

# Inchapter Exercise - I - Rahul Rai (RRP) <sup>①</sup>

$$\begin{aligned} 6. \quad & \frac{d}{dx} [(4x^2 - 7x + 5) \cdot \sec x] \\ &= \frac{d}{dx} (4x^2 - 7x + 5) \cdot \sec x + (4x^2 - 7x + 5) \frac{d}{dx} (\sec x) \\ &= (8x - 7) \sec x + (4x^2 - 7x + 5) \sec x \tan x \end{aligned}$$

$$\begin{aligned} 7. \quad & \frac{d}{dx} [x^4 (5 \sin x - 3 \cos x)] \\ &= \frac{d}{dx} (x^4) \cdot (5 \sin x - 3 \cos x) + x^4 \cdot \frac{d}{dx} (5 \sin x - 3 \cos x) \\ &= 4x^3 (5 \sin x - 3 \cos x) + x^4 (5 \cos x + 3 \sin x) \end{aligned}$$

$$\begin{aligned} 8. \quad \frac{d}{dx} \frac{\sqrt{x+1}}{\sqrt{x-1}} &= \frac{\frac{d}{dx}(\sqrt{x+1}) \cdot (\sqrt{x-1}) - (\sqrt{x+1}) \frac{d}{dx}(\sqrt{x-1})}{(\sqrt{x-1})^2} \\ &= \frac{\frac{1}{2\sqrt{x}} (\sqrt{x-1}) - \frac{1}{2\sqrt{x}} (\sqrt{x+1})}{(\sqrt{x-1})^2} \\ &= \frac{1}{2\sqrt{x}} \cdot \frac{(\sqrt{x-1} - \sqrt{x+1})}{(\sqrt{x-1})^2} = -\frac{1}{\sqrt{x} (\sqrt{x-1})^2} \end{aligned}$$

$$\begin{aligned} 9. \quad \frac{d}{dx} \cdot \frac{x \ln x}{e^x} &= \frac{\ln x}{e^x} \frac{d}{dx} (x) + x \ln x \frac{d}{dx} (e^{-x}) + \frac{x}{e^x} \frac{d}{dx} (\ln x) \\ &= \frac{\ln x}{e^x} - \frac{x \ln x}{e^x} + \frac{1}{e^x} \\ &= \frac{1 + \ln x - x \ln x}{e^x} \end{aligned}$$

10. (i)  $y = \frac{x^3}{3} - x$

Maxima or minima exist where  $\frac{dy}{dx} = 0$ .

$$\frac{dy}{dx} = x^2 - 1 = 0$$

$$(x-1)(x+1) = 0$$

$$\text{at } x = 1, -1$$

~~at  $x = 1, -1$~~

(ii)  $y = \sin x$   $x \in (0, 2\pi)$

$$\frac{dy}{dx} = \cos x$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

11. (i)  $y = x^3 - 3x + 10$

$$\frac{dy}{dx} = 0 \Rightarrow$$

$$3x^2 - 3 = 0 \text{ or } 3(x-1)(x+1) = 0$$

$$x = \pm 1$$

$$\frac{d^2y}{dx^2} = 6x$$

$$\frac{d^2y}{dx^2} > 0 \text{ at } x = +1 \text{ (minima)}$$

$$\frac{d^2y}{dx^2} < 0 \text{ at } x = -1 \text{ (maxima)}$$

Local maxima at  $x = -1, y = 12$

Local minima at  $x = +1, y = 8$ .

(ii)  $y = \frac{x^2}{2} + \frac{1}{x}$ ,  $\frac{dy}{dx} = x - \frac{1}{x^2}$   $\frac{dy}{dx} = 0 \Rightarrow \frac{x^3 - 1}{x^2} = 0$   
 or  $x = \underline{1}$ .

$$\frac{d^2y}{dx^2} = 1 + \frac{2}{x^3}$$

$$\frac{d^2y}{dx^2} > 0 \text{ at } x = 1.$$

Local minimum value is  $\frac{3}{2}$   
 at  $x = 1$ .

# Inchapter Exercise II

1.  $\int (x^5 + \frac{2}{x^2} - \frac{1}{x} - \frac{4}{\sqrt{x}} + 10) dx$   
 $= \int x^5 dx + 2 \int x^{-2} dx - \int x^{-1} dx - 4 \int x^{-1/2} dx + 10 \int dx$   
 $= \frac{x^6}{6} - \frac{2}{x} - \ln x + 8\sqrt{x} + 10x + C$

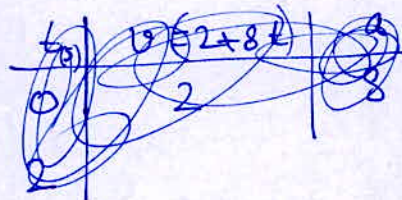
~~2.  $\int (7e^x + 4\sin x - \frac{9}{x^3} + e^x) dx$   
 $= 7 \int e^x dx + 4 \int \sin x dx - 9 \int x^{-3} dx + \int e^x dx$   
 $= 7e^x +$~~

2.  $\int (7e^x + 4\sin x - \frac{9}{x^3} + e) dx$   
 $= 7 \int e^x dx + 4 \int \sin x dx - 9 \int x^{-3} dx + e \int dx$   
 $= 7e^x - 4\cos x + \frac{9}{2x^2} + ex + C$

3.  $\int_1^5 (3+2t) dt = 36$   
 $= [3t + t^2]_1^5 = (3 \times 5 + 5^2) - (3 \times 1 + 1^2)$   
 $= 40 - 4 = 36$

4.  $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2\sin x - \cos x) dx$   
 $= [-2\cos x - \sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = [2\cos x + \sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$   
 $= (\sqrt{2} + \frac{1}{\sqrt{2}}) - 1 = \frac{3}{\sqrt{2}} - 1$

5.  $s = 2t + 4t^2$   
 $v = \frac{ds}{dt} = 2 + 8t$   
 $a = \frac{dv}{dt} = 8$



$t(s)$	$v (= 2+8t)$ in m/s	$a$ (in $\frac{m}{s^2}$ )
0	2	8
2	18	8
10	82	8

6.  $v = 3t^2$

$$a = \frac{dv}{dt} = 6t$$

$t$	$a (m/s^2)$
0	0
2	12
10	60

$$v = \frac{ds}{dt}$$

$$\frac{ds}{dt} = 3t^2$$

$$ds = 3t^2 dt$$

$$\int_0^s ds = \int_0^t 3t^2 dt$$

$$s = t^3$$

$t(s)$	$s(m)$
0	0
2	8
10	1000

7.  $v = u + at$

$$\frac{ds}{dt} = v$$

$$\text{or } ds = v dt$$

$$\text{or } \int_0^s ds = \int_0^t (u + at) dt$$

$$s = u \int_0^t dt + a \int_0^t t dt$$

$$s = ut + \frac{1}{2} at^2$$

8.  $v \frac{dv}{ds} = a$  or  $\int_u^v v dv = \int_0^s a ds$

$$\frac{v^2 - u^2}{2} = as$$

$$\text{or } v^2 = u^2 + 2as$$

# Subjective Questions

(5)

$$1. \quad y = (x^2 - 3x + 3)(x^2 + 2x - 1)$$

$$\frac{dy}{dx} = \left[ \frac{d}{dx} (x^2 - 3x + 3) \right] [x^2 + 2x - 1] + (x^2 - 3x + 3) \frac{d}{dx} (x^2 + 2x - 1)$$

$$= (2x - 3)(x^2 + 2x - 1) + (x^2 - 3x + 3)(2x + 2)$$

$$= 2x^3 + 4x^2 - 2x - 3x^2 - 6x + 3 + 2x^3 - 6x^2 + 6x + 2x^2 - 6x + 6$$

$$= 4x^3 - 3x^2 - 8x + 9$$

$$2. \quad y = \frac{x+1}{x-1}; \quad \frac{dy}{dx} = \frac{(x-1) \frac{d}{dx} (x+1) - (x+1) \frac{d}{dx} (x-1)}{(x-1)^2}$$

$$= \frac{x-1 - x-1}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

$$3. \quad y = \frac{x}{x^2+1}; \quad \frac{dy}{dx} = \frac{(x^2+1) \frac{d}{dx} x - x \frac{d}{dx} (x^2+1)}{(x^2+1)^2}$$

$$= \frac{x^2+1 - 2x^2}{(x^2+1)^2} = \frac{(1-x^2)}{(1+x^2)^2}$$

$$4. \quad y = \frac{ax+b}{cx+d}; \quad \frac{dy}{dx} = \frac{(cx+d) \frac{d}{dx} (ax+b) - (ax+b) \frac{d}{dx} (cx+d)}{(cx+d)^2}$$

$$= \frac{a(cx+d) - c(ax+b)}{(cx+d)^2}$$

$$= \frac{acx + ad - acx - bc}{(cx+d)^2} = \frac{ad - bc}{(cx+d)^2}$$

Ques 5.  $z = \frac{x^2+1}{3(x^2-1)} + (x^2+1)(1-x)$

$$\frac{dz}{dx} = \frac{\left[\frac{d}{dx}(x^2+1)\right](x^2-1) - (x^2+1)\frac{d}{dx}(x^2-1)}{3(x^2-1)^2} + (x^2+1)\frac{d}{dx}(1-x) + (1-x)\frac{d}{dx}(x^2+1)$$

$$= \frac{(x^2-1)2x - (x^2+1)\cdot 2x}{3(x^2-1)^2} + (x^2+1)(-1) + (1-x)\cdot 2x$$

$$= \frac{2x(x^2-1-x^2-1)}{3(x^2-1)^2} - x^2-1 + 2x-2x^2$$

$$= \frac{-4x}{3(x^2-1)^2} - 3x^2 + 2x - 1$$

5.  $z = \frac{x^2+1}{3(x^2-1)} + (x^2-1)(1-x)$

$$\frac{dz}{dx} = \frac{1}{3} \frac{(x^2-1)\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x^2-1)}{(x^2-1)^2} + (x^2-1)\frac{d}{dx}(1-x) + (1-x)\frac{d}{dx}(x^2-1)$$

$$= \frac{2x(x^2-1) - 2x(x^2+1)}{3(x^2-1)^2} - (x^2-1) + 2x(1-x)$$

$$= \frac{2x(x^2-1-x^2-1)}{3(x^2-1)^2} - x^2+1 + 2x-2x^2$$

$$= -\frac{4x}{3(x^2-1)^2} - 3x^2 + 1 + 2x$$

$$\begin{aligned}
 6. \quad & \frac{d}{dx} \left( \frac{1-x^3}{1+x^3} \right) \\
 &= \frac{(1+x^3) \frac{d}{dx} (1-x^3) - (1-x^3) \frac{d}{dx} (1+x^3)}{(1+x^3)^2} \\
 &= \frac{-3x^2(1+x^3) - 3x^2(1-x^3)}{(1+x^3)^2} \\
 &= \frac{-3x^2(1+x^3+1-x^3)}{(1+x^3)^2} \\
 &= \frac{-6x^2}{(1+x^3)^2}
 \end{aligned}$$

$$7. \quad y = \frac{2}{x^3-1} \quad \text{putting } t = x^3-1, \quad y = \frac{2}{t}$$

$$\boxed{\frac{dy}{dx} = \frac{2}{(x^3-1)^2}}$$

$$\frac{dt}{dx} = 3x^2, \quad \frac{dy}{dt} = \frac{-2}{t^2}$$

On applying chain rule,

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= \frac{-2}{t^2} \cdot 3x^2 = \frac{-6x^2}{(x^3-1)^2}$$

$$\begin{aligned}
 8. \quad & \frac{dy}{dx} = \frac{1}{a^3-3} \cdot \frac{d}{dx} (x^2-x+1) \\
 &= \frac{2x-1}{(a^3-3)}
 \end{aligned}$$

$$9. \quad \frac{dy}{dx} = \frac{1}{\sqrt{\pi}} \frac{d}{dx} (1-x^3)$$

$$= \frac{-3x^2}{\pi}$$

$$10. \quad \frac{d}{dx} (\sin x + \cos x) = \frac{d}{dx} \sin x + \frac{d}{dx} \cos x$$

$$= \cos x - \sin x$$

$$11. \quad \frac{dy}{dx} = x \cdot \frac{d}{dx} \sin x + \sin x \cdot \frac{d}{dx} x + \frac{d}{dx} \cos x$$

$$= x \cos x + \cancel{\sin x} - \cancel{\sin x}$$

$$= x \cos x$$

$$12. \quad \frac{d}{dx} (\cos^2 x) = 2 \cos x \cdot \frac{d}{dx} (\cos x)$$

$$= -2 \cos x \cdot \sin x$$

$$13. \quad y = 3 \sin^2 x - \sin^3 x$$

$$\frac{dy}{dx} = 3 \cdot 2 \sin x \cdot \frac{d}{dx} (\sin x) - 3 \sin^2 x \cdot \frac{d}{dx} (\sin x)$$

$$= 6 \sin x \cdot \cos x - 3 \sin^2 x \cdot \cos x$$

$$= 3 \sin x \cos x (2 - \sin x)$$

$$14. \quad \frac{d}{dx} (x \ln x) = \ln x \cdot \frac{d}{dx} x + x \cdot \frac{d}{dx} \ln x$$

$$= \ln x + x \cdot \frac{1}{x} = 1 + \ln x$$

$$15. \quad y = \ln^2 x$$

putting  $\ln x = t$ ;  $\frac{dt}{dx} = \frac{1}{x}$

$$y = t^2; \quad \frac{dy}{dt} = 2t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= 2t \cdot \frac{1}{x}$$

$$= \frac{2 \ln x}{x}$$



$$16. \quad y = \ln x^2$$

$$\text{Let } t = x^2, \quad \frac{dt}{dx} = 2x$$

$$y = \ln t \quad \frac{dy}{dt} = \frac{1}{t}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{1}{t} \cdot 2x = \frac{2x}{x^2} = \frac{2}{x}$$

$$17. \quad \int \sqrt{x} \, dx \\ = \int x^{1/2} \, dx = \frac{x^{1+1/2}}{\frac{1}{2}+1} + C = \frac{2}{3} x^{3/2} + C$$

$$18. \quad \int \sqrt[m]{x^n} \, dx = \int x^{n/m} \, dx \\ = \frac{x^{\frac{n}{m}+1}}{\frac{n}{m}+1} + C = \frac{m}{m+n} \cdot x^{\frac{n+m}{m}} + C$$

$$19. \quad \int \frac{dx}{x^2} = \int x^{-2} \, dx = \frac{x^{-2+1}}{-2+1} + C \\ = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$$

$$20. \quad \int \frac{dx}{2\sqrt{x}} = \frac{1}{2} \int x^{-1/2} \, dx = \frac{1}{2} \cdot \frac{x^{-1/2+1}}{-1/2+1} + C \\ = \sqrt{x} + C$$



# Level-I

(11)

1.  $v = at + \frac{b}{t+c}$

$$\left[ \begin{array}{l} [at] = [v] \\ [a] T = LT^{-1} \\ [a] = LT^{-2} \end{array} \right. \quad \left[ \begin{array}{l} [t] = [c] \\ \Rightarrow [c] = T \end{array} \right. \quad \left[ \begin{array}{l} \frac{[b]}{[t+c]} = LT^{-1} \\ \frac{[b]}{T} = LT^{-1} \\ [b] = L \end{array} \right.$$

2.  $[F] = MLT^{-2}$

V 
$$U = (10g)(10cm)(0.1s)^{-2}$$

$$= 10^{-2} \text{ kg} \times 0.1 \text{ m} \times (0.01)^{-2} \text{ s}^{-2} = 10^{-2} \times 10^{-1} \times 10^4 \text{ kg m/s}^2$$

$$= 10 \text{ N}$$
~~$$= 10^{-5} \text{ kg m/s}^2$$~~

3.  $P = P_0 e^{(-\alpha t^2)}$

$$[\alpha t^2] = M^0 L^0 T^0$$

$$[\alpha] T^2 = T^0$$

$$[\alpha] = T^{-2}$$

5.  $S_{nth} = u + \frac{a}{2} [2n-1]$

$S_{nth}$  is the displacement in  $n^{\text{th}}$  second.

So  $[S_{nth}] = L$

6.  $[F] = MLT^{-2}$

$$T^{-2} = F M^{-1} L^{-1}$$

$$T = F^{-1/2} M^{1/2} L^{1/2}$$

7. Intensity =  $\frac{\text{Power}}{\text{Area}}$

$$[I] = \frac{ML^2T^{-3}}{L^2} = \text{[scribble]}$$

$$= M^1T^{-3}$$

$$\boxed{\begin{matrix} \rho = \rho V \\ c = \frac{a}{V} \end{matrix}}$$

8.  $F = \frac{\alpha}{\text{Density} + \beta^3}$

$$[\text{Density}] = [\beta]^3$$

$$ML^{-3} = [\beta]^3$$

$$\Rightarrow [\beta] = M^{1/3} L^{-1}$$

$$[\alpha] = [F][\text{Density}]$$

$$= MLT^{-2} \cdot ML^{-3}$$

$$= M^2 L^{-2} T^{-2}$$

9.  $[P] = ML^{-1}T^{-2}$

14. Impulse = change in momentum.  
 $\vec{J} = \Delta \vec{p}$

16.  $F = p t^{-1} + \alpha t$

$$[p t^{-1}] = [F]$$

$$\frac{[P]}{T} = MLT^{-2}$$

$$[P] = MLT^{-1} = [\text{momentum}]$$

~~$\rho V = \rho$   
 $[\rho] = \frac{MLT^{-2}}{LT^{-1}L}$   
 $= \frac{ML^{-1}T^{-1}}{L}$   
 $= M^1 L^{-2} T^{-1}$   
 $[F] = \frac{MLT^{-2}}{T}$   
 $= ML^{-1}T^{-2}$   
 $MT^{-1} L^{-1} T^{-1} = V$   
 $MLT^{-3} I^{-1} = [V]$~~

18.  $[Calorie] = [Energy] = ML^2 T^{-2}$  (13)

$$1C = 4.2 \text{ Kg m}^2 / \text{s}^2, \quad 1J = 1\text{Kg} \times (1\text{m})^2 \times (1\text{s})^{-2}$$

$$1U = \frac{(\alpha \text{ Kg}) \times (\beta \text{ m})^2}{(\gamma \text{ s})^2} = \frac{\alpha \beta^2}{\gamma^2} \frac{\text{Kg m}^2}{\text{s}^2} = \frac{\alpha \beta^2}{\gamma^2} \text{ J}$$

$$\text{or } 1J = \frac{\gamma^2}{\alpha \beta^2} U$$

$$\text{So } 1C = \frac{4.2 \gamma^2}{\alpha \beta^2} U$$

19.  $[\eta] = ML^{-1} T^{-1}$

$$1 \text{ poise} = \frac{\text{gm}}{\text{cm} \cdot \text{s}} \Rightarrow \frac{\text{gm}}{\text{cm} \cdot \text{s}} = x \frac{(2 \text{ kg})}{(2 \text{ m})(4 \text{ s})}$$

$$\text{or } \frac{\text{gm}}{\text{cm} \cdot \text{s}} = \frac{x}{4} \cdot \frac{1000 \text{ gm}}{100 \text{ cm} \cdot \text{s}}$$

$$\text{or } x = 0.4$$

20. We know  $E = \frac{hc}{\lambda}$  or  $h = \frac{E\lambda}{c}$

$$\text{So } \frac{h}{e} = \frac{E\lambda}{ec} \text{ or } \left[ \frac{h}{e} \right] = \frac{ML^2 T^{-2} \cdot L}{I T \cdot LT^{-1}} = ML^2 T^{-2} I^{-1}$$

which is same as dimension of magnetic flux.

21. Since  $BLV = V$  (I)  $B \rightarrow$  magnetic field  $E = qV$  (III)

$\& V = \frac{q}{C}$  (II)  $L \rightarrow$  length  $E \rightarrow$  Energy

$v \rightarrow$  speed

$V \rightarrow$  potential difference

$q \rightarrow$  charge

$C \rightarrow$  capacitance

$$B^2 L^2 C = (BL)^2 \cdot \frac{q}{V} = \frac{V^2}{v^2} \cdot \frac{q}{V}$$

$$= \frac{Vq}{v^2} = \frac{\text{Energy}}{v^2} \Rightarrow [B^2 L^2 C] = \frac{ML^2 T^{-2}}{L^2 T^{-2}} = M$$

23.  $y = a \cos(\omega t - kx)$

$$[\omega t] = [kx]$$

$$[\omega] T = [k] L = M^0 L^0 T^0$$

$$[k] = L^{-1}$$

24.  $x(t) = \frac{v_0}{\alpha} (1 - e^{-\alpha t})$

$$\Rightarrow \left[ \frac{v_0}{\alpha} \right] = L \quad \& \quad [\alpha t] = M^0 L^0 T^0$$

~~$$\frac{L}{T} = [\alpha] L \quad \& \quad [\alpha] T = M^0 L^0 T^0$$~~

$$[\alpha] = T^{-1}$$

Since,  $[\alpha] = T^{-1}$

$$\& \quad [v_0] = [\alpha][L]$$

$$[v_0] = L T^{-1}$$

25.  $A = B + \frac{C}{D+E}$

$$\Rightarrow [A] = [B] = \left[ \frac{C}{D+E} \right] \quad \& \quad [D] = [E]$$

$$\text{So } [A] = M^0 L T^{-1} \quad \& \quad [D] = \frac{[C]}{[B]} = \frac{M^0 L T^0}{M^0 L T^{-1}} = T$$

$$[E] = T.$$

26.  $\int \frac{dv}{\sqrt{2bv - v^2}} = a^n \sin^{-1} \left( \frac{v}{a} - 1 \right)$

$$[x] = [a] = L$$

$$\& \quad [a]^n = \frac{[dv]}{[v]} = M^0 L^0 T^{-1}$$

$$L^n = M^0 L^0 T^0$$

$$\Rightarrow n = 0$$

$$28. E \propto h^a c^b \lambda^d$$

$$\text{or } E = k h^a c^b \lambda^d$$

$$[E] = [h]^a [c]^b [\lambda]^d$$

$$E = \frac{hc}{\lambda}$$

$$[h] = \frac{ML^2T^{-2} \cdot L}{LT^{-1}} = ML^2T^{-1} \quad (15)$$

$$ML^2T^{-2} = [ML^2T^{-1}]^a [LT^{-1}]^b L^d$$

$$ML^2T^{-2} = M^a L^{2a+b+d} T^{-a-b}$$

$$\Rightarrow a=1, \quad 2a+b+d=2, \quad a+b=2$$

$$\therefore b=1 \quad \& \quad d=-1$$

$$29. \quad d = 10.1 \text{ cm.}$$

Least count is 0.1 cm

$$\text{So uncertainty} = \frac{0.1}{10.1} \times 100 = \pm 1\%$$

$$30. \quad T = 25 \text{ s.}$$

$$\Delta T (\text{error in } T) = 0.2 \text{ s}$$

$$\% \text{ error} = \frac{0.2}{25} \times 100 = 0.8\%$$

## Level-II

1.  $K \propto p^a m^b$   
 $[K] = [p]^a [m]^b$   
 $ML^2 T^{-2} = [MLT^{-1}]^a M^b$   
 $ML^2 T^{-2} = M^{a+b} L^a T^{-a}$

$a+b=1, a=2$  so  $b=-1$

3.  $F \propto \rho^a v^b A^c$   
 $[F] = [\rho]^a [v]^b [A]^c$   
 $MLT^{-2} = [ML^{-3}]^a [LT^{-1}]^b [L^2]^c$   
 $MLT^{-2} = M^a L^{-3a+b+2c} T^{-b}$

$a=1, b=2, -3a+b+2c=1$   
 which means  $c=1$

so  $F \propto \rho v^2 A$

4.  $r_2 = 2.12 \text{ cm}$   
 $A = \pi r_2^2$   
 $= \pi (2.12)(2.12)$   
 $= 14.1 \text{ cm}^2$

[must have same no. of significant digit - Product Rule]

5.  $[F] = MLT^{-2}$   
 $[L] = FM^{-1}T^2$

8.  $U = \frac{\alpha \sqrt{y}}{y+\beta}$  From here we can conclude that  $[y] = [\beta] = L$   
 and  $[U] = \frac{[\alpha] [y]^{1/2}}{[y]}$

$ML^2 T^{-2} = [\alpha] L^{-1/2}$  or  $[\alpha] = ML^{5/2} T^{-2}$

So  $[\alpha \cdot \beta] = [\alpha] [\beta] = ML^{5/2} T^{-2} \cdot L = ML^{7/2} T^{-2}$



9. (a)  $t = 2\pi \sqrt{\frac{ml^2}{E}}$

$[LHS] = T$  ;  $[RHS] = [2\pi] \left[ \frac{ml^2}{E} \right]^{1/2}$   
 $= \left[ \frac{ML^2}{ML^2T^{-2}} \right]^{1/2} = T.$

Since  $[LHS] = [RHS]$ , it can be dimensionally correct.

(b)  $[LHS] = L$  ,  $[RHS] = \frac{ML\cancel{x}}{M\cancel{x}^2} = L$  [correct]

(c)  $[RHS] = \left[ \frac{ML^2}{MLT^{-2}} \right]^{1/2} = L^{1/2} T \neq [LHS]$  [incorrect]

(d)  $[LHS] = LT^{-2}$  ;  $[RHS] = \frac{L(LT^{-1})^2}{M^{-1}L^3T^{-2} \cdot M} = \frac{L^3T^{-2}}{L^3T^{-2}} = M^0L^0T^0$

Since  $[LHS] \neq [RHS]$  ; incorrect.

11.  $\frac{d}{dx} \left( \frac{1}{\sqrt{x}} \right) = \frac{d}{dx} x^{-1/2} = \left( -\frac{1}{2} \right) x^{-1/2-1} = -\frac{1}{2} x^{-3/2}$

12.  $\frac{d}{dx} (ax+b)^{-2} = (-2) \cdot (ax+b)^{-3} \cdot \frac{d}{dx} (ax+b) = -2a (ax+b)^{-3}$

13.  $\frac{d}{dx} (x^3 + x^{-3} + 8) = 3x^2 - \frac{3}{x^4}$

14.  $\frac{d}{dx} \sin(x^3) = \cos(x^3) \cdot \frac{d}{dx} (x^3) = 3x^2 \cos(x^3)$

15.  $\frac{d}{dx} (4x^3-5)^{1/2} = \frac{1}{2} (4x^3-5)^{-1/2} \cdot \frac{d}{dx} (4x^3-5)$   
 $= \frac{1}{2} \cdot (4x^3-5)^{-1/2} \cdot (12x^2)$

$= \frac{6x^2}{\sqrt{4x^3-5}}$

$$16. \frac{d}{dx} \sin(\ln x) = \cos(\ln x) \cdot \frac{d}{dx} \ln x = \frac{\cos(\ln x)}{x}$$

$$17. \frac{d}{dx} (2x^2+1)^{1/2} = \frac{1}{2} \cdot (2x^2+1)^{\frac{1}{2}-1} \cdot \frac{d}{dx} (2x^2+1)$$

$$= \frac{1}{2} \cdot (2x^2+1)^{-1/2} \cdot 4x = 2x (2x^2+1)^{-1/2}$$

$$18. \frac{d}{dx} e^{\sqrt{2x}} = e^{\sqrt{2x}} \cdot \frac{d}{dx} \sqrt{2x} = e^{\sqrt{2x}} \cdot \sqrt{2} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{2x}} e^{\sqrt{2x}}$$

$$19. \frac{d}{dx} (x^4 - 2 \sin x + 3 \cos x)$$

$$= 4x^3 - 2 \cos x - 3 \sin x$$

$$20. \frac{d}{dx} (x^2 \sin x \cdot nx) = \frac{d}{dx} (x^2) (\sin x \cdot nx) + x^2 \cdot \frac{d}{dx} (\sin x) \cdot nx + x^2 \sin x \cdot \frac{d}{dx} (nx)$$

$$= 2x \sin x \cdot nx + x^2 \cdot \cos x \cdot nx + x^2 \sin x \cdot n$$

$$21. \frac{d}{dx} \left( \frac{x^2+1}{x+1} \right) = \frac{(x+1) \frac{d}{dx} (x^2+1) - (x^2+1) \frac{d}{dx} (x+1)}{(x+1)^2} = \frac{2x(x+1) - (x^2+1)}{(x+1)^2}$$

$$= \frac{2x^2+x-x^2-1}{(x+1)^2} = \frac{x^2+x-1}{(x+1)^2}$$

$$22. \frac{dV}{dr} : \frac{d}{dr} \left( \frac{4}{3} \pi r^3 \right) = \frac{4\pi}{3} \frac{d}{dr} (r^3) = \frac{4\pi}{3} \cdot 3r^2 = 4\pi r^2$$

$$23. y = \frac{c^2}{x} \quad \frac{dy}{dx} = \frac{-c^2}{x^2} = \frac{-xy}{x^2} = -\frac{y}{x} \quad [\text{as } c^2 = xy]$$

$$24. \quad x = at^2 \quad y = 2at$$

$$\frac{dx}{dt} = 2at$$

$$\frac{dy}{dt} = 2a$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

$$25. \int x^{5/2} dx = \frac{x^{5/2+1}}{\frac{5}{2}+1} + C = \frac{2}{7} x^{7/2} + C$$

$$25. \int x^{1/5} dx = \frac{x^{1/5+1}}{\frac{1}{5}+1} + C = \frac{x^{6/5}}{6/5} + C = \frac{5}{6} x^{6/5} + C$$

$$26. \int \frac{1}{(ax+b)^2} dx \quad \text{put } ax+b=t \\ dt = a dx \quad \text{or } dx = \frac{dt}{a}$$

$$= \frac{1}{a} \int \frac{dt}{t^2} = \frac{-1}{at} = \frac{-1}{a(ax+b)} + C$$

$$27. \int \sin x \cdot \cos x dx$$

$$\text{putting } \sin x = t; \quad dt = \cos x dx$$

$$I = \int t dt = \frac{t^2}{2} + C = \frac{\sin^2 x}{2} + C$$

or

$$28. \int \frac{x dx}{x^2+a^2} \quad \text{putting } x^2+a^2=t \\ 2x dx = dt$$

$$= \int \frac{dt}{2t} = 2 \ln|t| + C = 2 \ln|x^2+a^2| + C$$

$$29. \int_{-\pi/2}^{\pi/2} \cos x dx = [\sin x]_{-\pi/2}^{\pi/2} = \sin \frac{\pi}{2} - \sin \left(-\frac{\pi}{2}\right) \\ = 1 + 1 = 2$$

$$30. \int_0^{\pi/2} \sqrt{1+\cos x} dx = \sqrt{2} \int_0^{\pi/2} \cos\left(\frac{x}{2}\right) dx = 2\sqrt{2} \left[\sin \frac{x}{2}\right]_0^{\pi/2} = 2$$

$$\begin{aligned}
 31. \quad & \int \sqrt{x} \, dx - \int x\sqrt{x} \, dx \\
 &= \int x^{1/2} \, dx - \int x^{3/2} \, dx \\
 &= \frac{2}{3} x^{3/2} - \frac{2}{5} x^{5/2} + C
 \end{aligned}$$

$$\begin{aligned}
 32. \quad & \int_0^{\pi/2} \sin x \, dx + \int_0^{\pi/2} \cos x \, dx \\
 &= \left[ \sin x - \cos x \right]_0^{\pi/2} = (1 - 0) - (0 - 1) = 2
 \end{aligned}$$

$$\begin{aligned}
 33. \quad f &= \frac{m}{V} & \frac{\Delta m}{m} \times 100 &= 0.3 \quad (\text{Given}) \\
 & & \& \frac{\Delta l}{l} \times 100 &= 0.2.
 \end{aligned}$$

$$f = \frac{m}{l^3}$$

$$\ln f = \ln m - 3 \ln l$$

$$\frac{df}{f} = \frac{dm}{m} - 3 \frac{dl}{l}$$

$$\frac{\Delta f}{f} = \frac{\Delta m}{m} + 3 \frac{\Delta l}{l} \quad [\text{for maximum error}]$$

$$\frac{\Delta f}{f} \times 100 = \frac{\Delta m}{m} \times 100 + 3 \frac{\Delta l}{l} \times 100$$

$$= 0.3 + 3(0.2) = 0.9$$

$$34. \quad KE = \frac{1}{2} m v^2 \quad \frac{\Delta v}{v} \times 100 = 40 \Rightarrow \Delta v = 0.4 v$$

$$\Delta KE = \frac{m}{2} [(v + \Delta v)^2 - v^2] = \frac{m}{2} v^2 (1.4^2 - 1) = 0.96 \frac{m}{2} v^2$$

$$\Delta KE = 0.96 KE ; \& \frac{\Delta KE}{KE} = 0.96 \text{ or } \frac{\Delta KE}{KE} \times 100 = 96$$

123.  $T_1 = 2.63 \text{ s}$

$$T_2 = 2.56 \text{ s}$$

$$T_3 = 2.42 \text{ s}$$

$$T_4 = 2.71 \text{ s}$$

$$T_5 = 2.80 \text{ s}$$

$$T_{\text{mean}} = \frac{T_1 + T_2 + T_3 + T_4 + T_5}{5} = 2.62 \text{ s}$$

$$\Delta T_1 = |T_1 - T_{\text{mean}}| = 0.01 \text{ s}$$

$$\Delta T_2 = 0.06 \text{ s}$$

$$\Delta T_3 = 0.20 \text{ s}$$

$$\Delta T_4 = 0.09 \text{ s}$$

$$\Delta T_5 = 0.18 \text{ s}$$

$$\Delta T_{\text{mean}} = \frac{\Delta T_1 + \Delta T_2 + \Delta T_3 + \Delta T_4 + \Delta T_5}{5} = 0.11 \text{ s}$$

124.  $\rho = \frac{m}{V}$ ,  $V = \pi r^2 l$

$$\rho = \frac{m}{\pi r^2 l}$$

$$\ln \rho = \ln m - \ln\left(\frac{1}{\pi}\right) - 2 \ln r - \ln l$$

$$\frac{d\rho}{\rho} = \frac{dm}{m} - 2 \frac{dr}{r} - \frac{dl}{l}$$

$$\text{or } \frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + 2 \frac{\Delta r}{r} + \frac{\Delta l}{l} \quad [\text{For maximum error}]$$

$$\frac{\Delta \rho}{\rho} \times 100 = \frac{0.003}{0.3} \times 100 + 2 \times \frac{0.005}{0.5} \times 100 + \frac{0.006}{0.6} \times 100$$

$$= 4 \%$$

$$\text{or } -0.1 + \frac{\Delta P}{P} + \frac{\Delta P}{P} (-0.1) = 0$$

$$\frac{\Delta P}{P} (1 - 0.1) = 0.1$$

$$0.9 \frac{\Delta P}{P} = 0.1$$

$$\text{or } \frac{\Delta P}{P} \times 100 = \frac{100}{9} = 11.1\%$$

120.

$$KE = \frac{1}{2} m v^2$$

$$\ln(KE) = \ln\left(\frac{1}{2}\right) + \ln m + 2 \ln v$$

$$\frac{d(KE)}{KE} = \frac{dm}{m} + 2 \frac{dv}{v}$$

$$\text{or } \frac{\Delta KE}{KE} = \frac{\Delta m}{m} + 2 \frac{\Delta v}{v}$$

$$\frac{\Delta KE}{KE} \times 100 = \frac{\Delta m}{m} \times 100 + 2 \frac{\Delta v}{v} \times 100$$

$$= 2 + 2 \times 3$$

$$= 8\%$$

121.

$$V = \frac{4}{3} \pi R^3$$

$$\ln V = \ln\left(\frac{4\pi}{3}\right) + 3 \ln R$$

$$\frac{dV}{V} = 3 \frac{dR}{R}$$

$$\frac{\Delta V}{V} = 3 \frac{\Delta R}{R}$$

$$\frac{\Delta V}{V} \times 100 = 3 \frac{\Delta R}{R} \times 100 = 6\%$$

$$[\text{Given } \frac{\Delta R}{R} \times 100 = 2]$$

17.  $a^3 = 6a^2$   
 $a^3 - 6a^2 = 0$   
 $a^2(a-6) = 0$   
 $a = 6$   
∴  $V = a^3 = 216 \text{ m}^3$

Dimension

47.  $P = \frac{\alpha}{\beta} e^{-\frac{qz}{k\theta}}$

$$[\alpha z] = [k\theta] = [ML^2 T^{-2}]$$

$$[\alpha] \cdot L = ML^2 T^{-2}$$

$$[\alpha] = ML T^{-2}$$

$$[P] = \frac{[\alpha]}{[\beta]} \Rightarrow ML^{-1} T^{-2} = \frac{ML T^{-2}}{[\beta]}$$

$$\Rightarrow [\beta] = L^2$$

49.  $P = \frac{RT}{V-b} e^{-\frac{\alpha V}{RT}}$

$$\Rightarrow [\alpha][V] = [R][T]$$

We know  $[P][V] = [R][T]$

$$\text{So } [\alpha] = [P]$$

50.  $V = at^2 + bt + c$

$$[v] = [at^2] = [bt] = [c]$$

$$LT^{-1} = [a]T^2 = [b]T = [c]$$

From here,  $[a] = LT^{-3}$

$$[b] = LT^{-2}$$

$$[c] = LT^{-1}$$

51. We know  $[E] = \frac{[G][M]}{[R]} \Rightarrow \frac{[E]}{[G]} = \frac{M}{L}$

$$\frac{E^2 L^2}{M^5 G^2} = \left[ \frac{E}{G} \right]^2 \frac{M^2 L^2 T^{-2}}{M^5}$$

$$= \frac{M^2}{L^2} \frac{M^2 L^2 T^{-2}}{M^5} = M^{-1} T^{-2}$$

55.  $y = A \sin(Bx + Ct + D)$

$$[y] = [A] = L, \quad [B][x] = [C][t] = [D] = M^0 L^0 T^0$$

$$\Rightarrow [B] = L^{-1}, \quad [C] = T^{-1}, \quad [D] = M^0 L^0 T^0$$

$$[ABCD] = [A][B][C][D] = T^{-1}$$

56.  $[P]^x [Q]^y [C]^z = (ML^{-1}T^{-2})^x (ML^0T^{-3})^y (LT^{-1})^z$

$$= M^{x+y} L^{-x+z} T^{-2x-3y-z}$$

$$x+y=0$$

$$y=-x$$

$$-x+z=0$$

$$z=x$$

$$-2x-3y-z=0$$

$$-2x+3x-x=0$$

So  $(x, y, z)$  is  $(x, -x, x)$

Correct option is (b).



62.  $y = At^2 - Bt^3$   
 $[y] = [A]T^2 = [B]T^3 = L$   
 $[A] = LT^{-2}, [B] = LT^{-3}$

68. Use  $E = h\nu$

69. Impulse is same as change in momentum.

74. Use  $\tau$  (torque) = Force  $\times$  distance

75. Use  $F = 6\pi\eta r v$   
 $F \rightarrow$  force,  $r \rightarrow$  radius,  $v \rightarrow$  speed  
 $\eta \rightarrow$  coefficient of viscosity.

87. Use  $q = it$   
 $\begin{matrix} \swarrow & \text{time} \\ \searrow & \text{current} \\ \text{charge} \end{matrix}$

96.  $[W] = ML^2T^{-2}$   
 $[W] = [k]L^2$   
 $[k] = MT^{-2}$

106. Pressure gradient is same as  $\frac{P}{l}$

111.  $E = h\nu$   
 $h = E \cdot T$

$[I] = [\text{moment of inertia}] = ML^2$   
 $\left[\frac{h}{I}\right] = \frac{ML^2T^{-1}}{ML^2} = T^{-1}$  (same as frequency)

112. EMF of a battery is the potential difference across terminals of a battery when there is no current drawn from it.

114.  $\rho = 0.625 \text{ gm/cm}^3$

$$= \frac{0.625 \times 10^{-3} \text{ kg}}{(10^{-2} \text{ m})^3}$$

$$= \frac{0.625 \times 10^{-3}}{10^{-6}} \frac{\text{kg}}{\text{m}^3}$$

$$= 0.625 \times 10^3 \text{ kg/m}^3$$

$$= 625 \text{ kg/m}^3$$

# Previous Year's Questions (Error)

115.  $v = \sqrt{2gh}$

$$p = mv = m\sqrt{2g} \sqrt{h}$$

$$p' = m\sqrt{2g} \sqrt{2h} = \sqrt{2} m\sqrt{2gh} = p\sqrt{2}$$

$$\Delta p = p' - p = (\sqrt{2} - 1) p$$

$$\frac{\Delta p}{p} \times 100 = (1.41 - 1) \times 100 = 41\%$$

116.  $\lambda = \frac{h}{p}$

$$d\lambda = -\frac{h}{p^2} dp$$

or  $\Delta\lambda = -\frac{h}{p^2} \Delta p$  or  $\Delta\lambda = -\frac{\lambda}{p} \cdot \Delta p$

$$\frac{\Delta\lambda}{\lambda} \times 100 = 0.5 \text{ [given].}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta p}{p} \text{ where } \Delta p \text{ is change in momentum}$$

&  $p$  is initial momentum.

Here momentum is changed by 'p'  
we have to find new initial momentum  $p'$ .

$$\frac{\Delta\lambda}{\lambda} \times 100 = \frac{p}{p'} \times 100$$

$$0.5 = \frac{p}{p'} \times 100$$

$$p' = \frac{100p}{0.5} = \underline{\underline{200P}}$$

117.  $PV = nRT$

$$\ln P + \ln V = \ln (nRT)$$

$$\frac{dP}{P} + \frac{dV}{V} = 0 \quad [\text{as } T \text{ is constant}]$$

$$\frac{\Delta P}{P} = - \frac{\Delta V}{V}$$

$$\frac{\Delta P}{P} \times 100 = - \frac{\Delta V}{V} \times 100$$

$$\text{Given } \frac{\Delta V}{V} \times 100 = -4.$$

$$\text{then } \frac{\Delta P}{P} \times 100 = 4$$

so increase is 4%.

119. At constant temperature  $P_1 V_1 = P_2 V_2$

$$P V = (P + \Delta P)(V + \Delta V)$$

$$P V = P V + P \Delta V + V \Delta P + \Delta V \cdot \Delta P$$

on dividing both sides by  $P V$ , we get

$$1 = 1 + \frac{\Delta V}{V} + \frac{\Delta P}{P} + \frac{\Delta V}{V} \cdot \frac{\Delta P}{P}$$

$$\therefore \frac{\Delta V}{V} + \frac{\Delta P}{P} + \frac{\Delta V}{V} \cdot \frac{\Delta P}{P} = 0$$

$$\text{Given, that } \frac{\Delta V}{V} \times 100 = -10 \quad \text{or } \frac{\Delta V}{V} = -0.1$$

~~$$0.1 + \frac{\Delta P}{P} + \frac{\Delta P}{P} \times 0.1 = 0$$

$$\frac{\Delta P}{P} (1.1) = -0.1$$

$$\text{or } \frac{\Delta P}{P} \times 100 = -\frac{100}{11}$$~~

125.  $V = l \times b \times h$   
 $= 12 \text{ cm} \times 6 \text{ cm} \times 2.45 \text{ cm}$   
 $= 176.4 \text{ cm}^3$   
 $= 2 \times 10^2 \text{ cm}^3$  [upto 1 significant digit]

130.  $X = M^a L^b T^c$   
 $\ln X = a \ln M + b \ln L + c \ln T$

$$\frac{dX}{X} = a \frac{dM}{M} + b \frac{dL}{L} + c \frac{dT}{T}$$

$$\frac{\Delta X}{X} = a \frac{\Delta M}{M} + b \frac{\Delta L}{L} + c \frac{\Delta T}{T}$$

$$\frac{\Delta X}{X} \times 100 = a \left( \frac{\Delta M}{M} \times 100 \right) + b \left( \frac{\Delta L}{L} \times 100 \right) + c \left( \frac{\Delta T}{T} \times 100 \right)$$

$$= a\alpha + b\beta + c\gamma$$

131.  $A = \frac{a^2 b^3}{c \sqrt{d}}$        $\ln A = 2 \ln a + 3 \ln b - \ln c - \frac{1}{2} \ln d$

$$\frac{dA}{A} = 2 \frac{da}{a} + 3 \frac{db}{b} - \frac{dc}{c} - \frac{1}{2} \frac{dd}{d}$$

~~$$\frac{\Delta A}{A} = 2 \frac{\Delta a}{a} + 3 \frac{\Delta b}{b} - \frac{\Delta c}{c} - \frac{1}{2} \frac{\Delta d}{d}$$~~

$$\frac{\Delta A}{A} = 2 \frac{\Delta a}{a} + 3 \frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{1}{2} \frac{\Delta d}{d} \quad [\text{For maximum error}]$$

$$\frac{\Delta A}{A} \times 100 = 2 \times 1 + 3 \times 3 + 2 + \frac{1}{2} \cdot 2$$

$$= 14.$$

