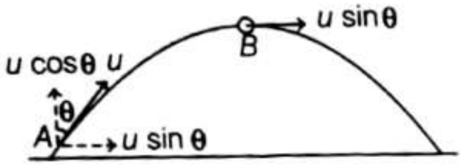
PART (A) : PHYSICS

SOLUTIONS

1. (B)
Let speed of projection is u. Initial velocity components will be as shown in the figure. We know that, at highest point on the trajectory for a projectile, there is only horizontal component of velocity as shown below.



Assuming potential energy at B (highest point) equals to U. By conservation of energy for motion from A to B,
Energy at A= Energy at B

$$\Rightarrow \frac{1}{2}mu^2 = \frac{1}{2}mu^2\sin^2\theta + U$$

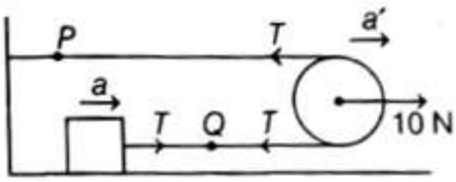
$$\Rightarrow U = \frac{1}{2}mu^2\left(1-\sin^2\theta\right)$$

$$= \frac{1}{2}mu^2\cos^2\theta$$

$$U = K\cos^2\theta$$

As initial kinetic energy is given as K in the question.

2. (A)
The free body diagram for the given system/ arrangement is as shown below



Acceleration of P=0
(as attached to wall)
From the FBD of pulley

$$2T = 10$$

$$\Rightarrow T = 5N$$

Limiting friction between block and ground $= \mu N$

$$= \mu mg$$

$$= 0.2 \times 1 \times 10$$

$$= 2N$$

As tension acting on block is greater than $f_{limiting}$, block will move and experience kinetic friction as shown
So, a= acceleration of block

$$= \frac{5-2}{1} = 3\,m/s^2 \qquad \ldots\ldots(i)$$

As acceleration of pulley is average of acceleration of ends of thread going over, it,

$a' = \frac{a_P + a_Q}{2} = \frac{0+a}{2} = \frac{3}{2} m/s^2$ ....(ii)

Relative acceleration,

$a_{rel} = a - a' = 3 - \frac{3}{2}$

$= 1.5 m/s^2$ [from Eqs. (i) and (ii)]

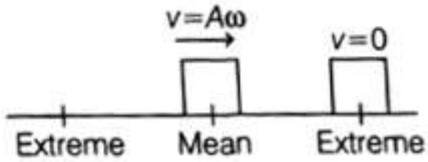
3. (C)

Velocity of a particle executing SHM at mean position , $v_{mean} = A\omega$

Velocity of the particle at extreme position, $v_{extreme} = 0$

Time of motion for the particle from mean to extreme $= \frac{T}{4}$

$$= \frac{\pi}{2\omega}\left(\because T = \frac{2\pi}{\omega}\right)$$



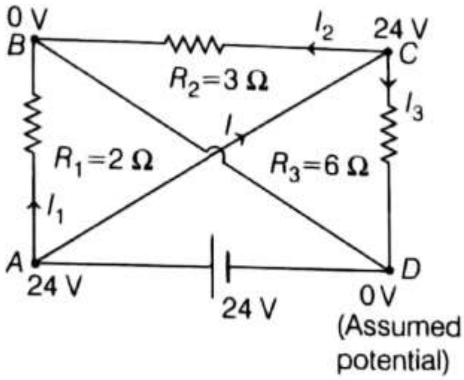
As, average acceleration is defined as

$a_{av} = \frac{\Delta v}{\Delta t}$

$\Rightarrow |a_{av}| = \frac{|A\omega - 0|}{\frac{\pi}{2\omega}} = \frac{2A\omega^2}{\pi}$

4. (D)

Assume potential at point D is equal to zero. As we know potential of all points on a plane wire are equal, so potential at B will also be equal to 0 V. Also potential difference across cell is 24 V, so potential at A will be 24 V. similarly, potential at C is also 24 V.



Using Ohm's law for branch AB,

$I_1 = \frac{V_A - V_B}{R_1}$

$= \frac{24-0}{2} = 12A$

for branch CB,

$l_2 = \frac{V_C - V_B}{R_2} = \frac{24-0}{3} = 8A$

For branch CD,

$l_3 = \frac{V_C - V_D}{R_3} = \frac{24-0}{6} = 4A$

By KCL for junction C,

$l = l_2 + l_3 = 8 + 4 = 12A$

5. (C)

Number of nuclei decayed as per Rutherford's and Soddy decay law is $N = N_0\left(1-e^{-\lambda t}\right)$

Where, $N_0 =$ initial number of nuclei,

$\lambda =$ decay constant and $t =$ time

Therefore, p= probability of decay

$p = \frac{N}{N_0} = 1 - e^{-\lambda t}$ ……(i)

As one mean life is equal to $\frac{1}{\lambda}$.

Putting $t = \frac{1}{\lambda}$ in Eq. (i),

$p = 1 - e^{-1} = 1 - 0.37$

$= 0.63 = 63\%$

6. (A)

Given that $T^3V^2 =$ constant

$\Rightarrow \left(\frac{pV}{nR}\right)^3 V^2 =$ constant

($\because$ using ideal gas equation, $pV = nRT$ or $T = \frac{pV}{nR}$)

$\Rightarrow p^3V^5 =$ constant $\Rightarrow pV^{\frac{5}{3}} =$ constant ……(i)

Differentiating Equation (i) with respect to volume V on both sides, we get

$\Rightarrow \frac{dp}{dv}.v^{\frac{5}{3}} + p.\frac{5}{3}.v^{\frac{2}{3}} = 0$

$\Rightarrow \frac{dp}{dv} = -\frac{5}{3}\frac{p}{v}$ ……..(ii)

Bulk modulus is defined as,

$B = \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$

$= \frac{dp}{-\frac{dv}{v}} = -v\frac{dp}{dv}$

$= -v\left(-\frac{5}{3}\frac{p}{v}\right)$ [Using Eq. (ii)]

$= \frac{5}{3}p$

7. (B)
Average energy (electric or magnetic) in electromagnetic wave is given by

$\frac{1}{4}\varepsilon_0 E_0^2 = \frac{B_0^2}{4\mu_0}$ ……..(i)

$\Rightarrow \varepsilon_0 E_0^2 = \frac{B_0^2}{\mu_0}$ ………(ii)

Consider option(B)

$\frac{1}{8}\varepsilon_0 E_0^2 + \frac{B_0^2}{8\mu_0}$

$\Rightarrow \frac{1}{8}\varepsilon_0 E_0^2 = \frac{1}{8}\varepsilon_0 E_0^2$ [using Eq. (ii)]

$= \frac{1}{4}\varepsilon_0 E_0^2$

Which is average electric energy as per Eq. (i)

8. (D)
As we know,
Range of tower , $r = \sqrt{2Rh}$ …….(i)
Where, R= radius of earth,
And $h =$ height of tower
Also, as volume $= \frac{\text{mass}}{\text{density}}$
Since, shape of Earth is considered as spherical
So, $\frac{4}{3}\pi R^3 = \frac{m}{p}$

As mass is constant, $R = \frac{k}{\rho^{\frac{1}{3}}}$ ………(ii)

Where , k is proportionality constant. Putting value of R from Eq. (ii) in Eq. (i) we get

$r = \sqrt{\frac{2kh}{\rho^{\frac{1}{3}}}}$

As k and h are constants, $r = \frac{C}{\rho^{\frac{1}{6}}}$ …….(iii)

Where, C is proportionality constant. Differentiating the above relation w.r.t ρ, we get

$\frac{dr}{d\rho} = -\frac{1}{6}C\rho^{-\frac{7}{6}}$ ………..(iv)

Dividing Eq. (iv) by Eq. (iii), we get

$\frac{dr}{r} = -\frac{1}{6}\frac{d\rho}{\rho}$

$\Rightarrow$ % change in the range of telecommunication,

$$\frac{\Delta r}{r}\times 100=-\frac{1}{6}\times\frac{\Delta\rho}{\rho}\times 100$$

$$=-\frac{1}{6}\times 3=-0.5\%$$

The range of a telecommunication tower on earth's surface will decreases by 0.5%.

9. (B)

If n_1 and n_2 are the number of moles of air in the two bubbles while n is number of moles in resulting bubble. By conservation of mass, $n=n_1+n_2$

$$\Rightarrow \frac{pV}{RT}=\frac{p_1V_1}{RT}+\frac{p_2V_2}{RT}$$

(using ideal gas equation)

$$\Rightarrow pV=p_1V_1+p_2V_2 \qquad \text{........(i)}$$

As bubbles are in vacuum, so outside pressure is zero $(p_0=0)$. As excess pressure inside bubble is given by

$$p-p_0=\frac{4T}{R}$$

$$\Rightarrow p=\frac{4T}{R} \qquad \text{......(ii)}$$

Also, $V=\frac{4}{3}\pi R^3$ ....(iii)

Using Eqs. (ii) and (iii), Eq. (i) can be written as

$$\frac{4T}{R},\frac{4}{3}\pi R^3=\frac{4T}{R_1}.\frac{4}{3}\pi R_1^3+\frac{4T}{R_2}.\frac{4}{3}\pi R_2^3$$

$$\Rightarrow R^2=R_1^2+R_2^2$$

$$\Rightarrow \pi R^2=\pi R_1^2+\pi R_2^2$$

$$\Rightarrow A=A_1+A_2=A+2A=3A$$

10. (D)

Given, potential at point C is zero, this means potential at O is zero as surface of metal bodies is equipotential in nature

$$\Rightarrow V_0=0\Rightarrow V_{dipole}=V_{point}=0$$

$$\Rightarrow \frac{Kp}{(2R)^2}\cos 180^\circ+\frac{Kq}{(2R)}=0$$

$$\Rightarrow p=2qR \qquad \text{.......(i)}$$

Now, as net field inside metal ball $=0$, So, at O

$$\Rightarrow E_{dipole}+E_{point}+E_{induced}=0$$

$$\Rightarrow=-\frac{2Kp}{(2R)^3}\hat{i}-\frac{Kq}{(2R)^2}\hat{i}+E_{induced}=0 \qquad \text{.....(ii)}$$

$$\Rightarrow E_{induced}=\frac{3}{8}\frac{Kp}{R^3}\hat{i}=\frac{3Kq}{4R^2}\hat{i} \quad \text{[using Eqs. (i) and (ii)]}$$

11. (B)
Since, force applied is same
$\Rightarrow k_1x_1 = k_2x_2 = k_{eq}x_0$, where
$$k_{eq} = \frac{k_1k_2}{k_1+k_2}$$
$$\Rightarrow x_2 = \frac{k_1x_1}{k_2} \text{ and } x_0 = \left(\frac{k_1+k_2}{k_2}\right)x_1$$
W_1 =Work done by F on block,
$$W_1 = \frac{1}{2}k_{eq}x_0^2$$
$$= \frac{1}{2}\frac{k_1k_2}{k_1+k_2}\times\left(\frac{k_1+k_2}{k_2}\right)^2 x_1^2$$
$$= \frac{1}{2}\frac{k_1\left(k_1+k_2\right)x_1^2}{k_2}$$
As change in KE of block is zero, work done by S_2 on block
$$= -W_1 = -\frac{1}{2}\frac{k_1\left(k_1+k_2\right)x_1^2}{k_2}$$
Work done by S_2 on $S_1 =$ Change in PE of $S_1 = \frac{1}{2}k_1x_1^2$
As displacement of wall is zero, work done by S_1 on wall $= 0$

12. (A)
If $v_1, v_2, v_3, ..., v_n$ are molecular speeds, then average molecular kinetic (translational) energy will be
$$K = \frac{\frac{1}{2}m\left(v_1^2+v_2^2+v_3^2+.......+v_n^2\right)}{n}$$
$$= \frac{1}{2}m\left[\sqrt{\frac{v_1^2+v_2^2+.....+v_n^2}{n}}\right]^2 = \frac{1}{2}mv_{rms}^2$$

13. (B)
Given, $f_0 = 2\text{cm}, f_e = 5\text{cm}$
$$|v_0| + |u_e| = 20\,\text{cm}$$
$$v_e = -25\text{cm}$$
From lens formula, $\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$
$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{5}$$
$$\therefore u_e = -\frac{25}{6}\text{cm}$$
Distance of real image from objective,

$$v_0 = 20 - |u_e| = 20 - \frac{25}{6}$$

$$= \frac{120 - 25}{6} = \frac{95}{6}\text{cm}$$

Now, $\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$

$$\Rightarrow \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{(95/6)} - \frac{1}{2}$$

$$= \frac{6}{95} - \frac{1}{2} = \frac{12 - 95}{190} = -\frac{83}{190}$$

$$\therefore u_0 = -\frac{190}{83} = -2.3\text{cm}$$

Magnifying power,

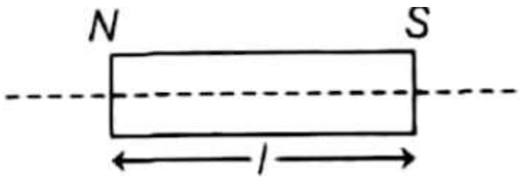
$$M = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$$

$$= -\frac{95/6}{2.3}\left(1 + \frac{25}{5}\right) = -41.5$$

It is close to 40.

14. (A)

On dividing a bar magnet along its length, as its pole strength of m will be halved, so dipole moment μ will also be halved.



$(\because \mu = ml)$

Also, moment of inertia of the magnet,

$$l = \frac{ml^2}{12}$$

As mass m is halved, so l is also halved.

Time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{l}{\mu B}} \qquad .....(i)$$

Time period of half of the magnet,

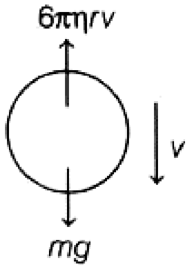
$$T' = 2\pi\sqrt{\frac{\frac{l}{2}}{\frac{\mu}{2}B}} = 2\pi\sqrt{\frac{l}{\mu B}} \qquad .....(ii)$$

From Eqs. (i) and (ii), we get T' = T

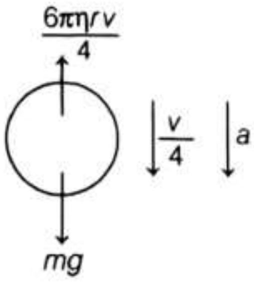
So, time period of oscillation on slight disturbance will still be T.

15. (C)
As per Einstein's photoelectric
Equation, $V_s = \frac{hf}{e} - \frac{\phi}{e}$ .....(i)
If f is doubled, new stopping potential,
$V_s' = \frac{2hf}{e} - \frac{\phi}{e}$ .....(i)
$= 2\left(\frac{hf}{e} - \frac{\phi}{e}\right) + \frac{\phi}{e}$
$= 2V_s + \frac{\phi}{e}$ [from Eq.(i)]
Which is more than double or an increase of more than 100%

16. (B)
Free body diagram of rain drop falling with terminal speed is as shown alongside.

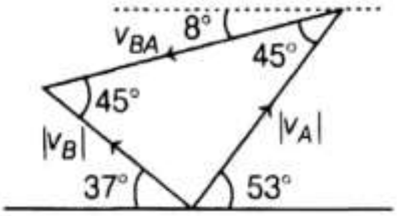


In this condition, its acceleration and net force are zero, i.e.,
$6\pi\eta rv(\text{drag force}) = mg(\text{weight})$ .....(i)
Where, η = Coefficient of viscosity and r = radius of the rain drop
For a rain drop of mass (m) falling with (1/4)th speed, free body diagram is as shown.



Net force on the drop,
$F = mg - \frac{6\pi\eta rv}{4}$
$= mg - \frac{mg}{4}$ [using Eq. (i)]
$= \frac{3mg}{4}$

17. (B)
As per triangle law, relative velocity (V_{BA}) of B w.r.t. A will be as shown in the figure.

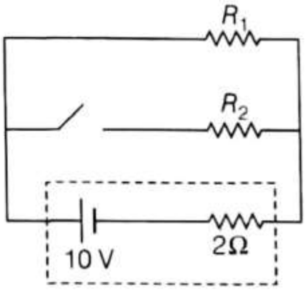


From the triangle using sine rule,
We get

$$\frac{V_B}{\sin 45^\circ} = \frac{60}{\sin 45^\circ}$$

$\Rightarrow \ V_B = 60\,\text{m/s}$

18. (D)
Diode 1 is forward biased as its p-side is connected to higher potential terminal of cell. So, it behaves as plane wire while Diode 2 is reverse biased. So, it will act like an open switch. Therefore, equivalent circuit is as shown.

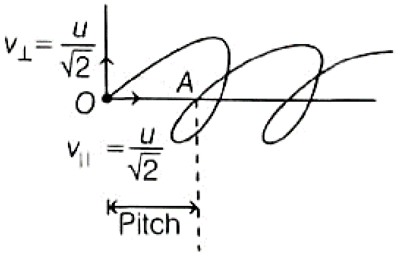


So, effective resistance of external circuit is R_1. For maximum power consumption, $R_{\text{extrenal}} = R_{\text{internal}}$ as per maximum power theorem

$\Rightarrow \ R_1 = 2\Omega$

And R_2 may be any value, hence option (d) is correct.

19. (B)
When a charged particle enters a region of a uniform magnetic field, such that the angle it makes with the direction of $B \neq 90^\circ$, then it follows a helical path as shown below



Radius of helix,

$$R = \frac{mv}{qB} = \frac{mu}{\sqrt{2}qB} \qquad .....(i)$$

Where, $V_\perp$ = Component of velocity perpendicular to field
Also, pitch, P = Displacement along field in one revolution (OA)

$= V_{\parallel} \times T = \frac{u}{\sqrt{2}} \times \frac{2\pi m}{qB}$ .....(ii)

Dividing Eq. (ii) by Eq. (i), $\frac{P}{R} = 2\pi$

20. (A)

In a series L- C – R circuit at resonance,
Capacitive reactance = Inductive reactance

$\Rightarrow \; X_C = X_L \; \Rightarrow \; \omega L = \frac{1}{\omega C}$

Where, ω is angular frequency, C is capacitance and L is inductance.
As, $\omega = 2\pi f$
Where, f is frequency
$\Rightarrow$ At resonance, frequency is given as

$f_r = \frac{1}{2\pi\sqrt{LC}}$

Now, the frequency of the source is half of f_r

$\Rightarrow \; f = \frac{1}{2} f_r = \frac{1}{4\pi\sqrt{LC}}$

$\Rightarrow \; f < f_r$

$\Rightarrow \; f < \frac{1}{2\pi\sqrt{LC}}$

$\Rightarrow \; \left(2\pi\sqrt{LC}\right) f < 1$

$\Rightarrow \; (2\pi f L)(2\pi f C) < 1$

$\Rightarrow \; X_L < X_C$

This means that the nature of the circuit is essentially capacitive.

21. (12)

Wavelength of photon/line for transition of electron from $n = n_2$ to $n = n_1$ is given by

$\lambda = \frac{R^{-1}}{\frac{1}{n_1^2} - \frac{1}{n_2^2}}$

Where, R is Rydberg's constant.
For first line of Lyman series, $n_1 = 1$ and $n_2 = 2$

$\Rightarrow \; \lambda_1 = \frac{R^{-1}}{1 - \frac{1}{2^2}} = \frac{4}{3R}$ ......(i)

For series limit of Brackett series,
$n_1 = 4$ and $n_2 = \infty$

$\Rightarrow \lambda_2 = \frac{R^{-1}}{\frac{1}{4^2} - \frac{1}{\infty^2}} = \frac{16}{R}$ .....(ii)

Dividing Eq. (ii) by Eq.(i), we get

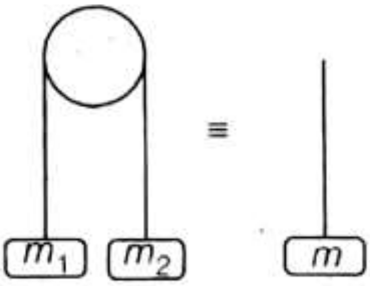
$$\frac{\lambda_2}{\lambda_1}=\frac{\frac{16}{R}}{\frac{4}{3R}}=\frac{16}{R}\times\frac{3R}{4}=12$$

22. (4.16)
As per Stefan's law, power emitted
$P=\sigma AT^4$
Where, σ is Stefan's Constant, A is surface area and T is temperature of black body.
$$\Rightarrow\ P=\sigma\left(4\pi r^2\right)\left(\frac{b}{\lambda}\right)^4$$
As black body is spherical in nature , so surface area = $4\pi r^2$ and according to Wien's law = $\lambda T=b$,
Where, b is Wien's Constant.
$$\Rightarrow\ P\ \alpha\ \frac{r^2}{\lambda^4}\Rightarrow\frac{P_A}{P_B}=\left(\frac{r_A}{r_B}\right)^2\left(\frac{\lambda_B}{\lambda_A}\right)^4$$
Substituting the given values, we get
$$\frac{P_A}{P_B}=2^2\left(1.01\right)^4$$
$$=4\left(1+0.01\right)^4$$
$$\approx 4\left(1+0.04\right)=4.16 \qquad \text{(Using Binomial approximation)}$$

23. (2)
As we know,
Frequency of string $\alpha\ \sqrt{\text{Tension}}$
$$\Rightarrow\ \frac{\Delta f}{f}\times100=\frac{1}{2}\frac{\Delta T}{T}\times100=\frac{1}{2}\times4 \qquad \left(\because\ \text{given},\frac{\Delta T}{T}\times100=4\%\right)$$
$=2\%$
Here, $f=100\text{Hz}\Rightarrow\Delta f=2\text{Hz}$
$\Rightarrow$ Frequency of string after tightening
$=100+2=102\text{Hz}$
Beat frequency = Difference in frequency of sources
$=120-100=2$

24. (1)
Planck's length will have dimensional formula of length,
$$\Rightarrow\ [L]=\left[M^0LT^0\right]=\left[\sqrt{\frac{hG}{c^3}}\right]=\left[M^xL^yT^z\right]$$
Comparing the powers of M. L and T on both sides, we get
$\Rightarrow\ x=0, y=1, z=0$
So, x + y + z = 1

25. (4.8)
A system of pulleys attached with two blocks of masses m_1 and m_2 is as shown below,

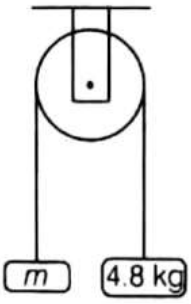


Equivalent mass of the above mentioned system, $m_{eq} = \frac{4m_1m_2}{m_1+m_2}$

So, equivalent of 2 kg and 3 kg system given in the question.

$m_{eq} = \frac{4\times2\times3}{2+3} = 4.8$

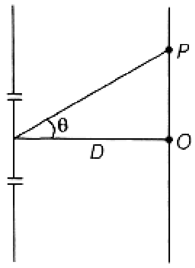
Given system can therefore be represented as



Therefore, for equilibrium m must equal to 4.8 kg.

26. (0.57 to 0.59)

For maxima, path difference = $n\lambda$



$\Rightarrow \quad d\sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n\lambda}{d}$

Substituting the given values, we get

$\sin\theta = \frac{2000\times5000\times10^{-10}}{2\times10^{-3}} = \frac{1}{2}$

$\Rightarrow \theta = 30^\circ$

According to the figure,

$OP = D\tan\theta = 1.\tan30^\circ = \frac{1}{\sqrt{3}}$

$= 0.58\,m$

27. (1)

As sphere is not slipping, instantaneous displacement of point of contact of sphere with incline is zero. Work is done only by $mg = mg \times \text{vertical dispalcement} = mgh$

So, by work – energy theorem,

$$W_{friction} + W_{normal} + W_{mg} = \Delta KE$$

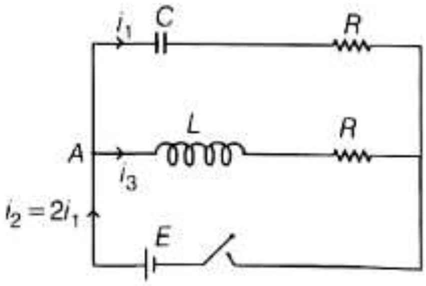
$$0 + 0 + mgh = \Delta KE$$

$$\Rightarrow \Delta KE = mgh$$

So, by comparing with the given value in equation, the value of x is 1.

28. (0.69)

The given circuit can be shown as follows



Applying KCL at node A,

$$2i_1 = i_1 + i_3$$

$$\Rightarrow i_1 = i_3$$

$$\Rightarrow \frac{E}{R} e^{-\frac{t}{RC}} = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

Substituting the given values, we get

$$\Rightarrow e^{-t} = 1 - e^{-t}$$

$$\Rightarrow e^{-t} = \frac{1}{2}$$

$$\Rightarrow t = \ln 2 = 0.69$$

29. (7.5)

Acceleration due to gravity at height h << R is given by

$$g_1 = g\left(1 - \frac{2h}{R}\right) \qquad .....(i)$$

Where, $g = 9.8\,m/s^2$ and R is the radius of the earth.

Also, acceleration due to gravity at depth d is given by

$$g_2 = g\left(1 - \frac{d}{R}\right) \qquad .....(ii)$$

Given that, $g_1 = g_2$

From Eqs. (i) and (ii), we get

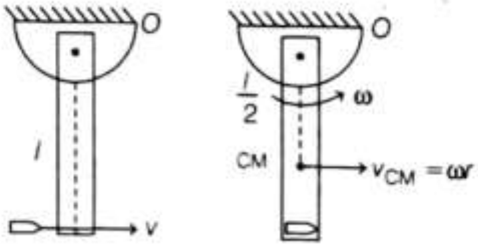
$$g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$$

$$\Rightarrow h = \frac{d}{2} = \frac{15}{2}$$

$$= 7.5\,km$$

30. (8)

The position of the bullet and rod during its motion is as shown below



By conservation of angular momentum about hinge O,

$L_{final} = L_{initial}$

$\Rightarrow I\omega = mvl$

$\Rightarrow \left(\frac{ml^2}{3} + ml^2\right)\omega = mvl$

$\Rightarrow \omega = \frac{3v}{4l}$

$\Rightarrow V_{CM} = \omega r = \left(\frac{3v}{4l}\right).\frac{l}{2} = \frac{3v}{8}$

$\Rightarrow x = 8$

PART (B) : CHEMISTRY

SOLUTIONS

31. (C)
Let total volume = 1000 mL = 1L
total mass of solution = 1460 g

$$\text{mass of HCl} = \frac{35}{100} \times 1460$$

$$\text{moles of HCl} = \frac{35 \times 1460}{100 \times 36.5}$$

$$\text{So molarity} = \frac{35 \times 1460}{100 \times 36.5} = 14\text{ M}$$

32. (D)
Mass of liquid = 135 – 40 = 95g

$$\text{Volume of liquid} = \frac{\text{mass}}{\text{density}} = \frac{95}{0.95}\text{ mL} = 100\text{ mL} = 0.1\text{ L}$$

Mass of ideal gas = 40.5 – 40g = 0.5 g
PV = nRT

$$0.82 \times 0.1 = \left(\frac{0.5}{M}\right) \times 0.082 \times 250$$

M = 125

33. (B)

$$r = 0.529 \times \frac{n^2}{Z}\text{ Å}$$

$$r_3 = 0.529 \times \frac{3^2}{1}$$

$$r_4 = 0.529 \times \frac{4^2}{1}$$

$$\frac{r_4}{r_3} = \frac{4^2}{3^2} = \frac{16}{9}$$

$$r_4 = \frac{16}{9} r_3$$

34. (A)

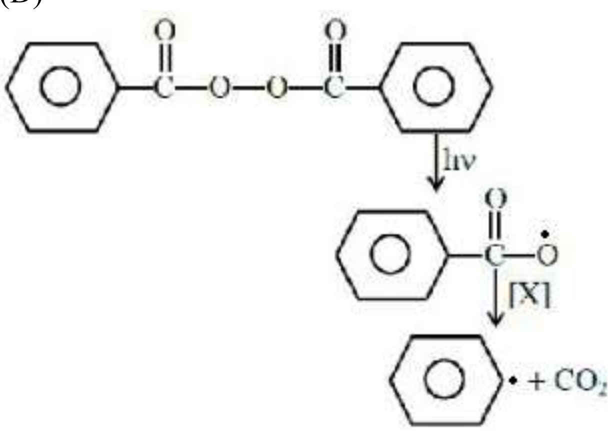
Bonder order $O_2^{2-} < O_2^- < O_2 < O_2^+$

Ion/molecule	Number of e^- in BMO	Number of e^- in ABMO	Bond order
O_2^+	10	5	2.5
O_2	10	6	2
O_2^-	10	7	1.5
O_2^{2-}	10	8	1

35. (C)

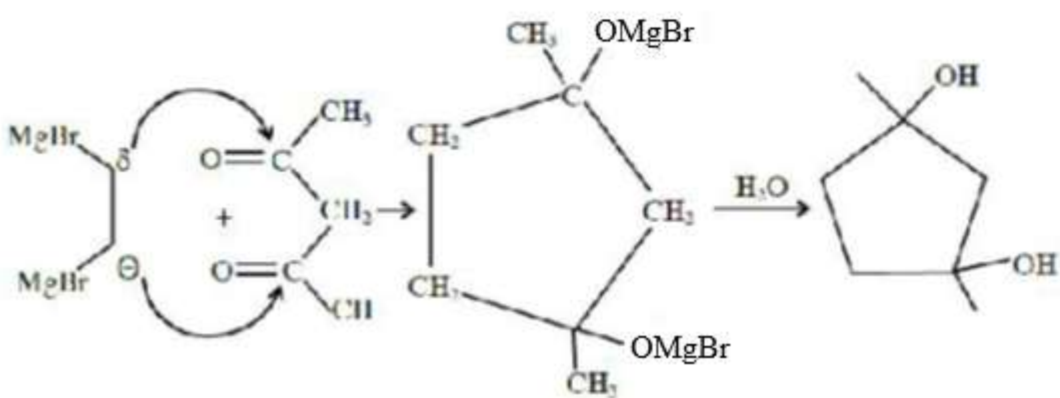
A cell with less variation in EMF with temperature is preferred as reference electrode because it can be used for wider range of temperature without much derivation from standard value so a cell with less $\left(\frac{\partial E}{\partial T}\right)_P$ is preferred.

36. (D)



37. (A)

Although Acetyl Acetone predominantly gives Acid base reaction with G.R due to Activemethylene group but according to given option ans. should be based on nucleophilic addition reaction(NAR).



38. (C)

Moles of $Fe_3O_4 = \frac{4.640\times10^3}{232} = 20$

Moles of $CO = \frac{2.52\times10^3}{28} = 90$

So limiting Reagent $= Fe_3O_4$

So moles of Fe formed = 60

Weight of $Fe = 60\times56 = 3360$ gms

39. (D)

(1) $Cr = [Ar]3d^5 4s^1$

(2) $m = -l$ to $+l$

(3) According to Aufbau principle, orbitals are filled in order of their increasing energies.

(4) Total nodes = n – 1

40. (C)

LiCl > NaCl > KCl > CsCl (Covalent character)

41. (D)

In de-ionized water no common ion effect will take place so maximum solubility

42. (D)

So equal & similar charge particle will repel each other, hence will never precipitate.

43. (A)

$_{78}Pt = [Xe]4f^{14}5d^9 6s^1$ (Exceptional electronic configuration)

44. (D)

For ZnS, KCN is used as depressant.

For Gold and Silver $\Rightarrow$ leaching (Cyanide process)

45. (C)

In option (A) and (C) reducing action of hydrogen peroxide is shown.

In option (A) it is in acidic medium, in option (B) it is in basic medium

Or

For reducing ability H_2O_2 changes to O_2, i.e. oxidize, so in option ‘A’ & ‘C’, O_2 is formed but ‘A’ is in acidic medium so option ‘C’ is correct.

46. (B)

$P_4 + 3NaOH + 3H_2O \rightarrow PH_3 + 3NaH_2PO_2$

Oxoacid = H_3PO_2 (hypo phosphorus acid) or (phosphinic acid)

47. (A)

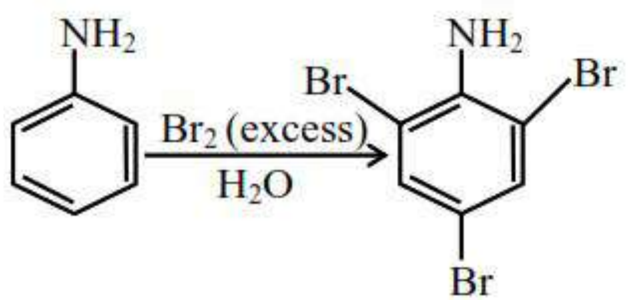
$$CaCO_3 + H_2SO_4 \rightarrow CaSO_4 + H_2O + CO_2$$

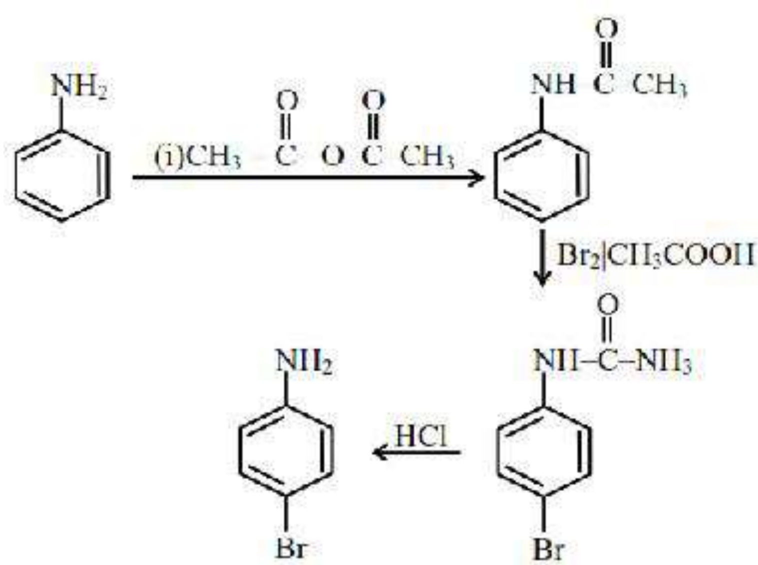
48. (B)

$$Ph{-}Br + Mg \xrightarrow{EtOEt} Ph{-}Mg{-}Br$$

$$Ph{-}Mg{-}Br \xrightarrow[(ii)\ H_3O^+]{(i)\ CO_2} Ph{-}COOH + MgBr(OH)$$

49. (C)





50. (C)

Buna- N is synthetic rubber which can be stretched and retains its original status on releasing the force.

51. (727)

$$\Delta U = -726\ \text{kJ/mol}$$

$$\Delta n_{(g)} = 1 - \frac{3}{2} = -\frac{1}{2}$$

$$\Delta H = \Delta U + \Delta n_{(g)}RT$$

$$= -726 - \frac{1}{2} \times \frac{8.3 \times 300}{1000}$$

$$= -727.245$$

52. (98)

If mass of $H_2O = 99.5$

$$m = \frac{0.5}{74.5} \times \frac{1}{0.0995}$$

$$i = \frac{0.24 \times 74.6 \times 0.0995}{0.5 \times 1.80} = 1.979$$

$$1.979 = 1 + \alpha$$

$$\alpha = 0.979$$

$$\%\alpha = 97.9\% = 98\%$$

53. (3)

Most basic oxide is V_2O_3

$$V^{+3} \rightarrow [Ar]3d^2$$

$$\mu = \sqrt{2(2+2)} = 2.84 \text{ BM} \approx 3$$

54. (18)

$$C_xH_yO_z + \left(x + \frac{y}{4} - \frac{z}{2}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

0.3 g   0.2 g   0.1 g

$$\frac{n_{CO_2}}{n_{H_2O}} = \frac{x}{\frac{y}{2}} = \frac{\frac{0.2}{44}}{\frac{0.1}{18}} \Rightarrow \frac{2x}{y} = \frac{36}{44} = \frac{9}{11}$$

$$x = \frac{9y}{22}$$

$$\frac{n_{C_xH_yO_z}}{n_{CO_2}} = \frac{1}{x}a$$

$$\frac{0.3}{12x + y + 16z} \times \frac{44}{0.2} = \frac{1}{x}$$

$$66x = 12x + y + 16z \Rightarrow 54x = y + 16z$$

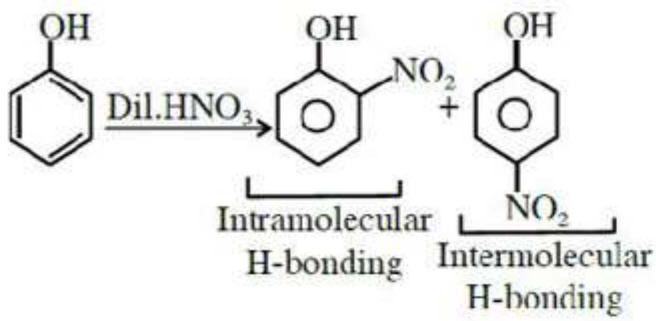
$$\frac{54 \times 9y}{22} - y = 16z \Rightarrow z = \frac{29y}{22}$$

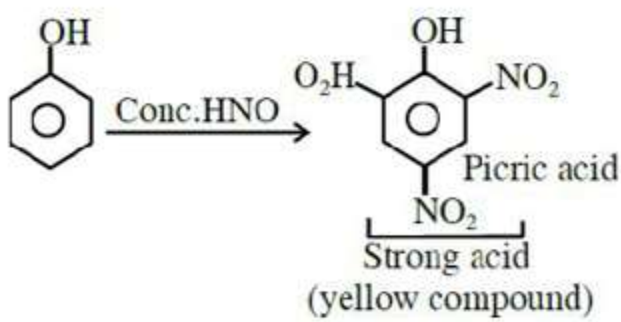
$$C_xH_yO_z = C_xH_yO_z$$

$$C_{\frac{9y}{22}}H_yO_{\frac{29y}{22}}$$

$$\% \text{ of } C=\frac{12\times 9}{\left(12\times 9+22+29\times 26\right)}\times 100=\frac{108}{594}\times 100=18.18\%$$

55. (7)





56. (152)

Assuming ideal behaviour $P=\frac{dRT}{M}$

$P=\frac{100}{760}\text{ atm, } T=257+273=530\text{ K}$

$d=0.46\text{ gm/L}$

So, $M=\frac{0.46\times 0.082\times 530}{100}\times 760$

$=151.93\approx 152$

57. (127)

At anode

$2H_2O\rightarrow O_2\left(g\right)+4H^{+}+4e^{-}$

At cathode

$2H^{+}+2e\rightarrow H_2\left(g\right)$

Now number of gm eq. $=\frac{i\times t}{96500}$

$=\frac{0.1\times 2\times 60\times 60}{96500}=0.00746$

$$V_{O_2} = \frac{0.00746}{4} \times 2.27 = 0.0423$$

$$V_{H_2} = \frac{0.00746}{2} \times 22.7 = 0.0846$$

$$V_{total} \approx 127 \text{ ml or cc}$$

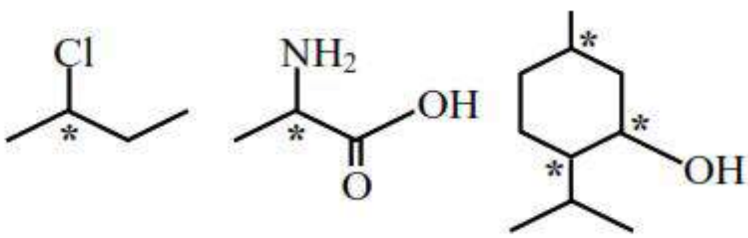
58. (0)

$$3MnO_4^{2-} + 4H^+ \rightarrow 2Mn^{+7}O_4^- + Mn^{+4} + 2H_2O$$

$Mn^{+7} =$ no. of unpaired electron is '0'

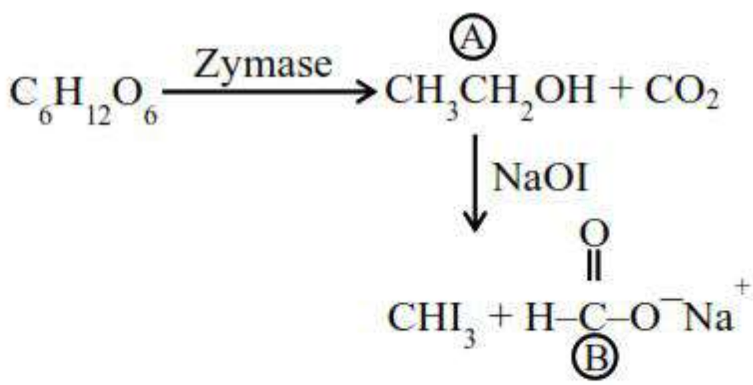
$\mu = 0$ B.M.

59. (3)



No. of compounds containing asymmetric carbons are three.

60. (1)



No. of carbon atoms present in B is 1.

PART (C) : MATHEMATICS

SOLUTIONS

61. (B)

$$f'(x)-\frac{2x(x+1)}{x+1}f(x)=\frac{e^{x^2}}{(x+1)^2}$$

$$IF=e^{\int -2x dx}=e^{-x^2}$$

$$\Rightarrow f(x).\left(e^{-x^2}\right)=\int\frac{dx}{(x+1)^2}$$

$$\Rightarrow f(x).e^{-x^2}=\frac{-1}{x+1}+C$$

At $x=0, f(0)=5 \qquad \Rightarrow C=6$

$$\therefore f(x)=\left(\frac{6x+5}{x+1}\right).e^{x^2}$$

Hence, (B) is the correct answer

62. (D)

Clearly, R is reflexive and symmetric

Let z_1Rz_2 and z_2Rz_3

$\Rightarrow \frac{z_1-z_2}{z_1+z_2}$ is real, say k

And $\frac{z_2-z_3}{z_2+z_3}$ is real, say I

Now, $\frac{z_1-z_2}{z_1+z_2}=k \Rightarrow \frac{z_1}{z_2}=\frac{1+k}{1-k}$

And $\frac{z_2-z_3}{z_2+z_3}=1 \Rightarrow \frac{z_2}{z_3}=\frac{1+I}{1-I}$

Thus, $\frac{z_1}{z_3}=\frac{(1+k)(1+I)}{(1-k)(1-I)}$

$$\frac{z_1-z_3}{z_1+z_3}=\frac{(1+k)(1+I)-(1-k)(1-I)}{(1+k)(1+I)+(1-k)(1-I)}$$

Which is real

63. (A)

Since $A.M \geq G.M.$, We get

$$\frac{1}{2}\left[\sqrt{x^2+x}+\frac{\tan^2\alpha}{\sqrt{x^2+x}}\right]\geq\sqrt{\sqrt{x^2+x}\times\frac{\tan^2\alpha}{\sqrt{x^2+x}}}=|\tan\alpha|$$

64. (D)
General term
$$T_{r+1} = \frac{\lfloor 10}{\lfloor r_1 \lfloor r_2 \lfloor r_3} (3)^{r_1} (-2)^{r_2} (5)^{r_3} (x)^{3r_1+2r_2-5r_3}$$
$$3r_1 + 2r_2 - 5r_3 = 0 \qquad ...(1)$$
$$r_1 + r_2 + r_3 = 10 \qquad ...(2)$$
From equation (1) and (2)
$$r_1 + 2(10 - r_3) - 5r_3 = 0$$
$$r_1 + 20 = 7r_3$$
$$(r_1, r_2, r_3) = (1, 6, 3)$$
Constant term $= \frac{\lfloor 10}{\lfloor 1 \lfloor 6 \lfloor 3} (3)^1 (-2)^6 (5)^3$
$$= 2^9.3^2.5^4.7^1$$
$$l = 9$$

65. (C)
$$\therefore \left(\left(\vec{a} \times \vec{b}\right) \cdot \vec{c}\right)^2 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$$
$$\Rightarrow \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = |a|^2 |b|^2 |c|^2 \sin^2 \frac{\pi}{6}$$
$$= \frac{\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)}{4}$$

66. (D)
Put $\frac{1}{2}\cos x = t$, so that $-\sin x dx = 2dt$ and $I = \int_{1/2}^{-1/2} e^{|t|} (2\sin t + 3\cos t)(-2) dt$
As $e^{|t|}$ sin t is an odd function, and $e^{|t|}$ cos t is an even function,
$$I = 12\int_0^{1/2} e^t \cos t \, dt \;\Rightarrow\; I = 6\sqrt{e}\left(\cos\frac{1}{2} + \sin\frac{1}{2} - 1\right)$$

67. (A)
Let $t = x + \sqrt{x^2 + b^2}$
$$\Rightarrow \frac{1}{t} = \frac{1}{x + \sqrt{x^2 + b^2}} = \frac{\sqrt{x^2 + b^2} - x}{b^2}$$
$$\therefore t - \frac{b^2}{t} = 2x \text{ and } t + \frac{b^2}{t} = 2\sqrt{x^2 + b^2}$$
Thus $2(a - x)\left(x + \sqrt{x^2 + b^2}\right) = \left(2a - t + \frac{b^2}{t}\right)(t)$

$= 2at - t^2 + b^2 = a^2 + b^2 - \left(a^2 - 2at + t^2\right)$

$= a^2 + b^2 - (a - t)^2$

Therefore, $y = 2(a - x)\left(x + \sqrt{x^2 + b^2}\right) \le a^2 + b^2$

Hence, the maximum value of y is $a^2 + b^2$. This value is attained at $t = a$ or $x = \dfrac{\left(a^2 - b^2\right)}{2a}$

68. (C)

$\tan^{-1}\dfrac{1}{1+n(n+1)} = \tan^{-1}\dfrac{n+1-n}{1+n(n+1)}$

$= \tan^{-1}(n+1) - \tan^{-1}(n)$

So that L.H.S. of the given equation is

$\tan^{-1}2 - \tan^{-1}1 + \tan^{-1}3 - \tan^{-1}2 + ... + \tan^{-1}(n+1) - \tan^{-1}n$

$= \tan^{-1}(n+1) - \tan^{-1}1 = \tan^{-1}\dfrac{n+1-1}{1+(n+1)}$

$= \tan^{-1}\dfrac{n}{n+2}$

So that $\tan^{-1}\dfrac{n}{n+2} = \tan^{-1}\theta = \theta = \dfrac{n}{n+2}$

69. (C)

Let $x_0 = \cos\theta$, then $x_1 = \sqrt{\dfrac{1}{2}(1+\cos\theta)} = \cos\dfrac{\theta}{2}, x_2 = \cos\left(\dfrac{\theta}{2^2}\right), x_3 = \cos\left(\dfrac{\theta}{2^3}\right)$….. and so on

So that $\left[\dfrac{\sqrt{1-x_0^2}}{x_1x_2x_3.....\text{to infinite}}\right]$

$$= \frac{\sin\theta}{\cos\frac{\theta}{2}\cos\frac{\theta}{2^2}.....\cos\frac{\theta}{2^n}.....\text{infinite}}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}\cos\frac{\theta}{2^2}......\cos\frac{\theta}{2^n}.......\text{infinite}}$$

$$= \frac{2^2\sin\frac{\theta}{2^2}\cos\frac{\theta}{2^2}}{\cos\frac{\theta}{2^2}\cos\frac{\theta}{2^3}.......\cos\frac{\theta}{2^n}......\text{infinite}}$$

$$= \lim_{n\to\infty}\frac{2^n\sin\frac{\theta}{2^n}}{\cos\frac{\theta}{2^{n+1}}} = \lim_{n\to\infty}\theta\left(\frac{\sin\frac{\theta}{2^n}}{\frac{\theta}{2^n}}\right)\frac{1}{\cos\frac{\theta}{2^{n+1}}}\theta$$

So that $\cos\left[\dfrac{\sqrt{1-x_0^2}}{x_1x_2.....\text{inf}}\right]=\cos\theta=x_0$

70. (D)

We have $f(x)=x^2+2bx+2c^2=(x+b)^2+2c^2-b^2$

$\Rightarrow \min f(x)=2c^2-b^2$

Also $g(x)=-x^2-2cx+b^2$

$=b^2+c^2-(x+c)^2$

So $\max(g)=b^2+c^2$

Since, $\min f(x)>\max g(x)$,

So $2c^2-b^2>b^2+c^2$

$\Rightarrow c^2>2b^2=|c|>\sqrt{2}|b|$.

71. (D)

$|\vec{a}|=1=|\vec{b}|$

$\phi(\theta)=\int_{-\cos^2\theta}^{\sin^2\theta} f^2(x).dx$

$2f(x)=\dfrac{2x}{x^2}\Rightarrow \boxed{\dfrac{1}{x}=f(x)}$

$\phi(\theta)=\int_{-\cos^2\theta}^{\sin^2\theta}\dfrac{1}{x^2}.dx$

$=\left[\dfrac{-1}{x}\right]_{-\cos^2\theta}^{\sin^2\theta}$

$=(-\operatorname{cosec}^2\theta)-(\sec^2\theta)$

$=-\operatorname{cosec}^2\theta.\sec^2\theta$

$\phi(\theta)=\dfrac{-4}{\sin^2 2\theta}=-4\operatorname{cosec}^2 2\theta$

Fundamental period is $\left(\dfrac{\pi}{2}\right)$

72. (D)

$x=-10$ is the directrix of the given parabola. Hence the circle $S=0$ will touch it. P_1P_2 is the diameter. So $\angle P_1PP_2=90^\circ$ and hence ΔP_1PP_2 is right angled and its orthocentre is point P.

Tangents at extremities of focal chord are perpendicular.

Hence in their combined equation (coefficient x^2) + $\left(\text{coefficieint } y^2\right)=0$

73. (C)

5^{5^5} is an odd natural number

$\therefore x = 5^{5^{5^5}} = 5^{2m+1} = (25^m).5$

$m \in N$

$x = (24+1)^m .5 = 5 + a$ multiple of 24

Hence Remainder $= 5$

74. (B)

$f(g(x)) = f(|3x+4|)$

$-5 \le 3x+4 \le 5$

$x \in \left[-3, \frac{1}{3}\right]$

75. (D)

$y = (\sin^{-1} x)^4 + (\cos^{-1} x)^4$

$\frac{dy}{dx} = \frac{4(\sin^{-1} x)^3}{\sqrt{1-x^2}} - \frac{4(\cos^{-1} x)^3}{\sqrt{1-x^2}} = 0$

$\Rightarrow \sin^{-1} x = \cos^{-1} x$

$\Rightarrow x = \frac{1}{\sqrt{2}}$

$f(1) = \frac{\pi^4}{16}$

$f(-1) = \frac{\pi^4}{16} + \pi^4 = \frac{17\pi^4}{16}$ (Maximum)

$F\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi^4}{128}$ (Minimum)

76. (D)

Let $f(x) = 3\tan x + x^3 - 2$

Then $f'(x) = 3\sec^2 x + 3x^2 > 0$

Hence $f(x)$ increases.

Also $f(0) = -2$ and $f\left(\frac{\pi}{4}\right) > 0$

So, by intermediate value theorem, $f(c) = 2$ for some c is $\left(0, \frac{\pi}{4}\right)$

Hence $f(x) = 0$ has only one root.

77. (B)

$\vec{b}.\vec{c} = 0$

$\tan\alpha = -2, 3$

$\tan\alpha = -2$

$\Rightarrow \alpha \to 2^{nd}$ or 4^{th} quadrant

$\tan\alpha = 3$

$\Rightarrow \alpha \to 1^{st}$ or 3^{rd} quadrant
Now,
$\vec{a}.\hat{k} < 0$
$\sin 2\alpha < 0$
$(2n-1)\pi < 2\alpha < 2n\pi$
$\frac{(2n-1)\pi}{2} < \alpha < n\pi$ & $\tan\frac{\alpha}{2} > 0$
$\Rightarrow \alpha \to 2^{nd}$ quadrant
Or, $\tan\alpha = -2$
Or, $\alpha = (2n+1)\pi - \tan^{-1} 2$

78. (C)
$y = u^m$
$\frac{dy}{dx} = m.u^{m-1}\frac{du}{dx}$
The given differential equation becomes $2x^4u^m.mu^{m-1}\frac{du}{dx} + u^{4m} = 4x^6$ or $\frac{du}{dx} = \frac{4x^6 - u^{4m}}{2mx^4x^{2m-1}}$
For homogenous equation, degree should be same for numerator and denominator, hence
$6 = 4m = 4 + 2m - 1 \Rightarrow m = \frac{3}{2}$

79. (A)
$(x^2 - 7x + 6)(x^2 - 7x + 12) + 10$
Taking first and last terms together and middle terms together. Now make quadratic in
$x^2 - 7x = t$
We get, $(t+6)(t+12)+10$
$= t^2 + 18t + 82$
$D = -4 < 0$
Hence value of always positive $\forall x \in R$

80. (C)
Second diagonal will pass through the centre of the hyperbola

81. (0.00)
$$|A(\operatorname{adj} A)|^{1/3} = |A| \Rightarrow \frac{d^2}{dx^2}\left(|A(\operatorname{adj} A)|^{1/3}\right) = \begin{vmatrix} 3 & 2 & 1 \\ 12 & 12x & 12x^2 \\ 1 & a & a^2 \end{vmatrix} = 0 \text{ at } x = a$$
$= 0$

82. (5.00)
The system has a non zero solution
$\therefore |A| = 0$

$$\begin{vmatrix} 2 & 3 & -1 \\ 3 & 2 & k \\ 4 & 1 & 1 \end{vmatrix} = 0$$

The solution is $(2a, -3a, -5a)$

Lest positive integer value of z is 5.

83. (0.00)

$$f(x) = \int_{\frac{\pi}{2}}^{x} (\cos t - 2\sin t)dt$$

$$f'(x) = \cos x - 2\sin x < 0 \text{ in} \left[\frac{\pi}{2}, \pi\right]$$

$\therefore f(x)$ is decreasing function

$\therefore$ Greatest value $= f\left(\frac{\pi}{2}\right) = 0$

84. (1.00)

$$dy = x\tan y(\sin y - 1)dx$$

$$\frac{dy}{dx} + x\tan y = x\tan y.\sin y$$

$$\cos y\frac{dy}{dx} + x\sin y = x\sin^2 y$$

Let $\sin y = t$

$$\frac{dt}{dx} + xt = xt^2$$

$$\frac{1}{t^2}\frac{dt}{dx} + x\left(\frac{1}{t}\right) = x$$

$$\frac{1}{t} = u$$

$$= -\frac{du}{dx} + xu = x$$

$$\frac{du}{dx} + (-x)u = -x$$

$$ue^{-\int xdx} = \int -xe^{-\int xdx}dx$$

$$ue^{-\frac{x^2}{2}} = -\int xe^{-\frac{x^2}{2}}dx = e^{-\frac{x^2}{2}} + c$$

$$u = +1 + ce^{\frac{x^2}{2}}$$

$$t = \frac{1}{1 + ce^{\frac{x^2}{2}}}$$

$$\sin y = \frac{1}{1 + ce^{\frac{x^2}{2}}}$$

85. (1.60)
Distance between the foci
$2ae = 4\sqrt{2}$
$ae = 2\sqrt{2}$
Product of distances of foci from tangent $= b^2$
$2 \times 6 = b^2$
$b^2 = a^2 - a^2e^2$
$12 = a^2 - 8$
$a^2 = 20$
$a = \sqrt{20}$
Major axis $2a = \sqrt{80}$
$[2a] = 8$
$\frac{[x]}{5} = \frac{8}{5} = 1.6$

86. (1.00)
In the 1st case
P(they match) $= \frac{^3C_2 + ^nC_2}{^{n+3}C_2} = \frac{1}{2}; \frac{6 + n(n-1)}{(n+3)(n+2)} = \frac{1}{2}$
$\Rightarrow 2(n^2 - n + 6) = n^2 + 5n + 6$
$\Rightarrow n^2 - 7n + 6 = 0$
$\Rightarrow n = 1 \text{ or } 6$
In the 2nd case,
$\frac{3}{n+3} \times \frac{3}{n+3} + \frac{n}{n+3} \times \frac{n}{n+3} = \frac{5}{8}$
Solving, $n^2 - 10n + 9 = 0$
$n = 9 \text{ or } 1$ ……..(ii)
From equation (i) and (ii), $n = 1$

87. (9.00)
Given,
$a + 19d = \log_{10} 20$ …….(1)
$a + 31d = \log_{10} 32$ ………(2)
(2)- (1)
$12d = \log_{10} \frac{32}{20} = \log_{10} 16 - 1$
$12d = 4\log_{10} 2 - 1$
$\log_{10} 2 = \frac{12d + 1}{4}$ …….(A)
Again (2) + (1)
$2a + 50d = \log_{10} 640 = 6\log_{10} 2 + 1$
$\log_{10} 2 = \frac{2a + 50d - 1}{6}$ …….(B)

$$\therefore \frac{12d+1}{4}=\frac{2a+50d-1}{6} \Rightarrow 36d+3=4a+100d-2$$

$$4a+64d=5$$

$$\underbrace{a+16d}_{17^{th}\text{ term}}=\frac{5}{4}$$

Hence, 17^{th} term is rational and its value is $\frac{5}{4}=\frac{p}{q} \Rightarrow (p+q)=9$

88. (2.50)

$$a-1=0 \Rightarrow a=1$$

$$L=\lim_{x\to 0}\frac{ae^x+2\sin 2x+b\cos bx}{2x}$$

$$1+b=0 \Rightarrow b=-1$$

$$L=\lim_{x\to 0}\frac{ae^x+4\cos 2x-b^2\sin bx}{2}=\frac{a+4}{2}=\frac{5}{2}=2.5$$

89. (32.00)

$$3\cos^2 2\theta+6\cos 2\theta-10\cos^2\theta+5=0 \Rightarrow 3\cos^2 2\theta+6\cos 2\theta-5(1+\cos 2\theta)+5=0$$

$$3\cos^2 2\theta+\cos 2\theta=0 \Rightarrow \cos 2\theta=0 \text{ or } \cos 2\theta=-\frac{1}{3}$$

$$\theta=[-4\pi, 4\pi] \Rightarrow 2\theta=(2n+1).\frac{\pi}{4}$$

$$\therefore \theta=\pm\frac{\pi}{4},\pm\frac{3\pi}{4}......\pm\frac{15\pi}{4}$$

Similarly $\cos 2\theta=-\frac{1}{3}$ gives 16 solution.

90. (2.07)

$$R=SH=5$$

Sides are $5\sqrt{2},5\sqrt{2},10$

$$r=\frac{\Delta}{S}$$