IIT – JEE: 2025 **TW TEST (MAIN)** **DATE: 22/10/23**

TOPIC: CIRCULAR MOTION & WPE

SOLUTIONS

1. (A)

Here $\vec{F}=\left(4\hat{i}+\hat{j}+3\hat{k}\right)N$

Displacement of the particle, $\vec{S}=\left(\vec{r}_2-\vec{r}_1\right)$

$$=\left(14\hat{i}+13\hat{j}-9\hat{k}\right)-\left(3\hat{i}+2\hat{j}-6\hat{k}\right)$$

$$=\left(11\hat{i}+11\hat{j}-3\hat{k}\right)m$$

$\therefore$ Work done.

$W=\vec{F}.\vec{S}=\left(4\hat{i}+\hat{j}+3\hat{k}\right).\left(11\hat{i}+11\hat{j}-3\hat{k}\right)$

$=44+11-9=46J$

$\left[\because \hat{i}.\hat{i}=\hat{j}.\hat{j}=\hat{k}.\hat{k};\hat{i}.\hat{j}=\hat{j}.\hat{k}=\hat{k}.\hat{i}=0\right]$

2. (A)

Work done by a force = area under $F-x$ graph

Work done in the internal $0\le x\le 2m$

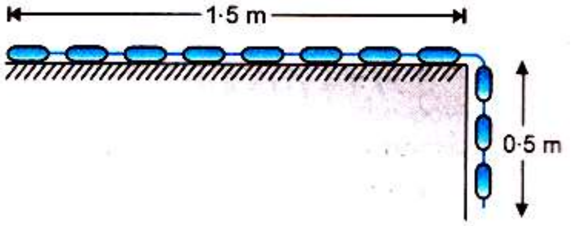
$=$ Area of triangle $OAB=\frac{1}{2}\times 2\times 2=2J$

3. (B)

Mass of chain, $m=100g=0.1kg$

$\therefore$ Mass per unit length of the chain $=\frac{m}{l}$

$=\frac{0.1}{2}=\frac{1}{20}kg\,m^{-1}$



$\therefore$ Mass of the hanging portion of the chain

$m=\frac{1}{20}\times 0.5=\frac{1}{40}kg$

Weight of hanging part of chain $=mg$

$=\frac{1}{40}\times 10=\frac{1}{4}N$

This weight acts at the centre of gravity of the hanging part of chain i.e. at $h=\frac{0.5}{2}m$

$\therefore$ P.E. of the hanging part of the chain w.r.t. the table

$$=-mgh=-\frac{1}{4}\times\frac{0.5}{2}=-\frac{1}{16}J$$

As the chain leaves the table, let its velocity be υ

$\therefore$ K.E. of the chain $=\frac{1}{2}m\upsilon^2$

$$=\frac{1}{2}\times 0.1\upsilon^2=\frac{\upsilon^2}{20}J$$

P.E. of the falling chain $=-mg\frac{l}{2}$

$$=-0.1\times 10\times\frac{2}{2}=-1J$$

According to the law of conservation of energy

$-\frac{1}{16}=\frac{\upsilon^2}{20}-1$

Or $\frac{\upsilon^2}{29}=1-\frac{1}{16}=\frac{15}{16}$

$\therefore \upsilon^2=\frac{15}{16}\times 20=\frac{75}{4}$

Or $\upsilon=\sqrt{\frac{75}{4}}=4.44ms^{-1}$

4. (D)

5. (B)

6. (C)

$\upsilon=\sqrt{2gh}$. Since h is same in both the cases, so

$\upsilon_1=\upsilon_2$

$a_1=g\sin\theta_1$ and $a_2=g\sin\theta_2$

Since $\theta_2>\theta_1$. Therefore, $a_2>a_1$

$t=\frac{\upsilon-u}{a}=\frac{\upsilon}{a}$ or $t\propto\frac{1}{a}$

$\therefore t_1>t_2$

7. (B)

$W=\int m\frac{d\upsilon}{dt}.dx$

$=\int m\upsilon d\upsilon \qquad \left(\because \frac{dx}{dt}=\upsilon\right)$

$\upsilon=ax^{3/2}=5x^{3/2}$

When $x=0,\upsilon=0$

When $x=2,\upsilon=10\sqrt{2}\ ms^{-1}$

$\therefore W=m\int_0^{10\sqrt{2}}\upsilon d\upsilon=0.5\left[\frac{v^2}{2}\right]_0^{10\sqrt{2}}$

$= \frac{0.5}{2} \times 100 \times = 50\text{J}$

8. (D)

$W = \int \vec{F}.\overrightarrow{dx} = \int A\left(y^2\hat{i} + 2x^2\hat{j}\right)\cdot\left(dx\hat{i} + dy\hat{j}\right)$

$= A\int\left(y^2dx + 2x^2dy\right)$

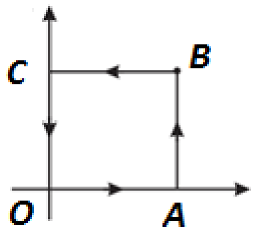
$W_{OA} = 0$

$W_{AB} = A\left[0 + 2d^3\right] = 2Ad^3$

$WBC = -Ad^3$

$W_{CD} = A(0+0) = 0$

$\therefore W = Ad^3$



9. (A)

Let v be the velocity with which the bullet will emerge. Now, change in kinetic energy = work done.

In the first case :

$\frac{1}{2}\text{m}(100)^2 - \frac{1}{2}\text{m}(0)^2 = \text{F} \times 1 \qquad \text{.....(i)}$

In the second case :

$\frac{1}{2}\text{m}(100)^2 - \frac{1}{2}\text{mv}^2 = \text{F} \times 0.5 \qquad \text{.......(ii)}$

Dividing eqn.(ii) by eqn.(i), we get;

$\frac{(100)^2 - \text{v}^2}{(100)^2} = \frac{0.5}{1} = \frac{1}{2}$

Or $1 - \left(\frac{\text{v}}{100}\right)^2 = \frac{1}{2}$ or $\left(\frac{\text{v}}{100}\right)^2 = \frac{1}{2}$

$\therefore \qquad \text{v} = \frac{100}{\sqrt{2}} = 50\sqrt{2}\ \text{m/s}$

10. (A)

Initial kinetic energy of the car = $\frac{1}{2}\text{mv}^2$

Work done against friction = μmgs

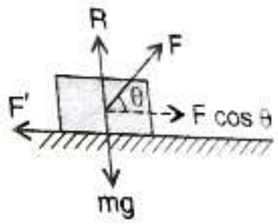
From conservation of energy

$\mu\text{mgs} = \frac{1}{2}\text{mv}^2$ or $\text{s} = \left(\text{v}^2 / 2\mu\text{g}\right)$

$\left[\text{Note: s} = \frac{\text{v}^2}{2\mu\text{g}} = \frac{\frac{1}{2}\text{mv}^2}{\mu\text{mg}} = \frac{\text{K}}{\text{F}}\right]$

11. (B)

Because the block moves with a uniform velocity, the resultant force is zero. Resolving F into horizontal component $\text{F}\cos\theta$ and vertical component $\text{F}\sin\theta$, we get .



$R + F\sin\theta = mg$

Or $\quad R = mg - F\sin\theta \qquad$ .....(i)

Also, $F' = \mu R = \mu(mg - F\sin\theta)$

But, $F\cos\theta = F'$

Or $\quad F\cos\theta = \mu(mg - F\sin\theta)$

Or $\quad F(\cos\theta + \mu\sin\theta) = \mu mg$

$\therefore \quad F = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta}$

Work, $W = Fs\cos\theta$

$\therefore \quad W = \dfrac{\mu mgd\cos\theta}{\cos\theta + \mu\sin\theta} \qquad (\because s = d)$

12. (C)

According to law of conservation of energy, elastic potential energy stored in spring = gravitational potential energy of shot

$\frac{1}{2}Kx^2 = mgh$

Or, $\quad h = \dfrac{Kx^2}{2mg}$

13. (A)

14. (B)

The magnitude of force is given by :

$|F| = \dfrac{dU}{dx}$

Thus, maximum force will occur at $x = x_2$

15. (D)

16. (B)

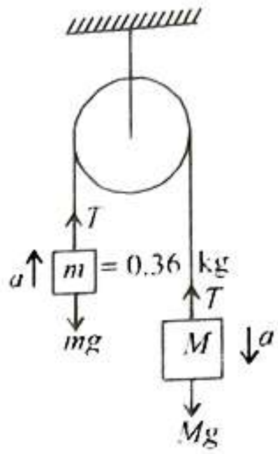
17. (A)

18. (D)

19. (B)

20. (D)

21. (8)



When the system is released,

$T = mg = ma$ ......(i)

$Mg - T = Ma$ .....(ii)

From eq. (i) & (ii)

$a = \dfrac{(M-m)g}{M+m} = g/3$

And T = 4 mg/3

For block m = 0.36 kg

$u = 0, a = g/3, \quad t = 1, s = ?$

$s = ut + \dfrac{1}{2}at^2 = 0 + \dfrac{1}{2} \times \dfrac{g}{3} \times 1^2 = g/6 \quad (m = 0.72\,kg)$

$\therefore$ Work done by the string on m

$T s \cos 0^\circ = 4\dfrac{mg}{3} \times \dfrac{g}{6} \times 1 = \dfrac{4 \times 0.36 \times 10 \times 10}{3 \times 6} = 8\,J$

22. (2)

Work done by A = Work done by B

$F_A d \cos 45^\circ = F_B d \cos 60^\circ$

$\Rightarrow \quad F_A \times \dfrac{1}{\sqrt{2}} = F_B \times \dfrac{1}{2} \quad \Rightarrow \quad \dfrac{F_A}{F_B} = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}} \quad \Rightarrow \quad x = 2$

23. (450)

Force, $F = (5y + 20)\hat{j} N$

Work done, $W = \int F.dy$

$\Rightarrow \quad W = \int_0^{10} (5y+20)dy = \left[\dfrac{5y^2}{2} + 20y\right]_0^{10}$

$= \dfrac{5}{2} \times 100 + 20 \times 10 = 450\,J$

24. (6.5)

We know area under F-x graph gives the work done by the body

$\therefore W = \dfrac{1}{2} \times (3+2) \times (3-2) + 2 \times 2 = 2.5 + 4 = 6.5J$

Using work energy theorem,

$\Delta K.E =$ work done

$\Delta K.E = 6.5J$

25. (6)

Here kinetic energy of ball is equal to P.E. stored in spring i.e., $\frac{1}{2}mv^2 = \frac{1}{2}k(\Delta x)^2$

$\Rightarrow \frac{1}{2} \times 4 \times (10)^2 = \frac{1}{2} \times 100 \times (\Delta x)^2 \Rightarrow \Delta x = 2m$

Therefore length of the compressed spring

$x = 8 - 2 = 6m$

26. (4)

$OP = 2R\cos 60^\circ = \frac{2R}{2} = R$

$OQ = 2R$

$\upsilon_{OP} = \sqrt{2a_{OP}(OP)} = \sqrt{2 \times \frac{g}{2} \times R} = \frac{1}{2}\sqrt{2gR} = \sqrt{gR}$

$\upsilon_{OQ} = \sqrt{2g \times 2R} = \sqrt{4gR} = 2\sqrt{gR}$

$\frac{V_{OP}}{V_{OQ}} = \frac{1}{2}$

27. (11)

Here centripetal force is provided by friction so

$\frac{mv^2}{r} \le \mu mg \Rightarrow v_{max} = \sqrt{\mu rg} = \sqrt{120} \approx 11 m/s$

28. (18)

In case of banking $\tan\theta = \frac{v^2}{rg}$. Here $v = 60\,km/h = 60 \times \frac{5}{18} m/s = \frac{50}{3} m/sr = 0.1km = 100m$

So $\tan\theta = \frac{50/3 \times 50/3}{100 \times 10} = \frac{5}{18} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{18}\right)$

29. (3)

$P = F\upsilon$

$P = m\upsilon\frac{dv}{dt}$

$\Rightarrow \upsilon \propto \sqrt{t}$

Also, $x \propto t^{3/2}$

30. (4)

$2mg = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{4g\ell} = 2\sqrt{g\ell}$

IIT – JEE: 2025 **TW TEST (MAIN)** **DATE: 22/10/23**

TOPIC: GASEOUS STATE

SOLUTIONS

31. (C)
Boyle's Law
Temperature and mass = constant
$PV = K$
$\log P = \log K - \log V$

32. (C)
PV = nRT

33. (A)
$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$
$$\frac{P \times V}{348} = \frac{(2P) \times (0.85\ V)}{T_2}$$
$T_2 = 592\ K = 319°C$

34. (A)
$P \propto n$, initial mole $n_1 = \frac{PV}{RT} = 0.1\,\text{mole}$
$$\frac{P_1}{P_2} = \frac{n_1}{n_2}$$
$$\frac{1}{0.75} = \frac{0.1}{n_2} \Rightarrow n_2 = 0.075$$
Escape mole = 0.025

35. (B)
The moles of the gas in the bubble remains constant, so that $n_1 = n_2$.
To calculate the final volume, V_2,
$$V_2 = V_1 \times \frac{P_1}{P_2} \times \frac{T_2}{T_1}$$
$$= 2.0\ \text{mL} \times \frac{6.0\ \text{atm}}{1.0\ \text{atm}} \times \frac{298\ \text{K}}{281\ \text{K}}$$
$$= 12.72\ \text{mL}$$

36. (B)
$$PV = \frac{W}{M}RT$$

37. (C)

$n_1 = n_2$

$\frac{1}{32} = \frac{2.375}{M_2}$

$M_2 = 76$

38. (A)

Moles of C_2H_6 and C_2H_4 are a and b respectively.

$a + b = \frac{28}{22.4} = 1.25$

$\underset{a}{C_2H_6}(g) + \underset{3.5a}{3.5O_2}(g) \longrightarrow 2CO_2(g) + 3H_2O(\ell)$

$\underset{b}{C_2H_4}(g) + \underset{3b}{3O_2}(g) \longrightarrow 2CO_2(g) + 2H_2O(\ell)$

Moles of O_2 = 3.5a + 3b = 4

$a + b = 1.25$

$a = 0.5$

$b = 0.75$

39. (B)

$P_{gas} = P_{dry\,gas} + P_{moisture}$ at T K

Or $P_{dry} = 830 - 30 = 800$

Now at $T_2 = 0.99T_1$;

At constant volume $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$p_{dry} = \frac{800 \times 0.99T}{T} = 792\,mm$

$\therefore P_{gas} = P_{dry} + P_{moisture}$

$= 792 + 25 = 817\,mm$

40. (B)

$KE_{avg} = \frac{3}{2}KT$

41. (C)

$P_{O_2} = \frac{3}{10} \times P_T$;

After removing 2 mole of O_2,

$P'_{O_2} = \frac{1}{8} \times P_T$

Decrease in partial pressure of O_2

$= \frac{\frac{3P_T}{10} - \frac{P_T}{8}}{\frac{3P_T}{10}} \times 100$

$= 58.33$

42. (B)

$$\frac{r_{O_2}}{r_{CH_4}} = \frac{n_{O_2}}{n_{CH_4}} \cdot \sqrt{\frac{M_{CH_4}}{M_{O_2}}}$$

$$= \frac{3}{2} \times \frac{16}{32} \times \sqrt{\frac{16}{32}} = \frac{3}{4\sqrt{2}}$$

43. (C)

$$P_{Real} = P_i - \frac{n^2 a}{V^2}$$

$a \uparrow P_r \downarrow$

$P_2 < P_3 < P_1$

44. (A)

$$P(V - nb) = nRT$$

$$PV - Pnb = nRT \qquad d = \frac{Mass}{Volume}$$

$$PV = nRT + Pnb \qquad n = \frac{Mass}{M}$$

$$P\frac{Mass}{d} = nRT + Pnb$$

$$\frac{P}{d} = \frac{n}{Mass}RT + \frac{Pnb}{Mass}$$

$$\frac{P}{d} = \frac{RT}{M} + \frac{Pb}{M}$$

45. (B)

If attraction are disappear than pressure of real gas $\uparrow$

46. (C)

At low pressure,

$$\left(p + \frac{a}{V^2}\right)(V) = RT$$

i.e., $PV^2 - RTV + a = 0$

$$V = \frac{RT \pm \sqrt{R^2T^2 - 4Pa}}{2P} = \frac{RT}{2P}$$

$(\because 4a.P = R^2T^2)$

47. (C)

$Z > 1$ real gas shows positive deviations from ideal behaviour when repulsive forces dominate.

48. (C)

$$\left(P + \frac{a}{V^2}\right)(V_m - b) = RT$$

49. (C)

$$T_C = \frac{8a}{27Rb} \qquad P_C = \frac{a}{27b^2} \qquad V_C = 3b$$

50. (B)

$$\frac{T_C}{P_C} = \frac{8a \times 27b^2}{27Rb \times a} = \frac{8b}{R}$$

$$V_{Excluded} = nb$$

51. (273)

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

52. (40)

$V_1 = V, T_1 = 300K, T_2 = 500\ K,\ V_2 = ?$

At constant pressure $V_1T_2 = V_2T_1$

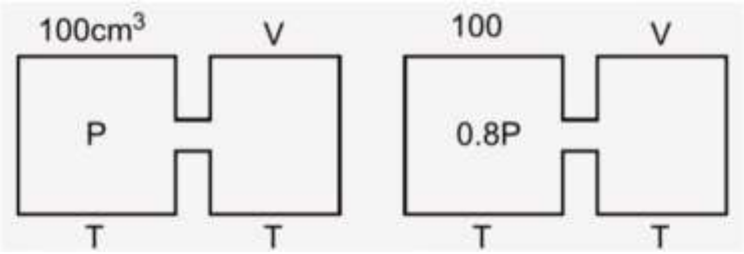
$$\therefore\ V_2 = \frac{V_1T_2}{T_1} = \frac{V \times 500}{300} = \frac{5V}{3}$$

$\therefore$ Volume of air escaped = final volume – initial volume

$$= \frac{5V}{3} - V = \frac{2V}{3}$$

$$\therefore\ \%\ \text{of air escaped} = \frac{2V/3}{5V/3} \times 100 = 40\%$$

53. (400)



$$\frac{0.2P \times (100 + V)}{RT} = \frac{100P}{RT}$$

$$0.2(100 + V) = 100$$

$$V = 400\ cm^3$$

54. (72)

$$\frac{r_1}{r_{H_2}} = \sqrt{\frac{2}{M_1}} = \frac{1}{6}$$

$$M = 72$$

55. (9)

$$2NH_3(g) \longrightarrow N_2(g) + 3H_2(g)$$

	$2NH_3(g)$	$N_2(g)$	$3H_2(g)$
Before sparking	76	0	0
After sparking at eqm	$76 - 2x$	x	3x

Increase in pressure $2x = 6;\ x = 3\ cm\ Hg$

Partial pressure of $H_2 = 3 \times 3 = 9\ cm\ Hg$

56. (1)

$$U_{rms} = \sqrt{\frac{3RT}{Mw}}$$

$$\frac{u_{H_2}}{U_{O_2}} = \sqrt{\frac{50}{2} \times \frac{32}{800}} = 1$$

57. (11)

$$KE_T = \frac{3}{2} nRT = \frac{3}{2} \frac{W}{M} RT$$

$$KE \propto \frac{T}{M}$$

$$\frac{KE_1}{KE_2} = \frac{T_1}{M_1} \frac{M_2}{T_2} = \frac{300}{2} \times \frac{44}{600} = \frac{11}{1}$$

58. (900)

$$\sqrt{\frac{2RT}{M_{SO_2}}} = \sqrt{\frac{3R \times T_1}{M_{O_2}}} \Rightarrow \frac{2 \times T}{64} = \frac{3 \times 300}{32}$$

$$T = 900\,K$$

59. (240)

$$200 = \sqrt{\frac{2RT}{2 \times 10^{-3}}}$$

$$RT = 40$$

$$\text{Total K.E.} = \frac{3}{2} nRT = \frac{3}{2} \times \frac{8}{2} \times 40 = 240J$$

60. (4)

$$\frac{r_{H_2}}{r_{H.C}} = \sqrt{\frac{M}{2}} = 3\sqrt{3}$$

$$\frac{M}{2} = 9 \times 3$$

$$M = 54$$

$$n = 4$$

SOLUTIONS

61. (D)

$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225 \quad \Rightarrow \quad 3(a_1 + a_{24}) = 225$

(sum of terms equidistant from beginning and end are equal) $a_1 + a_{24} = 75$

Now $a_1 + a_2 + \ldots\ldots + a_{23} + a_{24} = \frac{24}{2}[a_1 + a_{24}] = 12 \times 75 = 900$

62. (B)

$a_1 = 2; a_{n+1} = \frac{a_n}{3}; a_2 = \frac{a_1}{3} = \frac{2}{3}; a_3 = \frac{a_2}{3} = \frac{2}{3^2}$

$a_1 + a_2 + \ldots\ldots + a_{20} = 2 + \frac{2}{3} + \frac{2}{3^2} + \ldots\ldots = \frac{2.\left[1 - \left(\frac{1}{3}\right)^{20}\right]}{1 - \frac{1}{3}} = 3\left(1 - \frac{1}{3^{20}}\right)$

63. (B)

$S = \frac{a}{1-r} \Rightarrow r = \frac{S-a}{S}; S' = \frac{a[1-r^n]}{1-r} = S\left[1 - \left(\frac{S-a}{S}\right)^n\right]$

64. (C)

$\alpha + \beta = 3 \quad , \alpha\beta = a \quad ; \quad \gamma + \delta = 12 \quad , \gamma\delta = b$

$\alpha, \beta, \gamma, \delta$ are in G.P. Let r be the common ratio so $\alpha(1 + r) = 3$

$\alpha r^2(1 + r) = 12 \quad \Rightarrow \quad r^2 = 4 \quad \Rightarrow \quad r = 2$

so $\alpha = 1 \quad \Rightarrow \quad$ so $a = 2, \quad b = 32$ **Ans**

65. (D)

If a, G_1, G_2, G_3, b are in G.P. with common ratio equal to 'r' then $G_1 - a$, $G_2 - G_1$, $G_3 - G_2$, $b - G_3$ are also in G.P. with same common ratio $\Rightarrow \frac{G_3 - G_2}{G_2 - G_1} = r = 2 \Rightarrow \frac{b}{a} = r^4 = 16$

66. (A)

$T_3 = \frac{1}{3}$, $T_6 = \frac{1}{5}$. $T_n = \frac{3}{203}$

then 3rd , 6th term of A.P. series are 3, 6, $\frac{203}{3}$

a + 2d = 3 ⇒ a = 5d = 5

$d = \frac{2}{3}$, $a = \frac{5}{3}$

$a + (n-1)d = \frac{203}{3} \Rightarrow \frac{5}{3} + (n-1) = \frac{203}{3}$

$(n-1)^2 = 198$

n = 100

67. (D)

a = Rs. 200 ; d = Rs. 40 ⇒ savings in first two months = Rs. 400

remained savings = 200 + 240 + 280 + ... upto n terms

$= \frac{n}{2}[400 + (n-1)\ 40] = 11040 - 400 \Rightarrow 200n + 20n^2 - 20n = 10640$

$20n^2 + 180n - 10640 = 0 \Rightarrow n^2 + 9n - 532 = 0$

$(n + 28)(n - 19) = 0 \Rightarrow n = 19$

∴ no. of months = 19 + 2 = 21.

68. (B)

$$T_n = \frac{\frac{n^2(n+1)^2}{4}}{n^2}$$

$T_n = \frac{1}{4}(n+1)^2$

$T_n = \frac{1}{4}[n^2 + 2n + 1]$

$S_n = \sum_{n=1}^{n} T_n$

$$S_n = \frac{1}{4}\left[\frac{n(n+1)\ (2n+1)}{6} + n(n+1)\ + n\right]$$

n = 9

$$S_9 = \frac{1}{4}\left[\frac{9\times10\times19}{6} + 9\times10 + 9\right] = \frac{1}{4}[285 + 90 + 9] = \frac{384}{4} = 96.$$

69. (C)

$$S = \frac{3}{2} + \frac{15}{2^2} + \frac{35}{2^3} + \frac{63}{2^4} + ...\infty$$

$$\frac{1}{2}S = \frac{3}{2^2} + \frac{15}{2^3} + \frac{35}{24} + ...\infty$$

$$\frac{S}{2} = \frac{3}{2^2} + \frac{12}{2^2} + \frac{20}{2^3} + ...\infty$$

again use same concept S = 23

70. (B)

AM = A = $\frac{a+b+c}{3}$; GM = G = $(abc)^{1/3}$

HM = H = $\frac{3abc}{ab+bc+ca} = \frac{3G^3}{ab+bc+ca}$.

Equation whose roots are a,b,c ⇒ $x^3 - (a+b+c)x^2 + (\Sigma ab)x - abc = 0$

⇒ $x^3 - 3Ax^2 + \frac{3G^3}{H}.x - G^3 = 0$ **Ans**

71. (A)

Taking A.M. and G.M. of number, $\frac{a}{2}, \frac{a}{2}$, b, c

72. (D)

S = 1 + 2 + 4 + 7 + 11 + 16...............T_n ...(i)

S = 1 + 2 + 4 + 7 + 11 ....T_n ...(ii)

(i) – (ii) we get

O = 1+(1+2+3+4+5............(n–1) term) – T_n ⇒ $T_n = 1 + \frac{(n-1)\ n}{2} = \frac{n^2}{2} - \frac{n}{2} + 1$

∴ General term = $T_n = an^2 + bn + c$ here a = 1/2, b= –1/2, c = 1

$S_n = \sum T_n = \frac{n(n+1)\ (2n+1)}{12} - \frac{n(n+1\ )}{4} + n$

$S_{30} = \frac{30.\ 31.\ 61}{12} - \frac{30.\ 31}{4} + 30 = 4727.5 - 232.5 + 30 = 4525$ **Ans (D)**

73. (A)

$\frac{x_1}{x_1+1} = \frac{x_2}{x_2+3} = \frac{x_3}{x_3+5} = ... = \frac{x_{2013}}{x_{2013}+4025} = \frac{1}{\lambda}$

⇒ $x_1 = \frac{1}{\lambda-1}, x_2 = \frac{3}{\lambda-1}, x_3 = \frac{5}{\lambda-1}, ..., x_{2013} = \frac{4025}{\lambda-1}$

⇒ $x_1, x_2, x_3, ..., x_{2013}$ are in A.P. with common difference = $\frac{2}{\lambda-1}$ = d

$x_1, x_2, x_3, ..., x_{2013} = \frac{2}{\lambda-1}$ = d

74. (D)

$2b = a + c$ and $b^2 = \pm ac$

case-I

if $b^2 = ac$ and $a + c + b = \frac{3}{2}$ $\Rightarrow$ $b = \frac{1}{2}$

$a + c = 1$ $\Rightarrow$ $ac = \frac{1}{4} \Rightarrow$ $(1 - c)\, c = \frac{1}{4}$

$c^2 - c + \frac{1}{4} = 0 \Rightarrow$ $c = \frac{1}{2}$ $\Rightarrow$ $a = \frac{1}{2}$

a = b = c so not valid

case-II

$b^2 = - ac$ and $b = \frac{1}{2}$; $a + c = 1$ $\Rightarrow$ $ac = -\frac{1}{4}$

$(1 - c)\, c = -\frac{1}{4}$ $\Rightarrow$ $c^2 - c - \frac{1}{4} = 0$

$\Rightarrow$ $c = \frac{1 \pm \sqrt{1+1}}{2} = \frac{1 \pm \sqrt{2}}{2}$

$c = \frac{1+\sqrt{2}}{2}$ $\Rightarrow$ $a = \frac{1-\sqrt{2}}{2}$

75. (A)

$S_1 + S_2 + S_3 + ... + S_p$

$\Rightarrow$ $S_1 = \frac{n}{2}\,[2.1 + (n - 1)\, 1]$ $S_2 = \frac{n}{2}\,[2.2 + (n - 1)\, 3]$

$S_3 = \frac{n}{2}\,[2.3 + (n - 1)\, 5]$

$\vdots$

$\vdots$

$S_p = \frac{n}{2}\,[2.p + (n - 1)\,(2p - 1)]$

So $S_1 + S_2 + ... + S_p = \frac{n}{2}\,[2(1 + 2 + ... p) + (n - 1)\,(1 + 3 + 5 + ... (2p - 1))]$

$= \frac{n}{2}\left[2\ .\ \frac{p\ (p+1)}{2} + (n-1)\ p^2\right] = \frac{n\ \ p}{2}\ \ (n\,p + 1)$ **Ans.**

76. (A)

$x = \frac{1}{1-a}$, $y = \frac{1}{1-b}$, $z = \frac{1}{1-c}$ $\Rightarrow$ a, b, c are in A.P.

$\Rightarrow$ $1 - a$, $1 - b$, $1 - c$ are also in A.P. $\Rightarrow$ $\frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c}$ are in H.P.

77. (A)

$a_1 \cdot a_2 \cdot a_3 \cdot ....a_n = c$ $\Rightarrow \frac{a_1 + a_2 + a_3 + ...2a_n}{n} \ge (a_1a_2a_3 \ldots 2a_n)^{1/n} \ge (2c)^{1/n}$

$\Rightarrow$ $a_1 + a_2 + a_3 + ...2a_n \ge n(2\,c)^{1/n}$

78. (A)

If $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + 2003^2 = (2003)(4007)(334)$

$(1)(2003) + (2)(2002) + (3)(2001) + \ldots\ldots\ldots + (2003)(1) = (2003)(334)(x)$

$\Rightarrow \sum_{r=1}^{2003} r\ (2003 - r + 1) = (2003)(334)(x) \Rightarrow 2004.\sum_{r=1}^{2003} r - \sum_{r=1}^{2003} r^2 = (2003)(334)(x)$

$\Rightarrow 2004\left(\frac{2003\ .\ 2004}{2}\right). - 2003.(4007).334 = (2003)(334)(x)$

$\Rightarrow x = 2005$ **Ans.**

79. (A)

$\sum_{r=1}^{n} t_r = S_n \Rightarrow \sum_{r=1}^{n-1} t_r = S_{n-1} \Rightarrow t_n = S_n - S_{n-1} = \frac{n(n+1)(n+2)}{2}$

$$\sum_{r=1}^{n}\frac{1}{t_r} = \sum_{r=1}^{n}\frac{2}{r(r+1)(r+2)} = \sum\left(\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right) = -\left(\frac{1}{(n+1)(n+2)} - \frac{1}{2}\right)$$

80. (C)

$\because\ I = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots \infty = \frac{\pi^2}{6}$

Let $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots = A$

$\because\ I = \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\infty\right) + \frac{1}{2^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\infty\right)$

$\Rightarrow I = A + \frac{I}{4} \Rightarrow A = \frac{3I}{4} = \frac{3}{4} \times \frac{\pi^2}{6} \Rightarrow A = \frac{\pi^2}{8}$

83. (51)

Let first installment be = 'a' and the common difference of the A.P. be 'd'

So $a + (a + d) + (a + 2d) + \ldots\ldots. + (a + 39d) = 3600$

$\Rightarrow \frac{40}{2}[2a + 39d] = 3600$

$\Rightarrow 2a + 39d = 180$ ..... (1)

and $\frac{30}{2}[2a + 29d] = 2400$

$\Rightarrow 2a + 29d = 160$ ..... (2)

By equations (1) & (2), we get

$d = 2$ and $a = 51$ **Ans.**

82. (33)

$a + 6d = 9$; $T_1 T_2 T_7 = a(a + d)(a + 6d) = 9a(a + d) = 9(9 - 6d)(9 - 5d)$

$\because\ T_7 = a + 6d = 9.$

Let $A = T_1 T_2 T_7$

$\frac{dA}{d(d)} = 9[-45 - 54 + 60d] = 0 \Rightarrow 60d = 99$

$\Rightarrow d = \frac{33}{20}$ Ans.

83. (13)

Given series 63 + 65 + 67 + 69 + …… Eq. (i)

and 3 + 10 + 17 + 24 + ……. Eq. (ii)

Now from (i), m^{th} term = (2m + 61) and m^{th} term of (ii) series = (7m – 4)

Under condition, $\Rightarrow 7m-4=2m+61$

$\Rightarrow 5m=65$

$\Rightarrow m=13$

84. (40)

$a, a_1, a_2 \ldots\ldots, a_{2n}$, b are in AP and $a, g_1, g_2, \ldots\ldots g_{2n}$, b are in GP and $h=\frac{2ab}{a+b}$

$$\therefore \frac{a_1+a_{2n}}{g_1 \ \ g_{2n}}+\frac{a_2+a_{2n-1}}{g_2 \ \ g_{2n-1}}+\ldots\ldots+\frac{a_n+a_{n+1}}{g_n \ \ g_{n+1}}=\frac{a+b}{ab}+\frac{a+b}{ab}+\ldots\ldots+\frac{a+b}{ab}=2n\left(\frac{a+b}{2ab}\right)=\frac{2n}{h}$$

85. (225)

$$\frac{(a_1+a_2)+(a_3+a_4)}{2}\geq\sqrt{(a_1+a_2)(a_3+a_4)}$$

$$\Rightarrow (a_1+a_2)(a_3+a_4)\leq\frac{225}{4}$$

86. (8)

A.M. ≥ G.M.

$$\frac{\frac{1}{a^5}+\frac{1}{a^4}+\frac{1}{a^3}+\frac{1}{a^3}+\frac{1}{a^3}+1+a^8+a^{10}}{8}\geq\left(\frac{1}{a^5}\cdot\frac{1}{a^4}\cdot\frac{1}{a^3}\cdot\frac{1}{a^3}\cdot\frac{1}{a^3}\cdot 1\cdot a^8\cdot a^{10}\right)^{1/8}$$

$$\Rightarrow \frac{1}{a^5}+\frac{1}{a^4}+\frac{3}{a^3}+1+a^8+a^{10}\geq 8(1)^{1/8}$$

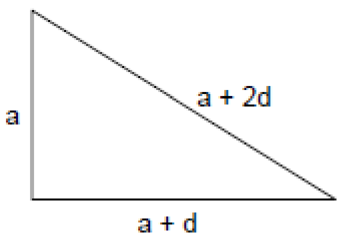
$\Rightarrow$ minimum value of $\frac{1}{a^5}+\frac{1}{a^4}+\frac{3}{a^3}+1+a^3+a^{10}=8$, at a = 1

87. (4)

Let b = ar, c = ar² ⇒ r is Integers. Also $\frac{a+ar+ar^2}{3}$ = ar + 2 ⇒ a + ar² = 2ar + 6 ⇒ a(r – 1)² = 6

$\Rightarrow$ r must be 2 and a = 6. Thus $\frac{a^2+a-14}{a+1}=\frac{36+6-14}{7}$ = 4 Ans.

88. (6)



$\frac{1}{2}a(a+d)=24 \quad\Rightarrow\quad a(a+d)=48$ ....(1)

$a^2+(a+d)^2=(a+2d)^2 \Rightarrow \quad 3d^2+2ad-a^2=0 \qquad (3d-a)(a+d)=0$

$\Rightarrow$ 3d = a $(\because\ a+d\neq 0)$ $\Rightarrow$ d = 2 a = 6 so smallest side = 6

89. (2)

a, b, c are in H.P., then $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P. $S=\frac{b+a}{b-a}+\frac{b+c}{b-c}=\frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a}-\frac{1}{b}}+\frac{\frac{1}{c}+\frac{1}{b}}{\frac{1}{c}-\frac{1}{b}}$

Let $\frac{1}{a}-\frac{1}{b}=\frac{1}{b}-\frac{1}{c}=d$

$$S=\frac{\left(\frac{1}{a}+\frac{1}{b}\right)-\left(\frac{1}{c}+\frac{1}{b}\right)}{d}=\frac{\left(\frac{1}{a}-\frac{1}{c}\right)}{d}=\frac{2d}{d}=2$$

90. (9)

$3+\frac{1}{4}(3+d)+\frac{1}{4^2}(3+2d)+ \ldots\ldots\ldots\ldots\ldots\ldots + \infty = 8$

$S = 3 + (3+d) + (3+2d) + \ldots\ldots\ldots\ldots\ldots\ldots + \infty$ ...... (i)

$\frac{1}{4}S=\frac{3}{4}+\frac{1}{4^2}(3+d)+\ldots\ldots\ldots\ldots\infty$ ...... (ii)

(i) – (ii) we get

$\frac{3}{4}S=3+\frac{1}{4}d+\frac{1}{4^2}d+\ldots\ldots\ldots\infty$; $\frac{3}{4}S=3+\frac{\frac{1}{4}d}{1-\frac{1}{4}}$

$\frac{3}{4}S=3+\frac{d}{3}$; $S=\frac{12}{3}+\frac{4}{9}d=8=4+\frac{4}{9}d=8\Rightarrow\frac{4}{9}d=4\Rightarrow d=9$ **Ans**