Level-1

1) Among all the 4 options only angular momentum follows vector law of addition

~~So~~ (d) angular momentum

2) (d)
When vector is slid parallel to itself Its magnitude and direction doesnt Change. So, vector is not changed

3) (d)
The maximum magnitude of $|\vec{A}+\vec{B}|$ is $|\vec{A}|+|\vec{B}|$ and minimum magnitude Possible is $||\vec{A}|-|\vec{B}||$

~~4)~~ $\vec{A} = \vec{B}+\vec{C}$

$\cancel{|\vec{A}|^2 = |\vec{B}+\vec{C}|^2} \Rightarrow A^2 = B^2 + \cancel{C^2 + 2BC\cos\theta}$

$25 = 16 + 9 + \cancel{2 \times 4 \times 3 \times \cos\theta}$

4) (a)

$\bar{A} = \bar{B} + \bar{C}$

$|\bar{A}-\bar{C}|^2 = |\bar{B}|^2$

$A^2 + C^2 - 2AC\cos\theta = B^2 \Rightarrow 25 + 9 - 2 \times 5 \times 3\cos\theta = 16$

$\Rightarrow 34 - 30\cos\theta = 16$

$\Rightarrow \cos\theta = \frac{18}{30} \Rightarrow \cos\theta = \frac{3}{5}$

$\Rightarrow \theta = \cos^{-1}(3/5)$

5) (C)

$\Rightarrow \vec{C} = \vec{A} + \vec{B}$

$|\vec{C}|^2 = |\vec{A}+\vec{B}|^2 \Rightarrow C^2 = A^2 + B^2 + 2AB\cos 120$

$= A^2 + B^2 - AB \quad (\because \cos 120 = \frac{-1}{2})$

$\Rightarrow C^2 > A^2 + B^2 - 2AB$

$\Rightarrow C^2 > (A-B)^2$

$\Rightarrow C > |A-B|$

6) $\vec{A} + \vec{B} + \vec{C} = 0$ (d)

let $|\vec{B}| = |\vec{C}| = P$

$\Rightarrow |\vec{A}| = \sqrt{2}P$

$|\vec{B} + \vec{C}|^2 = |-\vec{A}|^2$

$\Rightarrow B^2 + C^2 + 2BC\cos\theta = A^2$

$\Rightarrow P^2 + P^2 + 2P^2\cos\theta = (\sqrt{2}P)^2$

$\Rightarrow 2P^2 + 2P^2\cos\theta = 2P^2$

$\Rightarrow 2P^2\cos\theta = 0$

$\Rightarrow \cos\theta = 0$

or $\theta = 90°$ $\therefore$ angle between $\vec{B}$ and $\vec{C}$ is $90°$

$|\vec{A} + \vec{B}|^2 = |-\vec{C}|^2$

$A^2 + B^2 + 2AB\cos\theta = C^2$

$\Rightarrow 2P^2 + P^2 + 2\sqrt{2}P^2\cos\theta = P^2$

$\Rightarrow 3P^2 + 2\sqrt{2}P^2\cos\theta = P^2 \Rightarrow 2\sqrt{2}P^2\cos\theta = -2P^2$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$

$\Rightarrow \theta = 135^\circ$

$\therefore$ Angle between $\vec{A}$ and $\vec{B}$ is 135°

Similarly we get angle between $\vec{A}$ and $\vec{C}$ is 135°

7)(a) $(2F)^2 + (\sqrt{2}F)^2 + 2\times 2F \times \sqrt{2}F \cos\theta = (F\sqrt{10})^2$

$\Rightarrow 4F^2 + 2F^2 + 4\sqrt{2}F^2\cos\theta = 10F^2$

$\Rightarrow 4\sqrt{2}F^2\cos\theta = 4F^2$

$\Rightarrow F^2\cos\theta = \frac{F^2}{\sqrt{2}}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$

$\therefore$ angle between $2F$ and $\sqrt{2}F$ is 45°

8) (d)

Let us take first vector to be along +ve x axis

then $\vec{F_1} = F\hat{i}$

$\vec{F_N} = F\cos\left(\frac{2\pi(N-1)}{N}\right)\hat{i} + F\sin\left(\frac{2\pi(N-1)}{N}\right)\hat{j}$

$\sum_{i=1}^{i=N} \vec{F_i} = F\left(\sum_{N=1}^{N=n} \cos\frac{2\pi(N-1)}{N}\right)\hat{i} + F\left(\sum_{N=1}^{N=n} \sin\frac{2\pi(N-1)}{N}\right)\hat{j}$

$\sum_{N=1}^{N=n} \sin\frac{2\pi(N-1)}{N} = \frac{\sin n\times\frac{2\pi}{n\times 2}}{\sin\frac{\pi}{n}} \cdot \sin\left(\frac{(n-1)\pi}{n}\right)$

$$= \frac{\sin \pi}{\sin \frac{\pi}{n}} \times \sin\left(n-1\frac{\pi}{n}\right)$$

$$= 0$$

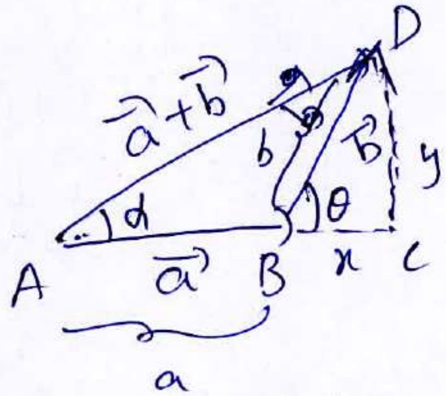
$$\sum_{N=1}^{N=n} \cos \frac{2\pi(N-1)}{N} = \frac{\sin n \times \frac{2\pi}{2n}}{\sin \frac{\pi}{n}} \cdot \times \cos\left(n-1\frac{\pi}{n}\right)$$

$$= \frac{\sin \pi}{\sin \frac{\pi}{n}} \times \cos\left(\left(\frac{n-1}{n}\right)\pi\right)$$

$$= 0$$

$$\therefore \sum_{i=1}^{i=N} \vec{F_i} = \vec{0}$$

9) c



In right angled triangle BDC

$\cos\theta = \frac{x}{b}$ $\qquad$ $\sin\theta = \frac{y}{b}$

$\Rightarrow x = b\cos\theta$ $\qquad$ $\Rightarrow y = b\sin\theta$

$\tan\alpha = \frac{y}{a+x} = \frac{b\sin\theta}{a+b\cos\theta}$

$\Rightarrow \tan\alpha = \frac{b\sin\theta}{a+b\cos\theta}$

10) d

10) $\vec{r} = \vec{r_2} - \vec{r_1}$

$= (-2i-2j) - (4i-4j)$

$\vec{r} = -6i+2j$

$\Rightarrow |\vec{r}| = \sqrt{6^2+4}$

$= \sqrt{40} = 2\sqrt{10}$

11) (b)

$|\vec{A}| = 1$

$\Rightarrow \sqrt{(0.4)^2 + (0.3)^2 + c^2} = 1$

$\Rightarrow 0.16 + 0.09 + c^2 = 1$

$\Rightarrow c^2 = 1 - 0.25$

$\Rightarrow c^2 = 0.75$

$\Rightarrow c = \sqrt{0.75}$

12) (c)

$|\vec{A}| = \sqrt{3^2 + 2^2 + 1}$

$= \sqrt{14}$

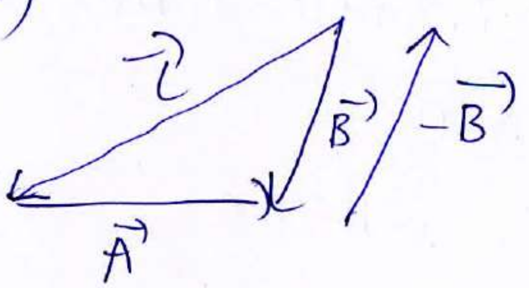
$|\vec{B}| = \sqrt{1+9+25} = \sqrt{35}$

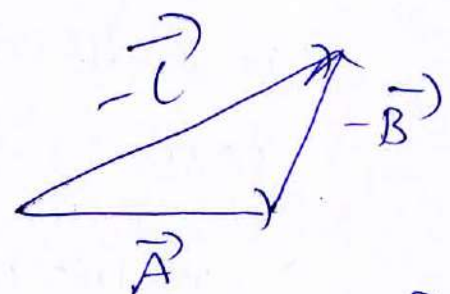
$|\vec{C}| = \sqrt{16+4+1} = \sqrt{20}$

$\Rightarrow A^2 + C^2 = B^2$

$\Rightarrow \vec{A} \perp \vec{C}$

13) (c)





$\Rightarrow \vec{A} + (-\vec{B}) = -\vec{C}$

$\Rightarrow \vec{A} + \vec{C} = \vec{B}$

14) (c)

let $|\vec{a}| = |\vec{b}| = |\vec{c}| = p$

$\vec{a} + \vec{b} = \vec{c}$

$|\vec{a} + \vec{b}|^2 = |\vec{c}|^2$

$\Rightarrow a^2 + b^2 + 2ab\cos\theta = c^2 \Rightarrow p^2 + p^2 + 2p^2\cos\theta = p^2$

$\Rightarrow 2p^2\cos\theta = -p^2$

$\Rightarrow \cos\theta = -\frac{1}{2}$

$\Rightarrow \theta = 120^\circ$

$\therefore$ angle between $\vec{a}$ & $\vec{b}$ is 120°

15) (d)

$\vec{A}\cdot\vec{B}=0 \quad \vec{A}\cdot\vec{C}=0$

$\Rightarrow \vec{A}\perp\vec{B} \Rightarrow \vec{A}\perp\vec{C} \quad \therefore \vec{A}\perp\vec{B}\ \&\ \vec{C}$

$\Rightarrow$ ~~A~~ $\vec{B}\times\vec{C}$ is also ~~to~~ to perpendicular to $\vec{B}\ \&\ \vec{C}$

$\therefore \vec{A} \parallel$ to $\vec{B}\times\vec{C}$

16) (d)

~~U Ca~~ ∵ we cannot divide two vectors

17) (a)

dot product is distributive over addition

$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$

$= 0+0$

$= 0$

18)

$\vec{A}+\vec{B} = 2\hat{i}+3\hat{j}+\hat{i}+4\hat{j}+\hat{k}$

$= 3\hat{i}+7\hat{j}+\hat{k}$

Unit vector along $\vec{A}+\vec{B}$ is $= \dfrac{\vec{A}+\vec{B}}{|\vec{A}+\vec{B}|}$

$$= \frac{3i+7j+k}{\sqrt{9+49+1}}$$

$$= \frac{3i+7j+k}{\sqrt{59}}$$

19) (a)

By right hand thumb rule

20) (c)

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{i} \times (\hat{j} \times \hat{k}) = \hat{i} \times \hat{i}$$

$$= \vec{0}$$

= null vector

21) (d)

$$(\hat{i}+\hat{j}) \cdot (\hat{i}-\hat{j}) = 1-1$$

$$= 0$$

as dot product is zero they are perpendicular

22) (b), $\vec{A} \times \vec{B} = 0 \Rightarrow \vec{A} \parallel \vec{B}$, $\vec{B} \times \vec{C} = 0 \Rightarrow \vec{B} \parallel \vec{C}$

A $\vec{A} \parallel \vec{B}$, $\vec{B} \parallel \vec{C} \Rightarrow \vec{A} \parallel \vec{C} \Rightarrow \vec{A} \times \vec{C} = \vec{0}$ zero vector

23) (c)

$\vec{r} = \vec{r_2} - \vec{r_1}$

$= 3i+4j+5k - (2i+3j+5k)$

$= i+j$

24) (a)

$\vec{A} = A\cos\alpha\,\hat{i} + A\cancel{\sin\alpha}^{\cos\beta}\,\hat{j} + A\cos\gamma\,\hat{k}$

$A = \sqrt{2^2+4^2+5^2}$

$= \sqrt{4+16+25}$

$= \sqrt{45}$

given

$A\cos\alpha = 2,\quad A\cos\beta = 4,\quad A\cos\gamma = -5$

$\Rightarrow \cos\alpha = \frac{2}{\sqrt{45}},\quad \cos\beta = \frac{4}{\sqrt{45}},\quad \cos\gamma = \frac{-5}{\sqrt{45}}$

$\cos\alpha, \cos\beta, \cos\gamma$ are direction cosines

25) (b)

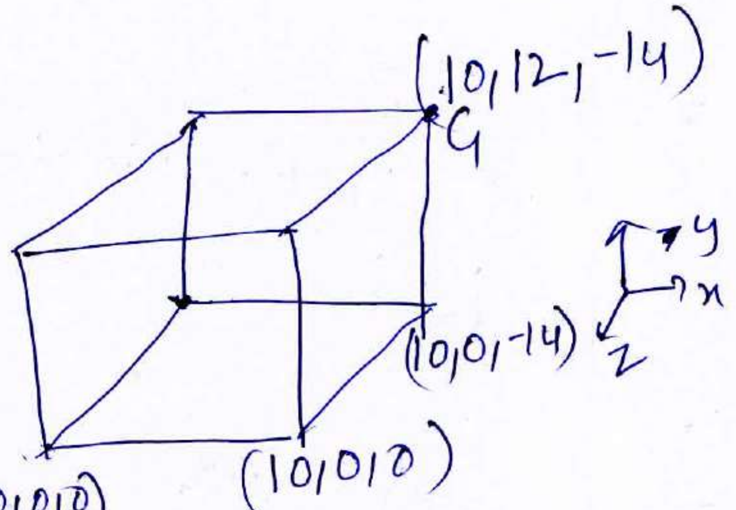
$i - 3j + 2\hat{k} + 3\hat{i} + 6j - 7k = 4\hat{i} + 3j - 5k$

Let $\vec{P}$ be a vector when added gives a unit vector along ~~+ve~~ y axis

$4\hat{i} + 3\hat{j} - 5\hat{k} + \vec{P} = \hat{j}$ or $4\hat{i} + 3\hat{j} - 5\hat{k} + \vec{P} = -\hat{j}$

$\Rightarrow \vec{P} = -4\hat{i} - 2\hat{j} + 5\hat{k}$ or $\vec{P} = -4\hat{i} - 4\hat{j} - 5\hat{k}$

26)



$\therefore AG = \sqrt{10^2 + 12^2 + 14^2}$
$= \sqrt{440}$

27) (a)

refer solution of problem. 8 in level 1

28) (d)

because $|\vec{P} + \vec{Q}| \neq |\hat{P} + \hat{Q}|$

29) (b)

His displacement is $\sqrt{(300)^2 + (400)^2}$
$= 500m$

30) (a)

$$\vec{A}+\vec{B} = 4\hat{i}+3\hat{j}+6\hat{k}+(-i+3j-8k)$$
$$= 3i+6j-2K$$

unit vector in direction of $\vec{A}+\vec{B}$ is $= \dfrac{\vec{A}+\vec{B}}{|\vec{A}+\vec{B}|}$

$$= \frac{3i+6j-2k}{\sqrt{9+\cancel{25}+36+4}}$$

$$= \frac{(3i+6j-2k)}{7}$$

32) (a)

θ is angle with vertical

$$\vec{V_1} = -V\cos\theta\hat{j} + V\sin\theta\hat{i}$$
$$\vec{V_2} = V\cos\theta\hat{j} + V\sin\theta\hat{i}$$

Change in momentum $= m(\vec{V_2}-\vec{V_1})$

$$\cancel{= m(2V\sin\theta\hat{i})}$$
$$\cancel{= 2mV\sin\theta}$$
$$= 2mV\,\cancel{\sin\theta}\cos\theta\hat{j}$$
$$= 2mV\cos\theta\hat{j}$$

$\therefore$ magnitude in change in momentum

$$= 2mV\cos\theta$$

33) (d)

maximum magnitude of resultant of $\vec{A}$ & $\vec{B}$ is $|\vec{A}|+|\vec{B}|$

and minimum is $||\vec{A}|-|\vec{B}||$

35) (a)

35) Let $|\vec{A}-\vec{B}| = |\vec{A}| = |\vec{B}| = P$

$\Rightarrow |\vec{A}-\vec{B}|^2 = |\vec{A}|^2$

$\Rightarrow A^2+B^2-2AB\cos\theta = A^2$

$\Rightarrow P^2-2P^2\cos\theta = 0$

$\Rightarrow P^2 = 2P^2\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 60^\circ$

36) (c)

refer to problem 5 level 1

37) (c)

$6i+7j+3i+4j=9i+\cancel{13j}\ 11\hat{j}$

$\Rightarrow |9\hat{i}+13\hat{j}| = \sqrt{9^2+11^2}$

$= \sqrt{\cancel{169}\ 121+81}$

$= \sqrt{202}$

38) ~~|A|~~ (c)

$|\vec{A}| = \sqrt{9+4+1} = \sqrt{14}$

$|\vec{B}| = \sqrt{1+9+25} = \sqrt{35}$

$|\vec{C}| = \sqrt{4+1+16} = \sqrt{21}$

$A^2+C^2=B^2$

$\therefore$ It is a right angled triangle

39) ~~$\vec{P}$~~

$\vec{P}+\vec{Q}=\vec{R}$

$\vec{P}=\vec{R}-\vec{Q}$

$|\vec{P}|^2=|\vec{R}-\vec{Q}|^2$

$\Rightarrow P^2 = R^2+Q^2-2QR\cos\theta$

$\Rightarrow 25 = 144+169-2\times12\times13\cos\theta$

$288 = 2 \times 12 \times 13 \cos\theta$

$\Rightarrow \cos\theta = \frac{12}{13}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{12}{13}\right)$

40) (C) $|\vec{F_1}| = F$

$|\vec{F_2}| = 2F$

given $|\vec{F_1} + \vec{F_2}|^2 = (2F)^2$

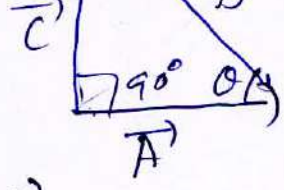
$\Rightarrow F^2 + 4F^2 + 2 \times F \times 2F \cos\theta = 4F^2$

$\Rightarrow F^2 = -4F^2 \cos\theta$

$\Rightarrow \cos\theta = -\frac{1}{4}$

$\Rightarrow \theta = \cos^{-1}\left(-\frac{1}{4}\right)$

41) (C)



$B^2 = A^2 + C^2$

$B = \sqrt{A^2 + A^2}$

$= \sqrt{2}A$

$\cos\theta = \frac{A}{B}$

$= \frac{A}{\sqrt{2}A}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 45^\circ$

angle between $\vec{A}$ & $\vec{B}$

is $180 - \theta = 180 - 45$

$= 135^\circ$

$= \frac{3\pi}{4}$ radian

42) (a)

$5i+8j+2i+7j=7i+15j$

$\therefore |7i+15j| = \sqrt{225+49}$

$= \sqrt{274}$

43) (d)

$\vec{A}+\vec{B}=\vec{A}-\vec{B}$

$2\vec{B}=\vec{0}$

$\Rightarrow \vec{B}=$ nullvector

45) (d)

Work done by forle

$= \vec{F}\cdot\vec{S}$

$= |F||S|\cos\theta$

$= 50\times10\times\cos60$

$= 50\times10\times 1/2$

$= 250J$

44)

$2i+3j-K$

and $-4i-6j-\lambda K$ arelled

$\Rightarrow \frac{2}{-4}=\frac{3}{-6}=\frac{1}{\lambda}$

$\Rightarrow \frac{-1}{2}=\frac{1}{\lambda}$

$\Rightarrow \lambda=-2$

46) (a)

$(\vec{A}+\vec{B}) \perp$ to $(\vec{A}-\vec{B})$

$(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B})=0$

$\Rightarrow \vec{A}\cdot\vec{A} \mp \cancel{\vec{A}\cdot\vec{B}} + \cancel{\vec{A}\cdot\vec{B}} - \vec{B}\cdot\vec{B}=0$

$\Rightarrow A^2-B^2=0$

$\Rightarrow A=B$

$\Rightarrow \frac{A}{B}=1$

47) (b)

$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$

$= i(4+4) - j(12-4) + k(-6-2)$

$\vec{A}\times\vec{B} = 8i - 8j - 8K$

$|\vec{A}\times\vec{B}| = \sqrt{8^2+8^2+8^2}$

$= 8\sqrt{3}$

48) (d)

If $\vec{C} = \vec{A}\times\vec{B}$

$\vec{C}$ is parallel to $\vec{A}\times\vec{B}$

49) (c)

If $\vec{P}$ is perpendicular to $\vec{F}$

then $\vec{P}\cdot\vec{F} = 0$

$7\hat{k}\cdot(4\hat{i}-3\hat{j}) = 0$

50) (b)

10m along y axis

$\Rightarrow$ displacement

$(\vec{S}) = 10\hat{j}$

$\therefore$ work done

$= \vec{F}\cdot\vec{S}$

$= (-2\hat{i}+15\hat{j}+6\hat{k})\cdot 10\hat{j}$

$= 150J$

51) (d)

If $\vec{F_1}$ is along positive x axis

Let it be $= K\hat{i}$

If $\vec{F_2}$ is in the form of $P_2\hat{j}$

then vector product of $\vec{F_1}$ and $\vec{F_2}$ is zero

52) (b)

$\because \vec{A}\times\vec{B} = 0 \Rightarrow |\vec{A}\times\vec{B}| = 0$

$\Rightarrow |\vec{A}||\vec{B}|\sin\theta = 0$

$\Rightarrow \sin\theta = 0$

$\Rightarrow \theta = 0°$

$\therefore \vec{A} \parallel \vec{B}$

53) (b)

Let vertically upward direction $= \hat{k}$

and north be $\hat{j}$

$\therefore \vec{A} \times \vec{B} = \hat{k} \times \hat{j}$

$= -\hat{i}$

$\Rightarrow$ Its towards west

54) (c)

$\vec{AB} = \vec{B} - \vec{A}$

$= \hat{i} + \hat{j} + \hat{k}$

$\vec{CD} = \vec{D} - \vec{AC}$

$= -3i - 3j - 3k$

$= -3(\hat{i} + \hat{j} + \hat{k})$

$\vec{CD} = -3(\vec{AB})$

$\therefore \vec{AB}$ & $\vec{CD}$ are antiparallel

55) (c)

vector perpendicular to $\vec{A}$ & $\vec{B}$ is $\vec{A} \times \vec{B}$

$\therefore \vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 2 & 2 & -1 \\ 6 & -3 & 2 \end{vmatrix}$

$= i(1) - j(4+6) + k(-6-12)$

$= i - 10j - 18k$

$\therefore$ unit vector will be $= \dfrac{i - 10j - 18k}{\sqrt{1 + 100 + 324}}$

$= \dfrac{i - 10j - 18k}{5\sqrt{17}}$

56) $\vec{F} = m\vec{a}$ (a)

$|\vec{F}| = m|\vec{a}|$

$\Rightarrow \sqrt{6^2 + 8^2 + 10^2} = m \times 1$

$\Rightarrow m = 10\sqrt{2}$ kg

57) (c)

When 2 vectors are equal in magnitude and opposite in direction, If one is $\vec{A}$ other will be $-\vec{A}$

$\Rightarrow \vec{A} + (-\vec{A}) = 0$

58) (c)

If $\vec{B}$ is normal to $\vec{A}$

then $\vec{A}\cdot\vec{B}=0$

$\vec{A}=A\cos\theta\,\hat{i}+A\sin\theta\,\hat{j}$

Take $\vec{B}=B\sin\theta\,\hat{i}-B\cos\theta\,\hat{j}$

then $\vec{A}\cdot\vec{B}=AB\cos\theta\sin\theta-AB\cos\theta\sin\theta$

$=0$

$\therefore$ this Value of $\vec{B}$ is satisfied.

59) (a)

$\vec{V}=\vec{\omega}\times\vec{r}$

$\Rightarrow \vec{\omega}\times\vec{r}=\begin{vmatrix} i & j & k \\ 1 & -2 & 2 \\ 0 & 4 & -3 \end{vmatrix}$

$\Rightarrow \cancel{i(6-8)-j(4)+k(8)}$

$=i(6-8)-j(-3)+k(4)$

$=-2i+3j+4k$

$\therefore |\vec{\omega}\times\vec{r}|=\sqrt{4+9+16}$

$\Rightarrow |\vec{V}|=\sqrt{29}$ units

60) $\cancel{\vec{r}=(a\cos\omega t)\hat{i}}$

(b)

$\vec{r}=(a\cos\omega t)\hat{i}+(a\sin\omega t)\hat{j}$

$\frac{d\vec{r}}{dt}=-a\omega\sin\omega t\,\hat{i}+a\omega\cos\omega t\,\hat{j}$

$\Rightarrow \vec{V}=a\omega(-\sin\omega t\,\hat{i}+\cos\omega t\,\hat{j})$

$\vec{r}\cdot\vec{V}=-a\omega^2\cos\omega t\sin\omega t+a\omega^2\cos\omega t\sin\omega t$

$=0$

$\therefore \vec{r}$ is perpendicular to $\vec{V}$

Level-II

1) (c)

$\vec{OA} = R\hat{i}$

$\vec{OB} = \frac{R}{\sqrt{2}}\hat{i} + \frac{R}{\sqrt{2}}\hat{j}$

$\vec{OC} = R\hat{j}$

$\vec{OA} + \vec{OB} + \vec{OC} = \left(R + \frac{R}{\sqrt{2}}\right)\hat{i} + \left(R + \frac{R}{\sqrt{2}}\right)\hat{j}$

$\therefore |\vec{OA} + \vec{OB} + \vec{OC}| = \sqrt{\left(R\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\right)^2 + \left(R\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\right)^2}$

$= \sqrt{\frac{1}{2}\left(R^2(3+2\sqrt{2}) + R^2(3+2\sqrt{2})\right)}$

$= R\sqrt{3+2\sqrt{2}}$

$= R\sqrt{(\sqrt{2}+1)^2}$

$= R(\sqrt{2}+1)$

2) $\cancel{\vec{a}+\vec{b}=\vec{c}}$ (d)

$\vec{a} + \vec{b} = \vec{c}$

given

$c = \sqrt{a^2+b^2}$

$\Rightarrow c^2 = a^2 + b^2$

$\vec{a} + \vec{b} = \vec{c}$

$\Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{c}|^2$

$\Rightarrow a^2 + b^2 + 2ab\cos\theta = c^2$

$\Rightarrow a^2 + b^2 + 2ab\cos\theta = a^2 + b^2$

$\Rightarrow 2ab\cos\theta = 0$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = 90^\circ$

$\therefore$ angle between $\vec{a}$ & $\vec{b}$ is 90°

3)

Let to $\vec{a}$

$\Delta\vec{a}$ is added

$\Rightarrow$ new vector be $(\vec{a}+\Delta\vec{a})$

$\Rightarrow |\vec{a}+\Delta\vec{a}| = |\vec{a}|^2 + |\Delta\vec{a}|^2 + 2|\vec{a}||\Delta\vec{a}|\cos\theta$

$= a^2 + |\Delta\vec{a}|^2 + 2a|\Delta\vec{a}|\cos\theta$ — ①

given increment of $\vec{a}$ is Δa

$\Rightarrow |\vec{a}+\Delta\vec{a}|^2 = (a+\Delta a)^2$

$\Rightarrow a^2 + \Delta a^2 + 2a\Delta a$ — ②

① = ②

from that we can say that

$|\Delta\vec{a}| \geq \Delta a$

4) (b)

As length of vector doesnt change magnitude of vector doesnt change

$\Delta a = 0$

Let $\vec{a} = a\hat{i}$

now when ~~it~~ it is rotated the new vector will be $a\cos d\theta\,\hat{i} + a\sin d\theta\,\hat{j}$

$\Rightarrow \Delta\vec{a} = (a - a\cos d\theta)\hat{i} + a\sin d\theta\,\hat{j}$

as $d\theta$ is very small we can take $\sin d\theta = d\theta$

$\cos d\theta = 0$

$\Rightarrow \Delta\vec{a} = a\,d\theta\,\hat{j}$

$\Rightarrow |\Delta\vec{a}| = a\,d\theta$

5)(c) $\vec{A}\cdot\vec{B} = 0$ and $\vec{A}\cdot\vec{C} = 0$

$\Rightarrow$ ~~$\vec{A} \parallel \vec{B}$~~

$\Rightarrow \vec{A}$ is $\perp$er to $\vec{B}$

and

$\vec{A}$ is $\perp$er to $\vec{C}$

$\Rightarrow \vec{A}$ will be parallel to $\vec{B}\times\vec{C}$

6) (a)

from triangle law

$\vec{OA} + \vec{AM} = \vec{OM}$

$\Rightarrow \vec{AM} = -\vec{MA}$

$\Rightarrow \vec{OA} - \vec{MA} = \vec{OM}$

7) (b)

$\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$

They need not be unit vectors
for example $\vec{a}=5i$ $\vec{c}=-5i$
$\vec{b}=10i$ $\vec{d}=-10i$

8) (d)

division of 2 vectors doesnt have any meaning

9) (b)

$\vec{A}+\vec{B}=\vec{C}$

$\Rightarrow$ ~~$\vec{A},\vec{B},\vec{C}$ are~~ n

$\Rightarrow \vec{C}$ is not in plane with $\vec{A}$ & $\vec{B}$

$\Rightarrow \vec{A}+\vec{B}+\vec{C}\neq 0$

10) (d)

$\vec{A}\times\vec{A}=\vec{0}$ because $\sin 0=0$

$\vec{A}\cdot(\vec{A}\times\vec{B})=0$ because $\vec{A}\perp\vec{A}\times\vec{B}$

$\vec{B}\times(\vec{A}\times(\vec{A}\times\vec{B}))=(\vec{B}\times\vec{A})\times(\vec{A}\times\vec{B})$
$=0$ angle between $\vec{B}\times\vec{A}$ and $\vec{A}\times\vec{B}$ is 180°

$\sin\pi=0^\circ$

11) (d)

when vector is rotated its direction changes so, now it will be a new vector

12) (a)

$\alpha = \beta = r$

$\Rightarrow \cos\alpha = \cos\beta = \cos r$

$\Rightarrow$ given

$A^2\cos^2\alpha + A^2\cos^2\beta + A^2\cos^2 r = A^2$

$\Rightarrow 3A^2\cos^2\alpha = A^2$

$\Rightarrow \cos^2\alpha = \frac{1}{3}$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

$\therefore$ every component $= A\cos\alpha$

$= \frac{A}{\sqrt{3}}$

13) (c)

$3i + j + 12k$

Its length in xy plane will be only because of $\hat{i}$ and $\hat{j}$

$\Rightarrow = \sqrt{3^2+1}$

$= \sqrt{10}$

15) (b)

resultant of $\vec{A}$ & $\vec{B}$ is $\vec{A}+\vec{B}$

$$\vec{A}+\vec{B} = \cancel{2i+3}\ 4i-3j+8i+8j$$

$$= 12i+5j$$

Unit vector $= \cancel{\phi A}\ \dfrac{\vec{A}+\vec{B}}{|\vec{A}+\vec{B}|}$

$$= \frac{12i+5j}{13}$$

16) (a)

Component of $\vec{A}$ along $\vec{B}$ is $\dfrac{\vec{A}\cdot\vec{B}}{|\vec{B}|}$

$$= \frac{(2i+3j)\cdot(i+j)}{\sqrt{\cancel{2^2+3^2}}\ \sqrt{1+1}}$$

$$= \frac{5}{\sqrt{2}}$$

17) (b)

$\hat{a}$ and $\hat{b}$ be unitvectors

and $\hat{c}$ be their resultant & $\hat{a}+\hat{b}=\hat{c}$

$|\hat{a}+\hat{b}|^2 = |\hat{c}|^2$

$|\hat{a}|^2+|\hat{b}|^2+2|\hat{a}||\hat{b}|\cos\theta = |\hat{c}|^2$

$\Rightarrow 1+1+2\cos\theta = 1$

$\Rightarrow 2+2\cos\theta = 1$

$\Rightarrow 1+2\cos\theta = 0$

$\Rightarrow \cos\theta = \frac{-1}{2}$

$|\hat{a}-\hat{b}|^2 = |\hat{a}|^2+|\hat{b}|^2-2|\hat{a}||\hat{b}|\cos\theta$

$= 1+1-2\cos\theta$

$= 1+1-2\times\frac{-1}{2}$

$|\hat{a}-\hat{b}|^2 = 3$

$\Rightarrow |\hat{a}-\hat{b}| = \sqrt{3}$

18) (a)

maximum will be $P+Q$

minimum is $P-Q$

$$\frac{P+Q}{P-Q}=\frac{3}{1}$$

$\Rightarrow P+Q=3P-3Q$

$\Rightarrow 4Q=2P$

$\Rightarrow P=2Q$

19) (d)

$R=\sqrt{3^2+2^2+2\times3\times2\cos\theta}$

$\Rightarrow R^2=13+12\cos\theta$ — (1)

$2R=\sqrt{\cancel{3^2+2}\, 6^2+2^2+2\times6\times2\cos\theta}$

$4R^2=40+24\cos\theta$ — (2)

Substituting (1) in (2)

$4(13+12\cos\theta)$
$=40+24\cos\theta$

$52+48\cos\theta$
$=40+24\cos\theta$

$\Rightarrow 12=-24\cos\theta$

$\Rightarrow \cos\theta=\frac{-1}{2}$

$\Rightarrow \theta=120^\circ$

20) (b)

Torque is

$\vec{r}\times\vec{F}$

$=(3i+2j+3k)\times(2i-3j+4k)$

$=\begin{vmatrix} i & j & k \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$

$= i(8+9)-j(12-6)+k(-9-4)$

$= 17i-6j-13k$

21) (C)

$$|\vec{v_1} + \vec{v_2}|^2 = |\vec{v_1} - \vec{v_2}|^2$$

$$v_1^2 + v_2^2 + 2v_1v_2\cos\theta = v_1^2 + v_2^2 - 2v_1v_2\cos\theta$$

$$\Rightarrow 4v_1v_2\cos\theta = 0$$

$$\Rightarrow \cos\theta = 0$$

$$\Rightarrow \theta = 90^\circ$$

22) (C)

$$\vec{P} = P_x\hat{i} + P_y\hat{j}$$

$$= 2\cos t\hat{i} + 2\sin t\hat{j}$$

$$\frac{d\vec{P}}{dt} = -2\sin t\,\hat{i} + 2\cos t\,\hat{j}$$

$$\Rightarrow \vec{F} = -2\sin t\,\hat{i} + 2\cos t\,\hat{j}$$

$$\vec{P}\cdot\vec{F} = -4\cos t\sin t + 4\cos t\sin t$$

$$= 0$$

$\Rightarrow \vec{P}$ is ⊥er to $\vec{F}$

$$\theta = 90^\circ$$

23) (c)

If $\vec{A}$ & $\vec{B}$ be adjacent sides of a parallelogram one diagonal will be $\vec{A}+\vec{B}$

other diagonal will be $\vec{A}-\vec{B}$

$$(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$$
$$= 2(\vec{B}\times\vec{A})$$

area of parallelogram $= |\vec{B}\times\vec{A}|$

$$|2i\times 2j| = 2|(\vec{B}\times\vec{A})|$$
$$\Rightarrow 4 = 2|\vec{B}\times\vec{A}|$$
$$\Rightarrow |\vec{B}\times\vec{A}| = 2$$

24) (b)

area of parallelogram $= |\vec{A}\times\vec{B}|$

$$\vec{A}\times\vec{B} = \begin{pmatrix} i & j & k \\ 1 & 2 & 3 \\ 3 & -2 & 1 \end{pmatrix}$$
$$= i(8) - j(1-9) + k(-8)$$
$$= 8i + 8j - 8k$$
$$\Rightarrow |\vec{A}\times\vec{B}| = 8\sqrt{3}$$

25) (a)

$\vec{F}\cdot\vec{S} = -12+2c-6$
$= 2c-18$

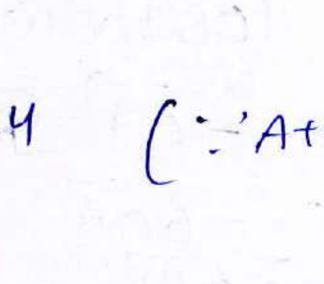
$2c-18=6$ or $2c-18=-6$

~~when~~ $\cancel{2c-18=-6}$
$\cancel{\Rightarrow 2c=18-6}$
$\cancel{\Rightarrow c=6}$

but $\vec{F}\cdot\vec{S}$ should be positive as per given

$2c-18=6$
$\Rightarrow 2c=24$
$c=12$

26) (a)



$R=|\vec{A}+\vec{B}|=12$
$A+B=18$
$A^2+R^2=B^2$
$\Rightarrow B^2-A^2=R^2$
$\Rightarrow (B-A)(A+B)=144$ $\quad (\because A+B=18)$
$\Rightarrow (B-A)\times 18=144$
$\Rightarrow B-A=8$
$A+B=16$
$B-A=8$
$\Rightarrow B=12,\ A=5$

27) (c)

we know

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$$

$$\Rightarrow 2 = \sin^2\alpha + \sin^2\beta + \sin^2\gamma$$

28) (d)

we know

$$\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} + \vec{OE} + \vec{OF} = 0$$

$$\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF}$$

$$= \vec{OB} - \vec{OA} + \vec{OC} - \vec{OA} + \vec{OD} - \vec{OA}$$

$$+ \vec{OE} - \vec{OA} + \vec{OF} - \vec{OA}$$

$$= (\vec{OB} + \vec{OC} + \vec{OD} + \vec{OE} + \vec{OF}) - 5\vec{OA}$$

$$= -\vec{OA} - \cancel{4\vec{OA}}\ 5\vec{OA}$$

$$= -6\vec{OA}$$

$$= 6\vec{AO}$$

29) (d)

Speed of seconds hand $= \cancel{2\pi}\ \frac{\text{distance}}{\text{time}}$

$= \frac{2\pi r}{60}$

$= \frac{\pi r}{30} = \frac{\pi}{30}$ cm/s $(\because r = 1cm)$

In 15 seconds it moves 90°

$\therefore$ Change in velocity $= |\vec{v_2} - \vec{v_1}|$

$= \sqrt{\left(\frac{\pi}{30}\right)^2 + \left(\frac{\pi}{30}\right)^2}$

$= \frac{\pi\sqrt{2}}{30}$ cm/sec

30) (a)

angular momentum $= \vec{r} \times \vec{p}$

$$\vec{r} \times \vec{p} = \begin{vmatrix} i & j & K \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix}$$

$= i(-4+4) - j(-2+3) + K(4-6)$

$= -j - 2K$

$\therefore$ $-j-2K$ is perpendicular to x axis

~~Assertion reason~~

Assertion and reasons

1) (a)

explanation is given in the reason itself

2) (a)

$$\cos\theta = \frac{(i+j)\cdot i}{\sqrt{2}\times 1}$$
$$= \frac{1}{\sqrt{2}}$$
$$\Rightarrow \theta = 45^\circ$$

3)

$$\sin\theta = |\vec{A}\times\vec{B}|$$
$$\cos\theta = \vec{A}\cdot\vec{B}$$
$$\Rightarrow \tan\theta = \frac{|\vec{A}\times\vec{B}|}{\vec{A}\cdot\vec{B}}$$

$\vec{A}\cdot\vec{B}$ is a scalar, $\vec{A}\times\vec{B}$ is a vector

~~So~~ a vector can never be ~~to~~ perpendicular to scalar

4) (b)

$$|\bar{A}+\bar{B}|^2 = |\vec{A}-\vec{B}|^2$$
$$\Rightarrow A^2+B^2+2AB\cos\theta = A^2+B^2-2AB\cos\theta$$
$$\Rightarrow 4AB\cos\theta = 0$$
$$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^\circ$$

5) (c)

$\vec{V} = \vec{\omega} \times \vec{r}$

6)

no need for explanation

7) no need for explanation

8) (c)

$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A}) \Rightarrow \vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

9) (d)

null vector has magnitude zero

10) (a)

explanation is given in reason itself

11) (c)

Vector quantities have a specific direction except null vector

12) (a)

$\vec{A}\cdot\vec{B} = |A||B|\cos\theta$

when $\cos\theta = 0$ dot product will be zero

$\Rightarrow \theta = 90^\circ$

(3) (b)

no need for explanation

14) (c)

when two vectors of equal magnitude and opposite direction their resultant is a null vector

15) (b)

$\vec{A}\times\vec{B} = \vec{0} \Rightarrow |\vec{A}\times\vec{B}| = 0$

$\Rightarrow AB\sin\theta = 0$ - (1)

$\vec{A}\cdot\vec{B} = 0 \Rightarrow AB\cos\theta = 0$ - (2)

from (1) & (2)

$AB = 0$

$\Rightarrow$ ~~A is 0 or B is~~ $A = 0$ or $B = 0$

16) (b)

$\vec{A} \times \vec{A} = \vec{0}$

17) (c)

let $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ be non coplanar vectors
then $\vec{C}$ will be not be in plane
of $\vec{A}$ & $\vec{B}$

$\Rightarrow \vec{A} + \vec{B} + \vec{C} \neq 0$ all the time

$\therefore$ minimum 4 vectors are required

resultant of two vectors to be
zero they should have equal magnitude

$\vec{A} + \vec{B} = 0$

$\Rightarrow \vec{A} = -\vec{B}$

$\Rightarrow |\vec{A}| = |-\vec{B}|$

(18) (a)

Two unequal vectors may make
Same angle with one vector

19) (d)

Current has magnitude and direction but it doesnot follow vector law of addition. So it is not a vector

20) (a)

explanation is given in reason itself

29) · Previous year questions

1) (a)

$$5 = -a$$
$$\Rightarrow a = -5$$

(2) (c)

Component of A along B $= \dfrac{\vec{A}\cdot\vec{B}}{|\vec{B}|}$

$$= \frac{(2i+3j)\cdot(i+j)}{\sqrt{2}}$$

$$= \frac{5}{\sqrt{2}}$$

3) (a)

$$A + 0 = A$$
$$A \times 0 = 0$$
$$\Rightarrow A + 0 + A \times 0 = A$$

4)

$$\bar{A}\cdot\bar{B} = |\bar{A}\times\bar{B}|$$

$$AB\cos\theta = AB\sin\theta$$

$$\Rightarrow \tan\theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

$$\Rightarrow |\bar{A}+\bar{B}| = \sqrt{A^2+B^2+2AB\cos\frac{\pi}{4}} = \sqrt{A^2+B^2+\sqrt{2}AB} \quad \left(\because \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}\right)$$

5) (d)

$$\bar{a} + \bar{b} = \bar{c}$$

$$\Rightarrow |\bar{a} + \bar{b}|^2 = |\bar{c}|^2$$

$$\Rightarrow a^2 + b^2 + 2ab \cos\theta = c^2 \quad -(1)$$

given

$$(a + b = c$$

$$\Rightarrow (a+b)^2 = c^2$$

$$\Rightarrow a^2 + b^2 + 2ab = c^2 \quad -(2)$$

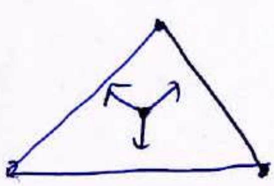
from (1) & (2)

$$\Rightarrow a^2 + b + 2ab\cos\theta = a^2 + b^2 + 2ab$$

$$\Rightarrow \cos\theta = 1$$

$$\Rightarrow \theta = 0^\circ$$

6) (d)



Three forces equal of magnitude and angle between any two adjacent forces is 120°

$\therefore$ The resultant will be zero

7) (c)

If $\vec{a} \perp \vec{b}$

$\vec{a}$ ~~is $\perp$ to $\vec{b}$~~

then

$\vec{a} \cdot \vec{b} = 0$

$5 - 2\lambda + 1 = 0$

$\Rightarrow 6 = 2\lambda$

$\Rightarrow \lambda = 3$

8)

given

$$\sqrt{(x+y)^2 + (x-y)^2 + 2(x+y)(x-y)\cos\theta} = \sqrt{x^2+y^2}$$

$$\Rightarrow \sqrt{x^2+y^2+x^2+y^2+2(x^2-y^2)\cos\theta} = \sqrt{x^2+y^2}$$

$$\Rightarrow 2(x^2+y^2) + 2(x^2-y^2)\cos\theta = x^2+y^2$$

$$\Rightarrow \cos\theta = \frac{-(x^2+y^2)}{2(x^2-y^2)}$$

$$\theta = \cos^{-1}\left(-\frac{(x^2+y^2)}{2(x^2-y^2)}\right)$$

9) (d)

Torque $= \vec{r} \times \vec{F}$

$$\vec{r} \times \vec{F} = \begin{pmatrix} i & j & k \\ 8 & 2 & 3 \\ -3 & 2 & 1 \end{pmatrix}$$

$$= i(2-6) - j(8+9) + k(16+6)$$

$$= -4i - 17j + 22k$$

10) (c)

work done by force F = Change in kinetic energy

$\cancel{\vec{S} = \vec{r_2} - \vec{r_1}}$

$\cancel{= 3i - (2i+3j)}$

$\cancel{= i - 3j}$

$\cancel{\therefore \vec{F}\cdot\vec{s} = (3x^2\hat{i}+4\hat{j})\cdot(\hat{i}-3\hat{j})}$

$d\vec{s} = dx\hat{i} + dy\hat{j}$

$$\int_{(2,3)}^{(3,0)} \vec{F}\cdot d\vec{s} = \int_{(2,3)}^{(3,0)} (3x^2\hat{i}+4\hat{j})\cdot(dx\hat{i}+dy\hat{j})$$

$$= \int_{(2,3)}^{(3,0)} 3x^2dx + 4dy$$

$$= \left| \frac{3x^3}{3} + 4y \right|_{(2,3)}^{(3,0)}$$

$$= \left(x^3+4y\right)_{(2,3)}^{(3,0)}$$

$$= 3^3 - 2^3 + 4(0-3)$$

$$= 27-8-12$$

$$= 7J$$

11) (b)

iet velocity of motor boat in still water = V_m

velocity of river = V_r

Let distance be d

$\frac{d}{V_m+V_r} = 6$ -(1) $\quad \frac{d}{V_m-V_r} = 10$ -(2)

$\Rightarrow \frac{V_m+V_r}{V_m-V_r} = \frac{10}{6} = \frac{5}{3}$

$\Rightarrow 3V_m + 3V_r = 5V_m - 5V_r$

$\Rightarrow 8V_r = 2V_m$

$\Rightarrow V_m = 4V_r$

$\Rightarrow V_r = \frac{V_m}{4}$

In (1)

$\frac{d}{V_m + \frac{V_m}{4}} = 6 \Rightarrow \frac{d4}{5V_m} = 6$

$\Rightarrow \frac{d}{V_m} = \frac{30}{4}$

$= 7.5 hr$

12) (c)

given $\vec{a_1}, \vec{a_2}$ are unit vector $\Rightarrow a_1 = 1$
$a_2 = 1$

$|\vec{a_1} + \vec{a_2}|^2 = (\sqrt{3})^2$

$\Rightarrow a_1^2 + a_2^2 + 2a_1a_2\cos\theta = 3$

$2+2\cos\theta=3$

$\Rightarrow 2\cos\theta=1$

$\Rightarrow \cos\theta=1/2$

$(a_1-a_2)\cdot(2a_1+a_2)=2a_1^2+a_1\cdot a_2-2a_2\cdot a_1-a_2^2$

$=2-1\cancel{+}-a_1\cdot a_2$

$=1-\cos\theta$

$=1-1/2$

$=1/2$

13) (c)

refer problem 8 level-1

14) (b)

let $\vec{A}=2i-j+k$

$\vec{B}=i+2j-3k$

$\vec{C}=3i+pj+5k$

as, 3 vectors are coplanar

$\vec{C}\perp\vec{A}\times\vec{B}$

$\vec{A}\times\vec{B}=\begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 1 & 2 & -3 \end{vmatrix}=i(3-2)-j(\overset{-6-1}{\cancel{4+1}})+k(4+1)$

$=i+7\cancel{8}j+5k$

$(\vec{A} \times \vec{B}) \cdot \vec{C} = 3 + 7p + 25 = 0$

$\Rightarrow 28 + 7p = 0$

$\Rightarrow p = -4$

15) (a)

$\vec{A} + \vec{B} + \vec{C} = 0$

$\Rightarrow \vec{C} = -(\vec{A} + \vec{B})$

$= -(3i + 4k)$

16) (d)

unit vector in direction of $\vec{A} = \frac{\vec{A}}{|\vec{A}|}$

$= \frac{\vec{A}}{A}$

17) (c)

The maximum of $|\vec{A} + \vec{B}|$ is when $\cos\theta = 1$

$= |\vec{A}| + |\vec{B}|$

$= 12 + 8$

$= 20\ N$

18) (d)

$\vec{A}+\vec{B} = -2i+3j+k$

let $\vec{C} = \hat{i}+2\hat{j}-4\hat{k}$

$(\vec{A}+\vec{B})\cdot\vec{C} = |\vec{A}+\vec{B}||\vec{C}|\cos\theta$

$\Rightarrow \cos\theta = \dfrac{(\vec{A}+\vec{B})\cdot\vec{C}}{|\vec{A}+\vec{B}||\vec{C}|}$

$= \dfrac{-2+6-4}{|\vec{A}+\vec{B}||\vec{C}|}$

$\cos\theta = 0$

$\Rightarrow \theta = 90^\circ$

19) (c)

when

$(\vec{a}+\vec{b})$ ~~is~~ $\perp$ $\vec{a}-\vec{b}$

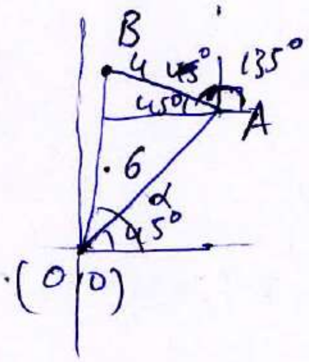
$\Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 0$

$\Rightarrow |a|^2 + \vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{b} - |b|^2 = 0$

$\Rightarrow |a|^2 = |b|^2$

$\Rightarrow$ ~~|a=b~~ $|a| = |b|$

20) (c)



$A = 6\cos 45\, i + 6\sin 45\, j$

$B = 6\cos 45 - 4\cos 45\, i + 6\sin 45 j + 4\sin 45\, j$

$= \frac{2}{\sqrt{2}} i + \frac{10}{\sqrt{2}} j$

$= \sqrt{2} i + 5\sqrt{2} j \qquad \Rightarrow \vec{OB} = \sqrt{2} i + 5\sqrt{2} j$

$\Rightarrow |\vec{OB}| = \sqrt{2+50}$

$= \sqrt{52}$ km

$\therefore \tan\alpha = \frac{y}{x}$

$= \frac{5\sqrt{2}}{\sqrt{2}}$

$= 5$

$\Rightarrow \alpha = \tan^{-1}(5)$

21) (d)

velocity of train $= 10 j$

velocity of Parrot $= -5 j$

$\Rightarrow$ velocity of Parrot with respect to train

$= -15 j$

$\therefore$ time $= \frac{150}{15} = 10$ s

22) (d)

Velocity of rain $= -4\hat{j}$

velocity of man $= 3\hat{i}$

$\therefore$ velocity of rain wrt to man $= -4j - 3i$

$\Rightarrow |-4j - 3i| = \sqrt{4^2+9}$

$= 5$

23) (c)

Let velocity of police wrt to thief $= v$

$\therefore v = \cancel{\frac{50}{10}}\ \frac{6}{10}$

$= \cancel{\frac{4}{10}}\ \frac{3}{5}$

In 25 seconds poleice moves $25 \times \frac{3}{5}$ wrt to thief $= 15$

$\therefore$ distance between them $= 50 - 15$

$= 35m$

24) (a)

Let velocity of rain $= xi - yj$

$\therefore$ velocity of rain wrt to man

$= (x-2)i - yj$

It appears vertically

$\Rightarrow x-2=0$

$\Rightarrow x=2$

and value of rain wrt man $=2$

$\Rightarrow y=2$

$\therefore$ velocity of rain is $= 2i-2j$, $|2i-2j| = 2\sqrt{2}$

$\Rightarrow \tan\theta = \frac{2}{2}$

$\theta = 45^\circ$

25) (a)

average acceleration $= \frac{|\vec{v_2}-\vec{v_1}|}{t}$

$= \frac{|40\hat{i}-30\hat{j}|}{20}$ $\quad |40i-30j|$

$= \sqrt{40^2+30^2}$

$= 50$

$= \frac{50}{20}$

$= 2.5 \text{ km/s}^2$

26) (b)

magnitude of Projection of A on B $= \frac{\vec{A}\cdot\vec{B}}{|\vec{B}|}$

$= \frac{-2+9-4}{\sqrt{3^2+4^2+1}}$

$= \frac{3}{\sqrt{26}}$

28) a

Area of parallelogram $= |\bar{A} \times \bar{B}|$

$= |\bar{p} \times \bar{q}|$

$$\bar{p} \times \bar{q} = \begin{vmatrix} i & j & k \\ 5 & -4 & 3 \\ 3 & 2 & -1 \end{vmatrix}$$

$= i(4-6) - j(-5-9) + k(10+12)$

$= -2i + \cancel{18} \overset{14j}{} + 22k$

$\Rightarrow |\bar{p} \times \bar{q}| = \sqrt{4+196+484}$

$= \sqrt{684}$ units

29) (b)

$\vec{S} = \vec{r_2} - \vec{r_1} = 11i + 11j + 3k$

Work done $= \vec{F} \cdot \vec{S}$

$= (4i + j + 3k) \cdot (11i + 11j + 3k)$

$= 44 + 11 + 9$

$= 64$

$= 64J$

30) (d)

If two vectors A, B are perpendicular then $\vec{A} \cdot \vec{B} = 0$

$\left(a\cos\theta i + b\sin\theta j\right) \cdot \left(b\sin\theta \hat{i} - a\cos\theta \hat{j}\right)$

$= ab\cos\theta\sin\theta - ab\cos\theta\sin\theta$

$= 0$

$\left(a\cos\theta i + b\sin\theta j\right) \cdot \left(\frac{1}{a}\sin\theta i - \frac{1}{b}\cos\theta\right)$

$= \cos\theta\sin\theta - \cos\theta\sin\theta$

$= 0$

$\left(a\cos\theta \hat{i} + b\sin\theta \hat{j}\right) \cdot 5\vec{k} = 0$

$\therefore$ all are ~~pendi~~ perpendicular

31) (b)

as $\frac{6}{0.3} = \frac{8}{0.4}$

32)

34)

$\vec{a} \times (\vec{b} \cdot \vec{c})$ cant exist
because $\vec{b} \cdot \vec{c}$ is a scalar
and you cannot crossproduct scalar and vector

35) .(b)

$|(B \times A) \times A|$

$\vec{B} \times \vec{A}$ is ~~per to~~ perpendicular to $\vec{A}$

$\therefore$

$\Rightarrow \quad |(B \times A) \times A| = |B \times A|\,|A| \sin 90$

$= |B||A| \sin\theta\, |A|$

$= BA^2 \sin\theta$

36) (a)

Area of $\Delta le = \frac{1}{2}(\vec{OA} \times \vec{OB})$

$\vec{OA} = 3i - 6j + 2k$

$\vec{OB} = 2i + j - 2k$

$\vec{OA} \times \vec{OB} = \begin{vmatrix} i & j & k \\ 3 & -6 & 2 \\ 2 & 1 & -2 \end{vmatrix}$

$= i(12-2) - j(-6-4) + k(3+12)$

$= 10i + 10j + 15k$

$|\vec{OA} \times \vec{OB}| = \sqrt{100+100+225}$

$= \sqrt{425}$

$= 5\sqrt{17}$

$\therefore$ area $= \frac{1}{2} \times 5\sqrt{17}$

$= \frac{5}{2}\sqrt{17}$

37) (b)

$\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$

$\frac{d\vec{r}}{dt} = -a\omega\sin\omega t\, i + a\omega\cos\omega t\, j$

$\vec{v} = -a\omega\sin\omega t\, i + a\omega\cos\omega t\, j$

$\vec{v}\cdot\vec{r} = -a^2\omega\cos\omega t\sin\omega t + a^2\omega\cos\omega t\sin\omega t$

$= 0$

$\therefore$ $\vec{v}$ is perpendicular to $\vec{r}$

38) (a)

$\cos\theta = \frac{(\sqrt{3}i+j)\cdot i}{\sqrt{3+1}\ \sqrt{1}}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 30^\circ$

39) (c)

Power $= \vec{F}\cdot\vec{v}$

$= 50 - 30 + 120$

$= 140\, J/s$

40) (a)

$|\vec{A} \times \vec{B}| = \sqrt{3}\,\vec{A} \cdot \vec{B}$

$AB\sin\theta = \sqrt{3}AB\cos\theta$

$\Rightarrow \tan\theta = \sqrt{3}$

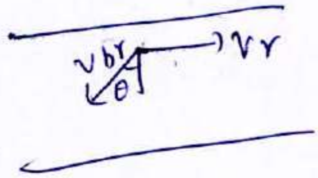
$\Rightarrow \theta = 60^\circ$

$\therefore |A+B| = \sqrt{A^2+B^2+2AB\cos 60}$

$= \sqrt{A^2+B^2+AB}$

$= (A^2+B^2+AB)^{1/2}$

41) (d)



$V_{br} \sin\theta = V_r$

$\Rightarrow \sin\theta = \frac{V_r}{V_{br}}$

$= \frac{5}{10}$

$= \frac{1}{2}$

$\theta = 30^\circ$

42) (c)

$$R = \sqrt{P^2 + P^2 + 2P^2 \cos 120}$$
$$= \sqrt{P^2 + P^2 - P^2}$$
$$= P$$

43) (c) ~~$\vec{A} \times \vec{B} = B$~~

$$\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$$
$$\Rightarrow -(\vec{B} \times \vec{A}) = \vec{B} \times \vec{A}$$
$$\Rightarrow 2(\vec{B} \times \vec{A}) = \vec{0}$$
$$\Rightarrow 2|\vec{B} \times \vec{A}| = 0$$
$$\Rightarrow |B||A| \sin\theta = 0 \quad \Rightarrow \sin\theta = 0$$
$$\Rightarrow \theta = 0 \text{ or } \theta = \pi$$

44) (c)

$$\vec{A} \cdot \vec{B} = -|\vec{A}||\vec{B}|$$
$$\Rightarrow |\vec{A}||\vec{B}| \cos\theta = -|\vec{A}||\vec{B}|$$
$$\Rightarrow \cos\theta = -1$$
$$\Rightarrow \theta = 180^\circ$$

$\therefore$ They are in opposite direction

45) (a)

$$\vec{a}+\vec{b}+\vec{c} = i+j-K$$

$$\therefore \hat{r} = \frac{\vec{a}+\vec{b}+\vec{c}}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$= \frac{i+j-k}{\sqrt{3}}$$

46) (a)

let initial be 0,0

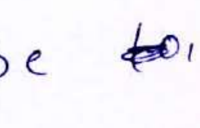
then final will be $10j+20i$

$$\therefore |10j+20i| = \sqrt{100+400}$$

$$= 10\sqrt{5} \text{ km}$$

$$= 22.36 \text{ Km}$$

47) (a)



$$\tan\theta = \frac{y}{x}$$

$$= \frac{20}{10}$$

$$\tan\theta = 2$$

$$\theta = \tan^{-1}2$$

48) (a)

$\vec{P} \perp \vec{Q}$

$\Rightarrow \vec{P} \cdot \vec{Q} = 0$

$\Rightarrow a^2 - 2a - 3 = 0$

$\Rightarrow a = 3$ or $a = -1$

- positive value of $a = 3$