Inchapter Exercise

1. $\theta = 45°$

Let two forces be $\vec{F_1}$ and $\vec{F_2}$, $|\vec{F_1}| = F_1 = 2F$, $|\vec{F_2}| = F_2 = F\sqrt{2}$

Let angle between them be $\theta = ?$

Let $\vec{F_R} = \vec{F_1} + \vec{F_2}$

$\Rightarrow |\vec{F_R}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

Given $F_R = F\sqrt{10} = \sqrt{(2F)^2 + \left(F\sqrt{2}\right)^2 + 2(2F)\left(F\sqrt{2}\right)\cos\theta}$

$$F\sqrt{10} = \sqrt{4F^2 + 2F^2 + 4F^2\sqrt{2}\cos\theta}$$

Squaring both sides

$$\left(F\sqrt{10}\right)^2 = 4F^2 + 2F^2 + 4F^2\sqrt{2}\cos\theta$$

$\Rightarrow 10F^2 = 6F^2 + 4F^2\sqrt{2}\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^0$

2. **30 N, 40 B**

Let force be $\vec{F_1}, |\vec{F_1}| = F_1, \vec{F_2}, |\vec{F_2}| = F_2$

Let $F_1 > F_2$

Case – 1 $\vec{F_1}$ oppo. to $\vec{F_2}$

ie if $\left.\begin{matrix}\vec{F_1} \rightarrow \\ \vec{F_2} \leftarrow\end{matrix}\right\}\theta = 180^0$

$|\vec{F_1} + \vec{F_2}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 180^0}$

Case 1

$\downarrow \quad \sqrt{F_1^2 + F_2^2 + 2F_1F_2(-1)}$

$10N = \sqrt{(F1 - F2)^2} = F_1 - F_2 (\text{as } F_1 > F_2)$

$\Rightarrow F_1 - F_2 = 10N$ …….(1)

Case – 2 - $\vec{F_1} \perp^r$ to $\vec{F_2}$ i.e $\theta = 90^0$

$\Rightarrow |\vec{F_1} + \vec{F_2}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2}\ \underset{=0}{\underset{\downarrow}{\cos 90}}$

$$\text{So} = \sqrt{F_1^2 + F_2^2}$$

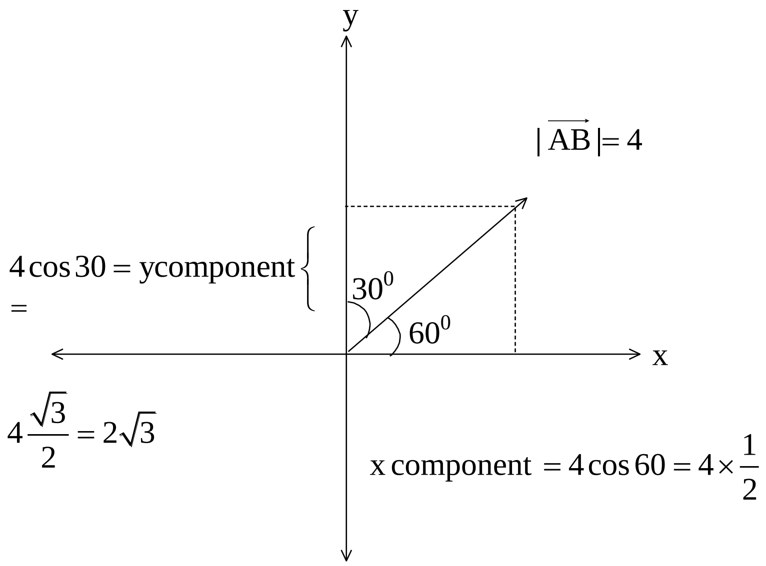
Squaring both sides

$(50)^2 = F_1^2 + F_2^2$ or $F_1^2 + F_2^2 = 2500$ ……. (2)

Solve equation (1) and (2) to get F_1 and F_2

Inchapter Exercise

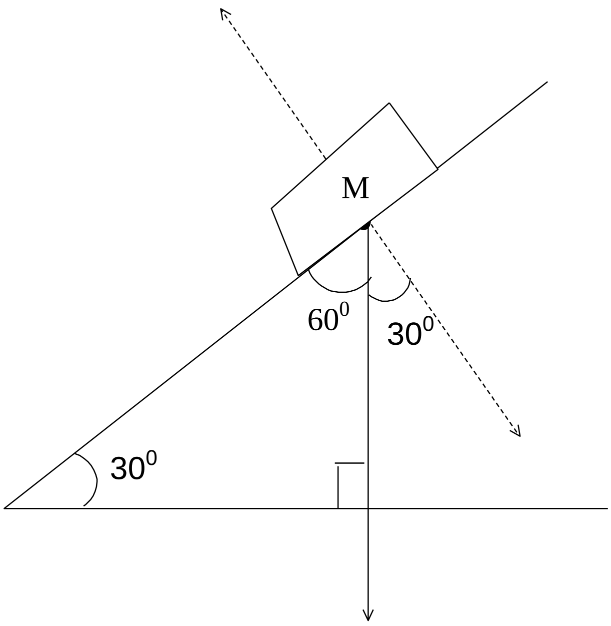
1. $|\overrightarrow{AB}| = 4, \theta_x = 60^0$ and $\overrightarrow{AB}$ lies in 1^{st} quadrant.



= 2

$\overrightarrow{AB} = 2\hat{i} + 2\sqrt{3}\hat{j}$

2. **(i)** 500 N **(ii)** 866 N



$w = \text{weight} = 1000\text{N}$

Component in direction to plane || = 1000 cos 60 = 500

Component in direction $\perp^r = 1000\cos 30 = 500\sqrt{3}$ to plane.

3. $\mathbf{2\vec{A} - 3\vec{B} = (4\hat{i} + 2\hat{j}) - (9\hat{j} - 3\hat{k})}$

$\vec{A} = 2\hat{i} + \hat{j},\ \vec{B} = 3\hat{j} - \hat{k}$

$\Rightarrow 2\vec{A} = 4\hat{i} + 2\hat{j}$

$\Rightarrow 3\vec{B} = 9\hat{j} - 3\hat{k}$

Hence $2\vec{A}-3\vec{B}=(4\hat{i}+2\hat{j})-(9\hat{j}-3\hat{k})$

$=4\hat{i}-7\hat{j}+3\hat{k}$

Inchapter Exercise

1. **$\theta = 30°$**

$$\left.\begin{array}{l}\vec{A}=2\hat{i}+\hat{j}-\hat{k}\\ \vec{B}=\hat{i}-\hat{k}\end{array}\right\}\theta=?$$

$$\cos\theta=\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}=\frac{(2+1)}{\sqrt{2^2+1^2+1^2}\sqrt{2}}$$

$$=\frac{3}{\sqrt{12}}=\frac{3}{2\sqrt{3}}=\frac{(\sqrt{3})(\sqrt{3})}{2\sqrt{3}}$$

$$\cos\theta=\frac{\sqrt{3}}{2}\Rightarrow\theta=30^0$$

2. **$m = 3/14$**

$$\left.\begin{array}{l}\vec{A}=4\hat{i}+\hat{j}-3\hat{k}\\ \vec{B}=2m\hat{i}+6m\hat{j}+\hat{k}\end{array}\right\}\vec{A}\perp^r\vec{B}\text{ given}$$

$$\Rightarrow\vec{A}\,.\,\vec{B}=|\vec{A}||\vec{B}|\cos 90=0$$

$$\Rightarrow 8m+6m-3=0$$

$$\Rightarrow m=\frac{3}{14}$$

3. **$\cos\theta = \frac{1}{\sqrt{3}}$**

$$\left.\begin{array}{l}\vec{A}=\hat{i}+\hat{j}+\hat{k}\\ \vec{B}=\hat{i}\end{array}\right\}\theta=?$$

$$\cos\theta=\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}=\frac{1}{\sqrt{1^2+1^2+1^2}\sqrt{1^2}}=\frac{1}{\sqrt{3}}$$

$$\cos\theta=\frac{1}{\sqrt{3}}$$

4. **2**

$\vec{A}=3\hat{i}-2\hat{j}+\hat{k},\ \vec{B}=12\hat{i}+3\hat{j}-4\hat{k}$

Magnitude of $\vec{A}=|\vec{A}|\cos\theta=\frac{|\vec{A}|\vec{A}\,.\,\vec{B}}{|\vec{A}||\vec{B}|}$

$$=\frac{36-6-4}{\sqrt{12^2+3^2+4^2}}=\frac{26}{\sqrt{169}}=\frac{26}{13}=2$$

Inchapter Exercise

1. $\hat{k}$ **or** $-\hat{k}$

$\vec{A} = 2\hat{i} + \hat{j}, \vec{B} = \hat{i} + 2\hat{j}$

If $\vec{c} = \vec{A} \times \vec{B}$

$\Rightarrow \vec{c}$ is $\perp^r$ to both $\vec{A}$ and $\vec{B}$

Hence unit vector

$\perp^r$ to both = $\hat{c}$

$\vec{A}$ and $\vec{B}$

Solve for $\vec{c}$ and find $\hat{c}$

2. **(A)**

Torque $(\vec{\tau}) = \vec{r} \times \vec{f}$

$\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}, F = -3\hat{i} + \hat{j} + 5\hat{k}$

Solve for $\vec{\tau} = \vec{r} \times \vec{F}$

3. $\boldsymbol{\theta = 60°}$

Given, $|\vec{A} \times \vec{B}| = \sqrt{3}\,\vec{A}.\vec{B}$

$|\vec{A}||\vec{B}|\sin\theta = \sqrt{3}\,|\vec{A}||\vec{B}|\cos\theta$

$\Rightarrow \tan\theta = \sqrt{3}$ or $\theta = 60^0$

4. $\boldsymbol{\sqrt{29}}$ **units**

$\vec{v} = \vec{w} \times \vec{r}$

$\vec{w} = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{r} = 4\hat{j} - 3\hat{k} = 0\hat{i} + 4\hat{j} - 3\hat{k}$

Solve for $\vec{v} = \vec{w} \times \vec{r}$

Then find $|\vec{v}|$

JEE Main Exercise

1. (D)

As the multiple of $\hat{j}$ in the given vector is zero therefore this vector lies in *xz* plane and projection of this vector on *y*-axis is zero.

2. (B)

If a point has coordinate (*x*, *y*, *z*) then its position vector $= x\hat{i} + y\hat{j} + z\hat{k}$.

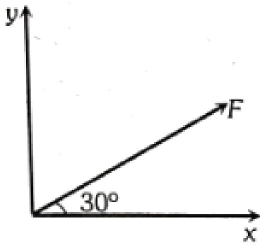
3. (B)

4. (D)

5. (C)

The X component of force F is

$$F_x = F\cos 30^{\circ} = F \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}F$$



The Y component of force F is

$$F_y = F\sin 30^{\circ} = F \times \frac{1}{2} = \frac{1}{2}F \,.$$

6. (A)

Resultant of vectors $\vec{A}$ and $\vec{B}$

$$\vec{R} = \vec{A} + \vec{B} = 4\hat{i} + 3\hat{j} + 3\hat{k} - \hat{i} + 3\hat{j} - 8\hat{k}$$

$$\vec{R} = 3\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{3\hat{i} + 6\hat{i} - 2\hat{k}}{\sqrt{3^2 + 6^2 + (-2)^2}} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$$

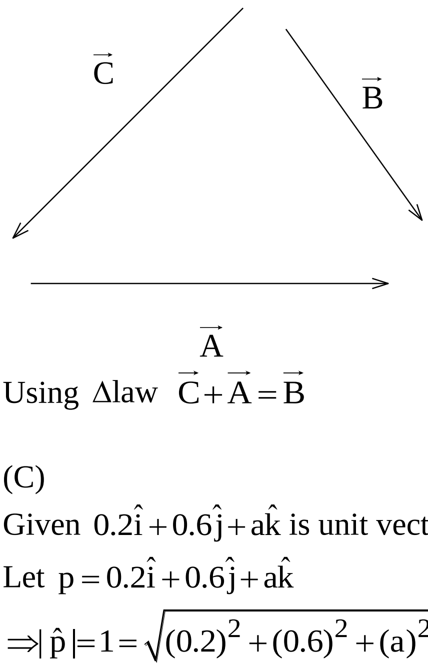
7. (D)

Unit vector in direction of $\vec{A} = \hat{A} = \frac{\vec{A}}{|\vec{A}|}$

$$\vec{A} = 5\hat{i} - 12\hat{j}$$

$$\Rightarrow \hat{A} = \frac{5\hat{i} - 12\hat{j}}{\sqrt{5^2 + (-12)^2}} = \frac{5\hat{i} - 12\hat{j}}{13}$$

8. (C)



Using Δlaw $\vec{C} + \vec{A} = \vec{B}$

9. (C)

Given $0.2\hat{i} + 0.6\hat{j} + a\hat{k}$ is unit vector

Let $p = 0.2\hat{i} + 0.6\hat{j} + a\hat{k}$

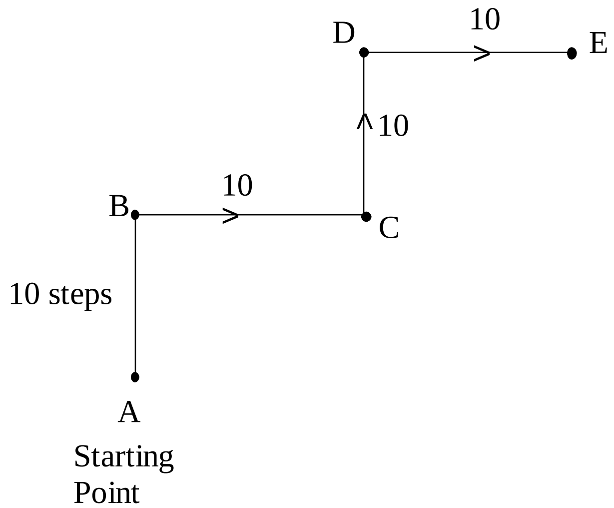
$$\Rightarrow |\hat{p}| = 1 = \sqrt{(0.2)^2 + (0.6)^2 + (a)^2}$$

On squaring both sides

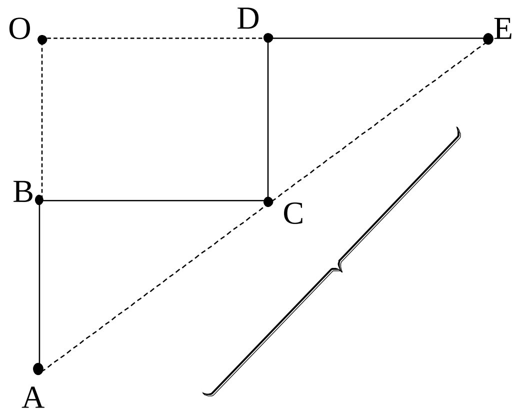
$$1^2 = 0.04 + 0.36 + a^2$$

$$1 - 0.4 = a^2 \text{ or } a = \sqrt{0.6}$$

10. (D)



(at C it u turn left. For max displacement as he should move away)
20 steps



$AE^2 = AO^2 + OE^2$
$AE^2 = (20)^2 + (20)^2$
$AE = \sqrt{800} = 20\sqrt{2}$ (Each step is 80 cm of 0.8 m long)
$\Rightarrow AE = 20\sqrt{2} \times 0.8 = 16\sqrt{2}$

11. (B)
Angle between $\vec{A}$ and $\vec{B}$ is acute

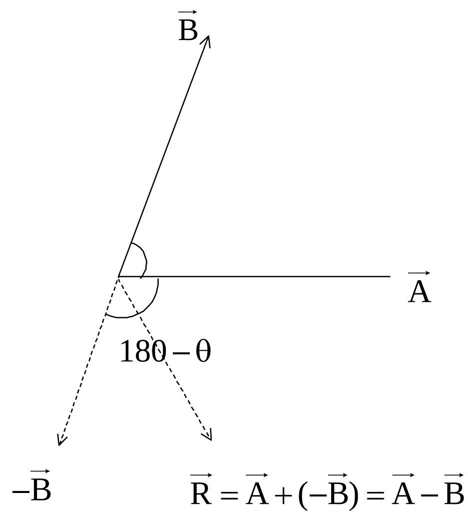
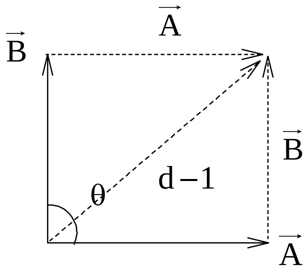
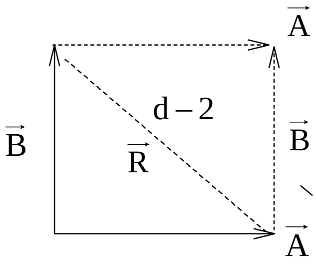


Fig – 1 Fig – 2





diagonal 1(d − 1) $= \vec{A} + \vec{B}$

↑

Major diagonal

Diagonal – 2 (d -2)

↓

$\vec{R}$

$\vec{B} + \vec{R} = \vec{A}$

$\Rightarrow \vec{R} = \vec{A} - \vec{B}$

↑

Minor diagonal

12. (D)

Let $\vec{R} = \vec{P} + \vec{Q}$

$\Rightarrow |P - Q| \le |\vec{R}| \le P + Q$

↑ ↓ ↑

When $\theta = 180^0$ When $\theta = 0^0$

R can lie anywhere between 0^0 and 180^0 both inclusive

13. (D)

When a vector is moved parallel to itself it remains same

14. (A)

$\underbrace{(\vec{A} + \vec{B}) + (\vec{A} - \vec{B}) = 2\vec{A}}$

As they are talking about resultant

$\Rightarrow |(\vec{A} + \vec{B}) + (\vec{A} - \vec{B})| = |2\vec{A}| = 2A$

15. (A)

For 17 N both the vector should be parallel i.e. angle between them should be zero.

For 7 N both the vectors should be antiparallel i.e. angle between then should be 180^o.

For 13 N both the vectors should be perpendicular to each other i.e. angle between them should be 90^o.

16. (D)

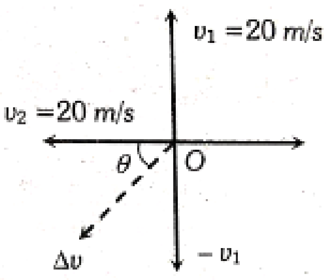
From figure

$\vec{\upsilon}_1 = 20\hat{j}$ and $\vec{\upsilon}_2 = -20\hat{j}$

$\Delta\vec{\upsilon} = \vec{\upsilon}_2 - \vec{\upsilon}_1 = -20\left(\hat{i} + \hat{j}\right)$

$|\Delta\vec{\upsilon}| = 20\sqrt{2}$ and direction

$\theta = \tan^{-1}(1) = 45^o$ i.e. S–W



17. (C)

According to problem there should be $\sum F_X = 0$

In the given figure net force along *x*-axis

$\vec{F} = 1\cos 60^o \hat{i} + 2\sin 30^o \hat{j} - 4\sin 30^o \hat{i} = \frac{1}{2}\hat{i} + 1\hat{i} - 2\hat{i}$

$\Rightarrow \vec{F} = -\frac{1}{2}\hat{i} = -0.5\,\hat{i}$

To cancel this force minimum additional force needed is 0.5 N along the positive direction of *x*-axis.

18. (A)

$\vec{P}_1 = mv\sin\theta\hat{i} - mv\cos\theta\hat{j}$ and $\vec{P}_2 = mv\sin\theta\hat{i} + mv\cos\theta\hat{j}$

So, charge in momentum

$\overrightarrow{\Delta P} = \vec{P}_2 - \vec{P}_1 = 2mv\cos\theta\hat{j}, \left|\Delta\vec{P}\right| = 2mv\cos\theta$.

19. (C)

$R_{\max} = A + B$ when $\theta = 0^o$

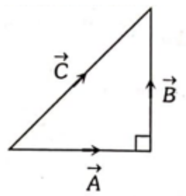
$\therefore R_{\max} = 12 + 8 = 20\,\text{N}$.

20. (A)

$C = \sqrt{A^2 + B^2}$

$= \sqrt{3^2 + 4^2} = 5$

$\therefore$ Angle between $\vec{A}$ and $\vec{B}$ is $\frac{\pi}{2}$.



21. (A)

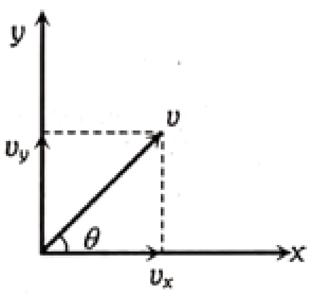
$\upsilon_y = 20$ and $\upsilon_x = 10$

$\therefore$ Velocity $\vec{\upsilon} = 10\hat{i} + 20\hat{j}$

Direction of velocity with *x* axis

$\tan\theta = \frac{\upsilon_y}{\upsilon_x} = \frac{20}{10} = 2$

$\therefore \theta = \tan^{-1}(2)$.



22. (D)

$R_{\max} = A + B = 17$ when $\theta = 0^o$

$R_{\min} = A - B = 7$ when $\theta = 180^o$

By solving we get $A = 12$ and $B = 5$

Now when $\theta = 90^o$ then $R = \sqrt{A^2 + B^2}$

$\Rightarrow R = \sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$.

23. (C)

$\vec{R} = \overrightarrow{S_1} + \overrightarrow{S_2} + \overrightarrow{S_3}$

$\Rightarrow \quad R = \sqrt{12^2 + 5^2 + 6^2} = \sqrt{144 + 25 + 36} = \sqrt{205} = 14.31\,\text{m}$

24. (D)

A = 3 N, B = 2N then $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$R = \sqrt{9 + 4 + 12\cos\theta}$ .....(i)

Now A = 6N, B = 2N then

$2R = \sqrt{36 + 4 + 24\cos\theta}$ .....(ii)

From (i) and (ii) we get $\cos = -\dfrac{1}{2}$

$\therefore\ \theta = 120^o$.

25. (A)

$a = \dfrac{|\vec{\upsilon}_f - \vec{\upsilon}_i|}{t} = \dfrac{\sqrt{30^2 + 40^2}}{10} = 5\,\text{m/s}^2$.

26. (A)

$\vec{R} = 2\hat{i} + 5\hat{j} + 3\hat{k}$

27. (D)

If $|\vec{F_1}| = F_1 = 4, |\overrightarrow{F_2}| = F_2 = 3$

And $\overrightarrow{F_R} = \vec{F_1} + \overrightarrow{F_2}$

$\Rightarrow 1 = |F_1 - F_2| \le |\overrightarrow{F_R}| \le F_1 + F_2 = 7$

28. (B)

$|\overrightarrow{F_1}| = 6, |\overrightarrow{F_2}| = 8$

$\overrightarrow{F_R} = \overrightarrow{F_1} + \overrightarrow{F_2}$

$\Rightarrow |F_1 - F_2| = 2 \le |\overrightarrow{F_R}| \le F_1 + F_2 = 6 + 8 = 17$

$\downarrow$

Hence only possible option is B ie 11 N

29. (C)

$\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = 0$

$\uparrow$

For this always add two smaller forces.

The result of above is then added to 3rd force and then check if it can give zero

Ans. (C)

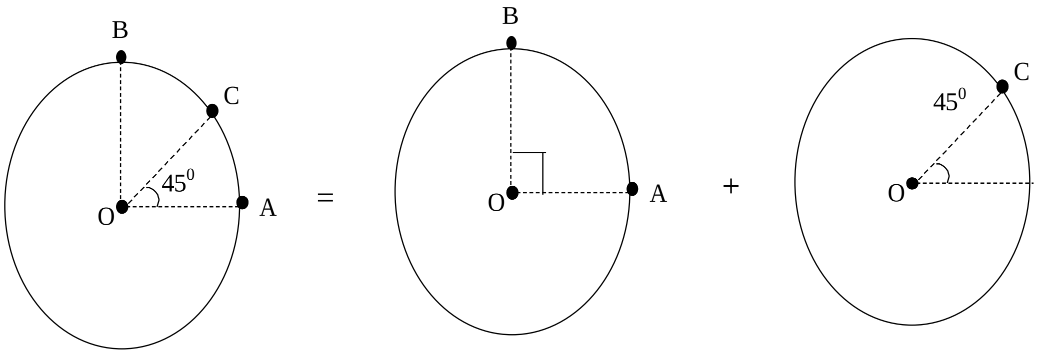
(1, 2, 1) $\Rightarrow$ 1+1 = 2 (Two smaller forces)

New 2 + 2 = 4 and 2 – 2 =0

$\uparrow$

3rd force

30. (D)



Let $\vec{P} = \overrightarrow{OA} + \overrightarrow{OB}$

$|\vec{P}| = \sqrt{R^2 + R^2 + 2(R)(R)\cos 90}$

$|\vec{P}| = R\sqrt{2}$

Angle between $\vec{A}$ and $\vec{P} \Rightarrow \text{tab}\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}$

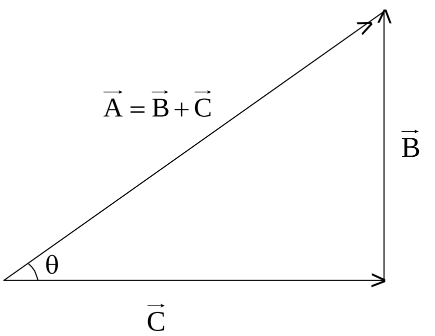
$\text{tab}\alpha = \dfrac{R\sin 90}{R + R\cos 90} = \dfrac{R}{R} = 1 \Rightarrow \alpha = 45^0$

c) $\vec{P}$ and $\overrightarrow{OC}$ both are along same direction ie angle between them = 0^0

$\Rightarrow |\vec{P} + \overrightarrow{OC}| = R\sqrt{2} + R = R\left(\sqrt{2} + 1\right)$

31. (A)

$\vec{A} = \vec{B} + \vec{C}, \underbrace{|\vec{A}| = 5, |\vec{B}| = 4, |\vec{C}| = 3}_{\text{Pythagoras Triplets}}$



$\cos\theta = \dfrac{|\vec{C}|}{|\vec{A}|} = \dfrac{3}{5} \Rightarrow \theta = \cos^{-1}\left(\dfrac{3}{5}\right)$

32. (D)

$\vec{A} + \vec{B} = \vec{A} - \vec{B}$

$\Rightarrow \vec{A} + \vec{B} - \vec{A} + \vec{B} = 0$

$2\vec{B} = 0$ or $\vec{B} = 0$

33. (B)

$|\vec{F_1}| = F, |\vec{F_2}| = F$

Let $\vec{F_R} = \vec{F_1} + \vec{F_2}$ then $|\vec{F_R}| = F$

Given $|\vec{F_R}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$F = \sqrt{F^2 + F^2 + 2(F)(F)\cos\theta}$

On squaring

$F^2 = 2F^2 + 2F^2\cos\theta = 2F^2(1+\cos\theta)$

$\Rightarrow \frac{F^2}{2F^2} = 1+\cos\theta$ or $1+\cos\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \frac{1}{2} - 1 = \frac{-1}{2}$

$\Rightarrow \theta = 120^0$ or 240^0

Ans θ is smaller of the two

34. (B)

$\vec{A} = 2\hat{i} - \hat{j} + \hat{k},\ \vec{B} = \hat{i} - 3\hat{j} - 5\hat{k}$

$\vec{C} = 3\hat{i} - 4\hat{j} - 4\hat{k}$ Find $|\vec{A}|, |\vec{B}|$ and $|\vec{C}|$ then use properties of Δ

Ans.: For equilateral $\Delta \Rightarrow |\vec{A}| = |\vec{B}| = |\vec{C}|$

In our case $|\vec{A}|^2 + |\vec{B}|^2 = |\vec{C}|^2$ RtΔ

35. (B)

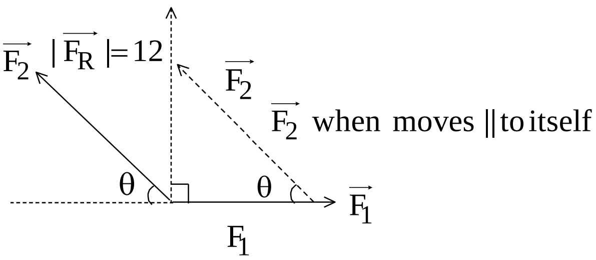
$|\vec{F_1}| = F_1, |\vec{F_2}| = F_2$ (Let $F_1 < F_2$)

Let $\vec{F_R} = \vec{F_1} + \vec{F_2}$

Given – $F_1 + F_2 = 18$, $|\vec{F_R}| = 12$

And $\vec{F_R} \perp^r \vec{F_1}$

$\vec{F_2}$



From rtΔ, $\tan\theta = \frac{F_R = 12}{F_1} \Rightarrow F_1 = \frac{12}{\tan\theta} = 2\cot\theta$

$\sin\theta = \frac{F_R = 12}{F_2} \Rightarrow F_2 = \frac{12}{\sin\theta} = 12\,\text{cosec}\,\theta$

As $F_1 + F_2 = 18$

$\downarrow$

$12\cot\theta + 12\,\text{cosec}\,\theta = 18$

$12\left[\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right] = 18$

$\Rightarrow \frac{\cos\theta + 1}{\sin\theta} = \frac{3}{2}$

$\Rightarrow 2\cos\theta + 12 = 3\sin\theta$

$2\cos\theta + 2 = 3\left(\sqrt{1-\cos^2\theta}\right)$

Squaring both sides $\left(\because \sin^2\theta+\cos^2\theta=1\right)$

$$(2\cos\theta+2)^2=\left(3\sqrt{1-\cos^2\theta}\right)^2$$

$$4\cos^2\theta+4+8\cos\theta=9(1-\cos^2\theta)$$

$$4\cos^2\theta+4+8\cos\theta-9+9\cos^2\theta=0$$

$$13\cos^2\theta+8\cos\theta-5=0$$

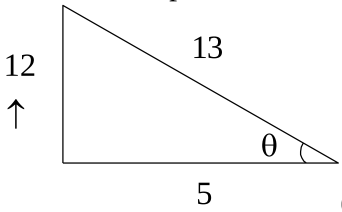
$$13\cos^2\theta+13\cos\theta-5\cos\theta-5=0$$

$$13\cos\theta[\cos\theta+1]-5(\cos\theta+1)=0$$

$$\Rightarrow(\cos\theta+1)(13\cos\theta-5)=0$$

$$\Rightarrow\cos\theta=-1 \text{ or } \cos\theta=\frac{5}{13}$$

$\theta=180^0$ not possible from figure $\theta=$ acute



(Pythagoras theorem)

$\Rightarrow$ from fig.

$$\sin\theta=\frac{12}{13}$$

$$\cos\theta=\frac{5}{13}$$

As $F_1=12\cot\theta=12\times\frac{\cos\theta}{\sin\theta}=12\times\frac{5/13}{12/13}$

$$\Rightarrow F_1=5$$

As $F_2=12\operatorname{cosec}\theta=\frac{12}{\sin\theta}=\frac{12}{12/13}=13$

36. (C)

$\vec{A}=2\hat{i}+3\hat{j}, \vec{B}=\hat{i}+\hat{j}$

Component of $\vec{A}$ along $\vec{B}=|\hat{A}|\cos\theta\hat{B}$

$$=|\vec{A}|\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}\hat{B}$$

$$=\frac{\vec{A}.\vec{B}}{|\vec{B}|}\hat{B}$$

$$=\left[\frac{(2\hat{i}+3\hat{j}).(\hat{i}+\hat{j})}{\sqrt{1^2+1^2}}\right]\frac{(\hat{i}+\hat{j})}{\sqrt{1^2+1^2}}$$

$$=\frac{(2+3)}{\sqrt{2}}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$$

Ans. $=\frac{5}{2}\hat{i}+\hat{j}$

37. (C)

Given vectors can be rewritten as $\vec{A} = 2\hat{i} + 3\hat{j} + 8\hat{k}$ and $\vec{B} = -4\hat{i} + 4\hat{j} + \alpha\hat{k}$

Dot product of these vectors should be equal to zero because they are perpendicular.

$\therefore\ \vec{A}.\vec{B} = -8 + 12 + 8\alpha = 0 \Rightarrow 8\alpha = -4 \Rightarrow \alpha = -\frac{1}{2}$.

38. (A)

$\vec{A} = \hat{i} + \hat{j} + \hat{k};\ \vec{B} = \hat{i} + \hat{j}$

$\therefore\ A = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3},\ B = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\vec{A}.\vec{B} = 2$

$\cos\theta = \frac{\vec{A}.\vec{B}}{AB} = \frac{2}{\sqrt{3}\sqrt{2}} = \left(\sqrt{\frac{2}{3}}\right)$

$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{2}{3}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$

$\therefore\ \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

39. (A)

$S = \vec{r_2} - \vec{r_1}$

$W = \vec{F}.\vec{S} = \left(4\hat{i} + \hat{j} + 3\hat{k}\right).\left(11\hat{i} + 11\hat{j} + 15\hat{k}\right)$

$= \left(4 \times 11 + 1 \times 11 + 3 \times 15\right) = 100\ \text{J}$.

40. (B)

Let $\vec{A}.\left(\vec{B} \times \vec{A}\right) = \vec{A}.\vec{C}$

Here $\vec{C} = \vec{B} \times \vec{A}$ which is perpendicular to both vector $\vec{A}$ and $\vec{B}$

$\therefore\ \vec{A}.\vec{C} = 0$.

41. (A)

$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix} = -\hat{j} - 2\hat{k}$$

i.e. the angular momentum is perpendicular to *x*-axis.

42. (C)

$\vec{A}.\vec{B} = 0 \qquad \therefore\ \theta = 90^{o}$.

43. (D)

$\vec{P} = 2\hat{i} + b\hat{j} + 2\hat{k}$

$\vec{Q} = \hat{i} + \hat{j} + \hat{k}$

Given $\vec{P}$ is $\perp^r$ to $\vec{Q}$

$\Rightarrow \vec{P}.\vec{Q} = |\vec{P}||\vec{Q}|\cos 90 = 0$

$2 + b + 2 = 0 \Rightarrow b = -4$

44. (D)

$\vec{A}, \vec{B}$ and $\vec{C}$ are three vectors

Concept – Vector or Dot product can be performed only between two vectors

45. (D)

Given $|\vec{A}| = |\vec{B}|$ and $\vec{A}$ is $\perp^r$ to $\vec{B}$

To find which of the following is $\perp^r$ to $\vec{A} + \vec{B}$

(A) $\vec{A} \times \vec{B} \leftarrow \because$ it will give a vector $\perp^r$ to both $\vec{A}$ and $\vec{B}$ i.e. $\perp^r$ to plane of $\vec{A}$ and $\vec{B}$.

(B) $\vec{A} - \vec{B} \leftarrow$ To check perform Dot product of $(\vec{A} + \vec{B}).(\vec{A} - \vec{B})$ if it is = 0 then yes.

(C) $3\vec{A} - 3\vec{B} \leftarrow$ Same as B option

(D) all of these

46. (D)

$\vec{A}.(\vec{B} \times \overline{A}) = 0$

$\vec{A} \bullet$ gives vector $\perp^r$ to $\vec{A}$ and $\vec{B}$

Dot Product = $0 (\because \theta = 90^0$ and $\cos 90 = 0)$

47. (A)

Given $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$

We know $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

Hence $-\vec{B} \times \vec{A} = \vec{B} \times \vec{A}$

Or $O = \vec{B} \times \vec{A} + \vec{B} \times \vec{A}$ or $2\vec{B} \times \vec{A} = 0$

$\Rightarrow \vec{B} \times \vec{A} = 0 \Rightarrow \theta = 0^0$ or π

$\because \vec{A} \neq 0, \vec{B} \neq 0$

48. (B)

$\vec{A}.\vec{B} = |\vec{A} \times \vec{B}|$ given

$|\vec{A}||\vec{B}|\cos\theta = |\vec{A}||\vec{B}|\sin\theta$

$\Rightarrow \cos\theta = \sin\theta \Rightarrow \theta = 45^0$

49. (A)

Let $\vec{A} = \hat{i}$, $\vec{B} = \hat{i} + \hat{j} + \hat{k}$

Hence $\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \dfrac{(\hat{i}).(\hat{i} + \hat{j} + \hat{k})}{\sqrt{1^2}\sqrt{1^2 + 1^2 + 1^2}}$

$\cos\theta = \dfrac{1}{\sqrt{1}\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

1. (c)

Given $\left|\vec{A}+\vec{B}\right| = \left|\vec{A}-\vec{B}\right| \times 2$

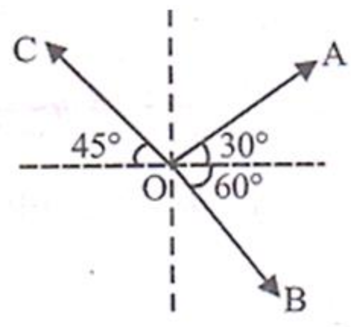
$\Rightarrow \left|\vec{A}+\vec{B}\right|^2 = \left|\vec{A}-\vec{B}\right|^2 \times 4$

$\Rightarrow 0 = 3|A|^2 + 3|B|^2 - |10|A||B|\cos\theta$

$\Rightarrow 0 - 3|A|^2 + 3|A|^2 - |10|A|^2\cos\theta \Rightarrow |10|A|^2\cos\theta = 6|A|^2$

$\Rightarrow \cos\theta = \dfrac{3}{5}$

2. (a)



Say, magnitudes of vectors is r.

$\overrightarrow{OA} = r\left[\cos 30^\circ \hat{i} + \sin 30 \hat{j}\right] = r\left[\dfrac{\sqrt{3}}{2}\hat{i} + \dfrac{1}{2}\hat{j}\right]$

$\overrightarrow{OC} = r\left[\cos 45^\circ\left(-\hat{i}\right) + \sin 45^\circ \hat{j}\right] = r\left[-\dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{j}\right]$

$\overrightarrow{OB} = r\left[\cos 60^\circ \hat{i} - \sin 60 \hat{j}\right] = r\left[\dfrac{1}{2}\hat{i} - \dfrac{\sqrt{3}}{2}\hat{j}\right]$

$\therefore \overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$

$= r\left[\left(\dfrac{\sqrt{3}+1}{2} + \dfrac{1}{\sqrt{2}}\right)i + \left(\dfrac{1}{2} - \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}}\right)\hat{j}\right]$

$\therefore$ Angle made by vector with x-axis

$\tan^{-1}\left[\dfrac{\dfrac{1}{2} - \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}}}{\dfrac{\sqrt{3}}{2} + \dfrac{1}{2} + \dfrac{1}{\sqrt{2}}}\right] = \tan^{-1}\left[\dfrac{1-\sqrt{3}-\sqrt{2}}{\sqrt{3}+1+\sqrt{2}}\right]$

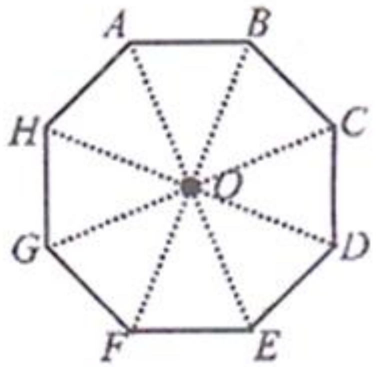
3. (d)

Projection of vector $\vec{A}$ on vector $\vec{B}$ is given by

$\left(A\cos\theta\right)\hat{B} = A\left(\dfrac{\vec{A}.\vec{B}}{AB}\right)\hat{B} = \dfrac{\vec{A}.\vec{B}}{B}\hat{B}$

$$= \frac{2}{\sqrt{2}}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \hat{i}+\hat{j}$$

4. (c)



From figure,

$\overline{AO}+\overline{OB}=\overline{AB}$ $\overline{AO}+\overline{OC}=\overline{AC}$ $\overline{AO}+\overline{OD}=\overline{AD}$

$\overline{AO}+\overline{OE}=\overline{AE}$ $\overline{AO}+\overline{OF}=\overline{AF}$ $\overline{AO}+\overline{OG}=\overline{AG}$

$\overline{AO}+\overline{OH}=\overline{AH}$

Now, adding we get

$8\overline{AO}=\left(\overline{AB}+\overline{AC}+\overline{AD}+\overline{AE}+\overline{AF}+\overline{AG}+\overline{AH}\right)$

$\Rightarrow 8\left(2\hat{i}+3\hat{j}-4\hat{k}\right)=\overline{AB}+\overline{AC}+\overline{AD}+\overline{AE}+\overline{AF}+\overline{AG}+\overline{AH}$

$\therefore \overline{AB}+\overline{AC}+\overline{AD}+\overline{AE}+\overline{AF}+\overline{AG}+\overline{AH}$

$=16\hat{i}+24\hat{j}-32\hat{k}$

5. (d)

Using

$R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\theta$

$5^2 = 3^2 + 5^2 + 2\times3\times5\cos\theta \text{ or } \cos\theta = -0.3$

$\left(2\vec{A}_1+3\overline{A}_2\right).\left(3\overline{A}_1-2\overline{A}_2\right) = 2A_1\times3A_1 + \left(3A_2\right)\left(3A_1\right)\cos\theta - \left(2A_1\right)\left(2A_2\right)\cos\theta - 3A_2\times2A_2$

$= 6A_1^2 + 9A_1A_2\cos\theta - 4A_1A_2\cos\theta - 6A_2^2$

$= 6A_1^2 - 6A_2^2 + 5A_1A_2\cos\theta$

$= 6\times3^2 - 6\times5^2 + 5\times3\times5\left(-0.3\right) = -118.5$

6. (c)

From figure, $\vec{r}_G = \frac{a}{2}\hat{i}+\frac{a}{2}\hat{k}$

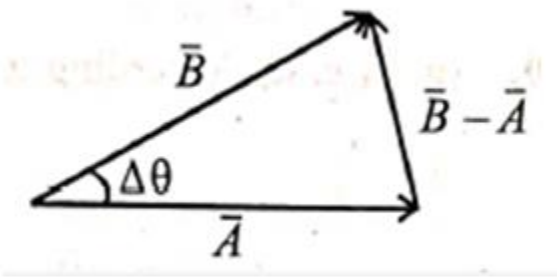
$\vec{r}_H = \frac{a}{2}\hat{j}+\frac{a}{2}\hat{k}$

$\therefore \vec{r}_H - \vec{r}_G = \left(\frac{a}{2}\hat{j}+\frac{a}{2}\hat{k}\right)-\left(\frac{a}{2}\hat{i}+\frac{a}{2}\hat{k}\right)=\frac{a}{2}\left(\hat{j}-\hat{i}\right)$

7. (a)

Arc length = radius $\times$ angle

So, $\left|\vec{B}-\vec{A}\right| = \left|\vec{A}\right|\Delta\theta$



$\left|\vec{A}\right| = \left|\vec{B}\right|$

8. (a)

If $\vec{C} = a\hat{i} + b\hat{j}$ then $\vec{A}.\vec{C} = \vec{A}.\vec{B}$

$a + b = 1$ …….(i)

$\vec{B}.\vec{C} = \vec{A}.\vec{B}$

$2a - b = 1$ ……(ii)

Solving equation (i) and (ii) we get

$a = \frac{1}{3}, b = \frac{2}{3}$

$\therefore$ Magnitude of coplanar vector, $\left|\vec{C}\right| = \sqrt{\frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{5}{9}}$

9. (a)

Here $\vec{P} + b\vec{R} = \vec{S} \Rightarrow \vec{R} = \frac{\vec{S} - \vec{P}}{b}$

Also $\vec{R} = \vec{Q} - \vec{P}$

$\therefore \frac{\vec{S} - \vec{P}}{b} = \vec{Q} - \vec{P} \Rightarrow \vec{S} - \vec{P} = b\vec{Q} - b\vec{P}$

$\therefore \vec{S} = b\vec{Q} + (1 - b)\vec{P}$

10. (5)

Here , $\vec{a}.\vec{b} = 0$

$\therefore \vec{a}.\vec{b} = \left(2\hat{i} + 4\hat{j} - 2\hat{k}\right).\left(\hat{i} + 2\hat{j} + \alpha\hat{k}\right) = 0$

$\therefore 2 \times 1 + 4 \times 2 - 2 \times \alpha = 0 \therefore \alpha = 5$

11. (2)

Given that

$\vec{A} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)m$ and $\vec{B} = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)m$

Component of vector $\vec{A}$ along vector $\vec{B} = \vec{A}.\hat{B}$

$= \frac{\vec{A}.\vec{B}}{\left|\vec{B}\right|} = \frac{2 + 6 - 2}{\sqrt{1^2 + 2^2 + 2^2}} \left(\because \hat{B} = \frac{\vec{B}}{\left|\vec{B}\right|}\right)$

$= \frac{6}{3} = 2m$

12. (3)

Direction of particle P,

$$\hat{v}_1 = \pm\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \pm\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$$

Direction of particle Q,

$$\hat{v}_2 = \pm\frac{\vec{A}\times\vec{C}}{|\vec{A}\times\vec{C}|} = \pm\frac{2\hat{k}}{2} = \pm\hat{k}$$

Angle between $\hat{v}_1$ and $\hat{v}_2$

$$\frac{\hat{v}_1.\hat{v}_2}{|\hat{v}_1||\hat{v}_2|} = \frac{\frac{\pm 1}{\sqrt{3}}}{(1)(1)} = \pm\frac{1}{\sqrt{3}}$$

Hence the angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\therefore$ value of $x = 3$

13. (180)

$\vec{P}\times\vec{Q} = \vec{Q}\times\vec{P}$ only if $\vec{P} = 0$ or $\vec{Q} = 0$

$\Rightarrow \vec{P}\times\vec{Q} = 0$

So, $\theta = 0^\circ$ or 180° and $0^\circ < \theta < 360^\circ$.

$\therefore \theta = 180^\circ$

14. (195)

Given: $\vec{F} = (\hat{i}+2\hat{j}+3\hat{k})N$

And, $\vec{r} = \left[(4\hat{i}+3\hat{j}-\hat{k})-(\hat{i}+2\hat{j}+\hat{k})\right] = 3\hat{i}+\hat{j}-2\hat{k}$

Torque, $\tau = \hat{r}\times\hat{F} = (3\hat{i}+\hat{j}-2\hat{k})\times(\hat{i}+2\hat{j}+3\hat{k})$

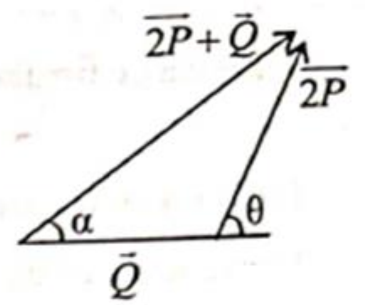
$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix} = 7\hat{i} - 11j + 5\hat{k}$$

Magnitude of torque, $|\vec{\tau}| = \sqrt{195}$

15. (90)

Given,

$|\vec{R}| = |\vec{P}| \Rightarrow |\vec{P}+\vec{Q}| = |\vec{P}|$



$P^2 + Q^2 + 2PQ.\cos\theta = P^2$

$\Rightarrow Q + 2P\cos\theta = 0$

$\Rightarrow \cos\theta = -\frac{Q}{2P} \quad \ldots(i)$

$\tan\alpha = \frac{2P\sin\theta}{Q+2P\cos\theta} = \infty\,(\because 2P\cos\theta + Q = 0)$

$\Rightarrow \alpha = 90^\circ$