Trigonometric Ratio

EXERCISE - 1 [A]

1. (B)

π radian = 180 degree $\Rightarrow$ 1 radian = $\frac{180}{\pi}$ degree $\simeq$ 57.3 degree, hence sin 1^c > sin 1^0.

2. (B)

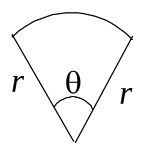
Length of arc of circle of radius r subtending θ at the center = rθ.

Hence $15 = r \times \frac{3}{4}$ or r = 20 cm

3. (D)

Perimeter $= r\theta + 2r = m \cdot r$

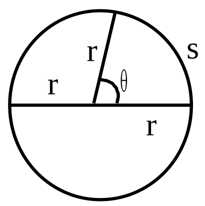
$\theta = (m-2)^c$



4. (C)

$\pi r = S + 2r$

$\Rightarrow S = (\pi - 2)r$



5. (B)

At half past 4, hour hand will be at $4\frac{1}{2}$ and minute hand will be at 6(for 30 minutes). In 12 hours, angle made by hour hand

$\frac{360}{12} \times \frac{9}{2} = 135^\circ$

In 60 minutes, angle made by minute hand

$= 360^\circ$

In 30 minutes, angle made by minute hand

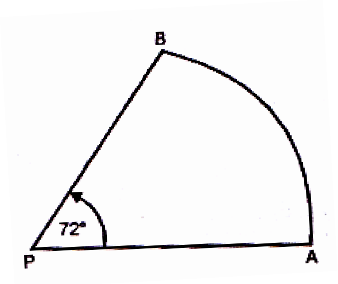
$= \frac{360^\circ}{60} \times 30 = 180^\circ$

$\therefore$ reqd. angle $= 180^\circ - 135^\circ = 45^\circ = \frac{\pi}{4}$

Hence (B) holds.

6. (B)

Let the post be at point P and let PA be the length of the rope in tight position. Suppose the horse moves along the arc AB so that $\angle APB = 72^\circ$ and arc $AB = 88m$. Let r be the length of the rope i.e $PA = r$ metres.



Now, $\theta = 72^\circ = \left(72 \times \frac{\pi}{180}\right)^c = \left(\frac{2\pi}{5}\right)^c$

And $s = 88m$ $\qquad \therefore \theta = \frac{\text{arc}}{\text{radius}}$

$\Rightarrow \frac{2\pi}{5} = \frac{88}{r} \Rightarrow r = 88 \times \frac{5}{2\pi} = 70m$

Hence (B) holds.

7. (C)

Since $180^\circ = \pi$ radians

$\therefore 1^\circ = \frac{\pi}{180}$ radians $\therefore p^\circ = \frac{p\pi}{180}$ radians

$\therefore \frac{p\pi}{180} = q$

$[\because p \text{ degrees} = q \text{ radians}]$

$\Rightarrow \frac{p}{180} = \frac{q}{\pi}$ $\qquad$ i.e. $\frac{p}{90} = \frac{2q}{\pi}$

Hence (C) holds.

8. (B)

Clearly length of the wire

= circumference of circle of radius 7

$= 2\pi(7) = 14\pi$

$\therefore l = 14\pi, r = 12cm$

Since $\theta = \frac{l}{r} = \frac{14\pi}{12} = \frac{7\pi}{6} = \frac{7}{6} \times 180^\circ = 210^\circ$

Hence angle subtended at the centre $= 210^\circ$

9. (A)

$\tan(90^0 - \theta) = \cot\theta \Rightarrow \tan 89^0 = \cot 1^0$, $\tan 88^0 = \cot 2^0$, $\tan 87^0 = \cot 3^0$, … etc.

Also, $\tan\theta \times \cot\theta = 1$, hence $\tan 1^0 \tan 2^0 \tan 3^0 \ldots \tan 89^0 = 1$

10. (A)

$\sin\theta - \cos\theta = 1 \Rightarrow \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = 1$

$\Rightarrow \sin\theta\cos\theta = 0$

11. (C)

$$\frac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta}=\frac{5\tan\theta-3}{5\tan\theta+2}$$

$$5\tan\theta=4\Rightarrow\frac{5\tan\theta-3}{5\tan\theta+2}=\frac{4-3}{4+2}=\frac{1}{6}$$

12. (A)

Sin x + cosec x = 2 $\Rightarrow$ $\sin^2 x - 2\sin x + 1 = 0$

$\Rightarrow$ sin x = 1 $\Rightarrow$ $\sin^n x + \text{cosec}^n x = 2$

13. (C)

$$x\sin 45^0\cos^2 60^0=\frac{\tan^2 60^0\,\text{cosec}\,30^0}{\sec 45^0\cot^2 30^0}\Rightarrow x\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)^2=\frac{\left(\sqrt{3}\right)^2(2)}{\left(\sqrt{2}\right)\left(\sqrt{3}\right)^2}\Rightarrow x=8$$

14. (A)

$$\tan^2 30+4\sin^2 45+\frac{1}{3}\cos^2 30=\frac{1}{3}+\frac{4}{2}+\frac{1}{3}\times\frac{3}{4}$$

$$=\frac{1}{3}+\frac{1}{4}+2$$

$$=2.\frac{7}{12}$$

15. (D)

$$\frac{\tan^2 60-2\tan^2 45+\sec^2 30}{3\sin^2 45-\sin 90+\cos^2 60.\cos^3 0}=\frac{3-2+\frac{4}{3}}{3.\frac{1}{2}+\frac{1}{4}}$$

$$=\frac{1+\frac{4}{3}}{\frac{1}{2}\left(3+\frac{1}{2}\right)}=\frac{\frac{7}{3}}{\frac{1}{2}.\frac{7}{2}}=\frac{4}{3}$$

16. (C)

$$2P_6-3P_4+1$$

$$=2\left(1-3\sin^2 x\cos^2 x\right)-3\left[\left(\sin^2 x+\cos^2 x\right)^2-2\sin^2 x\cos^2 x\right]+1$$

$$=2-6\sin^2 x\cos^2 x-3\left(1-2\sin^2 x\cos^2 x\right)+1=0$$

17. (A)

$90^0 < 130^0 < 135^0$ hence sin A > 0 & cos A < 0 & $|\sin A|>|\cos A|$

$\Rightarrow \sin A+\cos A>0$

18. (A)

$$\cos 24^0+\cos 5^0+\cos 175^0+\cos 204^0+\cos 300^0$$

$$=\cos 24^0+\cos 5^0+\cos\left(180^0-5^0\right)+\cos\left(180^0+24^0\right)+\cos\left(360^0-60^0\right)$$

$= \cos 24^0 + \cos 5^0 - \cos 5^0 - \cos 24^0 + \cos 60^0$

$= \cos 60^0 = \frac{1}{2}$

19. (C)

$\sin\theta < 0 \,\&\, \tan\theta > 0 \Rightarrow \theta$ lies in 3rd quadrant.

20. (C)

Given expression

$$= \frac{-\sin\left(660^\circ\right)\tan\left(1050^\circ\right)\sec\left(420^\circ\right)}{\cos\left(180^\circ + 45^\circ\right)\text{cosec}\left(360^\circ - 45^\circ\right)\cos\left(360^\circ + 150^\circ\right)}$$

$$= \frac{-\sin\left(7\times 90 + 30^\circ\right)\tan\left(3.360^\circ - 30^\circ\right)\sec\left(360^\circ + 60^\circ\right)}{\left(-\cos 45^\circ\right) - \cos ec\left(45^\circ\right)\cos 150^\circ}$$

$$= \frac{\cos\left(30^\circ\right)\left(-\tan 30^\circ\right)\sec 60^\circ}{-\left(\cos 45^\circ\right)\left(-\text{cosec}\, 45^\circ\right)\left(-\cos 30^\circ\right)}$$

$$= \frac{\cos 30^\circ \tan 30^\circ \sec 60^\circ}{\cos 45^\circ \,\text{cosec}\, 45^\circ \cos 30^\circ}$$

$$= \frac{\frac{1}{\sqrt{3}}.2}{\frac{1}{\sqrt{2}}.\frac{\sqrt{2}}{1}} = \frac{2}{\sqrt{3}}$$

Hence (C) holds

21. (C)

Given expression

$$= \frac{\left(-\sin\theta\right)\left(\sec\theta\right)\left(-\tan\theta\right)}{\left(\sin\theta\right)\left(-\sec\theta\right)\tan\theta} = -1$$

Hence (C) holds.

22. (B)

Since $\sec\theta$ i.e $\cos\theta$ is negative.

$\therefore \theta$ lies in II nd or III rd quadrant. Since $\sin\theta$ is positive.

$\therefore \theta$ lies in Ist or IInd quadrant. Hence θ lies in II nd quadrant.

23. (A)

$\cot A + \tan\left(180^\circ + A\right) + \tan\left(90^\circ + A\right) + \tan\left(360^\circ - A\right)$

$= \cot A + \tan A - \cot A - \tan A = 0$

24. (D)

$\sin\left[-870^\circ\right] = -\sin\left(870^\circ\right) = -\sin\left(9\times 90^\circ + 60^\circ\right)$

$= -\cos 60^\circ = -\frac{1}{2}$

25. (A)

$\tan\theta = -\frac{1}{\sqrt{5}}$

$\sec^2\theta = 1+\tan^2\theta = 1+\frac{1}{5} = \frac{6}{5}$

$\sec\theta = +\sqrt{\frac{6}{5}}$ [$\because \theta$ lies in the IV th quadrant]

$\therefore \cos\theta = \sqrt{\frac{5}{6}}$

26. (C)

$\sin 15^0 \cos 15^0 = \frac{1}{2}\sin 30^0 = \frac{1}{4}$

27. (A)

$\frac{\cos 17^0 + \sin 17^0}{\cos 17^0 - \sin 17^0} = \frac{1+\tan 17^0}{1-\tan 17^0}$

$= \tan\left(45^0 + 17^0\right) = \tan 62^0$

28. (A)

tan 75^0 – cot 75^0 = –2 cot 150^0

$= 2\tan 60^0 = 2\sqrt{3}$

29. (B)

$\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$

$\Rightarrow \cos^2 48^0 - \sin^2 12^0 = \cos 60^0 \cos 36^0 = \frac{\sqrt{5}+1}{8}$

30. (D)

As A & B are the acute angles hence,

$\sin A = \frac{1}{\sqrt{10}} \Rightarrow \cos A = \frac{3}{\sqrt{10}}$ & $\sin B = \frac{1}{\sqrt{5}} \Rightarrow \cos B = \frac{2}{\sqrt{5}}$

Now cos (A + B) = cos A cos B – sin A sin B

$\Rightarrow \cos(A+B) = \frac{3}{\sqrt{10}}\times\frac{2}{\sqrt{5}} - \frac{1}{\sqrt{10}}\times\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{2}}$

$\Rightarrow A+B = \frac{\pi}{4}$

31. (A)

$\cos^2\left(\frac{\pi}{6}+\theta\right) - \sin^2\left(\frac{\pi}{6}-\theta\right)$

$= \cos\left(\frac{\pi}{6}+\theta+\frac{\pi}{6}-\theta\right)\cos\left(\frac{\pi}{6}+\theta-\frac{\pi}{6}+\theta\right)$

$\left[\because \cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B\right]$

$$= \cos\left(\frac{\pi}{3}\right)\cos(2\theta) = \frac{1}{2}\cos 2\theta$$

Hence (A) holds.

32. (C)

$$(1+\tan A)(1+\tan B)$$

$$= (1+\tan A)\left(1+\tan\left(\frac{\pi}{4}-A\right)\right)$$

$$= (1+\tan A)\left(1+\frac{1-\tan A}{1+\tan A}\right)$$

$$= (1+\tan A)\left(\frac{1+\tan A+1-\tan A}{1+\tan A}\right) = 2$$

$\therefore$ (C) holds

33. (B)

$$\sin 163^\circ = \sin\left(180^\circ - 17^\circ\right) = \sin 17^\circ$$

$$\cos 347^\circ = \cos\left(360^\circ - 13^\circ\right) = \cos 13^\circ$$

$$\sin 73^\circ = \sin\left(90^\circ - 17^\circ\right) = \cos 17^\circ$$

$$\sin 167^\circ = \sin\left(180^\circ - 13^\circ\right) = \sin 13^\circ$$

$\therefore$ given expression

$$= \sin 17^\circ \cos 13^\circ + \cos 17^\circ \sin 13^\circ$$

$$= \sin\left(17^\circ + 13^\circ\right) = \sin\left(30^\circ\right) = \frac{1}{2}$$

34. (D)

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$$\therefore A + B = \frac{\pi}{4}$$

35. (B)

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

$$\left[\cos\theta = \sqrt{1-\frac{144}{165}} = \frac{5}{13} \because 0 < \theta < \frac{\pi^-}{2} \sin\phi = -\sqrt{1-\frac{9}{25}} = -\frac{4}{5} \because \pi < \theta < \frac{3\pi}{2}\right]$$

$$= \frac{12}{13}\left(-\frac{3}{5}\right) + \frac{5}{13}\left(-\frac{4}{5}\right)$$

$$= -\frac{36}{65} - \frac{20}{65} = -\frac{56}{65}$$

36. (D)

$A+B=45^\circ \Rightarrow \dfrac{\cot A\cot B-1}{\cot B+\cot A}=1$

$(\cot A-1)(\cot B-1)$

$=\cot A\cot B-(\cot A+\cot B)+1$

$=1+1=2$

$\therefore$ (D) holds

37. (C)

$32\sin\left(\dfrac{A}{2}\right)\sin\left(\dfrac{5A}{2}\right)=16(\cos 2A-\cos 3A)$

$=16\left(2\cos^2 A-1-4\cos^3 A+3\cos A\right)$

$=16\left(2\times\dfrac{9}{16}-1-4\times\dfrac{27}{64}+3\times\dfrac{3}{4}\right)=11$

38. (B)

$\sin 50^0-\sin 70^0+\sin 10^0=-2\cos 60^0\sin 10^0+\sin 10^0=0$

39. (D)

The given expression $=\cos 20^\circ+\cos(180^\circ-80^\circ)+\cos(180^\circ-40^\circ)$

$=\cos 20^\circ-(\cos 80^\circ+\cos 40^\circ)$

$=\cos 20^\circ-2\cos 60^\circ\cos 40^\circ=0$

$\therefore$ (D) holds

40. (A)

$\cos 52^\circ+\cos 68^\circ+\cos 172^\circ$

$=\cos 52^\circ+\cos 68^\circ+\cos(180^\circ-8^\circ)$

$=\cos 52^\circ+\cos 68^\circ-\cos 8^\circ$

$=2\cos\dfrac{52^\circ+68^\circ}{2}\cos\dfrac{68^\circ-52^\circ}{2}-\cos 8^\circ$

$=2\cos 60^\circ\cos 8^\circ-\cos 8^\circ$

$=\cos 8^\circ-\cos 8^\circ=0$

41. (C)

$\dfrac{\sin(2A+B)}{\sin B}=\dfrac{5}{1}$. By componendo and dividendo we shall get $\dfrac{\tan(A+B)}{\tan A}=\dfrac{3}{2}$

42. (C)

Given value $=(\cos 12^\circ+\cos 132^\circ)+(\cos 84^\circ+\cos 156^\circ)=-\dfrac{1}{2}$

43. (B)

$$\frac{\sin 70^{\circ}+\cos 40^{\circ}}{\cos 70^{\circ}+\sin 40^{\circ}}=\frac{\sin 70^{\circ}+\sin 50^{\circ}}{\cos 70^{\circ}+\cos 50^{\circ}}$$
$$=\frac{2\sin 60^{\circ}\cos 10^{\circ}}{2\cos 60^{\circ}\cos 10^{\circ}}=\tan 60^{\circ}=\sqrt{3}$$

44. (D)

$$\frac{\sin 55^{\circ}-\cos 55^{\circ}}{\sin 10^{\circ}}=\frac{\sin 55^{\circ}-\sin 35^{\circ}}{\sin 10^{\circ}}$$
$$=\frac{2\cos 45^{\circ}\sin 10^{\circ}}{\sin 10^{\circ}}=2\cos 45^{\circ}=2\frac{1}{\sqrt{2}}=\sqrt{2}$$

$\therefore$ (D) holds

45. (D)

$$1-2\sin^{2}\left(\frac{\pi}{4}+\theta\right)=\cos 2\left(\frac{\pi}{4}+\theta\right)$$
$$=\cos\left(\frac{\pi}{2}+2\theta\right)=-\sin 2\theta$$

46. (B)

$$2\cos^{2}\theta-2\sin^{2}\theta=1 \quad \Rightarrow \quad \cos 2\theta=\frac{1}{2}$$

$\therefore\ \theta=30^{o}$

47. (B)

$$2\sin A\cos^{3}A-2\sin^{3}A\cos A=2\sin A\cos A\left(\cos^{2}A-\sin^{2}A\right)$$
$$=\sin 2A\cos 2A=\frac{\sin 4A}{2}$$

48. (C)

$\sin A+\cos A=1$, we get $1+\sin 2A=1$

$\Rightarrow \sin 2A=0$

49. (A)

$$\frac{\tan 80^{\circ}-\tan 10^{\circ}}{\tan 70^{\circ}}=\frac{\cot 10^{\circ}-\tan 10^{\circ}}{\tan 70^{\circ}}$$
$$=\frac{2\cot 20^{\circ}}{\tan 70^{\circ}}=2$$

50. (A)

Use $\cos 2\theta=\dfrac{1-\tan^{2}\theta}{1+\text{tam}^{2}\theta}$

51. (D)

$$\tan 67\frac{1}{2}^{\circ} + \cot 67\frac{1}{2}^{2}$$

$$= \tan 67\frac{1}{2}^{\circ} + \frac{1}{\tan 67\frac{1}{2}^{\circ}}$$

$$= \frac{1+\tan^2 67\frac{1}{2}^{\circ}}{2\tan 67\frac{1}{2}^{\circ}}.2$$

$$= 2.\frac{1}{\sin 2\left(67\frac{1}{2}^{\circ}\right)}$$

$$\left[\because \sin 2A = \frac{2\tan A}{1+\tan^2 A}\right]$$

$$= \frac{2}{\sin\left(135^{\circ}\right)} = \frac{2}{\sin\left(180^{\circ}-45^{\circ}\right)}$$

$$= \frac{2}{\sin 45^{\circ}} = \frac{2}{\frac{1}{\sqrt{2}}} = 2\sqrt{2}$$

$\therefore$ (D) holds

52. (B)

$$\sqrt{2+\sqrt{2+2\cos 4\theta}} = \sqrt{2+\sqrt{2\left(1+\cos 4\theta\right)}}$$

$$= \sqrt{2+\sqrt{2.2\cos^2 2\theta}}$$

$$= \sqrt{2+2\cos 2\theta} = \sqrt{2\left(1+\cos 2\theta\right)}$$

$$= \sqrt{2\left(2\cos^2\theta\right)} = 2\cos\theta$$

$\therefore$ (B) holds

53. (B)

$$\cos 15^{\circ}\cos 7\frac{1}{2}^{\circ}\sin 7\frac{1}{2}^{\circ}$$

$$= \frac{1}{2}\cos 15^{\circ}\sin 15^{\circ} = \frac{1}{4}\sin 30^{\circ}$$

$$= \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$$

$\therefore$ (B) holds

54. (A)

$$\frac{1-t^2}{1+t^2} = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \cos\theta$$

55. (D)

Since $1+\cos x = K \qquad \therefore 2\cos^2\frac{x}{2} = K$

$\Rightarrow \cos^2\frac{x}{2} = \frac{K}{2} \qquad \Rightarrow 1-\sin^2\frac{x}{2} = \frac{K}{2}$

$\Rightarrow \sin^2\frac{x}{2} = 1-\frac{K}{2} = \frac{2-K}{2}$

$\Rightarrow \sin\frac{x}{2} = \sqrt{\frac{2-K}{2}}$

56. (A)

$$\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

$$\cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

57. (A)

$$\frac{\sin 3\theta - \cos 3\theta}{\sin\theta+\cos\theta}+1 = \frac{3\sin\theta - 4\sin^3\theta - 4\cos^3\theta + 3\cos\theta}{\sin\theta+\cos\theta}+1$$

$$= 4\frac{\sin\theta+\cos\theta-\sin^3\theta-\cos^3\theta}{\sin\theta+\cos\theta}$$

$$= 4\frac{\sin\theta+\cos\theta-(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)}{\sin\theta+\cos\theta}$$

$$= 4\sin\theta\cos\theta = 2\sin 2\theta$$

58. (C)

$\cos^2 A(3-4\cos^2 A)^2 + \sin^2 A\ (3-4\sin^2 A)^2 = (3\cos A - 4\cos^3 A)^2 + (3\sin A - 4\sin^3 A)^2$

$= \cos^2 3A + \sin^2 3A = 1$

59. (C)

$$\frac{3\cos\theta+\cos 3\theta}{3\sin\theta-\sin 3\theta} = \frac{3\cos\theta+(4\cos^3\theta-3\cos\theta)}{3\sin\theta-(3\sin\theta-4\sin^3\theta)} = \cot^3\theta$$

60. (C)

$\sin 20^0 \sin 40^0\ \sin 60^0 \sin 80^0 = (\sin(60^0-20^0)\sin 20^0 \sin(60^0+20^0))\sin 60^0$

$$= \frac{1}{4}\sin(3\times 20^0)\sin 60^0 = \frac{1}{4}\sin^2 60^0 = \frac{3}{16}$$

61. (D)

$$\cos 20^0\cos 40^0\cos 80^0 = \frac{2\sin 20^0\cos 20^0\cos 40^0\cos 80^0}{2\sin 20^0}$$

$$= \frac{2\sin 40^0\cos 40^0\cos 80^0}{4\sin 20^0} = \frac{2\sin 80^0\cos 80^0}{8\sin 20^0}$$

$= \frac{\sin 160^0}{8\sin 20^0}$, but sin 160^0 = sin ($180^0 – 20^0$) = sin 20^0, hence

$\cos 20^0 \cos 40^0 \cos 80^0 = \frac{1}{8}$

Alternately

cos 20^0 cos 40^0 cos 80^0 = cos ($60^0 – 20^0$) cos 20^0 cos ($60^0 + 20^0$)

$\frac{1}{4}\cos\left(3\times 20^0\right) = \frac{1}{8}$

62. (D)

$\frac{1}{4}\left(4\cos\theta\cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right)\right)$

$= \frac{\cos 3\theta}{4}$

63. (C)

$\tan\theta + \tan\left(\theta+\frac{\pi}{3}\right) + \tan\left(\theta-\frac{\pi}{3}\right)$

$= \tan\theta + \frac{\tan\theta+\sqrt{3}}{1-\tan\theta.\sqrt{3}} + \frac{\tan\theta-\sqrt{3}}{1+\tan\theta.\sqrt{3}}$

$= \tan\theta + \frac{\tan\theta+\sqrt{3}+\sqrt{3}\tan^2\theta+3\tan\theta+\tan\theta-\sqrt{3}-\tan^2\theta\sqrt{3}+3\tan\theta}{1-3\tan^2\theta}$

$= \frac{\tan\theta-3\tan^3\theta+8\tan\theta}{1-3\tan^2\theta}$

$= \frac{9\tan\theta-3\tan^3\theta}{1-3\tan^2\theta}$

$= \frac{3\left[3\tan\theta-\tan^3\theta\right]}{1-3\tan^2\theta} = 3\tan 3\theta$

$\therefore K = 3$

Hence (C) holds.

64. (A)

$x = \tan 15^\circ = 2-\sqrt{3}$

$y = \text{cosec}\,75^\circ = \sqrt{6}-\sqrt{2}$

$(z) = 4\sin 18^\circ = \sqrt{5}-1$

It follows that $x < y < z$

$\therefore$ (D) holds

65. (D)

$-\sqrt{3^2+4^2}+8 \le 3\cos x + 4\sin x + 8 \le \sqrt{3^2+4^2}+8$

$\Rightarrow 3 \le 3\cos x + 4\sin x + 8 \le 13$

66. (A)

$\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{6}\right)$ acquires maximum at $\alpha+\frac{\pi}{6}=\frac{\pi}{4}$ i.e. $\theta=\frac{\pi}{12}$

67. (D)

$5\sin^2\theta+4\cos^2\theta=4+\sin^2\theta\geq 4$

68. (B)

Maximum value of $\sin\theta\cos\theta$

$=$ Maximum value of $\left(\frac{\sin 2\theta}{2}\right)$

$=\frac{1}{2}$ Maximum Value of $\sin 2\theta=\frac{1}{2}(1)=\frac{1}{2}$

69. (C)

Minimum value of $\sin\theta\cos\theta$

$=\frac{1}{2}$ Minimum value of $\sin 2\theta=\frac{1}{2}(-1)=-\frac{1}{2}$

70. (C)

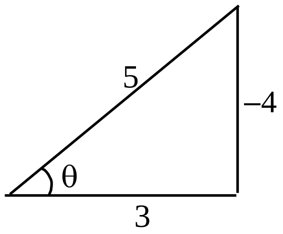
$\sin x+\cos x=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)\leq\sqrt{2}$

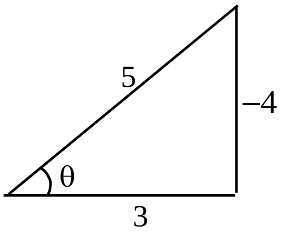
$\therefore$ Maximum value $=\sqrt{2}$

71. (D)

Put $-\left.\begin{array}{l}3=r\cos\alpha\\4=r\sin\alpha\end{array}\right]\begin{array}{l}\Rightarrow r^2=9+16=25\\\Rightarrow \qquad r=5\end{array}$

$\therefore 3\cos\theta+4\sin\theta=r(\cos\theta\cos\alpha+\sin\theta\sin\alpha)$





$=r\cos(\theta-\alpha)=5\cos(\theta-\alpha)$

Its Maximum value $=5(1)=5$

And Minimum value $=-5$

72. (A)

Given, $P=\frac{1}{2}\sin^2\theta+\frac{1}{3}\cos^2\theta$

$\frac{1}{2}(1-\cos^2\theta)+\frac{1}{3}\cos^2\theta=\frac{1}{2}-\frac{1}{6}\cos^2\theta$

Since, $0\leq\cos^2\theta\leq 1\Rightarrow -\frac{1}{6}\leq-\frac{1}{6}\cos^2\theta\leq 0$

$$\Rightarrow \frac{1}{3} \le \frac{1}{2} - \frac{1}{6}\cos^2\theta \le \frac{1}{2} \Rightarrow \frac{1}{3} \le P \le \frac{1}{2}$$

$\therefore$ (A) holds

73. (C)

$$A + B + C = \pi$$

$$\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$$

74. (A)

$$\frac{A+B+C}{2} = \frac{\pi}{2}$$

$$\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{c}{2}\right)$$

$$\frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}} = \frac{1}{\tan\frac{c}{2}}$$

$$\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{A}{2}\tan\frac{C}{2} = 1$$

$$\tan\frac{C}{2} = 1 - \frac{2}{9} = \frac{7}{9}$$

75. (D)

$$A + B + C = \pi$$

$$\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$$

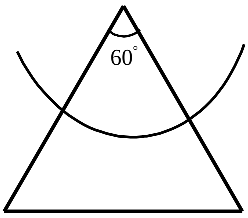
$$\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

$$\frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}} = \frac{1}{\tan\frac{C}{2}}$$

$$\Rightarrow \sum \tan\frac{A}{2}\tan\frac{B}{2} = 1$$

EXERCISE - 1 [B]

1. (A)



Area of $\Delta = \frac{\sqrt{3}}{4}.4^2 = 4\sqrt{3}$

2π _____ πr^2

1 ______ $\frac{\pi r^2}{2\pi}$

$\frac{\pi}{3}$ ____ $\frac{\pi r^2}{2\pi}.\frac{\pi}{3}$

Given, $\frac{\pi r^2}{6} = \frac{4\sqrt{3}}{2}$

$\pi r^2 = 12\sqrt{3}$

$r = \sqrt{\frac{12\sqrt{3}}{\pi}}$

2. (C)

Angle covered from 6 A.M. to 3.15 P.M.

$= 277\frac{1}{2}^{\circ} = \frac{555}{2}\times\frac{\pi}{180} = \frac{111}{2}\times\frac{\pi}{36}$

$\therefore \theta = \frac{37\pi}{24}$ radians

Length of hour hand $= 12\text{cm}$

i.e. $r = 12\text{cm}$

Since $\theta = \frac{l}{r}$ $\quad\quad \therefore l = r\theta = \frac{12\times 37}{24} = \frac{37\pi}{2}$

Hence reqd. distance $= \frac{37\pi}{2}$ cm.

3. (D)

An hour hand in 1 hour i.e., 60 minute traces $= \frac{360^{\circ}}{12} = 30^{\circ}$

$\therefore$ An hour hand in 1 minute traces $= \frac{30^{\circ}}{60^{\circ}} = \left(\frac{1}{2}\right)^{\circ}$

$\therefore$ An hour hand in 40 minutes traces

$= \left(\frac{1}{2}\times 40\right)^{\circ} = 20^{\circ}$

$\therefore$ reqd. angle $= 90^{\circ} - 20^{\circ} = 70^{\circ}$

$= \frac{70 \times \pi}{180} = \frac{7\pi}{18}$

$\therefore$ (D) holds

4. (B)

$\sin^2 x + \text{cosec}^2 x + 2 + \cos^2 x + \sec^2 x + 2 - \tan^2 x - \cot^2 x - 2$

$= 3 + \left(\text{cosec}^2 x - \cot^2 x\right) + \left(\sec^2 x - \tan^2 x\right)$

$= 5$

5. (D)

$\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta} = \text{cosec}^2\theta \sec^2 \theta$

6. (A)

Let $\lambda = 3\cos\beta - 5\sin\beta$

Let $S = 3\sin\beta + 5\cos\beta$

$\lambda^2 + S^2 = 9 + 25$

$\therefore \lambda^2 = 9$

7. (A)

$\tan^2 \alpha + \cot^2 \alpha + 2 = m^2$

$\left(\tan^2 \alpha + \cot^2 \alpha\right)^2 = \left(m^2 - 2\right)^2$

$\tan^4 \alpha + \cot^4 \alpha + 2 = m^4 + 4 - 4m^2$

$\tan^4 \alpha + \cot^4 \alpha = m^4 - 4m^2 + 2$

8. (B)

$\frac{\cot\theta + \text{cosec}\theta - 1}{\cot\theta - \text{cosec}\theta + 1}$

$= \frac{\frac{1}{\text{cosec}\theta - \cot\theta} - 1}{\cot\theta - \text{cosec}\theta + 1} \quad \left(\because \left(\text{cosec}\theta + \cot\theta\right)\left(\text{cosec}\theta - \cot\theta\right) = 1\right)$

$= \frac{1 - \text{cosec}\theta + \cot\theta}{\left(\text{cosec}\theta - \cot\theta\right)\left(\cot\theta - \text{cosec}\theta + 1\right)}$

$= \frac{1}{\text{cosec}\theta - \cot\theta} = \text{cosec}\theta + \cot\theta$

$= \frac{1 + \cos\theta}{\sin\theta} = \frac{\sin\theta}{1 - \cos\theta}$

9. (D)

Since $\sin x + \sin^2 x = 1$

$\therefore \sin x = 1 - \sin^2 x = \cos^2 x$

$\therefore \sin^2 x = \cos^4 x$

$\Rightarrow 1 - \cos^2 x = \cos^4 x$

$\Rightarrow 1 = \cos^4 x + \cos^2 x$

$\Rightarrow 1 = \left(\cos^4 x + \cos^2 x\right)^2$

$= \cos^8 x + 2\cos^6 x + \cos^4 x$

Thus $\cos^8 x + 2\cos^6 x + \cos^4 x = 1$

10. (C)

$\sin\left(-566^\circ\right) = -\sin 566^\circ = -\sin\left(6\times 90^\circ + 26^\circ\right)$

$= -\left(-\sin 26^\circ\right) = \sin 26^\circ$

11. (C)

$f(x) = 3\left[\sin^4\left(\frac{\pi}{2} - x\right) + \sin^4 x\right] - 2\left[\cos^6 x + \sin^6 x\right]$

$= 3\left[\left(\cos^2 x\right)^2 + \left(\sin^2 x\right)^2\right] - 2\left[\left(\sin^2 x\right)^3 + \left(\cos^2 x\right)^3\right]$

$= 3\left[1 - 2\sin^2 x\cos^2 x\right] - 2\left[1 - 3\sin^2 x\cos^2 x\right]$

$= 1$

12. (A)

$2\cos 10 + \sin(90 + 10) + \sin(3\times 360 - 80) + \sin(27\times 360 + 280)$

$= 2\cos 10 + \cos 10 - 2\sin 80$

$= \cos 10 + 2\left(\cos 10 - \sin(90 - 10)\right)$

$= \cos 10 + 2\left(\cos 10 - \cos 10\right) = \cos 10$

13. (C)

$\cos^2(90 - 17) + \cos^2 47 - \sin^2(90 - 47) + \sin^2(90 + 17)$

$= \sin^2 17 + \cos^2 47 - \cos^2 47 + \cos^2 17$

$= 1$

14. (D)

(i) $\sin(2\times 360 + 45) = \sin 45 = \frac{1}{\sqrt{2}}$

(ii) $\frac{1}{-\sin(3\times 360 + 330)} = \frac{1}{-\sin(360 - 30)} = \frac{1}{\sin 30} = 2$

(iii) $\tan\left(4\pi + \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

(iv) $\frac{-1}{\tan\left(4\pi - \frac{\pi}{4}\right)} = 1$

15. (C)

$\ell^2 = \left[\sin\theta - \sin\left(\frac{\pi}{2} - \theta\right)\right]^2 + \left[\cos\theta + \cos\left(\frac{\pi}{2} - \theta\right)\right]$

$= (\sin\theta - \cos\theta)^2 + (\cos\theta + \sin\theta)^2$

$= 1 + 1 = 2$

$\ell = \sqrt{2}$

16. (B)

$\sin(180+20)+\cos(180+20)$

$=-\sin 20-\cos 20<0$

17. (B)

$a=\sin 170^\circ+\cos 170^\circ$

$=\sin(180-10)+\cos(180-10)$

$=\sin 10-\cos 10$

$\Rightarrow a=-ve$

$\{\because \cos 10^\circ>\sin 10^\circ\}$

18. (C)

$\cos\left(\frac{\pi}{2}-x\right)=\sin x,\ \cos\left(\frac{3\pi}{2}+x\right)=\sin x$

$\cos(\pi-x)=-\cos x, \cos(2\pi-x)=\cos x$

$\therefore$ required $=\sin x\cdot\sin x+\cos x\cdot\cos x$

$= 1$

19. (B)

$\cos\frac{2\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\frac{\pi}{3},\cos\frac{4\pi}{3}=\cos\left(\pi+\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}$

$\cos\frac{2\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\frac{\pi}{3},\cos\frac{4\pi}{3}=\cos\left(\pi+\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}$

$\Rightarrow x=y\cos\frac{2\pi}{3}=z\cos\frac{4\pi}{3} \Rightarrow x=-\frac{y}{2}=-\frac{z}{2} \Rightarrow y=z=-2x$

Now, $xy + yz + zx = x(-2x) + (-2x)(-2x) + (-2x)x = 0$

20. (A)

$\cos^2\alpha+\cos^2(\alpha+120^0)+\cos^2(\alpha-120^0)$

$=\frac{1+\cos 2\alpha}{2}+\frac{1+\cos 2(\alpha+120^0)}{2}+\frac{1+\cos 2(\alpha-120^0)}{2}$

$=\frac{3+\cos 2\alpha+2\cos 2\alpha\cos 120^0}{2}=\frac{3+\cos 2\alpha-\cos 2\alpha}{2}=\frac{3}{2}$

21. (B)

$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}\Rightarrow \tan(A+B)=\frac{-\frac{1}{2}-\frac{1}{3}}{1-\left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)}$

$\Rightarrow \tan(A+B)=-1$

$\Rightarrow A+B=\frac{3\pi}{4}$

22. (D)

$\tan 45^0 = \tan\left(180^0 + 45^0\right) \Rightarrow \tan 225^0 = \tan\left(100^0 + 125^0\right)$

$\Rightarrow \dfrac{\tan 100^0 + \tan 125^0}{1 - \tan 100^0 \tan 125^0} = 1$

$\Rightarrow \tan 100^0 + \tan 125^0 + \tan 100^0 \tan 125^0 = 1$

23. (B)

$$\frac{\sin(B+A)+\cos(B-A)}{\sin(B-A)+\cos(B+A)} = \frac{\sin(B+A)+\sin\left(\frac{\pi}{2}-(B-A)\right)}{\cos\left(\frac{\pi}{2}-(B-A)\right)+\cos(B+A)}$$

$$= \frac{2\sin\left(\frac{\pi}{4}+A\right)\cos\left(\frac{\pi}{4}-B\right)}{2\cos\left(\frac{\pi}{4}+A\right)\cos\left(\frac{\pi}{4}-B\right)} = \tan\left(\frac{\pi}{4}+A\right)$$

$$= \frac{1+\tan A}{1-\tan A} = \frac{\cos A+\sin A}{\cos A-\sin A}$$

24. (C)

Tan 3A=tan(2A+A)

$= \dfrac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$

$\Rightarrow \tan 3A - \tan 3A \tan 2A \tan A$

$= \tan 2A + \tan A$

$\therefore \tan 3A - \tan 2A - \tan A$

$= \tan A \tan 2A \tan 3A$

25. (C)

$\sin^2 A \cos^2 B - \cos^2 A \sin^2 B$

$= \sin^2 A\left(1-\sin^2 B\right) - \sin^2 B\left(1-\sin^2 A\right)$

$= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$

$= \sin^2 A - \sin^2 B$

26. (B)

$\sin^2 A + \sin^2(A-B) + 2\sin A \cos B \sin(B-A)$

$= \sin^2 A + \sin^2(A-B) + \left[\sin(A+B)+\sin(A-B)\right]\sin(B-A)$

$= \sin^2 A + \sin^2(A-B) + \sin(A+B)\sin(B-A) - \sin^2(A-B)$

$= \sin^2 A + \sin^2 B - \sin^2 A$

$= \sin^2 B$

27. (A)

$$\frac{\dfrac{\sin\left(\dfrac{3\theta}{2}\right)}{\sin\left(\dfrac{\theta}{2}\right)}\sin\left(\theta+\dfrac{2\theta}{2}\right)}{\dfrac{\sin\left(\dfrac{3\theta}{2}\right)}{\sin\dfrac{\theta}{2}}\cos\left(\theta+\dfrac{2\theta}{2}\right)}=\frac{\sin\alpha}{\cos\alpha}$$

$$\tan 2\theta=\tan\alpha$$

$$\theta=\frac{\alpha}{2}$$

28. (B)

$$\sin\alpha+\sin\beta=a\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=a \qquad \ldots(i)$$

$$\&\ \cos\alpha-\cos\beta=b\Rightarrow 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}=-b \qquad \ldots(ii)$$

From (i) & (ii) $\tan\frac{\alpha-\beta}{2}=-\frac{b}{a}$

29. (C)

$$\frac{\sin 3\theta+\sin 5\theta+\sin 7\theta+\sin 9\theta}{\cos 3\theta+\cos 5\theta+\cos 7\theta+\cos 9\theta}=\frac{(\sin 3\theta+\sin 9\theta)+(\sin 5\theta+\sin 7\theta)}{(\cos 3\theta+\cos 9\theta)+(\cos 5\theta+\cos 7\theta)}$$

$$=\frac{2\sin 6\theta\cos 3\theta+2\sin 6\theta\cos\theta}{2\cos 6\theta\cos 3\theta+2\cos 6\theta\cos\theta}=\tan 6\theta$$

30. (D)

Given value

$$=\left(\sin 47^\circ+\sin 61^\circ\right)-\left(\sin 11^\circ+\sin 25^\circ\right)$$

$$=2\sin 54^\circ\cos 7^\circ-2\sin 18^\circ\cos 7^\circ$$

$$=2\cos 7^\circ\left(\sin 54^\circ-\sin 18^\circ\right)$$

$$=2\cos 7^\circ\frac{2\sin 18^\circ\cos 18^\circ}{\cos 18^\circ}\cos 36^\circ$$

$$=\cos 7^\circ\frac{2\sin 36^\circ\cos 36^\circ}{\cos 18^\circ}$$

$$=\cos 7^\circ\frac{\sin 72^\circ}{\cos 18^\circ}=\cos 7^\circ\left[\because \sin 72^\circ=\cos 18^\circ\right]$$

31. (D)

$$\cos 40^\circ+\cos 80^\circ+\cos 160^\circ+\cos 240^\circ$$

$$=2\cos 60^\circ\cos 20^\circ+\cos\left(180^\circ-20^\circ\right)+\cos\left(180^\circ+60^\circ\right)$$

$$=2\left(\frac{1}{2}\right)\cos 20^\circ-\cos 20^\circ-\cos 60^\circ$$

$= -\cos 60^\circ = -\frac{1}{2}$

32. (A)

$\cos 12^\circ + \cos 84^\circ + \cos 156^\circ + \cos 228^\circ$

$= \left(\cos 12^\circ + \cos 228^\circ\right) + \left(\cos 84^\circ + \cos 156^\circ\right)$

$= 2\cos\left(120^\circ\right)\cos\left(108^\circ\right) + 2\cos 120^\circ \cos 36^\circ$

$= 2\cos\left(120^\circ\right)\left[\cos\left(108^\circ + \cos 36^\circ\right)\right]$

$= 2\left(-\frac{1}{2}\right).2\cos 72^\circ \cos\left(36^\circ\right)$

$= -2\cos 72^\circ \cos 36^\circ = -2.\frac{\sqrt{5}-1}{4}.\frac{\sqrt{5}+1}{4}$

$= -2\left(\frac{5-1}{16}\right) = -\frac{1}{2}$

33. (B)

$\cot\theta - 2\cot 2\theta$

$\frac{1}{\tan\theta} - \frac{2\times\left(1-\tan^2\theta\right)}{2\tan\theta}$

$= \frac{\tan^2\theta}{\tan\theta} = \tan\theta$

34. (D)

$\frac{\cos 10 - \sqrt{3}\sin 10}{\sin 10.\cos 10}$

$= 2.2\frac{\left(\frac{1}{2}\cos 10 - \frac{\sqrt{3}}{2}\sin 10\right)}{2\sin 10\cos 10}$

$= \frac{4\sin\left(30-10\right)}{\sin 20} = 4$

35. (B)

$\cos\alpha + \cos\beta = 0 \Rightarrow \cos^2\alpha + \cos^2\beta = -2\cos\alpha\cos\beta$

$\& \sin\alpha + \sin\beta = 0 \Rightarrow \sin^2\alpha + \sin^2\beta = -2\sin\alpha\sin\beta$

Now $\cos 2\alpha + \cos 2\beta = 2\left(\cos^2\alpha + \cos^2\beta - 1\right) = 2\left(1 - \sin^2\alpha - \sin^2\beta\right)$

$\Rightarrow \cos 2\alpha + \cos 2\beta = 2\left(-2\cos\alpha\cos\beta - 1\right) \quad ...(i)$

$\& \cos 2\alpha + \cos 2\beta = 2\left(1 + 2\sin\alpha\sin\beta\right) \quad \ldots(ii)$

(i) + (ii)

$\Rightarrow \cos 2\alpha + \cos 2\beta = 2\cos\left(\alpha - \beta\right)$

36. (C)

$$\sqrt{3}\operatorname{cosec}20^0 - \sec 20^0 = \frac{\frac{\sqrt{3}}{2}\cos 20^0 - \frac{1}{2}\sin 20^0}{\frac{1}{2}\cos 20^0 \sin 20^0}$$

$$= \frac{4\sin\left(60^0 - 20^0\right)}{2\cos 20^0 \sin 20^0} = \frac{4\sin 40^0}{\sin 40^0} = 4$$

37. (C)

$$\sqrt{3}\left[\cot\theta + \tan\theta\right] = 4$$

$$\Rightarrow \sqrt{3}\left[\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right] = 4$$

$$\Rightarrow \frac{1}{\sin\theta\cos\theta} = \frac{4}{\sqrt{3}} \Rightarrow \frac{1}{2\sin\theta\cos\theta} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$$

$$\Rightarrow 2\theta = \frac{\pi}{3} \qquad \therefore \theta = \frac{\pi}{6}$$

38. (B)

Since, $\cos 20^\circ - \sin 20^\circ = p$

$$\Rightarrow \cos^2 20^\circ + \sin^2 20^\circ - 2\sin 20^\circ \cos 20^\circ = p^2$$

$$\Rightarrow 1 - p^2 = \sin 40^\circ$$

$$\Rightarrow 1 - p^2 = \sqrt{1 - \cos^2 40^\circ}$$

$$\Rightarrow \left(1 - p^2\right)^2 = 1 - \cos^2 40^\circ$$

$$\Rightarrow \cos^2 40^\circ = 1 - \left(1 + p^4 - 2p^2\right)$$

$$\Rightarrow \cos 40^\circ = \sqrt{2p^2 - p^4}$$

$$\Rightarrow \cos 40^\circ = p\sqrt{2 - p^2}$$

$\therefore$ (B) holds

39. (B)

$$\text{LHS} = \left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 + \cos\frac{5\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right)$$

$$= \left(1 - \cos^2\frac{\pi}{8}\right)\left(1 - \cos^2\frac{3\pi}{8}\right)$$

$$= \sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8} = \frac{1}{4}\left[\cos\frac{\pi}{4} - \cos\frac{\pi}{2}\right]^2 \frac{1}{8}$$

40. (C)

$$\sin\theta = \frac{2t}{1+t^2} = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} \text{ if } t = \tan\frac{\theta}{2}$$

$$\therefore \cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{1-t^2}{1+t^2}$$

41. (D)

$$\frac{1}{4}\left(4\cos\theta\cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right)\right)$$

$$= \frac{\cos 3\theta}{4}$$

42. (B)

$$\frac{\cos 20 + 4(\cos 40 - \cos 60)\sin 70}{\cos^2 10}$$

$$= \frac{\cos 20 + 4\cos 40\cos 20 - 2\cos 20}{\cos^2 10}$$

$$= \frac{2(\cos 20 + 2\cos 60 + 2\cos 20 - 2\cos 20)}{2\cos^2 10}$$

$$= \frac{2(\cos 20 + 1)}{(1+\cos 20)} = 2$$

43. (C)

$$\sin 12^0 \sin 48^0 \sin 54^0 = \sin(60^0 - 12)\sin 12^0 \sin(60^0 + 12^0)\frac{\sin 54^0}{\sin 72^0}$$

$$= \frac{\sin 36^0 \sin 54^0}{4\sin 72^0} = \frac{\sin 36^0 \sin 54^0}{8\sin 36^0 \cos 36^0}$$

$$= \frac{\sin 54^0}{8\cos 36^0}\text{, but } \sin 54^0 = \sin(90^0 - 36^0) = \cos 36^0$$

$$\therefore \sin 12^0 \sin 48^0 \sin 54^0 = \frac{1}{8}$$

44. (B)

$$x + \frac{1}{x} = 2\cos\theta \ \ \& \ \ x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 8\cos^3\theta - 6\cos\theta = 2\cos 3\theta$$

45. (C)

$$\sin\frac{\pi}{10}\sin\frac{13\pi}{10}$$

$$= \sin\frac{\pi}{10}\sin\left(\pi + \frac{3\pi}{10}\right)$$

$$= -\sin\frac{\pi}{10}\sin\frac{3\pi}{10} = -\sin 18^\circ \sin 54^\circ$$

$$= -\frac{\sqrt{5}-1}{4}.\frac{\sqrt{5}+1}{4} = -\frac{5-1}{16} = -\frac{1}{4}$$

46. (C)

$3\sin 2\theta = 2\sin 3\theta$

$\Rightarrow 6\sin\theta\cos\theta = 2\left(3\sin\theta - 4\sin^3\theta\right)$

$\Rightarrow 4\cos^2\theta - 3\cos\theta - 1 = 0$

$\Rightarrow \cos\theta = 1, \frac{1}{4}$

Since $\cos\theta \neq 1 \left(\because \theta \neq 0\right)$

$\therefore \cos\theta = \frac{1}{4} \Rightarrow \sin\theta = \frac{\sqrt{15}}{4}$

47. (C)

$f\left(\theta\right) = \sin\theta\left(\sin\theta + 3\sin\theta - 4\sin^3\theta\right)$

$f\left(\theta\right) = 4\sin^2\theta - 4\sin^4\theta$

$f\left(\theta\right) = 4\sin^2\theta\left(1 - \sin^2\theta\right) \geq 0 \quad \forall\theta$

48. (D)

$f\left(\theta\right) = 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$

$= 5\cos\theta + 3\cos\theta.\cos\frac{\pi}{3} - 3\sin\theta\sin\frac{\pi}{3} + 3$

$= \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$-\sqrt{\frac{13^2}{4} + \frac{27}{4}} + 3 \leq f\left(\theta\right) \leq \sqrt{\frac{13^2}{4} + \frac{27}{4}} + 3$

$-4 \leq f\left(\theta\right) \leq 10$

49. (C)

$\sin\theta + \cos\beta$

Maximum Value $= 2$

50. (C)

$$y = \frac{12}{9 + 5\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right)}$$

$$= \frac{12}{9 + 5\sin\left(\alpha + x\right)} \text{let} \left(\sin\alpha = \frac{3}{5}\right)$$

$$y_{\text{maximum}} = \frac{12}{4} = 3$$

51. (D)

$y_{minimum} = \frac{1}{9}$

52. (A)

$B = 2\sin^2 x - \cos 2x$

$= 2\sin^2 x - \left(1 - 2\sin^2 x\right)$

$= 4\sin^2 x - 1$

Since $0 \le \sin^2 x \le 1 \quad \therefore 0 \le 4\sin^2 x \le 4$

$\Rightarrow \quad 0 - 1 \le 4\sin^2 x - 1 \le 3 \Rightarrow -1 \le B \le 3$

53. (B)

$\sin\theta + \cos\theta = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$ is Max when $\theta + \frac{\pi}{4} = \frac{\pi}{2}$ i.e. when $\theta = \frac{\pi}{4} = 45^\circ$

54. (B)

$2\sin^2\theta + 3\cos^2\theta$

$= 2\left[\sin^2\theta + \cos^2\theta\right] + \cos^2\theta = 2 + \cos^2\theta$

Since least value of $\cos^2\theta = 0$

$\therefore$ least value of $2\sin^2\theta + 3\cos^2\theta = 2 + 0 = 2$

55. (C)

Let $y = \cos\theta - \cos 2\theta$

$\frac{dy}{dx} = \sin\theta + 2\sin 2\theta = 2.2\sin\theta\cos\theta - \sin\theta$

$= \sin\theta\left(4\cos\theta - 1\right)$

For Max or Min of $y, \frac{dy}{d\theta} = 0$

$\therefore \ \sin\theta = 0$ or $\cos\theta = \frac{1}{4}$

i.e. $\theta = 0, \pi, 2\pi ...... \text{or} \cos\theta = \frac{1}{4}$

$\frac{d^2y}{d\theta^2} = -\cos\theta + 4\cos 2\theta > 0$ for $\theta = 0, \pi 2\pi, ...$ and $4\cos 2\theta - \cos\theta = 4\left[2\cos^2\theta - 1\right] - \cos\theta$

$= 8\left(\frac{1}{4}\right)^2 - 4 - \frac{1}{4} = \frac{1}{2} - 4 - \frac{1}{4} < 0$

$\therefore$ y max. when $\cos\theta = \frac{1}{4}$

$\therefore$ max.y $= \cos\theta - \left(2\cos^2\theta - 1\right)$

$= \frac{1}{4} - \left[\frac{2}{16} - 1\right]$

$= \frac{1}{4} + 1 - \frac{1}{8} = \frac{1}{4} + \frac{7}{8}$

$= \frac{2+7}{8} = \frac{9}{8}$

$\therefore$ (C) holds

56. (D)

$\frac{8\sin x}{8} \cdot \frac{\cos x}{8} \cdot \frac{\cos x}{4} \cdot \frac{\cos x}{2}$

$= \theta = \frac{x}{7}$

$8\sin\theta \cdot \cos\theta \cdot \cos 2\theta \cdot \cos 4\theta$

$= 8\sin\theta \cdot \frac{\sin\left(2^3\theta\right)}{2^3 \sin\theta} = \sin 8\theta = \sin x$

57. (C)

We know that

$\cos A \cos 2A \cos 2^2 A \cos 2^3 A \ \cos 2^{n-1} = \frac{\sin 2^n A}{2^n \sin A}$

$\therefore \cos A \cos 2A \cos 2^2 A = \frac{\sin 2^3 A}{2^3 \sin A}$ [By putting $n = 3$]

$\therefore \cos A \cos 2A \cos 4A = \frac{\sin 8A}{8\sin A}$

$\therefore \cos\frac{\pi}{7} \cos\frac{2\pi}{7} \cos\frac{4\pi}{7} = \frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}}$

$= \frac{\sin\left(\pi + \frac{\pi}{7}\right)}{8\sin\frac{\pi}{7}} = \frac{-\sin\frac{\pi}{7}}{8\sin\frac{\pi}{7}} = -\frac{1}{8}$

Hence (C) holds

58. (D)

The given expression

$= \frac{1}{2\sin\frac{\pi}{65}}\left(2\sin\frac{\pi}{65}\cos\frac{\pi}{65}\right)\cos\frac{2\pi}{65}\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}$

$= \frac{1}{4\sin\frac{\pi}{65}}\left(2\sin\frac{2\pi}{65}\cos\frac{2\pi}{65}\right)\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}$

$= \frac{1}{8\sin\frac{\pi}{65}}\left(2\sin\frac{4\pi}{65}\cos\frac{4\pi}{65}\right)\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}$

$= \frac{1}{16\sin\frac{\pi}{65}}\left(2\sin\frac{8\pi}{65}\cos\frac{8\pi}{65}\right)\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}$

$$=\frac{1}{32\sin\frac{\pi}{65}}\left(2\sin\frac{16\pi}{65}\cos\frac{16\pi}{65}\right)\cos\frac{32\pi}{65}$$

$$=\frac{1}{64\sin\frac{\pi}{65}}\left(2\sin\frac{32\pi}{65}\cos\frac{32\pi}{65}\right)$$

$$=\frac{1}{64\sin\frac{\pi}{65}}\sin\frac{64\pi}{65}$$

$$=\frac{1}{64\sin\frac{\pi}{65}}\sin\left(\pi-\frac{\pi}{65}\right)=\frac{\sin\frac{\pi}{65}}{64\sin\frac{\pi}{65}}=\frac{1}{64}$$

59. (A)

$\cos A\cos 2A\cos 4A....\cos 2^{n-1}A$

$=\frac{\sin 2^n A}{2^n \sin A}$ [Standard Result]

$\therefore$ (A) hold.

60. (B)

$A+B+C=\frac{3\pi}{2}$

$\cos 2A+\cos 2B+\cos 2C+4\sin A\sin B\sin C$

$=2\cos(A+B)\cos(A-B)+2\cos^2 C-1+4\sin A\sin B\sin C$

$=2\cos\left(3\frac{\pi}{2}-c\right)\cos(A-B)+2\cos^2 C-1+4\sin A.\sin B.\sin C$

$=-2\sin C\cos(A-B)+1-2\sin^2 C+4\sin A\sin B\sin C$

$=-2\sin C\left[\cos(A-B)-\cos(A+B)\right]+4\sin A\sin B\sin C$

$=-4\sin A\sin B\sin C+4\sin A\sin B\sin C+1$

61. (C)

If $A+B+C=n\pi$ then

$\tan A+\tan B+\tan C=\tan A\tan B\tan C$

62. (B)

$\left(\frac{\cos B}{\cos B}+\frac{\cos C}{\sin C}\right)\left(\frac{\cos C}{\sin C}+\frac{\cos A}{\sin A}\right)\left(\frac{\cos A}{\sin A}+\frac{\cos B}{\sin B}\right)$

$=\frac{\sin(B+C).\sin(A+C).\sin(A+B)}{\sin^2 A.\sin^2 B\sin^2 C}$

$=\operatorname{cosec}A.\operatorname{cosec}B.\operatorname{cosec}C$

$A+B=\pi-C$

$\Rightarrow \sin(A+B)=\sin C$

63. (A)

$A+B=C=180^\circ \Rightarrow \frac{A}{2}+\frac{B}{2}=90^\circ-\frac{C}{2}$

$\therefore \tan\left(\frac{A}{2}+\frac{B}{2}\right)=\tan\left(90^\circ-\frac{C}{2}\right)=\cot\frac{C}{2}$

$\therefore \frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\frac{1}{\tan\frac{C}{2}}$

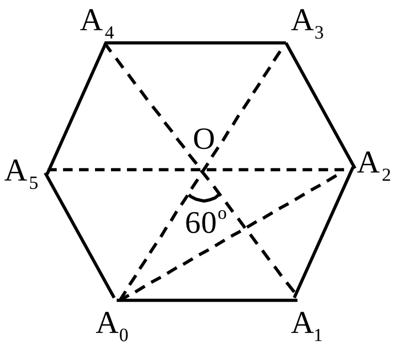
$\Rightarrow \tan\frac{A}{2}\tan\frac{C}{2}+\tan\frac{B}{2}\tan\frac{C}{2}=1-\tan\frac{A}{2}\tan\frac{B}{2}$

$\Rightarrow \tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}+\tan\frac{A}{2}\tan\frac{B}{2}=1$

Divide thro' out by $\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}$, we get

$\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$

64. (C)



OA_0A_1 is equilateral triangle

$A_0A_1=1$

$\cos120^\circ=\frac{1^2+1^2-A_0A_2{}^2}{2.1.1.}$ **(cosine law)**

$-\frac{2}{2}=2-A_0A_2{}^2$

$A_0A_2{}^2=3$

$A_0A_2=\sqrt{3}$

$A_0A_4=\sqrt{3}$

$A_0A_1\times A_0A_2+A_0A_4=1.\sqrt{3}.\sqrt{3}=3$

65. (C)

$\frac{b}{y}=\cot\theta$ & $\frac{a}{x}=\operatorname{cosec}\theta$

$\operatorname{cosec}^2\theta-\cot^2\theta=\frac{a^2}{x^2}-\frac{b^2}{y^2}$

$\frac{a^2}{x^2}-\frac{b^2}{y^2}=1$

66. (A)

Required number of sides $= \frac{360^\circ}{36^\circ} = 10^\circ$

$\therefore$ (A) holds

67. (A)

$\cos A = m\cos B \Rightarrow \frac{\cos A}{\cos B} = m$

Apply component to get $\frac{\cos A + \cos B}{\cos A - \cos B} = \frac{m+1}{m-1}$

$$\Rightarrow -\frac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{2\sin\frac{A+B}{2}\sin\frac{A-B}{2}} = \frac{m+1}{m-1}$$

$$\Rightarrow \cot\frac{A+B}{2} = \frac{m+1}{m-1}\tan\frac{B-A}{2}$$

68. (A)

$\frac{\sin 2x}{\sin 2y} = n \Rightarrow \frac{\sin 2x + \sin 2y}{\sin 2x - \sin 2y} = \frac{n+1}{n-1}$ {By componendo & dividendo}

$$\Rightarrow \frac{2\sin(x+y)\cos(x-y)}{2\cos(x+y)\sin(x-y)} = \frac{n+1}{n-1}$$

$$\Rightarrow \frac{\tan(x+y)}{\tan(x-y)} = \frac{n+1}{n-1}$$

69. (D)

$$\frac{\sin\alpha}{\sin\beta} = \frac{5}{3}$$

$$\frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \frac{8}{2} = 4\ (C \& D)$$

$$\frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)} = 4$$

$$\frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = 4$$

70. (B)

$\operatorname{cosec} A(\sin B\cos C + \cos B\sin C)$

$= \operatorname{cosec} A\sin(B+C)$

$= \operatorname{cosec} A\sin(180^\circ - A)$ $\quad [\because A+B+C = 180^\circ]$

$= \operatorname{cosec} A\sin A = 1$

Hence, (B) holds

1. (0.625)

$\sin 2\theta + \sin 2\phi = \frac{1}{2}$ & $\cos 2\theta + \cos 2\phi = \frac{3}{2}$

$\Rightarrow 2\sin(\theta+\phi)\cdot\cos(\theta-\phi) = \frac{1}{2}$ ...(1)

$2\cos(\theta+\phi)\cdot\cos(\theta-\phi) = \frac{3}{2}$ ...(2)

Square & add (1) & (2)

$\Rightarrow \ 4\cos^2(\theta-\phi)\cdot\left[\sin^2(\theta+\phi)+\cos^2(\theta+\phi)\right] = \frac{1}{4}+\frac{9}{4}$

$\Rightarrow \ \cos^2(\theta-\phi) = \frac{10}{16}$

2. (0.0625)

$\left[\cos 20^\circ \cdot \cos(60-20^\circ)\cdot\cos(60+20^\circ)\right]\cdot\cos 60^\circ$

$= \frac{1}{4}\cdot\cos 60^\circ\cdot\cos 60^\circ = \frac{1}{16}$

3. (0.78)

$\cos(A+B) = \cos A\cdot\cos B - \sin A\cdot\sin B$

$\sin A = \frac{1}{\sqrt{5}} \Rightarrow \cos A = \sqrt{1-\sin^2 A} = \frac{2}{\sqrt{5}}$

$\cos B = \frac{3}{\sqrt{10}} \Rightarrow \sin B = \sqrt{1-\cos^2 B} = \frac{1}{\sqrt{10}}$

$\therefore \ \cos(A+B) = \frac{2}{\sqrt{5}}\cdot\frac{3}{\sqrt{10}} - \frac{1}{\sqrt{5}}\cdot\frac{1}{\sqrt{10}} = \frac{1}{\sqrt{2}}$

$\therefore \ A+B = \frac{\pi}{4}$

4. (2)

$A-B = \frac{\pi}{4} \Rightarrow \tan(A-B) = 1$

$\Rightarrow \ \tan A - \tan B = 1 + \tan A\cdot\tan B$

$\therefore \ (1+\tan A)\cdot(1-\tan B) = 1 + (\tan A - \tan B) - \tan A\tan B$

$= 2$

5. (0.57)

Divide numerator & denominator by $\cos\theta$

$\frac{2\tan\theta-5}{4-5\tan\theta} = \frac{3-5}{4-5\cdot\frac{3}{2}} = \frac{-2}{-\frac{7}{2}} = \frac{4}{7}$

6. (0.26)

$$\frac{2\sin\left(x+\frac{\pi}{6}\right)\cdot\cos\left(x+\frac{\pi}{6}\right)}{2}$$

$$=\frac{\sin\left(2x+\frac{\pi}{3}\right)}{2}$$

Maximum $=\frac{1}{2}$ if $\sin\left(2x+\frac{\pi}{3}\right)=1$

$$\Rightarrow\ x=\frac{\pi}{12}$$

7. (0.75)

$$\cos^2 108^o+\cos^2 144^o$$

$$=\sin^2 18^o+\cos^2 36^o$$

$$=\left(\frac{\sqrt{5}-1}{4}\right)^2+\left(\frac{\sqrt{5}+1}{4}\right)^2=\frac{12}{16}=\frac{3}{4}$$

8. (1)

$$x\cdot 1\cdot\frac{1}{2}=\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{3}}$$

$$\Rightarrow\ x=1$$

9. (1.5)

$$\cos^2\frac{\pi}{12}+\cos^2\frac{\pi}{4}+\cos^2\frac{5\pi}{12}$$

$$=\cos^2 15^o+\cos^2 45^o+\cos^2 75^o$$

$$=\left(\sin^2 75^o+\cos^2 75^o\right)+\cos^2 45^o$$

$$=1+\frac{1}{2}=\frac{3}{2}$$

10. (1)

$$\frac{\sin 85^o-\sin 35^o}{\cos 65^o}$$

$$=\frac{2\sin 25^o\cdot\cos 60^o}{\cos 65^o}\qquad\left\{\sin 25^o=\cos 65^o\right\}$$

$$=1$$

11. (0.19)

$$\cos\left(\frac{7\pi}{6}\right)=\cos\left(\pi+\frac{\pi}{6}\right)=-\cos\frac{\pi}{6}=-\frac{\sqrt{3}}{2}$$

$$\cos\frac{2\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}=-\frac{1}{2}$$

$$\therefore\ \text{Ans}=\left(1+\frac{\sqrt{3}}{2}\right)\left(1+\frac{1}{2}\right)\cdot\left(1-\frac{1}{2}\right)\cdot\left(1-\frac{\sqrt{3}}{2}\right)$$

$$=\left(1-\frac{\sqrt{3}^2}{2^2}\right)\cdot\left(1-\frac{1}{2^2}\right)$$

$$=\left(1-\frac{3}{4}\right)\cdot\left(1-\frac{1}{4}\right)=\frac{3}{16}$$

12. (2)

$$\cos 54^\circ=\cos\left(90^\circ-36^\circ\right)=\tan 36^\circ$$

13. (1)

$$\cot\left(102^\circ\right)=\cot\left(90^\circ+12^\circ\right)=-\tan 12^\circ$$

$$\therefore \text{ Required }=\cot 12^\circ\left(-\tan 12^\circ\right)-\tan 12^\circ\cdot\cot 66^\circ+\cot 66^\circ\cot 12^\circ$$

$$=-1+\cot 66^\circ\cdot\left[\cot 12^\circ-\tan 12^\circ\right]$$

$$=-1+\cot 66^\circ\left(2\cot 24^\circ\right)$$

$$=-1+\left(\tan 24^\circ\right)\cdot\left(2\cot 24^\circ\right)$$

$$=-1+2=1 \qquad \left\{\cot 66^\circ=\tan 24^\circ\right\}$$

14. (1.69)

$$\tan 2\alpha=\tan\left(\alpha+\beta+\alpha-\beta\right)$$

$$=\frac{\tan\left(\alpha+\beta\right)+\tan\left(\alpha-\beta\right)}{1-\tan\left(\alpha+\beta\right)\cdot\tan\left(\alpha-\beta\right)} \qquad \ldots(1)$$

$$\cos\left(\alpha+\beta\right)=\frac{4}{5}$$

$$\Rightarrow \tan\left(\alpha+\beta\right)=\sqrt{\sec^2\left(\alpha+\beta\right)-1}$$

$$=\frac{3}{4} \qquad \ldots(2)$$

$$\sin\left(\alpha-\beta\right)=\frac{5}{13}$$

$$\Rightarrow \tan\left(\alpha-\beta\right)=\frac{\sin\left(\alpha-\beta\right)}{\sqrt{1-\sin^2\left(\alpha-\beta\right)}}$$

$$=\frac{5}{12} \qquad \ldots(3)$$

Put (2) & (3) in (1)

$$\tan 2\alpha=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}}=\frac{\frac{14}{12}}{1-\frac{5}{16}}=\frac{56}{33}$$

15. (0.40)

$$\cos^2 A-\sin^2 B=\cos\left(A+B\right)\cdot\cos\left(A-B\right)$$

$$=\cos\left(48+12\right)\cdot\cos\left(48-12\right)$$

$$=\cos 60^\circ\cdot\cos 36^\circ$$

$$=\frac{1}{2}\times\left(\frac{\sqrt{5}+1}{4}\right)$$

16. (1.73)

$$A=\frac{2\pi}{5},\ B=\frac{\pi}{15}$$

$$A-B=\frac{2\pi}{5}-\frac{\pi}{15}=\frac{\pi}{3}$$

$$\Rightarrow\ \tan(A-B)=\sqrt{3}$$

$$\Rightarrow\ \frac{\tan A-\tan B}{1+\tan A\cdot\tan B}=\sqrt{3}$$

$$\Rightarrow\ \tan A-\tan B=\sqrt{3}+\sqrt{3}\tan A\tan B$$

$$\Rightarrow\ \text{Ans.}=\sqrt{3}$$

17. (3.87)

$$\frac{\sin x}{\sin y}\cdot\frac{\cos y}{\cos x}=\frac{1/2}{3/2}\quad \&\quad \frac{\tan x}{\tan y}=\frac{1}{3}$$

$$\Rightarrow\ \tan x=\frac{\tan y}{3}\qquad ...(1)$$

Square & add

$$\sin^2 x+\cos^2 x=1$$

$$\Rightarrow\ \frac{\sin^2 y}{4}+\frac{9}{4}\cos^2 y=1$$

$$\Rightarrow\ 1-\cos^2 y+9\cos^2 y=4$$

$$\Rightarrow\ \cos^2 y=\frac{3}{8}$$

$$\Rightarrow\ \tan y=\sqrt{\sec^2 y-1}$$

$$\tan y=\sqrt{\frac{5}{3}}\qquad ...(2)$$

(1) & (2)

$$\therefore\ \tan x=\frac{1}{3}\sqrt{\frac{5}{3}}$$

$$\therefore\ \tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\cdot\tan y}$$

$$=\frac{\frac{4\tan y}{3}}{\frac{1-\tan^2 y}{3}}=\frac{4\tan y}{3-\tan^2 y}$$

$$=\frac{4\sqrt{5}/3}{3-\frac{5}{3}}=\sqrt{15}$$

18. (1.5)

$$\sin\frac{7\pi}{8}=\sin\left(\pi-\frac{\pi}{8}\right)=\sin\frac{\pi}{8}$$

$$\sin\frac{5\pi}{8}=\sin\left(\pi-\frac{3\pi}{8}\right)=\sin\frac{3\pi}{8}$$

Further $\sin\frac{3\pi}{8}=\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)$

$=\cos\frac{\pi}{8}$

$\therefore$ Required $=\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}$

$=2$

19. (0.5)

$A+B=90^o$

$\therefore \sin A\sin B$

$=\sin A\cdot\sin(90-A)$

$=\sin A\cos A$

$=\frac{1}{2}\sin 2A$

$\therefore$ Maximum $=\frac{1}{2}$

20. (2)

$\frac{\cot x-\tan x}{\cot 2x}$

$=\frac{\cos^2 x-\sin^2 x}{\left(\frac{2\cos x\cdot\sin x}{2}\right)\cdot\cos 2x}=\frac{2\cos 2x}{\sin 2x\cot 2x}=2$

JEE Main : PYQ

1. (c)

Take $\cos 72°=\frac{\sqrt{5}-1}{4}$

Let $\sin 36°=\alpha$

$\Rightarrow\ 1-2\sin^2 36°=\frac{\sqrt{5}-1}{4}\Rightarrow 4-8\alpha^2=\sqrt{5}-1$

$\Rightarrow\ 5-8\alpha^2=\sqrt{5}$

Take square both sides,

$\Rightarrow\ \left(5-8\alpha^2\right)^2=5\ \Rightarrow\ 25+64\alpha^4-80\alpha^2=5$

$\Rightarrow\ 64\alpha^4-80\alpha^2+20=0\ \Rightarrow\ 16\alpha^4-20\alpha^2+5=0$

2. (c)

Given, $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

$=2\sin^2\frac{\pi}{8}\sin^2\frac{2\pi}{8}\sin^2\frac{3\pi}{8}=2\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}\cdot\frac{1}{2}$

$$=\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}=\sin^2\frac{\pi}{8}\sin^2\left(\frac{\pi}{2}-\frac{\pi}{8}\right)$$

$$=\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}=\frac{1}{4}\sin^2\left(\frac{\pi}{4}\right)=\frac{1}{8}$$

3. (b)

$$3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4\sin^6\theta$$

$$=3(1-2\sin\theta\cos\theta)^2+6(1+2\sin\theta\cos\theta)+4\sin^6\theta$$

$$=9+12\sin^2\theta\cos^2\theta+4\sin^6\theta$$

$$=9+12\cos^2\theta(1-\cos^2\theta)+4(1-\cos^2\theta)^3$$

$$=9+12\cos^2\theta-12\cos^4\theta+4(1-\cos^6\theta-3\cos^2\theta+3\cos^4\theta)$$

$$=9+4-4\cos^6\theta$$

$$=13-4\cos^6\theta$$

4. (b)

Given $2\cos\theta+\sin\theta=1$

Squaring both sides, we get $(2\cos\theta+\sin\theta)^2=1^2$

$$\Rightarrow\quad 4\cos^2\theta+\sin^2\theta+4\sin\theta\cos\theta=1$$

$$\Rightarrow\quad 3\cos^2\theta+(\cos^2\theta+\sin^2\theta)+4\sin\theta\cos\theta=1$$

$$\Rightarrow\quad 3\cos^2\theta+1+4\sin\theta\cos\theta=1$$

$$\Rightarrow\quad 3\cos^2\theta+4\sin\theta\cos\theta=0$$

$$\Rightarrow\quad \cos\theta(3\cos\theta+4\sin\theta)=0$$

$$\Rightarrow\quad 3\cos\theta+4\sin\theta=0$$

$$\Rightarrow\quad 3\cos\theta=-4\sin\theta$$

$$\Rightarrow\quad \frac{-3}{4}=\tan\theta=-\sqrt{\sec^2\theta-1}=\frac{-3}{4}\qquad\left(\because\ \tan\theta=\sqrt{\sec^2\theta-1}\right)$$

$$\Rightarrow\quad \sec^2\theta-1=\left(\frac{-3}{4}\right)^2=\frac{9}{16}$$

$$\Rightarrow\quad \sec^2\theta=\frac{9}{16}+1=\frac{25}{16}\quad\Rightarrow\quad\sec\theta=\frac{5}{4}$$

$$\text{or}\quad \cos\theta=\frac{4}{5}\qquad\ldots(\text{i})$$

Now, $\sin^2\theta+\cos^2\theta=1\ \Rightarrow\ \sin^2\theta+\left(\frac{4}{5}\right)^2=1$

$$\sin^2\theta+\left(\frac{4}{5}\right)^2=1\ \Rightarrow\sin^2\theta+\left(\frac{4}{5}\right)=1$$

$$\sin\theta=\pm\frac{3}{5}\qquad\ldots(\text{ii})$$

Taking $\left(\sin\theta = -\frac{3}{5}\right)$ because $\left(\sin\theta = \frac{3}{5}\right)$ cannot satisfy the given equation,

Therefore; $7\cos\theta + 6\sin\theta = 7\times\frac{4}{5} - 6\times\frac{3}{5} = \frac{28}{5} - \frac{18}{5} = 2$

5. (b)

Given : $\sin\theta = \frac{1}{2}$ and θ is acute angle

$\therefore \quad \theta = \frac{\pi}{6}$

Also given, $\cos\phi = \frac{1}{3}$ and ϕ is acute angle.

$\therefore \quad 0 < \frac{1}{3} < \frac{1}{2}$

$\Rightarrow \quad \cos\frac{\pi}{2} < \cos\phi < \cos\frac{\pi}{3}$ or $\frac{\pi}{3} < \phi < \frac{\pi}{2}$

$\therefore \quad \frac{\pi}{3} + \frac{\pi}{6} < \theta < \phi < \frac{\pi}{2} + \frac{\pi}{6}$ or $\frac{\pi}{2} < \theta + \phi < \frac{2\pi}{3}$

$\Rightarrow \quad \theta + \phi \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

6. (b)

Since, $\sin\left(\frac{\pi}{22}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{22}\right) = \cos\frac{5\pi}{11} = -\cos\frac{16\pi}{11}$

$\sin\frac{3\pi}{22} = \cos\frac{4\pi}{11}, \sin\frac{5\pi}{22} = \cos\frac{3\pi}{11} = -\cos\frac{8\pi}{11}$

$\sin\frac{7\pi}{22} = \cos\frac{2\pi}{11}, \sin\frac{9\pi}{22} = \cos\frac{\pi}{11}$

Now, $2\sin\frac{\pi}{22}\cdot\sin\frac{3\pi}{22}\cdot\sin\frac{5\pi}{22}\cdot\sin\frac{7\pi}{22}\cdot\sin\frac{9\pi}{22}$

$= 2\cos\frac{\pi}{11}\cdot\cos\frac{2\pi}{11}\cdot\cos\frac{4\pi}{11}\cdot\cos\frac{8\pi}{11}\cos\frac{16\pi}{11}$

$= \dfrac{2\times\sin 2^5\frac{\pi}{11}}{2^5\sin\frac{\pi}{11}} = \frac{1}{16} \qquad \left(\because \sin 2^5\frac{\pi}{11} = \sin\frac{\pi}{11}\right)$

7. (a)

We are given that

$\cot\alpha = 1, \sec\beta = \frac{-5}{3},$

$\Rightarrow \quad \cos\beta = -\frac{3}{5}, \tan\beta = -\frac{4}{3}, \tan\alpha = 1$

$$\Rightarrow \quad \tan(\alpha+\beta)=\frac{1-\frac{4}{3}}{1+\frac{4}{3}\times 1}=\frac{-1}{7}$$

$$\because \quad \tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta}$$

$$\Rightarrow \quad \tan(\alpha+\beta)=\frac{-1}{7} \text{ lies in IVth quadrant}$$

8. (b)

Given trigonometric equation is

$$\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)$$

$$=\left(\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)\right).$$

$$=\cos\left(\frac{4\pi}{7}\right)\left[2\cos\left(\frac{2\pi}{7}\right)+1\right]$$

$$=\cos\left(\frac{4\pi}{7}\right)\left[2\left(1-2\sin^2\left(\frac{\pi}{7}\right)+1\right]\right.$$

$$=\cos\left(\frac{4\pi}{7}\right)\left[3-4\sin^2\left(\frac{\pi}{7}\right)\right]=\frac{\sin\left(3\times\frac{\pi}{7}\right)}{\sin\frac{\pi}{7}}\times\cos\left(\frac{4\pi}{7}\right)$$

Multiply both sides by 2

$$=\frac{2\sin\left(\frac{3\pi}{7}\right)}{2\sin\frac{\pi}{7}}\times\cos\left(\frac{4\pi}{7}\right)$$

Apply $2\sin A\cos B=\sin(A+B)+\sin(A-B)$

$$=\frac{\sin\left(\frac{7\pi}{7}\right)+\sin\left(\frac{-\pi}{7}\right)}{2\sin\frac{\pi}{7}}, \text{ Here, } \sin(\pi)=0.$$

$$=\frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}}=-\frac{1}{2}$$

9. (b)

$16\sin 20^\circ\sin 40^\circ\sin 80^\circ$

$=16\sin 40^\circ\sin 20^\circ\sin 80^\circ$

$=4\left(4\sin(60-20)\sin(20)\sin(60+20)\right)$

$=4\times\sin(3\times 20^\circ) \qquad \left[\because\ \sin 3\theta=4\sin(60-\theta)\times\sin\theta\times\sin(60+\theta)\right]$

$= 4 \times \sin 60°$

$= 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$

10. (d)

Given trigonometric equation is

$\sin 12° + \sin 12° - \sin 72°$

$= \sin 12° - 2\cos 42° \sin 30°$

$= \sin 12° - \sin(90° - 48°) = \sin 12° - \sin 48°$

$= -2\cos 30° \sin 18° = -2 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{5}-1}{4} = \frac{\sqrt{3}}{4}\left(1-\sqrt{5}\right)$

11. (d)

$x = \sum_{n=0}^{\infty} \cos^{2n}\theta = 1 + \cos^2\theta + \cos^4\theta + ...$ (G.P.)

$= \frac{1}{1-\cos^2\theta} = \text{cosec}^2\theta \Rightarrow \sin^2\theta = \frac{1}{x}$ and $y = \sum_{n=0}^{\infty} \sin^{2n}\phi = 1 + \sin^2\phi + \sin^4\phi + ...$ (G.P.)

$= \frac{1}{1-\sin^2\phi} = \sec^2\phi \Rightarrow \cos^2\phi = \frac{1}{y}$

Now, $z = \sum_{n=0}^{\infty} \cos^{2n}\theta \sin^{2n}\phi$

$= 1 + \cos^2\theta\sin^2\phi + \left(\cos^2\theta\sin^2\phi\right)^2 + ...$ (G.P.)

$= \frac{1}{1-\cos^2\theta\sin^2\phi}.$

$\Rightarrow \quad x = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)} = \frac{xy}{xy-(x-1)(y-1)}$

$= \frac{xy}{x+y-1} \Rightarrow z(x+y) - z = xy \Rightarrow z(x+y) = xy + z$

12. (d)

$L + M = 1 - 2\sin^2\frac{\pi}{8} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ ...(i)

and $L - M = -\cos\frac{\pi}{8}$ ...(ii)

From equations (i) and (ii),

$L = \frac{1}{2}\left(\frac{1}{\sqrt{2}} - \cos\frac{\pi}{8}\right) = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\frac{\pi}{8}$ and $M = \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \cos\frac{\pi}{8}\right) = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

13. (b)

$$\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$$

$$= \left(\frac{1+\cos 20°}{2}\right) + \left(\frac{1+\cos 100°}{2}\right) - \frac{1}{2}(2\cos 10° \cos 50°)$$

$$= 1 + \frac{1}{2}(\cos 20° + \cos 100°) - \frac{1}{2}[\cos 60° + \cos 40°]$$

$$= \left(1 - \frac{1}{4}\right) + \frac{1}{2}[\cos 20° + \cos 100° - \cos 40°]$$

$$= \frac{3}{4} + \frac{1}{2}[2\cos 60° \times \cos 40° - \cos 40°] = \frac{3}{4}$$

14. (a)

$$f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x); \quad f_4(x) = \frac{1}{4}[\sin^4 x + \cos^4 x]$$

$$= \frac{1}{4}\left[(\sin^2 x + \cos^2 x)^2 - \frac{(\sin 2x)^2}{2}\right] = \frac{1}{4}\left[1 - \frac{(\sin 2x)^2}{2}\right]$$

$$f_6(x) = \frac{1}{6}[\sin^6 x + \cos^6 x]$$

$$= \frac{1}{6}\left[(\sin^2 x + \cos^2 x)^3 - \frac{3}{4}(\sin 2x)^2\right] = \frac{1}{6}\left[1 - \frac{3}{4}(\sin 2x)^2\right]$$

Now $f_4(x) - f_{(6)}(x) = \frac{1}{4} - \frac{1}{6} - \frac{(\sin 2x)^2}{8} + \frac{1}{8}(\sin 2x)^2 = \frac{1}{12}$

15. (a)

We have

$$5\tan^2 x - 5\cos^2 x = 2(2\cos^2 x - 1) + 9$$

$$\Rightarrow \quad 5\tan^2 x - 9\cos^2 x = -2 + 9$$

$$\Rightarrow \quad 5\tan^2 x = 9\cos^2 x + 7$$

$$\Rightarrow \quad 5(\sec^2 x - 1) = 9\cos^2 x + 7$$

Let $\cos^2 x = t \Rightarrow \frac{5}{t} - 9t - 12 = 0$

$$\Rightarrow \quad 9t^2 + 12t - 5 = 0 \;\Rightarrow\; 9t^2 + 15t - 3t - 5 = 0$$

$$\Rightarrow \quad (3t-1)(3t+5) = 0 \;\Rightarrow\; t = \frac{1}{3} \text{ as } t \neq -\frac{5}{3}.$$

$$\cos 2x = 2\cos^2 x - 1 = 2\left(\frac{1}{3}\right) - 1 = -\frac{1}{3}$$

$$\cos 4x = 2\cos^2 2x - 1 = 2\left(-\frac{1}{3}\right)^2 - 1 = -\frac{7}{9}$$

16. (a)

$$4+\frac{1}{2}\sin^2 2x-2\cos^4 x$$

$$4+2\left(1-\cos^2 x\right)\cos^2 x-2\cos^4 x=-4\left\{\cos^4 x-\frac{\cos^2 x}{2}-1+\frac{1}{16}-\frac{1}{16}\right\}$$

$$=-4\left\{\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\right\};\ 0\le\cos^2 x\le 1$$

$$\Rightarrow\ -\frac{1}{4}\le\cos^2 x-\frac{1}{4}\le\frac{3}{4}\ \Rightarrow\ 0\le\left(\cos^2 x-\frac{1}{4}\right)^2\le\frac{9}{16}$$

$$-\frac{17}{16}\le\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\le\frac{9}{16}-\frac{17}{16}$$

$$\frac{17}{4}\ge-4\left\{\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\right\}\ge\frac{4}{2}$$

$$M=\frac{17}{4}\ \Rightarrow\ m=2;\ M-m=\frac{17}{4}-2=\frac{9}{4}$$

17. (b)

Let $\cos\alpha+\cos\beta=\frac{3}{2}$

$$\Rightarrow\ 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=\frac{3}{2}\qquad\ldots(\text{i})$$

and $\sin\alpha+\sin\beta=\frac{1}{2}$

$$\Rightarrow\ 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=\frac{1}{2}\qquad\ldots(\text{ii})$$

On dividing (ii) by (i), we get $\tan\left(\frac{\alpha+\beta}{2}\right)=\frac{1}{3}$

Given: $0=\frac{\alpha+\beta}{2}\ \Rightarrow\ 2\theta=\alpha+\beta$

Consider $\sin 2\theta+\cos 2\theta=\sin\left(\alpha+\beta\right)+\cos\left(\alpha+\beta\right)$

$$=\frac{\frac{2}{3}}{1+\frac{1}{9}}+\frac{1-\frac{1}{9}}{1+\frac{1}{9}}=\frac{6}{10}+\frac{8}{10}=\frac{7}{5}$$

18. (9)

Let $f\left(x\right)=\frac{4}{\sin x}+\frac{1}{1-\sin x}$

$$\Rightarrow\ f'\left(x\right)=0\ \Rightarrow\ \sin x=\frac{2}{3}\qquad\left(\because\ x\in\left(0,\frac{\pi}{2}\right)\cos x\ne 0\right)$$

$\therefore \quad f(x)_{\min} = \dfrac{4}{2/3} + \dfrac{1}{1-2/3} = 9; \; f(x)_{\max.} \to \infty$

(x) is continuous function $\therefore \; \alpha_{\min} = 9$

19. (80)

$$\sin 10°\left(\frac{1}{2}.2\sin 20°\sin 40°\right).\sin 10°\sin(60°-10°)\sin(60°+10°)$$

$$= \sin 10°\frac{1}{2}(\cos 20° - \cos 60°)\cdot\frac{1}{4}\sin 30°$$

$$= \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{2}\cdot\sin 10°\left(\cos 20° - \frac{1}{2}\right)$$

$$= \frac{1}{16}\cdot\frac{1}{2}(2\sin 10°\cos 20° - \sin 10°)$$

$$= \frac{1}{32}(\sin 30° - \sin 10° - \sin 10°)$$

$$= \frac{1}{32}\left(\frac{1}{2} - 2\sin 10°\right) = \frac{1}{64}(1 - 4\sin 10°)$$

$$= \frac{1}{64} - \frac{1}{16}(\sin 10°)$$

Hence, $\alpha = \dfrac{1}{64} \Rightarrow \alpha^{-1} = 64$

Hence, $16 + \alpha^{-1} = 80$

20. (1)

$$\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7} \text{ and } \sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$$

$$\Rightarrow \quad \frac{\sqrt{2}\sin\beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$$

$$\therefore \quad \tan\alpha = \frac{1}{7} \text{ and } \sin\beta = \frac{1}{\sqrt{10}}; \; \tan\beta = \frac{1}{3}$$

$$\therefore \quad \tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2\cdot\frac{1}{3}}{1-\frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}\text{ r}$$

$$\tan(\alpha+2\beta) = \frac{\tan\alpha + \tan 2\beta}{1-\tan\alpha\tan 2\beta} = \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\cdot\frac{3}{4}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1$$

EXERCISE - 2 [A]

1. (B)

$\tan A = \sqrt{2} \qquad \tan^2 A = 2$

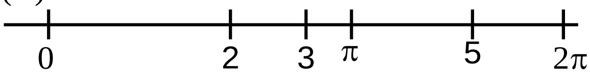
$\dfrac{\sin^4 A - 3\sin^2 A\cos^2 A + 7\cos^4 A}{1+\sin^2 A\cos^2 A + 5\cos^4 A}$

$= \dfrac{\sin^4 A - 3\sin^2 A\cos^2 A + 7\cos^4 A}{\left(\sin^2 A + \cos^2 A\right)^2 + \sin^2 A\cos^2 A + 5\cos^4 A}$

$= \dfrac{\tan^4 A - 3\tan^2 A + 7}{\left(\tan^2 A + 1\right)^2 + \tan^2 A + 5}$ (divide N & D by $\cos^4 x$)

$= \dfrac{4-6+7}{9+2+5} = \dfrac{5}{16}$

2. (A)



$\sin 2 > 0$

$\sin 3 > 0$

$\sin 5 < 0$

$\sin 2 \sin 3 \sin 5 < 0$

3. (A)

$f(x) = \sec x - \tan x$

$g(x) = \sec x + \tan x$

$f(x).g(x) = 1$

$f(A).f(B).f(C) = g(A).g(B).g(C)$

$f^2(A).f^2(B).f^2(C) = f(A)g(A)f(B).g(B).f(C)g(C)$

$f(A)f(B)f(C) = \pm 1$

4. (B)

$\cos(A) = \cos B\cos C$

$\cos(\pi - (B+C)) = \cos B\cos C$

$-[\cos B\cos C - \sin B\sin C] = \cos B\cos C$

$\sin B\sin C = 2\cos B\cos C$

$\tan B\tan C = 2$

5. (D)

$\sum_{r=0}^{10}\cos^3\left(\dfrac{\pi r}{3}\right)$

$1+\left(\dfrac{1}{2}\right)^3+\left(-\dfrac{1}{2}\right)^3+(-1)^3+\left(-\dfrac{1}{2}\right)^3+\left(\dfrac{1}{2}\right)^3+(1)^3+\left(\dfrac{1}{2}\right)^3+\left(-\dfrac{1}{2}\right)^3+(-1)^3+\left(-\dfrac{1}{2}\right)^3+(-1)^3+\left(-\dfrac{1}{2}\right)^3$

$= -\dfrac{1}{8}$

6. (B)

$U_n = 2\cos n\theta$

$U_1U_n - U_{n-1}$

$= 2\left(2\cos n\theta.\cos\theta\right) - 2\cos\left(n-1\right)\theta$

$= 2\left(\cos\left(n+1\right)\theta + \cos\left(n-1\right)\theta\right) - 2\cos\left(n-1\right)\theta$

$= 2\cos\left(n+1\right)\theta$

$= U_{n+1}$

7. (C)

$\cos 5\theta = a\cos 5\theta + b\cos 3\theta + c\cos\theta$

$\theta = 0 \qquad 1 = a+b+c$

$\theta = \frac{\pi}{3} \qquad \frac{1}{2} = \frac{a}{32} + \frac{b}{8} + \frac{c}{2}$

$\theta = \frac{\pi}{4} \qquad \frac{-1}{\sqrt{2}} = \frac{a}{4\sqrt{2}} + \frac{b}{2\sqrt{2}} + \frac{c}{\sqrt{2}}$

Solve: $a = 16, b = -20, c = 5$

8. (D)

$f\left(x\right) = \frac{\sin 3x}{\sin x} \qquad \sin x \neq 0$

$3 - 4\sin^2 x$

Range in $\left[-1, 3\right)$

9. (C)

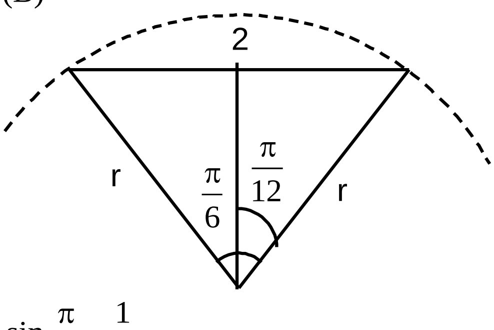
$\cos\left(\theta+\phi\right) = m\left(\cos\left(\theta-\phi\right)\right)$

$\frac{\cos\theta\cos\phi - \sin\theta\sin\phi}{\cos\theta\cos\phi + \sin\theta\sin\phi} = \frac{m}{1}$

$\frac{\sin\theta\sin\phi}{\cos\theta\cos\phi} = \frac{1-m}{1+m}$

$\tan\theta = \left(\frac{1-m}{1+m}\right)\cot\phi$

10. (B)



$\sin\frac{\pi}{12} = \frac{1}{r}$

$r = \frac{1}{\sin\frac{\pi}{12}} = \sqrt{6} + \sqrt{2}$

11. (B)

$$\frac{1-2\sin^2\frac{\alpha}{2}}{1+\sin\alpha}=\frac{\cos\alpha}{1+\sin\alpha}$$

$$=\frac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)^2}$$

$$=\frac{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}$$

$$=\frac{1-m}{1+m}$$

12. (C)

$A=\sin x\cos x$

$A=\frac{1}{2}\sin 2x$

$A\le\frac{1}{2}$

13. (D)

$\sin^8 75^\circ-\cos^8 75^\circ$

$=\left(\sin^4 75^\circ+\cos^4 75^\circ\right)\left(\sin^2 75^\circ-\cos^2 75^\circ\right)(1)$

$=\left(1-\frac{1}{2}\sin^2 150\right)\left(-\cos 150^\circ\right)$

$=\left(1-\frac{1}{8}\right)\left(+\frac{\sqrt{3}}{2}\right)$

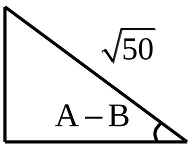
$=\frac{7\sqrt{3}}{16}$

14. (D)

$\tan A=3$

$\tan B=2$

$\tan(A-B)=\frac{3-2}{1+6}=\frac{1}{7}$



$$\sin(A-B)=\frac{1}{\sqrt{50}}$$

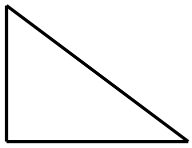
$$\sin 2(A-B)=2\times1\frac{1}{\sqrt{50}}\times\frac{7}{\sqrt{50}}$$

$$=\frac{7}{25}$$

15. (A)

$$\tan A+\tan B=a$$
$$\tan A.\tan B=b$$

$$\tan(A+B)=\frac{a}{1-b}$$



$$\sin(A+B)=\frac{a}{\sqrt{a^2+(1-b)^2}}$$

$$\sin^2(A+B)=\frac{a^2}{a^2+(1-b)^2}$$

16. (A)

$$\tan B=\frac{n\sin A\cos A}{1-n\cos^2 A}$$

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$=\frac{\frac{\sin A}{\cos A}+\frac{n\sin A\cos A}{1-n\cos^2 A}}{1-\frac{\sin A}{\cos A}\frac{n\sin A\cos A}{1-n\cos^2 A}}$$

$$=\frac{\frac{\sin A-n\cos^2 A\sin A+n\cos^2 A\sin A}{\cos A\left(1-n\cos^2 A\right)}}{\frac{1-n\cos^2 A-n\sin^2 A}{1-n\cos^2 A}}=\frac{\sin A}{\cos A(1-n)}$$

17. (B)

$$\tan\beta=\frac{1}{3}$$

$$\tan 2\beta=\frac{\frac{2}{3}}{1-\frac{1}{9}}=\frac{\frac{2}{3}}{\frac{8}{9}}=\frac{3}{4}$$

$$\tan\alpha=\frac{1}{7}$$

$$\tan(\alpha+2\beta)=\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{3}{28}}=\frac{\frac{4+21}{28}}{\frac{25}{28}}=1$$

$$\alpha+2\beta=\frac{\pi}{24}$$

18. (C)

$\sin A=\sin B$

$\cos A=\cos B$

$\Rightarrow A=B+2n\pi$

$\sin\left(\frac{A-B}{2}\right)=0$

19. (C)

$1-\frac{1}{\sin\theta+\cos\theta}\left(\sin^3\theta+\cos^3\theta\right)$

$=1-\frac{1}{\sin\theta+\cos\theta}(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)$

$=\sin\theta\cos\theta$

20. (B)

$\frac{\cos x\cos y+\sin x\sin y}{\cos x\cos y-\sin x\sin y}=\frac{a}{1}$

$\cot x\cot y=\frac{1+a}{a-1}$

21. (D)

$2\sin A=\sqrt{3}\sin B$ ________(1)

$2\cos A=\sqrt{5}\cos B$ _________-(2)

Square and add (1) & (2)

$3\sin^2 B+5\cos^2 B=4$

$3\tan^2 B+5=4\sec^2 B \text{ or } \tan B=1$

$\tan A=\frac{\sqrt{3}}{\sqrt{5}}$

$\tan A+\tan\beta=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}}$

22. (B)

$P_n-P_{n-2}=\cos^n\theta+\sin^n\theta-\cos^{n-2}\theta-\sin^{n-2}\theta$

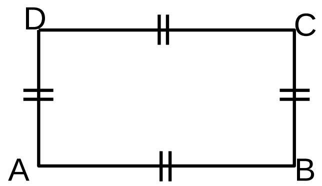
$=\cos^{n-2}\theta\left(-\sin^2\theta\right)+\sin^{n-2}\theta\left(-\cos^2\theta\right)$

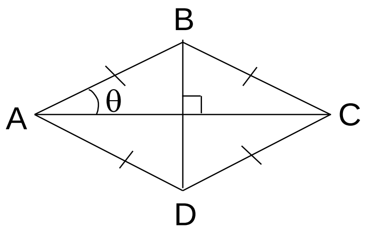
$=-\sin^2\theta\cos^2\theta\left(\cos^{n-4}\theta+\sin^{n-4}\theta\right)$

$=k\,P_{n-4}$

$K=-\sin^2\theta\cos^2\theta$

23. (C)
Given





$AB^2 = AC \times BD$

$AC^2 + BD^2 = 4AB^2$ & $\tan\theta = \dfrac{BD}{AC}$

$\Rightarrow AC^2 + BD^2 = 4AC\,BD$

$\Rightarrow \left(\dfrac{BD}{AC}\right)^2 + 1 = 4\left(\dfrac{BD}{AC}\right)$

$\tan^2\theta + 1 = 4\tan\theta$

$\sin 2\theta = \dfrac{1}{2} \Rightarrow \theta = 15^\circ$

Ans. $2\theta = 30^\circ$

24. (A)

$\sin\left(\dfrac{A}{2} + \left(\dfrac{A}{2} + \dfrac{C}{2}\right)\right) = K\sin\left(\dfrac{\pi - (A+B)}{2}\right)$

$\sin\left(\dfrac{A}{2} + \dfrac{\pi - B}{2}\right) = K\cos\left(\dfrac{A+B}{2}\right)$

$\cos\left(\dfrac{A-B}{2}\right) = K\cos\left(\dfrac{A+B}{2}\right)$

$$\frac{\cos\frac{A}{2}\cos\frac{B}{2} + \sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{B}{2} - \sin\frac{A}{2}\sin\frac{B}{2}} = \frac{K}{1}$$

$\dfrac{K-1}{K+1} = \tan\dfrac{A}{2}\tan\dfrac{B}{2}$

25. (A)

$$\frac{3 + \cot 76^\circ \cot 16^\circ}{\cot 76^\circ + \cot 16^\circ}$$

$\cot 76^\circ + \cot 16^\circ = \dfrac{\sin 92^\circ}{\sin 76^\circ \sin 16} = \dfrac{\sin 88}{\sin 76^\circ \sin 16^\circ}$

$1 + \cot 76 \cot 16 = \dfrac{1}{2\sin 76 \sin 16}$

$$\text{LHS} = \frac{3 + \cot 17 \cot 16}{\cot 76 + \cot 16} = \frac{2 + \dfrac{1}{2\sin 76 \sin 16}}{\dfrac{\sin 88}{\sin 76 \sin 16}} = \frac{2(\cos 60 - \cos 92) + 1}{2\sin 88^\circ}$$

$$= \frac{2\sin^2 46}{2\sin 44^\circ \cos 44} = \cot 44^\circ = \tan 46^\circ$$

26. (A)

$$\frac{2\left(\sin 1^\circ + \sin 2^\circ --------- \sin 89^\circ\right)}{2\left(\cos 1^\circ + \cos 2^\circ -------- \cos 44^\circ\right)+1}$$

$$N^r = 2\left(\sin 1^\circ + \sin 2^\circ ------ \sin 44^\circ\right) = \frac{2\sin\left(\frac{89}{2}\right)\sin 45}{\sin\left(\frac{1}{2}\right)}$$

$$D^r = 2\left(\cos 1^\circ + \cos 2^\circ -------- \cos 44^\circ\right) = \frac{2\sin 22^\circ \cos\frac{45}{2}}{\sin\frac{1^\circ}{2}} + 1$$

$$= \frac{-\sin\frac{1^\circ}{2} + \sin\frac{89^\circ}{2}}{\sin\frac{1^\circ}{2}} + 1$$

$$\frac{N^r}{D^r} = \frac{\frac{2\sin\frac{89}{2}\sin 45}{\sin\frac{1}{2}}}{\frac{\sin\frac{89}{2}}{\sin\frac{1}{2}}} = \sqrt{2} \qquad = \frac{\sin\frac{89}{2}}{\sin\frac{1}{2}}$$

27. (A)

$$x \in \left(\frac{5\pi}{4}, \frac{9\pi}{4}\right)$$

$$\sin x < -\frac{1}{2}$$

$$\cos x > -\frac{1}{2}$$

$$\sin x - \cos x < 0$$

$$\sin x - \cos x = t$$

$$1 - \sin 2x = t^2$$

$$t^2 = 1 - \frac{2024}{2025}$$

$$t^2 = \frac{1}{2025}$$

$$t = \frac{-1}{45}$$

28. (D)

$$\left(\tan 4\theta + \tan 2\theta\right)\left(1 - \tan^2 3\theta \tan^2 \theta\right)$$

$$= \frac{\sin 6\theta}{\cos 4\theta \cos 2\theta}\left(1 - \tan 3\theta \tan \theta\right)\left(1 + \tan 3\theta \tan \theta\right)$$

$$= \frac{\sin 6\theta}{\cos^2 3\theta \cos^2 \theta}$$

$= 2\tan 3\theta \sec^2\theta$

29. (D)

$\frac{1}{\tan x}+\frac{1-\sqrt{3}\tan x}{\sqrt{3}+\tan x}+\frac{1+\sqrt{3}\tan x}{\tan x-\sqrt{3}}$

$=\frac{3}{\tan 3x}=\frac{3\left(1-3\tan^2 x\right)}{3\tan x-\tan^3 x}$

30. (B)

$\frac{a\cos\alpha-b\sin\alpha}{\sin\alpha\cos\alpha}$

$=2\sqrt{a^2+b^2}\frac{\left(\sin\theta\cos\alpha-\cos\theta\sin\alpha\right)}{\sin 2\alpha}$

$=2\sqrt{a^2+b^2}\quad\frac{\sin\left(\theta-\alpha\right)}{\sin 2\alpha}\qquad\left(\theta=3\alpha\right)$

$=2\sqrt{a^2+b^2}$

31. (D)

$2\sin\frac{A}{2}=\sin\frac{A}{2}+\cos\frac{A}{2}-\left(\cos\frac{A}{2}-\sin\frac{A}{2}\right)$

$=-\sqrt{1+\sin A}+\sqrt{1-\sin A}$

32. (A)

$p^2=1-\sin 40^\circ$

$\sin 40^\circ=1-p^2 \qquad\qquad \left(p<0\right)$

$\cos 40^\circ=\sqrt{1-\left(1-p^2\right)^2}$

$=-p\sqrt{2-p^2}$

33. (C)

$\tan\frac{\alpha}{2}+\cot\frac{\alpha}{2}$

$=\frac{2}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$

$=\frac{2}{p}$

$x^2-\frac{2}{p}+1=0$

$px^2-2x+p=0$

34. (B)

$\frac{1}{\cos 290^\circ}+\frac{1}{\sqrt{3}\sin 250^\circ}$

$\frac{1}{\sin 20^\circ}-\frac{1}{\sqrt{3}\cos 20^\circ}$

$$\frac{2\left(\sqrt{3}\cos 20^{\circ} - \sin 20^{\circ}\right)}{\sqrt{3}\,2\sin 20^{\circ}\cos 20^{\circ}}$$

$$= \frac{4}{\sqrt{3}}\frac{\sin\left(60^{\circ} - 20^{\circ}\right)}{\sin 40^{\circ}} = \frac{4}{\sqrt{3}}$$

35. (B)

$$\sin x + \sin y = a$$

$$\cos x + \cos y = b$$

$$= 2$$

$$a = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$b = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\frac{a}{b} + \frac{b}{a} = \frac{1}{\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x+y}{2}\right)}$$

$$\frac{a^2 + b^2}{ab} = \frac{2}{\sin\left(x+y\right)}$$

$$\sin\left(x+y\right) = \frac{2ab}{a^2 + b^2}$$

36. (C)

$$\sqrt{2}\sin\frac{3\pi}{20} + \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\frac{\pi}{10} - \frac{1}{\sqrt{2}}\cos\frac{\pi}{10}\right)$$

$$= \sqrt{2}\sin\frac{3\pi}{20\pi} - \sqrt{2}\left(\sin\left(\frac{\pi}{4} - \frac{\pi}{10}\right)\right)$$

$$= 0$$

37. (A)

If $A + B = \frac{\pi}{4}$

$$\tan\left(A+B\right) = 1$$

$$\tan A + \tan B + \tan A \tan B + 1 = 2$$

$$\left(1 + \tan A\right)\left(1 + \tan\right) = 2$$

$A = 11^{\circ}$ $\qquad$ $B = 34^{\circ}$

$$\left(1 + \tan A\right)\left(1 + \tan 34^{\circ}\right) = 2$$

$A = 17^{\circ}$ $\qquad$ $B = 28^{\circ}$

$\left(1 + \tan 17^{\circ}\right)$ $\qquad$ $\left(1 + \tan 28^{\circ}\right) = 2$

Hence $\frac{\left(1 + \tan 11^{\circ}\right)\left(1 + \tan 34^{\circ}\right)}{\left(1 + \tan 17^{\circ}\right)\left(1 + \tan 28^{\circ}\right)} = 1$

38. (C)

$$\frac{4\sin 9^{\circ}\sin 51^{\circ}\sin 39^{\circ}\sin 51^{\circ}\sin 69^{\circ}\sin 81^{\circ}}{\sin 54^{\circ}}$$

$$=\frac{4\left(\sin 9^{\circ}\sin 51^{\circ}\sin 69^{\circ}\right)\left(\sin 21^{\circ}\sin 39^{\circ}\sin 81^{\circ}\right)}{\sin 54^{\circ}}$$

$$=\frac{4\frac{\left(\sin 27\right)}{4}\left(\frac{\sin 63}{4}\right)}{\sin 54}$$

$$=\frac{1}{4}\frac{\sin 27^{\circ}\cos 27^{\circ}}{\sin 54}=\frac{1}{8}$$

39. (C)

$$\left(\sin\frac{\pi}{10}+\sin\frac{13\pi}{10}\right)\cdot\left(\cos^2\frac{\pi}{6}-\cos^2\frac{\pi}{10}\right)$$

$$=\sin\frac{13\pi}{10}=\sin\left(\pi+\frac{3\pi}{10}\right)=-\sin\frac{3\pi}{10}$$

$$\therefore\ \left(\sin 18^{\circ}-\sin 54^{\circ}\right)\cdot\left(\frac{3}{4}-\cos^2\frac{\pi}{10}\right)$$

$$=\left(\frac{\sqrt{5}-1}{4}\right)-\left(\frac{\sqrt{5}+1}{4}\right)\cdot\frac{3\cos\frac{\pi}{10}-4\cos^3\frac{\pi}{10}}{4\cos\frac{\pi}{10}}$$

$$=-\frac{1}{2}\cdot\frac{-\cos\frac{3\pi}{10}}{4\cos\frac{\pi}{10}}=\frac{\cos 54^{\circ}}{8\cos 18^{\circ}}=\frac{\sin 36^{\circ}}{8\cos 18^{\circ}}$$

$$=\frac{1}{4}\sin 18^{\circ}=\frac{\sqrt{5}-1}{16}$$

40. (A)

$$\sin^2\theta=\frac{1}{2}\left(\frac{x}{y}+\frac{y}{x}\right)$$

Possible only when $x=y$ $\left(\frac{x}{y}+\frac{y}{x}\in(-\infty,-2]\cup[2,\infty)\right)$

41. (B)

$$\cos\frac{6\pi}{7}=-\cos\frac{\pi}{7}$$

$$\cos\frac{5\pi}{7}=-\cos\frac{2\pi}{7}$$

$$\cos\frac{3\pi}{7}=-\cos\frac{4\pi}{7}$$

$$\left(\cos\frac{\pi}{7}+\cos\frac{6\pi}{7}\right)+\left(\cos\frac{2\pi}{7}+\cos\frac{5\pi}{7}\right)+\left(\cos\frac{3\pi}{7}+\cos\frac{4\pi}{7}\right)+\cos\pi=-1$$

42. (D)

$\cot(\alpha+\beta)=0$

$\alpha+\beta=\frac{\pi}{2}$

$\sin(\alpha+2\beta)=\sin\left(\frac{\pi}{2}+\beta\right)=\cos\beta$

43. (C)

$\sin x=1-\sin^2 x$

$\sin x=\cos^2 x$

$=\sin^6 x+3\sin^5 x+3\sin^4 x+\sin^3 x-2$

$=\left(\sin x+\sin^2 x\right)^3-2$

$1-2=-1$

44. (D)

$\frac{1}{4}\left[\sqrt{3}\cos 23^\circ-\sin 23^\circ\right]$

$\frac{1}{2}\left[\sin 60^\circ\cos 23-\cos 60\sin 23\right]$

$\frac{1}{2}\sin 37^\circ$

45. (B)

$\tan 60^\circ=\sqrt{3}$

$\tan\left(20^\circ+40^\circ\right)=\sqrt{3}$

$\tan 20^\circ+\tan 40^\circ=\sqrt{3}-\sqrt{3}\tan 20^\circ\tan 40^\circ$

$\tan 20^\circ+\tan 40^\circ+\sqrt{3}\tan 20^\circ\tan 40^\circ=\sqrt{3}$

46. (A)

$\tan(\alpha+\beta)=1$

$\tan\alpha+\tan\beta=1-\tan\alpha\tan\beta$

$(1+\tan\alpha)(1+\tan\beta)=2$

$f(\theta)=\frac{1}{1+\tan\theta}$

$f(\alpha)f(\beta)=\frac{1}{(1+\tan\alpha)(1+\tan\beta)}=\frac{1}{2}$

47. (A)

If $A+B+C=\pi$ $\quad$ $\tan A+\tan B+\tan C=\tan A+\tan B+\tan C$

$\tan A\tan B\tan C=6$ $\quad$ $\tan A+\tan B=3$

$\tan A\tan B=2$ $\quad$ $\tan A=1$

$\Rightarrow\tan C=3$ $\quad$ $\tan B=2$

48. (D)

$$\frac{(\sin\alpha+\cos\alpha)^2}{\cos 2\alpha\left(\frac{\tan\alpha+1}{\tan\alpha-1}\right)}-\frac{1}{4}\sin 2\alpha\left[\cot\frac{\alpha}{2}-\tan\frac{\alpha}{2}\right]$$

$$\frac{(\sin\alpha+\cos\alpha)^2(\sin\alpha-\cos\alpha)}{(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)(\sin\alpha+\cos\alpha)}-\frac{2}{4}\sin\alpha\cos\alpha\frac{\cos\alpha}{\left(\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right)}$$

$$1-\cos^2\alpha=\sin^2\alpha$$

49. (C)

$$x=\sqrt{2+\sqrt{2-\sqrt{2+x}}}$$
$$x=2\cos\theta$$
$$2\cos\theta=\sqrt{2+\sqrt{2-2\cos\frac{\theta}{2}}}$$
$$=\sqrt{2+\sqrt{2\left(1-\cos\left(\frac{\theta}{2}\right)\right)}}$$
$$=\sqrt{2+2\sin\frac{\theta}{4}}$$
$$=\sqrt{2\left(1+\cos\left(\frac{\pi}{2}-\frac{\theta}{4}\right)\right)}$$
$$2\cos\theta=2\cos\left(\frac{\pi}{4}-\frac{\theta}{8}\right)$$
$$\theta=\frac{\pi}{4}-\frac{\theta}{8}$$
$$\frac{9\theta}{8}=45^\circ$$
$$\theta=40^\circ$$
$$\text{Hence } x=2\cos 40^\circ$$

50. (B)

$$\frac{1}{\sin 1^\circ}\left(\frac{\sin(46^\circ-45^\circ)}{\sin 46\sin 45}+\frac{\sin(48-47)}{\sin 48\sin 47}-------\right)$$

$$\frac{1}{\sin 1^\circ}(\cot 45-\cot 46+\cot 47-\cot 48------)$$

$$=\frac{1}{\sin 1^\circ}=\operatorname{cosec}1^\circ$$

51. (A)

$$3\sin x+4\cos x+12\sin y+5\cos y=18$$

$$5\cos\left(x-\tan^{-1}\frac{3}{4}\right)+13\cos\left(y-\tan^{-1}\frac{12}{5}\right)=18$$

Possible $x=\tan^{-1}\frac{3}{4}$

$$y = \tan^{-1}\frac{12}{5}$$

$$\tan(x+y) = \frac{\frac{3}{4}+\frac{12}{5}}{1-\frac{3}{4}\times\frac{12}{5}} = -\frac{63}{16}$$

52. (D)

$$\sum \tan B \tan C = x$$

$$(1-\tan B \tan C) = \frac{\cos(B+C)}{\cos B \cos C} = \frac{-\cos A}{\cos B \cos C}$$

$$\tan B \tan C = 1 + \frac{\cos A}{\cos B \cos C}$$

$$\sum \tan B \tan C = 3 + \sum \frac{\cos A}{\cos B \cos C}$$

$$= 3 + \sum \frac{\cos^2 A}{\cos A \cos B \cos C}$$

$$\sum \cos^2 A = \frac{3+\cos 2A+\cos 2B+\cos 2C}{2}$$

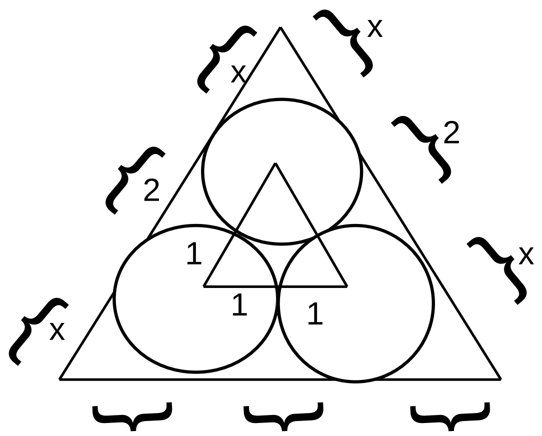
$$\frac{2-4\pi\cos A}{2} = 1-2\pi(\cos A)$$

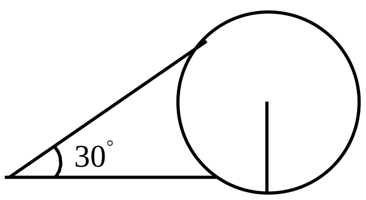
$$\sum \tan B \tan C = 3 + \frac{1-2\pi\cos A}{\pi\cos A}$$

$$= 1 + \frac{1}{\pi\cos A} = \frac{q+1}{q}$$

53. (B)

$$\tan 30^\circ = \frac{1}{x} = \frac{1}{\sqrt{3}}$$





$x = \sqrt{3}$

Side of equilateral triangle

$$=2+2\sqrt{3}$$

$$\text{Area } = \frac{\sqrt{3}}{4}\left(2+2\sqrt{3}\right)^2$$

$$=4\sqrt{3}+6$$

54. (C)

$$\sum_{m=1}^{6}\operatorname{cosec}\left(\theta+\frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta+\frac{m\pi}{4}\right)=4\sqrt{2}$$

$$\frac{1}{\sin\frac{\pi}{4}}\sum_{m=1}^{6}\frac{\sin\frac{\pi}{4}}{\sin\left(\theta+(m-1)\frac{\pi}{4}\right)\sin\left(\theta+\frac{M\pi}{4}\right)}=4\sqrt{2}$$

$$=\sqrt{2}\sum_{M=1}^{6}\frac{\sin\left(\theta+m\frac{\pi}{4}-\left(\theta+(m-1)\frac{\pi}{4}\right)\right)}{\sin\left(\theta+(m-1)\frac{\pi}{4}\right)\sin\left(\theta+\frac{m\pi}{4}\right)}$$

$$\frac{\sin(A-B)}{\sin A\sin B}=\cot B-\cot A$$

$$\sqrt{2}\sum_{m=1}^{6}\left(\cot\left(\theta+(m-1)\frac{\pi}{4}\right)-\cot\left(\theta+\frac{m\pi}{4}\right)\right)$$

$$\sqrt{2}\left(\cot\theta-\cot\left(\theta+\frac{3\pi}{2}\right)\right)=4\sqrt{2}$$

$$\cot\theta+\tan\theta=4$$

$$\sin 2\theta=\frac{1}{2}$$

$$2\theta=\frac{\pi}{6} \quad \text{or} \quad \frac{5\pi}{6}$$

$$\theta=\frac{\pi}{12} \quad \text{or} \quad \frac{5\pi}{12}$$

55. (B)

$$3\sin^2 A+2\sin^2 B=1$$

$$\Rightarrow 3\sin^2 A=\cos 2B \qquad \ldots\ldots (1)$$

Also $3\sin 2A=2\sin 2B$ …… (2)

From (1) & (2)

$$\frac{3\sin^2 A}{3(2\sin A\cos A)}=\frac{\cos 2B}{2\sin 2B}$$

$$\Rightarrow \tan A=\cot 2B$$

$$\tan A=\tan\left(\frac{\pi}{2}-2B\right)$$

$$A+2B=\frac{\pi}{2}$$

56. (A)

$$\tan 20^\circ\tan 80^\circ\cot 50^\circ$$

$$=\tan 20\tan(60+20)\tan(60-20)$$

$= \tan\left(3\times 20^{\circ}\right)$

$= \sqrt{3}$

57. (B)

$a\cos x + b\sin x = c$

Put $\cos x = \frac{1-t^2}{1+t^2}$ and $\sin x = \frac{2t}{1+t^2}$

Where $t = \tan\frac{x}{2}$

$$a\left(\frac{1-t^2}{1+t^2}\right) + b\left(\frac{2t}{1+t^2}\right) = c$$

$$(a+c)t^2 - 2bt + c - a = 0$$

Roots of this equation are $\tan\frac{x_1}{2}$ and $\tan\frac{x_2}{2}$

$$\tan\frac{x_1}{2} + \tan\frac{x_2}{2} = \frac{2b}{a+c}$$

$$\tan\frac{x_1}{2}, \tan\frac{x_2}{2} = \frac{c-a}{a+c}$$

$$\tan\left(\frac{x_1+x_2}{2}\right) = \frac{\frac{2b}{a+c}}{1-\frac{c-a}{c+a}} = \frac{2b}{2a} = \frac{b}{a}$$

58. (B)

$$\sin x + \cos x = \frac{\sqrt{7}}{2}$$

$$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = \frac{\sqrt{7}}{2} \qquad \text{Where } t = \tan\frac{x}{2}$$

$$\left(\sqrt{7}+2\right)t^2 - 4t + \left(\sqrt{7}-2\right) = 0$$

$$t = \frac{4 \pm \sqrt{16 - 4\left(\sqrt{7}+2\right)\left(\sqrt{7}-2\right)}}{2\left(\sqrt{7}+2\right)} = \frac{1}{3}\left(\sqrt{7}-2\right) \text{ or } \left(\sqrt{7}-2\right)$$

$$\tan\frac{x}{2} < \tan\frac{\pi}{8}$$

$$x \in \left[0, \frac{\pi}{4}\right] \quad \frac{x}{2} \in \left[0, \frac{\pi}{8}\right]$$

So $\tan\frac{x}{2}$ will have lower value

$$\tan\frac{x}{2} = \frac{\sqrt{7}-2}{3}$$

59. (C)

$$\cos x + \cos y - \cos(x+y) = \frac{3}{2}$$

$$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - 2\cos^2\left(\frac{x+y}{2}\right) + 1 = \frac{3}{2}$$

$$4\cos^2\left(\frac{x+y}{2}\right) - 4\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) + 1 = 0$$

$D \geq 0$

$$16\cos^2\left(\frac{x-y}{2}\right) - 16 \geq 0$$

$$-16\sin^2\left(\frac{x-y}{2}\right) \geq 0$$

Possible only at $x = y$

60. (D)

$\tan x = n \tan y \quad (\tan y = t)$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$= \frac{nt - t}{1 + nt^2}$$

$$\tan(x-y) = \frac{t(n-1)}{1+nt^2} = \alpha$$

$\sec^2(x-y)$ will be max if $\tan(x-y)$ is max

$\alpha + n\alpha t^2 = (n-1)t$

$D \geq 0$

$(n-1)^2 - 4n\alpha^2 \geq 0$

$$\alpha^2 \leq \frac{(n-1)^2}{4n}$$

$$\tan^2(x-y) \leq \frac{(n-1)^2}{4n}$$

$$\sec^2(x-y) \leq \frac{(n-1)^2}{4n} + 1$$

$$\leq \frac{(n+1)^2}{4n}$$

1. **(ABC)**

$$a=\cos x+\cos\left(x+\frac{2\pi}{3}\right)+\cos\left(x+\frac{4\pi}{3}\right)$$
$$=\cos x+2\cos x\cdot\cos(2\pi/3)$$
$$=\cos x-\cos x=0$$
$$b=\sin x+\sin\left(x+\frac{2\pi}{3}\right)+\sin\left(x+\frac{4\pi}{3}\right)$$
$$=\sin x+2\sin x\cdot\cos\left(\frac{2\pi}{3}\right)$$
$$=\sin x-\sin x=0$$

2. **(BD)**

$$\left.\begin{aligned}&\sin 1\approx\sin 57^o\\&\sin 2\approx\sin 114^o\approx\sin 66^o\\&\sin 3\approx\sin 171^o\approx\sin 9^o\end{aligned}\right\}\sin 2>\sin 1>\sin 3$$

$$\left.\begin{aligned}&\cos 1\approx\cos 57^o\\&\cos 2\approx\cos 114^o\approx-\cos 66^o\\&\cos 3\approx\cos 171^o\approx-\cos 9^o\end{aligned}\right\}\cos 1>\cos 2>\cos 3$$

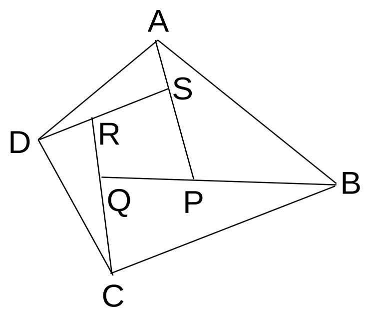
3. **(AB)**

$$x=r\sin A\cos B$$
$$y=r\sin A\sin B$$
$$z=r\cos A$$
$$x^2+y^2+z^2=r^2\sin^2 A+r^2\cos^2 A=r^2$$

4. **(BCD)**

Quadrilateral is cyclic hence $\left.\begin{aligned}P+R=\pi\\Q+S=\pi\end{aligned}\right\}$

$$P=\frac{A}{2}+\frac{B}{2}$$
$$R=\frac{C}{2}+\frac{D}{2}$$



5. **(ABD)**

$\tan\frac{x}{2} \in \theta$

(i) $\sin x = \dfrac{2\tan x/2}{1+\tan^2 x/2} \in \theta$

(ii) $\cos x = \dfrac{1-\tan^2 x/2}{1+\tan^2 x/2} \in \theta$

(iii) $\sec\frac{x}{2} = \sqrt{1+\tan^2\frac{x}{2}}$ (not necessarily)

(iv) $\tan x = \dfrac{2\tan x/2}{1-\tan^2 x/2} \in \theta$

6. **(ABCD)**

$2\sin A/2 = \pm\sqrt{1+\sin A} \pm \sqrt{1-\sin A}$

7. **(AD)**

$\left(4\cos^2 9^o - 3\right)\left(4\cos^2 27^0 - 3\right)$

$= \dfrac{\left(4\cos^3 9^o - 3\cos 9^o\right)\left(4\cos^3 27^o - 3\cos 27^0\right)}{\cos 9^o \cdot \cos 27^o}$

$= \dfrac{\cancel{\cos 27^o} \cdot \cos 81^o}{\cos 9^o \cdot \cancel{\cos 27^o}} = \dfrac{\sin 9^o}{\cos 9^o} = \tan 9^o = \cot 81^o$

8. **(AD)**

$\cos\beta = \sqrt{\sin\alpha \cdot \cos\alpha}$

$\Rightarrow \cos^2\beta = \sin\alpha \cdot \cos\beta$

$\Rightarrow \dfrac{1+\cos 2\beta}{2} = \dfrac{\sin 2\alpha}{2} \Rightarrow \cos 2\beta = -\left(1-\sin 2\alpha\right)$

$= -\left(1-\cos\left(\pi/2 - 2\alpha\right)\right)$

$= -2\sin^2\left(\pi/4 - \alpha\right)$

$= -2\cos^2\left(\pi/4 + \alpha\right)$

9. **(BC)**

$\dfrac{\sqrt{1+\cos\alpha}+\sqrt{1-\cos\alpha}}{\sqrt{1+\cos\alpha}-\sqrt{1-\cos\alpha}}$; $\alpha \in (\pi, 2\pi)$

$= \dfrac{\cancel{\sqrt{2}}\left(\left|\cos\frac{\alpha}{2}\right| + \left|\sin\frac{\alpha}{2}\right|\right)}{\cancel{\sqrt{2}}\left(\left|\cos\frac{\alpha}{2}\right| - \left|\sin\frac{\alpha}{2}\right|\right)}$

$$= \frac{-\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}}{-\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}} = \frac{1 - \tan\frac{\alpha}{2}}{1 + \tan\frac{\alpha}{2}} = \tan\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$$

$$= \cot\left(\pi/4 + \alpha/2\right)$$

10. **(CD)**

$$\cos ce\theta - \cot\theta = q$$

$$\cos ec\theta + \cot\theta = \frac{1}{q}$$

So, $\cos ec\theta = \frac{\left(q + \frac{1}{q}\right)}{2}$

$$\cot\theta = \frac{\left(\frac{1}{q} - q\right)}{2}$$

11. **(AD)**

$$\cos A \cdot \cos B + \sin A \sin B = \frac{3}{5} \qquad \ldots(1)$$

$$\sin A \sin B = 2\cos A \cos B \qquad \ldots(2)$$

So, $\cos A \cos = \frac{1}{5}$

$$\sin A \sin B = \frac{2}{5}$$

12. **(BCD)**

$1^c \approx 57°$

sin 1, sin 2, sin 3 $\rightarrow$ positive

sin 4, sin 5 $\rightarrow$ negative

cos 1, cos5 $\rightarrow$ positive

cos 2, cos 3, cos 4 $\rightarrow$ negative

tan 1, tan 4 $\rightarrow$ positive

tan 2, tan 3, tan 5 $\rightarrow$ negative

Ans. B, C, D

13. **(ABD)**

$$\frac{\sin 22^o \cos 8^o + \cos 158^o \cos 98^o}{\sin 23^o \cos 7^o + \cos 157 \cos 97^o}$$

$$= \frac{\frac{1}{2}\left(\sin 30^o + \sin 14^o + \cos 256^o + \cos 60^o\right)}{\frac{1}{2}\left(\sin 30^o + \sin 16^o + \cos 254^o + \cos 60^o\right)}$$

$$\sin 30^o = \cos 60^o = \frac{1}{2}$$

$$\cos 254^o = \cos\left(270^o - 16^o\right) = -\sin 16^o$$

$$\cos 256^o = \cos\left(270^o - 14^o\right) = -\sin 14^o$$

$$\therefore = \frac{\frac{1}{2}\left(\frac{1}{2} + \sin 14^o - \sin 14^o + \frac{1}{2}\right)}{\frac{1}{2}\left(\frac{1}{2} + \sin 16^o - \sin 16^o + \frac{1}{2}\right)}$$

$\sec(-100\pi) = 1$

$\operatorname{cosec}\left(\frac{-3\pi}{2}\right) = \operatorname{cosec}\left(\frac{\pi}{2}\right) = 1$ ...(B)

$\sin\left(\frac{7\pi}{2}\right) = \sin\left(\frac{3\pi}{2}\right) = -1 \neq 1$

$\cot\left(\frac{5\pi}{4}\right) = 1$ ...(D)

14. **(AD)**

$\sqrt{\frac{1-\sin A}{1+\sin A}} + \frac{\sin A}{\cos A} = \frac{1}{\cos A}$

$\sqrt{\frac{1-\sin A}{1+\sin A}}$

$\Rightarrow \sqrt{\left(\frac{1-\sin A}{1+\sin A}\right)\left(\frac{1+\sin A}{1+\sin A}\right)}$

$\Rightarrow \sqrt{\frac{1-\sin^2 A}{(1+\sin A)^2}} = \sqrt{\frac{\cos^2 A}{(1+\sin A)^2}}$

$\Rightarrow \left|\frac{\cos A}{1+\sin A}\right|$ now 1 + sin A is always positive

$\Rightarrow \left|\frac{\cos A}{(1+\sin A)}\frac{(1-\sin A)}{(1-\sin A)}\right|$

$\Rightarrow \left|\frac{\cos A(1-\sin A)}{(1-\sin^2 A)}\right| \Rightarrow \left|\frac{1-\sin A}{\cos A}\right|$

L.H.S

$\Rightarrow \frac{1-\sin A}{|\cos A|} + \frac{\sin A}{\cos A} = \frac{1}{\cos A}$

$\frac{1-\sin A}{|\cos A|} = \frac{1-\sin A}{\cos A}$

$\Rightarrow \sin A = 1$ or $|\cos A| = \cos A$

But $\sin A \neq 1$

$\because$ then $\cos A = 0$

$|\cos A| = \cos A$ means cos A is positive i.e. 1st and 4th Quadrant

15. **(ABCD)**

$\sec A = \frac{17}{8}$ $\operatorname{cosec} B = \frac{5}{4}$

$\cos A = \frac{8}{17}$ $\sin B = \frac{4}{5}$

$\sin A = \pm\frac{15}{17}$ $\cos B = \pm\frac{3}{5}$

$$\therefore \sec(A+B) = \frac{1}{\cos(A+B)} = \frac{1}{\cos A\cos B - \sin A\sin B}$$

$$= \frac{1}{\left(\frac{8}{17}\right)\left(\pm\frac{3}{5}\right) - \left(\pm\frac{15}{17}\right)\left(\frac{4}{5}\right)}$$

$$= \frac{85}{\pm 24 \pm 60}$$

Possible answer are

$\frac{85}{84}, \frac{85}{-84}, \frac{85}{36}, \frac{85}{-36}$

16. **(ABD)**

(A) $\frac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$

$$= \frac{\cos 2\alpha}{2\tan\left(\frac{\pi}{4}-\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$$

$$= \frac{\cos 2\alpha}{2\sin\left(\frac{\pi}{4}-\alpha\right)\cos\left(\frac{\pi}{4}-\alpha\right)}$$

$$= \frac{\cos 2\alpha}{\sin\left(2\left(\frac{\pi}{4}-\alpha\right)\right)}$$

$$= \frac{\cos 2\alpha}{\sin\left(\frac{\pi}{2}-2\alpha\right)} = \frac{\cos 2\alpha}{\cos 2\alpha} = 1$$

(B) $\frac{\sin(\pi-\alpha)}{\sin\alpha - \cos\alpha\tan\frac{\alpha}{2}} + \cos(\pi-\alpha)$

$$= \frac{\sin\alpha}{\frac{\left(\sin\alpha\cos\frac{\alpha}{2} - \cos\alpha\sin\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}}} - \cos\alpha$$

$$= \frac{\sin\alpha\cos\frac{\alpha}{2}}{\sin\left(\alpha-\frac{\alpha}{2}\right)} - \cos\alpha$$

$$= \frac{\left(2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right)\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} - \cos\alpha$$

$$= 2\cos^2\frac{\alpha}{2} - \cos\alpha$$

$= 1 + \cos\alpha - \cos\alpha = 1$

(C) $\dfrac{\left(1 - \tan^2\alpha\right)}{4\tan^2\alpha}$

$= \left(\dfrac{1 - \tan^2\alpha}{2\tan\alpha}\right)^2 = \cot^2 2\alpha$

$\dfrac{1}{4\sin^2\alpha\cos^2\alpha} = \operatorname{cosec}^2 2\alpha$

$\therefore \quad \operatorname{cosec}^2 2\alpha - \cot^2 2\alpha = 1$

(D) $1 + \sin 2\alpha = \left(\sin\alpha + \cos\alpha\right)^2$

$\therefore \quad \dfrac{1 + \sin 2\alpha}{\left(\cos\alpha + \sin\alpha\right)^2} = 1$

17. **(AD)**

$\dfrac{\cos A}{\sin B \sin C} + \dfrac{\cos B}{\sin C \sin A} + \dfrac{\cos C}{\sin A \sin B}$

$= \dfrac{\sin A\cos A + \sin B\cos B + \sin C\cos C}{\sin A\sin B\sin C}$

$A + B + C = \pi$

$\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

$\therefore$ Numerator = 2sin A sin B sin C

L.H.S = 2.

18. **(ABC)**

$\dfrac{\sqrt{3}\sin\left(\alpha + \beta\right) - \dfrac{2}{\cos\left(\dfrac{\pi}{6}\right)}\cos\left(\alpha + \beta\right)}{\sin\alpha}$

$= \dfrac{\sqrt{3}\sin\left(\alpha + \beta\right) - \dfrac{4}{\sqrt{3}}\cos\left(\alpha + \beta\right)}{\sin\alpha}$

$\sin\beta = \dfrac{4}{5}$

If $\beta \in \left(0, \dfrac{\pi}{2}\right)$ and $\tan\beta > 0$

Then $\cos\beta = \dfrac{3}{5}$

$\sin\left(\alpha + \beta\right) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \dfrac{3}{5}\sin\alpha + \dfrac{4}{5}\cos\alpha$

$\cos\left(\alpha + \beta\right) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \dfrac{3}{5}\cos\alpha - \dfrac{4}{5}\sin\alpha$

$= \dfrac{1}{\sqrt{3}\sin\alpha}\left(3\sin\left(\alpha + \beta\right) - 4\cos\left(\alpha + \beta\right)\right)$

$= \dfrac{1}{\sqrt{3}\sin\alpha}\left(\dfrac{9}{5}\sin\alpha + \dfrac{12}{5}\cos\alpha - \dfrac{12}{5}\cos\alpha + \dfrac{16}{5}\sin\alpha\right)$

$$= \frac{1}{\sqrt{3}\sin\alpha}\left(\frac{25}{5}\sin\alpha\right) = \frac{5}{\sqrt{3}}$$

(A) & (B) true

For $\tan\beta < 0 \quad \cos\beta = \frac{-3}{5}$

$$\sin(\alpha+\beta) = \frac{-3}{5}\sin\alpha + \frac{4}{5}\cos\alpha$$

$$\cos(\alpha+\beta) = \frac{-3}{5}\cos\alpha - \frac{4}{5}\sin\alpha$$

$$\text{L.H.S.} = \frac{1}{\sqrt{3}\sin\alpha}\left(3\sin(\alpha+\beta) - 4\cos(\alpha+\beta)\right)$$

$$= \frac{1}{\sqrt{3}\sin\alpha}\left(\frac{-9}{5}\sin\alpha + \frac{12}{5}\cos\alpha + \frac{12}{5}\cos\alpha + \frac{16}{5}\sin\alpha\right)$$

$$= \frac{1}{\sqrt{3}\sin\alpha}\left(\frac{24}{5}\cos\alpha + \frac{7}{5}\sin\alpha\right)$$

$$= \left(\frac{24\cos\alpha + 7\sin\alpha}{5\sqrt{3}\sin\alpha}\right)$$

$$= \frac{\sqrt{3}}{15}\left(24\cot\alpha + 7\right)$$

$\therefore$ (A) (B) (C)

19. **(ACD)**

$$\cos 20^o + \cos 40^o + \cos 60^o + \ldots.. + \cos 140^o$$

$$= \left(\cos 40^o + \cos 140^o\right) + \left(\cos 60^o + \cos 120^o\right) + \ldots\ldots + \left(\cos 80^o + \cos 100^o\right) + \cos 20^o$$

As $\cos(\pi-\theta) = -\cos\theta$

$\therefore \cos(\pi-\alpha) + \cos\theta = 0$

L.H.S

$= 0 + \cos 20^o$ (A)

$= \cos 20^o$

$= \sin 70^o$ (D)

$= \cos 20^o$

$= \cos\left(30^o - 10^o\right)$

$= \frac{\sqrt{3}}{2}\cos 10^o + \frac{1}{2}\sin 10^o$ (C)

20. **(ABC)**

$x\cos\alpha + y\sin\alpha = K$ where

$\alpha = A, B$ are roots of this equation

$x\cos\theta = k - y\sin\theta$

Square

$$x^2\cos^2\theta = \left(k - y\sin\theta\right)^2$$

$$x^2\left(1 - \sin^2\theta\right) = k^2 + y^2\sin^2\theta - 2ky\sin\theta$$

$$x^2 - x^2\sin^2\theta = k^2 + y^2\sin^2\theta - 2ky\sin\theta$$

$$0 = \left(x^2 + y^2\right)\sin\theta - 2ky\sin\theta + \left(k^2 - x^2\right)$$

Quadratic in $\sin\theta$ roots are sin A, sin B

$$\sin A \sin B = \frac{k^2 - x^2}{x^2 + y^2}$$

$$\sin A + \sin B = \frac{2ky}{x^2 + y^2}$$

$$y \sin\theta = k - x\cos\theta$$

Square

$$y^2 \sin^2\theta = (k - x\cos\theta)^2$$

$$y^2\left(1 - \cos^2\theta\right) = k^2 + x^2\cos^2\theta - 2kx\cos\theta$$

$$\left(x^2 + y^2\right)\cos^2\theta - 2kx\cos\theta + k^2 - y^2 = 0$$

Quadratic in cos B having roots cos A, cos B

$$\cos A + \cos B = \frac{2kx}{x^2 + y^2}$$

$$\cos A \cos B = \frac{k^2 - y^2}{x^2 + y^2}$$

21. **(AC)**

$$\frac{2\sin 2^o + 4\sin 4^o + ...+180\sin 180^o}{90} = A$$

$$A = \frac{\left(2\sin 2^o + 4\sin 4^o + ...+176\sin 176^o + 178\sin 178^o\right)}{90}$$

Supplementary angles have the same sign value

$$\sin 2^o = \sin 178^o$$

$$\sin 4^o = \sin 176^o \text{etc}$$

$$90A = \left(2\sin 2^o + 178\sin 178^o\right) + \left(4\sin 4^o + 176\sin 176^o\right) + ... + \left(88\sin 88^o + 92\sin 92^o\right) + 90\sin 90^o$$

$$= \left(2\sin 2^o + 178\sin 2^o\right) + \left(4\sin 4^o + 176\sin 176^o\right) + ... + \left(88\sin 88^o + 92\sin 88^o\right) + 90$$

$$= \left(180\sin 2^o + 180\sin 4^o + ... + 180\sin 88^o\right) + 90$$

$$90A = 180\left(\sin 2^o + \sin 4^o + ... + \sin 88^o\right) + 90$$

$$\sin 2^o + \sin 4^o + ... + \sin 88^o$$

$$= \frac{\sin\left(44^o\right)\sin\left(45^o\right)}{\sin 1^o}$$

$$90A = 180\frac{\sin 44^o \sin 45^o}{\sin 1^o} + 90^o$$

$$= 90\left(\frac{2\sin 44^o \sin 45^o}{\sin 1^o} + 1\right)$$

$$= 90\left(\frac{\cos 1^o - \cos 89^o}{\sin 1^o} + 1\right)$$

$$= 90\left(\cot 1^o - \frac{\cos 89^o}{\sin 1^o} + 1\right)$$

$$\cos 89^o = \sin 1^o$$

$$\therefore 90A = 90\left(\cot 1^o - 1 + 1\right)$$

$$= 90\cot 1^o$$

$A = \cot 1^o$ (A)

(B) is $\tan 1^o$

(C) $= \dfrac{\cos\left(39^o\right)\cos 1^o}{\sin 51^o \sin 1^o}$

(D) is $\tan 1^o$

22. **(ABCD)**

$\sin\theta + \cos\theta = -\dfrac{1}{5}$

$\dfrac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} + \dfrac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \dfrac{-1}{5}$ Let $\tan\dfrac{\theta}{2} = x$

$\dfrac{2x+1-x^2}{1+x^2} = \dfrac{-1}{5}$

$5\left(2x+1-x^2\right)+1\left(1+x^2\right)=0$

$10x+5-5x^2+1+x^2=0$

$6+10x-4x^2=0$

$2x^2-5x-3=0$ (C)

$x = \dfrac{5\pm\sqrt{25+4(2)(3)}}{2}$

$= \dfrac{5\pm 7}{2} = 6-1$

Now question is value of $\tan\dfrac{\theta}{2}$ is a root of which equation. If we see the other equations putting $x=-1$, or 6 will satisfy them. So All (A) (B) (C) (D) Have roots either -1 or 6

23. **(AD)**

$\cos(A-B)=\dfrac{3}{5},\ \cos(A+B)=\dfrac{2}{5}$

Add $2\cos A\cos B = 1$

Subtract $2\sin A\sin B = \dfrac{1}{5}$

24. **(ABCD)**

$\sin a + \sin b = \dfrac{1}{\sqrt{2}}$ …(1)

$\cos a + \cos b = \dfrac{\sqrt{3}}{\sqrt{2}}$ …(2)

Square and add the equations we get

$2+2\cos a\cos b + 2\sin a\sin b = \dfrac{1}{2}+\dfrac{3}{2}$

$\therefore 2\cos\left(a-b\right)=0$

$\cos\left(a-b\right)=0$

$2\cos^2\left(\dfrac{a-b}{2}\right)-1=0$

$\cos^2\left(\dfrac{a-b}{2}\right)=\dfrac{1}{2}$

$$\sec^2\left(\frac{a-b}{2}\right)=2$$

$$\tan^2\left(\frac{a-b}{2}\right)=1$$

$$\therefore \cot^2\left(\frac{a-b}{2}\right)=1 \qquad \text{(C)}$$

$$\sin a+\sin b=\frac{1}{\sqrt{2}}$$

$$2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)=\frac{1}{\sqrt{2}}$$

$$\cos a+\cos b=\frac{\sqrt{3}}{\sqrt{2}}$$

$$2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)=\frac{\sqrt{3}}{\sqrt{2}}$$

Hence $\tan\left(\frac{a+b}{2}\right)=\frac{1}{\sqrt{3}}$ (D)

$$\sin(a+b)=\frac{2\tan\left(\frac{a+b}{2}\right)}{1+\tan^2\left(\frac{a+b}{2}\right)}=\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}=\frac{\sqrt{3}}{2}$$

$$\sin(a+b)=\frac{\sqrt{3}}{2} \qquad \text{(A)}$$

25. **(AC)**

$$\frac{\sin A}{\sin B}=p,\frac{\cos A}{\cos B}=q$$

Dividing we get $\frac{\tan A}{\tan B}=\frac{p}{q}$

Now $\sin A=p\sin B$ …(1)

$\cos A=q\cos B$ …(2)

$$\sin^2 A+\cos^2 A=1$$

$$p^2\sin^2 B+q^2\cos^2 B=1$$

$$p^2\sin^2 B+q^2\left(1-\sin^2 B\right)=1$$

$$\left(p^2-q^2\right)\sin^2 B=1-q^2$$

$$\sin^2 B=\left(\frac{1-q^2}{p^2-q^2}\right)$$

$$\cos^2 B=1-\sin^2 B$$

$$=\left(\frac{p^2-1}{p^2-q^2}\right)$$

$$\tan^2 B=\left(\frac{1-q^2}{p^2-1}\right)=\frac{q^2-1}{1-p^2} \qquad \text{(C)}$$

From (1) & (2)

$$\sin^2 A=p^2\sin^2 B$$

$$= p^2\left(\frac{1-q^2}{p^2-q^2}\right)$$

$$\cos^2 A = q^2\left(\frac{p^2-1}{p^2-q^2}\right)$$

$$\tan^2 A = \frac{p^2\left(1-q^2\right)}{q^2\left(p^2-1\right)} \qquad \text{(A)}$$

26. **(ABCD)**

$0 \le \theta \le \pi$

$81^{\sin^2\theta} + 81^{\cos^2\theta} = 30$

$81^{\sin^2\theta} + 81^{1-\sin^2\theta} = 30$

Let $81^{\sin^2\theta} = x$

$x + \frac{81}{x} = 30$

$x^2 + 81 = 30x$

$x^2 - 30x + 81 = 0$

$x = 27$ or 3

$\therefore\ 81^{\sin^2\theta} = 27, 3$

$3^{4\sin^2\theta} = 3^3, 3^1$

$\therefore 4\sin^2\theta = 3, 1$

$\sin^2\theta = \frac{3}{4}, \frac{1}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}, \pm\frac{1}{2}$

For $0 \le \theta \le \pi$ $\qquad \theta = 30^o, 60^o, 120^o, 150^o$

27. **(BCD)**

For a cyclic Quadrilateral B + D = π or B = π – D

sin B = sin D $\Rightarrow$ cosec B = cosec D

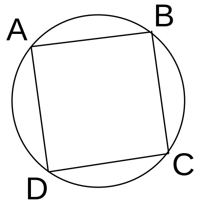
cot B = – cot D or tan B = – tan D

similarly A = π – C

cot A = – cot C

cot A + cot C = 0

sec B = –sec D



28. **(ABCD)**

$$\sin\left(\frac{11\pi}{12}\right)\sin\left(\frac{5\pi}{12}\right)$$

$$= \frac{1}{2}\left(\cos\left(\frac{6\pi}{12}\right) - \cos\left(\frac{16\pi}{12}\right)\right)$$

$$= \frac{1}{2}\left(0 - \cos\left(\frac{4\pi}{3}\right)\right)$$

$$= \frac{1}{2}\left(+\frac{1}{2}\right) = \frac{1}{4} \qquad \ldots \text{(A)}$$

$$\cos ec\left(\frac{9\pi}{10}\right)\sec\left(\frac{4\pi}{5}\right)$$

$$=\frac{1}{-\sin\frac{\pi}{10}\cos\left(\frac{\pi}{5}\right)}=\frac{1}{(-1)\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right)}$$

= (–1) (4) … (B)

$$\sec^4\theta+\cos^4\theta$$

$$=1-2\sin^2\theta\cos^2\theta$$

$$=1-\frac{\sin^2 2\theta}{2} \quad \theta=\frac{\pi}{8}$$

$$=1-\frac{\sin^2\frac{\pi}{4}}{2}$$

$$=1-\frac{\frac{1}{2}}{2}$$

$$=\frac{3}{4}$$ …(C)

$$\left(1+\cos\frac{2\pi}{9}\right)\left(1+\cos\frac{4\pi}{9}\right)\left(1+\cos\frac{8\pi}{9}\right)$$

$$=2\cos^2\frac{\pi}{9}\times 2\cos^2\frac{2\pi}{9}\times 2\cos^2\frac{4\pi}{9}$$

$$=8\left(\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\right)$$

$$=8\left(\frac{1}{8}\frac{\sin\left(\frac{8\pi}{9}\right)}{\sin\frac{\pi}{9}}\right)^2$$

$$=\frac{8}{64}=\frac{1}{8}$$ …(D)

29. **(ABC)**

(A) $\tan\alpha\tan 2\alpha\tan 3\alpha$

$\tan 3\alpha-\tan 2\alpha-\tan\alpha$

Is true for all angles

Hint: $2\alpha+\alpha=3\alpha$

$\therefore$ take tan on both sides to get the answer

(B) $\cos ec2\alpha+\cos ec4\alpha-\cos ec\alpha$

$$=\frac{1}{\sin\frac{2\pi}{7}}+\frac{1}{\sin\frac{4\pi}{7}}-\frac{1}{\sin\frac{\pi}{7}}$$

$$=\frac{2\sin\left(\frac{3\pi}{7}\right)\cos\left(\frac{\pi}{7}\right)}{\left[2\sin\left(\frac{\pi}{7}\right)\cos\left(\frac{\pi}{7}\right)\right]\sin\frac{4\pi}{7}}-\frac{1}{\sin\frac{\pi}{7}}$$

$$=\frac{1}{\sin\frac{\pi}{7}}=\frac{1}{\sin\frac{\pi}{7}}=0$$

(D) $8\cos\alpha\cos 2\alpha\cos 4\alpha$

$$=\frac{\sin 8\alpha}{\sin\alpha}=\frac{\sin\frac{8\pi}{7}}{\sin\frac{\pi}{7}}=-1$$

(C) $\cos\alpha+\cos 3\alpha-\cos 2\alpha$

$$=2\cos 2\alpha\cos\alpha-\cos 2\alpha$$

$$=\cos 2\alpha\left(2\cos\alpha-1\right)$$

$$=\cos\left(\frac{2\pi}{7}\right)\left(2\cos\frac{\pi}{7}-1\right)$$

$$=\cos\left(\frac{2\pi}{7}\right)\left(2\cos\frac{\pi}{7}-1\right)\frac{\sin\frac{\pi}{7}}{\sin\frac{\pi}{7}}$$

$$=\cos\left(\frac{2\pi}{7}\right)\left(\frac{2\sin\frac{\pi}{7}\cos\frac{\pi}{7}-\sin\frac{\pi}{7}}{\sin\frac{\pi}{7}}\right)$$

$$=\frac{\cos\left(\frac{2\pi}{7}\right)}{\sin\frac{\pi}{7}}\left(\sin\frac{2\pi}{7}-\sin\frac{\pi}{7}\right)$$

$$=\frac{\cos\left(\frac{2\pi}{7}\right)\left(2\sin\frac{\pi}{14}\right)\left(\cos\frac{3\pi}{14}\right)}{2\sin\frac{\pi}{14}\cos\frac{\pi}{14}}$$

$$=\frac{\cos\frac{3\pi}{14}\cos\frac{2\pi}{7}}{\left(\cos\frac{\pi}{14}\right)}$$

$$=\frac{1}{2}\frac{\left(2\cos\left(\frac{3\pi}{14}\right)\sin\frac{3\pi}{14}\right)}{\cos\frac{\pi}{14}}=\frac{1}{2}\frac{\sin\left(\frac{6\pi}{14}\right)}{\cos\frac{\pi}{14}}$$

$\frac{6\pi}{14},\frac{\pi}{14}$ are complementary

$\therefore$ L.H.S. $=\frac{1}{2}\times 1=\frac{1}{2}$

30. **(AC)**

$\cot\alpha - \tan\alpha = 2\cot 2\alpha$ ...(A)

(A) is true

$\tan\alpha + \cot\alpha = 2\operatorname{cosec}2\alpha$ ...(D)

(D) is wrong

$\tan\left(45^0 + \alpha\right) - \tan\left(45^0 - \alpha\right)$

$= \cot\left(45^0 - \alpha\right) - \tan\left(45^0 - \alpha\right)$

$= 2\cot\left(2\left(45^0 - \alpha\right)\right)$

$2\cot\left(90^0 - 2\alpha\right)$

$= \tan 2\alpha$

(B) (C) wrong

31. **(A)**

$45^{\circ} = \frac{45}{180} \times 200 = 50^{g}$

32. (D)

$\frac{23\pi^{C}}{4} = \frac{23 \times 180^{\circ}}{4} = 1035^{\circ}$

33. (B)

$200^{g} = \frac{200}{200} \times 180^{\circ} = 180^{\circ}$

$= \frac{200}{200} \times \pi^{C} = \pi^{C}$

34. (B)

Sum of angles of a hexagon $= 180\left(n - 2\right)$

$= 180 \times 4 = 720^{\circ} = 800^{g} = 4\pi^{C}$

35. (A)

$\sin\alpha + \sin\beta = \frac{1}{3}$

$\cos\alpha + \cos\beta = \frac{1}{4}$

36. (B)

$2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{3}$ ______(1)

37. (D)

$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{4}$ _______(2)

$\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{4}{3}$

Hence, $\sin(\alpha+\beta) = \dfrac{2\tan\left(\dfrac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\dfrac{\alpha+\beta}{2}\right)} = \dfrac{2\times\dfrac{4}{3}}{1+\dfrac{16}{9}} = \dfrac{8\times 3}{25} = \dfrac{24}{25}$

$$\cos(\alpha+\beta) = \frac{1-\tan^2\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)} = \frac{1-\frac{16}{9}}{1+\frac{16}{9}} = \frac{-7}{25}$$

$$\tan(\alpha+\beta) = \frac{24}{25}\times\frac{25}{7} = -\frac{24}{7}$$

38. (B)

$P_n - P_{n-2}$

$= \sin^n\theta + \cos^n\theta - \sin^{n-2}\theta - \cos^{n-2}\theta$

$= \sin^{n-2}\theta(\sin^2\theta - 1) + \cos^{n-2}(\cos^2\theta - 1)$

$= -\sin^2\theta\cos^2\theta(\sin^{n-4}\theta + \cos^{n-4}\theta)$

$= -\sin^2\theta.\cos^2\theta.P_{n-4}$

$\Rightarrow k = -\sin^2\theta.\cos^2\theta$

39. (B)

$\sin\theta + \cos\theta = m \Rightarrow \sin\theta.\cos\theta = (m^2-1)/2$

$4(1-P_6) = 4(1-(\sin^6\theta + \cos^6\theta))$

$= 4(1-(1-3\sin^2\theta.\cos^3\theta))$

$= 12\sin^2\theta.\cos^2\theta = 12\left(\dfrac{m^2-1}{2}\right)^2$

$= 3(m^2-1)^2$

40. (D)

$P_{n-2} - P_n = \sin^2\theta.\cos^2\theta P_\lambda$

$\lambda = n-4$

41. (C)

$2P_6 - 3P_4 + 10 = 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 10$

$= 2(1-3\sin^2\theta\cos^3\theta) - 3(1-2\sin^2\theta\cos^2\theta) + 10$

$= 2 - 3 + 10 = 9$

42. (B)

$\sin\dfrac{\pi}{7} + \sin\dfrac{2\pi}{7} + \sin\dfrac{3\pi}{7}$

$= \dfrac{2\sin\dfrac{2\pi}{7}.\sin\dfrac{3\pi}{14}}{2.\sin\dfrac{\pi}{14}} = \dfrac{\cos\left(\dfrac{\pi}{14}\right) - \cos\left(\dfrac{\pi}{2}\right)}{2\sin\dfrac{\pi}{14}}$

$$=\frac{1}{2}\cot\left(\frac{\pi}{14}\right)$$

43. (B)

$$\sin\frac{\pi}{n}+\sin\frac{3\pi}{n}+\sin\frac{5\pi}{n}+------n \text{ terms}$$

$$=\frac{\sin\left(\frac{\pi}{n}+\frac{(n-1)}{2}\frac{2\pi}{n}\right).\sin\left(\frac{2\pi.n}{2n}\right)}{\sin\left(\frac{2\pi}{2n}\right)}$$

$$=\frac{\sin\pi.\sin\pi}{\sin\frac{\pi}{4}}=0$$

44. (C)

$$\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}$$

$$=\frac{\cos\left(\frac{\pi}{11}+\frac{(5-1)}{2}\frac{2\pi}{11}\right).\sin\left(\frac{5}{2}\frac{2\pi}{11}\right)}{\sin\left(\frac{2\pi}{11}/2\right)}$$

$$=\frac{2\cos\frac{5\pi}{11}.\sin\frac{5\pi}{11}}{2\sin\frac{\pi}{11}}=\frac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}}$$

$$=\frac{\sin\frac{\pi}{11}}{2\sin\frac{\pi}{11}}=\frac{1}{2}$$

45. (C)

$$\sum_{r=0}^{n}\sin^2\frac{r\pi}{n}=\frac{1}{2}\sum_{r=0}^{n}\left(1-\cos\frac{2r\pi}{n}\right)$$

$$=\frac{n+1}{2}-\frac{1}{2}\left(\cos 0+\cos\frac{2\pi}{n}+\cos\frac{4\pi}{n}+........\cos 2\pi\right)$$

$$=\frac{n+1}{2}-\frac{\cos\left(0+\frac{2\pi}{2}\right).\sin\left(\frac{2\pi}{n}.\frac{(n+1)}{2}\right)}{2\sin\left(\frac{2\pi}{n.2}\right)}$$

$$=\frac{n+1}{2}-\frac{1}{2}\frac{\sin\left(\pi+\frac{\pi}{n}\right)(-1)}{\sin\frac{\pi}{n}}=\frac{n+1}{2}-\frac{1}{2}=\frac{n}{2}$$

46. (B)

$$\cos 7\approx\cos(6.28+0.72)$$

$$\approx\cos(0.72)$$

So, $\cos 7 > \cos 1$
(but not a correct explanation)

47. (A)

$\left(27^{\cos 2x}.81^{\sin 2x}\right)_{max}$

$=\left(3^{3\cos 2x+4\sin 2x}\right)_{min}$

$=3^{-5}=\frac{1}{3^5}$

48. (B)

$\sin 3 \simeq \sin 9^{\circ}$

$\sin 1 \simeq \sin 57$

$\sin 2 \simeq \sin 66^{\circ}$

Sin x is possible but and in clearly as decreasing
(Not a correct explanation)

49. (A)

$\sin^2 A+\sin^2 B+\sin^2 C=2$

$\Rightarrow 2\sin^2 A+2\sin^2 B+2\sin^2 C=4$

$\Rightarrow 1-\cos 2A+1-\cos 2B+1-\cos 2C=4$

$\Rightarrow \cos 2A+\cos 2B+\cos 2C=-1$

$\Rightarrow -1-4\cos A\cos B\cos C=-1$

$\Rightarrow \cos A\cos B\cos C=0$

Hence are of them is 90°

$\sin^2 A+\sin^2 B+\sin^2 C=2+2\cos A\cos B\cos C$

50. (B)

$xy+yz+zx=1$

Let $x=\tan\frac{A}{2}, y=\tan\frac{B}{2}, z=\tan\frac{C}{2}$

$xy+yz+zx=\tan\frac{A}{2}.\tan\frac{B}{2}+\tan\frac{B}{2}.\tan\frac{C}{2}+\tan\frac{C}{2}.\tan\frac{A}{2}=1$

Hence, $\left(\frac{A}{2}+\frac{B}{2}+\frac{C}{2}\right)=n\pi+\frac{\pi}{2}\Rightarrow A+B+C=2n\pi+\pi$

We know $\frac{2x}{1+x^2}=\sin A, \frac{2y}{1+y^2}=\sin B, \frac{2z}{1+z^2}=\sin C$

$\sin A+\sin B+\sin C=4\cos\frac{A}{2}.\cos\frac{B}{2}\cos\frac{C}{2}$

$\Rightarrow \frac{2x}{1+x^2}+\frac{2y}{1+y^2}+\frac{2z}{1+z^2}=4\frac{1}{\sqrt{1+x^2}}.\frac{1}{\sqrt{1+y^2}}.\frac{1}{\sqrt{1+z^2}}$

The identify $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$ is true but not used here

51. (A) - Q, (B) - S, (C) - P, (D) - R

(A) $A=\sin^2\theta+\cos^4\theta$

$=\sin^2\theta+\left(1-\sin^2\theta\right)^2$

$=\sin^4\theta-\sin^2\theta+1$

$A \in \left[\frac{3}{4}, 3\right]$ (minimum at $\sin^2\theta = \frac{1}{2}$, maximum of $\sin^2\theta = 1$)

Ans: (Q)

(B) $A = 3\cos^2\theta + \sin^4\theta$

$= \sin^4\theta - 3\sin^2\theta + 3$

$A \in [1, 3]$

(min of $\sin^2\theta = 0,$ max of $\sin^2\theta = 1$)

Ans: (S)

(C) $A = \sin^2\theta - \cos^4\theta$

$= \left(1 - \cos^2\theta\right) - \cos^4\theta$

$= -\left(\cos^4\theta + \cos^2\theta\right) + 1$

$A \in [-1, 1]$

Ans: (P)

(D) $A = \tan^2\theta + 2\cot^2\theta$

By A.M $\geq$ G.M., $A \geq 2\sqrt{2}$

$A \in \left[2\sqrt{2}, \infty\right)$

52. (A) - R, (B) - P, (C) - Q, (D) - S

$\cos\alpha + \cos\beta = \frac{1}{2}$; $\sin\alpha + \sin\beta = \frac{1}{3}$

$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{2}$ ……..(i)

$2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{3}$ ……..(ii)

(C) $\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{1/3}{1/2} = \frac{2}{3}$

(A) $\cos\left(\frac{\alpha+\beta}{2}\right) = \pm\frac{3}{\sqrt{13}}$

$(i)^2 + (ii)^2$

$4\cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{4} + \frac{1}{9} = \frac{13}{36}$

$\cos\left(\frac{\alpha-\beta}{2}\right) = \pm\frac{\sqrt{13}}{12}$

$\tan\left(\frac{\alpha-\beta}{2}\right) = \pm\frac{\sqrt{131}}{\sqrt{13}}$

53. (A) - S, (B) - R, (C) - P, (D) - Q

(A) $\cos 20^\circ + \cos 80^\circ - \sqrt{3}\cos 50^\circ$

$= 2\cos 50^\circ \cos 30^\circ - \sqrt{3}\cos 50^\circ$

$= 0$

(B) $1 + \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + ... + \cos\frac{6\pi}{7}$

$$=1+\frac{\cos\left(\frac{\pi}{7}+\frac{5}{2}.\frac{\pi}{7}\right).\sin\left(\frac{6}{2}.\frac{\pi}{7}\right)}{\sin\left(\frac{\pi}{14}\right)}$$

$$=1+\frac{\cos\left(\frac{\pi}{2}\right).\sin\left(\frac{3\pi}{7}\right)}{\sin\frac{\pi}{14}}=1+0=1$$

(C) $\cos 20^\circ+\cos 40^\circ+\cos 60^\circ-4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ$

$=2\cos 30^\circ.\cos 10^\circ+2\cos^2 30^\circ-1-4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ$

$=2\cos 30^\circ\left(\cos 10^\circ+\cos 30^\circ\right)-1-4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ$

$=4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ-1-4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ$

$=-1$

(D) $\frac{1}{2}\left[2\cos 20^\circ.\cos 100^\circ+2\cos 100^\circ.\cos 140^\circ-2\cos 140^\circ.\cos 200^\circ\right]$

$=\frac{1}{2}\left[\cos 120^\circ+\cos 80^\circ+\cos 240^\circ+\cos 40^\circ-\cos 340^\circ-\cos 60^\circ\right]$

$=\frac{1}{2}\left[-\frac{1}{2}-\frac{1}{2}-\frac{1}{2}+\cos 80^\circ+\cos 40^\circ-\cos 20^\circ\right]$

$=-\frac{3}{4}+\frac{1}{2}\left(2\cos 60^\circ.\cos 20^\circ-\cos 20^\circ\right)$

$=-\frac{3}{4}$

54. (A) - R, (B) - P, (C) - S, (D) - Q

(A) $\tan\theta\left(\cot\theta.\cos\theta+\sin\theta\right)$

$=\tan\theta.\cot\theta.\cos\theta+\frac{\sin\theta}{\cos\theta}\sin\theta$

$=\frac{\cos^2\theta+\sin^2\theta}{\cos\theta}=\sec\theta$ **Ans: (R)**

(B) $\frac{\tan\theta.\operatorname{cosec}^2\theta}{1+\tan^2\theta}$

$=\frac{\sin\theta}{\cos\theta}.\frac{1}{\sin^2\theta}.\cos^2\theta=\cot\theta$ **Ans: (P)**

(C) $\frac{\operatorname{cosec}^2\theta-1}{\operatorname{cosec}\theta.\cos^2\theta}=\frac{\cot^2\theta}{\frac{1}{\sin\theta}.\cos^2\theta}$

$=\frac{\cos^2\theta}{\sin^2\theta.\cos^2\theta}.\sin\theta$

$=\operatorname{cosec}\theta$ **Ans: (S)**

(D) $\frac{\cot\theta.\sec^2\theta}{1+\cot^2\theta}=\frac{\frac{\cos\theta}{\sin\theta}.\frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta}}$

$=\frac{\sin\theta}{\cos\theta}=\tan\theta$ **Ans: (Q)**

55. (A) - RS, (B) - RT, (C) - PQ

(A) $3\sin 2\theta + 4\cos 2\theta + 3$

$\in [-5+3, 5+3]$ $\quad\lambda + k = 6$ **(R)**

$\in [-2, 8]$ $\quad\lambda - k = 10$ **(S)**

(B) $5\cos\theta + 3\cos\theta.\cos\frac{\pi}{3} - 2\sin\theta.\sin\frac{\pi}{3} + 3$

$$= \left(\frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta\right) + 3$$

$$\in \left[\sqrt{\frac{169}{4} + \frac{27}{4}} + 3, \sqrt{\frac{169}{4} + \frac{27}{4}} + 3\right]$$

$\in [-4, 10]$ $\quad\lambda + k = 6$ **(R)**

$\lambda - k = 14$ **(T)**

(C) $1 + \sin\theta.\frac{1}{\sqrt{2}} + \cos\theta.\frac{1}{\sqrt{2}} + 2.\frac{\cos\theta}{\sqrt{2}} + \frac{2\sin\theta}{\sqrt{2}}$

$$= 1 + \frac{3}{\sqrt{2}}\cos\theta + \frac{3}{\sqrt{2}}\sin\theta$$

$$= \in \left[1 - \frac{3\sqrt{2}}{\sqrt{2}}, 1 + \frac{3\sqrt{2}}{\sqrt{2}}\right]$$

$\in [-2, 4]$ $\quad\lambda + k = 2$ **(P)**

$\lambda - k = 6$ **(Q)**

56. (A) - R, (B) - S, (C) - Q, (D) – P

$\cos A = \frac{1}{3}$

$A \in (1350^\circ, 1440^\circ)$

$\Rightarrow \in (270^\circ, 360^\circ)$ IV quad

$\sin A = \frac{-2\sqrt{2}}{3}$

$\frac{A}{2} \in (675^\circ, 720^\circ)$ IV quad

$$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1-\frac{1}{3}}{2}} = -\frac{1}{\sqrt{3}}$$

$$\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}} = \sqrt{\frac{2}{3}} \qquad \tan\frac{A}{2} = -\frac{1}{\sqrt{2}}$$

57. (A) - R, (B) - P, (C) - S, (D) - Q

(A) $\sqrt{3}\sin x - \cos x \quad \in [-2, 2]$ **(R)**

(B) $4\cos^2 x - 4\cos x + 3$

$= (2\cos x - 1)^2 + 2$

$\in [0, 9] + 2$

$\in [2, 11]$ **(P)**

(C) $\frac{2\tan x}{\tan^2 x+1}=\sin 2x\in[-1,1]$ **(S)**

(D) $\sin^4 x+\cos^4 x=1-2\sin^2 x.\cos^2 x$

$=1-\frac{1}{2}\sin^2 2x\ \in[0,1]$

So, $\in\left[\frac{1}{2},1\right]$ **(Q)**

58. (A) - Q, (B) - P, (C) - R, (D) – S
Standard solutions
$\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$
$\sin^2 A+\sin^2 B+\sin^2 C=2+2\cos A\cos B\cos C$
$\cos 2A+\cos 2B+\cos 2C=-1-4\cos 2A\cos B\cos C$
$\cos^2 A+\cos^2 B+\cos^2 c=1-2\cos A\cos B\cos C$

59. (A) - S, (B) - P, (C) - Q, (D) – R
$\sin 2\theta=k$

(A) $\operatorname{cosec} 20+\cot 2\theta-\cos 2\theta$

$=\frac{1}{\sin 2\theta}+\frac{1-\sin\theta}{\sin 2\theta}=\frac{1}{k}+\frac{1-k^2}{k}=\frac{2-k^2}{k}$ **(S)**

(B) $\left(\frac{1+\tan\theta}{1-\tan\theta}\right)^2=\frac{1+\sin 2\theta}{1-\sin 2\theta}=\frac{1+k}{1-k}$ **(P)**

(C) $\sin 2\theta-\frac{1}{2}\left(2\cos^2 2\theta\right)$

$=k-\frac{b}{2}\left(1-k^2\right)=k^2+k-1$ **(Q)**

(D) $\sin 60=3\sin\theta-4\sin^3 2\theta$

$=3k-4k^3$ **(R)**

60. (A) - R, (B) - S, (C) - P, (D) - Q

(A) $\sin B=\frac{4}{5}$, $\tan\left(\frac{A}{2}\right)=1\Rightarrow A=90^\circ$

$B+C=90^\circ$

$\cos C=\frac{4}{5}$

$\frac{a}{b}=\frac{\sin A}{\sin B}=\frac{1}{\frac{4}{5}}=\frac{5}{4}$ **(R)**

(B) $\frac{b}{c}=\frac{\sin B}{\sin C}=\frac{\frac{4}{5}}{\frac{3}{5}}=\frac{4}{3}$ **(S)**

(C) $\frac{c+b}{a}=\frac{\sin C+\sin B}{\sin A}=\frac{3}{5}+\frac{45}{}=\frac{7}{5}$ **(P)**

(D) $\sqrt{\frac{a^2+c^2-b^2}{20c}}=\sqrt{\frac{\sin^2 A+\sin^2 C-\sin^2 B}{2\sin A\sin C}}=\sqrt{\frac{3}{5}}$ **(Q)**

EXERCISE - 2 [C]

1. **(3)**

$$\frac{\sin 2\alpha + \sin 4\alpha - \sin 3\alpha}{\cos 2\alpha + \cos 4\alpha - \cos 3\alpha}$$

$$= \frac{2\sin 3\alpha \cos\alpha - \sin 3\alpha}{2\cos 3\alpha \cos\alpha - \cos 3\alpha} = \frac{\sin 3\alpha}{\cos 3\alpha} = \tan 3\alpha$$

$k = 3$

2. **(1)**

$$\tan 45^\circ = 1 \Rightarrow \tan\left(27^\circ + 18^\circ\right) = 1$$

$$\Rightarrow \frac{\tan 27^\circ + \tan 18^\circ}{1 - \tan 27^\circ + \tan 18^\circ} = 1$$

$$\Rightarrow \tan 27^\circ + \tan 18^\circ = 1 - \tan 27^\circ \tan 18^\circ$$

$$\Rightarrow \tan 27^\circ + \tan 18^\circ + \tan 27^\circ \tan 18^\circ = 1$$

3. **(3)**

LHS $\dfrac{\cos 96^\circ + \sin 96^\circ}{\cos 96^\circ - \sin 96^\circ} = \dfrac{1 + \tan 96^\circ}{1 - \tan 96^\circ}$

$$\Rightarrow \frac{\tan 45^\circ + \tan 96^\circ}{1 - \tan 45^\circ \tan 96^\circ \Rightarrow} = \tan(141)$$

Or $\tan(180 + 141) = \tan(321)$

$\therefore n = 3$

4. **(3)**

In ABC $\sum \tan A = \pi \tan A$

$6 = 2 \times \tan C$

$\Rightarrow \tan C = 3$

5. **(0)**

$$a\sin\theta - \frac{a}{\sin\theta} = \frac{b}{\cos\theta} - \cos\theta$$

$$a\frac{\left(\sin^2\theta - 1\right)}{\sin\theta} = b\frac{\left(1 - \cos^2\theta\right)}{\cos\theta}$$

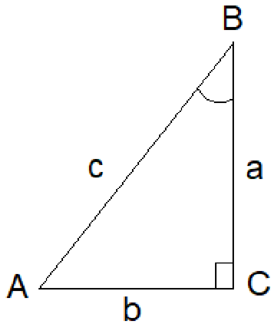
$$\Rightarrow -a\cos^3\theta = b\sin^3\theta \Rightarrow a\cos^3\theta + b\sin^3\theta = 0$$

Square both side

$$a^2\cos^6\theta + b^2\sin^6\theta + 2ab\cos^3\theta\sin^3\theta = 0$$

6. **(8)**

$$\left(\frac{c}{a} + \frac{c}{b}\right)^2$$



$= (\sec\theta + \operatorname{cosec}\theta)^2$

$= 4\dfrac{(\sin\theta+\cos\theta)^2}{(\sin 2\theta)^2} \Rightarrow 4\left(\dfrac{1}{\sin^2 2\theta}+\dfrac{1}{\sin 2\theta}\right)$

$\therefore$ **For** $\min(\sin 2\theta)\ \max = 1$

Hence $\min = 8$

7. **(1)**

$\sqrt{2\cot\alpha + \operatorname{cosec}^2\alpha} + \cot\alpha + 2$

$\sqrt{(1+\cot\alpha)^2} + \cot\alpha + 2$

$|1+\cot\alpha| + \cot\alpha + 2$

Since $\dfrac{3\pi}{4} < \alpha < \pi \Rightarrow 1+\cot\alpha < 0$

$\Rightarrow 2 - 1 - \cot\alpha + \cot\alpha$

Ans: 1

8. **(3)**

Let $\alpha \,\&\, \beta$ be the roots

$\alpha+\beta\ = -\cos\theta + 1, \alpha\beta = \dfrac{1}{2}\cos^2\theta$

$\therefore \alpha^2+\beta^2 = (1-\cos\theta)^2 - 2\times\dfrac{1}{2}\cos^2\theta$

$= 1 - 2\cos\theta$

Maximum as 3

9. **(1)**

$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

$= 1 - \dfrac{\sin^2 2\theta}{2}$

For maximum $\sin 2\theta = 0$

i.e. 1

10. **(0)**

If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$

That is only possible when all

$\sin\theta = 1 \qquad \Rightarrow \cos\theta_1 = 0$

$\therefore \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$

11. **(1)**

$\tan\frac{\pi}{4n}\tan\frac{2\pi}{4n}----\tan\frac{n\pi}{4\pi}------\tan\left(\frac{\pi}{2}-\frac{2\pi}{4n}\right)\tan\left(\frac{\pi}{2}-\frac{\pi}{4n}\right)$

(use $\left[\text{Use}\tan\left(\frac{\pi}{2}-\alpha\right)=\cot\alpha\right]$

OR

$\tan\frac{4\pi}{4n}.\tan\frac{2\pi}{4n}-------1------\cot\frac{2\pi}{4n}.\cot\frac{\pi}{4n}$

$(\text{use}\tan\theta\cot\theta=1)$

Ans: 1

12. **(5)**

$\sin x=1-\sin^2 x$ or $\sin x=\cos^2 x$ of $\cos^2 x+\cos^4 x=1$

Expression

$\cos^6 x\left[\cos^6 x+3\cos^4 x+3\cos^4 x+1\right]+\cos^4 x+\cos^2 x+3$

$=\cos^6 x\left[\cos^2 x+1\right]^3+4$

$=\left(\cos^4 x+\cos^2 x\right)^3+4$

$=1+4=5$

13. **(1)**

$\tan(100+125°)=\frac{\tan 100°+\tan 125°}{1-\tan 100°\cdot\tan 125°}$

$\tan 225°=1-\tan 100°\tan 125°=\tan 100°+\tan 125°$

14. **(1)**

On simplification the expression reduces to

$\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}$

When is minimum when all angles equal $=60$

$\therefore$ Min value is 1

15. **(0)**

$\frac{x}{\cos\theta}=\frac{y}{\cos\left(\theta-\frac{2\pi}{3}\right)}=\frac{z}{\cos\left(\theta+\frac{2\pi}{3}\right)}=k$

$\Rightarrow x+y+z=k\left(\cos\theta+\cos\left(\theta-\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{2\pi}{3}\right)\right)$

$=k\left(\cos\theta+2\cos\theta\cos\frac{2\pi}{3}\right)$

$=k\left(\cos\theta-2\cos\theta\times\frac{1}{2}\right)$

$=0$

16. **(1)**

$$\frac{\tan 6^\circ \tan 54^\circ \tan 66^\circ \tan 42^\circ \tan 78^\circ}{\tan 54^\circ}$$

$$\frac{\tan 18^\circ \tan 42^\circ \tan 78^\circ}{\tan 54^\circ}$$

Use $\tan\theta\tan(60-\theta)\tan(160^\circ+\theta)=\tan 3\theta$

17. **(2)**

Angle moved in 60 min $=360^\circ$

Angle moved in 20 min $=120^\circ$

$\therefore\ d=10\times 120^\circ\times\frac{\pi}{180}$

$d=\frac{20\pi}{3}$

Ans : 2

18. **(7)**

During every motion particle rotates by 1 radian angle (Approx. 57.16°)

$\therefore$ To rotate 360° it must be in its 7^{th} round

19. **(1)**

$\sec^4 A-\tan^4 A-2\tan^2 A$

$=(\sec^2 A-\tan^2 A)(\sec^2 A+\tan^2 A)-2\tan^2 A$

$=\sec^2 A+\tan^2 A-2\tan^2 A$

$=\sec^2 A-\tan^2 A$

$=1$

20. **(1)**

We know $\sin^2\theta\ge\sin^8\theta$ ….. (1)

$\cos^2\theta\ge\cos^{14}\theta$ ….. (2)

Add (1) & (2)

$\Rightarrow\sin^8\theta+\cos^{14}\theta\le 1$

$0<A\le 1$

$a=0, b=1$

21. **(3)**

We know $x+\frac{1}{x}$ lies in interval

$(-\infty,-2]\text{ or }[2,\infty)$

$\therefore$ For is to be equal to $2\cos\theta$

$\cos\theta$ must be ± 1

$\therefore$ in $[0,2\pi]$ only 3 possible angle $0,\pi,2\pi$

22. **(2)**

Use $\sin^2 A-\sin^2 B=\sin(A+B)\sin(A-B)$

$\therefore\ \sin^2 24-\sin^2 6=\sin 30\sin 18$

$= \frac{1}{2} \times \frac{\sqrt{5}-1}{4}$

$= \frac{\sqrt{5}-1}{8}$

$\therefore a = 5, b = 1, c = 8$

Ans: 2

23. **(2)**

$A + B = 225^o$ (Similar to question 2)

$\Rightarrow \tan A \tan B + \tan A + \tan B = 1$

Now $\frac{(1+\cot A)(1+\cot B)}{\cot A \cot B}$

Simplifying to $1 + \tan A + \tan B + \tan A \tan B = 2$

24. **(7)**

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Or $\cos^3\theta = \frac{\cos 3\theta + 3\cos\theta}{4}$

$\therefore$ Expression is

$\frac{\cos 3\theta + 3\cos\theta}{4} + \frac{\cos(360+3\theta) + 3\cos(120+\theta)}{4} + \frac{\cos(720+3\theta) + 3\cos(240+\theta)}{4}$

$= \frac{3}{4}\cos 3\theta + \left(\frac{\cos\theta + \cos(120+\theta) + \cos(240+\theta)}{4}\right) \cdot 3$

$= \frac{3}{4}\cos 3\theta + 0$

$a + b = 7$

25. **(2)**

$\frac{\sin(A+B)\cos A}{\cos(A+B)\sin A} = 3$

Apply Componendo & dividendo

$\frac{\sin(2A+B)}{\sin(B)} = \frac{4}{2} = 2$

26. **(2)**

$\cos A = \cos(\pi - (B+C))$

$= -\cos(B+C)$

$\therefore \frac{\cos A}{\sin B \sin C} = -\frac{\cos B \cos C + \sin B \sin C}{\sin B \sin C}$

$\frac{\cos A}{\sin B \sin C} = (1 - \cot B \cot C)$

$\therefore$ Expression becomes

$= 3 - (\cot A \cot B + \cot B \cot C + \cot C \cot A)$

In $\Delta ABC \sum \cot A \cot B = 1$

$= 2$

27. **(4)**

$$\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\left(-\frac{\cos\pi}{15}\right)$$

$$=-\left[\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right]$$

$$=-\left[\frac{\sin\left(\frac{16\pi}{15}\right)}{16\sin\frac{\pi}{15}}\right]$$

Use formula

$$\prod_{r=0}^{n-1}\cos 2^r\theta=\frac{\sin 2^n\theta}{2^n\sin\theta}$$

$$=\frac{1}{16}$$

$$\frac{1}{\sqrt{x}}=4$$

28. **(3)**

$\sin A\cos B=\frac{1}{4}$ $\qquad$ $3\sin A\cos B=\cos A\sin B$

$\therefore\ \cos A\sin B=\frac{3}{4}$

$=\sin(A+B)=1,\sin(A-B)=-\frac{1}{2}$

$=A+B=90$ $\qquad$ & $A-B=-30^\circ$

$A=30^\circ,B=60^\circ$

$=\cot^2 A=3$

29. **(4)**

$a^2+b^2=2-2\sin A\sin B+2\cos A\cos B$

$=2+2\cos(A+B)$

$=4\cos^2\left(\frac{A+B}{2}\right)$

Maximum $=4$

30. **(8)**

$$\tan\frac{\pi}{3}+2\tan\frac{2\pi}{3}+4\tan\frac{4\pi}{3}+8\tan\frac{8\pi}{3}$$

$$=\sqrt{3}+2\left(\tan\left(\pi-\frac{\pi}{3}\right)\right)+4\tan\left(\pi+\frac{\pi}{3}\right)+8\tan\left(3\pi-\frac{\pi}{3}\right)$$

$$=\sqrt{3}-2\sqrt{3}+4\sqrt{3}-8\sqrt{3}$$

$$=5\sqrt{3}-10\sqrt{3}=-5\sqrt{3}$$

31. **(8)**

For minimum in ΔABC, $A=B=C=60^\circ$

$\therefore\ |\sec A\sec B\sec C|=8$

32. **(1)**

$\cot 6^\circ \cot 42^\circ \cot 66^\circ \cot 78^\circ$

$= \left(\dfrac{\cot 6^\circ \cdot \cot 54^\circ \cot 66^\circ}{\cot 54^\circ}\right) \cdot \left(\dfrac{\cot 18^\circ \cot 42^\circ \cot 78^\circ}{\cot 18^\circ}\right)$

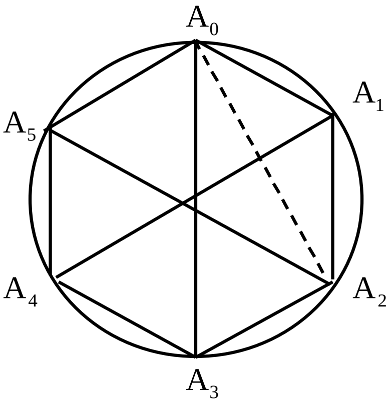
$= \dfrac{\cot 18^\circ}{\cot 54^\circ} \cdot \dfrac{\cot 54^\circ}{\cot 18^\circ} = 1$

33. **(3)**

Use cosine Rule

$A_0A_4 = A_0A_2 = \sqrt{1^2 + 1^2 - 2\cos 120^\circ}$

$= \sqrt{3}$



$\therefore A_0A_1 = 1$

$A_0A_2 = \sqrt{3}$

$A_0A_4 = \sqrt{3}$

$\therefore$ Ans. 3

34. **(8)**

Expression is

$3 + 2\sin^2\theta - 3\sin 2\theta$

Or $3 + 1 - \cos 2\theta - 3\sin 2\theta$

$4 - [\cos 2\theta + 3\sin 2\theta]$

Minimum $4 \pm \sqrt{10}$

Ans:-8

35. **(0)**

$\left|9\tan^2 A + 4\cot^2 B\right|$

For min put both $\tan^2 A = 0$ & $\cot^2 B = 0$

36. **(2)**

Use $AM \geq GM$

$\therefore \left|\tan A + \cot A\right| \geq 2$

37. **(2)**

$\dfrac{A}{2} + \dfrac{B}{2} = 45^\circ$

$\Rightarrow \left(1 + \tan\dfrac{A}{2}\right)\left(1 + \tan\dfrac{B}{2}\right) = 2$

38. **(1)**

$\cos 20^{\circ} + 1 - \cos 110^{\circ} - \sqrt{2}\sin 65^{\circ}$

$1 + \cos 20^{\circ} - \cos 110^{\circ} - \sqrt{2}\sin 65^{\circ}$

$1 + 2\sin 65 \sin 45^{\circ} - \sqrt{2}\sin 65^{\circ}$

$1 + \sqrt{2}\sin 65^{\circ} - \sqrt{2}\sin 65^{\circ}$

Ans: 1

39. **(3)**

$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

OR $\dfrac{3\tan\frac{\pi}{9} - \tan^3\frac{\pi}{9}}{1 - 3\tan^2\frac{\pi}{9}} = \sqrt{3}$

Square

$9\tan^2\frac{\pi}{9} + \tan^6\frac{\pi}{9} - 6\tan^4\frac{\pi}{9} = 3 + 27\tan^4\frac{\pi}{9} - 18\tan^2\frac{\pi}{9}$

Solving to get 3

40. **(3)**

$\sum \cos A = 0$

$\sum \sin A = 0$

$\Rightarrow B = 120 + A, C = 240 + A$

Expression become $\dfrac{3\cos 3A}{\cos 3A} = 3$

JEE Advanced : PYQ

1. (b)

$A = \sin^2\theta + \cos^4\theta = \sin^2\theta + \left(1 - \sin^2\theta\right)^2$

$= \sin^4\theta - \sin^2\theta + 1 \Rightarrow A = \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4}$

Now, $0 \le \left(\sin^2\theta - \frac{1}{2}\right)^2 \le \frac{1}{4}$

$\Rightarrow \frac{3}{4} \le \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4} \le 1$

$\Rightarrow \frac{3}{4} \le A \le 1$

2. (a)

$\alpha + \beta + \gamma = 2\pi \Rightarrow \frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = \pi$

$\therefore \quad \tan\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)=\tan\left(\pi-\frac{\gamma}{2}\right)=-\tan\frac{\gamma}{2}$

$\Rightarrow \quad \tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2}=\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

3. (b)

Given expression can be written as

$$\frac{\sin A}{\cos A}\times\frac{\sin A}{\sin A-\cos A}+\frac{\cos A}{\sin A}\times\frac{\cos A}{\cos A-\sin A} \qquad \left(\because \tan A=\frac{\sin A}{\cos A} \text{ and } \cot A=\frac{\cos A}{\sin A}\right)$$

$$=\frac{1}{\sin A-\cos A}\left\{\frac{\sin^3 A-\cos^3 A}{\cos A\sin A}\right\}$$

$\because \; a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$

$$=\frac{\sin^2 A+\sin A\cos A+\cos^2 A}{\sin A\cos A}=1+\sec A\operatorname{cosec} A$$

4. (c)

$$\sum_{k=1}^{13}\frac{1}{\sin\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$$

$$=\sum_{k=1}^{13}\frac{1}{\sin\frac{\pi}{6}}\left[\frac{\sin\left\{\frac{\pi}{6}+\frac{k\pi}{6}-\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\right\}}{\sin\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}\right]$$

$$\sum_{k=1}^{13}2\left[\cot\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)\right]$$

$$=2\left[\left\{\cot\frac{\pi}{4}-\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\right\}+\left\{\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{2\pi}{6}\right)\right\}+....+\left\{\cot\left(\frac{\pi}{4}+\frac{12\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right\}\right]$$

$$=2\left[\cot\frac{\pi}{4}-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right]=2\left[1-\cot\frac{5\pi}{12}\right]$$

$$=2\left[1-\frac{\sqrt{3}-1}{\sqrt{3}+1}\right]=2\left[1-\left(2-\sqrt{3}\right)\right]=2\left(\sqrt{3}-1\right)$$

5. (b)

Given : $\theta\in\left(0,\frac{\pi}{4}\right) \Rightarrow \tan\theta<1$ and $\cot\theta>1$

Let $\tan\theta=1-x$ and $\cot\theta=1+y$, where $x, y>0$ and are very small, then

$\therefore \quad t_1=(1-x)^{1-x}, t_2=(1-x)^{1+y}, \; t_3=(1+y)^{1-x}, t_4=(1+y)^{1+y}$

Clearly, $t_4>t_3$ and $t_1>t_2$ also, $t_3>t_1$

$\therefore \quad t_4>t_3>t_1>t_2$.

6. (c)

Given : $\alpha+\beta=\frac{\pi}{2} \Rightarrow \alpha=\frac{\pi}{2}-\beta$

$\Rightarrow \tan\alpha=\tan\left(\frac{\pi}{2}-\beta\right)=\cot\beta=\frac{1}{\tan\beta}$

$\Rightarrow \tan\alpha\tan\beta=1 \Rightarrow 1+\tan\alpha\tan\beta=2$.

Now, $\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

$\Rightarrow \tan\gamma=\frac{\tan\alpha-\tan\beta}{2}$

$\Rightarrow 2\tan\gamma=\tan\alpha-\tan\beta \Rightarrow \tan\alpha=2\tan\gamma+\tan\beta$

7. (a)

Given : $(\cot\alpha_1).(\cot\alpha_2).....(\cot\alpha_n)=1$

$\Rightarrow (\cos\alpha_1)(\cos\alpha_2)....(\cos\alpha_n)$

$=(\sin\alpha_1)(\sin\alpha_2).....(\sin\alpha_n)$ … (i)

Let $y=(\cos\alpha_1)(\cos\alpha_2)....(\cos\alpha_n)$ (to be max.)

$\Rightarrow y^2=(\cos^2\alpha_1)(\cos^2\alpha_2).....(\cos^2\alpha_n)$

$=\cos\alpha_1\sin\alpha_1\cos\alpha_2\sin\alpha_2.....\cos\alpha_n\sin\alpha_n$ [From (i)]

$=\frac{1}{2^n}\left[\sin 2\alpha_1\sin 2\alpha_2.....\sin 2\alpha_n\right]$

Now, $0\le\alpha_1, \alpha_2, ... a_n\le\frac{\pi}{2}$

$\therefore 0\le 2\alpha_1, 2\alpha_2, ..., 2\alpha_n\le\pi$

$\Rightarrow 0\le\sin 2\alpha_1, \sin 2\alpha_2, ..., \sin 2\alpha_n\le 1$

$\therefore y^2\le\frac{1}{2^n}\cdot 1 \Rightarrow y\le\frac{1}{2^{n/2}}$

$\therefore$ Max. value of y i.e. $(\cos\alpha_1).(\cos\alpha_2).....(\cos\alpha_n)=\frac{1}{2^{n/2}}$

8. (c)

$f(\theta)=\sin\theta(\sin\theta+\sin 3\theta)$

$=\sin\theta\left(\sin\theta+3\sin\theta-4\sin^3\theta\right)$

$=\sin\theta\left(4\sin\theta-4\sin^3\theta\right)=\sin^2\theta\left(4-4\sin^2\theta\right)$

$=4\sin^2\theta\left(1-\sin^2\theta\right)$

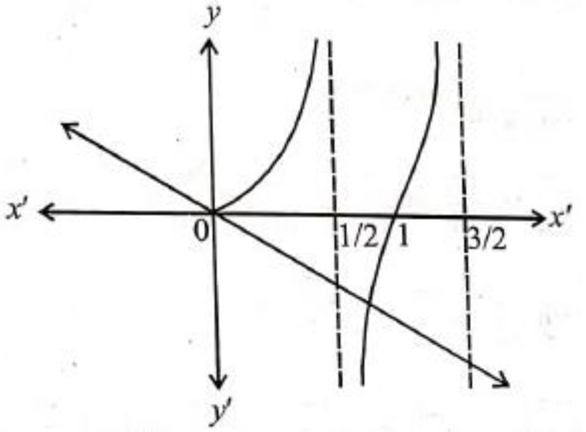
$=4\sin^2\theta\cos^2\theta=(2\sin\theta\cos\theta)^2=(\sin 2\theta)^2\ge 0$, which is true for all θ.

9. (b, c)

Given : $f(x) = x \sin \pi x, x > 0$

$\Rightarrow \quad f'(x) = \sin \pi x + x\pi \cos \pi x$

Now, $f'(x) = 0 \quad \Rightarrow \quad \tan \pi x = -\pi x$



From graph of $y = \tan \pi x$ and $y = -\pi x$, it is clear that they intersect each other at unique point in the intervals

$(n, n+1)$ and $\left(n + \frac{1}{2}, n+1\right)$

10. (a, c, d)

As $\tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}, \theta \in [0, 2\pi]$

Hence $\frac{3\pi}{2} < \theta < \frac{5\pi}{3}$

Now $2\cos\theta(1 - \sin\varphi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\varphi - 1$

$\Rightarrow \quad 2\cos\theta(1 - \sin\varphi) = 2\sin\theta\cos\varphi - 1$

$\Rightarrow \quad 2\cos\theta + 1 = 2\sin(\theta + \varphi)$

As $\quad \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right), 1 < 2\sin(\theta + \varphi) < 2$

As $\quad \theta + \varphi \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ or $(\theta + \varphi) \in \left(\frac{13\pi}{6}, \frac{17\pi}{6}\right)$

We have $\varphi \in \cup\left(\frac{2\pi}{3}, \frac{7\pi}{6}\right)$

11. (a, b)

Given : $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5} \quad \Rightarrow \quad 3\sin^4 x + 2\cos^4 x = \frac{6}{5}$

$\Rightarrow \quad \sin^4 x + 2\left[\sin^4 x + \cos^4 x\right] = \frac{6}{5}$

$$\Rightarrow \quad \sin^4 x + 2\left[1 - 2\sin^2 x\cos^2 x\right] = \frac{6}{5}$$

$$\Rightarrow \quad \sin^4 x + 2 - 4\sin^2 x\left(1 - \sin^2 x\right) = \frac{6}{5}$$

$$\Rightarrow \quad 5\sin^4 x - 4\sin^2 x + 2 - \frac{6}{5} = 0$$

$$\Rightarrow \quad 25\sin^4 x - 20\sin^2 x + 4 = 0$$

$$\Rightarrow \quad \left(5\sin^2 x - 2\right)^2 = 0 \quad \Rightarrow \sin^2 x = \frac{2}{5}$$

$$\Rightarrow \quad \cos^2 x = \frac{3}{5} \text{ and } \tan^2 x = \frac{2}{3}$$

Also $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{625} + \frac{3}{625} = \frac{5}{625} = \frac{1}{125}$

12. (a, b, c, d)

Note that multiplicative loop is very important approach in IIT Mathematics

$$\left(\tan\frac{\theta}{2}\right)\left(1 + \sec\theta\right) = \frac{\sin\theta/2}{\cos\theta/2} \cdot \left[1 + \frac{1}{\cos\theta}\right]$$

$$= \frac{\left(\sin\theta/2\right)2\cos^2\theta/2}{\left(\cos\theta/2\right)\cos\theta}$$

$$= \frac{\left(2\sin\theta/2\right)\cos\theta/2}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\therefore \quad f_n\left(\theta\right) = \left(\tan\theta/2\right)\left(1 + \sec\theta\right)$$

$$\left(1 + \sec 2\theta\right)\left(1 + \sec 2^2\theta\right)....\left(1 + \sec 2^n\theta\right)$$

$$= \left(\tan\theta\right)\left(1 + \sec 2\theta\right)\left(1 + \sec 2^2\theta\right)....\left(1 + \sec 2^n\theta\right)$$

Now, $f_2\left(\frac{\pi}{16}\right) = \tan\left(2^2 \cdot \frac{\pi}{16}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

Therefore, (a) is the correct option.

$$f_3\left(\frac{\pi}{32}\right) = \tan\left(2^3 \cdot \frac{\pi}{32}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

Therefore, (b) is the correct option.

$$f_4\left(\frac{\pi}{64}\right) = \tan\left(2^4 \cdot \frac{\pi}{64}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

Therefore, (c) is the correct option.

$$f_5\left(\frac{\pi}{128}\right) = \tan\left(2^5 \cdot \frac{\pi}{128}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

Therefore, (d) is the correct option.

13. (A) $\rightarrow$ r; B $\rightarrow$ p

(p) If $\frac{13\pi}{48} < \alpha < \frac{14\pi}{48}$ then $\frac{13\pi}{16} < 3\alpha < \frac{14\pi}{16}$ and $\frac{13\pi}{24} < 2\alpha < \frac{14\pi}{24}$

$\Rightarrow$ $3\alpha \in$ II quad and $2\alpha \in$ II quad $\Rightarrow$ $\sin 3\alpha = +\text{ve}$

$\cos 2\alpha = -\text{ve}$ $\quad\therefore\ \dfrac{\sin 3\alpha}{\cos 2\alpha} = -\text{ve}$

$\therefore$ (B) corresponds to (p).

(q) If $\alpha \in \left(\dfrac{14\pi}{48}, \dfrac{18\pi}{48}\right)$ then $\dfrac{14\pi}{16} < 3\alpha < \dfrac{18\pi}{16}$ and $\dfrac{14\pi}{24} < 2\alpha < \dfrac{18\pi}{24}$

$\Rightarrow$ $3\alpha \in$ II or III quad and $2\alpha \in$ II quad

$\Rightarrow$ Nothing can be said about the sing of $\dfrac{\sin 3\alpha}{\cos 2\alpha}$ over this interval.

(r) If $\alpha \in \left(\dfrac{18\pi}{48}, \dfrac{23\pi}{48}\right)$ then $\dfrac{18\pi}{16} < 3\alpha < \dfrac{23\pi}{16}$ and $\dfrac{18\pi}{24} < 2\alpha < \dfrac{23\pi}{24}$

$\Rightarrow$ $3\alpha \in$ III quad and $2\alpha \in$ II quad

$\Rightarrow$ $\sin 3\alpha = -\text{ve}$, $\cos 2\alpha = -\text{ve}$, $\therefore\ \dfrac{\sin 3\alpha}{\cos 2\alpha} = +\text{ve}$

$\therefore$ (A) corresponds to (r)

(s) If $\alpha \in \left(0, \dfrac{\pi}{2}\right)$, then $0 < 3\alpha < \dfrac{3\pi}{2}$ and $0 < 2\alpha < \pi$

$\Rightarrow$ Nothing can be said about the sing of $\dfrac{\sin 3\alpha}{\cos 2\alpha}$ over the given interval.

14. (True)

$$\tan A = \frac{1-\cos B}{\sin B} = \frac{2\sin^2 B/2}{2\sin B/2\cos B/2} = \tan B/2$$

Now, $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A} = \dfrac{2\tan B/2}{1-\tan^2 B/2} = \tan B$

Hence, statement is true.

15. (1)

Rearrange the given expression

$$\left(\frac{\sin\alpha}{\cos\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\cos\alpha}{\sin\beta} + \frac{\sin\beta}{\cos\alpha}\right)^2$$

$$= \left(\frac{\cos(\alpha-\beta)}{\sin\beta\cos\beta} + \frac{\cos(\alpha-\beta)}{\sin\alpha\cdot\cos\alpha}\right)^2$$

$$= \left(\frac{4}{3}\left\{\frac{1}{\sin 2\beta} + \frac{1}{\sin 2\alpha}\right\}\right)^2 \qquad \left[\because\ \cos(\alpha-\beta) = \frac{2}{3}\right]$$

$$= \frac{16}{9}\left[\frac{\sin 2\alpha + \sin 2\beta}{\sin 2\alpha.\sin 2\beta}\right]$$

$$= \frac{64}{9}\left(\frac{2\sin(\alpha+\beta).\cos(\alpha-\beta)}{2\sin 2\alpha.\sin 2\beta}\right)^2$$

$$=\frac{64}{9}\left(\frac{2\cdot\frac{1}{3}\cdot\frac{2}{3}}{\cos(2\alpha-2\beta)-\cos(2\alpha+2\beta)}\right)^2$$

$$=\frac{64}{9}\left(\frac{\frac{4}{9}}{2\cos^2(\alpha-\beta)-1-1+2\sin^2(\alpha+\beta)}\right)^2$$

$$=\frac{64}{9}\left(\frac{\frac{4}{9}}{\frac{8}{9}-2+\frac{2}{9}}\right)^2=\frac{64}{9}\left(\frac{1}{-2}\right)^2=\left[\frac{16}{9}\right]=[1.7]=1$$

16. (2)

Let $f(\theta)=\frac{1}{g(\theta)}$, where $g(\theta)=\sin^2\theta+3\sin\theta\cos\theta+5\cos^2\theta$

Clearly, f is maximum when g is minimum

Now, $g(\theta)=\frac{1-\cos 2\theta}{2}+\frac{3}{2}\sin 2\theta+\frac{5}{2}(1+\cos 2\theta)$

$=3+2\cos 2\theta+\frac{3}{2}\sin 2\theta\ge 3+\left(-\sqrt{4+\frac{9}{4}}\right)$

$\therefore\quad g_{\min}=3-\frac{5}{2}=\frac{1}{2}\quad\Rightarrow\quad f_{\max.}=2.$

17. (1)

Let $\pi x-\frac{\pi}{4}=\theta\in\left[\frac{-\pi}{4},\frac{7\pi}{4}\right]$

$\because\quad f(x)\ge 0$

So, $\left(3-\sin\left(\frac{\pi}{2}+2\theta\right)\right)\sin\theta\ge\sin(\pi+3\theta)$

$\Rightarrow\quad(3-\cos 2\theta)\sin\theta\ge-\sin 3\theta$

$\Rightarrow\quad\sin\theta\left[3-4\sin^2\theta+3-\cos 2\theta\right]\ge 0$

$\Rightarrow\quad\sin\theta\left(6-2(1-\cos 2\theta)-\cos 2\theta\right)\ge 0$

$\Rightarrow\quad\sin\theta(4+\cos 2\theta)\ge 0\quad\Rightarrow\quad\sin\theta\ge 0$

$\Rightarrow\quad\theta\in[0,\pi]\Rightarrow 0\le\pi x-\frac{\pi}{4}\le\pi\quad\Rightarrow\quad x\in\left[\frac{1}{4},\frac{5}{4}\right]$

$\Rightarrow\quad[\alpha,\beta]=\left[\frac{1}{4},\frac{5}{4}\right];\ \therefore\ \beta-\alpha=\frac{5}{4}-\frac{1}{4}=1$

18. (1/2)

Given : $A+B=\frac{\pi}{3} \Rightarrow \tan(A+B)=\sqrt{3}$

$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\tan B}=\sqrt{3} \Rightarrow \frac{\tan A+\frac{y}{\tan A}}{1-y}=\sqrt{3}$ [Let $y=\tan A\tan B$]

$\Rightarrow \tan^2 A+\sqrt{3}(y-1)\tan A+y=0$

For real value of $\tan A, 3(y-1)^2-4y\geq 0$

$\Rightarrow 3y^2-10y+3\geq 0 \Rightarrow (y-3)\left(y-\frac{1}{3}\right)\geq 0$

$\Rightarrow y\leq\frac{1}{3}$ or ≥ 3

But $A, B>0$ and $A+B=\frac{\pi}{3} \Rightarrow A, B<\frac{\pi}{3}$

$\Rightarrow \tan A\tan B<3$

$\Rightarrow y\leq\frac{1}{3} \Rightarrow$ Max. value of y is $\frac{1}{3}$.

19. $[-\pi/2, -\pi/10]\cup[3\pi/10, \pi/2]$

Given : $2\sin t=\frac{1-2x+5x^2}{3x^2-2x-1}$, $t\in[-\pi/2, \pi/2]$

$\Rightarrow (6\sin t-5)x^2+2(1-2\sin t)x-(1+2\sin t)=0$

The given equation will hold, if x be some real number, and hence, $D\geq 0$

$\Rightarrow 4(1-2\sin t)^2+4(6\sin t-5)(1+2\sin t)\geq 0$

$\Rightarrow 16\sin^2 t-8\sin t-4\geq 0$

$\Rightarrow (4\sin^2 t-2\sin t-1)\geq 0$

$\Rightarrow \sin t\leq -\left(\frac{\sqrt{5}-1}{4}\right)$ or $\sin t\geq\frac{\sqrt{5}+1}{4}$

$\Rightarrow \sin t\leq\sin\left(-\frac{\pi}{10}\right)$ or $\sin t\geq\sin\left(\frac{3\pi}{10}\right)$

$\Rightarrow t\leq -\frac{\pi}{10}$ or $t\geq\frac{3\pi}{10}$ [Note that $\sin x$ is an increasing function from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$]

$\therefore$ range of t is $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right]\cup\left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$.

20. (1/8)

$K=\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$

$=\cos\left(\frac{\pi}{2}-\frac{\pi}{18}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)\cos\left(\frac{\pi}{2}-\frac{7\pi}{18}\right)$

$$= \cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9} = \frac{1}{2^3 \sin\frac{\pi}{9}} \cdot \sin\frac{8\pi}{9}$$

$$[\because \cos\alpha\cos 2\alpha\cos 2^2\alpha......\cos 2^{n-1}\alpha = \frac{1}{2^n \sin\alpha} \cdot \sin(2^n\alpha)]$$

$$= \frac{1}{8\sin\frac{\pi}{9}} \cdot \sin\frac{\pi}{9} = \frac{1}{8}$$

21. (1/64)

$$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$$

$$= \left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)^2 \times 1 \qquad [\because \sin\frac{13\pi}{14} = \sin\frac{\pi}{14}, \sin\frac{11\pi}{14} = \sin\frac{3\pi}{14} \text{ and } \sin\frac{3\pi}{14} = \sin\frac{5\pi}{14}]$$

$$= \left[\cos\left(\frac{\pi}{2} - \frac{\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{5\pi}{14}\right)\right]^2$$

$$= \left[\cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}\right]^2 = \left[\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\right]^2 \qquad [\because \cos\alpha\cos 2\alpha\cos 2^2\alpha......\cos 2^{n-1}\alpha]$$

$$= \left[\frac{1}{2^n \sin\alpha} \cdot \sin\left(2^n\alpha\right)\right]^2$$

$$= \left(\frac{1}{8\sin\frac{\pi}{7}}\sin\frac{8\pi}{7}\right)^2 = \left(\frac{\sin\left(\pi + \pi/7\right)}{8\sin\pi/7}\right)^2$$

$$= \left(\frac{-\sin\pi/7}{8\sin\pi/7}\right)^2 = \left(\frac{1}{8}\right)^2 = \frac{1}{64}$$