## JEE Main Exercise

1. (b)

$$W = \int F\,dx$$

$$W = \int_0^d (a+bx)\,dx$$

$$= \left[ax + \frac{bx^2}{2}\right]_0^d = ad + \frac{db^2}{2}$$

2. (b)

$$W = \Delta K$$

$$W = K_2 - K_1$$

$$W = \frac{1}{2}\times 2\times 0^2 - \frac{1}{2}\times 2\times (20)^2$$

$$W = -400\text{ J}$$

3. (a)

$$W = \mathbf{F}\cdot\mathbf{s}$$

$$W = F\,s\cos\theta$$

$$W = FR$$

4. (a)

$$v = a\sqrt{s}$$

$$\frac{ds}{dt} = a\sqrt{s}$$

$$\Rightarrow \int_0^s \frac{ds}{\sqrt{s}} = a\int_0^t dt$$

$$\Rightarrow 2\sqrt{s} = at$$

$$\Rightarrow s = \frac{1}{4}a^2t^2$$

$$W = \Delta K = \frac{1}{2}m\left(a\sqrt{s}\right)^2 - \frac{1}{2}m(0)^2$$

$$= \frac{1}{2}ma^2s - 0 = \frac{1}{2}ma^2\left(\frac{1}{4}a^2t^2\right)$$

$$= \frac{1}{8}ma^4t^2$$

5. (a)

Applying work-energy theorem,

$$W_{mg}+W_N+W_{\text{friction}}+W_{\text{spring}}=\Delta K$$

$$0+0-(\mu mg)x+\frac{1}{2}K\left(0^2-x^2\right)=0-\frac{1}{2}mu^2$$

$$\Rightarrow \quad -50x-50x^2=-\frac{1}{2}\times 50\times 2^2$$

$$\Rightarrow \quad x^2+x-2=0$$

$$\Rightarrow \quad x=1\text{m}$$

6. (b)

From $A$ to $B$, applying work-energy theorem,

$$W_{mg}+W_N=\Delta K$$

$$\Rightarrow \quad -mg(R-h)+0=0-\frac{1}{2}mu^2$$

$$u=\sqrt{2g(R-h)}$$

7. (d)

$U=2x+5y-xy$

$$\mathbf{F}=\left(-\frac{\partial U}{\partial x}\right)\hat{\mathbf{i}}+\left(-\frac{\partial U}{\partial y}\right)\hat{\mathbf{j}}$$

$$\Rightarrow \quad \mathbf{F}=(-2+y)\hat{\mathbf{i}}+(-5+x)\hat{\mathbf{j}}$$

At (2, –2)

$$\mathbf{F}=-4\hat{\mathbf{i}}-3\hat{\mathbf{j}}$$

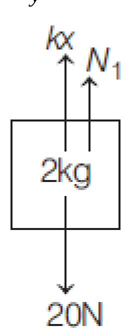
$$\Rightarrow \quad \mathbf{a}=-4\hat{\mathbf{i}}-3\hat{\mathbf{j}}$$

$$\Rightarrow \quad a=5\text{ m/s}^2$$

8. (b)

For 2 kg block

$$\sum F_y=0$$




$$\Rightarrow \quad Kx+N_1=20$$

When 2 kg leaves contact $(N_1=0)$

$$\Rightarrow \quad Kx=20\Rightarrow x=0.5\text{ m}$$

Applying work-energy theorem for 5 kg block

$$W_{mg} + W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad +5\times 10\times 0.5 + \frac{1}{2}\times 40\times\left(0^2 - (0.5)^2\right)$$

$$= \frac{1}{2}\times 5\times v^2 - 0$$

$$\Rightarrow \quad v = 2\sqrt{2}\ \text{m/s}$$

9. (a)

Applying work-energy theorem for (2 kg + 1 kg) system

$$W_{mg} + W_T = \Delta K_{\text{system}}$$

$$\Rightarrow \quad (+2\times 10\times 0.6 - 1\times 10\times 0.6) + 0 = \left(\frac{1}{2}\times 2\times v^2 + \frac{1}{2}\times 1\times v^2\right) - (0+0)$$

$$\Rightarrow \quad v = 2\ \text{m/s}$$

10. (c)

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad v = x\sqrt{\frac{k}{m}} = 0.05\sqrt{\frac{600}{15\times 10^{-3}}}$$

$$\Rightarrow \quad v = 10\ \text{m/s}$$

$$R_{\max} = \frac{v^2}{g} = \frac{(10)^2}{10} = 10\ \text{m}$$

11. (c)

$$P = \frac{3t^2}{2}$$

$$\Rightarrow \quad \frac{dK}{dt} = \frac{3t^2}{2}$$

$$\Rightarrow \quad \int_0^K dK = \int_0^t \frac{3t^2}{2}\,dt$$

$$\Rightarrow \quad K = \frac{t^3}{2}$$

At $t = 2$,

$$K = \frac{2^3}{2} = \frac{1}{2}\times 2\times v^2$$

$$\Rightarrow \quad v = 2\ \text{m/s}$$

12. (d)

$$U_1 = \frac{1}{2}kx^2$$

$$U_2 = \frac{1}{2}K(x+y)^2$$

$$W_{\text{ext}} = \Delta U + \Delta K = \left(\frac{1}{2}k(x+y)^2 - \frac{1}{2}kx^2\right) + 0$$
$$= \frac{1}{2}ky(2x+y)$$

13. (b)

$$\mathbf{v} = 2\hat{\mathbf{i}} + 4t\,\hat{\mathbf{j}}$$
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 4\hat{\mathbf{j}}$$
$$P = \mathbf{F}\cdot\mathbf{v} = (ma)\cdot\mathbf{v} = 2(16t)$$

At $t = 5$, $P = 160\,\text{W}$

14. (a)

$$W = \Delta K$$
$$W = \frac{1}{2}m\left(v^2 - u^2\right)$$

From rest to speed $v$,

$$W = \frac{1}{2}m\left(v^2 - 0^2\right) = \frac{1}{2}mv^2$$

From $v$ to $2v$,

$$W = \frac{1}{2}m\left((2v)^2 - v^2\right) = \frac{3}{2}mv^2$$

15. (d)

$$K_{\text{man}} = \frac{1}{2}k_{\text{boy}}$$
$$\Rightarrow \quad \frac{1}{2}mv_{\text{man}}^2 = \frac{1}{2}\left(\frac{1}{2}\frac{m}{2}v_{\text{boy}}^2\right)$$
$$\Rightarrow \quad v_{\text{man}} = \frac{v_{\text{boy}}}{2}$$

Now, $\frac{1}{2}m(v_{\text{man}}+1)^2 = \frac{1}{2}\left(\frac{m}{2}\right)v_{\text{boy}}^2$

$$\Rightarrow \quad (V_{\text{man}}+1)^2 = \frac{(2v_{\text{man}})^2}{2}$$
$$\Rightarrow \quad V_{\text{man}}^2 + 1 + 2v_{\text{man}} = 2v_{\text{man}}^2$$
$$\Rightarrow \quad v_{\text{man}}^2 - 2v_{\text{man}} - 1 = 0$$
$$\Rightarrow \quad v_{\text{man}} = \frac{2+\sqrt{8}}{2} = 1+\sqrt{2}$$

16. (a)

$$P = Fv = \text{ constant}$$
$$\Rightarrow \quad mav = \text{ constant}$$
$$\Rightarrow \quad m\frac{dv}{dt}v = \text{ constant}$$

$\Rightarrow \int v\,dv \propto \int dt$

$\Rightarrow v^2 \propto t$

$\Rightarrow v \propto \sqrt{t}$

$\frac{ds}{dt} \propto \sqrt{t}$

$\Rightarrow \int ds \propto \int \sqrt{t}\,dt$

$\Rightarrow s \propto t^{3/2}$

$\frac{s}{v} = \frac{t^{3/2}}{t^{1/2}} = t$

17. (c)

Speed of the block is maximum when acceleration of block is zero.

So, $mg\cos\theta = kx$

$\Rightarrow x = \frac{mg\cos\theta}{k}$

18. (a)

$W_{mg} = -mgh$

$W_{\text{friction}} = -\int(\mu mg\cos\theta)\,dl\cos 0^\circ$

$= -\mu mg\int dl\cos\theta$

$= -\mu mg\int_0^l dx$

$= -\mu mgl$

$W_N = 0$

$W_F = ?$

$W_{mg} + W_N + W_{\text{friction}} + W_F = \Delta K$

$\Rightarrow -mgh + 0 - \mu mgl + W_F = 0 - 0$

$\Rightarrow W_F = mg(h + \mu l)$

19. (d)

$K = \frac{p^2}{2m}$

$\Rightarrow \log_e K = \log_e\left(\frac{p^2}{2m}\right)$

$\Rightarrow \log_e K = \log_e p^2 - \log_e 2m$

$\Rightarrow \log_e K = 2\log_e p - \log_e 2m$

$\Rightarrow y = 2x - \log_e 2m$

20. (a)

When the object is lowered very slowly to its equilibrium position, then at equilibrium position

$$Kx = mg$$

$$\Rightarrow K(0.1) = 0.5 \times 10$$

$$\Rightarrow K = 50 \text{ N/m}$$

Applying work-energy theorem,

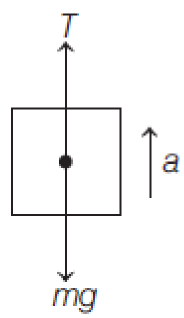
$$W_{mg} + W_{\text{spring}} = \Delta K$$

$$\Rightarrow +0.5 \times 10 \times 0.1 + \frac{1}{2} \times 50\left(0^2 - (0.1)^2\right)$$

$$= \frac{1}{2} \times 0.5v^2 - 0$$

$$\Rightarrow v = 1 \text{ m/s}$$

21. (a)




$$\sum F_y = ma_y$$

$$\Rightarrow T - mg = ma$$

$$\Rightarrow T = m(g + a)$$

$$W_T = Fs\cos\theta$$

$$= m(g + a)\frac{1}{2}at^2 \cos 0^\circ$$

$$= \frac{m}{2}(g + a)at^2$$

22. (c)

$P = Fv$ = constant

$mav$ = constant

$$\Rightarrow \frac{dv}{dt}v = \text{constant}$$

$$\Rightarrow \int v\,dv \propto \int dt$$

$$\Rightarrow v^2 \propto t$$

$$\Rightarrow v \propto \sqrt{t}$$

Now, $v = \dfrac{dx}{dt} \propto \sqrt{t}$

$$\Rightarrow \int dx \propto \int \sqrt{t}\,dt$$

$$\Rightarrow x \propto t^{3/2}$$

23. (d)

Applying work-energy theorem

$$W_{mg} + W_N + + W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad +mg\left(\frac{3R}{2}\right) + 0 + \frac{1}{2}k\left(\left(\frac{R}{2}\right)^2 - 0^2\right)$$

$$= \frac{1}{2}mv^2 - 0$$

$$\Rightarrow \quad v = \sqrt{4gR}$$

24. (b)

Lets say maximum elongation in the spring is $x$.

$$W_{mg} + W_N + W_F + W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad 0 + 0 + Fx + \frac{1}{2}k\left(0^2 - x^2\right) = 0 - 0$$

$$\Rightarrow \quad x = \frac{2F}{k}$$

$$W_F = Fx = F\left(\frac{2F}{k}\right) = \frac{2F^2}{k}$$

25. (a)

$$W_{\text{friction}} = \int -\mu mg\cos\theta dx = \int_0^5 -\mu_0\, x\, mg\cos\theta\, dx$$

$$= \frac{-\mu_0 mg\cos\theta\, s^2}{2}$$

Applying work-energy theorem

$$W_{mg} + W_N + W_{\text{friction}} = \Delta K$$

$$\Rightarrow \quad \left(mg\sin\theta\right)s + 0 - \frac{\left(\mu_0 mg\cos\theta\right)s^2}{2} = 0 - 0$$

$$\Rightarrow \quad s = \frac{2\tan\theta}{\mu_0}$$

26. (c)

Using work-energy theorem between $A$ and $B$.

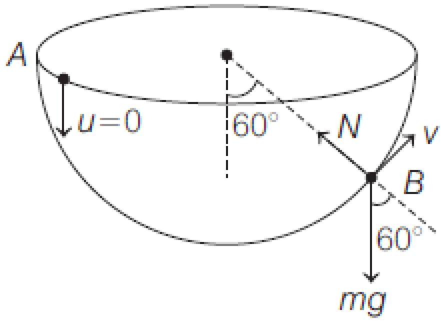
$$W_{mg} + W_N = \Delta K$$

$$\Rightarrow \quad +mgR\cos 60^\circ + 0 = \frac{1}{2}mv^2 - 0$$

$$\Rightarrow \quad v = \sqrt{gR}$$

Equation for centripetal force at $B$,

$$N - mg\cos 60^\circ = \frac{mv_B^2}{R}$$

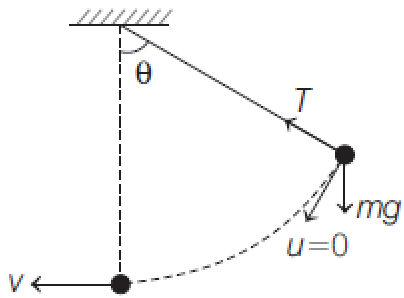



$N = 1.5mg$

27. (d)

In extreme position,

$a_{\text{centripetal}} = 0$




$a_{\text{tantential}} = g\sin\theta$

$a_1 = \sqrt{a_c^2 + a_T^2} = g\sin\theta$

Using work-energy theorem,

$W_{mg} + W_T = \Delta K$

$\Rightarrow \quad +mgl(1-\cos\theta) + 0 = \frac{1}{2}mv^2 - 0$

$\Rightarrow \quad v = \sqrt{2gl(1-\cos\theta)}$

$a_{\text{centriperal}} = \frac{v^2}{l} = 2g(1-\cos\theta)$

$a_{\text{tantential}} = 0$

$a_2 = \sqrt{a_c^2 + a_T^2} = 2g(1-\cos\theta)$

$\because \quad a_1 = a_2$

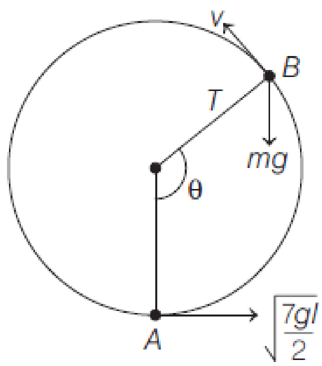
$\Rightarrow \quad g\sin\theta = 2g(1-\cos\theta)$

$\Rightarrow \quad \theta = 2\tan^{-1}\left(\frac{1}{2}\right)$

28. (c)

Using work-energy theorem between *A* and *B*,

$W_{mg} + W_T = \Delta K$




$$-mgl\left(1+\cos\left(180°-\theta\right)\right)+0=\frac{1}{2}mv^2-\frac{1}{2}m\left(\sqrt{\frac{7gl}{2}}\right)^2$$

$$v^2=\frac{7gl}{2}-2gl\left(1-\cos\theta\right) \qquad ...(i)$$

**At *B*,**

$$mg\cos\left(180°-\theta\right)+T=\frac{mv^2}{l}$$

$$\Rightarrow \quad -mg\cos\theta+0=\frac{m}{l}\left(\frac{7gl}{2}-2gl\left(1-\cos\theta\right)\right)$$

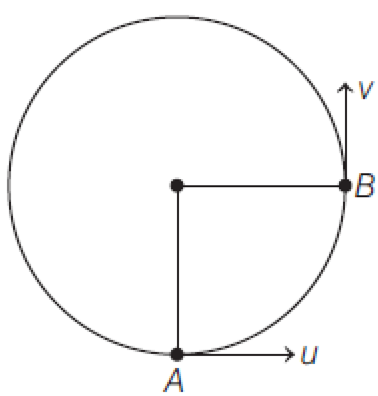
$$\Rightarrow \quad -2\cos\theta=7-4\left(1-\cos\theta\right)$$

$$\Rightarrow \quad \cos\theta=-\frac{1}{2}$$

$$\theta=120°$$

29. (d)

$$\mathbf{v}_A=u\,\hat{\mathbf{i}}$$




Using work-energy theorem between *A* and *B*,

$$W_{mg}+W_T=\Delta T$$

$$\Rightarrow \quad -mgL+0=\frac{1}{2}mv^2-\frac{1}{2}mu^2$$

$$\Rightarrow \quad v=\sqrt{u^2-2gL}$$

$$\Rightarrow \quad \mathbf{v}_B=\sqrt{u^2-2gL}\,\hat{\mathbf{j}}$$

$$\Rightarrow \quad \mathbf{v}_B - \mathbf{v}_A = \sqrt{u^2 - 2gL}\,\hat{\mathbf{j}} - u\,\hat{\mathbf{i}}$$

$$|\mathbf{v}_B - \mathbf{v}_A| = \sqrt{\left(\sqrt{u^2 - 2gL}\right)^2 + u^2} = \sqrt{2\left(u^2 - gL\right)}$$

30. (a)

Using work - energy theorem between $A$ and $B$,

$$W_{mg} + W_T = \Delta K$$

$$\Rightarrow \quad +mgl\cos\theta + 0 = \frac{1}{2}mv^2 - 0$$

$$\Rightarrow \quad v = \sqrt{2gl\cos\theta}$$

**At *B*,**

Centripetal acceleration $= \dfrac{v^2}{l} = \dfrac{\left(\sqrt{2gl\cos\theta}\right)^2}{l} = 2g\cos\theta$

Tangential acceleration $= \sqrt{a_c^2 + a_T^2}$

$$= \sqrt{(2g\cos\theta)^2 + (g\sin\theta)^2}$$

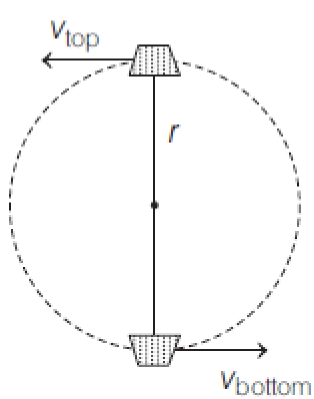
$$= g\left(\sqrt{4\cos^2\theta + \sin^2\theta}\right)$$

$$= g\sqrt{3\cos^2\theta + 1}$$

31. (a)

At the top most point,

$$T + mg = \frac{mv_{\text{top}}^2}{r}$$




For $v_{\text{top}}$ to be minimum

$$T = 0$$

$$\Rightarrow \quad v_{\text{top}} = \sqrt{rg}$$

Using work - energy theorem,

$$W_{mg} + W_T = \Delta K$$

$$\Rightarrow \quad +mg(2r) + 0 = \frac{1}{2}mv_{\text{bottom}}^2 - \frac{1}{2}m\left(\sqrt{rg}\right)^2$$

$$v_{\text{bottom}} = \sqrt{5rg} \text{ and } T - mg = \frac{mv_{\text{bottom}}^2}{r}$$

$$\Rightarrow \quad T - mg = \frac{m}{r}\left(\sqrt{5rg}\right)^2$$

$$\Rightarrow \quad T = 6\,mg$$

32. (a)

Using work-energy theorem between $A$ and $B$,

$$W_{mg} + W_N = \Delta K$$

$$\Rightarrow \quad +mg(2R) + 0 = \frac{1}{2}mv_2^2 - 0$$

$$\Rightarrow \quad v_2 = \sqrt{4gR}$$

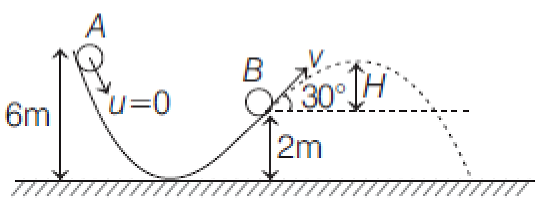
**At point 2,**

$$N + mg = \frac{mv_2^2}{R}$$

$$\Rightarrow \quad N + mg = 4mg$$

$$\Rightarrow \quad N = 3mg$$

33. (3)




Between $A$ and $B$

$$W_{mg} + W_N = \Delta K$$

$$\Rightarrow \quad +mg(6-2) + 0 = \frac{1}{2}mv^2 - 0$$

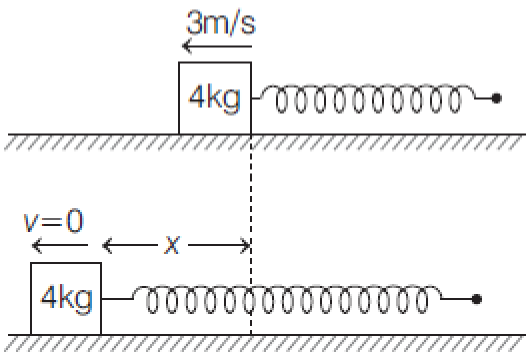
$$\Rightarrow \quad v = \sqrt{8g}$$

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{\left(\sqrt{8g}\right)^2 \sin^2 30^\circ}{2g} = 1\text{ m}$$

$$H_{\max} = 2 + 1 = 3\text{ m}$$

34. (6)

With respect to free end, velocity of block is 1 − (−2) = 3 m/s left.




Elongation in the spring will be maximum when velocity of block with respect to free end is zero.

$$W_{mg}+W_N+W_{\text{spring}}=\Delta K$$

$$\Rightarrow\quad 0+0+\frac{1}{2}\times\frac{100}{10^{-2}}\left(0^2-x^2\right)=0-\frac{1}{2}\times4\times3^2$$

$$x=0.06\text{ m}=6\text{ cm}$$

35. (8)

$$P=640-16v-8v^2$$

$$\Rightarrow\quad Fv=640-16v-8v^2$$

For velocity to be maximum, $a$ = 0

$$\Rightarrow\quad F=0$$

$$\Rightarrow\quad 0=640-16v-8v^2$$

$$\Rightarrow\quad v^2+2v-80=0$$

$$\Rightarrow\quad v=8\text{ m/s}$$

36. (2)

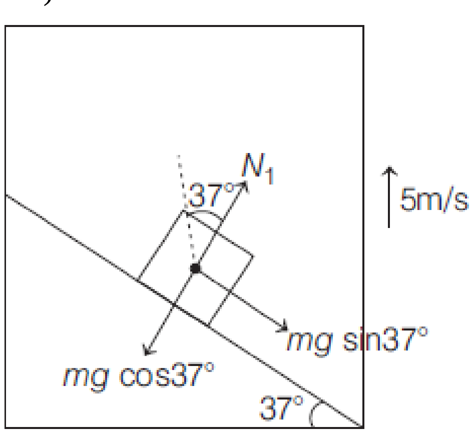
Using work-energy theorem between $A$ and $B$

$$W_{mg}+W_T+W_F=\Delta K$$

$$\Rightarrow\quad -mgl\left(1-\cos37^\circ\right)+0+\left(\frac{mg}{2}\right)\left(l\sin37^\circ\right)$$

$$=\frac{1}{2}mv_B^2-\frac{1}{2}mv_A^2\Rightarrow v_B=2\text{ m/s}$$

37. (320)



$$\sum F_y = 0$$

$\Rightarrow$ $N_1 - mg\cos 37° = 0$

$\Rightarrow$ $N_1 = mg\cos 37°$

$$W_{N_1} = Fs\cos\theta$$

$$= (mg\cos 37°)(vt)\cos 37° = 320 \text{ J}$$

38. (2)

Using work-energy theorem for block,

$$W_{mg} + W_N + W_{\text{friction}} + W_{\text{spring}} = \Delta K$$

$\Rightarrow$ $+mg\sin 37° \times 0.5 + 0 - \frac{1}{8}(mg\cos 37°)(0.5) + \frac{1}{2}\times 8\left(0^2 - (0.5)^2\right)$

$$= \frac{1}{2}mv^2 - 0$$

$\Rightarrow$ $v = 2 \text{ m/s}$

39. (1.5)

Using work-energy theorem for block,

$$W_{mg} + W_{\text{spring 1}} + W_{\text{spring 2}} + \Delta K$$

$\Rightarrow$ $+2\times 10\times(1+x) + \frac{1}{2}\times 24\left(0^2 - (1+x)^2\right) + \frac{1}{2}\times 24\left(0^2 - x^2\right) = 0 - 0$

$\Rightarrow$ $20 + 20x - 12 - 12x^2 - 24x - 12x^2 = 0$

$\Rightarrow$ $24x^2 + 4x - 8 = 0$

$\Rightarrow$ $6x^2 + x - 2 = 0$

$\Rightarrow$ $6x^2 + 4x - 3x - 2 = 0$

$\Rightarrow$ $2x(3x+2) - (3x+2) = 0$

$\Rightarrow$ $x = 0.5$

So, maximum extension in upper spring

$$= 1 + 0.5 = 1.50 \text{ m}$$

40. (730)

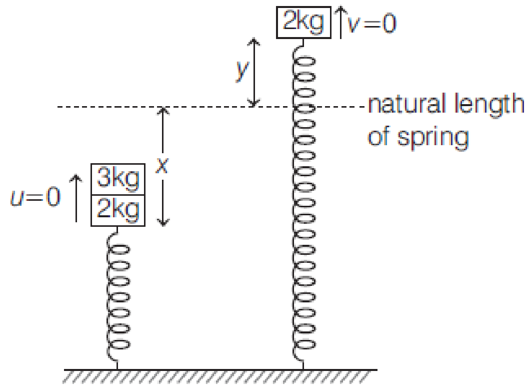
Using work-energy theorem for the object

$$W_{mg} + W_{\text{air}} = \Delta K$$

$\Rightarrow$ $+5\times 9.8\times 20 + W_{\text{air}} = \frac{1}{2}\times 5\times 10^2 - 0$

$$W_{\text{air}} = -730 \text{ J}$$

41. (1.50)




Initially lets take compression to be $x$.

$$\sum F_y = 0$$

$$\Rightarrow \quad 30 + 20 - 40x = 0$$

$$\Rightarrow \quad x = 1.25\,\text{m}$$

After removing 3 kg block, lets take maximum elongation in the spring to be $y$.

Using work-energy theorem for 2 kg block,

$$W_{mg} + W_{\text{spring}} = \Delta K$$

$$\Rightarrow \quad -2\times 10(1.25 + y) + \frac{1}{2}\times 40\left[(1.25)^2 - y^2\right]$$

$$= 0 - 0$$

$$\Rightarrow \quad y = 0.25\,\text{m}$$

So, the maximum height reached by 2 kg block

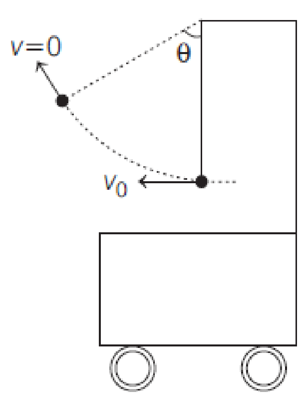
= 1.25 + 0.25

= 1.5 m

42. (7)

Using work - energy theorem,

$$W_{mg} + W_T = \Delta T$$




$$\Rightarrow \quad -mgl(1 - \cos 60°) + 0$$

$$= 0 - \frac{1}{2}mv_0^2$$

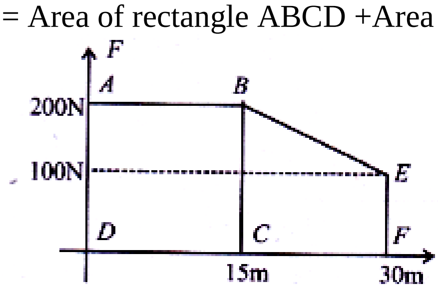
$$\Rightarrow \quad v_0 = \sqrt{gl} = \sqrt{9.8\times 5} = 7\text{ m/s}$$

1. (B)
By work-energy theorem
$W = \Delta K$
$\Rightarrow W = \frac{1}{2}m\left(v_f^2 - v_i^2\right) \Rightarrow W = \frac{1}{2} \times 0.5 \times \left(16^2 - 4^2\right)$
$\Rightarrow W = \frac{1}{4} \times 240 \Rightarrow W = 60J$

2. (D)
By work $-$energy theorem
$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2} \times 0.5 \times \left(b^2.4^5\right)$
$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{4^2} \times 4^5 = 16J$

3. (B)
From work-energy theorem,
$W_{Porter} + W_{mg} = \Delta K.E = 0 \quad (\because \text{velocity constant})$
Or, $W_{Porter} = -W_{mg} = -mgh$
$\therefore W_{Porter} = -80 \times 9.8 \times \frac{80}{100} = -627.2J$

4. (D)
The given situation can be drawn graphically as shown in figure.
Work done = Area under F-x graph
= Area of rectangle ABCD +Area of trapezium BCFE




$W = (200 \times 15) + \frac{1}{2}(100 + 200) \times 15 = 3000 + 2250$
$\Rightarrow W = 5250J$

5. (C)
Work done, $W = \int \vec{F}.\vec{ds} = \left(-x\hat{i} + y\hat{j}\right).\left(dx\hat{i} + dy\hat{j}\right)$
$\Rightarrow W = -\int_1^0 xdx + \int_0^1 ydy = \left(0 + \frac{1}{2}\right) + \frac{1}{2} = 1J$

6. (D)

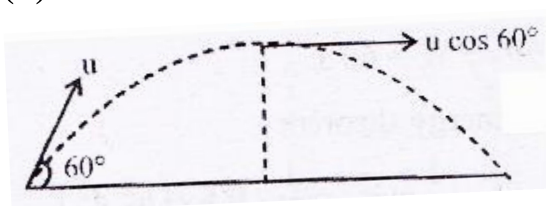
Here, $N - mg = ma = \frac{mg}{2} \Rightarrow N = \frac{3mg}{2}$

N-normal reaction

Now, work done by normal reaction 'N' on block in time, $W = \vec{N}\vec{S} = \left(\frac{3mg}{2}\right)\left(\frac{1}{2}g/2t^2\right)$

Or, $W = \frac{3mg^2t^2}{8}$

7. (C)




At maximum height, we only have horizontal component of velocity . So, Velocity $v = u\cos 60^\circ = \frac{u}{2}$

$\therefore$ K.E. at top most point $= \frac{1}{2}m\left(\frac{u}{2}\right)^2 = \frac{E}{4}$

8. (C)

Momentum of a body is increased by $P' = P + \frac{20}{100}P = 1.2P$

Percentage change in $KE = \frac{K' - K}{K} \times 100$

$$= \left(\frac{\frac{P'^2}{2m} - \frac{P^2}{2m}}{\frac{P^2}{2m}}\right) \times 100 = \left[(1.2)^2 - 1\right] \times 100 = 44\%$$

9. (A)

Using $mv = \sqrt{2mk} \Rightarrow v = \frac{1}{m}\sqrt{2mk}$

So, $u = \frac{1}{0.2}\sqrt{2 \times 0.2 \times 90} = 30 m/s$

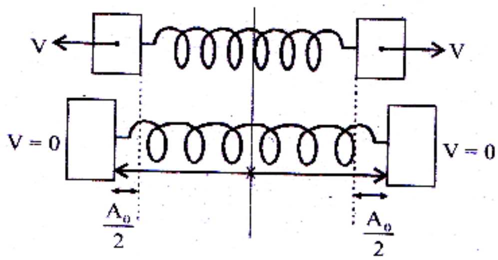
$v = \frac{1}{0.2}\sqrt{2 \times 0.2 \times 40} = 20\,m/s$

$a = \frac{20 - 30}{1} = -10\,m/s^2$ So, $s = \frac{u^2}{2a} = 45\,m$

10. (B)

Given, spring constant of spring , $K = 2\,Nm^{-1}$

Mass of block, $m = 250\,g = \frac{250}{1000}g = \frac{1}{4}kg$




Using energy conservation

$$\frac{1}{2}mv^2 \times 2 = \frac{1}{2}kx^2 \Rightarrow \frac{1}{4}v^2 = \frac{1}{2} \times 2 \times x^2$$

$$\therefore x = \frac{v}{2}$$

11. (B)

Kinetic energy, K.E. $\frac{p^2}{2m}$

$$\frac{K.E_1}{K.E_2} = \left(\frac{P_1}{P_2}\right)^2 \times \left(\frac{m_2}{m_1}\right) = \left(\frac{1}{2}\right)^2 \times \frac{8}{5} = \frac{2}{5}$$

12. (D)

By law of conservation of mechanical energy $\Delta k = -\Delta U$

$$\Rightarrow k_f - k_i = U_i - U_f \Rightarrow k_f = mgy - mg[y - y_0]$$

$$[\because k_i = 0, U_i = mgy \text{ and } U_f = mg(y - y_0)]$$

$$\Rightarrow k_f = mgy_0$$

13. (A)

At maximum height , $\nu = 0$

$$\Rightarrow m\nu = 0 \Rightarrow P = 0$$

14. (C)

By work-energy theorem,

$$\Delta k = W_{\text{all forces}} = \int \vec{F}.d\vec{r}$$

$$= \int \left(4x\hat{i} + 3y^2\hat{j}\right).\left(dx\hat{i} + dy\,\hat{j}\right) = \int_1^2 4x\,dx + \int_2^3 3y^2 dy$$

$$= 4\left[\frac{x^2}{2}\right]^2 + 3\left[\frac{y^3}{3}\right]_2^3 = 2\left[2^2 - 1^2\right] + \left[3^3 - 2^3\right]$$

$$= 6 + 19 = 25J$$

15. (A)

Work done by air friction= Final kinetic energy. Initial potential energy $W_{air-friction} = \frac{1}{2}mv^2 - mgh$

$$= \frac{1}{2}m\left(0.8\sqrt{gh}\right)^2 - mgh$$

$$W_{air-friction} = \frac{64}{2}mgh - mgh = -0.68\,mgh$$

16. (C)

We know area under F-x graph gives the work done by the body

$$\therefore W = \frac{1}{2}\times(3+2)\times(3-2)+2\times 2 = 2.5+4 = 6.5J$$

Using work energy theorem,

$w = \Delta KE$ = work done $\therefore \Delta K.E. = 6.5J$

17. (C)

$l_1 + l_2 = l$ and $l_1 = nl_2$ $\quad \therefore l_1 = \frac{nl}{n+1}$ and $l_2 = \frac{l}{n+1}$

As $k \propto \frac{1}{l}$, $\quad \therefore \frac{k_1}{k_2} = \frac{l/(n+1)}{(nl)/(n+1)} = \frac{1}{n}$

18. (E)

Velocity of 1 kg block just before it collides with 3 kg block $= \sqrt{2gh} = \sqrt{2000}\,m/s$

Using principle of conservation of linear momentum just after collision, we get

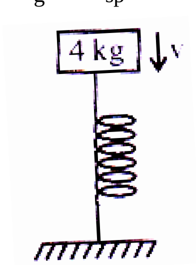
$$1\times\sqrt{2000} = 4v \Rightarrow v = \frac{\sqrt{2000}}{4}\,m/s$$

Initial compression of spring

$$1.25\times10^6 x_0 = 30 \Rightarrow x_0 \approx 0$$

Using work energy theorem,

$$W_g + W_{sp} = \Delta KE$$




$$\Rightarrow 40\times x + \frac{1}{2}\times1.25\times10^6\left(0^2 - x^2\right) = 0 - \frac{1}{2}\times4\times v^2$$

Solving $x \approx 2cm$

19. (A)

$$W = u_f - u_i = 0 - \left(-\frac{mg}{n}\times\frac{L}{2n}\right) = \frac{MgL}{2n^2}$$

20. (C)

$$m\nu = (m+M)V'$$

Or $\nu = \frac{m\nu}{m+M} = \frac{m\nu}{m+4m} = \frac{\nu}{5}$

Using conservation of ME, we have

$$\frac{1}{2}mv^2 = \frac{1}{2}(m+4m)\left(\frac{v}{5}\right)^2 + mgh \text{ or } h = \frac{2}{5}\frac{v^2}{g}$$

21. (D)

When force 'F' is applied, initially $F > F_s$. As $F_s$ will be increase, suppose after x distance $F = F_s$ and there is equilibrium. At this moment block has maximum velocity.

$$F = kx \Rightarrow x = \frac{F}{k}$$

From work-energy theorem,

$$W_F + W_{sp} = \Delta KE$$

$$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0 \Rightarrow F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}\frac{F^2}{K} = \frac{1}{2}mv^2 \text{ or, } v_{max} = \frac{F}{\sqrt{mk}}$$

22. (D)

Position, $x = 3t^2 + 5$

$$\therefore \text{ Velocity, } v = \frac{dx}{dt} \Rightarrow v = \frac{d(3t^2+5)}{dt}$$

$\Rightarrow v = 6t + 0$

At $t = 0$ $\quad v = 0$

And, at $t = 5\,\text{sec}$ $\quad v = 30\,m/s$

According to work-energy theorem , $w = \Delta KE$

Or $W = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(2)(30)^2 = 900J$

23. (C)

$$F = -\frac{\partial u}{\partial r}\hat{r} = \frac{K}{r^3}\hat{r}$$

Since particle is moving in circular path

$$F = \frac{mv^2}{r} = \frac{K}{r^3} \Rightarrow mv^2 = \frac{K}{r^2}$$

$$\therefore K.E. = \frac{1}{2}mv^2 = \frac{K}{2r^2}$$

Total energy $= P.E. + K.E.$

$$= -\frac{K}{2r^2} + \frac{K}{2r^2} = \text{zero}\left(\because P.E. = -\frac{K}{2r^2} \text{ given}\right)$$

24. (B)

As the particles moving in circular orbits. So $\frac{mv^2}{r} = \frac{16}{r} + r^2$

Kinetic energy, $KE_0 = \frac{1}{2}mv^2 = \frac{1}{2}\left[16 + r^4\right]$

For first particle, $r = 1, K_1 = \frac{1}{2}m(16+1)$

Similarly, for second particle, $r = 4, K_2 = \frac{1}{2}m(16+256)$

$$\therefore \frac{K_1}{k_2} = \frac{\frac{16+1}{2}}{\frac{16+256}{2}} = \frac{17}{272} \simeq \frac{17}{272} \simeq 6\times10^{-2}$$

25. (A)

Let $V_f$ is the final speed of the body. From questions,

$$\frac{1}{2}mV_f^2 = \frac{1}{8}mV_0^2 \Rightarrow V_f = \frac{V_0}{2} = 5m/s$$

$$F = m\left(\frac{dV}{dt}\right) = -kV^2 \qquad \therefore \left(10^{-2}\right)\frac{dV}{dt} = -kV^2$$

$$\int_{10}^{5}\frac{dV}{V^2} = -100\,K\int_{0}^{10}dt$$

$$\frac{1}{5} - \frac{1}{10} = 100K(10) \qquad \text{or, } K = 10^{-4}\,kgm^{-1}$$

26. (A)

$$h \propto \frac{V^2}{2g}$$

$h \propto$ K.E.

As K.E. becomes half after every collision. So height will also become half.

So, total distance $= h + 2\left(\frac{h}{2} + \frac{h}{4} + .....\right)$

$$= h + 2h\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right) = 3h$$

27. (C)

Using, $F = ma = m\frac{dV}{dt}$

$$6t = 1.\frac{dV}{dt} \qquad [\because m = 1kg\ \text{given}]$$

$$\int_0^v dV = \int 6t\,dtV = 6\left[\frac{t^2}{2}\right]_0^1 = 3ms^{-1}\,[\because t = 1\sec\ \text{given}]$$

From work energy theorem,

$$W = \Delta KE = \frac{1}{2}m\left(V^2 - u^2\right) = \frac{1}{2}\times1\times9 = 4.5J$$

28. (A)

Work done by friction at $QR = \mu mgx$

In triangle, $\sin 30^\circ = \frac{1}{2} = \frac{2}{PQ} \Rightarrow PQ = 4m$

Work done by friction at $PQ = \mu mg \times \text{Cos}30^\circ \times 4$

$= \mu mg \times \frac{\sqrt{3}}{2} \times 4 = 2\sqrt{3}\mu mg$

Since work done by friction on parts PQ and QR are equal, $\mu mgx = 2\sqrt{3}\mu mg \Rightarrow x = 2\sqrt{3} \cong 3.5m$

Using work energy theorem $4mg\sin 30^\circ = 2\sqrt{3}\mu mg + \mu mgx$

$\Rightarrow 2 = 4\sqrt{3}\mu \Rightarrow \mu = 0.29$

29. (B)

$n = \frac{W}{\text{input}} = \frac{mgh \times 1000}{\text{input}} = \frac{10 \times 9.8 \times 1 \times 1000}{\text{input}}$

Input $= \frac{98000}{0.2} = 49 \times 10^4 J$

Fat used $= \frac{49 \times 10^4}{3.8 \times 10^7} = 12.89 \times 10^{-3} kg$

30. (B)

As we know, $dU = F.dr$

$U = \int_0^r \alpha r^2 dr = \frac{\alpha r^3}{3}$ …..(i)

As, $\frac{mv^2}{r} = \alpha r^2 \Rightarrow m^2v^2 = m\alpha r^3$

Or, $2m(KE) = \frac{1}{2}\alpha r^3$ …..(ii)

Total energy $=$ Potential energy + kinetic energy

Now, from equation (i) and (ii)

Total energy $= K.E. + P.E. = \frac{\alpha r^3}{3} + \frac{\alpha r^3}{2} = \frac{5}{6}\alpha r^3$

31. (A)

Let u be the initial velocity of the bullet of mass m. After passing through a plank of width x, its velocity decreases ti v.

$\therefore u - v = \frac{u}{n} \text{ or}, v = u - \frac{u}{n} = \frac{u(n-1)}{n}$

If F be the retarding force applied by each plank, then using work – energy theorem.

$Fx = \frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{1}{2}mu^2 - \frac{1}{2}mu^2\frac{(n-1)^2}{n^2}$

$= \frac{1}{2}mu^2\left[\frac{1-(n-1)^2}{n^2}\right]$

$$Fx = \frac{1}{2}mu^2\left(\frac{2n-1}{n^2}\right)$$

Let P be the number of planks required to stop the bullet. Total distance travelled by the bullet before coming to rest $= Px$

Using work-energy theorem again,

$$F(Px) = \frac{1}{2}mu^2 - 0$$

Or, $P(Fx) = P\left[\frac{1}{2}mu^2\frac{(2n-1)}{n^2}\right] = \frac{1}{2}mu^2$

$$\therefore P = \frac{n^2}{2n-1}$$

32. (A)

Given: $k_A = 300\,N/m, k_B = 400\,N/m$

Let when the combination of springs is compressed by force F. Spring A is compressed by x. Therefore compression in spring B

$x_B = (8.75 - x)cm$. In series force is same across both spring

So, $F = 300 \times x = 400(8.75 - x)$

Solving we get, $x = 5cm$

$x_B = 8.75 - 5 = 3.75cm$

$$\frac{E_A}{E_B} = \frac{\frac{1}{2}k_A(x_A)^2}{\frac{1}{2}k_B(x_B)^2} = \frac{300\times(5)^2}{400\times(3.75)^2} = \frac{4}{3}$$

33. (D)

$$F_{thrust} = V_{rel}\frac{dm}{dt} = 5\times0.5 = 2.5N$$

So, Power $=$ Force $\times$Velocity $= 2.5\times5 = 12.5watt$

34. (B)

We know that

Power, $P = Fv$

But $F = max = m\frac{dv}{dt}$ $\quad\therefore P = mv\frac{dv}{dt} \Rightarrow P\,dt = mvdv$

Integrating both sides $\int_0^t P\,dt = m\int_0^v v\,dv$

$$P.t = \frac{1}{2}mv^2 \Rightarrow v = \left(\sqrt{\frac{2P}{m}}\right)t^{1/2}$$

Distance, $s = \int_0^t vdt = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt = \sqrt{\frac{2P}{m}}.\frac{t^{3/2}}{3/2}$

$\Rightarrow s=\sqrt{\frac{8P}{9m}}.t^{3/2} \Rightarrow s \propto t^{3/2}$

So, graph (B) is correct.

35. (B)

Total force required to lift maximum load capacity against frictional force = 4000 N

$F_{total} = Mg + friction$

$= 2000\times 10 + 4000 = 20,000 + 4000 = 4000N$

Using power, $P = F \times v$

$60 \times 746 = 24000 \times v \Rightarrow 1.86 m/s = 1.9 m/s$

Hence speed pf the elevator at full load is close to $1.9\,ms^{-1}$

36. (B)

Centripetal acceleration $a_c = n^2Rt^2$

$a_c = \frac{v^2}{R} = n^2Rt^2$

$v^2 = n^2R^2t^2$

$v = nRt$

Here power is delivered by tangential force only because power by centripetal force is zero. $\left[\text{Since}\,\vec{F}_c \perp \vec{V}\right]$

$a_t = \frac{dv}{dt} = nR$

Power $= ma_t v = m\,nR\,nRt = Mn^2R^2t$

37. (C)

Power, $P = \frac{w}{t} = \frac{E}{t} = \text{constant}$ $\quad\therefore \frac{\frac{1}{2}mv^2}{t} = \text{constant}$

From work-energy theorem, not work done =change in kinetic energy.

$\Rightarrow \frac{v^2}{t} = \text{constant}(k) \qquad \therefore kt^{1/2}\text{and}\,\frac{ds}{dt} = kt^{1/2}$

Or, $ds = kt^{1/2}dt$

By integrating, we get $\Rightarrow s = \frac{2kt^{3/2}}{3} + C \Rightarrow s \propto t^{3/2}$

i.e., Distance moved $S \propto t^{3/2}$

38. (2)

Work done by A=Work done by B

$F_A d\cos 45^\circ = F_B d\cos 60^\circ$

$\Rightarrow F_A \times \frac{1}{\sqrt{2}} = F_B \times \frac{1}{2} \Rightarrow \frac{F_A}{F_B} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \Rightarrow x = 2$

39. (450)

Given,

Force, $F=(5y+20)\hat{j}$

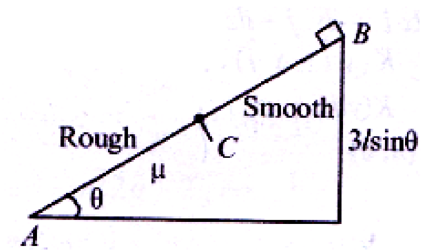
Work done, $E=\int F.dy$

$$\Rightarrow W=\int_0^{10}(5y+20)dy=\left[\frac{5y^2}{2}+20y\right]_0^{10}$$

$$=\frac{5}{2}\times100+20\times10=450J$$

40. (3)

If $AC=l$ then according to question, $BC=2l$ and $AB=3l$.




Here, work done by all the forces is zero.

$W_{friction}+W_{mg}=0$

$mg(3l)\sin\theta-\mu mg\cos\theta(l=0)$

$\Rightarrow \mu mg\cos\theta l=3mgl\sin\theta\Rightarrow\mu=3\tan\theta=k\tan\theta$

$\therefore k=3$

41. (24)

Using work-energy theorem, $W_{net}=(K_f-K_i)$

$$\Rightarrow\frac{1}{2}Kx^2=\frac{1}{2}m\left(\frac{v}{2}\right)^2-\frac{1}{2}mv^2=\frac{E}{4}-E$$

$$\Rightarrow\frac{1}{2}Kx^2=\frac{3E}{4}\Rightarrow K=\frac{3E}{2x^2}\Rightarrow K=\frac{3E}{2\times\left(\frac{1}{4}\right)^2}=24E$$

So, value of spring constant of used spring is 24 times of kinetic energy

$\therefore n=24$

42. (2)

Using energy conservation for plane AB

$\frac{1}{2}mu^2=mgh$ (Here, u= initial velocity of block)

$$\Rightarrow\frac{1}{2}\times m\times u^2=m\times10\times10\Rightarrow u=10\sqrt{2}$$

At point B

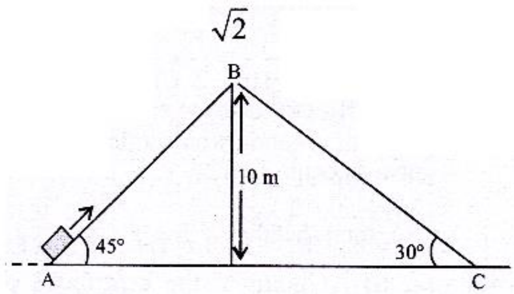
Acceleration, $a=-g\sin45^\circ=\frac{-10}{\sqrt{2}}$

Using $v=u+at_1$

$\Rightarrow 0 = 10\sqrt{2} - \frac{10}{\sqrt{2}} t_1$

$\Rightarrow t_1 = 2\,\text{sec}$

For plane BC




Using $s = ut_2 + \frac{1}{2}at_2^2$

$\Rightarrow \frac{10}{\sin 30^\circ} = \frac{1}{2}\left(10\sin 30^\circ\right)t_2^2 \quad \left(\because s = \frac{10}{\sin 30^\circ}\right)$

$\Rightarrow t_2 = 2\sqrt{2}$

So total time $T = t_1 + t_2 = 2\sqrt{2} + 2 = 2\left(\sqrt{2} + 1\right)\text{sec}$

43. (16)

Mass of engine − wagon system, $m = 40{,}000\,\text{kg}$ Velocity , $v = 72 \times 5/18 = 20\,\text{m/s}$

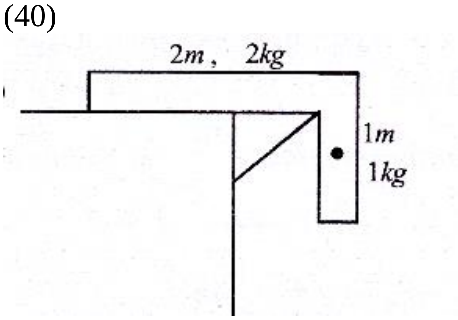
$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times (40{,}000) \times (20)^2 = 8000000\text{J}$

As 90% of K.E. of system lost in friction, only 10% is transferred to spring.

$\therefore \frac{1}{2}Kx^2 = \frac{10}{100} \times 8000000 \Rightarrow \frac{1}{2} \times K \times 1 \times 1 = 8 \times 10^5$

$\Rightarrow K = 16 \times 10^5\,\text{N/m}$

44. (40)




Loss in potential energy =gain in kinetic energy

Take zero potential energy at table, initial potential energy

$= -1 \times 10 \times \frac{1}{2} = -5\text{J}$

Final potential energy $= -3 \times 10 \times \frac{3}{2} = -45\text{J}$

Change in potential energy $= -5 - (-45)\,\text{J} = 40\text{J}$

$\therefore k = 40$

45. (10)

By mechanical energy conservation,

$T.E_A = T.E_B$

$PE_A + KE_A = PE_B + KE_B$

$mg(10) + 0 = mg(5) + \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2 \times g \times 5} = 10\,m/s$

$\therefore x = 10$

46. (6)

Here kinetic energy of ball is equal to P.E. stored in spring i.e., $\frac{1}{2}mv^2 = \frac{1}{2}k(\Delta x)^2$

$\Rightarrow \frac{1}{2} \times 4 \times (10)^2 = \frac{1}{2} \times 100 \times (\Delta x)^2 \Rightarrow \Delta x = 2m$

Therefore length of the compressed spring

$x = 8 - 2 = 6m$

47. (150)

From work energy theorem,

$W = F.s = \Delta KE = \frac{1}{2}mv^2$

Here $V^2 = 2gh$

$\therefore F.s = F \times \frac{2}{10} = \frac{1}{2} \times \frac{15}{100} \times 2 \times 10 \times 20$

$\therefore F = 150N$

48. (10)

Kinetic energy =change in potential energy of the particle.

$KE = mg\Delta h$

Given, $m = 1kg$.

$\Delta h = h_2 - h_1 = 2 - 1 = 1m$

$\therefore KE = 1 \times 10 \times 1 = 10J$

49. (18)

Given, Mass of the body, $m = 2kg$

Power delivered by engine, $P = 1J/s$

Time, t=9 seconds

Power, $P = Fv$

$\Rightarrow P = mav \qquad [\because F = ma]$

$\Rightarrow m\frac{dv}{dt}v = P \qquad \left(\because a = \frac{dv}{dt}\right),$

$\Rightarrow vdv = \frac{P}{m}dt$

Integrating both sides we get

$\Rightarrow \int_0^v vdv = \frac{P}{m}\int_0^t dt \Rightarrow \frac{v^2}{2} = \frac{Pt}{m} \Rightarrow v = \left(\frac{2Pt}{m}\right)^{1/2}$

$$\Rightarrow \frac{dx}{dt} = \sqrt{\frac{2P}{m}} t^{1/2} \qquad \left(\because v = \frac{dx}{dt}\right)$$

$$\Rightarrow \int_0^x dx = \sqrt{\frac{2P}{m}} \int_0^t t^{1/2} dt$$

$$\therefore \text{Distance, } x = \sqrt{\frac{2P}{m}} \frac{t^{3/2}}{3/2} = \sqrt{\frac{2P}{m}} \times \frac{2}{3} t^{3/2}$$

$$\Rightarrow x = \sqrt{\frac{2 \times 1}{2}} \times \frac{2}{3} \times 9^{3/2} = \frac{2}{3} \times 27 = 18\text{m}$$