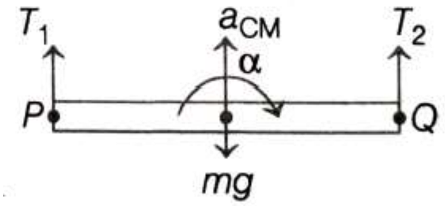
PART (A) : PHYSICS

SOLUTIONS

1. (A, B)

$$a_A = \frac{dv_A}{dt} = 3 \text{ and } a_B = \frac{dv_B}{dt} = 1$$



$$a_P = a_{CM} + (0.5)\alpha = 3 \qquad \text{..... (i)}$$

$$a_Q = a_{CM} - (0.5)\alpha = 1 \qquad \text{..... (ii)}$$

Solving we get, $a_{CM} = 2\ m/s^2$ and $\alpha = 2\ rad/s^2$

$$\sum F_{ext} = ma_{CM} \Rightarrow T_1 + T_2 - mg = ma_{CM} \qquad \text{..... (iii)}$$

$$\sum \tau = I_{CM}\alpha \Rightarrow T_1(0.5) - T_2(0.5) = \left(\frac{ml^2}{12}\right)\alpha \qquad \text{..... (iv)}$$

Solving Eqs. (iii) and (iv), we get

$$T_1 = \frac{185}{6}N \text{ and } T_2 = \frac{175}{6}N$$

2. (A, D)

$$F_G = \frac{GMm}{R^3}\left(\frac{R}{2}\right) = \frac{GMm}{2R^2}$$

$$N + F_G = m\omega^2\left(\frac{R}{2}\right)$$

$$\Rightarrow 2F_G + F_G = m\omega^2\left(\frac{R}{2}\right) \Rightarrow \omega = \sqrt{\frac{3GM}{R^3}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{3g}}$$

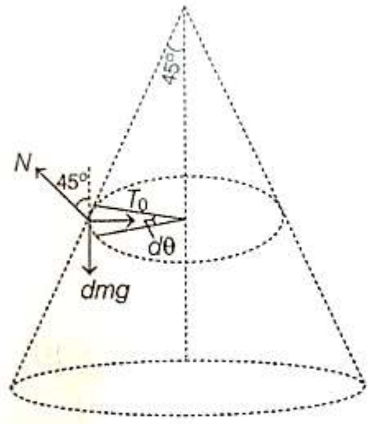
$$F_G = m\omega^2 r \Rightarrow \frac{GMmr}{R^3} = m\omega^2 r$$

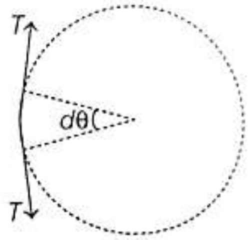
$$\Rightarrow \omega = \text{constant}$$

$$\Rightarrow T = \text{constant}$$

3. (A, B, C)

$$T_0 = 2T\sin\left(\frac{d\theta}{2}\right) = Td\theta$$





$$N\cos 45° = dmg$$
$$N\sin 45° = T_0 = Td\theta$$

From Eqs. (i) and (ii), we get

$$Td\theta = (dm)g$$

$$\Rightarrow \quad Td\theta = \left(\frac{m}{2\pi R} rd\theta\right)g \Rightarrow T = \frac{mg}{2\pi}$$

Extension in the ring $= \dfrac{FL}{AY} = \left(\dfrac{mg}{2\pi}\right)\dfrac{2\pi R}{aY} = \dfrac{mgR}{aY}$

Energy stored in the ring $= \dfrac{1}{2}\dfrac{(\text{Stress})^2}{Y} \times (\text{Volume})$

$$= \frac{1}{2}\left(\frac{mg}{2\pi a}\right)^2 \frac{1}{Y}(2\pi Ra)$$

$$= \frac{m^2g^2R}{4\pi Ya}$$

4. (A)

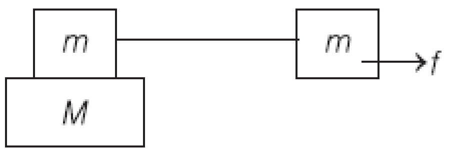
$$H_A = \frac{u^2\sin^2\alpha}{2g\cos\theta}$$

$$H_B = \frac{u^2}{2g\cos\theta}$$

$$H_C = \frac{u^2\cos^2\alpha}{2g\cos\theta}$$

$$\Rightarrow \quad H_A + H_C = H_B$$

5. (A)



For the above system,

$\sum F_x = ma_x$

$\Rightarrow$ $f = (M+2m)a$

and $f \le f_L$

$(M+2m)a \le \mu mg$

$a \le \dfrac{\mu mg}{M+2m}$

Taking all the four blocks as system,

$\sum F_x = ma_x$

$\Rightarrow$ $F = (2M+2m)a$

$\Rightarrow$ $F \le \dfrac{2(M+m)\mu mg}{M+2m}$

6. (A)

Speed is maximum when acceleration is zero.

$\Rightarrow$ $F - mg\sin\theta - \mu mg\cos\theta = 0$

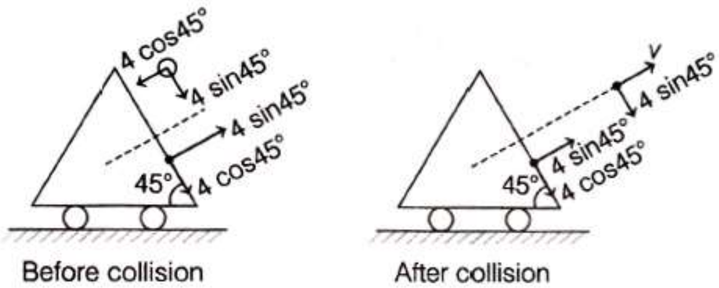
$\Rightarrow$ $F = mg\sin\theta + \mu mg\cos\theta$

$P = Fv$

$\Rightarrow$ $P = (mg\sin\theta + \mu mg\cos\theta)v_{max}$

$\Rightarrow$ $v_{max} = \dfrac{P}{mg\sin\theta + \mu mg\cos\theta}$

7. (D)



$e = 1 = \dfrac{v - 4\sin 45°}{4\cos 45° + 4\sin 45°}$

$\Rightarrow$ $v = 6\sqrt{2}$ m/s

$v_{ball} = \sqrt{(6\sqrt{2})^2 + (2\sqrt{2})^2} = 4\sqrt{5}$ m/s

8. (7)

Lets take a strip of thickness *dh* at depth *h*. Velocity of liquid flowing out of the strip

$= v = \sqrt{2gh}$

Volume flow rate

$= \dfrac{dV}{dt} = \int_{h_1}^{h_2} (\sqrt{2gh})(l\,dh) = (\sqrt{2g})\,l\left[\dfrac{h^{3/2}}{3/2}\right]_{h_1}^{h_2}$

$= 7\sqrt{2g}\ m^3/s$

9. (25)

Length of diagonal at time $t = 10\,t$

$$\phi = BA = B\left(\frac{10t}{\sqrt{2}}\right)^2 = 50Bt^2$$

$$\varepsilon = \frac{d\phi}{dt} = 100Bt = 10t$$

$$R = (0.01)4\left(\frac{10t}{\sqrt{2}}\right) = \frac{0.4t}{\sqrt{2}}$$

$$i = \frac{\varepsilon}{R} = 25\sqrt{2}\text{ A}$$

10. (3)

In absence of spring, $L\frac{di}{dt} = B\ell V$

$$\Sigma F = ma \Rightarrow mg - Bil = m\frac{dv}{dt}$$

Differentiating both sides w.r.t. t,

$$\Rightarrow \quad -B\frac{di}{dt}l = m\frac{d^2v}{dt^2}$$

$$\Rightarrow \quad \frac{d^2v}{dt} = -\left(\frac{B^2l^2}{mL}\right)v$$

$$\Rightarrow \quad v = A\omega\sin\omega t \text{ where, } \omega = \frac{Bl}{\sqrt{mL}}$$

$$a = \frac{dv}{dt} = A\omega^2\cos\omega t$$

At $t = 0; a = g = A\omega^2 \Rightarrow A = \frac{g}{\omega^2}$

So, $X_1 = \frac{g}{\omega^2} = \frac{mgL}{B^2l^2}$

In presence of spring, $L\frac{di}{dt} = Blv$

$$\Sigma F = ma \Rightarrow mg - Bil - kx = ma$$

$$\Rightarrow \quad -B\frac{di}{dt}l - k\frac{dx}{dt} = m\frac{d^2v}{dt^2}$$

$$\Rightarrow \quad \frac{d^2v}{dt^2} = -\left(\frac{B^2l^2}{mL} + \frac{k}{m}\right)v$$

$$\Rightarrow \quad v = A\omega\sin\omega t\text{, where } \omega = \sqrt{\frac{B^2l^2}{mL} + \frac{k}{m}}$$

$$a = \frac{dv}{dt} = A\omega^2\cos\omega t$$

At $t = 0; a = g = A\omega^2 \Rightarrow A = \frac{g}{\omega^2}$

So, $X_2 = \frac{g}{\omega^2} = \frac{mgL}{B^2l^2 + kL}$

$$\frac{X_1}{X_2} = \frac{B^2l^2 + kL}{B^2l^2} = 3$$

11. (4)

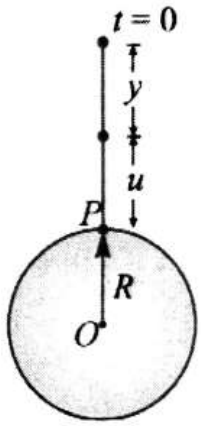
Loss in magnetic potential energy = Gain in gravitational potential energy

$$-MB\cos 90^\circ - (-MB\cos 0^\circ) = mgh$$

$$\Rightarrow \quad i\left(\frac{\sqrt{3}}{4}a^2\right)B = mg\left(\frac{a}{\sqrt{3}}\right) \Rightarrow B = \frac{4mg}{3la} = 4T$$

12. (2)

If y is the distance falls by the ball in time t, then $y = \frac{1}{2}gt^2$. The distance of the ball from the point P of the sphere, $u = -\left(y_0 - \frac{1}{2}gt^2\right)$ and velocity of ball, $v_0 = gt$.



By refraction formula

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R} \qquad ...(i)$$

$$\frac{\mu}{v} - \frac{1}{-\left(y_0 - \frac{1}{2}gt^2\right)} = \frac{\mu - 1}{+R}$$

$$\therefore \frac{\mu}{v} = \left[\frac{(\mu-1)\left(y_0 - \frac{1}{2}gt^2\right) - R}{R\left(y_0 - \frac{1}{2}gt^2\right)}\right]$$

Differentiating equation (i), we have

$$\mu\left(-\frac{1}{v^2}\right)\frac{dv}{dt} - \left(\frac{-1}{u^2}\right)\frac{du}{dt} = 0$$

$$\therefore \frac{dv}{dt} = \frac{1}{\mu}\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$$

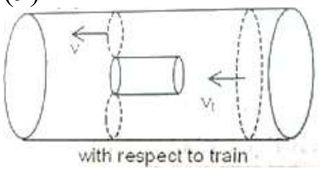
Or image velocity

$$v_i = \frac{1}{\mu}\frac{v^2}{u^2}v_0.$$

After substituting the values of u, v and v_0, we get

$$v_i = \frac{\mu R^2 gt}{\left[(\mu-1)\left(y_0 - \frac{1}{2}gt^2\right) - R\right]^2}$$

13. (9)



Applying Bernoulli's equation

$$P_0 + \frac{1}{2}\rho v_t^2 = P + \frac{1}{2}\rho v^2$$

$$P_0 - P = \frac{1}{2}\rho\left(v^2 - v_t^2\right) \quad ...(i)$$

From equation to continuity

Also, $4S_t v_t = v \times 3S_t \Rightarrow v = \frac{4}{3}v_t \quad ...(ii)$

From (i) and (ii)

$$P_0 - P = \frac{1}{2}\rho\left(\frac{16}{9}v_t^2 - v_t^2\right) = \frac{1}{2}\rho\frac{7v_t^2}{9}$$

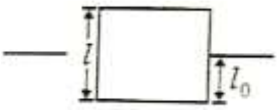
$\therefore\ N = 9$

14. (D)

15. (D)

16. (B)
(A) The angle of deviation, $\delta = 180° - 2i = 180° - 2\times 60° = 60°$
(B) Glass slab produces no deviation and so $\delta = 0$.
(C) Deviation in both the cases are 180°.
(D) $\delta = 60° - 30° = 30°$

17. (C)

(Q) 

$$Al_0 g \times r_w = mg$$

$$mg - A\left(r_0 + x\right) g\, r_w = ma_n$$

$$a_n = \frac{-A\rho_w gx}{m}$$

$$\omega = \sqrt{\frac{A\rho_w gx}{m}}$$

$$= \sqrt{\frac{\rho_w g}{\rho_w l}}$$

(R) $T = 2\pi\sqrt{\frac{I}{mgx}} = 2\pi\sqrt{\frac{\frac{1}{2}mR^2}{mgx}}$

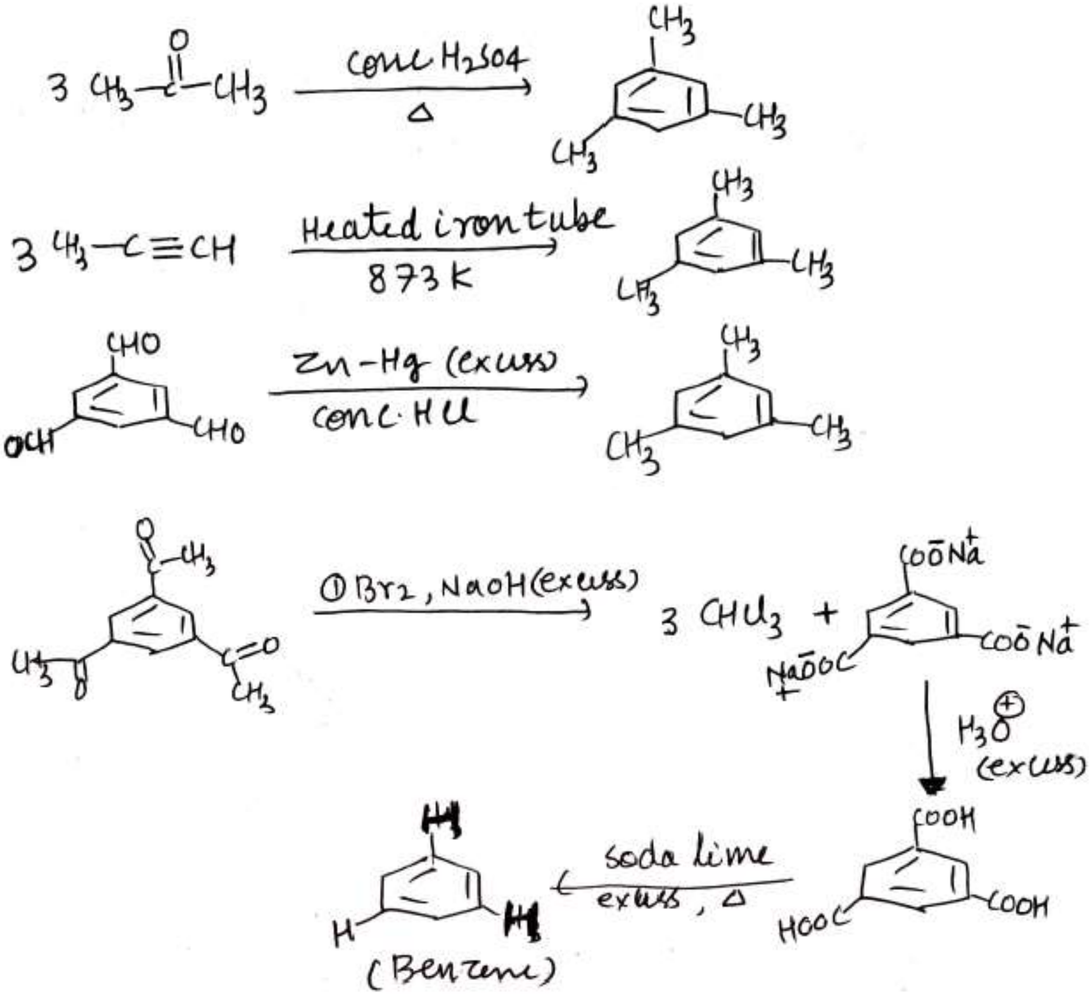
$$= 2n\sqrt{\frac{R}{2g}}$$

PART (B) : CHEMISTRY

SOLUTIONS

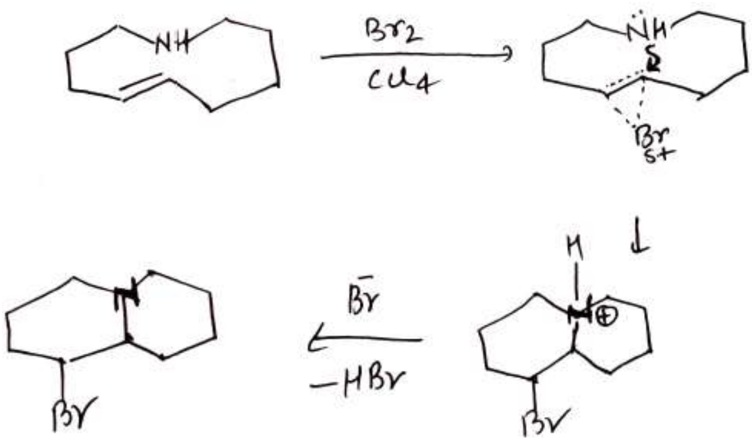
18. (A, D)

19. (A, B, D)



20. (B, C, D)

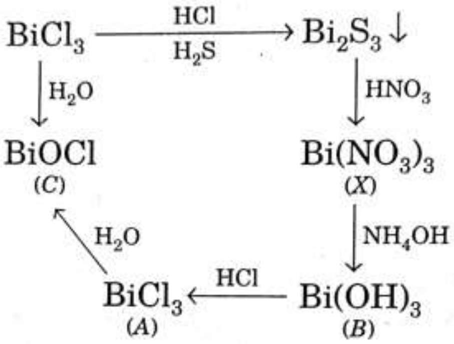
21. (D)



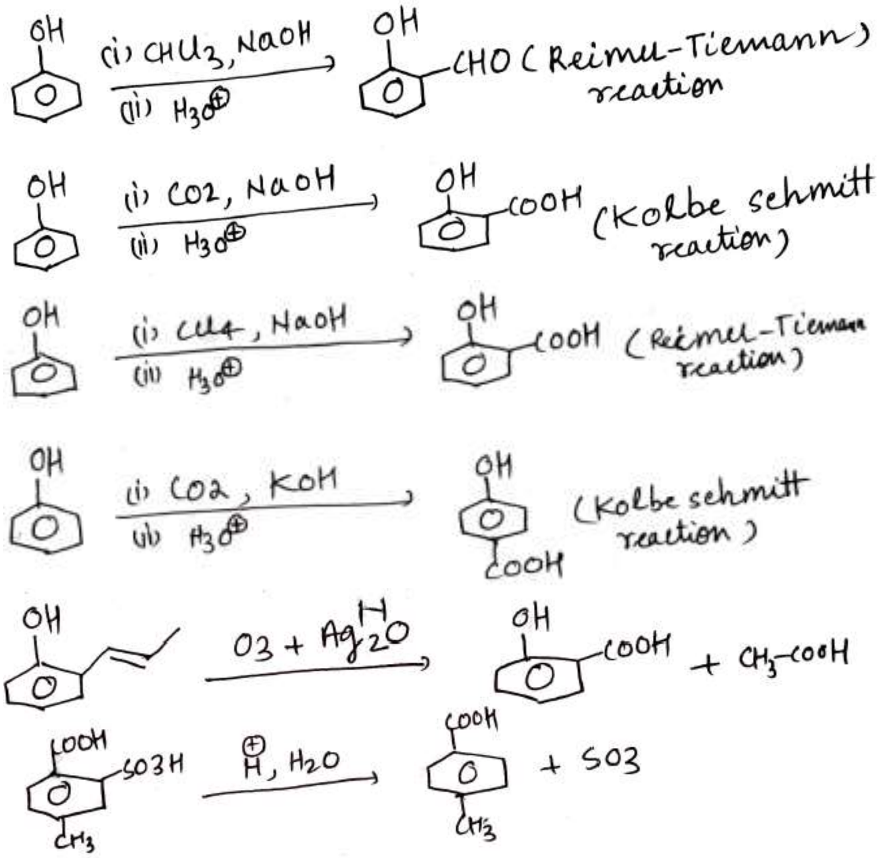
22. (A)

23. (D)
If vibrational degree are not activated, then γ of CH_4 & SO_3 will be equal.

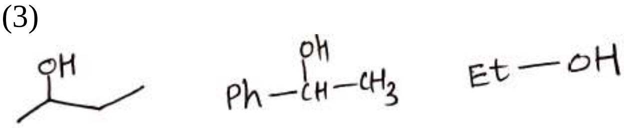
24. (A)



25. (3)



26. (3)



27. (5)
(2), (3), (6), (7), (8)

28. (3)

$$2H_2O(g)+2Cl_2(g) \rightleftharpoons 4HCl(g)+O_2(g)$$

i	0.5	2	2	2
e	0.5	2 – 2*x*	2 + 4*x*	2 + *x*

$2-2x \approx 0$

$$12\times10^8 = \frac{6^4\times3}{(0.5)^2(2-2x)^2}$$

$$2-2x = \frac{6\times6\times\sqrt{3}}{0.5\times10^4\times2\times\sqrt{3}}$$

$$= 36\times10^{-3}\text{ atm}$$

29. (9)

All have longer bond length than free CO molecule.

30. (8)

$$V_{\text{unit cell}} = \frac{4\times60.23}{50\times N_A}\text{cc}$$

$$= \frac{4\times6.23}{50\times6.023\times10^{23}}$$

$$= \frac{4}{5}\times10^{-23}\text{ cc}$$

$$\text{No. of unit cells } = \frac{64\times10^{-30}}{\frac{4}{5}\times10^{-23}\times10^{-6}}$$

$$= \frac{64\times5}{4\times10} = 8$$

No. of cation = 16

16 – 8 = 8

31. (A)

32. (C)

33. (B)

(P) $-\frac{d[A]}{dt}$ *vs* $[A]^3$

(Q) $[A] \propto -t;\ [A]=[A]_0 - Kt$

(R) $t_{1/2} \propto a^2$

(S) $-\frac{d[A]}{dt} \propto [A]^2$

34. (D)

PART (C) : MATHEMATICS

SOLUTIONS

35. **(A, D)**

Since, $A_\alpha A_\beta = A_{\alpha+\beta}$ & $A_0 = I$

$\Rightarrow A_\alpha A_{(-\alpha)} = I \Rightarrow \left(A_\alpha^{-1}\right) = A_{(-\alpha)}$

Also, $A_\pi A_{\frac{3\pi}{2}} A_{\frac{5\pi}{4}} ..... = A_s$;

Where $S = \pi + \frac{3\pi}{2} + \frac{5\pi}{4} + \frac{7\pi}{8} + ... = 6\pi$

Again, $Adj\left(A_\alpha\right) = A_{(-\alpha)}$

36. **(A, D)**

Let the plane be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Hence, $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = 1$

Volume of tetrahedron $= \frac{abc}{6} = V$

Since, A.M. $\geq$ G.M.

$$\Rightarrow \frac{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}{3} \geq \left(\frac{1}{abc}\right)^{2/3}$$

$\Rightarrow V \geq \frac{\sqrt{3}}{2}$ (from (1))

For $V_{\min}$; $a^2 = b^2 = c^2 = 3$

Also, s.d = Length of perpendicular distance from the origin to the line $x + y = \sqrt{3}$ in $x - y$ plane.

37. **(A, B)**

Let (h, k) be the circumcircle of ΔABC

Then, $h = \frac{4+3p}{2p}$; $k = \frac{4-3p}{2q}$

$\Rightarrow p = \frac{4}{2h-3}$; $q = \frac{4h-12}{k(2h-3)}$

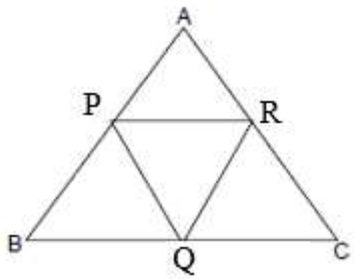
This (p, q) lies on $x^2 = 4ay$

$\Rightarrow$ Locus of (h, k) is $\frac{1}{2a}\left(y + \frac{9}{8}a\right) = \left(x - \frac{9}{4}\right)^2$

38. **(B)**

Now by property of triangles

$RQ = \frac{1}{2} BC = \frac{a}{2}$



Similarly $PQ = \frac{c}{2}, PR = \frac{b}{2}$

Area of $\Delta ABC = \frac{abc}{4R}$

That of $\Delta PQR = \frac{\frac{a}{2}\frac{b}{2}\frac{c}{2}}{4R'}$

Also area of $\Delta ABC = 4$ (area of ΔPQR)

$\Rightarrow \frac{abc}{32R'} = \frac{abc}{16R} = \left(\frac{R}{R'} = \frac{2}{1}\right)$

39. **(C)**

$\vec{r} = (\vec{a}\times\vec{b})\sin x + (\vec{b}\times\vec{c})\cos y + 2(\vec{c}\times\vec{a})$

$\vec{r}.(\vec{a}+\vec{b}+\vec{c}) = 0$

$\Rightarrow [\vec{a}\vec{b}\vec{c}](\sin x + \cos y + 2) = 0$

$\Rightarrow [\vec{a}\vec{b}\vec{c}] \neq 0$

$\Rightarrow \sin x + \cos y = -2$

This is possible only when $\sin x = -1$ and $\cos y = -1$

For $x^2 + y^2$ to be minimum $x = -\frac{\pi}{2}$ and $y = \pi$

$\Rightarrow$ minimum value of $(x^2 + y^2)$ is

$= \frac{\pi^2}{4} + \pi^2 = \frac{5\pi^2}{4}$

40. **(D)**

$\frac{1}{\sqrt{r+\sqrt{r^2-1}}} = \sqrt{\frac{r+1}{2}} - \sqrt{\frac{r-1}{2}}$

41. (C)

42. **(6)**

$\int_0^1 \ln(x)\ln(1-x)\,dx = -\int_0^1 \ln(x)\sum_{n=1}^{\infty}\frac{x^n}{n}\,dx$

$= -\sum_{n=1}^{\infty}\frac{1}{n}\int_0^1 x^n \ln(x)\,dx$

So, let $I_n = \int_0^1 x^n \ln(x)\,dx$.

Using integration by parts, we see $I_n = -\frac{1}{(n+1)^2}$,

So, $\int_0^1 \ln(x)\ln(1-x)\,dx = \sum_{n=1}^{\infty}\frac{1}{n(n+1)^2}$

$$\int_0^1 \ln(x)\ln(1-x)\,dx = \sum_{n=1}^{\infty}\frac{1}{n(n+1)^2}$$
$$= \sum_{n=2}^{\infty}\frac{1}{(n-1)n^2}$$
$$= \sum_{n=2}^{\infty}\frac{1}{n-1}-\frac{1}{n}-\frac{1}{n^2}$$
$$= \sum_{n=2}^{\infty}\left(\frac{1}{n-1}-\frac{1}{n}\right)-\sum_{n=2}^{\infty}\frac{1}{n^2}$$
$$= \frac{1}{2-1}-\left(\frac{\pi^2}{6}-1\right)$$
$$= 2-\frac{\pi^2}{6}$$

43. (7)

$(\mathrm{EM})^{\mathrm{T}} = 20\mathrm{I}$

Take transpose on both sides

$\mathrm{EM} = 20\mathrm{I}$

$(\mathrm{E}+\mathrm{M})^{\mathrm{T}} = 17(\mathrm{E}-\mathrm{M})^{\mathrm{T}}$

$\mathrm{E}^{\mathrm{T}}+\mathrm{M}^{\mathrm{T}} = 17\left(\mathrm{E}^{\mathrm{T}}-\mathrm{M}^{\mathrm{T}}\right)$

$16\mathrm{E}^{\mathrm{T}} = 18\mathrm{M}^{\mathrm{T}}$

Take transpose on both sides

$16\mathrm{E} = 18\mathrm{M}$

From Eqns. (1) and (2), we get

$$\mathrm{E}^2+\mathrm{M}^2 = \frac{725}{18}\mathrm{I} \Rightarrow \mathrm{a}+\mathrm{b} = 743$$

44. **(2)**

$$I_1 = \int_0^1 \frac{dx}{e^x(1+x)} \text{ and } I_2 = \int_0^{\pi/4} \frac{e^{\tan^2\theta}}{2-\tan^2\theta}\cdot\frac{\sin\theta}{\cos^3\theta}\,d\theta \xrightarrow{\tan^2\theta = x} I_2 = \frac{1}{2}\int_0^1 \frac{e^x}{2-x}\,dx$$

$$I_2 = \frac{1}{2}\int_0^1 \frac{e^x}{2-x}\,dx = \frac{1}{2}\int_0^1 \frac{e^{1-x}}{2-(1-x)}\,dx \qquad \text{[Apply } (a-x) \text{ property]}$$

$$I_2 = \frac{e}{2}\int_0^1 \frac{dx}{e^x(1+x)} \qquad \therefore \quad \frac{I_1}{I_2} = \frac{2}{e} \Rightarrow \frac{eI_1}{I_2} = 2$$

45. **(1)**

$t^2 f(x) - 2tf'(x) + f''(x) = 0$, has equal roots

Discriminant $= 4\left(f'(x)\right)^2 - 4f(x)\,f''(x) = 0$

The above equation can be expressed as: $\frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}$

Integrating both sides, we get : $\ln(f'(x)) = \ln f(x) - \ln c$ or $f(x) = cf'(x)$

$\Rightarrow f(0) = cf'(0) \Rightarrow c = \frac{1}{2}$ Hence, $\frac{f'(x)}{f(x)} = 2$

$\ln f(x) = 2x + k \xrightarrow{f(0)=1} \ln f(x) = 2x$ or $f(x) = e^{2x}$

$t^2e^{2x} - 4te^{2x} + 4e^{2x} = 0 \Rightarrow t^2 - 4t + 4 = 0$ or $t = 2$

Now, $\lim_{x\to 0} \frac{f(x)-1}{x} - \frac{t}{2} = \lim_{x\to 0} \frac{e^{2x}-1}{2x} \times 2 - \frac{2}{2} = 2 - 1 = 1$

46. **(0)**

At the point of maxima, $f'(x) = 0$ and $f''(x) < 0$

$\Rightarrow f''(x) = -9 + x^2 + f^2(x) < 0$ or $x^2 + f^2(x) < 9$

$\Rightarrow P(x, f(x))$ lies inside $x^2 + y^2 = 9$

No tangent is possible.

47. **(1)**

$B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $B_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\therefore B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |B| = 1$

So, $|\text{adj } B| = |B|^2 = (1)^2 = 1$

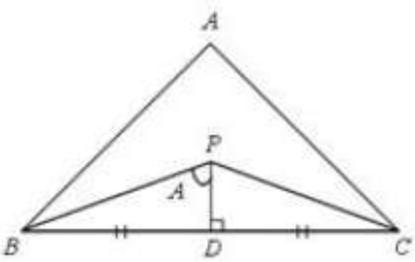
48. **(C)**

(P) – 4; (Q) – 3; (R) – 2; (S) - 1

(A) $\tan A \tan C = 3 \Rightarrow \sin A \sin C = 3\cos A \cos C$

(B) $\tan A = \frac{BD}{PD} = \frac{\frac{a}{2}}{2r_1} = \frac{a}{4r_1}$

Similarly, $\tan B = \frac{b}{4r_2} \tan C = \frac{c}{4r_3}$



So, $\frac{a^2}{r_1^2} + \frac{b^2}{r_2^2} + \frac{c^2}{r_3^2} = 16(\tan^2 A + \tan^2 B + \tan^2 C)$

But $\tan^2 A+\tan^2 B+\tan^2 C \ge 3[\tan A\tan B\tan C]^{\frac{2}{3}}$ (using $A.M. \ge G.M.$)

Also, in an acute angled ΔABC

$\tan A+\tan B+\tan C \ge 3(\tan A.\tan B.\tan C)^{\frac{1}{3}}$ (using $A.M. \ge G.M.$)

But $\tan A+\tan B+\tan C = \tan A\tan B\tan C$

So, $(\tan A\tan B\tan C)^{\frac{2}{3}} \ge 3$

Therefore, $\tan^2 A+\tan^2 A+\tan^2 C > 0$

So, $\frac{a^2}{r_1^2}+\frac{b^2}{r_2^2}+\frac{c^2}{r_3^2} \ge 144$

The minimum value of $\frac{1}{18}\left[\frac{a^2}{r_1^2}+\frac{b^2}{r_2^2}+\frac{c^2}{r_3^2}\right]$ is 8.

49. (A)

(P) $\frac{b^2}{a} = \frac{1}{3}(2a)$ ...(i)

And $b^2 = a^2(1 - e^2)$ ...(ii)

From Equations (i) and (ii), $e = \frac{1}{\sqrt{3}}$

(Q) Director circle, $x^2 + y^2 = 9$

(R) $M_{BF} = \frac{b}{-ae}$ and $M_{BF'} = \frac{b}{ae}$

$\because M_{BF} \times M_{BF'} = -1$

$\Rightarrow e = \frac{1}{\sqrt{2}}$

(D) By solving the equation, $x + y - 2 = 0$ and $x - y = 0$

$\therefore$ Required centre (1, 1).

50. (C)

(P) Equation of line passing through A(2, –3, –1) and B(8, –1, 2) is

$$\frac{x-2}{8-2} = \frac{y+3}{-1+3} = \frac{z+1}{2+1}$$

$$\Rightarrow \frac{x-2}{6} = \frac{y+3}{2} = \frac{z+1}{3} = \lambda \text{(say)}$$

$\therefore$ Any point on the line is $P(6\lambda + 2, 2\lambda - 3, 3\lambda - 1)$

According to question,

$PA = 14$

$\Rightarrow (PA)^2 = 196$

$\Rightarrow (6\lambda)^2 + (2\lambda)^2 + (3\lambda)^2 = 196$

$\Rightarrow 49\lambda^2 = 196$

$\Rightarrow \lambda^2 = 4$

$\Rightarrow \lambda = \pm 2$

$\therefore$ Required points are (14, 1, 5) and (–10, –7, –7)

Distance from origin of

$$(14,1,5)=\sqrt{14^2+1^2+5^2}=\sqrt{196+1+25}$$
$$=\sqrt{222}$$

Distance from origin of

$$(-10,-7,-7)=\sqrt{(-10)^2+(-7)^2+(-7)^2}=\sqrt{198}$$

The point nearer to origin is (–10, –7, –7).

(Q) Equation of the plane containing the lines

$$\frac{x-2}{3}=\frac{y+3}{5}=\frac{z+5}{7} \text{ and parallel to } \hat{\mathbf{i}}+4\hat{\mathbf{j}}+7\hat{\mathbf{k}} \text{ is}$$

$$\begin{vmatrix} x-2 & y+3 & z+5 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix}=0$$

$$\Rightarrow (x-2)(-7)-(-14)(y+3)+(-7)(z+5)=0$$
$$\Rightarrow (x-2)-2(y+3)+(z+5)=0$$
$$\Rightarrow \quad x-2y+z-3=0$$

Clearly, the point (–1, –2, 0) lies on the plane.

(R) The given point is $\hat{\mathbf{i}}+4\hat{\mathbf{k}}$ i.e., $P(1, 0, 4)$

The equation of line in Cartesian form passing through (0, –11, 3) and (2, –3, 1) is

$$\frac{x}{2}=\frac{y+11}{8}=\frac{z-3}{-2}=k \qquad \text{(say)}$$

If Q is the foot of perpendicular form P on this line, its coordinates are

$(2k, 8k - 11, -2k + 3)$

Direction ratios of PQ are

$2k - 1, 8k - 11, -1 - 2k$

$PQ \perp XY$

$\therefore \quad 2(2k - 1) + 8(8k - 11) + 2(1 + 2k) = 0$

$\Rightarrow 72k = 88$

$\Rightarrow k = 11/9$

$\therefore$ Q is the point $\left(\frac{22}{9},\frac{-11}{9},5/9\right)$

(R) The point is $\hat{\mathbf{i}}+7\hat{\mathbf{k}}$ i.e., $P(1, 0, 7)$

Let $P'(x, y, z)$ be the image of P on the line.

Then, PP' is perpendicular to the line and mid-point of PP' lies on line.

$\therefore \quad (x - 1).1 + y(2) + (z - 7)\, 3 = 0$

($\because$ the line is parallel to $\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$)

$\Rightarrow \quad x + 2y + 3z = 22 \qquad \dots$ (i)

$\left(\frac{x+1}{2},\frac{y}{2},\frac{z+7}{2}\right)$ lies on the line.

$$\therefore \quad \frac{(x+1)}{2}\hat{\mathbf{i}}+\frac{y}{2}\hat{\mathbf{j}}+\left(\frac{z+7}{2}\right)\hat{\mathbf{k}}=\hat{\mathbf{j}}$$

$$+2\hat{\mathbf{k}}+\lambda(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}})$$

$$\Rightarrow \lambda=\frac{x+1}{2}, 2\lambda+1=y/2, 3y+2=\frac{z+7}{2}$$

i.e., $\quad x=2\lambda-1, y=4\lambda+2, z=6\lambda-3$

Putting the values in Eq. (i), we get
$(2\lambda - 1) + 2(4\lambda + 2) + 3(6\lambda - 3) = 22$

$\Rightarrow$ $28\lambda = 28$

$\Rightarrow$ P' is (1, 6, 3)

51. (C)

(P) Let $\int_{-\pi}^{\pi} \frac{x^2}{1+\sin x+\sqrt{1+\sin^2 x}}dx$ (replace x by –x)

$I = \int_{-\pi}^{\pi} \frac{x^2}{1-\sin x+\sqrt{1+\sin^2 x}}dx; 2I = \int_{-\pi}^{\pi} x^2\left[\frac{1}{1+\sin x+\sqrt{1+\sin^2 x}}+\frac{1}{1-\sin x+\sqrt{1+\sin^2 x}}\right]dx$

$= \int_{-\pi}^{\pi} x^2dx; 2I = 2\int_0^{\pi} x^2dx;\ I = \frac{\pi^3}{3}$

(Q) Let $y = \frac{1}{x}, dy = -\frac{1}{x^2}dx$ then $\frac{dx}{x^2-x+1} = -\frac{dy}{y^2-y+1}; \tan^{-1}x+\tan^{-1}y = \frac{\pi}{2}; \tan^{-1}\frac{1}{x}; \cot^{-1}x$

$\therefore\ \frac{1}{2}\left(\int_{1/2}^{2}\frac{\tan^{-1}x}{x^2-x+1}dx+\int_{1/2}^{2}\frac{\frac{\pi}{2}-\tan^{-1}y}{y^2-y+1}\right) = \frac{\pi}{4}\int_{1/2}^{2}\frac{dx}{x^2-x+1} = \frac{\pi^2\sqrt{3}}{18}$

(R) $\int_0^{\pi/2}\frac{\frac{\cos x}{x}-\frac{\sin x}{x^2}}{1+\left(\frac{\sin x}{x}\right)^2}dx$; Put $\frac{\sin x}{x} = t$

$\frac{x\cos x-\sin x}{x^2}dx - dt; \int_1^{2/\pi}\frac{dt}{1+t^2}; \left[\tan^{-1}x\right]_1^{2/\pi}; \tan^{-1}\frac{2}{\pi}-\frac{\pi}{4}$

(S) As function is odd

PART (A) : PHYSICS

SOLUTIONS

1. (B)

$h_1 + h_2 = 0.29 \times 2 + 0.1$

$h_1 + h_2 = 0.68 \qquad ...(1)$

$\Rightarrow P_0 + \rho_k g(0.1) + \rho_w g(h_1 - 0.1) \qquad [\rho_k = \text{density of kerosene } \& \ \rho_w = \text{density of water}]$

$-\rho_w g h_2 = P_0$

$\Rightarrow \rho_k g(0.1) + \rho_w g h_1 - \rho_w g \times (0.1)$

$= \rho_w g h_2$

$\Rightarrow 800 \times 10 \times 0.1 + 1000 \times 10 \times h_1 - 1000 \times 10 \times 0.1 = 1000 \times 10 \times h_2$

$\Rightarrow 10000(h_1 - h_2) = 200$

$\Rightarrow h_1 - h_2 = 0.02 \qquad ...(2)$

$\boxed{\Rightarrow h_1 = 0.35}$

$\boxed{\Rightarrow h_2 = 0.33}$

So, $\boxed{\dfrac{h_1}{h_2} = \dfrac{35}{33}}$

2. (A)

$V = i|X_L - X_C| = 0, \ Z = \sqrt{R^2 + (X_L - X_C)^2} = 30\,\Omega$

$i = \dfrac{V_R}{R} = \dfrac{240}{30} = 8 \text{ A}$

3. (B)

$\phi = Mi = Mat$

$\varepsilon = \left|\dfrac{d\phi}{dt}\right| = Ma$

$i = -\dfrac{\varepsilon}{R}\left(1 - e^{\frac{-Rt}{L}}\right)$

4. (B)

5. (B, D)

$\vec{S} = \left[\vec{E} \times \vec{B}\right]\dfrac{1}{\mu_0}$

S is pointing vector denotes flow of energy per unit area per unit time

$\vec{S} = \dfrac{\text{watt}}{m^2}$

Hence, (B) & (D) are correct.

6. (A, C, D)

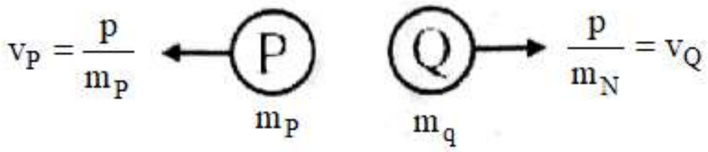
$N \rightarrow P + Q$

Energy released $= (m_N - m_P - m_Q)c^2 = \delta c^2$

This will be distributed kinetic energy of P and Q

$\Rightarrow E_P + E_q = \delta c^2$ ...(i)

By conservation of momentum



$\frac{v_P}{v_q} = \frac{M_q}{M_P}$ ...(ii)

Kinetic energy be written as $KE = \frac{p^2}{2m}$

Hence divided in inverse ratio of masses.

$E_P = \frac{M_q}{M_P + M_q} c^2 \delta$ ...(iii)

By equation $(i) \Rightarrow \frac{p^2}{2M_P} + \frac{p^2}{2M_q} = \delta c^2$

$\Rightarrow \frac{p^2}{2\mu} = \delta c^2 \Rightarrow p = c\sqrt{2\mu\delta}$

7. (B, C)

$u = Fr$

[Using U = Potential energy and v = velocity]

$\Rightarrow$ Force $= \frac{-dU}{dr} = -F$

$\Rightarrow F = \frac{mv^2}{R}$ ...(1)

$\Rightarrow mvR = \frac{nh}{2\pi}$ ...(2)

$\Rightarrow F = \frac{m}{R} \times \frac{n^2h^2}{4\pi^2} \times \frac{1}{m^2R^2}$

$\Rightarrow R = \left(\frac{n^2h^2}{4\pi^2 mF}\right)^{1/3}$ ...(3)

$\Rightarrow v = \frac{nh}{2\pi mR}$

$\Rightarrow v = \frac{nh}{2\pi m}\left(\frac{4\pi^2 mF}{n^2h^2}\right)^{1/3}$

$\Rightarrow v = \frac{n^{1/3}\, h^{1/3}\, F^{1/3}}{2^{1/3}\, \pi^{1/3}\, m^{2/3}}$ ...(4)

(B) is correct.

$\Rightarrow\ E = \frac{1}{2}mv^2 + U$

$\Rightarrow\ \frac{1}{2}mv^2 + FR$

$\Rightarrow\ E = \frac{1}{2}m\left(\frac{n^{2/3}h^{2/3}F^{2/3}}{2^{2/3}\pi^{2/3}m^{4/3}}\right) + F\times\left(\frac{n^2h^2}{4\pi^2 mF}\right)^{1/3}$

$\Rightarrow\ E = \left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{1/3}\left[\frac{1}{2}+1\right]$

$= \frac{3}{2}\left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{1/3}$

8. (3)

9. (6)

For P & Q

$E_1 - 4 = E_P$

$E_1 - 4.5 = E_Q$

$E_P = 2E_Q$

$E_1 - 4 = 2(E_1 - 4.5)$

$E_1 = 5\,eV$

$E_P = 1\,eV,\ E_Q = E_R = 0.5\,eV$

For $E_2 - 5.5 = 0.5$

$E_2 = 6\,eV$

10. (22)

The pressure exerted by the weight of the piston

$= \frac{mg}{A}$

$= \frac{1\times 10}{10\times 10^{-4}} = 1\times 10^4\ N/m^2$

Thus initial pressure of the gas

$P_1 = 100\times 10^3 + 10^4$

$= 110\times 10^3\ N/m^2$

$V_1 = A\ell_1$

$= A\times 0.20$

$P_2 = 10^4\ N/m^2$

Using $P_1V_1 = P_2V_2$

or $P_1(A\ell_1) = P_2(A\ell_2)$

$\therefore\ \ell_2 = \frac{P_1\ell_1}{P_2}$

$$= \frac{110 \times 10^3}{1 \times 10^4} \times 0.20 = 2.2\text{ m} = 220\text{ cm} = 22 \times 10\text{ cm}$$

11. (78)

Assuming the discharge to be divided equally between the two nozzles, so

$Q_A = Q_B = 3\text{ litre/minute}$

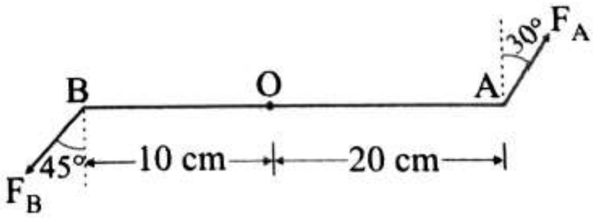
$$= \frac{3 \times 10^{-3}}{60} = 50 \times 10^{-6}\text{ m}^3/\text{s}$$

The force exerted by the discharging water on the nozzle is given by

$$F = \rho v Q = \rho \frac{Q^2}{A}$$

Thus $F_A = F_B = \dfrac{1000 \times \left(50 \times 10^{-6}\right)^2}{\frac{\pi}{4}(0.01)^2}$

$$= 31.85 \times 10^{-3}\text{ N}$$



The net torque at the hand (about)

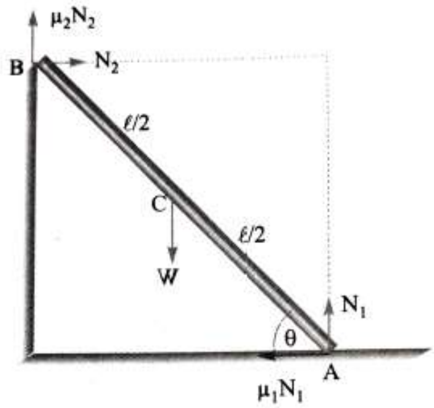
$\tau = F_A \cos 30° \times 0.20 + F_B \cos 45° \times 0.10$

$= 31.85 \times 10^{-3} \times \frac{\sqrt{3}}{2} \times 0.20 + 31.85 \times 10^{-3} \times \frac{1}{\sqrt{2}} \times 0.10$

$= 0.078$ N-m. $= 78 \times 10^{-3}$ N-m.

Hence, $x = 78$.

12. (2)



The FBD of the rod is shown in fig. Let θ be the angle the rod makes with the horizontal without slipping.

For horizontal equilibrium of rod, we have

$\sum F_H = 0;$ or $N_2 - \mu_1 N_1 = 0,$ ...(i)

And for vertical equilibrium of rod, we have

$\sum F_V = 0;$ or $N_1 + \mu_2 N_2 - W = 0,$ ...(ii)

Solving equations (i) and (ii), we get

$N_1 = \frac{W}{1+\mu_1\mu_2}$ and $N_2 = \frac{\mu_1 W}{1+\mu_1\mu_2}$

For rotational equilibrium of the rod, we have $\sum \tau = 0$; taking moment of all forces acting on rod about A (or may choose any other point).

$W \times \frac{\ell}{2}\cos\theta - N_2 \times \ell\sin\theta - \mu_2 N_2 \times \ell\cos\theta = 0$ ... (iii)

Substituting the value of N_2 in equation (iii) and solving, we get

$\tan\theta = \left(\frac{1-\mu_1\mu_2}{2\mu_1}\right)$

13. (61)

$\frac{P}{4\pi(4)^2} = 25 = I_1$

So, $\frac{P}{4\pi(5)^2} = I_2 = 16$

$\Delta x = 5 - 4 = 1\,\text{km}$

$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(\Delta\theta)$

$= 16 + 25 + 2(4)(5)\cos\left(\frac{2\pi}{(48/5)} \times 1000\right) = 41 + 20 = 61$

14. (5.00)

$Bil\sin\theta = mg \Rightarrow B\left(\frac{V}{R}\right)l\sin\theta = mg$

$\Rightarrow 12.5\left(\frac{V}{25}\right)(0.5)\sin 90° = 0.750 \times 10$

$\Rightarrow V = 30\,\text{V}$

$\therefore 6x = 30 \Rightarrow x = 5$

15. (5.00)

$i = \frac{30}{2} = 15\,\text{A}$

$Bil\sin\theta - mg = ma$

$\Rightarrow 12.5 \times 15 \times 0.5 - 0.750 \times 10 = 0.750\,a$

$\Rightarrow a = 115\,\text{m/s}^2$

$23x = 115 \Rightarrow x = 5$

Solution for 16, 17

16. (2.00)

17. (5.00)

At $t = 0; q_0 = CV = 10(1) = 10\,\mu C$

$$i_0 = \frac{\varepsilon}{A} = \frac{10}{100} = 0.1\,A$$

$$U = \frac{q_0^2}{2C} + \frac{1}{2}Li_0^2$$

$$= \frac{\left(10\times10^{-6}\right)^2}{2\times1\times10^{-6}} + \frac{1}{2}\times10\times10^{-3}\left(0.1\right)^2$$

$$= 100\,\mu J$$

$$\frac{q_{max}^2}{2C} = 100\times10^{-6} \quad\Rightarrow\quad q_{max} = 10\sqrt{2}\mu C$$

So, $q = q_{max}\sin\left(\omega t+\phi\right)$ where $\omega = \frac{1}{\sqrt{LC}} = 10^4$ rad/s

$$i = \frac{dq}{dt} = q_{max}\theta\cos\left(\omega t+\phi\right)$$

So, $i_{max} = q_{max}\omega = \left(10\sqrt{2}\times10^{-6}\right)\left(10^4\right) = \frac{1}{5\sqrt{2}}A$

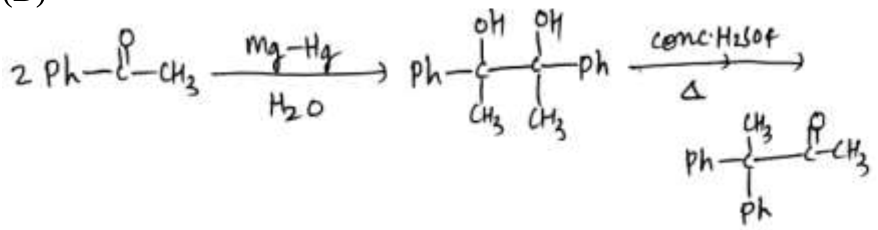
At $t = 0; i = -0.1 = \frac{1}{5\sqrt{2}}\cos\phi \Rightarrow \phi = \frac{3\pi}{4}$

So, $q = \left(10\sqrt{2}\mu C\right)\sin\left(10^4 t + \frac{3\pi}{4}\right)$

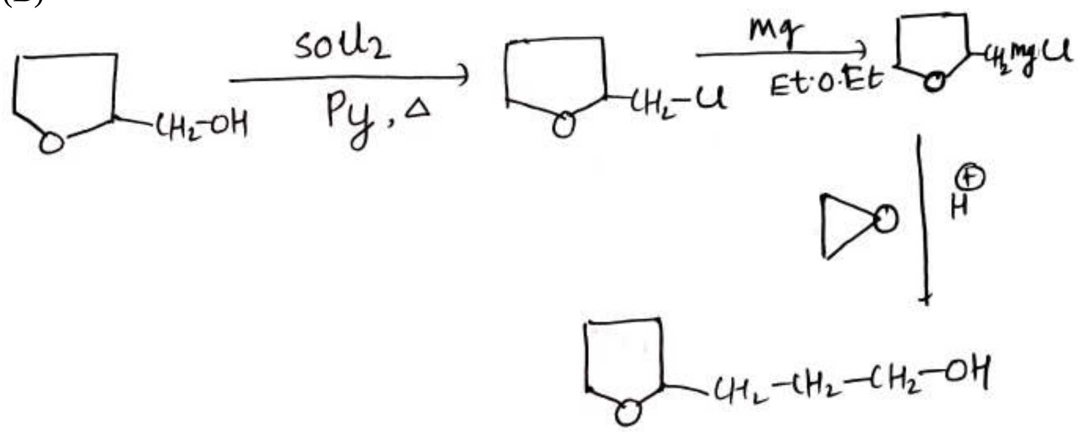
PART (B) : CHEMISTRY

SOLUTIONS

18. (D)



19. (B)



20. (C)

CrO_2Cl_2 is exhibiting colour because of charge transfer spectra.

21. (D)

$A \rightleftharpoons B$ if $\Delta H > 0$, as $T\uparrow$, reaction goes forward.

22. (B, C, D)

23. (A, B)

24. (A, C, D)

(A) $pH = \frac{1}{2}[5 - \log 4 - \log 0.4] = 2.4$

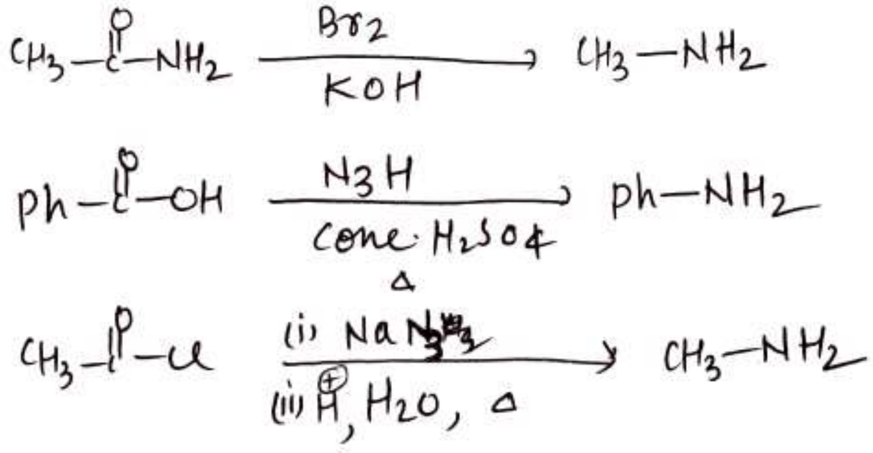
(B), (C) $pOH = 11.6;\ [OH^-] = 2.5 \times 10^{-12} = [H^+]_{H_2O}$

(D) $[H^+] = c \cdot \alpha = 10^{-24}$

$\Rightarrow \alpha = 1\%$

$\therefore$ % unionised = 99%

25. (3)



26. (4)

27. (6)

$N_2O_5, P_4O_{10}, Cr_2O_7^{2-}, Cl_2O_7, (HPO_3)_3, (SO_3)_3$

28. (7)

(a), (b), (c), (d), (e), (g), (h)

(f) $ZnSO_4 + 2C \xrightarrow{\Delta} ZnS + 4CO$

29. (2)

$$\left.\begin{array}{l}\frac{3}{1000} = K(100)^{1/n} \\ \frac{1.5}{1000} = K(25)^{1/n}\end{array}\right\} 2 = 2^{2/n} \quad \Rightarrow \frac{2}{n} = 1 \text{ or } n = 2$$

$$\log\left(\frac{x}{m}\right) = \log k + \frac{1}{n}\log P$$

$$\log P = n\log k + n\log\left(\frac{x}{m}\right)$$

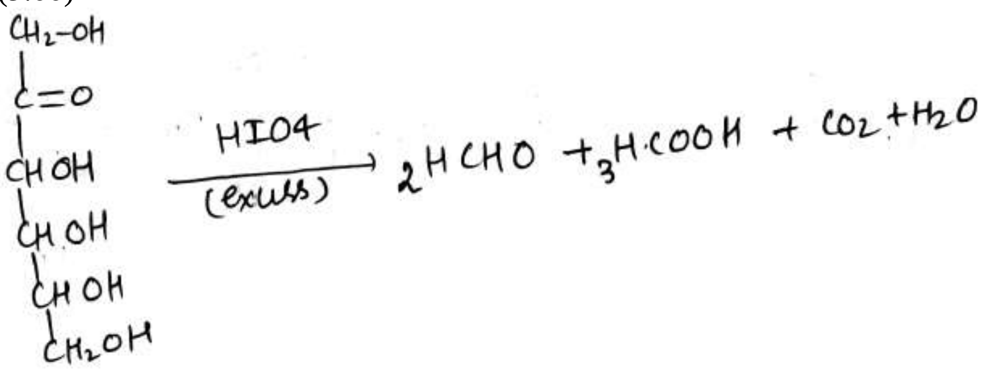
30. (1)

$$\frac{u_{mP_1} + fu_{mP_1}}{u_{mP_2} + fu_{mP_2}} = 1 \qquad \text{(Ratio of fraction of molecules)}$$

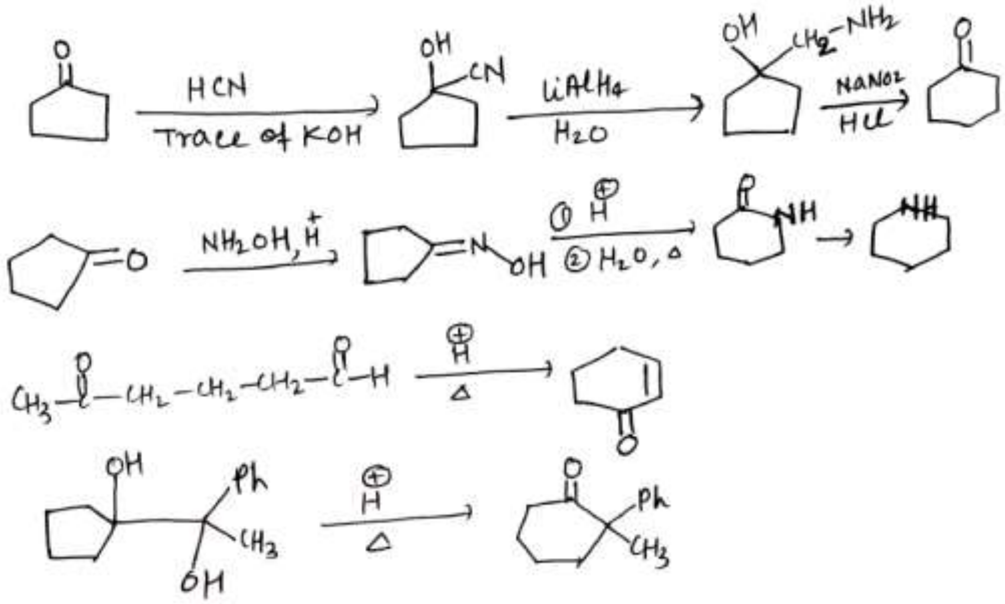
$u_{mP_1} = 500; \quad f \times 500 = 1$

$u_{mP_2} = 600; \quad f \times 600 = 1.2$

31. (3.00)



32. (1.00)



33. (2.00)

Conformationally fixed stereoisomers do not show reaction with HIO_4.

34. (1.00)

PART (C) : MATHEMATICS

SOLUTION

35. **(C)**

Parametric form for hyperbola is $(ct, c/t)$

$$c^2t^2+\frac{c^2}{t^2}+2gct+\frac{2gc}{t}-a^2=0 \quad\Rightarrow\quad c^2t^4+2gct^3-a^2t^2+2gct+c^2=0$$

(i) $\sum x_1 = c\sum t_1 = -2g \quad\Rightarrow\quad \left|\frac{\sum x_1}{g}\right| = 2$ **(ii)** $\sum x_1x_2 = c^2\sum t_1t_2 = -a^2 \quad\Rightarrow\quad \left|\frac{\sum x_1x_2}{a^2}\right| = 1$

(iii) $\frac{\sum x_1x_2x_3}{\sum y_1y_2y_3} = \frac{t_1t_2t_3t_4 \ \sum t_1t_2t_3}{\sum t_1} = 1$

(iv) G is the point $\left(\frac{\sum x_1}{4},\frac{\sum y_1}{4}\right)=\left(\frac{-g}{2},\frac{-g}{2}\right)$ and C is the point $(-g,-g)$

$\therefore$ M is the point $\left(\frac{-g}{2},\frac{-g}{2}\right)$. Thus M and G coincide $\therefore \frac{2OG}{OM}=2$

$$\therefore \left|\frac{\sum x_1}{g}\right| + \left|\frac{\sum x_1x_2}{a^2}\right| + \frac{\sum x_1x_2x_3}{\sum y_1y_2y_3} + \frac{2OG}{OM} = 6$$

36. **(D)**

$$f(x)=(1+2+3+...+n)+[\sin x]+\left[\sin\frac{x}{2}\right]+\left[\sin\frac{x}{3}\right]+....+\left[\sin\frac{x}{n}\right]$$

$$=\frac{n(n+1)}{2}+[\sin x]+\left[\sin\frac{x}{2}\right]+....+\left[\sin\frac{x}{n}\right]$$

$\forall x\in[0,\pi]\,;\,[\sin x]=0,1$ where 1 occurs at $x=\pi/2$

$\forall x\in[0,\pi]\,;\,\left[\sin\frac{x}{2}\right]=0,1$ where 1 occurs at $x=\pi$

$\forall x\in[0,\pi]\,;\,\left[\sin\frac{x}{3}\right]=0$

So, $f(x)=\frac{n(n+1)}{2}+(0 \text{ or } 1)\Rightarrow$ Range of $f(x)=\left\{\frac{n^2+n}{2},\frac{n^2+n+2}{2}\right\}$

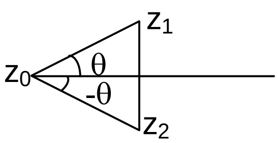
37. **(D)**

$$a\left(\frac{\bar z_1+\bar z_2}{2}\right)+\bar a\left(\frac{z_1+z_2}{2}\right)+b=0$$

$$\Rightarrow\quad a(\bar z_1+\bar z_2)+\bar a(z_1+z_2)+2b=0$$

$$\Rightarrow\quad \operatorname{Re} a(\bar z_1+\bar z_2)=-b$$

$$\frac{z_1-z_2}{\bar z_1-\bar z_2}-\frac{a}{\bar a}=0$$



38. **(D)**

$P(i)\alpha i^2 \Rightarrow p(i) = ki^2$ where $\sum_{i=1}^{6} p(i) = 1 \Rightarrow k = \frac{1}{\sum i^2} = \frac{1}{91}$

Let 'a' be outcome on one die and 'b' be outcome on other

$p(a<b) + p(a=b) + p(a>b) = 1$

Using symmetry, $p(a<b) = p(a>b)$

$\Rightarrow p(a<b) = \frac{1}{2}\left[1 - p(a=b)\right]$

$= \frac{1}{2}\left[1 - \sum_{i=1}^{6} ki^2 \ ki^2\right]$

$= \frac{1}{2}\left[1 - \frac{1}{91^2}(1^4 + ...6^4)\right] = \frac{1}{2}\left[1 - \frac{25}{91}\right] = \frac{33}{91}$

39. (A, B)

Hence only four solutions exist for $f'(x) = 0$.

Since f is differentiable and monotonic so $f'(x) = 0$ gives only at least 4 points of inflexion.

If $f(a)$ is integer (P)

$\therefore f(a^+) > P$ and $f(a^-) < P$ (or vice versa) as f is monotonic)

So, $[f(a^+)] = P$ but $[f(a^{-1})] = P - 1$, so limit does not exist.

If $g(x) = f(f(x))$

$\Rightarrow g'(x) = f'(f(x)) f'(x)$ will have eight roots.

Hence, (A) and (B).

40. **(ABD)**

(A) $1 + \alpha + \alpha^2 + \alpha^3 + ... + \alpha^{10} = 0$

$\overline{\alpha} = \alpha^{10}, \overline{\alpha^2} = \alpha^9, \overline{\alpha^3} = \alpha^8, \overline{\alpha^4} = \alpha^7, \overline{\alpha^5} = \alpha^6$

So, $\text{Re}(\lambda + \lambda^2 + \lambda^3 + \lambda^4 + \lambda^5) = \text{Re}(\alpha^6 + \alpha + \alpha^7 + \alpha^2 + \alpha^8)$

$= \frac{1}{2}\text{Re}(\alpha^6 + \alpha^5 + \alpha + \alpha^{10} + \alpha^7 + \alpha^4 + \alpha^2 + \alpha^9 + \alpha^8 + \alpha^3) = \frac{1}{2}(-1) = -\frac{1}{2}$

(B,C) $\frac{x^{11} - 1}{x - 1} = (x - \alpha)(x - \alpha^2).....(x - \alpha^{10})$

$= \left[(x-\alpha^2)(x-\alpha^4)(x-\alpha^6)(x-\alpha^8)(x-\alpha^{10})\right]\left[(x-\alpha)(x-\alpha^3)(x-\alpha^5)(x-\alpha^7)(x-\alpha^9)\right]$

$= \left[(x-\alpha^2)(x-\alpha^4)(x-\alpha^6)(x-\alpha^8)(x-\alpha^{10})\right]\left[(x-\alpha^{12})(x-\alpha^{14})(x-\alpha^{16})(x-\alpha^{18})(x-\alpha^{20})\right]$

$= \prod_{k=1}^{10}(x - \beta^k)$

Put $x = \alpha^7 = \mu$

$\frac{\alpha^{77} - 1}{\alpha^7 - 1} = \prod_{k=1}^{10}(\mu - \beta^k) = 0$

$$x = i \Rightarrow \frac{(i)^{11}-1}{i-1} = \prod_{k=1}^{10}\left(i-\beta^k\right) = \frac{i^3-1}{i-1} = i$$

(D) Put $x=\alpha \Rightarrow \prod_{k=1}^{10}\left(\alpha-\beta^k\right) = \frac{\alpha^{11}-1}{\alpha-1} = 0$

41. (A, C)

$1+x > 1+x^{\pi/2} > 1+x^2$

$$\int_0^1 \frac{dx}{1+x^2} > \int_0^1 \frac{dx}{1+x^{\pi/2}} > \int_0^1 \frac{dx}{1+x}$$

$$\frac{\pi}{4} > I > \ell n\, 2$$

42. **(4)**

$$I_1 = \int_0^\infty \frac{x^{2017}}{(1+x)^{4035}}dx = \int_0^1 \frac{x^{2017}}{(1+x)^{4035}}dx + \int_1^\infty \frac{x^{2017}}{(1+x)^{4035}}dx$$

Put $x = \frac{1}{t}$

$$I_1 = \int_0^\infty \frac{x^{2017}}{(1+x)^{4035}}dx + \int_0^1 \frac{t^{2016}}{(1+t)^{4035}}dx$$

$$I_1 = \int_0^1 \frac{\left(x^{2017}+x^{2016}\right)}{(1+x)^{4035}}dx = I_2 \Rightarrow \frac{I_1}{I_2} = 1$$

$$I_1 = \int_0^1 \frac{\left(x^{2017}+x^{2016}\right)}{(1+x)^{4035}}dx = I_2 \Rightarrow \frac{I_1}{I_2} = 1$$

Whereas $n = 3$ (when $a = 0, 1, -1$)

43. **(1)**

Apply log and apply limit as a sum to get answer as $e^{-1/4}$

44. **(2)**

Let

$$a_n = \sum_{i=0}^{n}\binom{n}{i}^{-1}$$

Assume that $n \geq 3$. It is clear that

$$a_n = 2 + \sum_{i=1}^{n-1}\binom{n}{i}^{-1} > 2$$

Also note that

$$a_n = 2 + 2/n + \sum_{i=2}^{n-2}\binom{n}{i}^{-1}.$$

Since $\binom{n}{i} \geq \binom{n}{2}$ for all I with $2 \leq i \leq n-2$,

$$a_n \le 2 + 2/n + (n-3)\binom{n}{2}^{-1} \le 2 + 2/n + 2/n = 2 + 4/n.$$

So we have show that for all $n \ge 3$,

$$2 < a_n \le 2 + 4/n$$

Thus, $\lim_{n\to\infty} a_n = 2$

45. **(1)**

For those who haven't taken enough physics. "rolling without slipping" means that the perimeter of the ellipse and the curve pass at the same rate, so all we're saying is that the perimeter of the ellipse equals the length of one period of the sine curve. So set up the integrals:

$$\int_0^{2\pi} \sqrt{(-a\sin\theta)^2 + (b\cos\theta)^2}\, d\theta$$

$$= \int_0^{2\pi a} \sqrt{1 + (c/a\cos x/a)^2}\, dx.$$

Let $\theta = \frac{x}{a}$ in the second integral and write 1 as $\sin^2\theta + \cos^2\theta$ and you get

$$\int_0^{2\pi} \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}\, d\theta$$

$$\int_0^{2\pi} \sqrt{a^2\sin^2\theta + (a^2 + c^2)\cos^2\theta}\, d\theta$$

Since the left side is increasing as a function of b,

We, have equality if and only if $b^2 = a^2 + c^2$.

46. (997)

Define $f^h = f(f(....f(f(x))...))$, where the function f is performed h times.

We find that $f(84) = f(f(89)) = f^2(89) = f^3(94) = ...f^y(1004)$, $1004 = 84 + 5(y-1) \Rightarrow y = 185$.

So we now need to reduce $f^{185}(1004)$.

Let's write out a coupe more iterations of this function:

$$f^{185}(1004) = f^{184}(1001) = f^{183}(998) = f^{194}(1003) = f^{183}(1000)$$

$$= f^{182}(997) = f^{183}(1002) = f^{182}(999) = f^{183}(1004)$$

So, this function reiterates with a period of 2 for x. It might be tempting at first to assume that $f(1004) = 1001$ is the answer. However, that is not true since the solution occurs slightly before that.

Start at $f^3(1004)$:

$$f^3(1004) = f^2(1001) = f(998) = f^2(1003) = f(1000) = \boxed{997}$$

47. **(9)**

$\lfloor 1 + \lfloor 2 + \lfloor 3 + \lfloor 4 = 33$

For $x \ge 4$, $k^2 = \sum_{r=1}^{x} r!$ has unit place =3.

But square of any no. can not have unit place 3.

It implies $x < 4$ is the only possibility.

For $x = 1$, $k^2 = 1 \Rightarrow k = 1$, for $x = 3$, $\lfloor 1 + \lfloor 2 + \lfloor 3 = k^2 \Rightarrow k = 3$

$\Rightarrow \alpha = 1, \beta = 3$

$$\left(a+\sqrt{b}\right)^{x^2-(1+2+3+4+5)} + \quad \left(a-\sqrt{b}\right)^{x^2-15} = 2a$$

$$\left(a+\sqrt{b}\right)^{x^2-15} + \left(a-\sqrt{b}\right)^{x^2-15} = 2a$$

As $a^2 - b = 1$, the only possibility for $\left(a+\sqrt{b}\right)^{x^2-15} + \left(a-\sqrt{b}\right)^{x^2-15} = 2a$ is that

$x^2 - 15 = 1 \Rightarrow x = \pm 4$ or $x^2 - 15 = -1$

Hence, $x = \pm 4, \; \pm \sqrt{14}$

$$\left|\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - \alpha_1\alpha_2\alpha_3\alpha_4\right| - 215 = \left|4 - 4 + \sqrt{14} - \sqrt{14} - (16)(14)\right| - 215 = 9$$

48. **(2.41)**

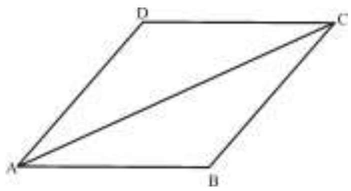
Area of $\Delta ABC \; (A_\Delta) = \frac{1}{2}$ Area of parallelogram $ABCD\left(A_{\|gm}\right)$

$(I.Q)_{\|\, gm} = (I.Q)_\Delta$

$$\frac{4\pi A_{\|gm}}{P_{\|gm}^2} = \frac{4\pi A_\Delta}{P_\Delta^2} \;\Rightarrow\; 2P_\Delta^2 = P_{\|gm}^2$$

$$\sqrt{2}\left[AB + BC + AC\right] = 2\left[AB + BC\right]$$

$$\left(\sqrt{2}-1\right)(AB + BC) = AC \;\Rightarrow\; \frac{AB+BC}{AC} = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1$$



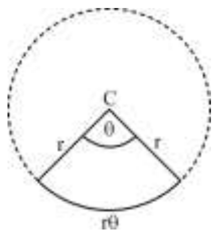
49. **(2.00)**

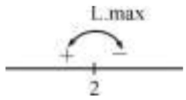
Area of sector = $\frac{\theta r^2}{2}$

$$I.Q. = \frac{4\pi\left(\frac{\theta r^2}{2}\right)}{(2r + \theta r)^2} = \frac{2\pi\theta}{(\theta+2)^2}$$

$$\frac{d}{d\theta}(I.Q) = 2\pi\left[\frac{(\theta+2)^2 - \theta 2(\theta+2)}{(\theta+2)^4}\right] = 2\pi\left(\frac{2-\theta}{(\theta+2)^3}\right)$$

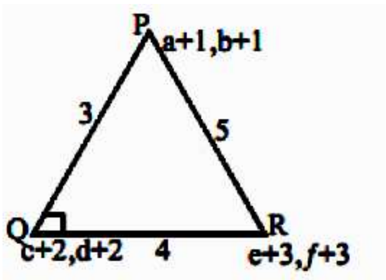
$\frac{d}{d\theta}(I.Q.) = 0 \;\Rightarrow\; \theta = 2$ Hence $I.Q.$ is maximum if $\theta = 2$





50. (0.00)

Consider a triangle with vertices $(a+1, b+1)$, $(c+2, d+2)$, $(e+3, f+3)$



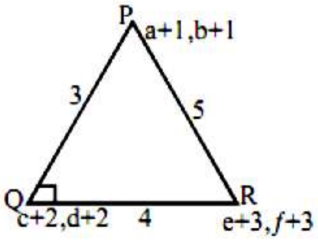
$m_{PQ} \times m_{QR} = 1$

$$\Rightarrow \left(\frac{d-b+1}{c-a+1}\right)\left(\frac{f-d+1}{e-c+1}\right) = -1$$

$$\Rightarrow \frac{d-b+1}{c-a+1}+\frac{e-c+2}{f-d+2}=0$$

51. (1.50)

Consider a triangle with vertices $(a+1, b+1), (c+2, d+2)$, $(e+3, f+3)$ then $|A|$ = 2Area of triangle PQR



$$|A| = 2\times\frac{1}{2}\times 3\times 4 = 12$$

$$\det\left(\frac{A}{2}\right)=\frac{|A|}{8}=\frac{12}{8}=\frac{3}{2}$$