is (nearly)

(a) 30 m/s (b) 10 m/s (c) 22 m/s (d) 42 m/s



73. A man moves in an open field such that after moving 10 m on a straight line, he makes a sharp turn of 60º to his left. The total displacement just at the start of 8th turn is equal to

(a) 12 m (b) 15 m (c) 17.32 m (d) 14.14 m

When a man moves 10m before turning by 60°, he makes a hexagon in six steps.

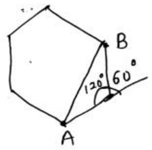
∴ In eight steps he will trace two additional steps of hexagon from starting point.

∴ Displacement is AB

$= \sqrt{10^2 + 10^2 + 2 \times 10 \times 10 \cos 60}$

$= \sqrt{100 + 100 + 200\left(\frac{1}{2}\right)}$

$= 10\sqrt{3} = 17.32\text{ m}$



LEVEL – II

DISTANCE AND DISPLACEMENT, SPEED AND VELOCITY, AVG. SPEED AND AVG. VELOCITY

1. A particle moving in a straight line covers half the distance with speed of 3m/s. The other half of the distance is covered in two equal time intervals with speed of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during this motion is

(a) 4.0 m/s (b) 5.0 m/s (c) 5.5 m/s (d) 4.8 m/s

$t_1 = \frac{\frac{S}{2}}{3} = \frac{S}{6}$

$\frac{S}{2} = 4.5 \times t_2 + 7.5 t_3 \Rightarrow 12 t_2 = \frac{S}{2}$

We have $t_2 = t_3$

Total time $t = t_1 + t_2 + t_3 = \frac{S}{6} + \frac{S}{24} + \frac{S}{24} = \frac{S}{4}$

Average of $\frac{S}{t} = \frac{S}{\frac{S}{4}} = 4$ m/s

2. The table shows the distance covered in successive seconds by a body accelerated uniformly from rest

Time interval (s)	I	II	III	IV
Distance (cm)	2	6	10	14

What is the speed of the body after 4 sec?

(a) 4 cm/sec (b) 8 cm/sec (c) 14 cm/sec (d) 16 cm/sec

3. A body moves in a straight line along x-axis, its distance from the origin is given by the equation $x = 8t - 3t^2$. The average velocity in the interval from t = 0 to t = 4 is

(a) 2 m/s (b) –16 m/s (c) –4 m/s (d) 5 m/s

4. The height y and distance x along the horizontal for a body projected in the vertical plane are given by

$y = 8t - 5t^2$ and $x = 6t$. The initial velocity of the projected body is

(a) 8 m/s (b) 9 m/s (c) 10 m/s (d) $\frac{10}{3}$ m/s

Solution:

$y = 8t - 5t^2$ $\quad x = 6t$

$v = \frac{dy}{dt} = 8 - 10t$ $\quad \frac{dx}{dt} = 6$

$v_y\big]_{t=0} = 8$ m/s $\quad v_x\big]_{t=0} = 6$ m/s

$v = \sqrt{(v_x)^2 + (v_y)^2}$

$v = \sqrt{(8)^2 + (6)^2}$

$v = \sqrt{64 + 36}$

$v = 10$ m/s

Hence the correct option is C.

5. Choose the correct statement from the following:
(a) The magnitude of velocity of a particle is equal to its speed.
(b) The magnitude of the average velocity in an interval is equal to its average speed in that interval.
(c) It is possible to have a situation in which the speed of the particle is never zero but the average speed in an interval is zero.
(d) It is possible to have a situation in which the instantaneous speed of particle is always zero but the average speed is not zero.
Velocity is speed in a certain direction. So, magnitude of velocity is speed.
But, average velocity is not the average speed in a certain direction. And, if a body is moving, its average speed is always non-zero. If a body is not moving, its average speed must also be zero.

6. The location of a particle has changed. What can we say about the displacement and the distance covered by the particle.
(a) Neither can be zero (b) One may be zero
(c) Both may be zero (d) One is +ve, other is -ve and vice versa
When location of a particle has changed, it must have covered some distance and undergone some displacement. As, initial and final position of body are different, so distance and displacement, both can neither be zero.

7. A train moving with a constant speed along a straight track takes a bend in a curve with the same speed. Due to this :
(a) its velocity is changed in magnitude (b) its velocity is not changed
(c) its speed only is changed (d) its velocity is changed.

8. A particle projected from origin moves in x-y plane with a velocity $\vec{v} = 3\hat{i} + 6x\hat{j}$, where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis. Find the equation of path followed by the particle
(a) $y = x^2$ (b) $y = \frac{1}{x^2}$ (c) $y = 2x^2$ (d) $y = \frac{1}{x}$

$V = 3i + 6xj$
$V = \frac{dx}{dt}i + \frac{dy}{dt}j$
$\frac{dx}{dt} = 3........(1)$
$\frac{dy}{dt} = 6x........(2)$
from equation (1)
$dx = sdt$
integrate both side
$x = 3t......(3)$
take equation (2)
$dy = 6xdt$
putting the value of x from 3
$dy = 6(st)dt$
$dy = 18tdt$
integrate both side
$y = 9t^2$
$y = 9\left(\frac{x}{3}\right)^2$.......from equation 3
we get
$y = x^2$

AVG. ACCELERATION AND APPLICATION OF CALCULAS

9. Choose the wrong statement .
   (a) Zero velocity of a particle does not necessarily mean that its acceleration is zero.
   (b) Zero acceleration of a particle does not necessarily mean that its velocity is zero.
   (c) If speed of a particle is constant, its acceleration must be zero.
   (d) none of these.

10. The initial velocity of a particle moving along *x*-axis is *u* (at *t* = 0 and *x* = 0) and its acceleration a is given by *a* = *kx*. Which of the following equation is correct between its velocity (*v*) and position (*x*)?
   (a) $v^2 - u^2 = 2kx$ (b) $v^2 = u^2 + 2kx^2$ (c) $v^2 = u^2 + kx^2$ (d) $v^2 + u^2 = 2kx$

$a = kx$

$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt}$

$Vdv = adx$

$= kx\,dx$

$\frac{V^2}{2} = \frac{kx^2}{2} + C$

at $t = 0$

$c = 0$

velocity = u

So

$C = \frac{U^2}{2}$

$V^2 = u^2 + kx^2$

11. A particle moving with a constant acceleration from A to B in the straight line AB has velocities u and v at A and B respectively. If C is the mid-point of AB then the velocity of particle while passing C will be
   (a) $\sqrt{\frac{v^2+u^2}{2}}$ (b) $\frac{v+u}{2}$ (c) $\frac{v-u}{2}$ (d) $\frac{\left(\frac{1}{v}+\frac{1}{u}\right)}{2}$

Let the distance between A and B=s and as mentioned C is the midpoint of A and B thus distance between A C and

C B= $\frac{s}{2}$

From equation of motion we get distance between AB

$v^2 = u^2 + 2as$

$s = \frac{v^2 - u^2}{2a}$ ----------(1)

Let speed of the vehicle be pm/s at point C

Again from equation of motion we get,

$p^2 = u^2 + 2a\frac{s}{2}$

as distance between A and C is $\frac{s}{2}$

$s = \frac{p^2 - u^2}{a}$ --------(2)

Computing 1 and 2 we get

$\frac{p^2 - u^2}{a} = \frac{v^2 - u^2}{2a}$

$p^2 = \frac{v^2 + u^2}{2}$

$p = \sqrt{\frac{v^2 + u^2}{2}}$

12. If a body travels half of its total path in the last second of its fall from rest then the total time of its fall will be

(a) 1.82 s (b) 3.4 s (c) 3.41 (d) 3.73

13. The displacement of a particle undergoing rectilinear motion along the x-axis is given by $x = 2t^3 - 21t^2 + 60t + 6$. The acceleration of the particle when its velocity is zero will be.

(a) 36 m/sec^2 (b) 9 m/sec^2 (c) –9m/sec^2 (d) –18m/sec^2

$X = 2t^3 + 21t^2 + 60t + 6m$

$\upsilon = \frac{dx}{dt} = 6t^2 + 42 + 60$

when $\upsilon = 0$, $6t^2 + 42t + 60 = 0$

$t = \frac{-42 \pm \sqrt{42^2 - 4 \times 6 \times 60}}{12}$

$= \frac{-42 \pm 18}{12} = -5, -2s$

$a = \frac{d\upsilon}{dt} = 12t + 42$

at $t = -2$

$a = 12(-2) + 42$

$= 18m / s2$

14. A particle, initially at rest, starts moving in a straight line with an acceleration a = 6t + 4 m/s^2. The distance covered by it in 3 s is

(a) 15 m (b) 30 m (c) 45 m (d) 60 m

15. The acceleration 'a' in m/s^2, of a particle is given by $a = 3t^2 + 2t + 2$ where 't' is the time. If the particle starts out with a velocity v = 2 m/s at t = 0, then the velocity at the end of 2 s is

(a) 12 m/s (b) 14 m/s (c) 16 m/s (d) 18 m/s

$\int dv = \int a\,dt$

$\int_{t=0}^{t=2} (3t^2 + 2t + 2)\,dt = v - u$

$\Rightarrow \left| t^3 + t^2 + 2t \right|_0^2 = v - 2$

$\Rightarrow 8 + 4 + 4 = v - 2$

$\Rightarrow v = 18$ m/s.

16. A balloon is ascending at the rate of 12 m/s. When it is at a height of 65 m from the ground, a packet is dropped from it. The packet will reach the ground after time (take g = 10 m/s^2)
(a) 10 s (b) 2.5 s (c) 5 s (d) 7.5 s

Using the second equation of motion,

$s = ut + \frac{1}{2}at^2$

Here, s = 65 m, u = −12 m/s (since it's moving upwards), a = g = 10 m/s^2

$65 = -12t + \frac{1}{2}10t^2$

$\Rightarrow 5t^2 - 12t - 65 = 0$

$\Rightarrow t = 5 \quad ; t = -13/5$

Since, time cannot be negative.

Thus, t = 5s

17. A body falls from a certain height. Two seconds later another body falls from the same height. How long after the beginning of motion of the first body is the distance between the bodies twice the distance at the moment the second body starts to fall?
(a) 3 s (b) 10 s (c) 15 s (d) 20

18. The velocity v of body moving along a straight line varies with time *t* as $v = 2t^2 e^{-t}$, where v is in m/s and t is in second. The acceleration of body is zero at t =
(a) 0 (b) 2s (c) 3 (d) Both (a) & (b)

It is given that

$a = \frac{dv}{dt} = \frac{d}{dt}(2t^2e^{-t})$

$= 2\left[t^2e^{-1}(-1) + e^{-1}2t\right]$

$= e^{-1}(4t - 2t^2)$

So, $a = 0$

$\Rightarrow 4t - 2t^2 = 0$

Or $2t(2 - t) = 0$

$\Rightarrow t = 0$ and $t = 2$

19. The velocity of a body depends on time according to the equation $v = \frac{t^2}{10} + 20$. The body is undergoing
(a) Uniform acceleration (b) Uniform retardation
(c) Non-uniform acceleration (d) Zero acceleration

$\vec{a} = \frac{dv}{dt} = \frac{d}{dt}(20 + 0.1t^2) = 0.2t$

Since, acceleration, a, does depend on time, t, it is non-uniform acceleration.

20. A body starts from origin and moves along *x*-axis so that its position at any instant is $x = 4t^2 - 12t$ where *t* is in second and *v* in m/s. What is the acceleration of particle?
(a) 4 m/s^2 (b) 8 m/s^2 (c) 24 m/s^2 (d) 0 m/s^2

$x = 4t^2 - 12t$

$\frac{dx}{dt} = v = 8t - 12$

$\frac{dv}{dt} = a = 8$

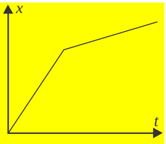
21. The relation between position (x) and time (t) are given below for a particle moving along a straight line. Which of the following equation represents uniformly accelerated motion? [where α and β are positive constants]

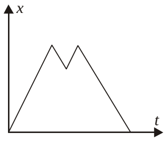
(a) $\beta x = \alpha t + \alpha\beta$ (b) $\alpha x = \beta + t$ (c) $xt = \alpha\beta$ (d) $\alpha t = \sqrt{\beta + x}$

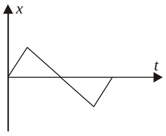
for a uniformly accelerated motion, acceleration must be a constant.\brSpace \brSpace In the forth option,\brSpace \brSpace

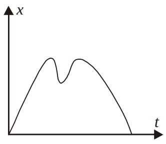
$$\alpha t = \sqrt{\beta + x} \ \backslash \text{brSpace} \Rightarrow x = \alpha^2 t^2 - \beta \ \backslash \text{brSpace} \Rightarrow \upsilon = \frac{dx}{dt} = 2\alpha^2 t \ \backslash \text{brSpace} \Rightarrow a = \frac{d\upsilon}{dt} = 2\alpha^2 = \text{constant} \ \backslash \text{brSpace}$$

22. Which of the following distance time graphs is possible :

(a) 

(b) 

(c) 

(d) 

23. If magnitude of average speed and average velocity over a time interval are same, then
(a) The particle must move with zero acceleration
(b) The particle must move with non-zero acceleration
(c) The particle must be at rest
(d) The particle must move in a straight line without turning back

24. When the velocity of body is variable, then
(a) Its speed may be constant
(b) Its acceleration may be constant
(c) Its average acceleration may be constant
(d) All of these

25. An object is moving with variable speed, then
(a) Its velocity may be zero (b) Its velocity must be variable
(c) Its acceleration may be zero (d) Its velocity may be constant

Velocity means instantaneous velocity which is nothing but direction + magnitude of speed at that instant.
So velocity must be variable.
Acceleration can't be zero as the magnitude of velocity is changing.

26. If the displacement of a particle varies with time as $\sqrt{x} = t + 7$, then
(a) Velocity of the particle is inversely proportional to t
(b) Velocity of the particle is proportional to t^2
(c) Velocity of the particle is proportional to $\sqrt{t}$
(d) The particle moves with constant acceleration

$\sqrt{x} = t + 7$

$\Rightarrow \quad x = (t+7)^2$

$\Rightarrow \quad v = \dfrac{dx}{dt} = 2(t+7)$

Thus, Velocity is proportional to time.

Also as $a = \dfrac{dv}{dt} = 2$

Thus particle moves with a constant acceleration.

27. A particle moves in a straight line and its position x at time t is given by $x^2 = 2 + t$. Its acceleration is given by

(a) $\dfrac{-2}{x^3}$ (b) $-\dfrac{1}{4x^3}$ (c) $-\dfrac{1}{4x^2}$ (d) $\dfrac{1}{x^2}$

Given $x^2 = 2 + t$ Now differentiating this equation with respect to time we get ,

$2x\dfrac{dx}{dt} = 1$

$v = \dfrac{dx}{dt} = \dfrac{1}{2x}$

Now again differentiating equation A with respect to dx we get $v\dfrac{dv}{dx}.\dfrac{dx}{dt} = \dfrac{1}{2x}dx$

$v\dfrac{dv}{dt} = -\dfrac{1}{2x^2}$

$\dfrac{dv}{dt} = -\dfrac{1}{2x^2}\left(\dfrac{1}{2x}\right) = -\dfrac{1}{4x^3}$

CONSTANT ACCELERATION AND FREE FALL

28. A, B, C and D are points in a vertical line such that AB = BC = CD. If a body falls from rest at A, then the times of descent through AB, BC and CD are in the ratio :

(a) $1 : \sqrt{2} : \sqrt{3}$ (b) $\sqrt{2} : \sqrt{3} : 1$

(c) $\sqrt{3} : 1 : \sqrt{2}$ (d) $1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$

for AB

$d = \frac{1}{2} g t_1^2$

for AC

$2d = \frac{1}{2} g (t_1 + t_2)^2$

for AD

$3d = \frac{1}{2} g (t_1 + t_2 + t_3)^2$

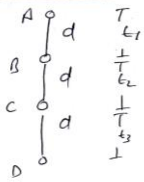
$\Rightarrow t_1 + t_2 + t_3 = \sqrt{\frac{6d}{g}}$

$t_1 + t_2 = \sqrt{\frac{4d}{g}}$

$t_1 = \sqrt{\frac{2d}{g}}$ — (i)

$t_2 = \sqrt{\frac{4d}{g}} - \sqrt{\frac{2d}{g}} = (\sqrt{2} - 1)\sqrt{\frac{2g}{d}}$ — (ii)

$t_3 = \sqrt{\frac{6d}{g}} - \sqrt{\frac{4d}{g}} = \sqrt{\frac{2d}{g}}(\sqrt{3} - \sqrt{2})$ — (iii)



taking ratio of (i) & (ii) & (iii)

$t_1 : t_2 : t_3 = 1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$

29. A ball is thrown upward with speed 10 m/s from the top of the tower reaches the ground with a speed 20 m/s. The height of the tower is [Take $g = 10$ m/s^2]

(a) 10 m (b) 15 m (c) 20 m (d) 25 m

Velocity at equal height remains equal in magnitude and hence

$v^2 = u^2 + 2as$

$20^2 = 10^2 + 2 \times 10 \times h$

$\therefore h = \frac{400 - 100}{2 \times 10}$

$= \frac{300}{20} = 15m$

30. A body thrown vertically up with initial velocity 52 m/s from the ground passes twice a point at h height above at an interval of 10 s. The height h is (g = 10 m/s^2)

(a) 22 m (b) 10.2 m (c) 11.2 m (d) 15 m

Initial velocity, V = 52m/s

Time taken, t = 10 sec

$\Rightarrow S = ut + \frac{1}{2}at^2$

$= 52(10) - \frac{1}{2}(10)(100)$

$= 20m$

As, S = 2h

$\Rightarrow h = 10m$

Hence, the answer is 10m.

31. When a particle is thrown vertically upwards, its velocity at one third of its maximum height is $10\sqrt{2}$ m/s. The maximum height attained by it is

(a) 20 2 m (b) 30 m (c) 15 m (d) 12.8 m

Solution:

Given that, $v = 10\sqrt{2}$

Let h be the maximum height reached

The velocity of the particle when it was reached its maximum height is zero.

Distance travelled is

$h - \frac{h}{3} = \frac{2h}{3}$.

Using equation of motion

$v^2 = u^2 + 2gh$

$(10\sqrt{2})^2 = 0 + 2 \times 9.8 \times \frac{2h}{3}$

$h = \frac{3 \times (10\sqrt{2})^2}{4 \times 9.8}$

$h = 15.3$

$h \approx 15 m$

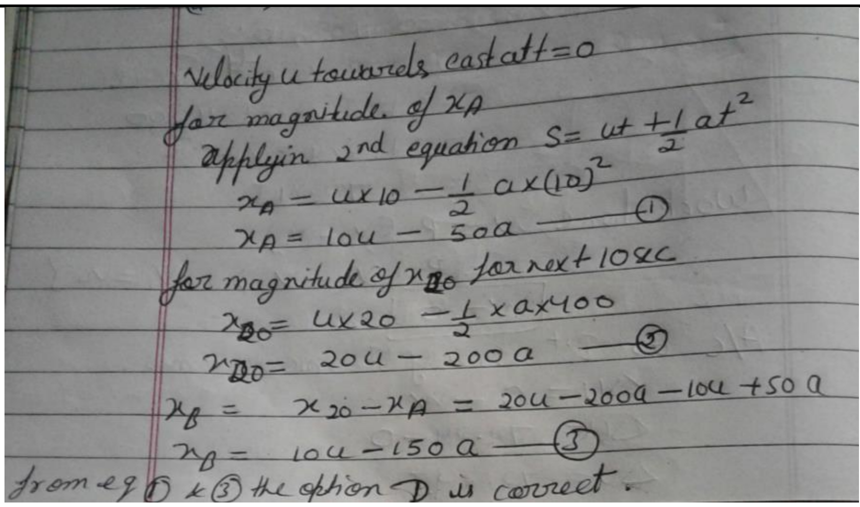
32. A particle has a velocity u towards east at t = 0. Its acceleration is towards west and is constant. Let x_A and x_B be the magnitude of displacement in the first 10 seconds and the next 10 seconds :

(a) $x_A < x_B$

(b) $x_A = x_B$

(c) $x_A > x_B$

(d) the information is insufficient to decide the relation of x_A and x_B.


Velocity u towards east at t = 0
for magnitude of x_A
Applying 2nd equation $S = ut + \frac{1}{2}at^2$
$x_A = u \times 10 - \frac{1}{2} a \times (10)^2$
$x_A = 10u - 50a$ ——— (1)
for magnitude of x_{20} for next 10 sec
$x_{20} = u \times 20 - \frac{1}{2} \times a \times 400$
$x_{20} = 20u - 200a$ ——— (2)
$x_B = x_{20} - x_A = 20u - 200a - 10u + 50a$
$x_B = 10u - 150a$ ——— (3)
from eq (1) & (3) the option D is correct.

33. A balloon rises from rest with a constant acceleration g/8. A stone is released from it when it has risen to a height h. The time taken by the stone to reach the ground is

(a) $\sqrt{\frac{h}{g}}$ (b) $\sqrt{\frac{2h}{g}}$ (c) $2\sqrt{\frac{h}{g}}$ (d) $4\sqrt{\frac{h}{g}}$

Upward velocity of stone at height H

$V^2 = \mu^2 + 2as \Rightarrow v^2 = 0 + 2\frac{g}{8} \times H = \frac{H}{4} v = \frac{\sqrt{Hg}}{2}$

Also,

$\vec{S} = \vec{\mu}t + \frac{1}{2}at^2$

$H = \frac{\sqrt{Hg}}{2}t + \frac{1}{2}gt^2$

$gt^2 - \sqrt{Hg} - 2H = 0$

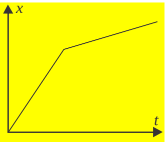
$t^2 - \sqrt{\frac{H}{g}} - \frac{2H}{g} = 0$

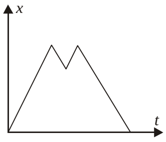
$t = \frac{\sqrt{\frac{H}{g}} \pm \sqrt{\frac{H}{g} + \frac{8H}{g}}}{2} = \frac{\sqrt{\frac{H}{g}} \pm 3\sqrt{\frac{H}{g}}}{2}$

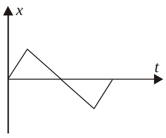
$t = 2\sqrt{\frac{H}{g}}$

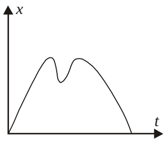
GRAPHICAL ANALYSIS

34. Which of the following distance time graphs is possible :

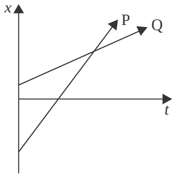
(a)


(b)


(c)


(d)


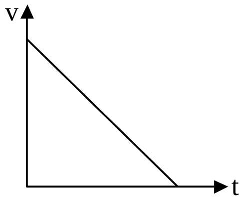
35. Figure shows the time-displacement curve of the particles P and Q. Which of the following statement is correct?

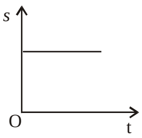


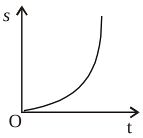
(a) Both P and Q move with uniform equal speed
(b) P is accelerated Q is retarded
(c) Both P and Q move with uniform speeds but the speed of P is more than the speed of Q
(d) Both P and Q move with uniform speeds but the speed of Q is more than the speed of P.

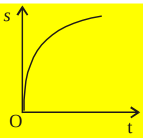
As the x−t graph is a straight line, in either case, the velocity of both the particles is uniform. As the slope of the x−t graph for P is greater, therefore, the velocity of P is greater than that of Q. (Slope of x−t graph represents velocity)

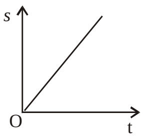
36. A ball is thrown vertically upwards and its velocity v varies with time t as shown in the figure. Which of the graphs (a), (b), (c), (d) shows the correct curve of displacement versus time?



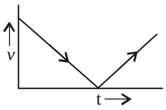
(a) 

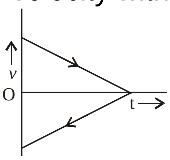
(b) 

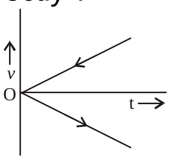
(c) 

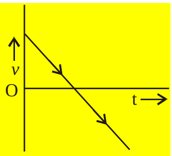
(d) 

37. A body is thrown vertically upwards with a velocity u. Which of the following graphs represents the variation of velocity with time correctly ?

(a) 

(b) 

(c) 

(d) 

Velocity of a body thrown vertically upwards can be given as:

$v_f = v_i + at$

where $a = -g$ (always acts in downward direction)

$\therefore v_f = v_i - gt$

for $t < \frac{v_i}{g}$, v_f is positive

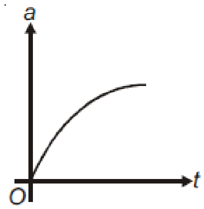
at $t = \frac{v_i}{g}$, we get $v_f = 0$

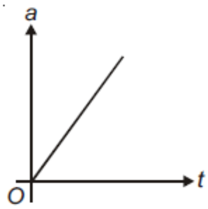
for $t > \frac{v_i}{g}$, v_f is negative.

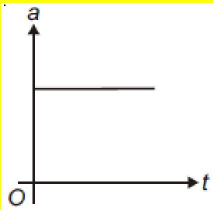
also the slope of v vs t graph is −g [Negative]

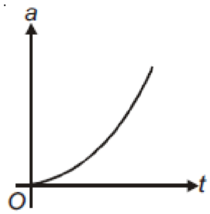
From the above explanations, we can conclude that the velocity starts from a positive value, starts deceasing, reaches zero and keeps on decreasing for be a negative value.

38. The velocity v of a particle moving along x-axis varies with its position (x) as $v = \alpha\sqrt{x}$; where α is a constant. Which of the following graph represents the variation of its acceleration (a) with time (t)?

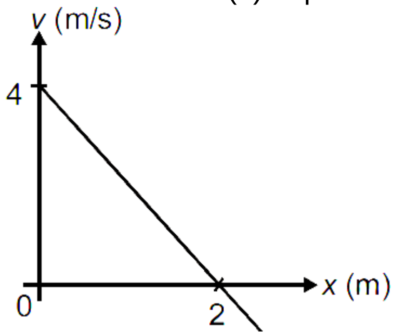
(a) 

(b) 

(c) 

(d) 

39. The velocity (v) of a particle moving along x-axis varies with its position x as shown in figure. The acceleration (a) of particle varies with position (x) as



(a) $a^2 = x + 3$ (b) $a = 2x^2 + 4$ (c) $2a = 3x + 5$ (d) $a = 4x - 8$

As shown in the given figure

equation of velocity and displacement is

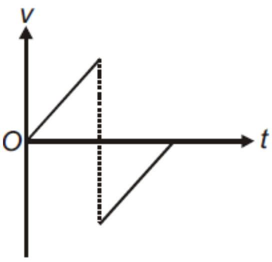
$V = -2x + 4$

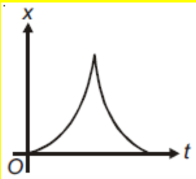
acceleration $a = V\dfrac{dv}{dx}$

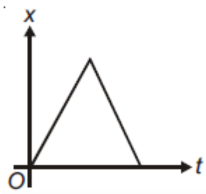
$= (-2x + 4)(-2)$

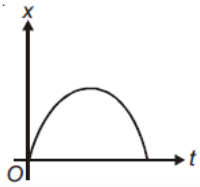
$= 4x - 8$

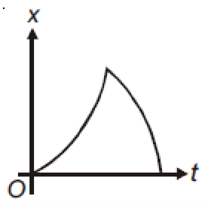
40. The velocity (v)-time (t) graph for a particle moving along x-axis is shown in the figure. The corresponding position (x)- time (t) is best represented by

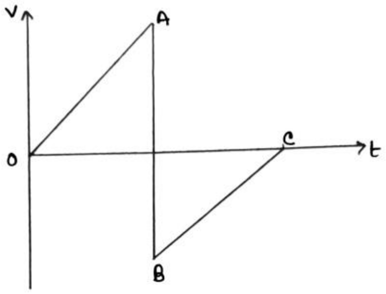


(a) 

(b) 

(c) 

(d) 



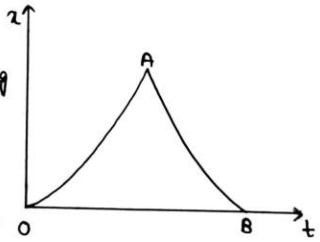
As we are looking at the figure,

velocity, $v = \frac{dx}{dt}$, i.e, slope of $(x-t)$ graph

From slope, OA is positive and increasing

BC is negative and increasing

For that option (A):

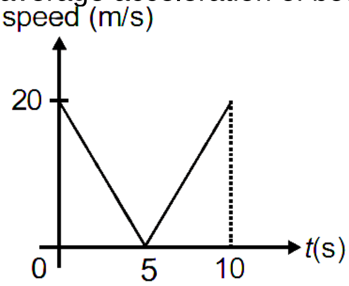


OA slope, it is increasing and positive

AB slope, it is negative and at every point, slope is increasing.

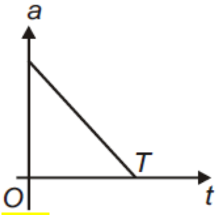
therefore, the corresponding position (x) – time (t) is represented by option (A).

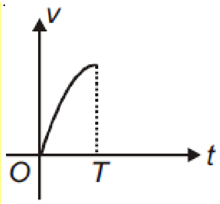
41. The speed-time graph for a body moving along a straight line is shown in figure. The average acceleration of body may be



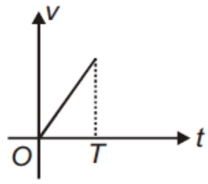
(a) 0 (b) 4 m/s^2 (c) -4 m/s^2 (d) All of these

42. The acceleration (a)-time (t) graph for a particle moving along a straight starting from rest is shown in figure. Which of the following graph is the best representation of variation of its velocity (v) with time (t)?

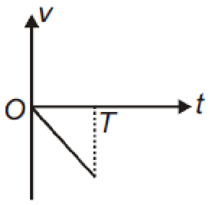


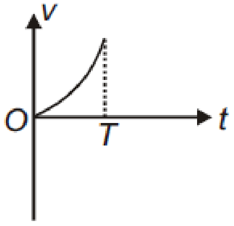


(a)

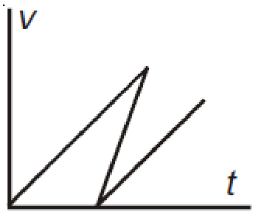


(b)

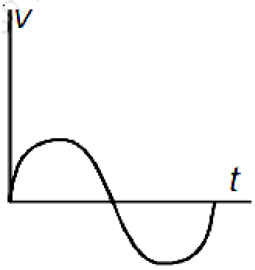
(c) 

(d) 

43. Which of the following speed-time (v - t) graphs is physically not possible?

(a) 

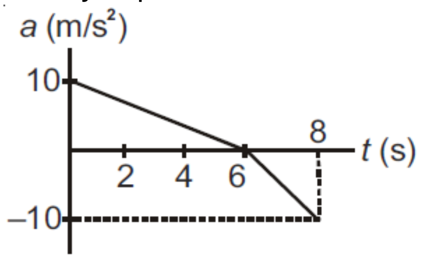
(b) 

(c) 

(d) All of these

(4)
Is also wrong because speed can't be negative
At a single instance of time these can't be two values of velocity
$\therefore$ (1) and (2) are false

44. The acceleration-time graph for a particle moving along x-axis is shown in figure. If the initial velocity of particle is –5 m/s, the velocity at t = 8 s is



(a) + 15 m/s (b) +20 m/s (c) -15 m/s (d) -20 m/s

Given :

$u = -5$ m/s

We know,

$v = u + at$

for $t = 0$ to $t = 6$s

$v = -5 + 0(6)$

$v = -5$ m/s

for $t = 6$s to $t = 8$ sec

$u = -5$ m/s

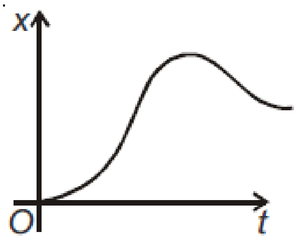
$t = 2$ s

$a = -10$ m/s^2

$\therefore v = -5 + (10) \times 2$

$v = 15$ m/s

45. The displacement (x) - time (t) graph of a particle is shown in figure. Which of the following is correct?



(a) Particle starts with zero velocity and variable acceleration
(b) Particle starts with non-zero velocity and variable acceleration
(c) Particle starts with zero velocity and uniform acceleration
(d) Particle starts with non-zero velocity and uniform acceleration

From the given graph, it is clear that

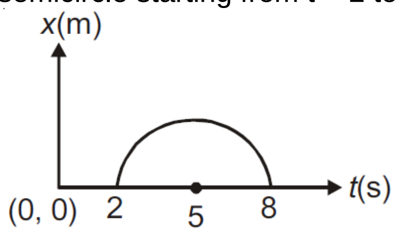
At $t = 0, \dfrac{dx}{dt} = 0$

$\therefore$ Velocity $v = \dfrac{dx}{dt} = 0$, at t = 0

As time passes $\dfrac{dx}{dt}$ increases

$\therefore$ Velocity and hence acceleration changes.

46. Position time graph of a particle moving along straight line is shown which is in the form of semicircle starting from t = 2 to t = 8 s. Select correct statement



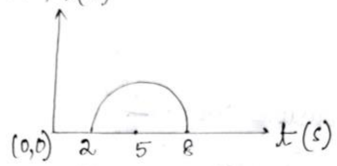
(a) Velocity of particle between t = 0 to t = 2 s is positive

(b) Velocity of particle is opposite to acceleration between $t = 2$ to $t = 5$ s

(c) Velocity of particle is opposite to acceleration between $t = 5$ to $t = 8$ s

(d) Acceleration of particle is positive between $t_1 = 2$ s to $t_2 = 5$ s while it is negative between $t_1 = 5$ s to $t_2 = 8$ s

Given, x(m)

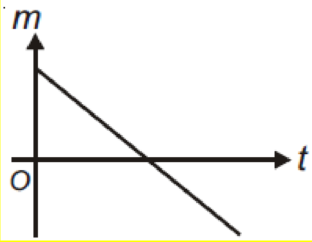


We know, in a position time graph of a particle, the slope of the graph gives the velocity of the particle at an instant.

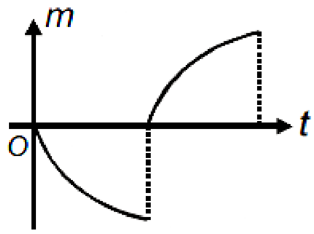
In the time interval $t = 2s$ to $t = 5s$, velocity of the particle decreases continuously and become 0 at the instant of $t = 5s$.

Hence velocity should be opposite to acceleration in between $t = 2s$ to $t = 5s$

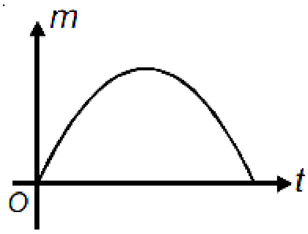
47. A particle is projected at angle θ with horizontal from ground. The slop (m) of the trajectory of the particle varies with time (t) as



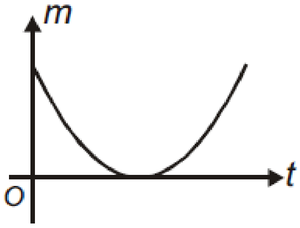
(a)



(b)



(c)



(d)

Equation of Trajectory:

$$y = x\tan\theta - \frac{g(x)^2}{2u^2\cos^2\theta}$$

$$\frac{dy}{dx} = \left[\frac{d}{dx}(x)\right]\tan\theta - \frac{g}{2u^2\cos^2\theta}\left[\frac{d}{dx}(x)^2\right]$$

$$m = \tan\theta - \frac{g}{u^2\cos^2\theta}\cdot(x)$$

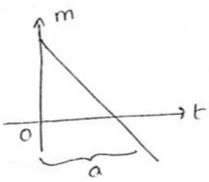
[Range $= x = (u\cos\theta)t$ $\Rightarrow t = (x/u\cos\theta)$]

$$m = \tan\theta - \frac{g}{(u\cos\theta)}\cdot(t)$$

let, $a = \tan\theta$, $b = g/u\cos\theta$

So, $(m = a - bt)$. Hence, option(A)

straight line with (−) negative slope and an intercept.



48. If H_1 and H_2 be the greatest heights of a projectile in two paths for a given value of range, then the horizontal range of projectile is given by

(a) $\frac{H_1+H_2}{2}$ (b) $\frac{H_1+H_2}{4}$ (c) $4\sqrt{H_1H_2}$ (d) $4[H_1+H_2]$

49. A particle projected from ground moves at angle 45º with horizontal one second after projection and speed is minimum two seconds after the projection. The angle of projection of particle is [Neglect the effect of air resistance]

(a) $\tan^{-1}(3)$ (b) $\tan^{-1}(2)$ (c) $\tan^{-1}(\sqrt{2})$ (d) $\tan^{-1}(4)$

50. A particle is projected with speed u at angle θ with horizontal from ground. If it is at same height from ground at time t_1 and t_2, then its average velocity in time interval t_1 to t_2 is

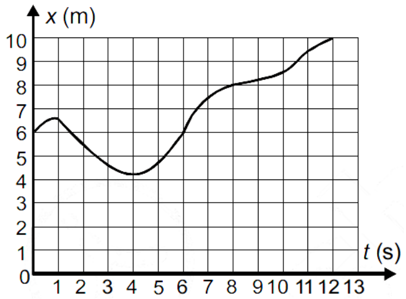
(a) Zero (b) $u\sin\theta$ (c) $u\cos\theta$ (d) $\frac{1}{2}[u\cos\theta]$

$$\text{Average velocity} = \frac{\text{total displacement in 'x'}}{\text{total time}}$$

$$= \frac{u\cos\theta(t_2-t_1)}{(t_2-t_1)}$$

$$= u\cos\theta$$

51. Position-time graph for a particle is shown in figure. Starting from t = 0, at what time t, the average velocity is zero?

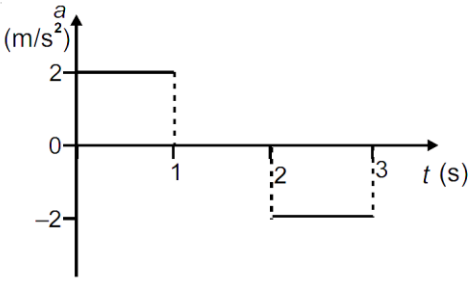


(a) 1 s (b) 3 s (c) 6 s (d) 7 s

$$\text{Average velocity, } V = \frac{\text{total displacement}}{\text{total time}}$$

From the graph, at time 6 sec displacement is zero.
Hence, at 6 sec, average velocity is zero.

52. Acceleration-time graph for a particle is given in figure. If it starts motion at t = 0, distance travelled in 3 s will be



(a) 4 m (b) 2 m (c) 0 (d) 6 m

1st second

distance traveled= $ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times 2 \times 1 \times 1 = 1m$

$v = u + at,\ v = 0 + 2 \times 1 = 2\,m/s$

2nd second

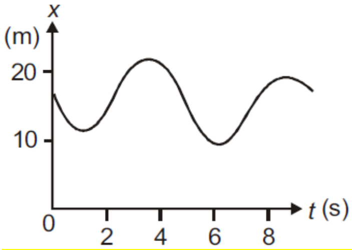
$s = ut = 2 \times 1 = 2\,m$

3rd second

$s = ut + \frac{1}{2}at^2 = 2 \times 1 \times -\frac{1}{2} \times 2 \times 1 \times 1 = 2 - 1 = 1\,m$

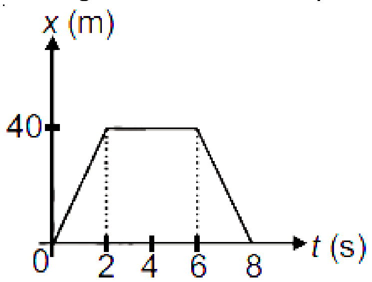
Total distance= 4 m

53. Figure shows the position of a particle moving on the x-axis as a function of time



(a) The particle has come to rest 4 times
(b) The velocity at t = 8 s is negative
(c) The velocity remains positive for t = 2 s to t = 6 s
(d) The particle moves with a constant velocity

54. The position (x) of a particle moving along x-axis varies with time (t) as shown in figure. The average acceleration of particle in time interval t = 0 to t = 8 s is



(a) 3 m/s^2 (b) -5 m/s^2 (c) -4 m/s^2 (d) 2.5 m/s^2

average acceleration $a = \frac{v_2 - v_1}{t}$ eq(1)..

intial velocity is the slop of the graph in between the 0 to 2sec

$v_1 = \frac{40}{2} = 20ms^{-1}$

final velocity os the slop of the graph in between the 6-8sec

$v_2 = -\frac{40}{2} = -20ms^{-1}$

total time is 8sec.

put v_{1}and v_{2} in eq(1)

$a = \frac{-20 - 20}{8} = \frac{-40}{8} = -5ms^{-2}$

PROJECTILE MOTION

55. The distance x and y along the horizontal plane of a projectile is given by x = 6t and y = 8t–5t^2 in metres where t is in seconds. The angle with the horizontal at which the projectile is projected is

(a) $\tan^{-1}\left(\frac{3}{4}\right)$ (b) $\tan^{-1}\left(\frac{4}{3}\right)$ (c) $\sin^{-1}\left(\frac{3}{4}\right)$ (d) $\cos^{-1}\left(\frac{3}{4}\right)$

56. Three particles, A, B, C are projected from the same point with same initial speeds making angles 30º, 45º and 60º respectively with the horizontal. Which of the following statement is correct?

(a) A, B and C have unequal ranges
(b) Ranges of A and C are equal and less than that of B
(c) Ranges of A and C are equal and greater than that of B
(d) A, B and C have equal ranges.

If two bodies thrown at same speed at angles such that their sum $= \pi/2$ then both of them will have the same range, so A and C will have same range. Also the range is maximum when angle of projection is 45^o.

57. A projectile is projected at an angle with an initial velocity u. The time t, at which its horizontal velocity will equal the vertical velocity for the first time

(a) $t = \frac{u}{g}(\cos\alpha - \sin\alpha)$ (b) $t = \frac{u}{g}(\cos\alpha + \sin\alpha)$

(c) $t = \frac{u}{g}(\sin\alpha - \cos\alpha)$ (d) $t = \frac{u}{g}(\sin^2\alpha - \cos^2\alpha)$

at time 't'

$V_y = V_x = u_x = u\cos\alpha$

$\Rightarrow u\cos\alpha = u\sin\alpha - gt$

$gt = u(\sin\alpha - \cos\alpha)$

$t = \frac{u(\sin\alpha - \cos\alpha)}{g}$

58. What is the path followed by a moving body, on which a constant force acts in a direction other than initial velocity (i.e. excluding parallel and antiparallel direction)?

(a) Straight line (b) Parabolic (c) Circular (d) Elliptical

This is the same as a projectile.
$x=u_2t+a_2t^2 \cdot y=u_yt+a_yt^2$
This gives rise to a parabolic trajectory.

59. Two stones are thrown with same speed u at different angles from ground in air. If both stones have same range and height attained by them are h_1 and h_2, then $h_1 + h_2$ is equal to

(a) $\frac{u^2}{g}$ (b) $\frac{u^2}{2g}$ (c) $\frac{u^2}{3g}$ (d) $\frac{u^2}{4g}$

Range will be same only when they are at an angle

So H_{max} for 1st particle

$= \frac{u^2\sin^2\theta}{2g}$

$= \frac{u^2\sin^2\theta}{2g}$

$H_2 = \frac{u^2\sin^2(90-\alpha)}{2g}$

$= u^2[\sin(90-\alpha)]^2$

$= \frac{u^2\cos^2\alpha}{2g}$

Now $H_1 + H_2 = \frac{u^2}{2g}(\sin^2\alpha + \cos^2\alpha)$

$= \frac{u^2}{2g}.$

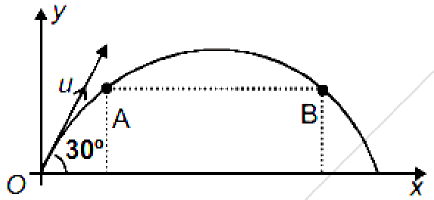
60. A projectile is projected with speed *u* at an angle θ with the horizontal. The average velocity of the projectile between the instants it crosses the same level is
(a) $u\cos\theta$ (b) $u\sin\theta$ (c) $u\cot\theta$ (d) $u\tan\theta$

displacement $= x = u\cos\theta \times t$

time $= t$

$\therefore v_{avg} = \frac{x}{t} = \frac{u\cos\theta \times t}{t} = u\cos\theta$

61. A particle is thrown with a velocity of u m/s. It passes A and B as shown in figure at time $t_1 = 1$ s and $t_2 = 3$ s. The value of u is ($g = 10$ m/s^2)



(a) 20 m/s (b) 10 m/s (c) 40 m/s (d) 5 m/s

62. Which one of the following statements is not true about the motion of a projectile?
(a) The time of flight of a projectile is proportional to the speed with which it is projected at a given angle of projection
(b) The horizontal range of a projectile is proportional to the square root of the speed with which it is projected
(c) For a given speed of projection, the angle of projection for maximum range is 45°
(d) At maximum height, the acceleration due to gravity is perpendicular to the velocity of the projectile

63. A projectile is thrown with speed 40 ms^{-1} at angle θ from horizontal. It is found that projectile is at same height at 1 s and 3 s. What is the angle of projection?
(a) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (b) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (c) $\tan^{-1}\left(\sqrt{3}\right)$ (d) $\tan^{-1}\left(\sqrt{2}\right)$

64. Bullets are fired with velocity u at a fixed angle to the horizontal in all the directions. The maximum area covered by the bullets is
(a) $\frac{\pi u^2}{g}$ (b) $\frac{\pi^2 u^2}{g}$ (c) $\frac{\pi^2 u^4}{g}$ (d) $\frac{\pi u^4}{g^2}$

65. A projectile is projected with kinetic energy K. If it has the maximum possible horizontal range, then its kinetic energy at the highest point will be
(a) 0.25 K (b) 0.5 K (c) 0.75 K (d) K

66. A stone projected from ground with certain speed at an angle θ with horizontal attains maximum height h_1. When it is projected with same speed at an angle θ with vertical attains height h_2. The horizontal range of projectile is

(a) $\frac{h_1+h_2}{2}$ (b) $2h_1h_2$ (c) $4\sqrt{h_1h_2}$ (d) h_1+h_2

So, $H1 = \frac{u^2\sin^2\theta}{2g}$ ---- (1)

Also, $H2 = \frac{u^2\sin^2(90-\theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$ ---- (2)

Range in case 1 :

$R_1 = \frac{u^2\sin 2\theta}{g}$

Range in case 2 :

$R_2 = \frac{u^2\sin 2(90-2\theta)}{g}$

$= \frac{u^2\sin 2\theta}{g}$

Here, $R_1 = R_2 = R$ (let)

From (1) & (2),

$H1 \cdot H2 = \frac{u^2\sin^2\theta \cdot \cos^2\theta}{4g^2}$

$H1 \cdot H2 = \frac{u^4(2\sin\theta \cdot \cos\theta)^2}{16g^2}$

Taking square root both sides,

$\sqrt{H1 \cdot H2} = \frac{u^2\sin 2\theta}{4g}$

$\sqrt{H_1 \cdot H_2} = \frac{R}{4}$

$R = 4\sqrt{H_1 \cdot H_2}$

67. From the top of a tower of height 40 m a ball is projected upwards with a speed of 20 m/sec at an angle of elevation of 30º. Then the ratio of the total time taken by the ball to hit the ground to its time of flight (time taken to come back to the same elevation) is (take g = 10 m/sec^2)

(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 4 : 1

Time of flight,

$$T = \frac{2v\sin\theta}{g} = \frac{2\times 20\times\sin 30^\circ}{10} = 2\text{sec}$$

Total time taken by the ball to hit the ground is

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{40}{20\sin 30^\circ} = \frac{40}{20\times\frac{1}{2}} = 4\text{sec}$$

So, the ratio is 4 : 2 = 2 : 1

68. Two seconds after projection, a projectile is moving at 30º above the horizontal; after one more second it is moving horizontally. The initial speed of the projectile is ($g = 10\ m/s^2$)

(a) $10\sqrt{3}$ m/s (b) 20 m/s (c) 10 m/s (d) $20\sqrt{3}$ m/s

- Let in 2s the body reaches upto point A and after 1s more, to point B.

$t = \frac{u\sin\theta}{g} = 3s \Rightarrow u\sin\theta = 30$ ——(1)

Horizontal component of velocity remains always constant, $u\cos\theta = v\cos 30°$

for vertical upward motion between O and A,

$v\sin 30° = u\sin\theta - g\times 2 = 30 - 20$

$v = 20\ m/s$

$\therefore u\cos\theta = v\cos 30° = 20\cos 30°$

$\frac{u\sin\theta}{u\cos\theta} = \tan\theta = \frac{30}{10\sqrt{3}} = \sqrt{3}$ $\theta = 60°$

$u = 20\sqrt{3}\ m/s$

RELATIVE MOTION IN ONE DIMENSION

69. To a man walking at the rate of 3 km/h the rain appears to fall vertically. When he increases his speed to 6 km/h it appears to meet him at an angle of 45º with vertical from the front. The actual speed of rain is

(a) 3 km/hr (b) 4 km/hr (c) $3\sqrt{2}$ km/h (d) $2\sqrt{3}$ km/h

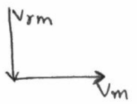
Let the velocity of man be v_m; velocity of rain = v_r

If $\bar{V}_r = a\hat{i} + b\hat{j}$

When man walks at $V_m = 3\ km/hr$, rains appears to fall vertically. Let velocity of rain w.r.t man be: V_{rm}

$\bar{V}_{rm} = \bar{V}_r - \bar{V}_m$

$\bar{V}_{rm} = (a-3)\hat{i} + b\hat{j}$



Since, rain appears to fall vertically,

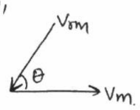
$a - 3 = 0$

$a = 3$.

Also, if man walks at $\bar{V}_m = 6\hat{i}\ km/h$, then,

$\bar{V}_{rm} = (a-6)\hat{i} + b\hat{j}$

$\bar{V}_{rm} = 3\hat{i} + b\hat{j}$



rain appears to fall at angle $\theta = 45°$. So,

$\tan\theta = \frac{b}{3} = \tan 45°$

or, $\frac{b}{3} = 1 \Rightarrow b = 3$.

Hence, velocity of rain is : $\bar{V}_r = 3\hat{i} + 3\hat{j}$

So, its speed $= |V| = \sqrt{3^2 + 3^2}$

Thus, we get $|V| = 3\sqrt{2}\ km/h$

70. Two cars are moving in the same direction with a speed of 30 km/h. These are separated from each other by 5 km. Another third car moving in the opposite direction meets the two cars after an interval of 4 minutes. The speed of the third car is

(a) 45 km/h. (b) 40 km/h (c) 35 km/h (d) 30 km/h.

Let the speed of third car be x km/hr

t = 4min = 1/15hr

D = 5km

Relative speed = 30 + x = D/t = 75

Thus x = 75 – 30 = 45 km/hr

71. A passenger sitting by the window of a train moving with a velocity of 72 km/h observes for 10 seconds that a train moving with a velocity of 32.4 km/h completely passes by it in 10 seconds. The length of the second train is
(a) 110 m (b) 145 m (c) 210 m (d) 290 m

72. A body is thrown up in a lift with a velocity u relative to a lift and its time of flight is found to be t. The acceleration with which the lift is moving up will be.
(a) $\frac{u-gt}{2}$ (b) $\frac{2u-gt}{t}$ (c) $\frac{u^2-gt}{t}$ (d) $\frac{u^2}{gt}$

73. A parachutist drops freely from an aeroplane for 10 s before the parachute opens out. Then he descends with a net retardation of 2.5 m/s^2. If he falls out of the plane at a height of 2495 m and g = 10 m/s^2, hit velocity on reaching the ground will be :
(a) 5 m/s (b) 10 m/s (c) 15 m/s (d) 20 m/s

Initial velocity (u) = 0

From t = 0 to t = 10 free fall

V = 0 + (10)(10)

V = 100m.s

Distance covered= $+\frac{1}{2} \times 10 \times 10^2$

= 500m

After this retardation of a = 2.5m/s^2

Now from here tall it reaches ground-

Distance covered = total distance–the distance covered in first 10sec

= (2495 – 500)m

= 1995m

Now, By 3rd equation of motion-

$v^2 = u^2 + 2as$

$v^2 = (100)^2 + 2(-2.5)(1995)$

= 10000 – 9975

= 25

⇒ v = 5m/s

74. A ball is dropped from an elevator moving upward with acceleration '*a*' by a boy standing in it. The acceleration of ball with respect to [Take upward direction positive]
(a) Boy is – g (b) Boy is – (g + a) (c) Ground is – g (d) Both (b) & (c)

75. A stone is released from an elevator going up with acceleration 5 m/s^2. The acceleration of the stone after the release is :
(a) 5 ms^{-2} (b) 4.8 ms^{-2} upward (c) 4.8 down ward (d) 9.8 ms^{-2} down ward.

76. A ball 'A' is thrown up vertically with speed u. At the same instant another ball 'B' is released from rest from a height h. At time t, the velocity of A relative to B is :
(a) u (b) u – 2gt (c) $\sqrt{u^2 - 2gh}$ (d) u – gt

velocity of ball A is at any t $v_A = u - gt$

velocity of ball B is released from height h at any time t $v_B = -gt$

speed of A relative to B is $v_{A/B} = v_A - v_B = u - gt - (-gt) = u$

77. Two cars *A* and *B* are moving in same direction with velocities 30 m/s and 20 m/s. When car *A* is at a distance *d* behind the car *B*, the driver of the car *A* applies brakes producing uniform retardation of 2 m/s^2. There will be no collision when

(a) $d < 2.5$ m (b) $d > 125$ m (c) $d > 25$ m (d) $d < 125$ m

78. A man can swim at a speed of 5 km/h w.r.t. water. He wants to cross a 1.5 km wide river flowing at 3 km/h. He keeps himself always at an angle of 60º with the flow direction while swimming. The time taken by him to cross the river will be :

(a) 0.25 hr. (b) 0.35 hr. (c) 0.45 hr. (d) 0.55 hr.

The man has to travel 1.5km perpendicular to the flow to cross the river.And the velocity of the man is at 60 degree with flow, so component of man's velocity perpendicular to flow is $v\sin 60 = 5 \times \frac{\sqrt{3}}{2}$

time taken will be $\frac{\text{distance}}{\text{speed}} = \frac{1.5}{5 \times \frac{\sqrt{3}}{2}} = 0.35\text{hr.}$