Inchapter Exercise

1. **$\theta = 45°$**

Let two forces be $\vec{F_1}$ and $\vec{F_2}$, $|\vec{F_1}| = F_1 = 2F$, $|\vec{F_2}| = F_2 = F\sqrt{2}$

Let angle between them be $\theta = ?$

Let $\vec{F_R} = \vec{F_1} + \vec{F_2}$

$\Rightarrow |\vec{F_R}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

Given $F_R = F\sqrt{10} = \sqrt{(2F)^2 + \left(F\sqrt{2}\right)^2 + 2(2F)\left(F\sqrt{2}\right)\cos\theta}$

$F\sqrt{10} = \sqrt{4F^2 + 2F^2 + 4F^2\sqrt{2}\cos\theta}$

Squaring both sides

$\left(F\sqrt{10}\right)^2 = 4F^2 + 2F^2 + 4F^2\sqrt{2}\cos\theta$

$\Rightarrow 10F^2 = 6F^2 + 4F^2\sqrt{2}\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^0$

2. **30 N, 40 B**

Let force be $\vec{F_1}, |\vec{F_1}| = F_1, \vec{F_2}, |\vec{F_2}| = F_2$

Let $F_1 > F_2$

Case – 1 $\vec{F_1}$ oppo. to $\vec{F_2}$

ie if $\left.\begin{matrix}\vec{F_1} \rightarrow \\ \vec{F_2} \leftarrow\end{matrix}\right\}\theta = 180^0$

$|\vec{F_1} + \vec{F_2}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos180^0}$

Case 1

$\downarrow \sqrt{F_1^2 + F_2^2 + 2F_1F_2(-1)}$

$10N = \sqrt{(F1 - F2)^2} = F_1 - F_2 (\text{as } F_1 > F_2)$

$\Rightarrow F_1 - F_2 = 10N$ …….(1)

Case – 2 - $\vec{F_1} \perp^r$ to $\vec{F_2}$ i.e $\theta = 90^0$

$\Rightarrow |\vec{F_1} + \vec{F_2}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\underbrace{\cos90}_{=0}}$

So $= \sqrt{F_1^2 + F_2^2}$

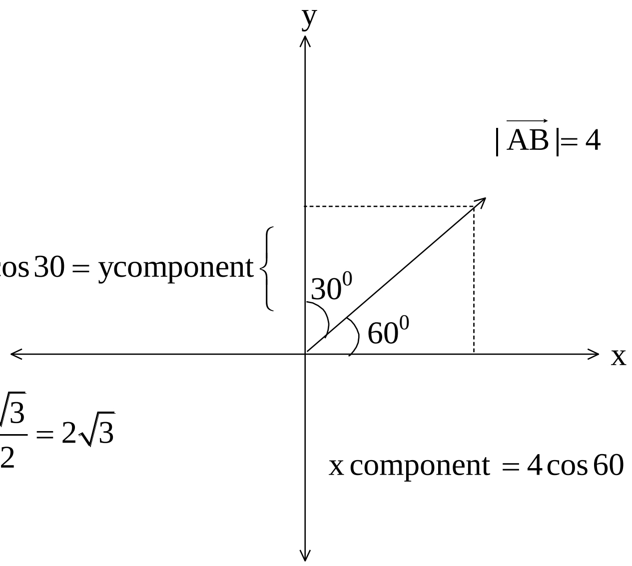
Squaring both sides

$(50)^2 = F_1^2 + F_2^2$ or $F_1^2 + F_2^2 = 2500$ ……. (2)

Solve equation (1) and (2) to get F_1 and F_2

Inchapter Exercise

1. $|\overrightarrow{AB}| = 4, \theta_x = 60^0$ and $\overrightarrow{AB}$ lies in 1^{st} quadrant.



$4\cos 30 = \text{ycomponent} =$

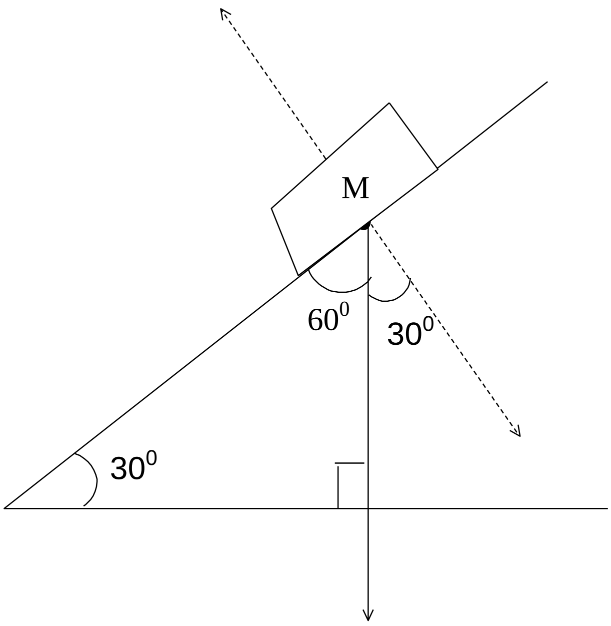
$4\frac{\sqrt{3}}{2} = 2\sqrt{3}$

$\text{x component} = 4\cos 60 = 4 \times \frac{1}{2}$

= 2

$\overrightarrow{AB} = 2\hat{i} + 2\sqrt{3}\hat{j}$

2. **(i)** 500 N **(ii)** 866 N



$w = \text{weight} = 1000\text{N}$

Component in direction to plane || = 1000 cos 60 = 500

Component in direction $\perp^r = 1000\cos 30 = 500\sqrt{3}$ to plane.

3. $\mathbf{2\vec{A} - 3\vec{B} = (4\hat{i} + 2\hat{j}) - (9\hat{j} - 3\hat{k})}$

$\vec{A} = 2\hat{i} + \hat{j},\ \vec{B} = 3\hat{j} - \hat{k}$

$\Rightarrow 2\vec{A} = 4\hat{i} + 2\hat{j}$

$\Rightarrow 3\vec{B} = 9\hat{j} - 3\hat{k}$

Hence $2\vec{A}-3\vec{B}=(4\hat{i}+2\hat{j})-(9\hat{j}-3\hat{k})$

$=4\hat{i}-7\hat{j}+3\hat{k}$

Inchapter Exercise

1. **$\theta = 30°$**

$$\left.\begin{aligned}\vec{A}&=2\hat{i}+\hat{j}-\hat{k}\\ \vec{B}&=\hat{i}-\hat{k}\end{aligned}\right\}\theta=?$$

$$\cos\theta=\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}=\frac{(2+1)}{\sqrt{2^2+1^2+1^2}\sqrt{2}}$$

$$=\frac{3}{\sqrt{12}}=\frac{3}{2\sqrt{3}}=\frac{\left(\sqrt{3}\right)\left(\sqrt{3}\right)}{2\sqrt{3}}$$

$$\cos\theta=\frac{\sqrt{3}}{2}\Rightarrow\theta=30^0$$

2. **$m = 3/14$**

$$\left.\begin{aligned}\vec{A}&=4\hat{i}+\hat{j}-3\hat{k}\\ \vec{B}&=2m\hat{i}+6m\hat{j}+\hat{k}\end{aligned}\right\}\vec{A}\perp^r\vec{B}\text{ given}$$

$$\Rightarrow\vec{A}\,.\,\vec{B}=|\vec{A}||\vec{B}|\cos 90=0$$

$$\Rightarrow 8m+6m-3=0$$

$$\Rightarrow m=\frac{3}{14}$$

3. **$\cos\theta=\frac{1}{\sqrt{3}}$**

$$\left.\begin{aligned}\vec{A}&=\hat{i}+\hat{j}+\hat{k}\\ \vec{B}&=\hat{i}\end{aligned}\right\}\theta=?$$

$$\cos\theta=\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}=\frac{1}{\sqrt{1^2+1^2+1^2}\sqrt{1^2}}=\frac{1}{\sqrt{3}}$$

$$\cos\theta=\frac{1}{\sqrt{3}}$$

4. **2**

$\vec{A}=3\hat{i}-2\hat{j}+\hat{k},\ \vec{B}=12\hat{i}+3\hat{j}-4\hat{k}$

Magnitude of $\vec{A}=|\vec{A}|\cos\theta=\frac{|\vec{A}|\vec{A}\,.\,\vec{B}}{|\vec{A}||\vec{B}|}$

$$=\frac{36-6-4}{\sqrt{12^2+3^2+4^2}}=\frac{26}{\sqrt{169}}=\frac{26}{13}=2$$

Inchapter Exercise

1. $\hat{k}$ **or** $-\hat{k}$

$\vec{A} = 2\hat{i} + \hat{j}, \vec{B} = \hat{i} + 2\hat{j}$

If $\vec{c} = \vec{A} \times \vec{B}$

$\Rightarrow \vec{c}$ is $\perp^r$ to both $\vec{A}$ and $\vec{B}$

Hence unit vector

$\perp^r$ to both = $\hat{c}$

$\vec{A}$ and $\vec{B}$

Solve for $\vec{c}$ and find $\hat{c}$

2. **(A)**

Torque $(\vec{\tau}) = \vec{r} \times \vec{f}$

$\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}, F = -3\hat{i} + \hat{j} + 5\hat{k}$

Solve for $\vec{\tau} = \vec{r} \times \vec{F}$

3. $\boldsymbol{\theta = 60°}$

Given, $|\vec{A} \times \vec{B}| = \sqrt{3}\vec{A}.\vec{B}$

$|\vec{A}||\vec{B}|\sin\theta = \sqrt{3}|\vec{A}||\vec{B}|\cos\theta$

$\Rightarrow \tan\theta = \sqrt{3}$ or $\theta = 60^0$

4. $\boldsymbol{\sqrt{29}}$ **units**

$\vec{v} = \vec{w} \times \vec{r}$

$\vec{w} = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{r} = 4\hat{j} - 3\hat{k} = 0\hat{i} + 4\hat{j} - 3\hat{k}$

Solve for $\vec{v} = \vec{w} \times \vec{r}$

Then find $|\vec{v}|$

JEE Main Exercise

1. (D)

As the multiple of $\hat{j}$ in the given vector is zero therefore this vector lies in *xz* plane and projection of this vector on *y*-axis is zero.

2. (B)

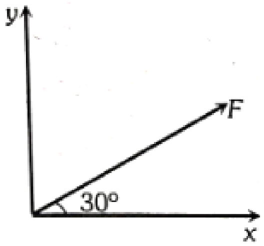
If a point has coordinate (*x*, *y*, *z*) then its position vector $= x\hat{i} + y\hat{j} + z\hat{k}$.

3. (B)

4. (D)

5. (C)

The X component of force F is



$$F_x = F\cos 30^{o} = F \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}F$$

The Y component of force F is

$$F_y = F\sin 30^{o} = F \times \frac{1}{2} = \frac{1}{2}F\ .$$

6. (A)

Resultant of vectors $\vec{A}$ and $\vec{B}$

$$\vec{R} = \vec{A} + \vec{B} = 4\hat{i} + 3\hat{j} + 3\hat{k} - \hat{i} + 3\hat{j} - 8\hat{k}$$

$$\vec{R} = 3\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{3\hat{i} + 6\hat{i} - 2\hat{k}}{\sqrt{3^2 + 6^2 + (-2)^2}} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$$

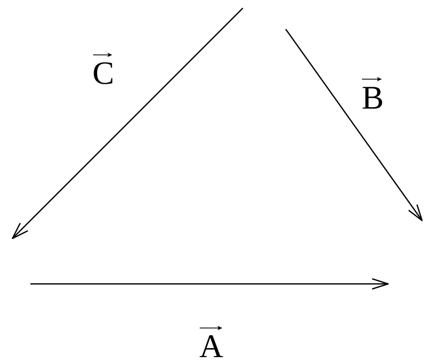
7. (D)

Unit vector in direction of $\vec{A} = \hat{A} = \frac{\vec{A}}{|\vec{A}|}$

$$\vec{A} = 5\hat{i} - 12\hat{j}$$

$$\Rightarrow \hat{A} = \frac{5\hat{i} - 12\hat{j}}{\sqrt{5^2 + (-12)^2}} = \frac{5\hat{i} - 12\hat{j}}{13}$$

8. (C)



Using Δlaw $\vec{C} + \vec{A} = \vec{B}$

9. (C)

Given $0.2\hat{i} + 0.6\hat{j} + a\hat{k}$ is unit vector

Let $p = 0.2\hat{i} + 0.6\hat{j} + a\hat{k}$

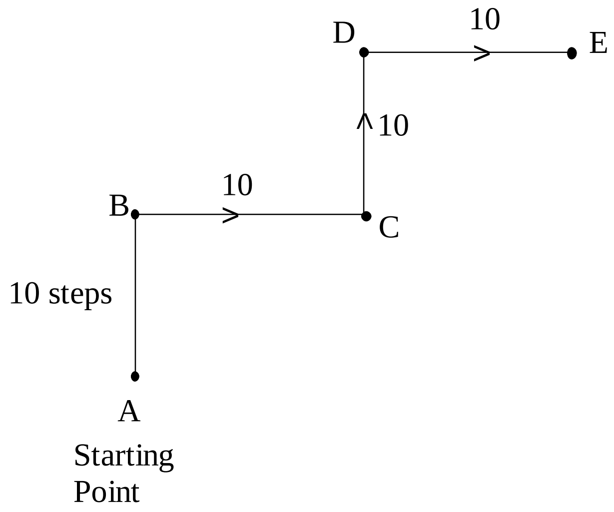
$$\Rightarrow |\hat{p}| = 1 = \sqrt{(0.2)^2 + (0.6)^2 + (a)^2}$$

On squaring both sides

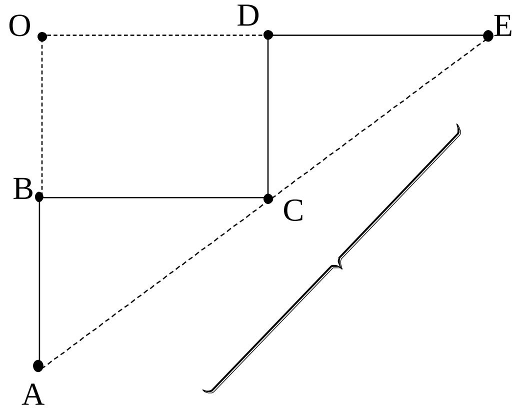
$$1^2 = 0.04 + 0.36 + a^2$$

$$1 - 0.4 = a^2 \text{ or } a = \sqrt{0.6}$$

10. (D)



(at C it u turn left. For max displacement as he should move away)
20 steps



$AE^2 = AO^2 + OE^2$
$AE^2 = (20)^2 + (20)^2$
$AE = \sqrt{800} = 20\sqrt{2}$ (Each step is 80 cm of 0.8 m long)
$\Rightarrow AE = 20\sqrt{2} \times 0.8 = 16\sqrt{2}$

11. (B)
Angle between $\vec{A}$ and $\vec{B}$ is acute

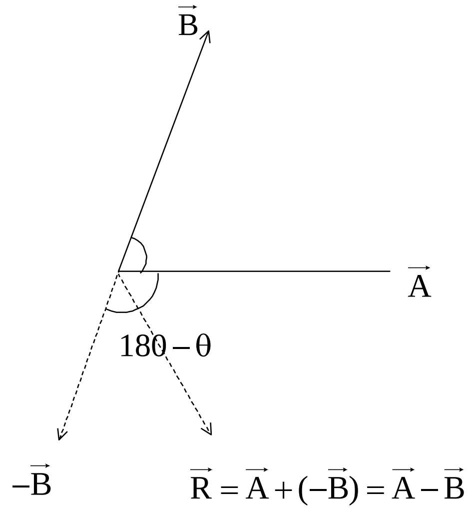
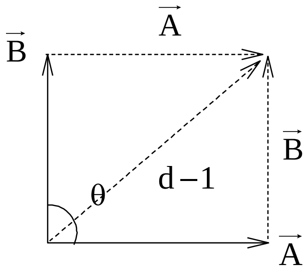


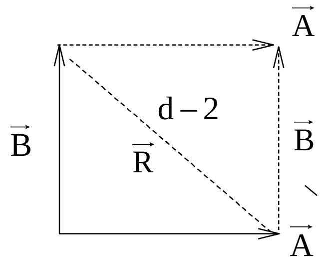
Fig – 1 Fig – 2



diagonal 1(d − 1) = $\vec{A} + \vec{B}$

↑

Major diagonal



Diagonal – 2 (d -2)

↓

$\vec{R}$

$\vec{B} + \vec{R} = \vec{A}$

$\Rightarrow \vec{R} = \vec{A} - \vec{B}$

↑

Minor diagonal

12. (D)

Let $\vec{R} = \vec{P} + \vec{Q}$

$\Rightarrow |P - Q| \le |\vec{R}| \le P + Q$

↑ ↓ ↑

When $\theta = 180^0$ When $\theta = 0^0$

R can lie anywhere between 0^0 and 180^0 both inclusive

13. (D)

When a vector is moved parallel to itself it remains same

14. (A)

$\underbrace{(\vec{A} + \vec{B}) + (\vec{A} - \vec{B}) = 2\vec{A}}$

As they are talking about resultant

$\Rightarrow |(\vec{A} + \vec{B}) + (\vec{A} - \vec{B})| = |2\vec{A}| = 2A$

15. (A)

For 17 N both the vector should be parallel i.e. angle between them should be zero.

For 7 N both the vectors should be antiparallel i.e. angle between then should be 180^o.

For 13 N both the vectors should be perpendicular to each other i.e. angle between them should be 90^o.

16. (D)

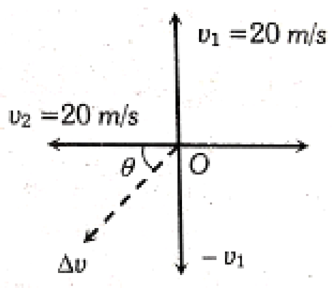
From figure

$\vec{\upsilon}_1 = 20\hat{j}$ and $\vec{\upsilon}_2 = -20\hat{j}$

$\Delta\vec{\upsilon} = \vec{\upsilon}_2 - \vec{\upsilon}_1 = -20(\hat{i} + \hat{j})$

$|\Delta\vec{\upsilon}| = 20\sqrt{2}$ and direction

$\theta = \tan^{-1}(1) = 45^o$ i.e. S–W



17. (C)

According to problem there should be $\sum F_X = 0$

In the given figure net force along x-axis

$$\vec{F} = 1\cos 60^o \hat{i} + 2\sin 30^o \hat{j} - 4\sin 30^o \hat{i} = \frac{1}{2}\hat{i} + 1\hat{i} - 2\hat{i}$$

$$\Rightarrow \quad \vec{F} = -\frac{1}{2}\hat{i} = -0.5\,\hat{i}$$

To cancel this force minimum additional force needed is 0.5 N along the positive direction of x-axis.

18. (A)

$\vec{P}_1 = mv\sin\theta\hat{i} - mv\cos\theta\hat{j}$ and $\vec{P}_2 = mv\sin\theta\hat{i} + mv\cos\theta\hat{j}$

So, charge in momentum

$\overrightarrow{\Delta P} = \vec{P}_2 - \vec{P}_1 = 2mv\cos\theta\hat{j}, \left|\Delta\vec{P}\right| = 2mv\cos\theta$.

19. (C)

$R_{\max} = A + B$ when $\theta = 0^o$

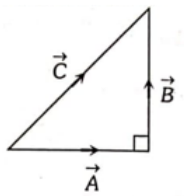
$\therefore\ R_{\max} = 12 + 8 = 20\,\text{N}$.

20. (A)

$C = \sqrt{A^2 + B^2}$

$\quad = \sqrt{3^2 + 4^2} = 5$

$\therefore$ Angle between $\vec{A}$ and $\vec{B}$ is $\frac{\pi}{2}$.



21. (A)

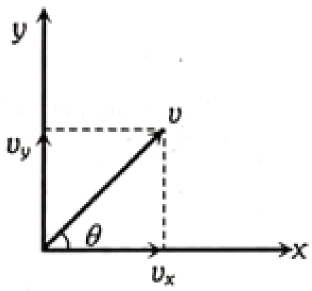
$\upsilon_y = 20$ and $\upsilon_x = 10$

$\therefore$ Velocity $\vec{\upsilon} = 10\hat{i} + 20\hat{j}$

Direction of velocity with x axis

$$\tan\theta = \frac{\upsilon_y}{\upsilon_x} = \frac{20}{10} = 2$$

$\therefore \quad \theta = \tan^{-1}(2)$.



22. (D)

$R_{\max} = A + B = 17$ when $\theta = 0^o$

$R_{\min} = A - B = 7$ when $\theta = 180^o$

By solving we get $A = 12$ and $B = 5$

Now when $\theta = 90^o$ then $R = \sqrt{A^2 + B^2}$

$$\Rightarrow \quad R = \sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13.$$

23. (C)

$\vec{R} = \vec{S_1} + \vec{S_2} + \vec{S_3}$

$\Rightarrow \quad R = \sqrt{12^2 + 5^2 + 6^2} = \sqrt{144 + 25 + 36} = \sqrt{205} = 14.31\,\text{m}$

24. (D)

A = 3 N, B = 2N then $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$R = \sqrt{9 + 4 + 12\cos\theta}$ .....(i)

Now A = 6N, B = 2N then

$2R = \sqrt{36 + 4 + 24\cos\theta}$ .....(ii)

From (i) and (ii) we get $\cos = -\frac{1}{2}$

$\therefore \; \theta = 120^o$.

25. (A)

$a = \frac{|\vec{\upsilon}_f - \vec{\upsilon}_i|}{t} = \frac{\sqrt{30^2 + 40^2}}{10} = 5\,\text{m/s}^2$.

26. (A)

$\vec{R} = 2\hat{i} + 5\hat{j} + 3\hat{k}$

27. (D)

If $|\vec{F_1}| = F_1 = 4, |\vec{F_2}| = F_2 = 3$

And $\vec{F_R} = \vec{F_1} + \vec{F_2}$

$\Rightarrow 1 = |F_1 - F_2| \le |\vec{F_R}| \le F_1 + F_2 = 7$

28. (B)

$|\vec{F_1}| = 6, |\vec{F_2}| = 8$

$\vec{F_R} = \vec{F_1} + \vec{F_2}$

$\Rightarrow |F_1 - F_2| = 2 \le |\vec{F_R}| \le F_1 + F_2 = 6 + 8 = 17$

$\downarrow$

Hence only possible option is B ie 11 N

29. (C)

$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$

$\uparrow$

For this always add two smaller forces.

The result of above is then added to 3rd force and then check if it can give zero

Ans. (C)

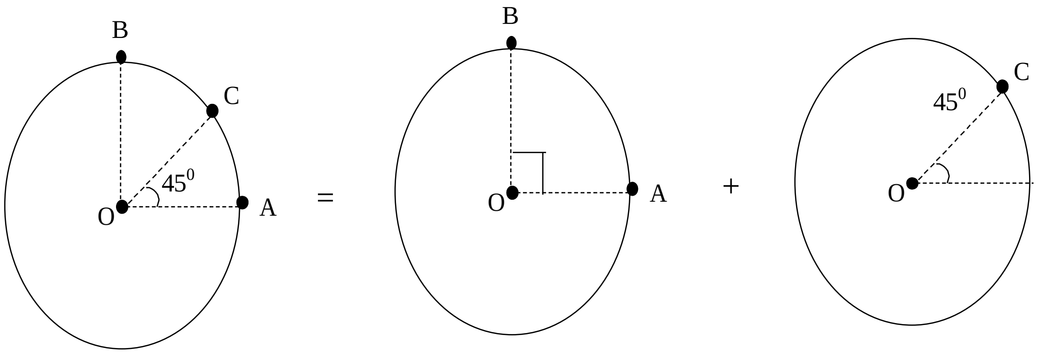
(1, 2, 1) $\Rightarrow$ 1+1 = 2 (Two smaller forces)

New 2 + 2 = 4 and 2 – 2 =0

$\uparrow$

3rd force

30. (D)



Let $\vec{P} = \overrightarrow{OA} + \overrightarrow{OB}$

$|\vec{P}| = \sqrt{R^2 + R^2 + 2(R)(R)\cos 90}$

$|\vec{P}| = R\sqrt{2}$

Angle between $\vec{A}$ and $\vec{P} \Rightarrow \text{tab}\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}$

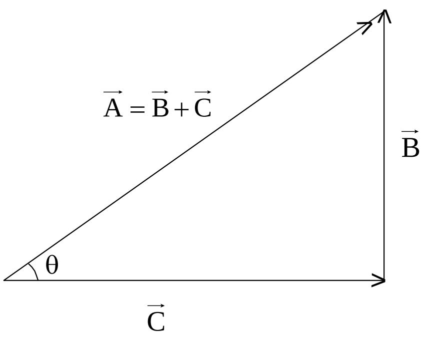
$\text{tab}\alpha = \dfrac{R\sin 90}{R + R\cos 90} = \dfrac{R}{R} = 1 \Rightarrow \alpha = 45^0$

c) $\vec{P}$ and $\overrightarrow{OC}$ both are along same direction ie angle between them = 0^0

$\Rightarrow |\vec{P} + \overrightarrow{OC}| = R\sqrt{2} + R = R\left(\sqrt{2} + 1\right)$

31. (A)

$\vec{A} = \vec{B} + \vec{C}, \underbrace{|\vec{A}| = 5, |\vec{B}| = 4, |\vec{C}| = 3}_{\text{Pythagoras Triplets}}$



$\cos\theta = \dfrac{|\vec{C}|}{|\vec{A}|} = \dfrac{3}{5} \Rightarrow \theta = \cos^{-1}\left(\dfrac{3}{5}\right)$

32. (D)

$\vec{A} + \vec{B} = \vec{A} - \vec{B}$

$\Rightarrow \vec{A} + \vec{B} - \vec{A} + \vec{B} = 0$

$2\vec{B} = 0$ or $\vec{B} = 0$

33. (B)

$|\vec{F_1}| = F, |\vec{F_2}| = F$

Let $\vec{F_R} = \vec{F_1} + \vec{F_2}$ then $|\vec{F_R}| = F$

Given $|\vec{F_R}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$F=\sqrt{F^2+F^2+2(F)(F)\cos\theta}$

On squaring

$F^2=2F^2+2F^2\cos\theta=2F^2(1+\cos\theta)$

$\Rightarrow \frac{F^2}{2F^2}=1+\cos\theta$ or $1+\cos\theta=\frac{1}{2}$

$\Rightarrow \cos\theta=\frac{1}{2}-1=\frac{-1}{2}$

$\Rightarrow \theta=120^0$ or 240^0

Ans θ is smaller of the two

34. (B)

$\vec{A}=2\hat{i}-\hat{j}+\hat{k},\ \vec{B}=\hat{i}-3\hat{j}-5\hat{k}$

$\vec{C}=3\hat{i}-4\hat{j}-4\hat{k}$ Find $|\vec{A}|,|\vec{B}|$ and $|\vec{C}|$ then use properties of Δ

Ans.: For equilateral $\Delta \Rightarrow |\vec{A}|=|\vec{B}|=|\vec{C}|$

In our case $|\vec{A}|^2+|\vec{B}|^2=|\vec{C}|^2$ RtΔ

35. (B)

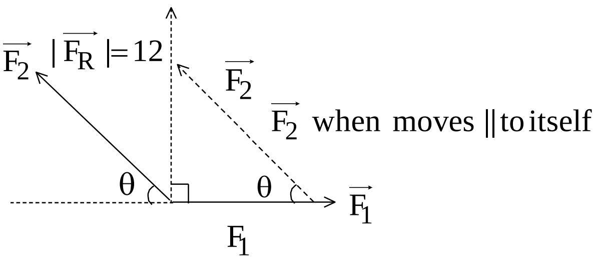
$|\vec{F_1}|=F_1, |\vec{F_2}|=F_2$ (Let $F_1 < F_2$)

Let $\vec{F_R}=\vec{F_1}+\vec{F_2}$

Given – $F_1 + F_2 = 18$, $|\vec{F_R}|=12$

And $\vec{F_R} \perp^r \vec{F_1}$

$\vec{F_2}$



From rtΔ, $\tan\theta=\frac{F_R=12}{F_1} \Rightarrow F_1=\frac{12}{\tan\theta}=2\cot\theta$

$\sin\theta=\frac{F_R=12}{F_2} \Rightarrow F_2=\frac{12}{\sin\theta}=12\operatorname{cosec}\theta$

As $F_1+F_2=18$

$\downarrow$

$12\cot\theta+12\operatorname{cosec}\theta=18$

$12\left[\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}\right]=18$

$\Rightarrow \frac{\cos\theta+1}{\sin\theta}=\frac{3}{2}$

$\Rightarrow 2\cos\theta+12=3\sin\theta$

$2\cos\theta+2=3\left(\sqrt{1-\cos^2\theta}\right)$

Squaring both sides $\left(\because \sin^2\theta+\cos^2\theta=1\right)$

$$(2\cos\theta+2)^2=\left(3\sqrt{1-\cos^2\theta}\right)^2$$

$$4\cos^2\theta+4+8\cos\theta=9(1-\cos^2\theta)$$

$$4\cos^2\theta+4+8\cos\theta-9+9\cos^2\theta=0$$

$$13\cos^2\theta+8\cos\theta-5=0$$

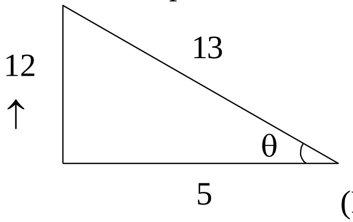
$$13\cos^2\theta+13\cos\theta-5\cos\theta-5=0$$

$$13\cos\theta[\cos\theta+1]-5(\cos\theta+1)=0$$

$$\Rightarrow(\cos\theta+1)(13\cos\theta-5)=0$$

$$\Rightarrow \cos\theta=-1 \text{ or } \cos\theta=\frac{5}{13}$$

$\theta=180^0$ not possible from figure $\theta=$ acute



(Pythagoras theorem)

$\Rightarrow$ from fig.

$$\sin\theta=\frac{12}{13}$$

$$\cos\theta=\frac{5}{13}$$

As $F_1=12\cot\theta=12\times\frac{\cos\theta}{\sin\theta}=12\times\frac{5/13}{12/13}$

$\Rightarrow F_1=5$

As $F_2=12\operatorname{cosec}\theta=\frac{12}{\sin\theta}=\frac{12}{12/13}=13$

36. (C)

$\vec{A}=2\hat{i}+3\hat{j}, \vec{B}=\hat{i}+\hat{j}$

Component of $\vec{A}$ along $\vec{B}=|\hat{A}|\cos\theta\hat{B}$

$$=|\vec{A}|\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}\hat{B}$$

$$=\frac{\vec{A}.\vec{B}}{|\vec{B}|}\hat{B}$$

$$=\left[\frac{(2\hat{i}+3\hat{j}).(\hat{i}+\hat{j})}{\sqrt{1^2+1^2}}\right]\frac{(\hat{i}+\hat{j})}{\sqrt{1^2+1^2}}$$

$$=\frac{(2+3)}{\sqrt{2}}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$$

Ans. $=\frac{5}{2}\hat{i}+\hat{j}$

37. (C)

Given vectors can be rewritten as $\vec{A} = 2\hat{i} + 3\hat{j} + 8\hat{k}$ and $\vec{B} = -4\hat{i} + 4\hat{j} + \alpha\hat{k}$

Dot product of these vectors should be equal to zero because they are perpendicular.

$\therefore \quad \vec{A}.\vec{B} = -8 + 12 + 8\alpha = 0 \Rightarrow 8\alpha = -4 \Rightarrow \alpha = -\frac{1}{2}$.

38. (A)

$\vec{A} = \hat{i} + \hat{j} + \hat{k}; \ \vec{B} = \hat{i} + \hat{j}$

$\therefore \ A = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}, \ B = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\vec{A}.\vec{B} = 2$

$\cos\theta = \frac{\vec{A}.\vec{B}}{AB} = \frac{2}{\sqrt{3}\sqrt{2}} = \left(\sqrt{\frac{2}{3}}\right)$

$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{2}{3}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$

$\therefore \quad \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

39. (A)

$S = \vec{r_2} - \vec{r_1}$

$W = \vec{F}.\vec{S} = \left(4\hat{i} + \hat{j} + 3\hat{k}\right).\left(11\hat{i} + 11\hat{j} + 15\hat{k}\right)$

$= \left(4 \times 11 + 1 \times 11 + 3 \times 15\right) = 100 \text{ J}$.

40. (B)

Let $\vec{A}.\left(\vec{B} \times \vec{A}\right) = \vec{A}.\vec{C}$

Here $\vec{C} = \vec{B} \times \vec{A}$ which is perpendicular to both vector $\vec{A}$ and $\vec{B}$

$\therefore \quad \vec{A}.\vec{C} = 0$.

41. (A)

$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix} = -\hat{j} - 2\hat{k}$$

i.e. the angular momentum is perpendicular to *x*-axis.

42. (C)

$\vec{A}.\vec{B} = 0 \qquad \therefore \quad \theta = 90^{\circ}$.

43. (D)

$\vec{P} = 2\hat{i} + b\hat{j} + 2\hat{k}$

$\vec{Q} = \hat{i} + \hat{j} + \hat{k}$

Given $\vec{P}$ is $\perp^{r}$ to $\vec{Q}$

$\Rightarrow \vec{P}.\vec{Q} = |\vec{P}||\vec{Q}|\cos 90 = 0$

$2 + b + 2 = 0 \Rightarrow b = -4$

44. (D)

$\vec{A}$, $\vec{B}$ and $\vec{C}$ are three vectors

Concept – Vector or Dot product can be performed only between two vectors

45. (D)

Given $|\vec{A}| = |\vec{B}|$ and $\vec{A}$ is $\perp^r$ to $\vec{B}$

To find which of the following is $\perp^r$ to $\vec{A} + \vec{B}$

(A) $\vec{A} \times \vec{B} \leftarrow \because$ it will give a vector $\perp^r$ to both $\vec{A}$ and $\vec{B}$ i.e. $\perp^r$ to plane of $\vec{A}$ and $\vec{B}$.

(B) $\vec{A} - \vec{B} \leftarrow$ To check perform Dot product of $(\vec{A} + \vec{B}).(\vec{A} - \vec{B})$ if it is = 0 then yes.

(C) $3\vec{A} - 3\vec{B} \leftarrow$ Same as B option

(D) all of these

46. (D)

$\vec{A}.(\vec{B} \times \overline{A}) = 0$

$\vec{A} \bullet$ gives vector $\perp^r$ to $\vec{A}$ and $\vec{B}$

Dot Product = $0 (\because \theta = 90^0$ and $\cos 90 = 0)$

47. (A)

Given $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$

We know $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

Hence $-\vec{B} \times \vec{A} = \vec{B} \times \vec{A}$

Or $O = \vec{B} \times \vec{A} + \vec{B} \times \vec{A}$ or $2\vec{B} \times \vec{A} = 0$

$\Rightarrow \vec{B} \times \vec{A} = 0 \Rightarrow \theta = 0^0$ or π

$\because \vec{A} \neq 0, \vec{B} \neq 0$

48. (B)

$\vec{A}.\vec{B} = |\vec{A} \times \vec{B}|$ given

$|\vec{A}||\vec{B}|\cos\theta = |\vec{A}||\vec{B}|\sin\theta$

$\Rightarrow \cos\theta = \sin\theta \Rightarrow \theta = 45^0$

49. (A)

Let $\vec{A} = \hat{i}$, $\vec{B} = \hat{i} + \hat{j} + \hat{k}$

Hence $\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \dfrac{(\hat{i}).(\hat{i} + \hat{j} + \hat{k})}{\sqrt{1^2}\sqrt{1^2 + 1^2 + 1^2}}$

$$\cos\theta = \frac{1}{\sqrt{1}\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

1. (c)

Given $\left|\vec{A}+\vec{B}\right| = \left|\vec{A}-\vec{B}\right| \times 2$

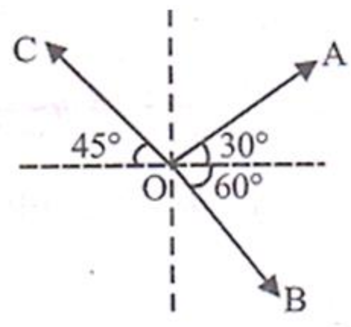
$\Rightarrow \left|\vec{A}+\vec{B}\right|^2 = \left|\vec{A}-\vec{B}\right|^2 \times 4$

$\Rightarrow 0 = 3|A|^2 + 3|B|^2 - |10|A||B|\cos\theta$

$\Rightarrow 0 - 3|A|^2 + 3|A|^2 - |10|A|^2 \cos\theta \Rightarrow |10|A|^2 \cos\theta = 6|A|^2$

$\Rightarrow \cos\theta = \frac{3}{5}$

2. (a)



Say, magnitudes of vectors is r.

$\overrightarrow{OA} = r\left[\cos 30^\circ \hat{i} + \sin 30 \hat{j}\right] = r\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right]$

$\overrightarrow{OC} = r\left[\cos 45^\circ\left(-\hat{i}\right) + \sin 45^\circ \hat{j}\right] = r\left[-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right]$

$\overrightarrow{OB} = r\left[\cos 60^\circ \hat{i} - \sin 60 \hat{j}\right] = r\left[\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right]$

$\therefore \overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$

$= r\left[\left(\frac{\sqrt{3}+1}{2} + \frac{1}{\sqrt{2}}\right)i + \left(\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right)\hat{j}\right]$

$\therefore$ Angle made by vector with x-axis

$\tan^{-1}\left[\frac{\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{\sqrt{2}}}\right] = \tan^{-1}\left[\frac{1-\sqrt{3}-\sqrt{2}}{\sqrt{3}+1+\sqrt{2}}\right]$

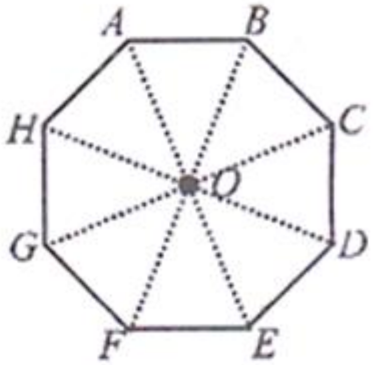
3. (d)

Projection of vector $\vec{A}$ on vector $\vec{B}$ is given by

$\left(A\cos\theta\right)\hat{B} = A\left(\frac{\vec{A}.\vec{B}}{AB}\right)\hat{B} = \frac{\vec{A}.\vec{B}}{B}\hat{B}$

$= \frac{2}{\sqrt{2}}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \hat{i}+\hat{j}$

4. (c)



From figure,

$\overline{AO}+\overline{OB}=\overline{AB}$ $\overline{AO}+\overline{OC}=\overline{AC}$ $\overline{AO}+\overline{OD}=\overline{AD}$

$\overline{AO}+\overline{OE}=\overline{AE}$ $\overline{AO}+\overline{OF}=\overline{AF}$ $\overline{AO}+\overline{OG}=\overline{AG}$

$\overline{AO}+\overline{OH}=\overline{AH}$

Now, adding we get

$8\overline{AO}=\left(\overline{AB}+\overline{AC}+\overline{AD}+\overline{AE}+\overline{AF}+\overline{AG}+\overline{AH}\right)$

$\Rightarrow 8\left(2\hat{i}+3\hat{j}-4\hat{k}\right)=\overline{AB}+\overline{AC}+\overline{AD}+\overline{AE}+\overline{AF}+\overline{AG}+\overline{AH}$

$\therefore \overline{AB}+\overline{AC}+\overline{AD}+\overline{AE}+\overline{AF}+\overline{AG}+\overline{AH}$

$=16\hat{i}+24\hat{j}-32\hat{k}$

5. (d)

Using

$R^2 = A_1{}^2 + A_2{}^2 + 2A_1A_2\cos\theta$

$5^2 = 3^2 + 5^2 + 2\times3\times5\cos\theta \text{ or } \cos\theta = -0.3$

$\left(2\vec{A}_1+3\overline{A}_2\right).\left(3\overline{A}_1-2\overline{A}_2\right) = 2A_1\times3A_1 + (3A_2)(3A_1)\cos\theta - (2A_1)(2A_2)\cos\theta - 3A_2\times2A_2$

$= 6A_1{}^2 + 9A_1A_2\cos\theta - 4A_1A_2\cos\theta - 6A_2{}^2$

$= 6A_1{}^2 - 6A_2{}^2 + 5A_1A_2\cos\theta$

$= 6\times3^2 - 6\times5^2 + 5\times3\times5(-0.3) = -118.5$

6. (c)

From figure, $\vec{r}_G = \frac{a}{2}\hat{i}+\frac{a}{2}\hat{k}$

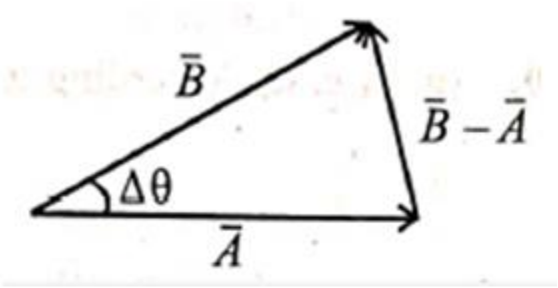
$\vec{r}_H = \frac{a}{2}\hat{j}+\frac{a}{2}\hat{k}$

$\therefore \vec{r}_H - \vec{r}_G = \left(\frac{a}{2}\hat{j}+\frac{a}{2}\hat{k}\right)-\left(\frac{a}{2}\hat{i}+\frac{a}{2}\hat{k}\right) = \frac{a}{2}\left(\hat{j}-\hat{i}\right)$

7. (a)

Arc length = radius × angle

So, $\left|\vec{B}-\vec{A}\right| = \left|\vec{A}\right|\Delta\theta$



$\left|\vec{A}\right| = \left|\vec{B}\right|$

8. (a)

If $\vec{C} = a\hat{i} + b\hat{j}$ then $\vec{A}.\vec{C} = \vec{A}.\vec{B}$

$a + b = 1$ …….(i)

$\vec{B}.\vec{C} = \vec{A}.\vec{B}$

$2a - b = 1$ ……(ii)

Solving equation (i) and (ii) we get

$a = \frac{1}{3}, b = \frac{2}{3}$

$\therefore$ Magnitude of coplanar vector, $\left|\vec{C}\right| = \sqrt{\frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{5}{9}}$

9. (a)

Here $\vec{P} + b\vec{R} = \vec{S} \Rightarrow \vec{R} = \frac{\vec{S} - \vec{P}}{b}$

Also $\vec{R} = \vec{Q} - \vec{P}$

$\therefore \frac{\vec{S} - \vec{P}}{b} = \vec{Q} - \vec{P} \Rightarrow \vec{S} - \vec{P} = b\vec{Q} - b\vec{P}$

$\therefore \vec{S} = b\vec{Q} + (1 - b)\vec{P}$

10. (5)

Here , $\vec{a}.\vec{b} = 0$

$\therefore \vec{a}.\vec{b} = \left(2\hat{i} + 4\hat{j} - 2\hat{k}\right).\left(\hat{i} + 2\hat{j} + \alpha\hat{k}\right) = 0$

$\therefore 2 \times 1 + 4 \times 2 - 2 \times \alpha = 0 \therefore \alpha = 5$

11. (2)

Given that

$\vec{A} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)m$ and $\vec{B} = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)m$

Component of vector $\vec{A}$ along vector $\vec{B} = \vec{A}.\hat{B}$

$= \frac{\vec{A}.\vec{B}}{\left|\vec{B}\right|} = \frac{2 + 6 - 2}{\sqrt{1^2 + 2^2 + 2^2}} \left(\because \hat{B} = \frac{\vec{B}}{\left|\vec{B}\right|}\right)$

$= \frac{6}{3} = 2m$

12. (3)

Direction of particle P,

$$\hat{v}_1 = \pm\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \pm\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$$

Direction of particle Q,

$$\hat{v}_2 = \pm\frac{\vec{A}\times\vec{C}}{|\vec{A}\times\vec{C}|} = \pm\frac{2\hat{k}}{2} = \pm\hat{k}$$

Angle between $\hat{v}_1$ and $\hat{v}_2$

$$\frac{\hat{v}_1.\hat{v}_2}{|\hat{v}_1||\hat{v}_2|} = \frac{\frac{\pm 1}{\sqrt{3}}}{(1)(1)} = \pm\frac{1}{\sqrt{3}}$$

Hence the angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\therefore$ value of $x = 3$

13. (180)

$\vec{P}\times\vec{Q} = \vec{Q}\times\vec{P}$ only if $\vec{P} = 0$ or $\vec{Q} = 0$

$\Rightarrow \vec{P}\times\vec{Q} = 0$

So, $\theta = 0^\circ$ or 180° and $0^\circ < \theta < 360^\circ$.

$\therefore \theta = 180^\circ$

14. (195)

Given: $\vec{F} = (\hat{i}+2\hat{j}+3\hat{k})\text{N}$

And, $\vec{r} = \left[(4\hat{i}+3\hat{j}-\hat{k})-(\hat{i}+2\hat{j}+\hat{k})\right] = 3\hat{i}+\hat{j}-2\hat{k}$

Torque, $\tau = \hat{r}\times\hat{F} = (3\hat{i}+\hat{j}-2\hat{k})\times(\hat{i}+2\hat{j}+3\hat{k})$

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix} = 7\hat{i}-11\text{j}+5\hat{k}$$

Magnitude of torque, $|\vec{\tau}| = \sqrt{195}$

15. (90)

Given,

$|\vec{R}| = |\vec{P}| \Rightarrow |\vec{P}+\vec{Q}| = |\vec{P}|$

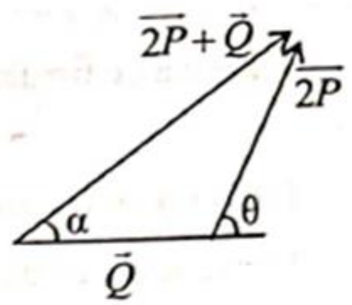
$P^2+Q^2+2PQ.\cos\theta = P^2$

$\Rightarrow Q+2P\cos\theta = 0$

$\Rightarrow \cos\theta = -\frac{Q}{2P} \quad \ldots(i)$

$\tan\alpha = \frac{2P\sin\theta}{Q+2P\cos\theta} = \infty\,(\because 2P\cos\theta+Q=0)$

$\Rightarrow \alpha = 90^\circ$



1. (A)

Let force be $\vec{F_1}, |\vec{F_1}| = F_1, \vec{F_2}, |\vec{F_2}| = F_2$

Let $F_1 > F_2$

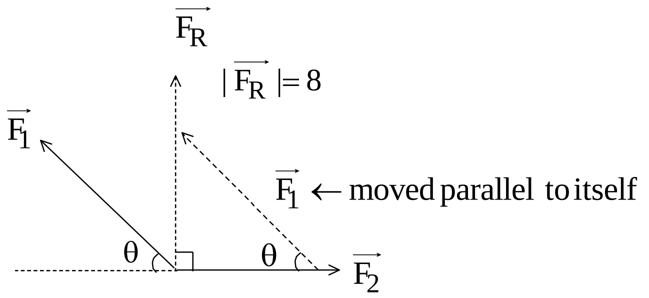
Let $\vec{F_R} = \vec{F_1} + \vec{F_2}$

Given (i) $F_1 + F_2 = 16$

(ii) $F_R = 8$

(iii) F_R is $\perp^r$ to $F_2 \leftarrow \perp^r$ to smaller force

$\vec{F_R}, |\vec{F_R}| = 8$



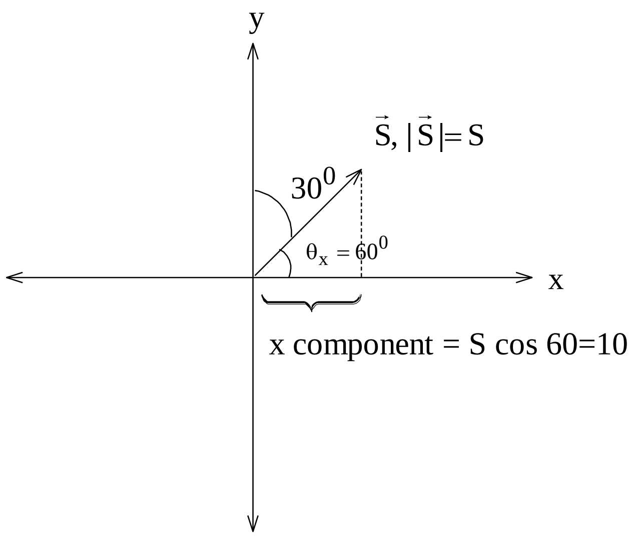
2. (D)

Component of $\vec{A}$ along x axis = $|\vec{A}| \cos\theta_x$

Now, $|\vec{A}| \cos\theta_x$ is always $\leq |\vec{A}|$

$(\because \cos\theta_x \leq 1)$

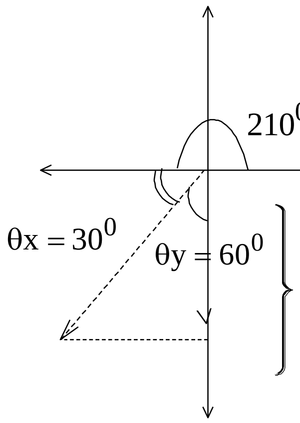
3. (D)



$\Rightarrow S \times \frac{1}{2} = 10$

$S = 20$

4. (D)



$y \text{ component } = 25\cos 60(-\hat{j}) = -12.5\hat{j}$

5. (B)

Add two smaller forces (10, 20)

min = |10 - 20|
= 10

max = 10 + 20
= 10

Now, left forces are 30, 40,

If we add min (10) with 30

min = |10 - 30|
= 20

max = 10 + 30
= 40

Now $\max'(=40)$ and 40 can add

$\min^{11}$ = |40 - 40|
= 0

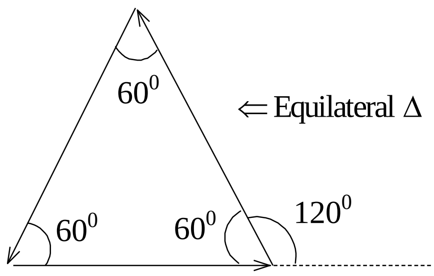
$\max^{11}$ = 40 +40
= 80

B option is possible

6. (C)

Direction are along the sides of equilateral Δ taken in same order [Head of one coincides tail of another]

i.e



* Vector are along directions of equilateral Δ hence angle between any two $=120^0$

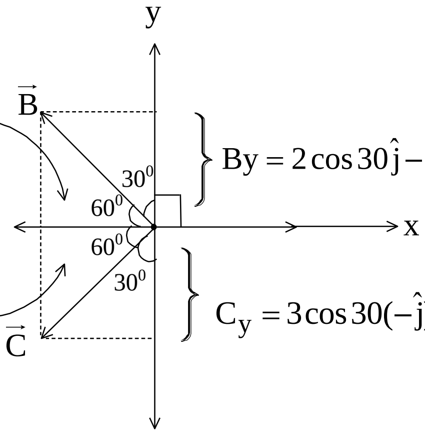
$\uparrow$ $(\neq 60^0)$

($\because$ angle is measured when they are joined by tails)

Let three vectors be $\vec{A}, \vec{B}$ and $\vec{C}$. They each make 120^0 with other

And $|\vec{A}| = 1, |\vec{B}| = 2, |\vec{C}| = 3$

Lets represent them in XY plane. Lets take $\vec{A}$ along X axis (we can take any)



$B_x = 2\cos 60(\hat{i}) = -\hat{i}$

$By = 2\cos 30\hat{j} - \sqrt{3}\hat{j}$

$C_x = 3\cos 60(-\hat{i})$

$C_x = -\frac{3}{2}\hat{i}$

$C_y = 3\cos 30(-\hat{j}) = \frac{-3\sqrt{3}}{2}\hat{j}$

From fig

* $\vec{A} = \hat{i} \leftarrow$ only along x axis with magnitude =1

* $\vec{B} = \vec{B}x + \vec{B}y$

$\vec{B} = -\hat{i} + \sqrt{3}\hat{j}$

* $\vec{C} = \overrightarrow{C_x} + \overrightarrow{C_y}$

$\vec{C} = \frac{-3}{2}\hat{i} - \frac{3\sqrt{3}}{2}\hat{j}$

$\Rightarrow \underbrace{A+B+C} = \hat{i}\left(1-1-\frac{3}{2}\right) + \hat{j}\left(0+\sqrt{3}-\frac{3\sqrt{3}}{2}\right)$

Let $\vec{R} = \frac{-3}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}$

$\Rightarrow |\vec{R}| = \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{-\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{9}{4}+\frac{3}{4}}$

$|\vec{R}| = \sqrt{3}$

7. (A)

In Q.6 1st vector is $\vec{A} = \hat{i}$ Resultant. Vector $\vec{R} = \frac{-3}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}$

Hence for angle between $\vec{R}$ and $\vec{A}$

We can use

$$\cos\theta = \frac{\vec{R}.\vec{A}}{|\vec{R}||\vec{A}|}$$

$$= \left(\frac{\left(\frac{-3}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right).(\hat{i})}{\sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{-\sqrt{3}}{2}\right)^2}\sqrt{(1)^2}}\right)$$

$$= \frac{-3/2}{\sqrt{\frac{9}{4}+\frac{3}{4}}\sqrt{1}} = \frac{-3}{2\sqrt{3}}$$

$$\cos\theta = \frac{-\sqrt{3}\times\sqrt{3}}{2\times\sqrt{3}} = \frac{-\sqrt{3}}{2}$$

$\Rightarrow \theta = 150^0$ or 210^0

Answer should be 150^0 but its not given in question. Hence we'll consider 210^0

8. (A)

As body can move only along y-axis

$\Rightarrow$ displacement $\vec{S} = 10\hat{j}$

$\downarrow$

(moves by 10m)

$\vec{F} = -2\hat{i} + 15\hat{j} + 6\hat{k}$

Work Done $(w) = \vec{F}\,.\,\vec{S}$

$= (-2\hat{i} + 15\hat{j} + 6\hat{k}).(10\hat{j})$

= 150 joule

9. (A)

$\vec{A} = \hat{i} + \hat{j},\ \vec{B} = \hat{i} - \hat{j}$

We want $|\vec{c}| = 3$ and $\vec{c}$ to be $\perp^r$ to $\vec{A}$ and $\vec{B}$

Let $0\vec{P} = \vec{A} \times \vec{B}$

$\vec{P} = (\hat{i} + \hat{j}) \times (\hat{i} - \hat{j})$

$$\vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}$$

$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(-1-1)$

$\vec{P} = -2\hat{k}$

So, $\vec{P}$ is $\perp^r$ to $\vec{A}$ and $\vec{B}$ from definition of cross product

$\vec{c} = 3(\hat{P})$

We want is magnitude to be 3 and direction $\perp^r$ to $\vec{A}$ and $\vec{B}$ that's why 3 times $(\hat{p}) \leftarrow$ magnitude = 1 direction $\perp^r$ to $\vec{A}$ and $\vec{B}$

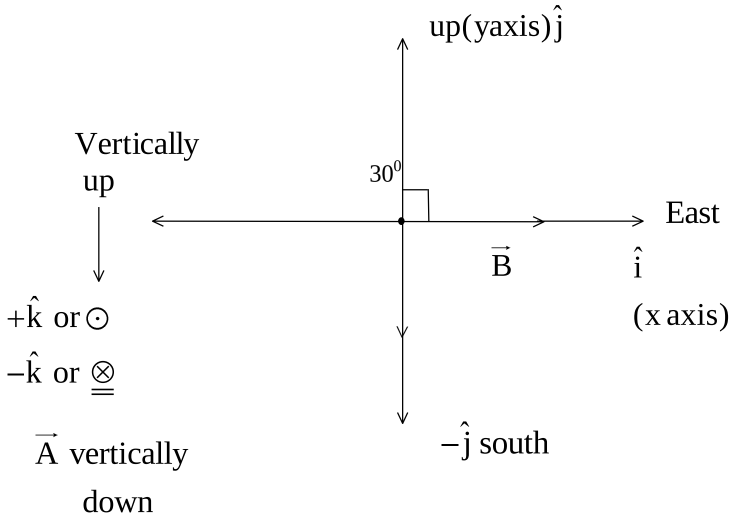
$$\vec{c} = 3\frac{(-2\hat{k})}{\sqrt{(-2)^2}} = \frac{-6\hat{k}}{2} = -3\hat{k}$$

If we perform $\vec{B} \times \vec{A}$ then we'll get $\vec{C} = +3\hat{k}$

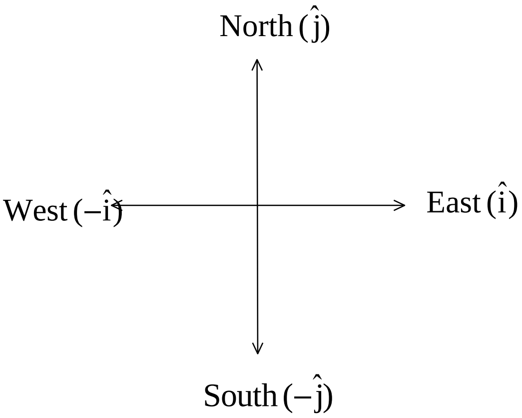
$\vec{A} \times \vec{B} = -\hat{k} \times \hat{i}$

$= -\hat{j}$

10. (D)



11. (B)



Initial Velocity $\vec{u} = 5\hat{i}$

Final velocity $\vec{v} = 5\hat{j}$

$\Rightarrow \Delta\vec{v} = \overrightarrow{V_f} - \overrightarrow{V_i} = \vec{v} - \vec{u} = 5\hat{j} - 5\hat{i}$

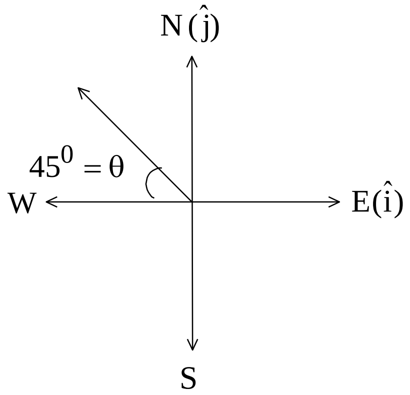
$\Rightarrow \vec{a} = \frac{\Delta\vec{v}}{t} = \frac{5\hat{j} - 5\hat{i}}{10} = \frac{(-\hat{i} - \hat{j})}{2}$

$\vec{a} = \frac{-1}{2}\hat{i} + \frac{1}{2}\hat{j}$

$\Rightarrow |\vec{a}| = \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$

$|\vec{a}| = \frac{1}{\sqrt{2}}$

$\vec{a} = \frac{-1}{2}\hat{i} + \frac{1}{2}\hat{j}$



-ve x and +ve y, $\vec{a}$ lies along

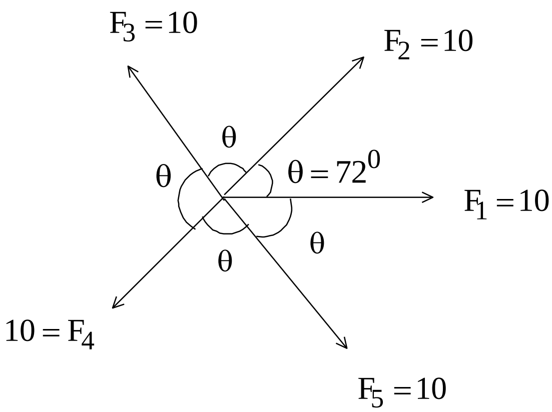
$\Rightarrow$ B option North west

When magnitude of x component and y component are equal then $\theta = 45^0$ always.

$\because$ from fig $\tan\theta = \dfrac{\text{y component}}{\text{x componet}} = 1$

$\Rightarrow \theta = 45^0$

12. (A)



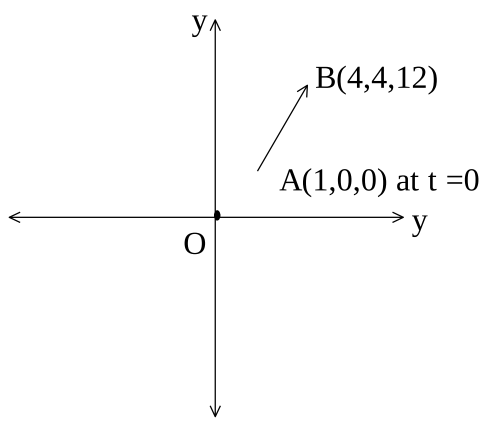
$\vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} + \vec{F_5} = 0$

Note

When force of equal magnitude using in same plane acts at a point such that angle between any two adjacent is equal. Then these form regular closed polygon.

As polygon is closed. Hence $\vec{F} = 0$ net

13. (D)



Direction of motion is A to B ie = AB

$\overrightarrow{AB} = (4-1)\hat{i} + (4-0)\hat{j} + (12-0)\hat{k}$

$= 3\hat{i} + 4\hat{j} + 12\hat{k}$

$= 3\hat{i} + 4\hat{j} + 12\hat{k}$

$\Rightarrow AB = \dfrac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \dfrac{3i + 4j + 12k}{\sqrt{3^2 + 4^2 + 12^2}} = \sqrt{169} = 13$

$\Rightarrow AB = \dfrac{3\hat{i} + 4\hat{j} + 12\hat{k}}{13}$

Velocity $= |\vec{v}|$ and direction AB

$= 65$

$= 5\dfrac{(3\hat{i} + 4\hat{j} + 12\hat{k})}{13}$

$\vec{v} = 15\hat{i} + 20\hat{j} + 60\hat{k}$

Displacement $\vec{S}$ = velocity $(\vec{v}) \times$ time

$\vec{S} = 2(15\hat{i} + 20\hat{j} + 60\hat{k})$

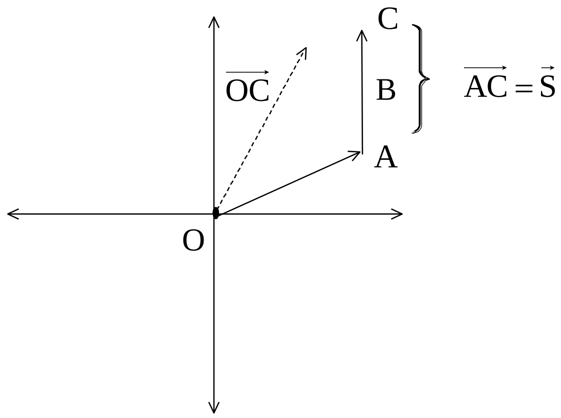
$\vec{S} = 30\hat{i} + 40\hat{j} + 60\hat{k}$

Displacement wrt to point $\vec{A}$

Hence position vector = $\overrightarrow{OA} + \vec{S}$

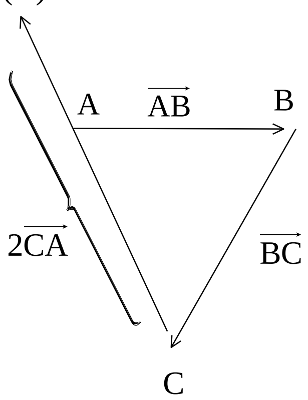
$(i + 0j + 0k) + (30\hat{i} + 40\hat{j} + 120\hat{k})$

$= 31\hat{i} + 40\hat{j} + 120\hat{k}$



Position $= \overrightarrow{OC} = 31\hat{i} + 40\hat{j} + 120\hat{k}$

14. (A)



$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ from and low AB

$$\Rightarrow \underbrace{\overrightarrow{AB}+\overrightarrow{BC}}_{\overrightarrow{AC}}+2\overrightarrow{CA}=\underbrace{\overrightarrow{AC}+2\overrightarrow{CA}}_{\overrightarrow{CA}}$$

$\because \overrightarrow{AC}=-\overrightarrow{CA}$

$=\overrightarrow{CA}$

15. (C)

Let $|\overrightarrow{F_1}|=3P, |\overrightarrow{F_2}|=2P$

Let $\overrightarrow{F_R}=\overrightarrow{F_1}+\overrightarrow{F_2}$

$\Rightarrow |\overrightarrow{F_R}|=R$

$|\overrightarrow{F_R}|=\sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$

$R=\sqrt{(3P)^2+(2P)^2+2(3P)(2P)\cos\theta}$

$R=\sqrt{3P^2+4P^2+12P^2\cos\theta}$

Squaring both sides

$R^2=13P^2+12P^2\cos\theta$

Now if $|\overrightarrow{F_1}|^1=2(3P)=6P$

$\Rightarrow |F_R|^1=2(R)=2R$

$|\vec{F_2}|$ =2P remains same

$|\overrightarrow{F_R}|^1=\sqrt{F_1'^2+F_2^2+2F_1^1F_2\cos\theta}$

$2R=\sqrt{(6P)^2+(2P)^2+2(6P)(2P)\cos\theta}$

Squaring both sides

$4R^2=36P^2+4P^2+24P^2\cos\theta$

$\Rightarrow 4R^2=40P^2+24P^2\cos\theta$

From equation

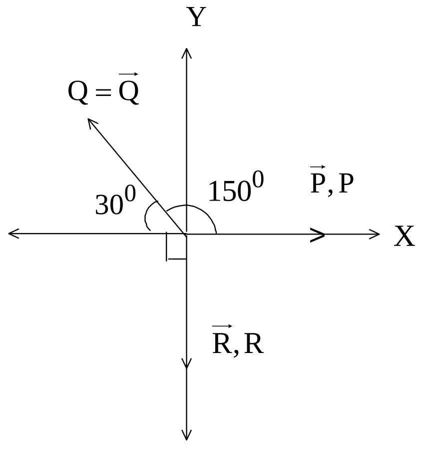
$4[13P^2+12P^2\cos\theta]=4(10P^2+6P^2\cos\theta)$

$6P^2\cos\theta=-3P^2$

$\Rightarrow \cos\theta=-\frac{1}{2}$

Or $\theta=120^0$ or 240^0

16. (D)



Resolve $\vec{P}, \vec{Q}$ and $\vec{R}$ along X and Y axis

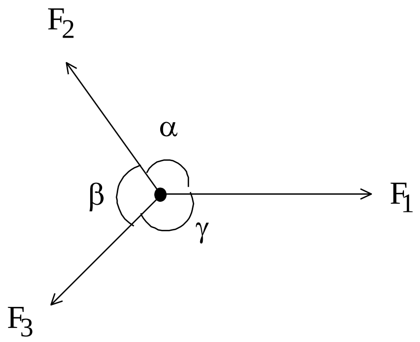
And $\vec{P} + \vec{Q} + \vec{R} = 0 \leftarrow \because$ they are in equilibrium

Solve to get relation between P, Q and R

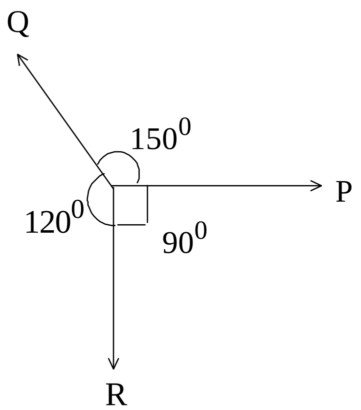
Alternate method $\rightarrow$Lami's theorem

Valid only when 3 vectors add to give resultant =0

Given



If $\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$ then from lami's theorem



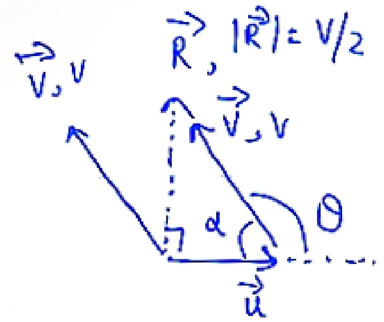
From lami's theorem

$$\frac{P}{\sin 120} = \frac{Q}{\sin 90} = \frac{R}{\sin 150} = K$$

$$\left.\begin{aligned} &\Rightarrow P = K\sin 120 = \frac{K\sqrt{3}}{2} \\ &\Rightarrow Q = K\sin 90 = k \\ &\Rightarrow R = K\sin 150 = \frac{k}{2} \end{aligned}\right\} P : Q : R = \sqrt{3} : 2 : 1$$

17. (D)

Let $\vec{R}=\vec{u}+\vec{v}$ and $\vec{R}$ is $\perp^r$ to $\vec{u}$



$$\sin\alpha = \frac{V/2}{V} = \frac{1}{2}$$

$\alpha = 30^o$

$\Rightarrow \ \theta = 180 - \alpha = 150^o$

18. (B)

let $\vec{R} = \vec{P} + \vec{Q}$

$\Rightarrow R_{min} = |P - Q|$

It will be 0 (zero)

If P = Q

19. (B)

$|\vec{F_1}| = 6,\ \vec{F_2} = 8,\ \text{let } \vec{F_R} = \vec{F_1} + \vec{F_2}$

$|F_1 - F_2| \le |\vec{F_R}| \le F_1 + F_2$

$|-2| \le F_R \le 14$

$2 \le F_R \le 14$

Ans. (B)

20. (D)

$6 = |2-8| \le |\vec{R}| \le (2+8) = 10$

4 doesn't lie between 6 and 10

21. (A)

$\vec{A} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{B} = 2\hat{i} + \hat{j} + 2\hat{k}$

Projection of $\vec{B}$ on $\vec{A} = |\vec{B}|\cos\theta\hat{A}$

$$= |\vec{B}| \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}\hat{A}$$

$$= \left[\frac{(\hat{i}+2\hat{j}+2\hat{k}).(2\hat{i}+\hat{j}+2\hat{k})}{\sqrt{1^2+2^2+2}}\right] = \frac{\vec{A}}{|\vec{A}|}$$

$$= \left(\frac{2+2+4}{\sqrt{9}}\right)\frac{(\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{1^2+2^2+2^2}} = \frac{8}{9}(\hat{i}+2\hat{j}+2\hat{k})$$

EXERCISE - 2

1. (A, D)

$\vec{A} = 3\hat{i} + 4\hat{j}, \vec{B} = \hat{i} + \hat{j}$

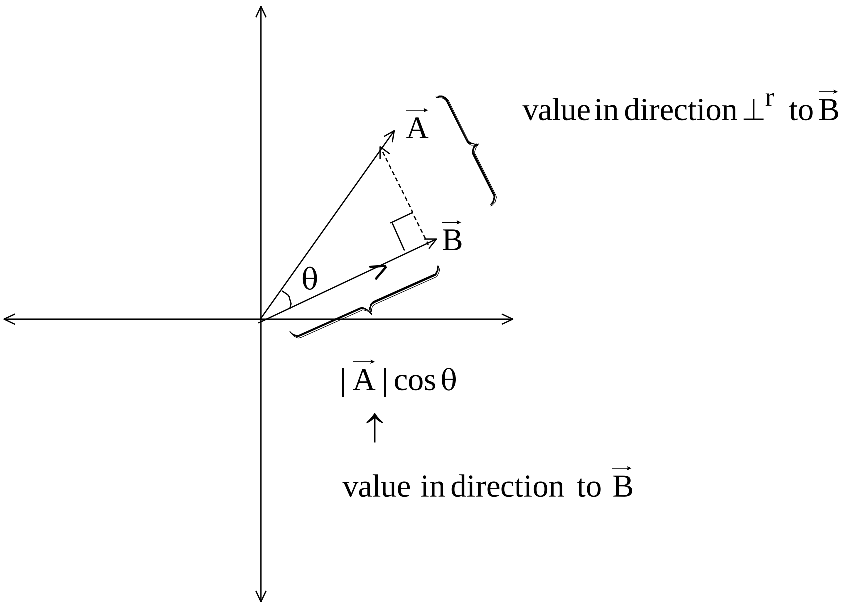
$\theta =$ angle between $\vec{A}$ and $\vec{B}$

Component of $\vec{A}$ along $\vec{B} = |\vec{A}| \cos\theta \hat{B}$

$= |\vec{A}| \cos\theta \hat{B} = |\vec{A}| \cos\theta \frac{\vec{B}}{|\vec{B}|}$

$|\vec{A}| \cos\theta \frac{(\hat{i} + \hat{j})}{\sqrt{1^2 + 1^2}}$

$|\vec{A}| \cos\theta \frac{(\hat{i} + \hat{j})}{\sqrt{2}}$



We can apply Δ law in out figure

$\Rightarrow \vec{A} = \vec{A}$ along $\vec{B} + \vec{A} \perp^r$ to $\vec{B}$

Learn always valid

As $\vec{A}$ along $\vec{B} = |\vec{A}| \cos\theta \hat{B}$

$|\vec{A}| \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} \frac{\vec{B}}{|\vec{B}|}$

$= \frac{(\vec{A}.\vec{B})(\vec{B})}{|\vec{B}|^2}$

$= \frac{(3\hat{i} + 4\hat{j}).(\hat{i} + \hat{j})(\hat{i} + \hat{j})}{\left[\sqrt{1^2 + 1^2}\right]^2}$

$\vec{A}_{\text{along}\,\vec{B}} = \frac{(3+4)}{2} \hat{i} + \hat{j} = \frac{7}{2}(\hat{i} + \hat{j})$

As $\vec{A} = \vec{A}_{\text{along}\,\vec{B}} + \vec{A}_{\perp^r \text{ to } \vec{B}}$

$\Rightarrow A_{\perp^r \text{ to B}} = \vec{A} - \vec{A}_{\text{along}\,\vec{B}}$

$= (3\hat{i} + 4\hat{j}) - \frac{7}{2}(\hat{i} + \hat{j})$

$= \frac{6\hat{i} + 8\hat{j} - 7\hat{i} - 7\hat{j}}{2}$

$\Rightarrow \vec{A}_{\perp^r \text{ to } \vec{B}} = \frac{-\hat{i} + \hat{j}}{2}$

Lets convert the above expression in given options form.

Clearly two options are possible B and D

B is incorrect as for B direction is $\frac{\hat{i} - \hat{j}}{2}$

* And direction of $\vec{A} \perp^r$ to $\vec{B}$ is $\frac{-\hat{i} + \hat{j}}{2}$

* For D option we have to check as direction matches

(D) $\vec{A} \perp^r$ to $\vec{B} = |\vec{A}| \sin\theta \left(\frac{\hat{j} - \hat{i}}{\sqrt{2}}\right)$

$|\vec{A}| = \sqrt{3^2 + 4^2} = 5$

$\sin\theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|} \quad \left[\because |\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin\theta\right]$

$\Rightarrow \vec{A} \times \vec{B} = 3(\hat{i} \times \hat{i}) + 3(\hat{i} \times \hat{j}) + 4(\hat{j} \times \hat{i}) + 4(\hat{j} \times \hat{j})$

$\qquad\qquad 0 \qquad\qquad \hat{k} \qquad\qquad -\hat{k} \qquad\qquad 0$

$\vec{A} \times \vec{B} = 3\hat{k} - 4\hat{k} = -\hat{k}$

$\Rightarrow |\vec{A} \times \vec{B}| = 1$

$\Rightarrow \sin\theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|} = \frac{1}{5\sqrt{2}}$

$\Rightarrow$ D option $= |\vec{A}| \sin\theta \frac{(\hat{j} - \hat{i})}{\sqrt{2}}$

$5 \frac{1}{5\sqrt{2}} \left[\frac{\hat{j} - \hat{i}}{\sqrt{2}}\right]$

D option = $\frac{-\hat{i} + \hat{j}}{\sqrt{2}} = \vec{A} \perp r$ to $\vec{B}$

Ans – A, D

2. (A, B, C)

$\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ $\vec{B} = \hat{i} + \hat{j} + \hat{k}$ then unit vector.

(i) First check which options are not unit vector and eliminate them

In out case all are unit vector (ii) solve options

(A) $\perp^r$ to $\vec{A}$ is $\frac{-\hat{j} + \hat{k}}{\sqrt{2}}$

$\Rightarrow$ A.$\frac{-\hat{j} + \hat{k}}{\sqrt{2}}$ must be =0

$(2\hat{i}+\hat{j}+\hat{k}).\left(\frac{-\hat{j}+\hat{k}}{\sqrt{2}}\right)=\frac{(0-1+1)}{\sqrt{2}}=0$

$\Rightarrow$ A is correct

(B) Parallel to $\vec{A}$ is $\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}$

Apply condition for which are ||

i.e. $\frac{Ax}{Bx}=\frac{Ay}{By}=\frac{Az}{Bz}$

In our case $\frac{A_x=2}{B_x=\frac{2}{\sqrt{6}}}=\frac{A_y=1}{B_y=\frac{1}{\sqrt{6}}}=\frac{Az=1}{Bz=\frac{1}{\sqrt{6}}}=\sqrt{6}$

Condition satisfies $\Rightarrow$B is correct

(C) $\perp^r$ to $\vec{B}$ is $\frac{-\hat{j}+\hat{k}}{\sqrt{2}}$

$\Rightarrow \vec{B}.\left[\frac{-\hat{j}+\hat{k}}{\sqrt{2}}\right]$must be =0

$\left(\hat{i}+\hat{j}+\hat{k}\right).\left[\frac{-\hat{j}+\hat{k}}{\sqrt{2}}\right]=\frac{0-1+1}{\sqrt{2}}=0$

$\Rightarrow$C option is correct

(D) In correct

$\because$ condition doesn't satisfy

$\left[\frac{A_x}{B_x}=\frac{A_y}{B_y}=\frac{A_z}{B_z}\right]$

Ans : A, B, C

3. (B, D)

If $(\vec{V}_1+\vec{V}_2)$ is $\perp^r$ to $(\vec{V}_1-\vec{V}_2)$

Dot product is zero $\because \theta=90^0$

$\Rightarrow\left[\vec{V}_1+\vec{V}_2\right].\left[\vec{V}_1-\vec{V}_2\right]=0$

$\vec{V_1}.\vec{V_1}-\vec{V_1}.\vec{V_2}+\vec{V_2}.\vec{V_1}-\vec{V_2}.\vec{V_2}=0$

$\Rightarrow|\vec{V_1}|^2-|\vec{V_2}|^2=0$

$\Rightarrow|\vec{V_1}|^2=|\vec{V_2}|^2$ or $|\vec{V_1}|=|\vec{V_2}|$

(A) Not necessary

(B) Derived

(C) Not possible

(D) yes they can have any angle between them

4. (A, D)

$\vec{A}$ and $\vec{B}$ lie in one plane $\leftarrow$let it be xy plane

$\vec{C}$ lies in different plane $\leftarrow$ cant be XY

Let it be XZ plane

Hence $\vec{A}+\vec{B}+\vec{C}$ will be a vector which can lie only in

XYZ (3dplane)

If X component gets cancelled

(A) can't be zero
(D) XYZ or YZ is different from XY and XZ

5. (A, C)

Dot or cross product is performed between 2 vectors

(A) $\vec{u}.\underbrace{(\vec{v}\times\vec{w})}$

$\vec{u}$.[cross product hence it u be a vector]

(B) $(\vec{u}.\vec{v}).\vec{w}$

scalar $.\vec{w}$ not possible

(C) $(\vec{u}.\vec{v}).\vec{w}$

scalar $\left(\vec{w}\right)$ possible no dot or cross product

(D) $\vec{u}\times\left(\vec{v}.\vec{w}\right)$

$\vec{u}.\times$ (scalar)

Not possible

6. (B, C)

Given $|\vec{A}.\vec{B}| = (\pm 8) = \left||\vec{A}||\vec{B}|\cos\theta\right|$

$|\vec{A}\times\vec{B}| = 8\sqrt{3} = |\vec{A}||\vec{B}|\sin\theta$

$\Rightarrow \frac{|\vec{A}\times\vec{B}|}{|\vec{A}.\vec{B}|} = \frac{|A||B|\sin\theta}{\left||A||B|\cos\theta\right|} = \pm\sqrt{3}$

$\tan\theta = \pm\sqrt{3}$

$\Rightarrow \theta = 60^0$ or 120^0

7. (A, C, D)

(B) Incorrect as on changing orientation of axis, the given vector will make different angle wrt new axis hence component will change.

8. (A, B, D)

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} \Rightarrow |\vec{A}| = \sqrt{A_x^2 + A_y^2}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} \Rightarrow |\vec{B}| = \sqrt{B_x^2 + B_y^2}$

$$\vec{R} = \vec{A} + \vec{B} = \underbrace{(A_x + B_y)}_{R_x}\hat{i} + \underbrace{(A_y + B_y)}_{R_y}\hat{j}$$

(A) $R_X = A_X + B_X$

(B) $R_x = A_x + B_x \le |\vec{A}| + |\vec{B}|$

Equal to when $A_y = B_y = 0$

(C) Proved from B

(D) Prove from B

9. (B, C, D)

$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin\theta$

And $\sin\theta \le 1$ always

$\Rightarrow X$ (a) $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin\theta \le |\vec{A}||\vec{B}|$

(b) for $\sin\ \theta = 1$

(c) for $\sin\theta < 1$

(d) for $\sin\theta = 0$

EXERCISE - 3

1. (I) $\rightarrow$ A, (II) $\rightarrow$ A, (III) $\rightarrow$ D, (IV) $\rightarrow$ C

$\vec{A} = 4\hat{i} + 4\hat{j}, \vec{B} = 4\hat{i} - 4\hat{j}$

Column A

(i) $|\vec{A} + \vec{B}| = |4\hat{i} + 4\hat{j} + 4\hat{i} - 4\hat{j}| = |8\hat{i}| = 8$

(ii) $|\vec{A} - \vec{B}| = |4\hat{i} + 4\hat{j} - (4\hat{i} - 4\hat{j})| = |8\hat{j}| = 8$

(iii) $\vec{A}.\vec{B} = (4\hat{i} + 4\hat{j}).(4\hat{i} - 4\hat{j})$ = 16 – 16 = 0

(iv) $|\vec{A} \times \vec{B}| = \left|(4\hat{i} + 4\hat{j}) \times (4\hat{i} - 4\hat{j})\right|$

$= |16\hat{i} \times \hat{i} - 16\hat{i} \times \hat{j} + 16\hat{j} \times \hat{i} + 16\hat{j} \times \hat{j}|$

$= |32\hat{k}|$

= 32

Ans. $I \rightarrow A, II \rightarrow A, III \rightarrow D, IV \rightarrow C$

2. (I) $\rightarrow$ D, (II) $\rightarrow$ A, (III) $\rightarrow$ C, (IV) $\rightarrow$ C

$|\vec{A}| = 1, |\vec{B}| = 2, \theta$ between $\vec{A}$ and $\vec{B} = 90^0$

Column A

(i) $\vec{A}.\vec{B} = 0 \ \ [\because \theta = 90 \Rightarrow \cos 90 = 0]$ A

(ii) $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin 90$

= (1) (2) (1)

= 2

(iii) $|A + B| = \sqrt{A^2 + B^2 + 2AB\cos 0}$

$= \sqrt{1^2 + 2^2 + 2(1)(2)(0)}$

$= \sqrt{5}$

(iv) $|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos 90}$

$= \sqrt{1^2 + 2^2 - 0}$

$= \sqrt{5}$

Ans. I $\to$D, II$\to$A, III$\to$ C, IV$\to$ C

EXERCISE - 4

1. Dot and cross product can occur between 2 vectors only
   * Scalar can't be added to vector using above two points
   Ans $\to$C, D, F, H (only)

2. $\vec{F} = q(\vec{V} \times \vec{B})$

   (A) $\left.\begin{array}{l}\vec{V} = -i \\ \vec{B} = \hat{j}\end{array}\right\} \vec{V} \times \vec{B} = -\hat{i} \times \hat{j} = -\hat{k}$

   (B) $\left.\begin{array}{l}\vec{V} = \hat{j} \\ \vec{B} = \hat{j}\end{array}\right\} \vec{V} \times \vec{B} = \hat{j} \times \hat{j} = 0$ no direction

   (C) $\left.\begin{array}{l}\vec{V} = -\hat{j} \\ \hat{B} = \hat{k}\end{array}\right\} \vec{V} \times \vec{B} = -\hat{j} \times \hat{k} = -\hat{i}$

3. $\vec{A} = 5\hat{i} + 4\hat{j} - 6\hat{k}$

   $\vec{B} = -2\hat{i} + 2\hat{j} + 3\hat{k}$

   $\vec{C} = 4\hat{i} + 3\hat{j} + 2\hat{k}$

   a) $\vec{R} = \vec{A} - \vec{B} + \vec{C}$

   $\vec{R} = 11\hat{i} + 5\hat{j} + -7\hat{k}$

   b) $\cos\theta_z = \frac{R_z}{|\vec{R}|} = \frac{-7}{\sqrt{11^2 + 5^2 + (-7)^2}}$

   They have asked angle wrt +z hence value with sign is substituted.

   $\Rightarrow \theta_z = \cos^{-1}\left[\frac{-7}{\sqrt{195}}\right]$

   c) magnitude of $\vec{A}$ along $\vec{B} = |\vec{A}|\cos\theta$

   $\Rightarrow |A|\cos\theta = |\vec{A}| \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$

   $= \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|} = \frac{(5\hat{i} + 4\hat{j} - 6\hat{k}).(-2\hat{i} + 2\hat{j} + 3\hat{k})}{\sqrt{(-2)^2 + (2)^2 + (3)^2}}$

   $= \frac{-10 + 8 - 18}{\sqrt{17}} = \frac{-20}{\sqrt{17}}$

4. $\vec{A} = 3\hat{i} + 5\hat{j}$ $\hat{B} = 2\hat{i} + 4\hat{j}$

   (a) $\vec{A} \times \vec{B} = (3\hat{i} + 5\hat{j}) \times (2\hat{i} + 4\hat{j})$

   $= \underset{0}{6\hat{i} \times \hat{i}} + 12\hat{i} \times \hat{j} + 10\hat{j} \times \hat{i} + \underset{0}{20\hat{j} \times \hat{j}}$

$12\hat{k}-10\hat{k}=2\hat{k}$

(b) $\vec{A}.\vec{B}=(3\hat{i}+5\hat{j}).(2\hat{i}+4\hat{j})$

$= 6 + 20 = 26$

(c) $(\vec{A}+\vec{B}).\vec{B}=(5\hat{i}+9\hat{j}).(2\hat{i}+4\hat{j})$

$= 10 + 36 = 46$

(d) magnitude of $\vec{A}$ along $\vec{B} = |\vec{A}|\cos\theta$

$$|\vec{A}|\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}=\frac{\vec{A}.\vec{B}}{|\vec{B}|}=\frac{26}{\sqrt{2^2+4^2}}=\frac{26}{\sqrt{20}}$$

$$=\frac{\cancel{26}\,13}{\cancel{2}\sqrt{5}}=\frac{13}{\sqrt{5}}$$

5. $\vec{A}=3\hat{i}+3\hat{j}-2\hat{k}$

$\vec{B}=-\hat{i}-4\hat{j}+2\hat{k}$

$\vec{C}=2\hat{i}+2\hat{j}+\hat{k}$

(a) $\vec{A}.(\vec{B}\times\vec{C})$

$$\vec{B}\times\vec{C}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & -4 & 2\\ 2 & 2 & 1\end{vmatrix}$$

$=\hat{i}(-4-4)-\hat{j}(-1-4)+\hat{k}(-2+8)$

$\vec{B}\times\vec{C}=-8\hat{i}+5\hat{j}+6\hat{k}$

$\Rightarrow \vec{A}.\vec{B}\times\vec{C}=(3\hat{i}+3\hat{j}-2\hat{k}).(-8\hat{i}+5\hat{j}+6\hat{k})$

$=-24+15-12$

$= -21$

(b) $\vec{A}.(\vec{B}+\vec{C})=(3\hat{i}+3\hat{j}-2\hat{k}).(\hat{i}-2\hat{j}+3\hat{k})$

$= 3 - 6 - 6 = 9$

(c) $\vec{a}\times(\vec{b}+\vec{c})$

$=(3\hat{i}+3\hat{j}-2\hat{k})\times(\hat{i}-2\hat{j}+3\hat{k})$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 3 & -2\\ 1 & -2 & 3\end{vmatrix}$$

$=\hat{i}(9-4)-\hat{j}(9+2)+\hat{k}(-6-3)$

$=5\hat{i}-11\hat{j}-9\hat{k}$

6. Let $\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$

Given $|\vec{B}|=8$

Given $\vec{A}$ lies along X axis $\Rightarrow \vec{A}=A_x\hat{i}$

Also $\vec{A}+\vec{B}=\vec{C}$ it lies along y axis and $|\vec{C}|=2|\vec{A}|$

Let $\vec{C}=Cy\,\hat{j}$

Hence $|\vec{C}|=C_y=2|\vec{A}|=2A_x$

i.e $Cy = 2A_x$ .....(1)

$\vec{A}+\vec{B}=(A_x+B_x)\hat{i}+B_y\hat{j}+B_z\hat{k}$

$\vec{C}=C_y\hat{j}=2A_x\hat{j}$ $(\because C_y=2A_x)$

$\Rightarrow 2A\times\hat{j}=(A_x+B_x)\hat{i}+B_y\hat{j}+B_z\hat{k}$ on comparing

$A_x+B_x=0$ $(\leftarrow \text{from}\,\hat{i})$

$\Rightarrow A_x=-B_x$ ....(2)

$*\,2A_x=B_y$ ......(3) (from $\hat{j}$)

*0 = B_z ....(4) (from $\hat{k}$)

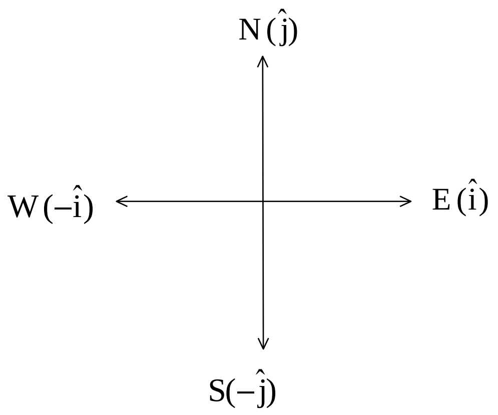
As $|\vec{B}|=\sqrt{B_x^2+B_y^2+B_z^2}$

$8=\sqrt{(-Ax)^2+(2Ax)^2+(0)^2}$

8 = $\sqrt{5A_x^2}=A_x\sqrt{5}$

$\Rightarrow A_x=\dfrac{8}{\sqrt{5}}$

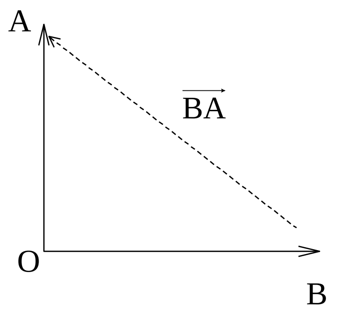
7.



$\overrightarrow{OA}=120\text{km North}=120\hat{j}$

$\overrightarrow{OB}=100\text{ km due east }=100\hat{i}$

$\overrightarrow{BA}=?$



Using Δ law $\overrightarrow{OB}+\overrightarrow{BA}=\overrightarrow{OA}$

$\Rightarrow\overrightarrow{BA}=\overrightarrow{OA}-\overrightarrow{OB}$

$=120\hat{j}-100\hat{i}$

8. $\vec{A}-\vec{B}=2\vec{C}$ …(1)

$\vec{A}+\vec{B}=4\vec{C}$ ….(2)

*equation (1) and (2)

$\vec{A}-\vec{B}+\vec{A}+\vec{B}=2\vec{C}+4\vec{C}$

$2\vec{A}=6\vec{C}\Rightarrow\vec{A}=3\vec{C}$

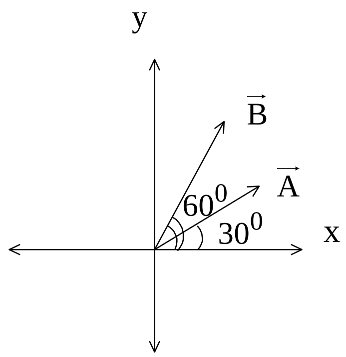
ie $\vec{A}=3(3\hat{i}-4\hat{j})=9\hat{i}+12\hat{j}$

$\vec{A}+\vec{B}-(\vec{A}-\vec{B})=4\vec{C}-2\vec{C}$

$\Rightarrow 2\vec{B}=2\vec{C}$

$\Rightarrow \vec{B}=\vec{C}=3\hat{i}+4\hat{j}$

9. $|\vec{A}|=|\vec{B}|=10$



a $\Rightarrow A_x=10\cos 30\hat{i}=5\sqrt{3}\hat{i}$

$A_y=10\cos 60\hat{j}=5\hat{j}$

$\Rightarrow B_x=10\cos 60\hat{i}=5\hat{i}$

$B_y=10\cos 30\hat{j}=5\sqrt{3}\hat{j}$

$\Rightarrow \vec{A}+\vec{B}=5\left(\sqrt{3}+1\right)\hat{i}+5\left(1+\sqrt{3}\right)\hat{j}$

$=5\left(\sqrt{3}+1\right)\left(\hat{i}+\hat{j}\right)$

10. $|\vec{v}|=50m/s$

Direction of velocity is along $\overrightarrow{AB}$

$\Rightarrow \vec{V}=|\vec{V}|\,AB$

$\overrightarrow{AB}=7\hat{i}+24\hat{j}\Rightarrow|\overrightarrow{AB}|=\sqrt{7^2+(24)^2}=25$

$\Rightarrow \vec{V}=50\,\dfrac{(7\hat{i}+24\hat{j})}{25}$

$=14\hat{i}+48\hat{j}$

11. $|\vec{A}|=A, |\vec{B}|=B$ and let $A>B$

$\Rightarrow A-B=17$ ….(1) given opposite direction

If $\vec{A}$ is $\perp^r$ to $\vec{B}$ and $\vec{R}=\vec{A}+\vec{B}$ then $|\vec{R}|=25$

$|\vec{R}|=\sqrt{A^2+B^2+2AB\cos 90}$

$25=\sqrt{A^2+B^2}\Rightarrow A^2+B^2=(25)^2=625$

Solve (1) and (2)

12. (A) No $\rightarrow$ eg current

(B) (i) No $\rightarrow$for two vectors resultant to be zero they should be equal in magnitude and opposite in direction

(ii) yes $\rightarrow$3 vectors can add to zero. They need not have same magnitude

$$\left.\begin{array}{l} A = 3\hat{i} \\ B = -2\hat{i} \\ C = -\hat{i} \end{array}\right\} \vec{A}+\vec{B}+\vec{C}=0$$

(C) To solve such question assume some magnitude of vectors.

let $|\vec{A}|=3, |\vec{B}|=4$

As $\vec{R}=\vec{A}+\vec{B}$

$\Rightarrow 1 = |A-B| \le |\vec{R}| \le A+B = 7$

So $|\vec{R}|$ can be $1 \le |\vec{A}|$ and $|\vec{B}|$

(D) $\vec{c}=\vec{a}+\vec{b}$

$$\Rightarrow \underset{\text{when } \theta=180^0}{\underset{\uparrow}{|a-b|}} \le |\vec{c}| \le \underset{\substack{\theta=0^0 \\ \text{between } \vec{A} \text{ and } \vec{B}}}{\underset{\uparrow}{a+b}}$$

So if $|\vec{C}|=|\vec{A}|+|\vec{B}| \Rightarrow \vec{A}$ is $\parallel$ to $\vec{B}$ (collinear)

(E) $|\vec{a}| \ne 0$ and $|b| \ne 0$

Given $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$

$\sqrt{a^2 b^2 + 2ab\cos\theta} = \sqrt{a^2+b^2-2ab\cos\theta}$

$\Rightarrow a^2+b^2+2ab\cos\theta = a^2+b^2-2ab\cos\theta$

$\Rightarrow 4ab\cos\theta = 0$

$\Rightarrow \cos\theta = 0$ as $a \ne 0$ and $b \ne 0$

$\Rightarrow \theta = 90^0$

(F) Time t doesn't have direction

(G) $\underset{\text{Vector}}{\vec{a}\times\vec{b}} = \underset{\text{Scalar}}{\vec{a}.\vec{b}}$

Not possible

(H) No $\rightarrow$ As they only denote direction

13. $|\vec{A}|=2, |\vec{B}|=3, \theta = 60^0$ between $\vec{A}$ and $\vec{B}$

(a) $\vec{A}.\vec{B}=|\vec{A}||\vec{B}|\cos(\theta=60^0)$

$= (2)(3)\frac{1}{2}$

$= 3$

(b) $|\vec{A}\times\vec{B}|=|\vec{A}||\vec{B}|\sin\theta$

$= (2)(3)\frac{\sqrt{3}}{2}$

$= 3\sqrt{3}$

14. (2.00)

Given: $\left|\vec{A}+\vec{B}\right| = \sqrt{3}\left|\vec{A}-\vec{B}\right|$

$\therefore \left|a\,\hat{i} + a\cos\omega t\,\hat{i} + a\sin\omega t\,\hat{j}\right| = \sqrt{3}\left|a\,\hat{i} - a\cos\omega t\,\hat{i} - a\sin\omega t\,\hat{j}\right|$

$\Rightarrow \left|(1+\cos\omega t)\,\hat{i} + \sin\omega t\,\hat{j}\right| = \sqrt{3}\left|(1-\cos\omega t)\,\hat{i} - \sin\omega t\,\hat{j}\right|$

$\sqrt{2+2\cos\omega t} = \sqrt{3}\sqrt{2-2\cos\omega t}$

$\therefore 1+\cos\omega t = 3(1-\cos\omega t)$

$\Rightarrow 4\cos\omega t = 2$ $\qquad \therefore \cos\omega t = \frac{1}{2}$ or, $\omega t = \frac{\pi}{3}$

$\therefore \frac{\pi}{6}\times\tau = \frac{\pi}{3}$ $\qquad \therefore \tau = 2.00\,\text{seconds}$