

①

physics  
Mechanical properties of Fluids

Level-01

1. c  $(\rho g h = P)$

2.  $P_1 V_1 = P_2 V_2$

$$(P_0 + \rho g L) \times \frac{4}{3} \pi R^3 = P_0 \times \frac{4}{3} \pi (2R)^3$$

$$P_0 + \rho g L = 8 P_0$$

$$\rho g L = 7 P_0$$

$$\rho g L = 7 \rho g H$$

$$L = 7H$$

3.  $P_1 V_1 = P_2 V_2$

$$(P_0 + \rho g L) V = P_0 \times 3V$$

$$\left( \rho_m g \times 75 + \frac{\rho_m}{10} \times g \times L \right) V = \rho_m g \times 75 \times 3V$$

$$75 + \frac{L}{10} = 225$$

$$L = 1500 \text{ cm}$$

4.  $P = \rho g h$

$$\frac{\Delta P \times 100}{P} = \frac{\Delta \rho}{\rho} \times 100 + \frac{\Delta h}{h} \times 100$$

$$0 = -2\% + \frac{\Delta h}{h} \times 100$$

$$\frac{\Delta h}{h} \times 100 = 2\% \quad \text{increases.}$$

5.  $P = P_0 + \rho gh$

pressure will be same at same depth.

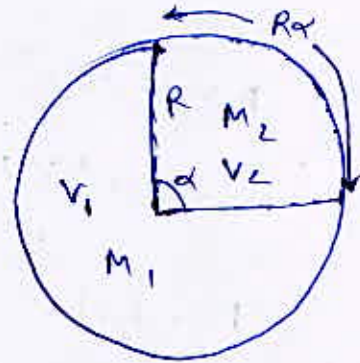
6. pressure will be same at partition

$$P = \frac{nRT}{V}$$

$$\frac{n_1}{V_1} = \frac{n_2}{V_2}$$

$$\frac{\frac{m}{M_1}}{V_1} = \frac{\frac{m}{M_2}}{V_2}$$

$$\frac{V_2}{V_1} = \frac{M_1}{M_2} = \frac{32}{28} = \frac{8}{7}$$



$$\frac{H \times R^2 \times (\alpha/2)}{H \times R^2 \times (180 - \alpha)/2} = \frac{8}{7}$$

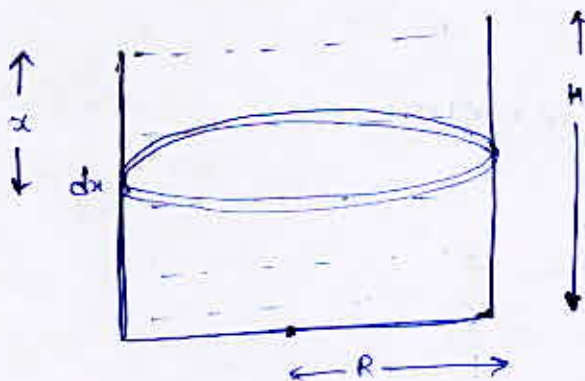
(Area of sector =  $\frac{1}{2} r^2 \theta$ )

$$\frac{\alpha}{360 - \alpha} = \frac{8}{7}$$

$$\frac{\alpha}{360} = \frac{8}{15}$$

$$\alpha = \frac{360 \times 8}{15} = 192$$

7.



Force on the side of vessel

$$dF = \rho g x \times (2\pi R dx)$$

$$F_s = 2\pi R \rho g \int_0^H x dx$$

$$= 2\pi R \times \rho g \frac{H^2}{2}$$

Force on side of the vessel = Force on Bottom

$$2\pi R \rho g \times \frac{H^2}{2} = \pi R^2 \times \rho g H$$

$$H = R$$

8. At same level of mercury, pressure will be same

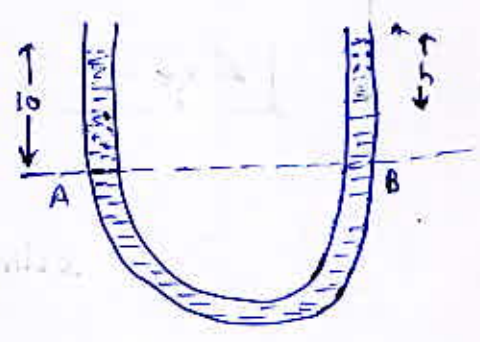
$$P_A = P_B$$

$$1.3 \times 10 \times g = h \times 0.8 \times g + (10-h) \times 13.6 \times g$$

$$13 = 0.8h + 13.6(10-h)$$

$$(13.6 - 0.8)h = 136 - 13$$

$$h = \frac{123}{12.8} = 9.6 \text{ cm}$$



9. -

10.  $P_1 = P_2$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\frac{10^2 \text{ N}}{10^4 \text{ cm}^2} = \frac{2000 \times 10 \text{ N}}{A_2 \text{ cm}^2}$$

$$A_2 = 2 \times 10^4 \text{ cm}^2$$

11.  $\frac{\rho_s}{\rho_w} = 7$

$$\rho_s = 7 \rho_w$$

Apparent weight =  $mg - \text{Buoyancy force}$

$$= \rho_s V g - \rho_w V g$$

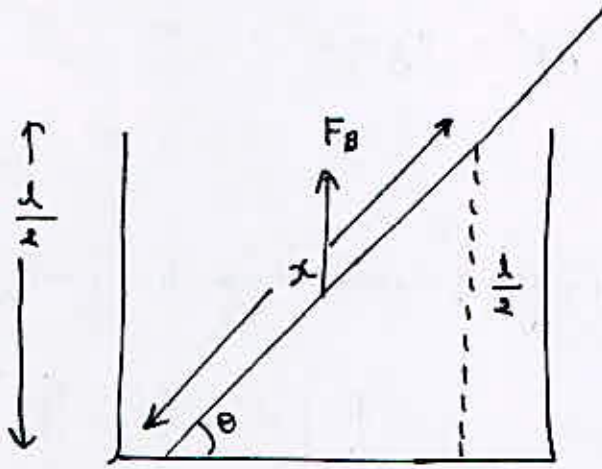
$$= V g (\rho_s - \rho_w)$$

$$= 5 \times 5 \times 5 \times 6 \rho_w \times g$$

12. -

4

13.



$$x \sin \theta = \frac{l}{2}$$

$$x = \frac{l}{2 \sin \theta}$$

$$F_B \times \frac{x}{2} \cos \theta = mg \times \frac{l}{2} \cos \theta$$

$$A \times \rho_0 \times g \times \frac{l}{4 \sin \theta} \times \cos \theta = A \times L \times \rho \times g \times \frac{l}{2} \cos \theta$$

$$\frac{x \rho_0}{4 \sin \theta} = \rho \frac{l}{2}$$

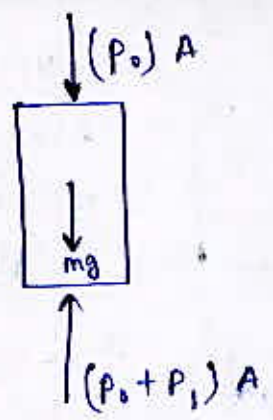
$$\frac{l}{4 \sin \theta} \times \frac{\rho_0}{4 \sin \theta} = \frac{\rho l}{2}$$

$$\sin^2 \theta = \sqrt{\frac{\rho_0}{4 \rho}}$$

$$\sin \theta = \frac{1}{2} \sqrt{\frac{\rho_0}{\rho}}$$



15.



$$P_1 = \rho_1 g h_1 + \rho_2 g h_2$$

$$P_1 = (0.6 \times 6 + 1 \times 4) g$$

$$= (3.6 + 4) g$$

$$= 7.6 g$$

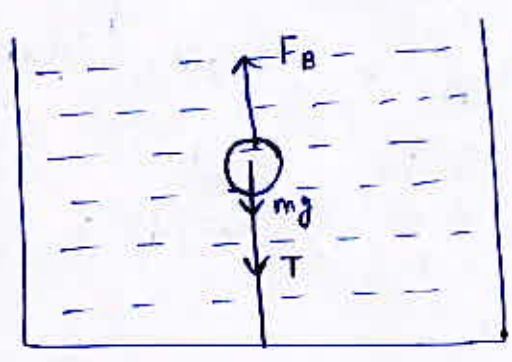
$$(P_0 + P_1) A - P_2 A = mg$$

$$7.6 g A = mg$$

$$m = 7.6 \times 10 \times 10$$

$$= 760 \text{ gms}$$

16.



$$T + mg = \text{Buoyancy force}$$

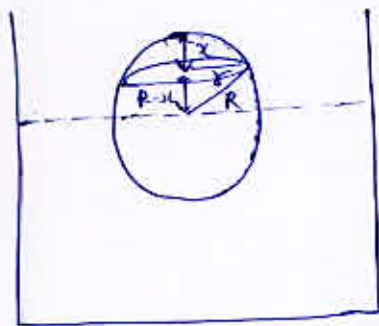
$$T = \cancel{mg} F_B - mg$$

$$= (\text{volume of displaced liquid}) \times \rho_w \times g - mg$$

$$= \frac{m}{\rho_w / \eta} \times \rho_w \times g - mg$$

$$= mg (\eta - 1)$$

17.

Take an element  $\alpha$  at  $x$  depth from top

$$dv \approx \pi [R^2 - (R-x)^2] dx = \pi r^2 dx$$

$$r = \sqrt{R^2 - (R-x)^2}$$

$$dv = \pi (2Rx - x^2) dx$$

$$\text{Buoyancy Force} = \rho dv \times g$$

$$dF = \rho \pi (2Rx - x^2) dx \times g$$

Work done by this constant force  $dF$  for distance  $x$ This is the  
work doneBy Buoyancy  
Force for upper  
part only.

$$dW = -dF \times x$$

$$dW = -\rho \pi g (2Rx^2 - x^3) dx$$

$$W = -\int_0^R \rho \pi g (2Rx^2 - x^3) dx$$

$$= -\rho \pi g \left[ \frac{2}{3} R^3 - \frac{R^4}{4} \right]$$

$$= -\rho \pi g \left[ \frac{5}{12} \right] R^4$$

$$\text{work done by gravity} = mgR$$

$$= \frac{4}{3} \pi R^3 \times \frac{\rho}{2} \times g R$$

$$= \frac{2}{3} \pi \rho g R^4$$

Work done by Buoyancy Force for half immersed part

$$= -F \times R$$

$$= -\frac{2}{3} \pi R^3 \times \frac{\rho}{2} \times g \times R$$

$$= -\frac{2}{3} \pi \rho g R^4$$

Work Energy theorem  $\Rightarrow$ 

$$W_{\text{gravity}} + W_{\text{external}} + W_{\text{Buoyancy force}} = \Delta K = 0$$

$$W_{\text{external}} - \left( \frac{5}{12} \pi \rho g R^4 + \frac{2}{3} \pi \rho g R^4 \right) + \frac{2}{3} \pi \rho g R^4 = 0$$

$$W_{\text{external}} = \frac{5}{12} \pi \rho g R^4$$

19.

$$v_1 = \frac{m}{\rho_1}$$

$$v_2 = \frac{m}{\rho_2}$$

$$V = v_1 + v_2$$

$$M = m + m = 2m$$

$$\rho = \frac{M}{V}$$

$$= \frac{2m}{\frac{m}{\rho_1} + \frac{m}{\rho_2}} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$$

20.

$$m_1 = V \times \rho_1$$

$$m_2 = V \times \rho_2$$

$$M = m_1 + m_2 = V(\rho_1 + \rho_2)$$

$$V = 2v$$

$$\rho = \frac{M}{V} = \frac{V(\rho_1 + \rho_2)}{2V} = \frac{\rho_1 + \rho_2}{2}$$

24.

$$A_1 v_1 = A_2 v_2$$

$$\frac{v_1}{v_2} = \frac{A_2}{A_1} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{3}{3}\right)^2 = \frac{4}{9}$$

25.

$$P + \rho gh + \frac{1}{2} \rho v^2 = \text{Constant}$$

$$A_1 v_1 = A_2 v_2$$



30.

$$\frac{1}{2} \rho v^2 = \rho_{Hg} \rho H$$

8

32.

From continuity equation

$$A_1 v_1 = A_2 v_2$$

$$A_1 = A_2 \Rightarrow v_1 = v_2$$

$$v_A = v_B$$

From Bernoulli's equation

$$P + \rho g h + \frac{1}{2} \rho v^2 = \text{constant}$$

$$P_A + \rho g h_A + \frac{1}{2} \rho v_A^2 = P_B + \rho g h_B + \frac{1}{2} \rho v_B^2$$

$$v_A = v_B$$

$$h_A > h_B$$

$$\text{so } P_A < P_B$$

33.

Horizontal velocity at depth D

$$= \sqrt{2gD}$$

time taken by man from height (H-D) to ground

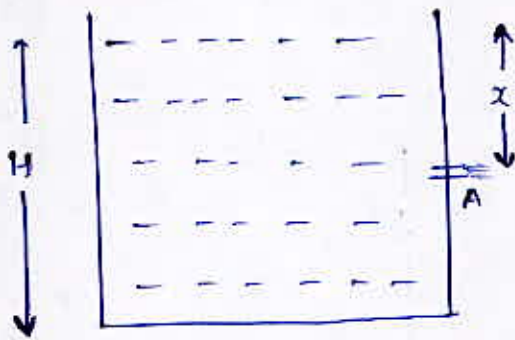
$$t = \sqrt{\frac{2(H-D)}{g}}$$

Range = Horizontal velocity (constant)  $\times$  time

$$= \sqrt{2gD} \times \sqrt{\frac{2(H-D)}{g}}$$

$$= 2\sqrt{D(H-D)}$$





$$\text{velocity at depth } x = \sqrt{2gx}$$

$$\text{time from A to ground} \Rightarrow H-x = ut + \frac{1}{2}at^2$$

$$H-x = \frac{1}{2}at^2 \quad (u=0 \text{ vertical})$$

$$t = \sqrt{\frac{2(H-x)}{g}}$$

$$\text{Horizontal Range} = \text{velocity (Horizontal)} \times \text{time}$$

$$= \sqrt{2gx} \times \sqrt{\frac{2(H-x)}{g}}$$

$$R = 2\sqrt{x(H-x)}$$

$$R^2 = 4x(H-x)$$

$R^2$  will be maximum when  $R$  is maximum

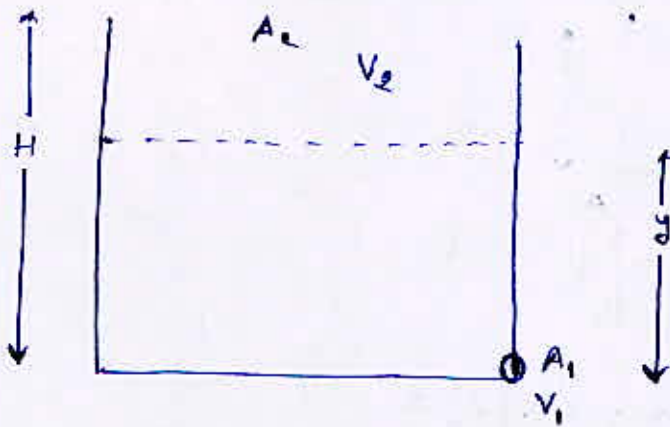
$$\frac{d(R^2)}{dx} = 4(H-x) - 4x = 0$$

$$x = \frac{H}{2}$$

$$\frac{d^2(R^2)}{dx} = -8 \quad \text{Maxima at } x = \frac{H}{2}$$

Horizontal distance will be maximum when hole is at half depth.

$$x = \frac{90}{2} = 45 \text{ cm}$$



$A_1$  = Area of orifice

$A_2$  = cross section Area of Rectangular vessel

From equation of continuity

$$A_1 V_1 = A_2 V_2$$

At any height  $y$

$$V_2 = -\frac{dy}{dt}$$

$$V_1 = \sqrt{2gy}$$

$$A_1 \sqrt{2gy} = -A_2 \times \frac{dy}{dt}$$

$$\int_H^0 \frac{dy}{\sqrt{y}} = \int_0^t -\frac{A_1}{A_2} \sqrt{2g} dt$$

$$(2\sqrt{y})_H^0 = \frac{A_1}{A_2} \sqrt{2g} \times t$$

$$t = \frac{A_2}{A_1} \times \sqrt{\frac{2H}{g}}$$

$$t \propto \sqrt{H}$$

$$\frac{t_2}{t_1} = \sqrt{\frac{h_2}{h_1}}$$

$$t_2 = \frac{t_1}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7 \text{ minutes}$$

1. volume of liquid remains constant

$$n \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3 = V \quad \text{--- (1)}$$

$$R = \left( \frac{3V}{4\pi} \right)^{\frac{1}{3}}$$

$$r = \left( \frac{3V}{4n\pi} \right)^{\frac{1}{3}}$$

$$\text{Released Energy} = T \times \Delta A$$

$$= T \times [n 4\pi r^2 - 4\pi R^2]$$

$$= \frac{T}{r} [n 4\pi r^3 - 4\pi r R^2] \quad \text{From (1)}$$

$$= \frac{T}{r} [4\pi r^3 - 4\pi r R^2]$$

$$= T \times 4\pi R^3 \left[ \frac{1}{r} - \frac{1}{R} \right]$$

$$= T \times 3V \left[ \frac{1}{r} - \frac{1}{R} \right] \quad \text{From (1)}$$

2.

$$n \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3 \quad (\text{Volume remains constant})$$

$$27 r^3 = \left( \frac{D}{2} \right)^3$$

$$\frac{D}{2} = 3r$$

$$r = \frac{D}{6}$$

From Q. (1)

$$\text{Released change in surface energy} = T \times 4\pi R^3 \left( \frac{1}{r} - \frac{1}{R} \right)$$

$$= T \times 4\pi \left( \frac{D}{2} \right)^3 \left( \frac{6}{D} - \frac{2}{D} \right)$$

$$= 2T\pi D^2$$

$$= 2\sigma\pi D^2$$

4.

Excess pressure in bubble =  $\frac{4T}{R}$ 

12

$$= \frac{4 \times 30}{\frac{8 \times 10^{-1}}{2}} = 300 \text{ dyne/cm}^2$$

5.

$$h = \frac{2T \cos \theta}{\rho g r}$$

$$h \propto \frac{1}{r}$$

6.

~~Work done~~ = ~~Increase in potential energy~~

+

$$\left( P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 \right) \times V_1 + W_{\text{motor}}$$

$$= \left( P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2 \right) \times V$$

$$(10^5 + 0 + 0) \times 4 + W_{\text{motor}} = (2 \times 10^5 + \rho g \times 20) \times 4$$

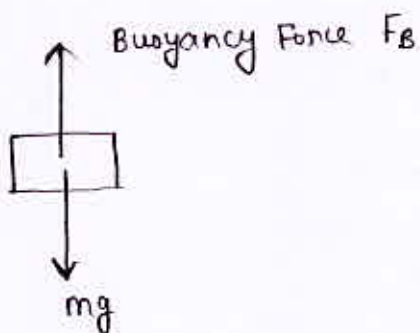
$$4 \times 10^5 + W_{\text{motor}} = 8 \times 10^5 + 10^3 \times 10 \times 20 \times 4$$

$$W_{\text{motor}} = 12 \times 10^5 \text{ J}$$



1.

$$\frac{S_{\text{stone}}}{\rho_w} = k$$



$$m = \rho_s V$$

$$mg - F_B = ma$$

$$\rho_s V g - V \rho_w g = \rho_s V a$$

$$\frac{\rho_s}{\rho_w} V g - \frac{V \rho_w g}{\rho_w} = \frac{\rho_s V a}{\rho_w}$$

$$kg - g = ka$$

$$a = g \left(1 - \frac{1}{k}\right)$$

2.

$$\text{volume of liquid } V = \frac{m_1}{d} = \frac{m}{d} \Rightarrow m = dV$$

$$m_1 g = m(dV)g - d_1 V g$$

$$m_1 = dV - d_1 V$$

$$V = \frac{m_1}{d - d_1}$$

$$m_2 g = dVg - d_2 Vg$$

$$V = \frac{m_2}{d - d_2}$$

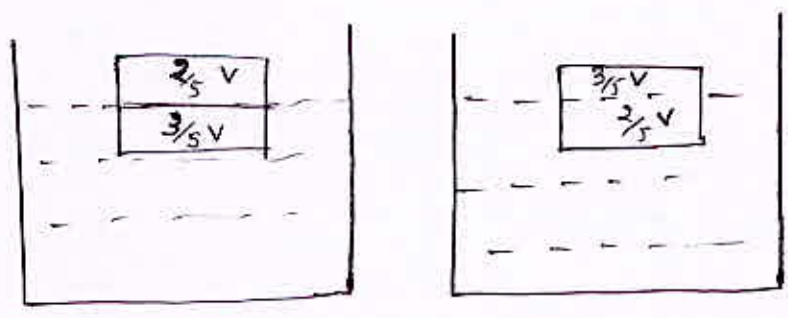
$$\frac{m_1}{d-d_1} = \frac{m_2}{d-d_2}$$

$$m_1 d - m_1 d_2 = m_2 d - m_2 d_1$$

$$d(m_1 - m_2) = m_1 d_2 - m_2 d_1$$

$$d = \frac{m_1 d_2 - m_2 d_1}{m_1 - m_2}$$

3.



$$\frac{3}{5} V \times d \times g = \frac{3}{5} V \times \rho_w \times g$$

$$\frac{3 \rho_w}{5} = d \Rightarrow \rho_w = \frac{5d}{3}$$

$$\frac{2}{5} V \times d \times g = \frac{2}{5} V \times \rho_o \times g$$

$$\frac{2 \rho_o}{5} = d \Rightarrow \rho_o = \frac{5d}{2}$$

$$\frac{\rho_o}{\rho_w} = \frac{\frac{5d}{2}}{\frac{5d}{3}} = \frac{3}{2} = 1.5$$

4.

change in potential energy = Amount of work done

$$W = MgH - Mg \frac{H}{2}$$

$$= (1 \times 1 \times 1 \times 10^3) g \times 1 - (1 \times 1 \times 1 \times 10^3) \times 10 \times \frac{1}{2}$$

$$= \frac{10000}{2} = 5000 \text{ J}$$

5.

$$\begin{aligned}
 F &= p \times A \\
 &= \rho g h \times A \\
 &= \rho g \times V \\
 F &= mg
 \end{aligned}$$

$$m_A = m_B = m_C$$

8.  
8.

$$10 \text{ mm Hg pressure} = \rho g H$$

$$(10 \times 10^{-1}) \times 13.6 \times g = 1 \times g \times H$$

$$H = 13.6 \text{ cm}$$

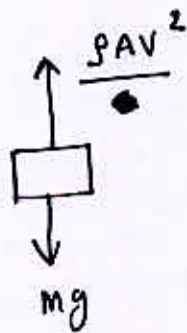
9.

$$\begin{aligned}
 p &= \rho g H \\
 p &\propto H
 \end{aligned}$$

10.

No figure given

11.



$$\begin{aligned}
 F &= \frac{dp}{dt} \\
 &= v \frac{dm}{dt} \\
 &= v \times (\rho A v)
 \end{aligned}$$

$$\frac{1}{2} \rho A v^2 = mg$$

$$\rho = \frac{mg}{A v^2}$$

$$= \frac{1.23 \times 10}{0.1 \times 100}$$

$$= 1.23 \text{ kg/m}^3$$

# Pascal's Law And Archimede's principle

(17)

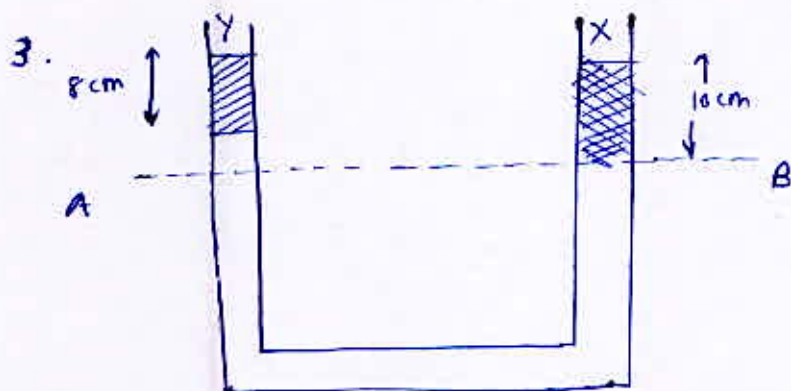
1.

$$\rho_{\text{wood}} \times V \times g = \frac{4}{5} V \times \rho_w \times g$$

$$\rho_{\text{wood}} = \frac{4}{5} \rho_w$$

$$\rho_{\text{wood}} \times V \times g = V \times \rho_L \times g$$

$$\rho_{\text{wood}} = \rho_L = \frac{4}{5} \rho_w = \frac{4}{5} \times 1000$$
$$= 800 \text{ kg/m}^3$$



pressure at AB line will be equal

$$\rho_y g \times 8 + \rho_{\text{Hg}} \times g \times 2 = \rho_x \times 10 \times g$$

$$\rho_y \times 8 + 13.6 \times 2 = 3.36 \times 10$$

$$\rho_y = \frac{33.6 - 27.2}{8}$$

$$= \frac{6.4}{8} = 0.8 \text{ gm/cm}^3$$

6.

volume of the body = V

$$m = 50 \text{ g}$$

$$40 \times g = 50 \times g - V \rho_w \times g$$

$$V = \frac{10}{\rho_w}$$



$$\begin{aligned}
 \text{Wt in liquid} &= mg - \rho_L \times V \times g \\
 &= 50g - 1.5 \rho_w \times \frac{10}{\rho_w} \times g \\
 &= 35g
 \end{aligned}$$

7.

density of body =  $\rho$ volume =  $V$ 

$$\rho V g = \frac{2}{3} V \times \rho_w \times g$$

$$\rho = \frac{2}{3} \rho_w$$

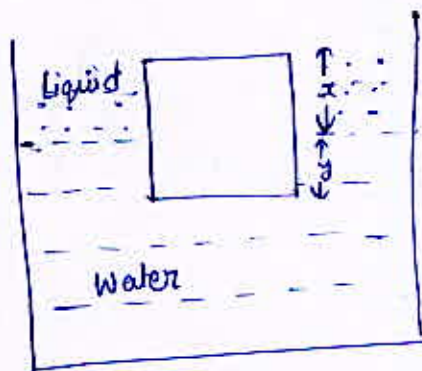
$$\text{oil} \Rightarrow \rho V g = \frac{1}{2} V \times \rho_o \times g$$

$$\rho = \frac{\rho_o}{2}$$

$$\rho = \frac{2}{3} \rho_w = \frac{\rho_o}{2}$$

$$\frac{\rho_o}{\rho_w} = \frac{4}{3}$$

8.

Area of cross section  
 $A = L^2$ 

$$L^3 = V$$

$$x + y = L \Rightarrow$$

$$Ax \rho_L g + Ay \rho_w g = mg = AxLg \times \rho$$

$$x \rho_L + y \rho_w = L \rho$$

$$x + y = L \Rightarrow x = L - y$$

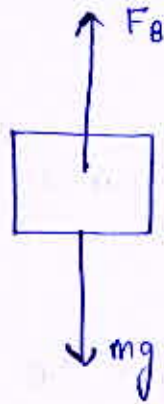
$$\rho_L (L - y) + \rho_w y = L \rho$$

$$y (\rho_w - \rho_L) = L (\rho - \rho_L)$$

$$\frac{y}{L} = \frac{\rho - \rho_L}{\rho_w - \rho_L} = \frac{0.9 - 0.7}{1 - 0.7} = \frac{2}{3}$$

10. Same as Q.8

12.



$$mg - F_B = 3 \text{ N}$$

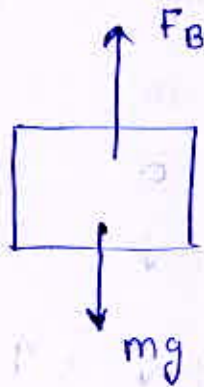
$$F_B = 4 - 3 = 1 \text{ N}$$

15.

velocity at the surface of water =  $\sqrt{2gH}$

$$= \sqrt{2 \times 9.8 \times 0.09}$$

=



$$\text{Retardation} = \frac{F_B - mg}{m}$$

$$= \frac{V \times \rho_w \times g - V \rho g}{V \times \rho}$$

$$= \frac{\rho_w - \rho}{\rho} g = \frac{1 - 0.4}{0.4} = \frac{3}{2} g$$

$$v^2 = u^2 + 2as$$

$$0 = \left( \sqrt{2 \times 9.8 \times 0.09} \right)^2 + \frac{3}{2} \times 2 \times 10 \times s$$

$$s = \frac{2 \times 10 \times 0.09 \times 2}{3 \times 2 \times 10} = 0.06 \text{ m} = 6 \text{ cm}$$

16. For both condition, same Amount of Buoyancy force ~~works~~ acts

$$\begin{aligned} \text{In Both cases } \Rightarrow mg &= F_B \\ mg &= \rho_w \times V \times g \end{aligned}$$

same Amount of volume is displaced

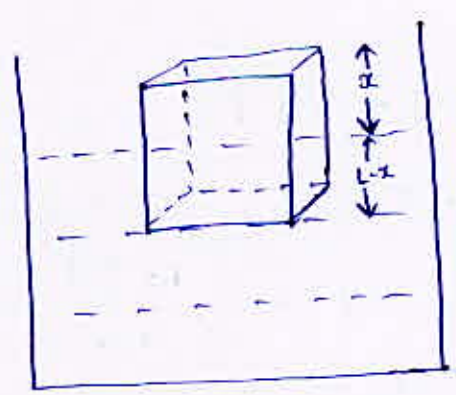
17.  $0^\circ \text{ to } 4^\circ \text{C} \rightarrow \text{density } \uparrow$

$$\begin{aligned} mg &= A \times x \times d \times g \\ x &= \frac{m}{Ad} \end{aligned}$$

$d \uparrow \quad x \downarrow$

$4^\circ \text{C} - 10^\circ \text{C} \Rightarrow \text{density } \downarrow \quad x \uparrow$

18.

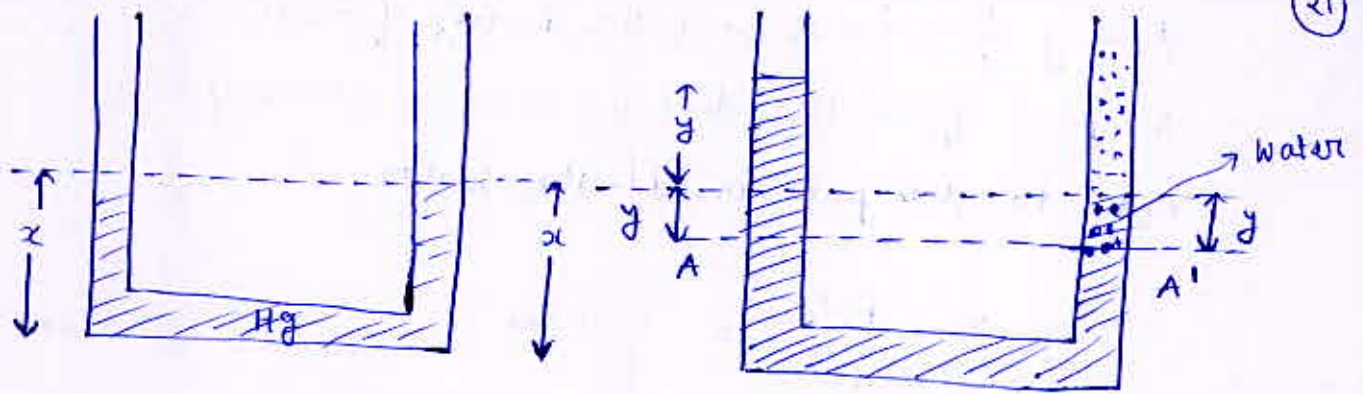


$$\begin{aligned} mg &= (L-x) \times L^2 \times \rho_w \times g \\ L^3 \times \rho &= (L-x) \times L^2 \times \rho_w \end{aligned}$$

$$\frac{L-x}{L} = \frac{\rho}{\rho_w}$$

$$1 - \frac{x}{L} = \frac{\rho}{\rho_w} \Rightarrow \frac{x}{L} = 1 - \frac{900}{1000} = \frac{1}{10} = 10\%$$

19.



(21)

At same level in Hg pressure will be equal

$$\rho_{Hg} \times g \times 2y = 11.2 \times \rho_w \times g$$

$$y = \frac{11.2 \times \rho_w}{2 \rho_{Hg}} = \frac{11.2 \times 1}{2 \times 13.6} = 0.41 \text{ cm}$$

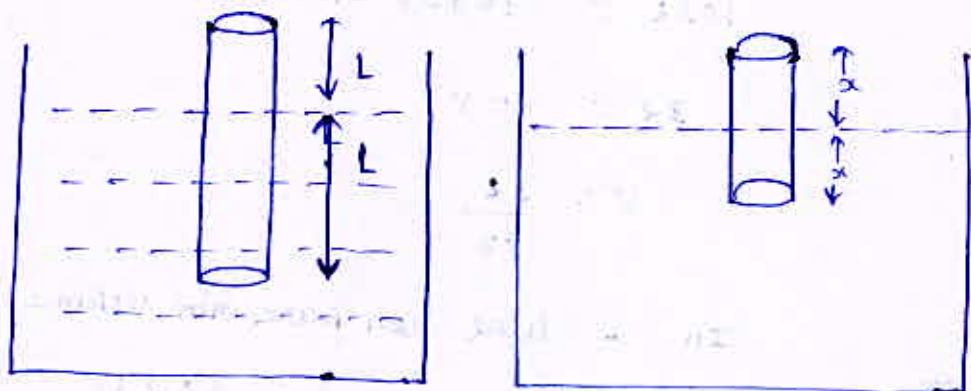
20.

$$\beta = - \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$\frac{\Delta V}{V} = - \frac{\Delta P}{\beta}$$

$$\frac{\Delta V}{V} \propto \frac{1}{\beta}$$

21.



$$\frac{d(2x)}{dt} = \frac{2 \text{ cm}}{\text{hr}}$$

From Figure (1)

$Mg = \text{Buoyancy Force}$

$$\frac{\pi d^2}{4} \times 2L \times \rho \times g = \frac{\pi d^2}{4} \times L \times \rho_w \times g$$

$$\rho_w = 2\rho$$



density of water is twice the density of candle.  
 so part of candle which is above water surface is  
 equal to the part inside the water.

$$\text{As } \frac{d(2x)}{dt} = 2 \text{ cm/hr}$$

$$\frac{dx}{dt} = 1 \text{ cm/hr}$$

upper top part will fall at the rate of  $\frac{dx}{dt} = 1 \text{ cm/hr}$   
 And lower part (lowest point) will go up at the rate  
 of  $\frac{dx}{dt} = 1 \text{ cm/hr}$

22.

$$\rho_{\text{mix}} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$

$$1032 = \frac{1080 \times V + (1-V) \times 1000}{V + (1-V)}$$

$$1032 = 1080V - 1000V + 1000$$

$$32 = 80V$$

$$V = \frac{32}{80}$$

In 1 litre pure mix volume =  $\frac{32}{80}$

In 10 litre =  $\frac{32 \times 10}{80}$

= 4 L

1.



$$F_B + F_V = mg$$

$$\text{Net force on sphere} = 0$$

$$a = 0$$

2.

Refer level 1, Q.33

3.

From equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{10 \times 1}{5} = 2 \text{ m/s}$$

$$p + \rho gh + \frac{1}{2} \rho v^2 = \text{Constant}$$

$$2000 + \rho gh + \frac{1}{2} \times 1000 \times (1)^2 = p + \frac{1}{2} \times 1000 \times 4 + \rho gh$$

$$p = 2000 + 500 - 2000 = 500 \text{ kPa}$$

4.

$$2 \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3 \quad (\text{Volume} = \text{Constant})$$

$$R = 2^{\frac{1}{3}} r$$

After Reaching terminal velocity

$$\text{Buoyancy force} = mg$$

$$6 \pi \eta r v = \frac{4}{3} \pi r^3 \times \rho \times g$$

$$v \propto r^2$$

$$\frac{v_2}{v} = \left( \frac{R}{r} \right)^2 \Rightarrow v_2 = v \times \frac{R^2}{r^2}$$

5.

For ~~From~~ From Q.4

24

$$V \propto r^2$$

$$M = \frac{4}{3} \pi r^3 \times \rho$$

$$8M = \frac{4}{3} \pi R^3 \times \rho$$

$$2r = R$$

$$\frac{V_2}{V} = \left(\frac{R}{r}\right)^2 = 4$$

$$V_2 = 4V$$

6.

$$P + \rho gh + \frac{1}{2} \rho v^2 = \text{Constant}$$

$$1 \times 13.6 \times g + \frac{1}{2} \times 1 \times 35^2 + \rho gh$$

$$= P + \frac{1}{2} \times 1 \times 65^2 + \rho gh$$

$$13.6 \times (1000 \text{ cm/s}^2) + \frac{1}{2} \times 1 \times (35-65)(35+65) = P$$

$$13600 - 1500 = P$$

$$12100 = 13.6 \times g \times h$$

$$h = \frac{12100}{13.6 \times 1000 \text{ cm/s}^2} = 0.89 \text{ cm of Hg}$$

7.

Level-1 Q. 35 same question

9.

For terminal velocity

(35)

Buoyancy Force = Viscous Force

$$\frac{4}{3} \pi r^3 \times \rho_L \times g = 6 \pi \eta r v$$

$$\eta = \frac{2}{9} \times \frac{\rho_L r^2 g}{v}$$

$$= \frac{2}{9} \times \frac{1.5 \times 1^2 \times (1000 \text{ gm/cm}^3)}{0.25}$$

$$= \frac{1}{3} \times 4000 = 1333 \text{ N/m}^2$$

$$= 1333 \times \left( \frac{\text{gm}}{\text{cm-s}^2} \right)$$

$$= 1333 \times \frac{10^{-3}}{10^{-2}} \text{ kg/m-s}^2$$

$$= 133.3 \text{ N/m}^2$$

10.

Sphere of volume =  $V'$ , velocity =  $V_1, V_2$ For terminal velocity  $\Rightarrow$ 

Buoyancy Force + viscous force = weight of the body

For gold

$$V' \times \rho_L \times g + 6 \pi \eta r V_1 = V' \times \rho_{\text{gold}} \times g$$

For silver

$$V' \times \rho_L \times g + 6 \pi \eta r V_2 = V' \times \rho_{\text{silver}} \times g$$

$$6 \pi \eta r V_1 = V' g (\rho_{\text{gold}} - \rho_L)$$

$$6 \pi \eta r V_2 = V' g (\rho_{\text{silver}} - \rho_L)$$

$$\frac{V_2}{V_1} = \frac{(\rho_{\text{silver}} - \rho_L)}{(\rho_{\text{gold}} - \rho_L)}$$

$$= \frac{10.5 - 1.5}{19.5 - 1.5} = \frac{9}{18} = \frac{1}{2}$$

$$V_2 = \frac{V_1}{2} = 0.1 \text{ m/s}$$



12.

$$\begin{aligned} \text{Pressure at the base} &= \rho g h \\ &= 900 \times 10 \times 0.4 \\ &= 3600 \text{ N/m}^2 \end{aligned}$$

$$\begin{aligned} \text{Force} &= P \times A \\ &= 3600 \times 2 \times 10^{-3} \\ &= 7.2 \text{ N} \end{aligned}$$

13. volume of water inflow = volume of water outflow

$$A_1 V_1 = A_2 V_2 + A_3 V_3$$

$$0.8 = 0.4 + 0.4 \times V_3$$

$$V_3 = 1 \text{ m/s}$$

15.

sphere falls with terminal velocity,

(27)

because Net force upward = Net force downward

$$mg + \text{Buoyancy Force} = \text{Viscous Force}$$

But in vacuum there is no viscous force, so  
its velocity will keep on increasing.

16.

$$\frac{1}{2} \rho_w v^2 = \rho_{Hg} g H$$

$$\frac{1}{2} \times 10^3 \times v^2 = 13.6 \times 10^3 \times 10 \times 40 \times 10^{-2}$$

$$v = 10.3 \text{ m/s}$$

17.

$$A_1 v_1 = A_2 v_2 = A v = \text{Constant}$$

~~$$A \propto \frac{1}{v}$$~~

$$v \propto \frac{1}{A}$$

18.

$$v = \frac{\pi p R^4}{8 \eta L}$$

$$v \propto R^4$$

$$\frac{v_2}{v} = \left( \frac{1.1 R}{R} \right)^4$$

$$v_2 = 1.46 v$$

$$\% \text{ Increase} = \frac{1.46v - v}{v} \times 100$$

$$= 46\%$$

19.

$$2 \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3$$

$$R = 2^{\frac{1}{3}} r$$

$$mg = \text{Stokes' viscous Force}$$

$$\frac{4}{3} \pi r^3 \rho g = 6 \pi \eta r v$$

$$v = \frac{2 r^2 \rho g}{9 \eta}$$

$$v \propto r^2$$

$$\frac{v_2}{v_1} = \left( \frac{r_2}{r_1} \right)^2 = \left( \frac{2^{1/3} r}{r} \right)^2$$

$$v_2 = v \times 2^{2/3}$$

22.

Refer question 19

23.

velocity head = A pressure difference

$$\frac{1}{2} \rho v^2 = 0.5 \times 10^5$$

$$v^2 = \frac{10^5}{10^3}$$

$$v = 10 \text{ m/s}$$

24.

$$mg = 6 \pi \eta r v$$

$$\rho v g = 6 \pi \eta r v$$

$$\rho \times \frac{4}{3} \pi r^3 \times g = 6 \pi \eta r v$$

$$\rho \times \frac{4}{3} \pi r^2 \times g = 6 \pi \eta v$$

$$v \propto r^2$$

$$\frac{v_2}{v_1} = \left( \frac{r_2}{r_1} \right)^2$$

$$v_2 = \frac{v}{4} = 5 \text{ m/s}$$

25.

$$A_1 V_1 = A_2 V_2$$

$$10^{-4} \times 8 \times 0.15 = 40 \times 10^{-8} \times V_2$$

$$V_2 = \frac{15 \times 8^{-2}}{4 \times 10^{-1}}$$

$$= 300 \text{ m/min}$$

$$= 300 \text{ m} / 60 \text{ sec}$$

$$= 5 \text{ m/sec.}$$

26.

Refer Level-1 Q.34

27.

$$sgH = 3 \times 10^5$$

$$H = \frac{3 \times 10^5}{10^3 \times 10} = 30$$

$$V = \sqrt{2gH}$$

$$= \sqrt{2 \times 10 \times 30}$$

$$= \sqrt{600}$$

30. 31.

$$\text{velocity} = \frac{\text{Volume Flow Rate}}{\text{Area of cross section}}$$

$$= \frac{200 \times 10^3}{0.5 \times 10^6}$$

$$= 0.4 \text{ mm/s}$$

32.

$$\text{change in pressure} = \frac{1}{2} \rho V_1^2 - \frac{1}{2} \rho V_2^2$$

$$\text{Force} = P \times A$$

$$= \frac{1}{2} \rho A (V_1^2 - V_2^2)$$



35.

using Bernoulli equation

30

$$P + \rho g H + \frac{1}{2} \rho v^2 = P_1 + \rho g H + \frac{1}{2} \rho (2v)^2$$

$$P_1 = P - \frac{3}{2} \rho v^2$$

38

$$A_A V_A = A_B V_B$$

$$\pi (2r)^2 v = \pi r^2 V_B$$

$$V_B = 4v$$

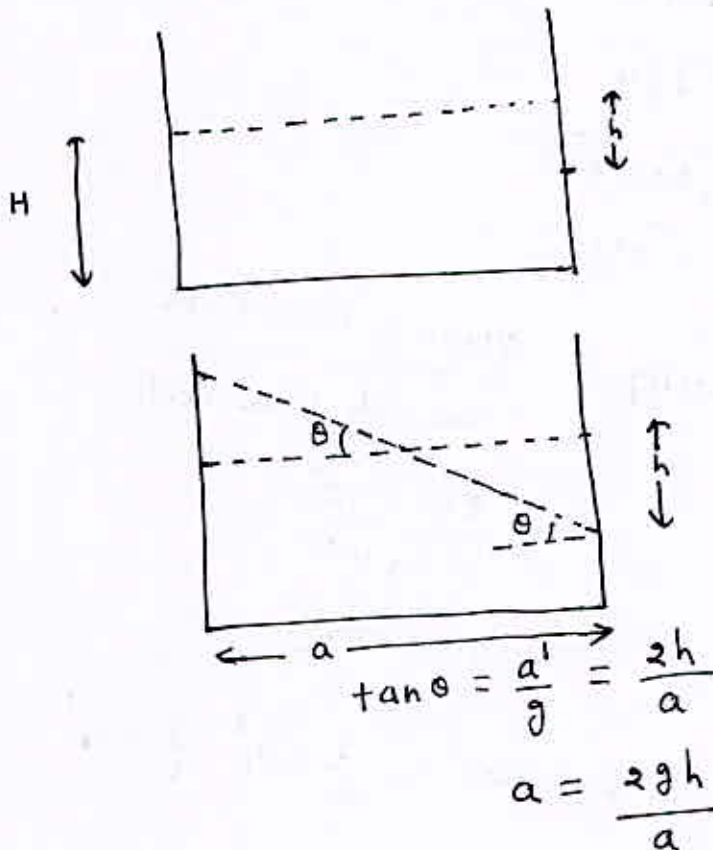
39.

same as Q.38

40.

viscous force acts on the ~~top~~ boundary

41.



43.

 $A \times V = \text{volume rate of flow}$ 

$$V = \frac{200 \times 10^3}{0.5 \times 10^6} = 0.4$$

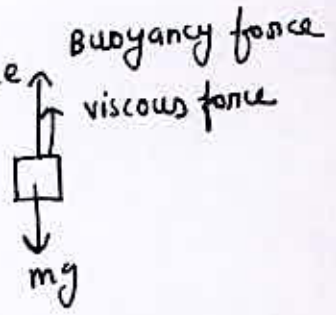
Q. 44

Refer Level 1 Q-35

Q. 46

First  $mg > \text{Buoyancy force} + \text{viscous force}$

velocity will increase



But As  $v \uparrow$ , viscous force  $= 6\pi\eta r v \uparrow$

so upward force increases.

After some time  $mg = \text{Buoyancy force} + 6\pi\eta r v$

Net Force = 0

will attain terminal velocity.

# Surface Tension And Surface Energy

33

1.  $\frac{4}{3} \pi R^3 = 64 \times \frac{4}{3} \pi r^3$

$$R = 4r$$

$$\therefore r = \frac{R}{4}$$

$$\begin{aligned} \text{change in surface Area} &= 4\pi r^2 \times 64 - 4\pi R^2 \\ &= \frac{4\pi \times R^2}{16} \times 64 - 4\pi R^2 \\ &= 12\pi R^2 \end{aligned}$$

$$\begin{aligned} \text{Energy needed} &= T \times \Delta A \\ &= 12\pi R^2 T \end{aligned}$$

3. Initial Energy =  $T \times A$

$$\text{Final Energy} = T \times \frac{A}{2}$$

$$\% \text{ change} = \frac{TA - \frac{TA}{2}}{TA} \%$$

$$= 50\%$$

4. same as q.1

5. soap bubble  $\Rightarrow$  Two surface

$$\begin{aligned} \Delta A &= 2 [4\pi (2R)^2 - 4\pi R^2] \\ &= 24\pi R^2 \end{aligned}$$

$$\begin{aligned} \text{Required surface Energy} &= T \Delta A \\ &= 24\pi R^2 T \end{aligned}$$

7.

$$2 \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3$$

$$R = 2^{1/3} r$$

$$\Rightarrow \frac{E_B}{E_S} = \frac{4\pi R^2 \times T}{4\pi r^2 \times T} = 2^{2/3}$$

8.

surface Tension is Acting on  $2l$  length

gravitational force = surface Tension force

$$mg = T \times 2l$$

$$\pi R^2 l \times \rho \times g = T \times 2l$$

$$R^2 = \frac{2T}{\pi \rho g}$$

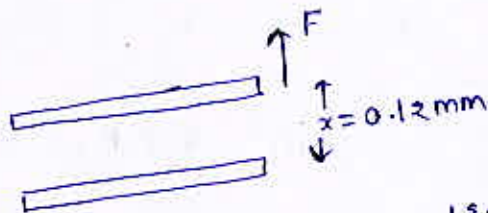
$$R = \sqrt{\frac{2T}{\pi \rho g}} = \approx 0.75 \text{ mm}$$

9.

with increase in temperature, surface Tension of water decreases

10.

$$\begin{aligned} \text{Increase in surface Energy} &= T \times 2A \\ &= 75 \times 2 \times 8 \\ &= 1200 \text{ dyne-cm} \end{aligned}$$



$$\begin{aligned} \text{So } F \times x &= 1200 \text{ dyne-cm} \\ F \times (0.12 \times 10^{-1} \text{ cm}) &= 1200 \text{ dyne-cm} \\ F &= 10^5 \text{ dyne} \end{aligned}$$



11. When drop splits up into a number of drops, surface Area Increases.

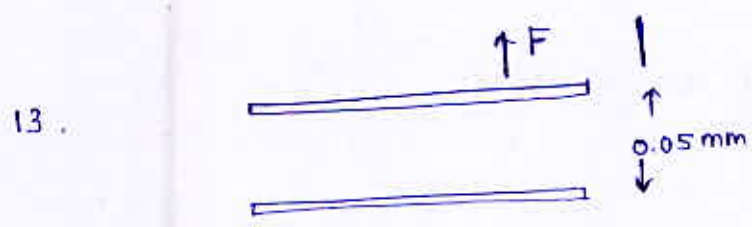
So surface Energy Increased =  $T \times \Delta A$

So Energy is being absorbed

12.  $W_1 = T \times 4\pi R^2 = T A$

$W_2 = T \times 4\pi (3R)^2$

$\frac{W_1}{W_2} = \frac{1}{9}$

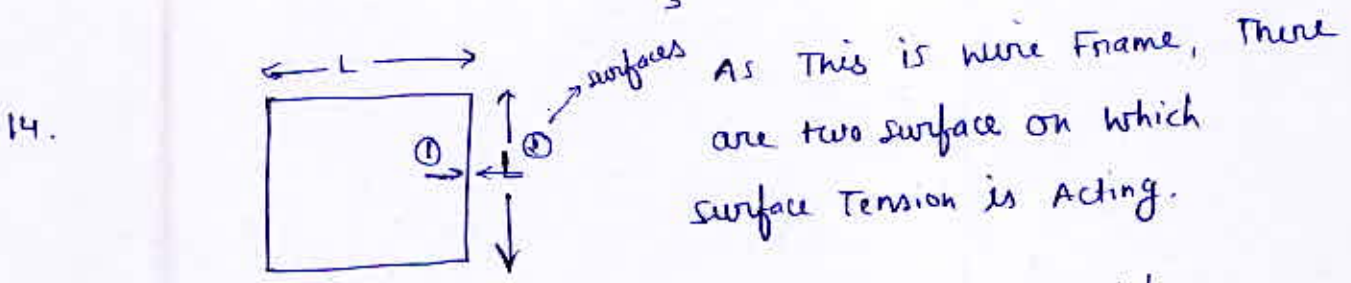


Increase in surface Energy =  $T \times 2A$   
=  $70 \times 10^{-3} \times 2 \times 10^{-2}$   
=  $14 \times 10^{-4} \text{ N-m}$

This Energy's work is done by External force F

Work done by Force F = Increase in surface Energy  
 $F \times 0.05 \times 10^{-3} = 14 \times 10^{-4} \text{ N-m}$

$F = \frac{140}{5} = 28 \text{ N}$



As This is wire Frame, There are two surface on which surface Tension is Acting.

Force Acting =  $2T \times 4L$   
=  $8TL$

15.

Volume will remain constant

$$\left( \begin{array}{l} m_1 = m_2 \\ V_1 \rho = V_2 \rho \\ V_1 = V_2 \end{array} \right)$$

36

$$\frac{4}{3} \pi R^3 = 1000 \times \frac{4}{3} \pi r^3$$

$$\frac{4}{3} \frac{R}{10} = r$$

$$r = \frac{0.1 \times 10^{-1}}{10} = 10^{-3} \text{ m}$$

$$\begin{aligned} \text{Increase in surface Area} &= 1000 \times 4\pi r^2 - 4\pi R^2 \\ &= (1000 \times 10^{-6} - 10^{-4}) 4\pi \\ &= 4\pi [0.001 - 0.0001] \end{aligned}$$

$$\begin{aligned} \text{Increase in surface Energy} &= T \times \Delta A \\ &= 7 \times 10^{-2} \times 4\pi [0.001 - 0.0001] \\ &= 7.9 \times 10^{-6} \text{ J} \end{aligned}$$

16.

$$P_{in} - P_{out} = \frac{4T}{R}$$

$$\text{Electrostatic pressure} = \frac{\sigma^2}{2\epsilon_0}$$

17.

$$V = \frac{4}{3} \pi R_1^3$$

$$2V = \frac{4}{3} \pi R_2^3$$

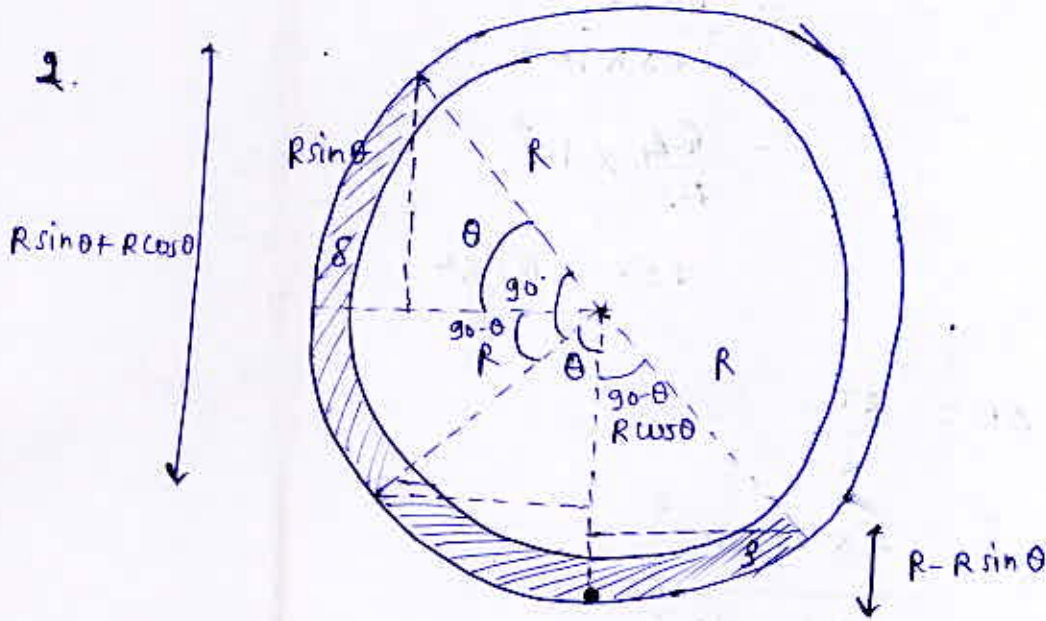
$$\left( \frac{R_2}{R_1} \right)^3 = 2$$

$$\left( \frac{R_2}{R_1} \right) = 2^{1/3}$$

$$\frac{W_2}{W_1} = \frac{4\pi R_2^2 \times T}{4\pi R_1^2 \times T} = \left( \frac{R_2}{R_1} \right)^2 = 2^{2/3}$$

$$W_2 = 2^{2/3} W = 4^{1/3} W$$

# pressure difference



$$\delta g (R \sin \theta + R \cos \theta) + \beta g (R - R \cos \theta) = \rho g (R - R \sin \theta)$$

$$\sin \theta (\delta + \beta) = \cos \theta (\beta - \delta)$$

$$\tan \theta = \frac{\beta - \delta}{\delta + \beta}$$

3.

$$\text{Excess pressure} = \frac{4T}{R}$$

$$\Delta P \propto \frac{1}{R}$$

$$\frac{\Delta P_1}{\Delta P_2} = \frac{R_2}{R_1} = \frac{1}{2}$$

4.

$$\frac{4}{3} \pi R^3 = 8 \times \frac{4}{3} \pi r^3$$

$$R = 2r$$

$$r = \frac{R}{2}$$

$$\text{Excess pressure} = \frac{8T}{R}$$

$$\Delta P \propto \frac{1}{R}$$

Big drop  
Radius double than small  
Excess pressure will  
be half.

5.

$$\begin{aligned}
 \text{Excess pressure} &= \frac{4T}{R} \\
 &= \frac{4 \times 1.6}{2.5 \times 10^{-3}} \\
 &= \frac{6.4}{2.5} \times 10^3 \\
 &= 2560 \text{ N/m}^2
 \end{aligned}$$

6.

$$\begin{aligned}
 \Delta P &= \frac{2T}{R} \\
 &= \frac{2 \times 70 \times 10^{-3}}{0.5 \times 10^{-3}} \\
 &= 280 \text{ N/m}^2
 \end{aligned}$$

8.

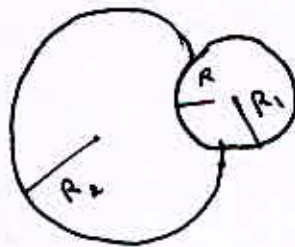
$$\Delta P \propto \frac{1}{R}$$

$$\frac{P_1}{P_2} = \frac{R_2}{R_1}$$

$$\frac{3P_2}{P_2} = \frac{R_2}{R_1} = 3$$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3} \pi R_1^3}{\frac{4}{3} \pi R_2^3} = \left( \frac{R_1}{R_2} \right)^3 = \frac{1}{27}$$

9.



$$\frac{4T}{R_1} - \frac{4T}{R_2} = \frac{4T}{R}$$

$$\frac{1}{0.03} - \frac{1}{0.04} = \frac{1}{R}$$

$$R = 0.12 \text{ m}$$



10. Excess pressure  $\propto \frac{1}{R}$   $\left( \frac{4T}{R} \right)$

11. Excess pressure =  $P_{in} - P_{out} = \frac{2T}{R}$

$$P_{in} = P_{out} + \frac{2T}{R}$$

$$= 1.013 \times 10^5 + \frac{2 \times 7.2 \times 10^{-2}}{0.1 \times 10^{-3}}$$

$$= 1.013 \times 10^5 + 0.014 \times 10^5$$

$$= 1.027 \times 10^5 \text{ N/m}^2$$

12.  $\frac{4T}{R_1} - \frac{4T}{R_2} = \frac{4T}{R}$

$$\frac{1}{R} = 0$$

$$R = \infty$$



Radius of curvature =  $\infty$   
Flat surface

13.  $\Delta P = \frac{4T}{R}$

15. Excess pressure  $\Delta P = \frac{4T}{R}$

$$\Delta P \propto \frac{1}{R}$$

pressure inside the soap bubble of small radius will be more

so air will flow from low radius bubble to higher radius bubble.

16.

$$\frac{4T}{R} = \rho g H$$

40

$$T = \frac{\rho g H R}{4}$$

$$= \frac{\left( \frac{0.8 \times 10^{-3}}{10^{-6}} \right) \times 10 \times (2 \times 10^{-3}) \times (10^{-2})}{4}$$

$$= 4 \times 10^{-2} \text{ N/m} = 3.9 \times 10^{-2} \text{ N/m}$$

$$\text{take } g = 9.8 \text{ m/s}^2$$

17.

$$\text{Excess pressure} \propto \frac{1}{R} \quad \left( \frac{4T}{R} \right)$$

$$\frac{P_1}{P_2} = \frac{R_2}{R_1}$$

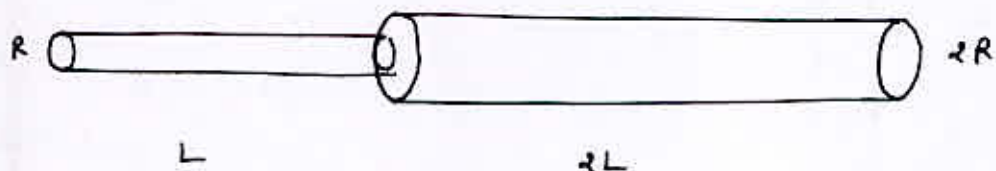
$$\frac{P_1}{P_2} = \frac{4}{1}$$

18.

$$\Delta P = \frac{4T}{R}$$

$$\Delta P \propto \frac{1}{R}$$

J.



volume of liquid flowing through both the tube is same.

For 1<sup>st</sup> tube

$$V = \frac{\pi P_1 R^4}{8 \eta L}$$

$$P_1 = V \left( \frac{8 \eta L}{\pi R^4} \right)$$

For 2<sup>nd</sup> tube

$$V = \frac{\pi P_2 (2R)^4}{8 \eta \times 2L}$$

$$P_2 = \frac{V \times 8}{\pi} \left( \frac{8 \eta L}{\pi R^4} \right)$$

$$P = P_1 + P_2$$

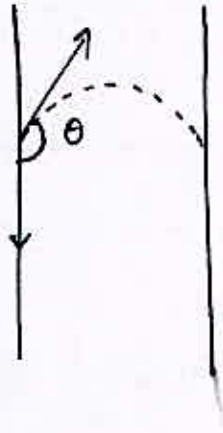
$$P = \frac{8 \eta L}{\pi R^4} \left( V + \frac{V}{8} \right)$$

$$P = \frac{8 \eta L}{\pi R^4} \times \frac{9V}{8}$$

$$V = \frac{8}{9} \times \frac{\pi P R^4}{8 \eta L}$$

$$V = \frac{8}{9} \times$$

2.



$$\theta > 90^\circ$$

$$F_a < \frac{F_c}{\sqrt{2}}$$

convex meniscus

Liquid does not wet the solid surface

3.

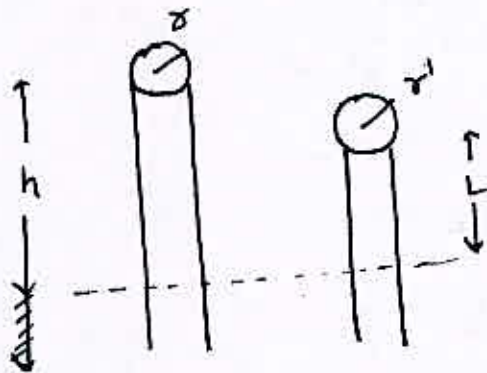
$$h = \frac{2T}{Rdg}$$

$$h \propto \frac{1}{g}$$

$$\frac{h_e}{h_m} = \frac{g_m}{g_e}$$

$$h_m = h_e \times \frac{g_e}{g_m} = \frac{h_e}{6} = 6 h_e$$

4.



$$hr = Lr'$$

$$\text{If } L < h \\ r' > r$$

5.

a

7



8.  $hdg = \frac{2T \cos \theta}{r}$

$h \propto \frac{1}{r}$

Radius half, height double

9.



Acute

10.

$h \propto \frac{1}{r}$

$\frac{h_2}{h_1} = \frac{r_1}{r_2}$   
 $h_2 = 3 \times \left( \frac{r_1}{r_1/3} \right) = 9 \text{ mm}$

$\left( R = \frac{r}{\cos \theta} \right)$

12.

$hdg = \frac{2T \cos \theta}{r}$

$T = \frac{hdgr}{2 \cos \theta}$

$T \propto \frac{hd}{\cos \theta}$

$\frac{T_w}{T_{Hg}} = \frac{h_w}{h_{Hg}} \times \frac{d_w}{d_{Hg}} \times \frac{\cos \theta_w}{\cos \theta_{Hg}}$   
 $= \frac{10}{-3.5} \times \frac{1}{13.6} \times \frac{-0.7}{1}$

$\frac{T_w}{T_{Hg}} = 0.15$