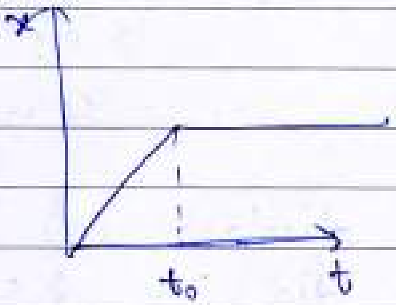
—: Kinematics :—

Level - I

Q. (1)



velocity $= \frac{dx}{dt} =$ Slope of x - t graph

upto time t_0

$\frac{dx}{dt} \neq 0$ and $t > t_0$

$\frac{dx}{dt} = 0$

(4) The Particle moves at a const velocity up to a time t_0 and then stops.

Q.(2) (1) magnitude of velocity of a particle is equal to its speed. → True

$|\vec{v}| =$ speed

(2)

average velocity $= \frac{dx}{dt} = \frac{x_2 - x_1}{t_2 - t_1}$

where x_2 - final displacement

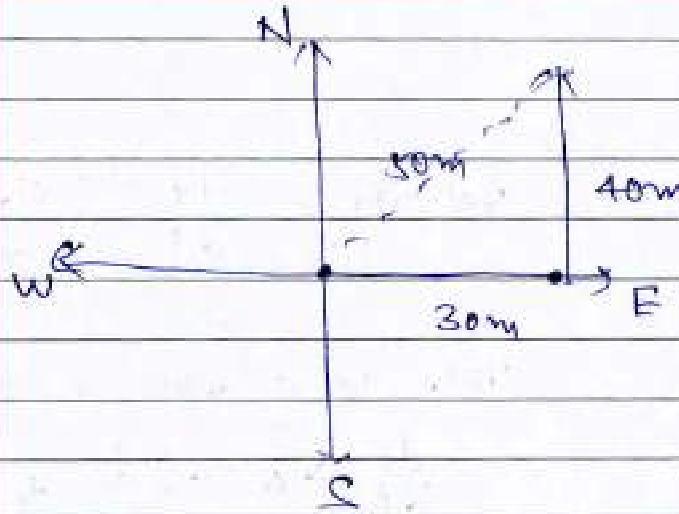
average velocity $= \frac{\text{Total displacement}}{\text{Total time}}$

average speed $= \frac{\text{Total distance}}{\text{Total time}}$

(3) Total distance $\neq 0$ in an Interval

(4) It is not possible to have a situation in which the instantaneous speed of particle is always zero but the avg speed is not zero.

Q. (3)



displacement in east dirn in 2 sec

$= 2 \times 15$

$= 30\text{ m}$

displacement in North dirn in 8 sec

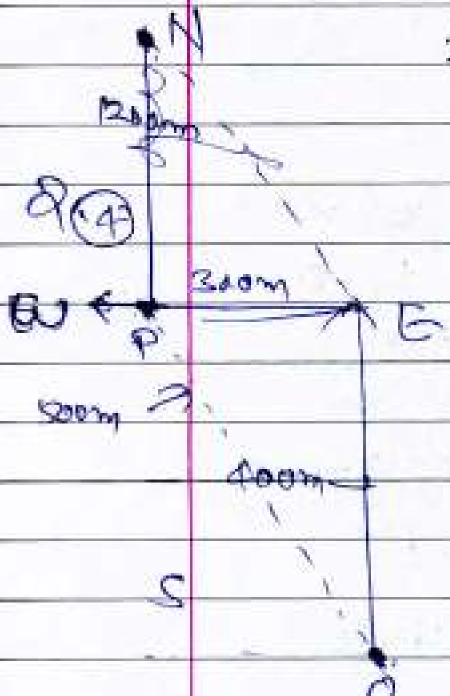
$= 8 \times 5 = 40\text{m}$

$$\text{avg velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$= \frac{\text{final Position} - \text{initial Position}}{\text{Total time}}$$

$$= \frac{50}{10} = 5\text{ m/sec}$$

Q(4)



So displacement

$$= \sqrt{(1200)^2 + (500)^2}$$

$$= \underline{1300\text{ m}}$$

Let take 'O' as a origin

$$\vec{OP} = -300\hat{i} + 400\hat{j}$$

$$|\vec{OP}| = 500\text{ m}$$

final Position in z-direction

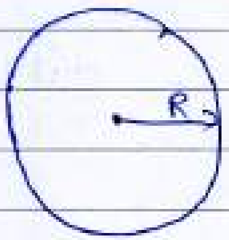
$= 1200\text{m}$

$= (-300\hat{i} + 400\hat{j} + 1200\hat{k})$

$\therefore$ finally $= \sqrt{(300)^2 + (400)^2 + (1200)^2}$

$= 1300\text{m}$

Q-5



Athelete completes one round in 40 sec.

given time = 2 min 20 sec
= 140 sec.

time taken for 3 round = $40 \times 3 = 120$ sec

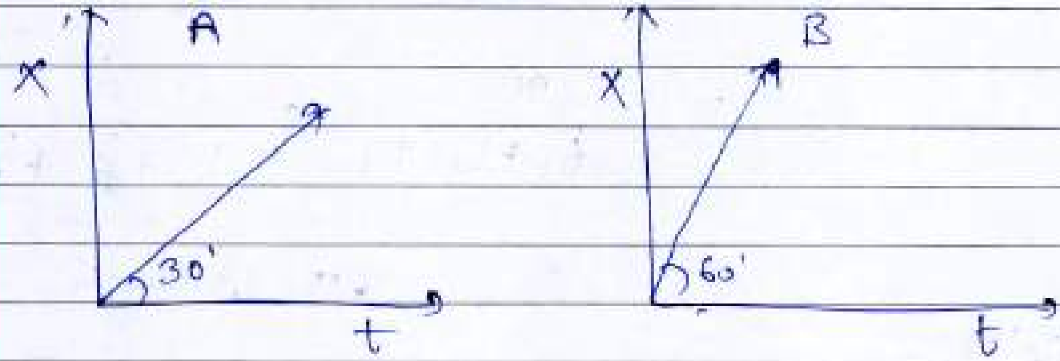
= $(140 - 120) = 20$ sec.

he will half circle in next 20 sec

so finally after 2 min 20 sec he will be at diagonally just opposite of initial position.

so displacement = $R + R = 2R$

Q.6



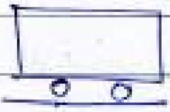
velocity = slope of x – t graph

$v_A = \frac{dx_A}{dt} = \tan 30° = 1/\sqrt{3}$

$v_B = \frac{dx_B}{dt} = \tan 60° = \sqrt{3}$

$\therefore v_A / v_B = 1/3$

Q. (7)



suppose Total distance $= x$

~~[illegible]~~ $x/3$ at 10 km/h

$x/4$ at 20 km/h

remaining $= \left(x - x/3 - x/4\right) = \frac{5x}{12}$ at 40 km/h

total time $= t_1 + t_2 + t_3$

$$= \frac{x/3}{10} + \frac{x/4}{20} + \frac{5x/12}{40}$$

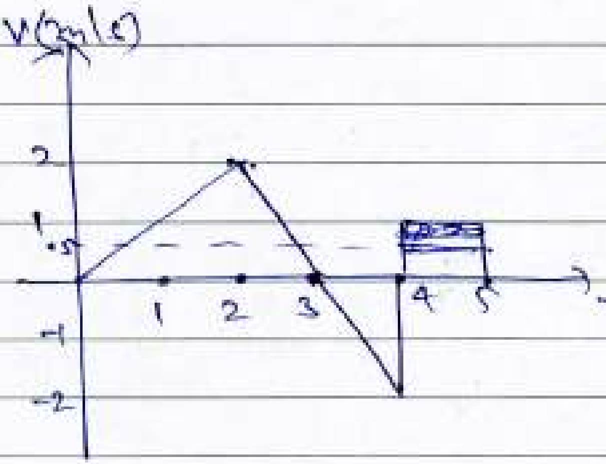
$$= \left(\frac{x}{30} + \frac{x}{80} + \frac{x}{96}\right) \text{hour}$$

average speed $= \frac{\text{total distance}}{\text{total time}}$

$$= \frac{x}{t_1+t_2+t_3} = \frac{1}{1/30 + 1/80 + 1/96} \approx 17.7 \text{ km/h}$$

$= 18$ km/h

Q. (8)



displacement $=$ Area under the v–t graph

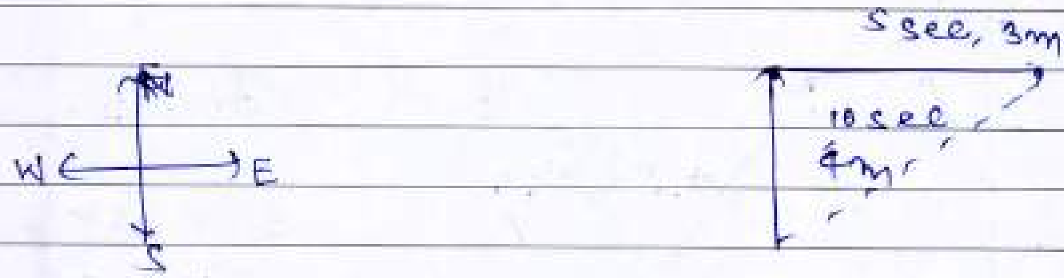
$\because x = \int v\,dt$

$\therefore$ displacement $= \frac{1}{2}\times 3\times 2 - \frac{1}{2}\times 2\times 1 + 1\times \frac{1}{2}$

$= 2.5$ m

Q ⑨ if the location of a particle has changed then displacement can not be zero and obviously distance will not zero.

Q ⑩



displacement = $\sqrt{3^2+4^2} = 5m$

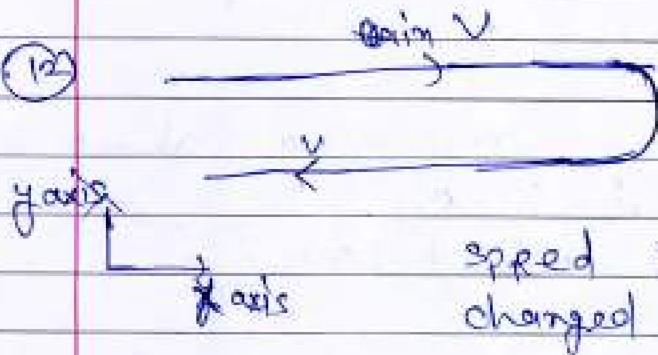
total time = 10 + 5 = 15 sec.

∴ average velocity = $5/15$ m/sec.

Q ⑪ ∵ after releasing stone will come downwards under the effect of gravity so acceleration of the stone = g

= $9.8 m/s^2$ ↓

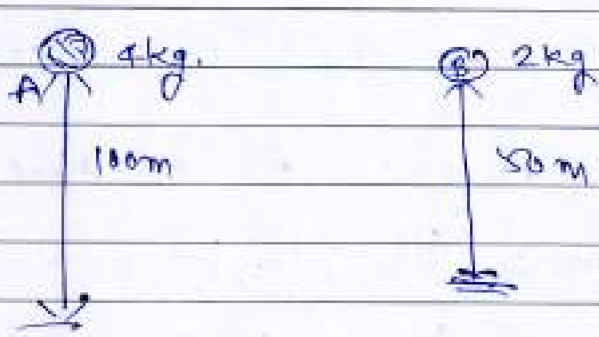
Q ⑫



initial velocity in +x-axis
final — " — in −x-axis

speed is not changed but direction changed so velocity is changed.

Q ⑬



$50 = \frac{1}{2} \times g \times t_B^2$

$t_B = \sqrt{10}$ sec.

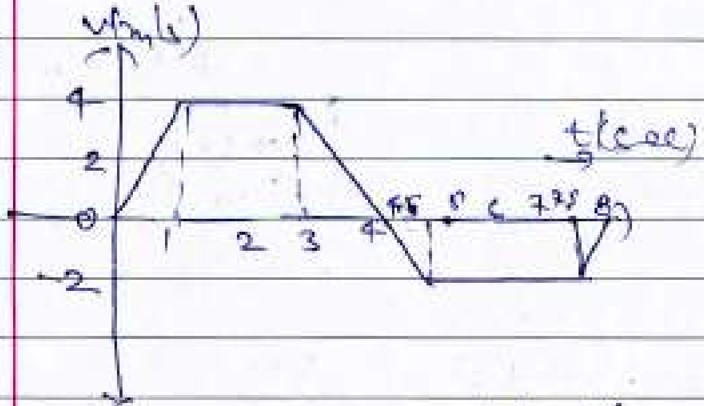
$100 = (t_A)^2 \times \frac{1}{2} \times g$

$t_A = \sqrt{20}$ sec.

$\boxed{t_A > t_B}$ $\boxed{t_A = \sqrt{2}\, t_B}$

Q.(14) Same as Q (1)

Q.(15)



displacement

= Area of v-t graph

$= \frac{1}{2}(4)(4+2) - \frac{1}{2}\times 2\times(3+4)$

$= 12 - 7 = \underline{5m}$

Q.(16) horizontal Range $= \frac{u^2 \sin 2\theta}{g}$

$R_A = \frac{u_A^2 \sin 30°}{g}$; $R_B = \frac{u_B^2 \sin 90°}{g}$

Here we dont know the values of u_A & u_B

So can not say anything about R_A & R_B.

→ The information is insufficient.

Q.(17) velocity of A = v_A (upwards)

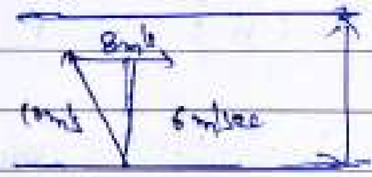
Velocity of A relative to B

$v_{AB} = (v_A - v_B)$

$= v_A = u$

Relative acceleration = 0 (zero)

Q.(18)



velocity of swimmer with r.t. ground = 10m/s

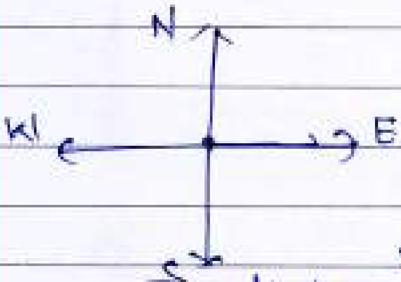
$$V_{sr} = V_s - V_r$$

$$|V_{sr}| = \sqrt{(10)^2 - (8)^2}$$
$$= 6 \text{ m/sec.}$$

width of the river = 480 m

$$\therefore \text{time} = \frac{480 \text{ m}}{6 \text{ m/sec}} = 80 \text{ sec.}$$

Q.(19)



→ velocity towards east
acceleration towards west and const

x_A & x_B be the magnitude of displacement in the first 10 sec and the next 10 seconds.

We can not calculate the displacement with knowing velocity u and acceleration.
So the information is insufficient to decide the relation of x_A and x_B.

Q.(20) (i) 

∵ angle b/w a and v < π/2

∴ v increase

(ii) 

here
∵ angle b/w a and v > π/2
so v will decrease and turning to left.

Q. (21) Initial velocity $= (a\hat{i} + b\hat{j})$ m/s.
given

Range is twice the max^m height

Range $= R = \dfrac{2u_x u_y}{g}$ & $H_{max} = \dfrac{u_y^2}{2g}$

$R = 2H_{max}$

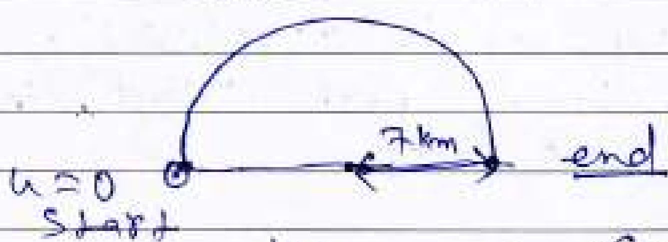
Here $u_x = a$ & $u_y = b$

$\therefore R = \dfrac{2ab}{g}$ & $H_{max} = \dfrac{b^2}{2g}$

$$\left(\frac{2ab}{g}\right) = \cancel{2}\left(\frac{b^2}{\cancel{2}g}\right)$$

$\boxed{b = 2a}$ An

Q.(22) Radius = 7 km



displacement = (final Position − Initial Position)

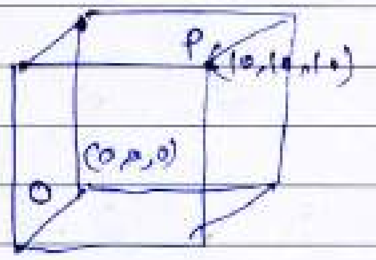
$= 2R = 14$ km

distance = total travelled distance

$= \pi R = 3.14 \times 7$

$= 22$ km

Q. (23) 10m × 10m × 10m



Let take 'O' as origin

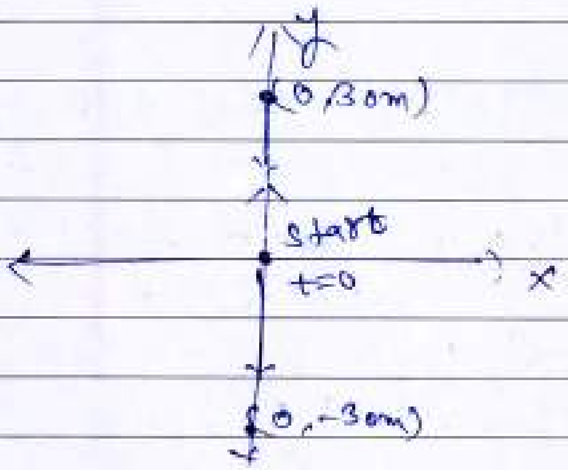
then

$\vec{OP} = 10\hat{i} + 10\hat{j} + 10\hat{k}$

displacement $= |\vec{OP}| = \sqrt{10^2+10^2+10^2}$

$= \underline{10\sqrt{3}\,m}$

Q (24)



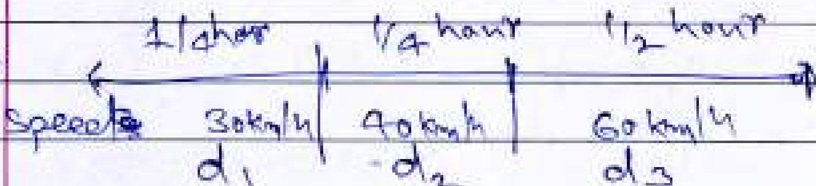
distance = total travelled distance

$= 30+30+30 = 90m$

displacement

$= \underline{-30m}$

Q (25)



total distance = (speed) × (time)

$= d_1 + d_2 + d_3$

$= 30/4 + 40/4 + 60/2$

$= 7.5 + 10 + 30 = 47.5\,km$

total time = 1 hour

$\therefore$ average speed $= \dfrac{\text{total distance}}{\text{total time}}$

$= \dfrac{47.5\,km}{1\,hour} = 47.5\,km/h$

Q. 26 Particle - 1 velocity $\vec{v}_1$
Particle - 2 velocity $\vec{v}_2$

Relative velocity $= \vec{v}_{12} = \vec{v}_1 - \vec{v}_2$

$$|\vec{v}_{12}| = \sqrt{|\vec{v}_1|^2 + |\vec{v}_2|^2 - 2\vec{v}_1 \cdot \vec{v}_2}$$

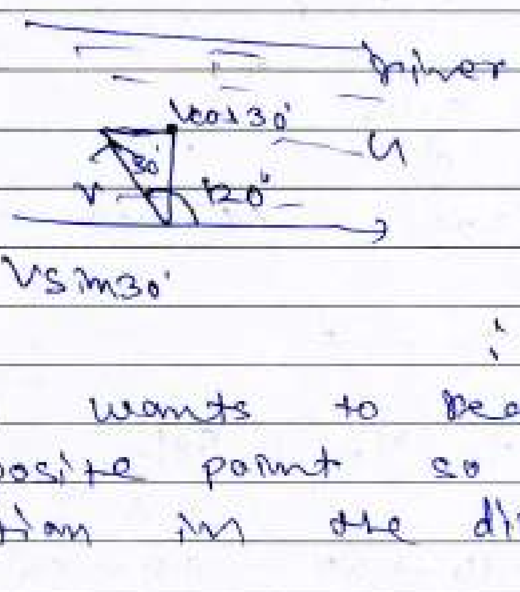
for max. $|\vec{v}_{12}|$

$$\boxed{\vec{v}_1 \cdot \vec{v}_2 < 0} \quad \therefore \cos\theta < 0$$

~~max~~
min value of $\cos\theta = -1$
at $\theta = \pi$

$\therefore |\vec{v}_{12}| = |\vec{v}_1 + \vec{v}_2|$
at $\boxed{\theta = \pi}$

Q. 27



v - velocity of swimmer in water

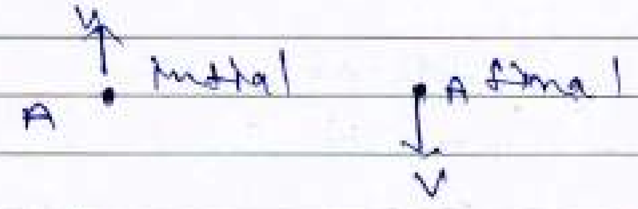
v ~~$\sin 30 = u$~~

$v\sin30' - u = 0$

$\therefore u = v\sin30' = \frac{v}{2} = 1\,km/h$

man wants to reach exactly opposite point so there will be no relative motion in the direction of flow of river

(28)



average speed $= \frac{x_f - x_i}{t}$

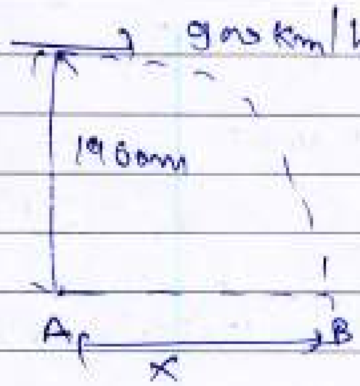
$=$ ~~$\frac{\ }{2}$~~

and avg velocity $= \frac{x_f - x_i}{t} = 0$

avg speed = $\dfrac{\text{total distance}}{\text{total time}}$

$= \dfrac{2\left(\frac{u^2}{2g}\right)}{\frac{2u}{g}} = \dfrac{u}{2} = \dfrac{2 \times \text{Max Height}}{\text{time of flight}}$

Q.29



900 km/h $= \dfrac{\overset{50}{\cancel{900}} \times 5}{\cancel{18}} = 250$ m/sec

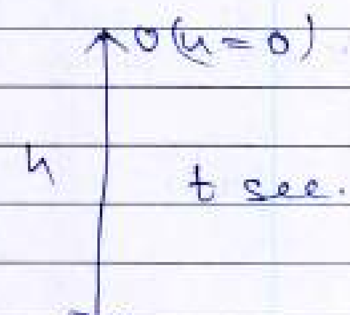
Initial velocity of body = velocity of aeroplane at that instant = 250m/sec in horizontal direction

~~time~~ $1960\,m = \frac{1}{2} \times 9.8 \times T^2$

$T = \sqrt{\dfrac{2 \times 19600}{9.8}} = 20$ sec.

$AB = x = (250\,m/sec) \times 20\,sec = 5000\,m$
$= 5\,km$

30



$h = \frac{1}{2} g t^2$

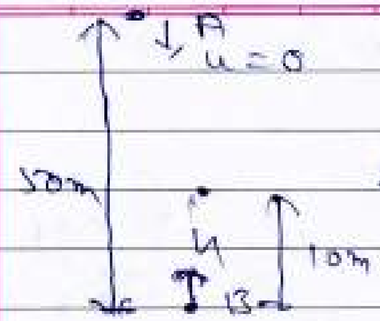
$t = \sqrt{\dfrac{2h}{g}}$ sec.

at $t' = t/2$

$y = \frac{1}{2} g t'^2 = \frac{1}{2} g \left(\frac{t^2}{4}\right) = \frac{h}{4}$ from Top.

$\therefore h - \frac{h}{4} = \frac{3h}{4}$ from the ground.

(31)



They meet at 10m height.

so body B travel 10 m upwards and body A travel 40 m downwards

$$t_A = \sqrt{\frac{2\times 40}{10}} = \sqrt{8} = \underline{2\sqrt{2}\,\text{sec.}}$$

$$10 = u_B(2\sqrt{2}) - \frac{1}{2}\times 10\times(2\sqrt{2})^2$$

$$\Rightarrow u_B(2\sqrt{2}) = 10 + 10\times 4 = 50$$

$$u_B = \frac{50}{2\sqrt{2}} = \underline{25/\sqrt{2}\ \text{m/s}}$$

$$= \underline{12.5\sqrt{2}\ \text{m/s}} \approx 17.7\ \text{m/s}$$

(32)

$$H_{max} = \frac{u_y^2}{2g}$$

time to reach $H_{max} = \frac{u_y}{g} = T$

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$y = gTt - \frac{1}{2} g t^2$$

$$y = gt\left(T - \tfrac{1}{2}t\right) \quad -(1)$$

$$H = u_y T - \frac{1}{2} g T^2 \quad -(2)$$

eq (1) − (2)

$$\cancel{y - H = (\ldots)T - \tfrac{1}{2}g(t^2 - T^2)}$$

$$y = H - \frac{1}{2}g(t^2 - T^2) + (gtT - u_y T)$$

$$= H - \frac{1}{2}g(t - T)^2 \quad \underline{\text{Ans.}}$$

(33) Const acceleration $= g/8$

$\uparrow g/8 \, m/s^2$

velocity of balloon ~~when reached~~ at height h

$v^2 = 2 \times g/8 \times h$

$= gh/4$

Initial velocity of stone $= v = \sqrt{gh/4}$ in upwards direction

$-h = vt - \frac{1}{2}gt^2 \rightarrow \frac{1}{2}gt^2 - vt - h = 0$

$t = \dfrac{+v \pm \sqrt{v^2 - 4 \frac{g}{2} \times h}}{g}$

$t = \dfrac{\frac{3}{2}\sqrt{gh} + \frac{\sqrt{gh}}{2}}{g} = 2\sqrt{h/g}$

(34) body starts from rest with uniform acceleration,

after n sec.

velocity $= v = \overset{0}{u} + at$

$\boxed{v = na}$ m/sec $\Rightarrow (a = v/n)$

displacement in n sec.

$s = \frac{1}{2}an^2$ —(1)

displacement in (n-2) sec

$s' = \frac{1}{2}a(n-2)^2$ —(2)

so displacement in last 2 sec.

$= (s - s') = \frac{1}{2}an^2 - \frac{1}{2}a(n-2)^2$

$= \cancel{\frac{1}{2}an^2} - \cancel{\frac{1}{2}an^2} - \frac{1}{2}a(2)^2 + \frac{1}{2}a(n)(2)(2)$

$= (2an - 2a)$

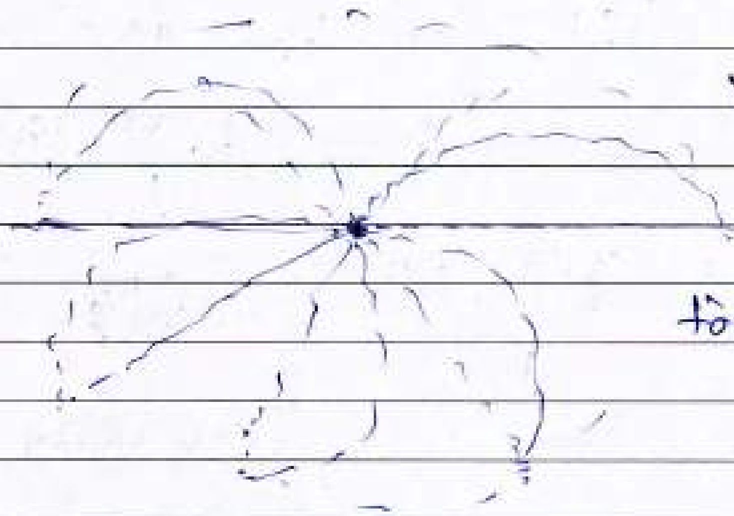
$= 2an - \frac{2v}{n} \Rightarrow 2(v - \frac{v}{n})$

$= 2a(n-1)$

$\Rightarrow$ put $a = u/n$

$\therefore \quad s = \frac{2u}{n}(n-1)$

(35)



velocity u

angle $= \theta$

for max^m area

$\theta = 45°$

Range $= \left(\frac{u^2}{g}\right)$

all bullets make a circle with the radius of Range $\left(\frac{u^2}{g}\right)$.

so Max^m area covered by the bullets

$= \pi(\text{Range})^2 = \pi\left(\frac{u^2}{g}\right) = \frac{\pi u^4}{g^2}$

(36)

Max Height $=$ Horizontal range

$\frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 (2)(\sin\theta)\cos\theta}{g}$

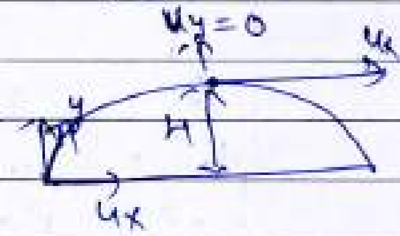
$\Rightarrow \frac{\sin\theta}{\cos\theta} = \tan\theta = 4$

$\therefore \theta = \tan^{-1}(4)$

(37) horizontal Range = 200m
time of flight = 5 sec. | $g = 10\,m/sec^2$

$$T = 5 = \frac{2u_y}{g}$$

$$\Rightarrow u_y = \frac{5 \times 10}{2} = 25\,m/s$$



$$\text{Range} = 200 = \frac{2u_x u_y}{g}$$

$$\Rightarrow 200 = \frac{2 \times (u_x)(25)}{10}$$

$$\Rightarrow u_x = \frac{200 \times 10}{2 \times 25} = 40\,m/s$$

then the horizontal component of velocity of the particle at the highest point is $u_x = 40\,m/s$

(38) Same as Q. (21)

(39) kinetic energy $k = \frac{1}{2}mv^2$

for max possible horizontal range ($u_x = u_y$) because $\theta = 45°$

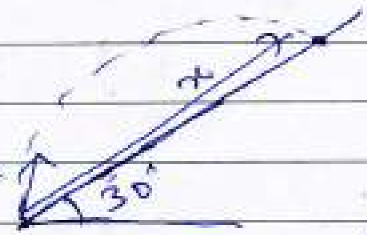
$$k = \frac{1}{2}m(u_x^2 + u_y^2) = \frac{1}{2}m(2u_x^2)$$

kinetic energy at highest point $k' = \frac{1}{2}mu_x^2$

$$k' = \frac{1}{2}mu_x^2 = k/2$$

(40) max^m Range $= 500m = \frac{u^2}{g}$

$$u^2 = 500g$$



given :-

A projectile has the max^m range of 500m. (it's not only horizontal max^m Range)

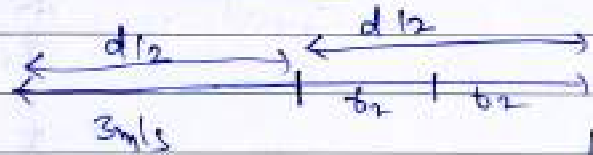
So if the projectile is thrown up an inclined plane with the same speed, it has same max^m Range that is the distance covered by it along the inclined plane.

= 500 m

→

Level - II

(1)



$t_1 = \frac{d/2}{3} = d/6$ sec.

$d/2 = t_2(4.5) + t_2(7.5)$

$d/2 = t_2(12)$

$t_2 = \frac{d}{24}$ sec.

average speed

$= \frac{\text{Total distance}}{\text{total time}}$

$= \frac{d}{t_1 + t_2 + t_2} = \frac{d}{t_1 + 2t_2}$

$= \frac{d}{d/6 + 2d/24} = \frac{1}{1/6 + 1/12} = \frac{12}{3} = 4$ m/sec

(2)

time interval (s)	I	II	III	IV
distance (cm)	2	6	10	14

Initial speed = 0

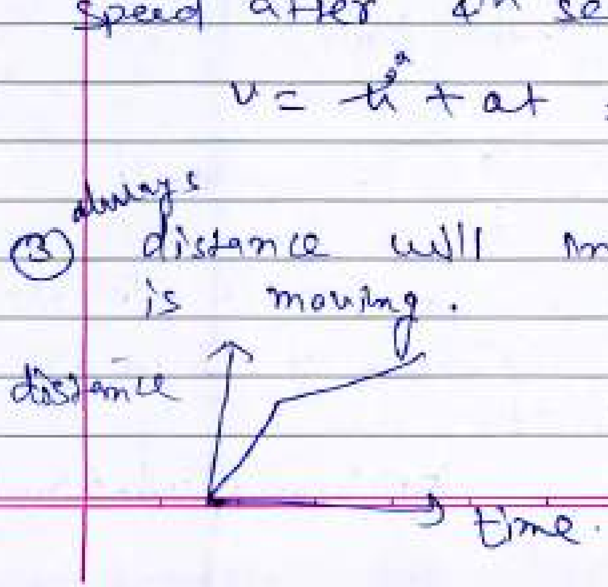
$.02 = \frac{1}{2} \times a \times (1)^2 \Rightarrow a = \cancel{4\,cm/sec^2}\ 0.04\ m/sec^2$

speed after 4th sec.

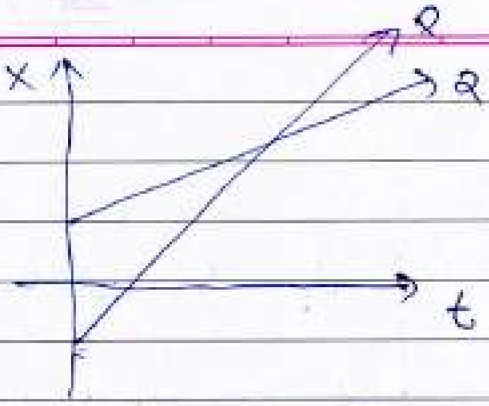
$v = \cancel{u} + at = 0.04 \times 4 = 0.16$ m/se

$= 16$ cm/sec

(3) always distance will increase with time if body is moving.



(4)



speed $= \frac{dx}{dt}$ = slope of x-t graph

slope for particle P is greater than particle Q

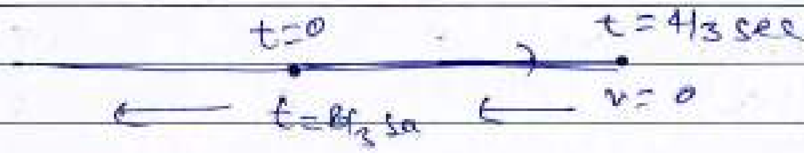
So speed of P > speed of Q.
and both have uniform speed.

(5) x-axis $x = 8t - 3t^2 = t(8 - 3t)$

$v = \frac{dx}{dt} = 8 - 6t$

So velocity v will be zero $t = 4/3$ sec

displacement x is zero at $t = 0$ & $t = 8/3$ sec

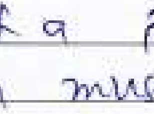


final position at $t = 4$ sec $= -16$ m
position at $t = 8/3 = 0$

avg velocity = $\frac{\text{Total displacement}}{\text{Total time}}$

$= \frac{-16}{4} = -4$ m/s

(6) if speed of a particle is const, doesn't mean its acceleration must be zero. if both are perpendicular then speed is const.



(uniform circular motion)

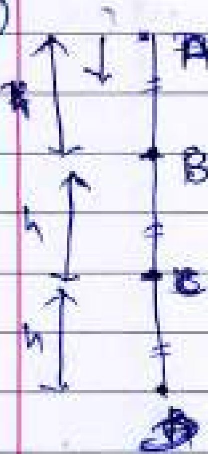
(7) $u = m/s \quad ; \quad a = m/s^2$

$t = 1\,sec$

$v = u + at \qquad = m/s + (m/s^2)\,sec$

$= m/s$

(8)



$AB = BC = CD$

time of descent through $AB = \sqrt{\frac{2h}{g}}$

— " — $BC = \sqrt{\frac{2(2h)}{g}} - \sqrt{\frac{2h}{g}}$

— " — $CD = \sqrt{\frac{2(3h)}{g}} - \sqrt{\frac{2(2h)}{g}}$

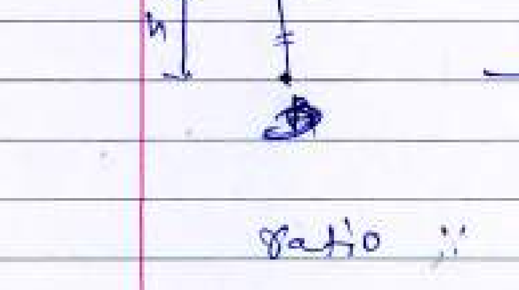
ratio :: $1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$

(9) speed of man w.r.t. water = 5 km/h

speed in direction of flow of river $= (3 + 5\sin 30°)$

Perpendicular to river $= 5\cos 30°$

$= \sqrt{3}(2.5)$ km/h



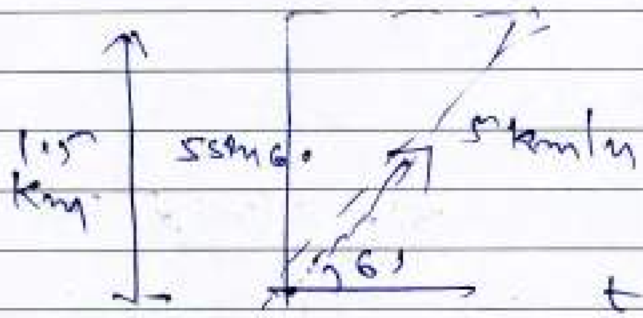
$\frac{x}{1.5} = \tan 30° = 1/\sqrt{3}$

$x = 1.5/\sqrt{3}$

time $= \frac{x}{(2.5)\sqrt{3}} = \frac{1.5}{(2.5)\sqrt{3}} = 0.346\,h$

time $= \frac{1.5}{5\sin 60}$

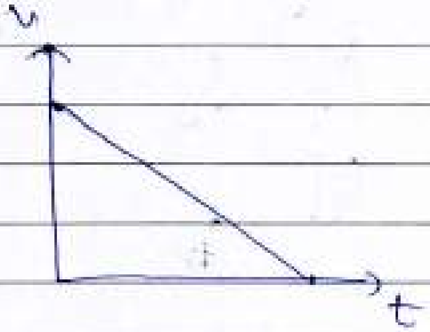
$\simeq 0.35\,h$



$$t = \frac{1.5\,km}{55 \sin 60^\circ}$$

$$= 0.35 \text{ hour}$$

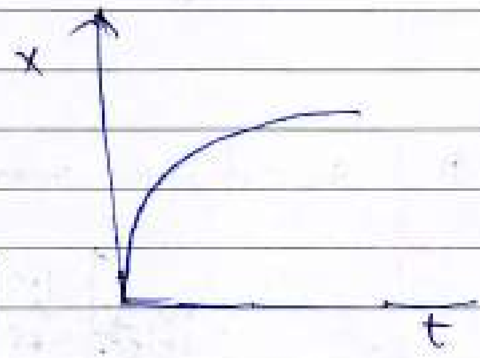
(10) ↑u $\quad v = u - gt$



$$\frac{dx}{dt} = (u - gt)$$

$$\int dx = \int (u - gt)\, dt$$

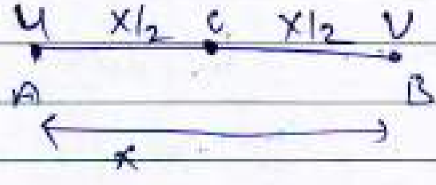
$$x = \left(ut - \tfrac{1}{2} g t^2\right)$$



at time $t = 0$

$\boxed{v \neq 0}$ $\quad \boxed{\frac{dx}{dt} \neq 0}$

(11)



$$v^2 = u^2 + 2ax \quad \text{—(1)}$$

$$v_c^2 = u^2 + 2a\frac{x}{2} \quad \text{—(2)}$$

$$v_c^2 = u^2 + \left(\frac{v^2 - u^2}{2}\right) = \frac{v^2 + u^2}{2}$$

$$v_c = \sqrt{\frac{u^2 + v^2}{2}}$$

(12) $u_m = 3\,km/h$

$$v_r = v_x\hat{i} + v_y\hat{j}$$

initial speed of rain w.r.t. man

$$v_{rm} = v_r - u_m = (v_x - 3)\hat{i} + v_y\hat{j}$$

given that rain appears to fall vertically.

so $v_x - 3 = 0 \qquad v_x = 3\ km/h$

after increase speed to $6\,km/h$

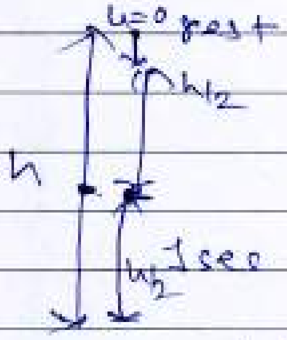
angle $45°$

$$\tan 45 = 1 = \frac{v_x'}{v_y'} = \frac{(v_x - 6)}{v_y} = 1$$

$$v_x = v_y + 6 \Rightarrow v_y = -3\ km/h$$

So $\vec{v_r} = 3\hat{i} - 3\hat{j} \Rightarrow |\vec{v_r}| = 3\sqrt{2}\ km/h$

(13)



$h/2 \rightarrow$ half of its path

$$T_{total} = \sqrt{\frac{2h}{g}}$$

last sec. its travels $h/2$

so initial $h/2$ travels in $(T_{total} - 1)$ sec.

$$v = \sqrt{2g \times \frac{h}{2}} = \sqrt{hg} \text{ at mid point}$$

$$h/2 = (\sqrt{gh})1 + \frac{1}{2} \times 10 \times 1^2$$

$$\Rightarrow h/2 = (\sqrt{gh} + 5) \Rightarrow (h/2 - 5)^2 = gh$$

$$\Rightarrow \frac{h^2}{4} - 5h + 25 = 10h$$

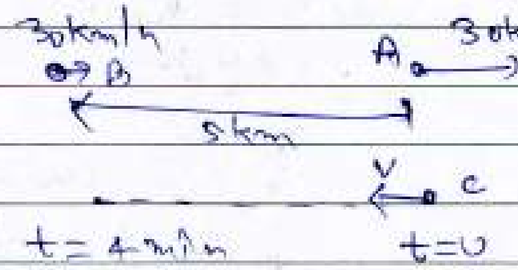
$$\Rightarrow h^2/4 - 15h + 25 = 0 \Rightarrow h^2 - 60h + 100 = 0$$

$$h = \frac{60 \pm \sqrt{3600 - 400}}{2} = 58.28\,m$$

$$T_{total} = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 58.28}{g}} =$$

$$= 3.41 \text{ sec}$$

(14) → 30km/h



velocity of car C w.r.t to car A

$$v_{CA} = (v + 30) \text{ km/h}$$

Relative travel distance = 5km

time = 4 min $= \frac{1}{15}$ hour

$$v_{CA} = (v+30) = \frac{5}{1/15} = 75 \text{ km/h}$$

$$\boxed{v = 45 \text{ km/h}}$$

(15)

$$x = 2t^3 - 21t^2 + 60t + 6$$

$$v = \frac{dx}{dt} = (6t^2 - 42t + 60)$$

when $v = 0 = 6t^2 - 42t + 60 = 0$

$$t^2 - 7t + 10 = 0$$

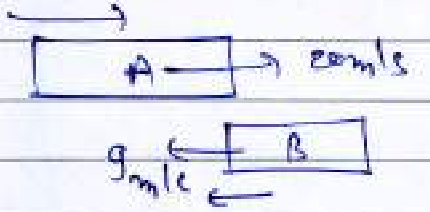
$$t = 2 \text{ sec} \quad \& \quad t = 5 \text{ sec}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = (12t - 42)$$

a at t = 2sec

$$a_{t=2sec} = -18 \text{ m/sec}^2$$

(16) $72\ km/h = \underline{20\ m/sec}$



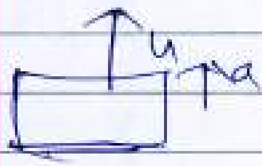
both trains are moving in opposite direction.

$v_{AB} = 29\ m/sec$

Crossing time $= 10\ sec.$

Length of train $= 29 \times 10 = \underline{290\ m}$

(17)



acceleration of body w.r.t lift $= (a+g)$

time of flight $t = \dfrac{2u}{a+g}$

$a = \dfrac{(2u - gt)}{t}$

(18) $y = 8t - 5t^2$ and $x = 6t$

$\dfrac{dy}{dt} = 8 - 10t = v_y$ $\qquad \dfrac{dx}{dt} = v_x = 6$

Initial velocity $(t=0)$ $v_y = 8$
$v_x = 6$

$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = 10\ m/sec.$

(19) acceleration $a = (6t+4)\ m/sec^2$

$\dfrac{dv}{dt} = (6t+4)$

$\Rightarrow \int_0^v dv = \int_0^t (6t+4)\,dt$

$v = (3t^2 + 4t) = dy/dt$

$$\int dx = \int (3t^2 + 4t)\,dt$$

$$x = \left(\frac{3t^3}{3} + \frac{4t^2}{2}\right)$$

x at t = 3 sec

$$x = \left(t^3 + \frac{4t^2}{2}\right)_{t=3sec} = (3)^3 + 2(3)^2$$

$$= 27 + 18 = 45 \text{ m}$$

(20) $a = (3t^2 + 2t + 2) = \frac{dv}{dt}$

$$\int_2^v dv = \int_0^2 (3t^2 + 2t + 2)\,dt$$

$$(v - 2) = \left[t^3 + t^2 + 2t\right]_0^2$$

$$= 8 + 4 + 4 = 16 \text{ m/s}.$$

$$v = (16 + 2) = \underline{18 \text{ m/sec}}$$

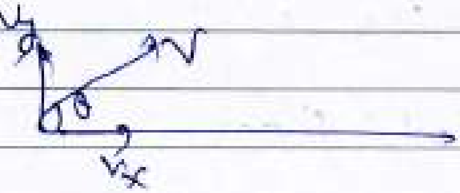
(21) $x = 6t$ and $y = 8t - 5t^2$

$\frac{dx}{dt} = v_x = 6$ and $\frac{dy}{dt} = v_y = 8 - 10t$

initial velocity $v_x = 6$

$v_y = 8$

$$\tan\theta = \frac{v_y}{v_x} = 8/6 = 4/3$$



$$\Rightarrow \theta = \tan^{-1}(4/3)$$

(22) Range $= \frac{u^2 \sin 2\theta}{g}$

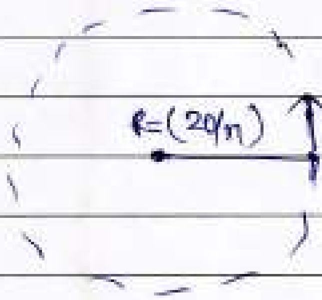
$\boxed{u_A = u_B = u_C}$ $\theta_A = 30°$, $\theta_B = 45°$; $\theta = 60°$

$R_A = \frac{u_A^2 \sin 60°}{g}$; $R_B = \frac{u_B^2 \sin 90°}{g}$; $R_C = \frac{u_C^2 \sin 120°}{g}$

$= \frac{\sqrt{3} u_A^2}{2g}$ $= \frac{u_B^2}{g}$ $= \frac{\sqrt{3} u_C^2}{2g}$

$\boxed{(R_A = R_C) < R_B}$

(23)



$\omega = \frac{\theta}{T} = v/R = \frac{80}{20/\pi} = (4\pi)$

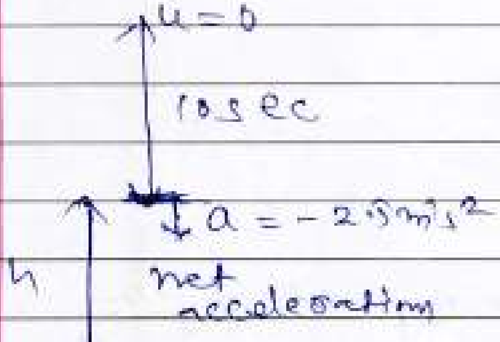
$a_t = \frac{dv}{dt} = \frac{(80 - 0)}{t}$

Time period = 1 sec.

so time taken for 2 revolution = 2 sec.

$a_t = \frac{80}{2} = 40$ m/sec.

(24)



before the parachute opens out distance travelled

$= \frac{1}{2} \times 10 \times 10 \times 10 = (500)$ m

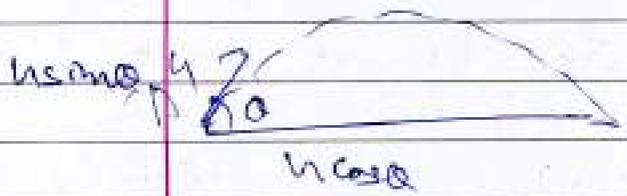
velocity = 10×10 = 100 m/sec

$h = 2495 - 500 = 1995$ m

$v^2 = (100)^2 - 2(2.5) \times 1995$

$v = 5$ m/s

(25)



after time t

$v\cos\theta' = u\cos\theta$

$v\sin\theta' = u\sin\theta - gt$

$$\tan\theta' = \frac{u\sin\theta - gt}{(u\cos\theta)}$$

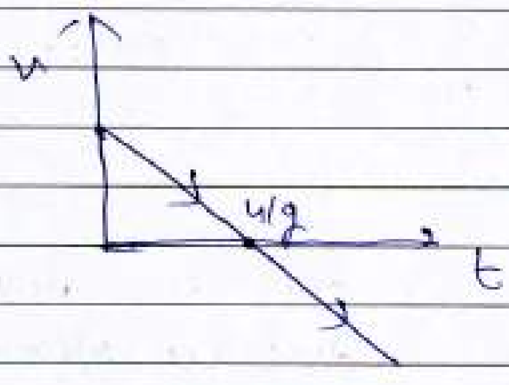
$v\cos\theta' = v\sin\theta' \rightarrow \underline{\tan\theta' = 1}$

$$\Rightarrow \frac{u\sin\theta - gt}{u\cos\theta} = 1$$

$$\Rightarrow t = \frac{u(\sin\theta - \cos\theta)}{g}$$

(26)

$\uparrow u$ $\boxed{v = u - gt}$



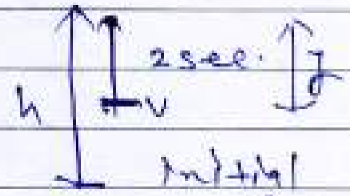
(27)

initial velocity of packet = velocity of balloon at that instant = 12 m/s

$-65 = 12t - \frac{1}{2}\times 10\times t^2$

$5t^2 - 12t - 65 = 0$ | $t = 5$ sec.

(28)



$y = \frac{1}{2} \times 10 (2)^2 = \underline{20m}$

Initial distance b/w them = 20m
(at t = 2sec)

velocity of 1st particle after 2sec
$= u + at$
$v = 20$ m/sec

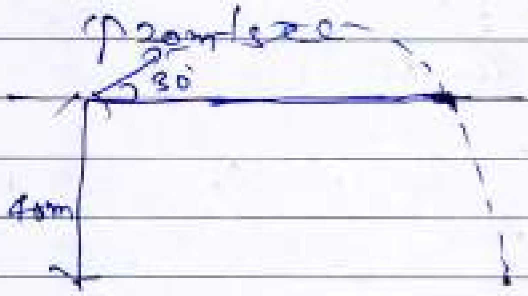
time when distance b/w them is 40m

$\cancel{2}0 = (20) T \Rightarrow \cancel{2sec}$

$T = \underline{1 sec}$

Total time = 2 + 1 = $\underline{3 sec}$

(29)



$-40 = (20 \sin 30°) T - \frac{1}{2} \times 10 \times T^2$

$-40 = 10T - 5T^2$

$T^2 - 2T - 8 = 0$

$\boxed{T = 4 sec}$

time of flight (same elevation)

$0 = (20 \sin 30°) t - \frac{1}{2} \times 10 \times t^2$

$t = 2 sec.$

(30)

$v \cos 30° = u \cos\theta$ — (1)

$v \sin 30° = u \sin\theta - g(2)$ — (2)

$0 = u \sin\theta - g(3)$ — (3)

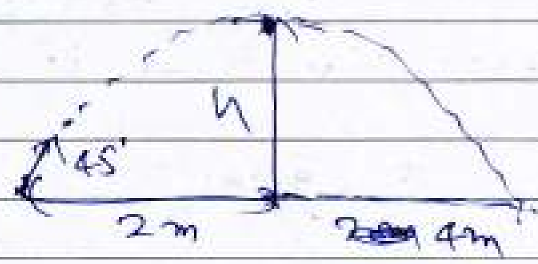
$u \sin\theta = 30$ m/s

$v = 2(30-20) = 20\,m/sec$

$u\cos\theta = v\cos 30^\circ = \underline{10\sqrt{3}}$

$\therefore\ u = \sqrt{u^2\cos^2\theta + u^2\sin^2\theta} = \sqrt{300+900} = 20\sqrt{3}\,m/s$

(31)



$R = \dfrac{u^2\sin 90^\circ}{g} = 6$

$u^2 = 60$

$\cancel{H = \dfrac{u^2\sin^2 45}{2g} = \dfrac{60\times 1/2}{2\times 10} = 1m}$

$=$

$y = x\tan\theta - \dfrac{\frac{1}{2}gx^2}{u^2\cos^2 45^\circ}$

$y = 2 - \dfrac{\frac{1}{2}\times 10\times 2}{60\times 1/2} = 2 - 2/3 = \underline{4/3\,m}$

(32) ranges are equal for angle θ & $(90^\circ-\theta)$

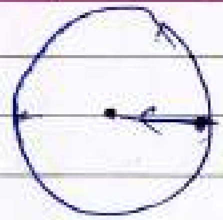
if $\theta_1 = \pi/3$ then $\theta_2 = \pi/6$

$H = \dfrac{u^2\sin^2\theta_1}{2g}$; $H' = \dfrac{u^2\sin^2\theta_2}{2g}$

$H = \dfrac{u^2\,3/4}{2g} = \dfrac{3u^2}{8g}$; $H' = \dfrac{u^2}{8g}$

$\underline{H' = H/3}$

(33)



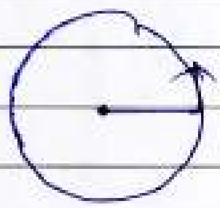
uniform circular motion

$\boxed{a_t = 0}$

$a_c = v^2/R$

in one complete revolution = null vector

(34)



uniform speed
velocity and acceleration both have direction. (vector quantity)
so vector will change in circular motion.

(35) if an object follow a curved path then centrifugal acceleration will change in direction but magnitude will const. during its motion.

— o —

~! ASSERTION - REASON !~

① A: A body can have acceleration even if its velocity is zero at a given instant of time.
R: A body is momentarily at rest when it reverses its direction of motion.

$v^0 = u + at$ both are true.

$$\boxed{a = -u/t}$$

② A: |displacement ≤ distance|

displacement = (final position - initial position)
distance → Total travelled distance, that is depend on actual path.

③

$$|\vec{V}_{avg}| \leq |V_{avg}| \text{ speed}$$

$$\left|\frac{\Delta \text{displacement}}{\Delta \text{time}}\right| \leq \left|\frac{\text{Total distance}}{\text{total time}}\right|$$

displacement ≤ distance

④ If speed is zero then velocity must be zero at that instant
for body falling
1st sec $h = \frac{1}{2}g(1)^2 = g/2$
2nd sec $= \frac{1}{2}\times g(2)^2 - \frac{1}{2}g(1)^2 = 3h$
3rd sec $= \frac{1}{2}g(3)^2 - \frac{1}{2}g(2)^2 = 5h$

(6) If speed is varying then velocity must be vary.

always $|\text{speed}| \geq |\text{velocity}|$

(7) $t_1 \rightarrow v_{1avg}$ $\qquad$ $\underline{v_{1avg} = v_{2avg}}$

$t_2 \rightarrow v_{2avg}$

$$\frac{\Delta x_1}{\Delta t} = \frac{\Delta x_2}{\Delta t}$$

$\boxed{\Delta x_1 = \Delta x_2}$

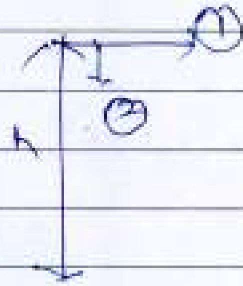
uniform velocity $\boxed{a = 0}$

$$\boxed{x = vt}$$

(8) for free falling body
velocity doesn't depends on mass of body

$\underline{v^2 = u^2 + 2gh}$

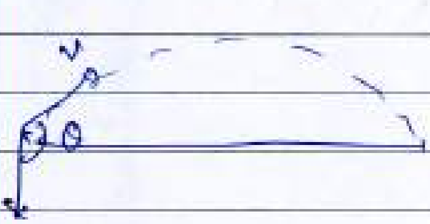
(9)



$t_1 = \sqrt{\frac{2h}{g}}$ $\qquad$ $t_1 = t_2$

$t_2 = \sqrt{\frac{2h}{g}}$

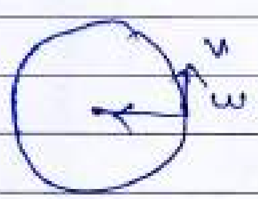
(10)



$\downarrow g$ $\qquad$ angle b/w a & v is greater than $\pi/2$ for ascending motion

So velocity will decrease.

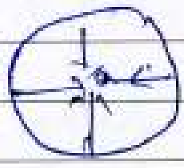
(11)



$a_t = 0$

$a_c = \omega^2 R = v^2/R$

$a_c \neq 0$ in circular motion

(12) 

If speed is const then $a_t = 0 = \frac{dv}{dt}$

$\therefore a_c = v^2/R$ gives centripetal acceleration.

(13) $w = \int \vec{F} \cdot d\vec{s}$ $\because$ $\vec{F}$ & $\vec{s}$ both are ⊥

$\boxed{w = 0}$

(14) acceleration is the rate of change of velocity

$a = dv/dt$

(15) always at rest

$\boxed{v = 0}$

(16)

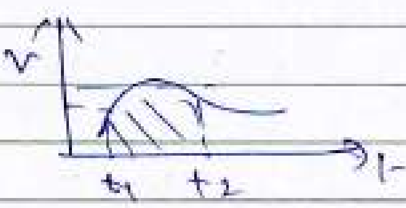
displacement $\leq$ distance

(17) The eqn of motion ~~eq~~ is applied when accelartion is constant.

$a = \frac{dv}{dt} = \frac{v\,dv}{dx}$

for uniform motion $\boxed{a = 0}$

(18) $x = \int_{t_1}^{t_2} v.dt$



(19) for uniformly motion

$v = const = \frac{dx}{dt} = const.$

(20) avg speed = $\frac{u_1 + u_2 + \ldots}{t_1 + t_2 + \ldots}$ = $\frac{\text{total distance}}{\text{total time}}$

(21) avg velocity = $\frac{\text{total displacement}}{\text{total time}}$

(22)

speed is always $\geq$ zero

(23)

$\frac{dx}{dt}$ = const. for uniform motion but it can be positive or negative.

(24) acceleration = $\vec{a}$ = $\frac{d\vec{v}}{dt}$

(25)

If speed is increase then acceleration and velocity both are in same direction.

(26) motion under gravity is independint of the mass of the body.

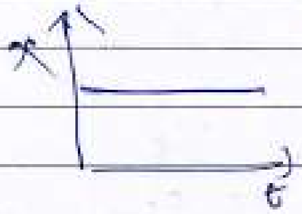
(27) velocity = $\frac{\text{displacement}}{\text{time}}$

speed = $\frac{\text{distance}}{\text{time}}$

(28) Rocket takes flight due to Combustion of fuel and does not move under the gravity effect alone.

(29) if distance travelled by a body is equal to the displacement then
avg speed = avg velocity = $\frac{\Delta x}{\Delta t}$

(30) for stationary object $v = 0$
$\frac{dx}{dt} = 0 \Rightarrow x$ is const w.r.t t.



(31) for uniformly accelerated motion
$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \neq 0$

so $\frac{dx}{dt} \neq$ const.

(32) $\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$
if they are moving in opp. direction
then $|\vec{v}_{12}| = (\vec{v}_1 + \vec{v}_2)$
for same direction $|\vec{v}_{12}| = (v_1 - v_2)$

(33) $\frac{d^2x}{dt^2} = a \neq 0$
$x \propto t^2$ for uniformly acceleration.

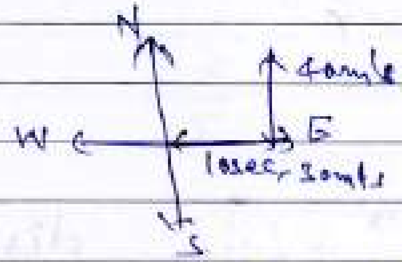
(34) uniform $\frac{dv}{dt} = 0 \Rightarrow v =$ const w.r.t t.

(35) for uniform
$a = 0 \Rightarrow \frac{dv}{dt} = 0$

Previous Year Questions

Kinematics Parameters

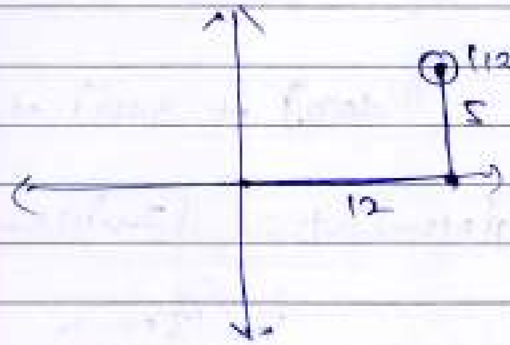
(1) → 30 m/s



avg acceleration

$$= \frac{v_f - v_i}{T} = \frac{40\hat{j} - 30\hat{i}}{10} = \frac{150}{10} = 5\ m/s^2$$

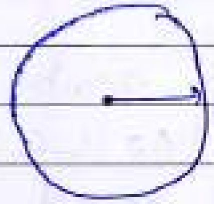
(2)



final position $= 12\hat{i} + 5\hat{j} + 6\hat{k}$

displacement $= \sqrt{12^2 + 5^2 + 6^2}$
$= \sqrt{169 + 36}$
$= \sqrt{205} = 14.31\ m$

(3)



distance travelled in one revolutions $(2\pi R)$

in 200 $\Rightarrow (200)(2\pi R)$
$\Rightarrow 400\pi R = 9.5 \times 10^3$

diameter $= 2R = \frac{9.5 \times 10^3}{200\pi} = \frac{95}{2\pi} = 15\ m$

(4) $S = 6t^2 - t^3$

$v = \frac{ds}{dt} = 12t - 3t^2 = 0$

$t(3t - 12) = 0 \Rightarrow t = 4\ sec$

(5) displacement can be zero for some time.

(6) $X \xrightarrow{v_u}$ (d) $\rightarrow Y$, $\leftarrow v_d$

$t_1 = d/v_u$ avg speed

$t_2 = d/v_d$

$$= \frac{2d}{t_1 + t_2} = \frac{2 v_u v_d}{v_d + v_u}$$

(16) $u_f = u/2$; u - initial velocity

$$\frac{u^2}{4} = u^2 - 2a(3)$$

$$\Rightarrow 6a = \frac{3u^2}{4}$$

$$\boxed{a = u^2/8}$$

$v^2 = u^2 + 2as \Rightarrow u^2 = 2\left(\frac{u^2}{8}\right)x$

$$x = 4\text{ cm}$$

It will further penetrate $= (4-3) = 1$ cm

(17) $\xleftarrow{d/2}$ 40 km/h $\times$ $\xrightarrow{d/2}$ 60 km/h avg velocity $= \frac{d}{t_1+t_2}$

$$= \frac{d}{\frac{d/2}{40} + \frac{d/2}{60}}$$

$$\Rightarrow \frac{1}{1/80 + 1/120} = \frac{240}{3+2} = 48\text{ km/h}$$

(18) total Length = 1000 m + 100 m
= 1100 m

velocity = 45 km/h

$$t = \frac{1100}{45000} \times 3600 = 88\text{ sec.}$$

(19) Train running on a straight track.

(20) $t = \frac{(850+150)}{45\times10^3} \times 3600 = \frac{400}{5} = 80$ sec.

Non - Uniform Motion

(1) $x = at^2 - bt^3$

$v = \frac{dx}{dt} = 2at - 3bt^2$

$a = \frac{d^2x}{dt^2} = 2a - 6bt = 0$

$t = a/3b$

(2) $v = u^{0} + na \Rightarrow \boxed{a = u/n}$

displacement in last 2 sec.

$s = \frac{1}{2}an^2 - \frac{1}{2}a(n-2)^2 = 2a(n-1)$

$\therefore s = \frac{2u}{n}(n-1)$

(3) $(20)^2 = (10)^2 + 2a(135)$

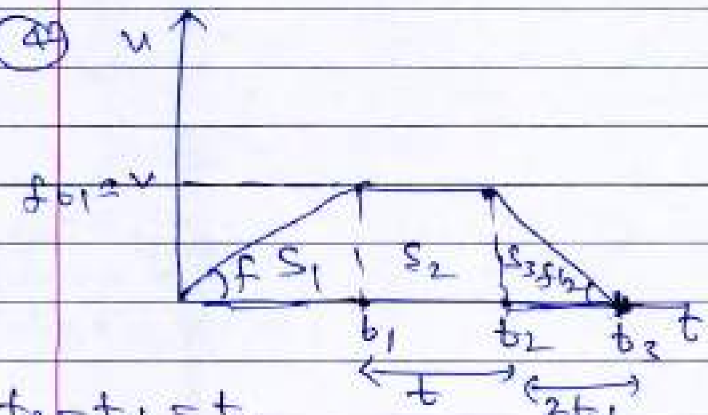
$a = \frac{300}{270} = 10/9 \ m/s^2$

$v = u + at$

$20 = 10 + (10/9)\,t$

$t = 9$ sec

(4)



Slope $= f = u/t_1$

$v = ft_1$

$S_1 = \frac{1}{2} \times t_1 \times ft_1$

$S_1 = \frac{1}{2} f t_1^2$

$\frac{ft_1}{(t_3 - t_2)} = f/2$

$t_2 - t_1 = t$

$\boxed{t_3 = t_2 + 2t_1}$

total distance

$= S_1 + S_2 + S_3 = 15S$

$= \boxed{S_2 = f t_1 t}$ & $S_1 = S = \frac{1}{2} f t_1^2$

$S_3 = \frac{1}{2} \times (2t_1) f t_1 = f t_1^2$

$S_3 = S/2$ & $S_2 = \frac{2St}{t_1}$

$S + S/2 + 2St/t_1 = 15S$

$\Rightarrow 2t/t_1 = 15 - 3/2 = 27/2$ 12

$\cancel{t/t_1 = 27/4}$

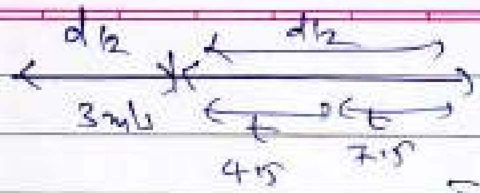
$\cancel{\Rightarrow t_1 = 4/27\, t}$

$\cancel{\therefore S = \frac{1}{2} f t_1^2 = \frac{1}{2} \times f \times \frac{16}{(27)^2} t^2} = \frac{8 f t^2}{(27)^2}$

$2t/t_1 = 12 \quad \Rightarrow \boxed{t/t_1 = 6}$

$\therefore S_1 = S = \frac{1}{2} f t_1^2 = \frac{1}{2} f (t/6)^2 = \frac{1}{72} f t^2$

(5)



avg speed $= \dfrac{d}{t_1+t_2+t_3}$

$= \dfrac{d}{\frac{d/2}{3} + 2t}$

$d/2 = 4.5t + 7.5t$

$t = d/24$

$= \dfrac{d}{d/6 + d/24} = \dfrac{24}{5} = 4.8\,m/s$

(6) $v^2 = u^2 + 2as$

$(50)^2 = 2(a)(6)\times10^{-3} \Rightarrow a = \dfrac{50\times50}{12\times10^{-3}}$

$u^2 = 2ax$

$(100)^2 = 2\left(\dfrac{50\times50}{12\times10^{-3}}\right)x$

$x = 2\times12\times10^{-3} = \underline{24\ m}$

(7) $x = \alpha t^3$ & $y = \beta t^3$

$v_x = \dfrac{dx}{dt} = 3\alpha t^2$ & $\dfrac{dy}{dt} = v_y = 3\beta t^2$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2t^4 + 9\beta^2t^4} = 3t^2\sqrt{\alpha^2+\beta^2}$

(8) $x = 9t^2 - t^3$

$v = \dfrac{dx}{dt} = 18t - 3t^2 \Rightarrow \dfrac{dv}{dt} = 18 - 2(t)(3) = 0$

$t = \underline{3\ sec}$

$x = 9(3)^2 - (3)^3$

$= 81 - 27$

$= \underline{54 m}$

(9) $(\sqrt{3}, 3)$

$$\tan\theta = \frac{y}{x} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\theta = \underline{60^\circ}$$

(10) $f = f_0\left(1 - t/T\right) = f_0 - f_0 t/T = \frac{dv}{dt}$

$$\int_0^v dv = \int_0^t \left(f_0 - f_0 t/T\right) dt$$

t when $f = 0 \Rightarrow \boxed{t = T}$

$$v = \left(f_0 t - f_0 t^2/2T\right)_0^T$$

$$= f_0 T - f_0 T/2 = f_0 T/2$$

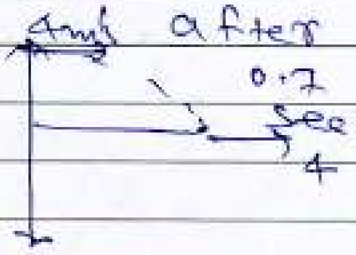
(11) $a = bt \Rightarrow \int_{v_0}^{v} dv = \int_0^t bt\,dt$

$$v - v_0 = bt^2/2$$

$$v = \left(v_0 + bt^2/2\right) = \frac{dx}{dt}$$

$$\int_0^x dx = \int_0^t \left(v_0 + bt^2/2\right) dt$$

$x = \left(v_0 t + bt^3/6\right)$ Ans.

(12)


$u_x = 4\text{ m/s}$

$v_y = 10 \times .7 = 7\text{ m/s}$

$v = \sqrt{7^2 + 4^2} = \sqrt{65}\text{ m/s}.$

(13) $(u/2)^2 = u^2 + 2a(30)$

$a = \dfrac{3u^2/4}{60} = u^2/80$

$u^2 = 2\left(u^2/80\right)(x+30)$

$x = 40 - 30 = \underline{10\,cm}$

(14) $a = (2t+5) = dv/dt$

$v = \int_0^5 (2t+5)\,dt$

$= \left(2t^2/2 + 5t\right)_0^5$

$= \underline{50\,m/s}$

(15) $u \xleftrightarrow[24m]{4sec} u' \xrightarrow[64m]{4sec}$

$24 = u(4) + \frac{1}{2}(a)(4)^2$

$64 = u'(4) + \frac{1}{2}(a)(4)^2$

$u' - u = \frac{40}{4}\,m/s$ — (1)

$u' = u + a(4)$ — (2)

$a = \underline{\frac{10}{4}\,m/s}$

$6 = u + \frac{1}{2} \times \frac{10}{4} \times 4 \Rightarrow u = \underline{1\,m/s}$

(16) $u \xleftrightarrow[200m]{2sec} u' \xleftrightarrow[4sec]{220m}$

$200 = u(2) + \frac{1}{2}(a)(2)^2$

$220 = u'(4) + \frac{1}{2}(a)(4)^2$

$u'^2 = u^2 + 2a(200)$

$u' = u + (2a)$

$\underline{(u'-u) = 2a}$

~~9000~~

$90 = (u-u') + \frac{1}{2}a\left(\frac{-12}{2}\right)$

$\Rightarrow -90 = \underline{(u' - u)} + 2a$

$a = -90/4 = \cancel{-45/2 \text{ m/sec}^2}$

$\Rightarrow 100 = u + \frac{1}{\cancel{2}}(-45/2)(\cancel{2})$

$u = 100 + 45/2 = \frac{245}{2}$ m/s

$\cancel{v = u + at}$

$\cancel{= \frac{245}{2} - \frac{45}{2} \times 7} =$

with uniform acceleration

$200 = u(2) + \frac{1}{2}a(2)^2$

$u + a = 100$ — ①

$\cancel{100} + 220 = u(6) + \frac{1}{2}a(6)^2$

$u + 3a = 70$ — ②

$a = -15$ m/s² $\quad u = 115$ m/s

v after 7 sec.

$v = u + at$

$v = 115 - 15 \times 7$

$v = \underline{10 \text{ m/s}}$

(17)
$16 = 10 + a(3)$

$a = 2\ m/sec^2$

~~10 = 10 + 2×2 = 14 m/s~~

$10 = u + 2(2)$

$\underline{u = 6\ m/s}$

(18)
$x = \frac{1}{2}a(10)^2 = 50a$

$v = \cancel{u} + at = 10a$

$y = (10a)10 + \frac{1}{2}a(10)^2$

$= 100a + 50a = 150a = 3\times(50a)$

$\boxed{y = 3x}$

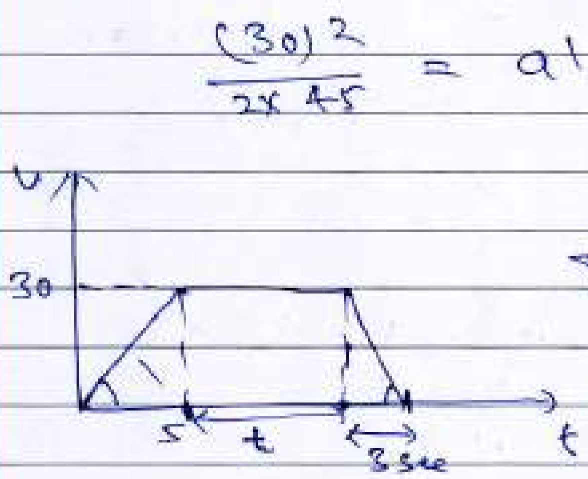
(19)
$108\ km/h = 30\ m/s$, $t = 5$ sec.

$v = \cancel{u}^0 + at$

$a = \frac{30}{5} = 6\ m/sec^2$

$\cancel{v}^0{}^2 = u^2 + 2a's$

$\frac{(30)^2}{2\times 45} = a' \Rightarrow a' = 10\ m/sec^2$ retardation.



Total distance

$\Rightarrow 395m = \frac{1}{2}\times\cancel{30}^{15}\times 5 + t\times 30 + \frac{1}{2}\times 3\times 30$

$\Rightarrow 30t = 395 - 75 - 45$

$t = \frac{275}{30} = 9.16$ sec

Total time

$= 5 + 9.16 + 3 = \underline{17.2\ sec}$

(20) $s = a + bt + ct^2$

$v = \frac{ds}{dt} = b + 2ct \Big|_{t=0}$ $\boxed{v = b}$

$a = \frac{dv}{dt} = 2c$

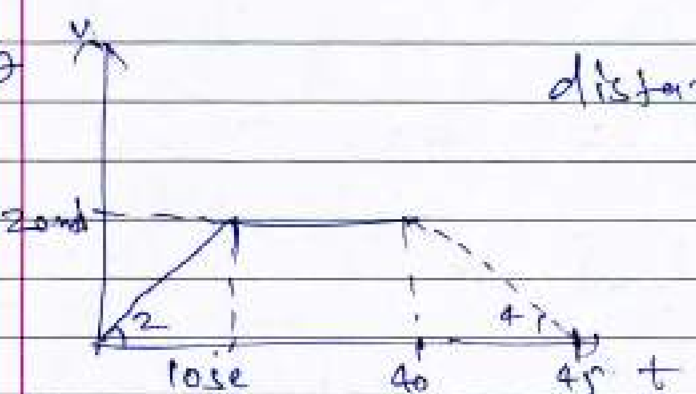
(21) $y = a + bt + ct^2 - dt^4$

$v = \frac{dy}{dt} = b + 2ct - 4t^3 d$

Initial v at $(t=0) = b$

Initial $a = \frac{dv}{dt} = 2c$

(22)



distance = Area under the v-t graph

$= \frac{1}{2} \times 10 \times 20 + 30 \times 20 + \frac{1}{2} \times 20 \times 5$

$= 100 + 600 + 50$

$= 750\,m$

(23) same as Q. (6)

$v^2 = u^2 + 2as$

$u^2 = 2a.s$

$(2u)^2 = 2a \times 1$

$\boxed{x = 4s}$

(24) $72\,km/h = 20\,m/s$

$\Rightarrow (20\times 20) = -2\times a\times 200$

$\Rightarrow a = -\dfrac{200\times 2}{2\times 200} = -1\, m/sec$

(25) $t = \alpha x^2 + \beta x$

$1 = \alpha\, 2x \dfrac{dx}{dt} + \beta \dfrac{dx}{dt}$ — (1)

$0 = 2\alpha\left(x\dfrac{d^2x}{dt^2} + \left(\dfrac{dx}{dt}\right)^2\right) + \beta\dfrac{d^2x}{dt^2}$

$\because \dfrac{d^2x}{dt^2} = a$ — (2)

$\Rightarrow 2\alpha(xa + v^2) + \beta a = 0$

$2\alpha v^2 = (2\alpha x - \beta)\, a$

$\Rightarrow a = \dfrac{2\alpha v^2}{(2\alpha x - \beta)}$ — (3)

from eqn (1) $1 = 2\alpha x v + \beta v$

$x = \dfrac{1-\beta v}{(2\alpha v)}$

Put in eqn (3)

$a = \dfrac{2\alpha v^2}{\frac{(1-\beta v)}{v} - \beta} = 2\alpha v^3$ Ans.

(26) $v^2 = 180 - 16x$

$2\underbrace{\left(v\dfrac{dv}{dx}\right)}_{a} = -16 \Rightarrow a = -8\, m/sec^2$

(27) $a = (2t-2) = \frac{dv}{dt}$

$v = \int_0^5 (2t-2)\,dt$

$= \left[(t^2-2t)\right]_0^5 = 15$ m/sec

(28) $x = ae^{-\alpha t} + be^{\beta t}$

$v = \frac{dx}{dt} = -a\alpha e^{-\alpha t} + \beta b e^{\beta t}$

v at $t=0 = -a\alpha + \beta b$

v at $t \to \infty = \beta b$

v is increasing with time t.

(29) $x = 4(t-2) + a(t-2)^2$

$v = \frac{dx}{dt} = 4 + 2a(t-2)$

$a = \frac{dv}{dt} = \underline{2a}$

(30) what determines the nature of the path followed by the Particle

→ none of these

only speed or only velocity or only Acceleration can not determines the nature of path.

Relative Motion

(1) Const. speed $= 2 m/s$

Speed of passenger $= 2 m/s$ in opp. direct.

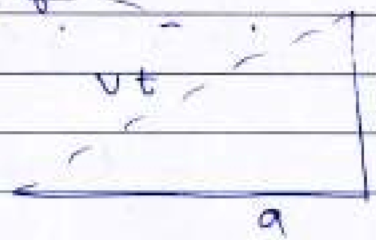
$V_{relative}$ w.r.t ground $= (2-2) = \underline{0}$

(2)

$$V_{BA} = u_1\hat{j} - (\vec{v})$$

$$V_R = \sqrt{u_1^2 + v^2}$$

after time t



$$v^2t^2 = (a^2 + u_1^2t^2)$$

$$t^2 = \frac{a^2}{(v^2 - u_1^2)}$$

$$t = \sqrt{a^2/(v^2-u_1^2)}$$

(3)

Simple time of flight $T = \frac{2 \times 10}{10} = \underline{2 sec}$

after 2 sec velocity w.r.t. ground $= (10-5) = 5 m/s$

(4)

$$V_{12} = (u_1 - v_2)$$

distance $= d$

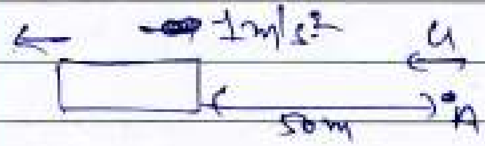
acceleration $= -a$

$$v^2 = u^2 + 2as$$

$$v_2^2 = (u_1 - v_2)^2 - 2ad \quad , \quad d \le \frac{v_2^2 - (u_1 - v_2)^2}{2a}$$

$\therefore d > \frac{(u_1 - u_2)^2}{2a}$

(5)



Relative distance = 50m
— " — acceleration = $1 m/s^2$
Initial " — velocity = u

$\cancel{500} \quad v^2 = u^2 + 2as$

$u^2 = 2 \times 1 \times 50$

$\underline{u = 10 \text{ m/sec}}$

(6)

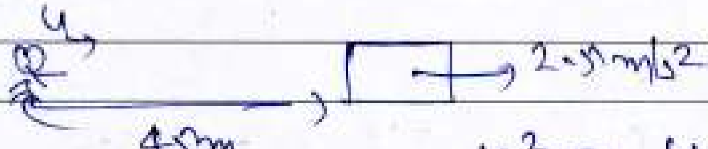
$x_B = vt$
$x_A = \frac{1}{2}at^2$

$\left.\begin{matrix} \\ \end{matrix}\right\}$ $x_A = x_B$

$vt = \frac{1}{2}at^{\cancel{2}}$

$\underline{t = 2v/a}$

(7)



$v_{rel}^2 = u_{rel}^2 + 2a_{rel}\, s_{rel}$

$0 = u_{rel}^2 + 2a_{rel}\, s_{rel}$

$\Rightarrow u_{rel}^2 = 2 \times 2.5 \times 45$
$= 25 \times 9$

$\vec{u}_{rel} = u_{man} = \underline{15 \text{ m/s}}$

(8) $\vec{v}_b = 3\hat{i} + 4\hat{j}$; $\vec{v}_w = -3\hat{i} - 4\hat{j}$

$\vec{v}_{bw} = \vec{v}_b - \vec{v}_w$
$= 6\hat{i} + 8\hat{j}$

(9) width = 1 km
V_{bw} = 5 km/h

$T = t_1 + t_2 = 2t$

$= 2\left(\frac{1}{4}\right) = \frac{1}{2}$ hour $= 30$ min

(10) If the velocity is doubled then

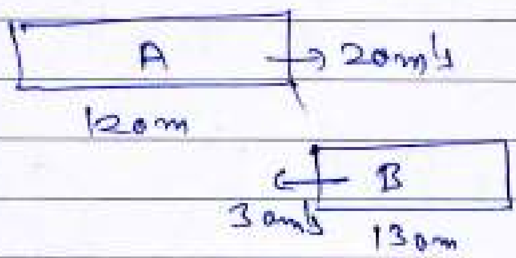
$P_i = mv$ $\qquad v_f' = (2v + v)'$

$P_i = \frac{mu}{\sqrt{1 - u^2/c^2}}$ $\qquad v_i' = (v + v')$

$\boxed{P_f > 2P_i}$

$P_f = \frac{2mu}{\sqrt{1 - u^2/c^2}}$ $\qquad \boxed{P_f > 2P_i}$

(11)



$v_{rel} = 50$ m/s

$d_{rel} = 120 + 130 = 250$

$T = \frac{d_{rel}}{v_{rel}} = 5$ sec

(12) Police → 45 km/h $= 12.5$ m/s $\qquad$ thief → 153 km/h $= 42.5$ m/s

bullet → 180 m/s

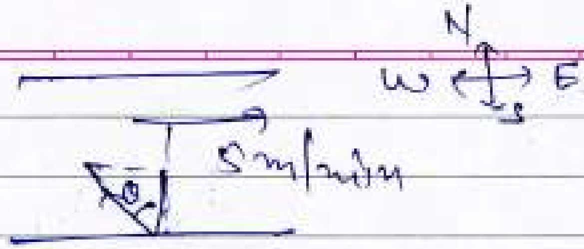
V_{bullet} w.r.t. to thief's car
$= 180$ m/s $+ 12.5 - 42.5 = 150$ m/s

(13)

$V_g = u$
$V_b = v = 8$ km/hr
resultant velocity $= \sqrt{v^2 + u^2}$

$\therefore u = 6$ km/hr

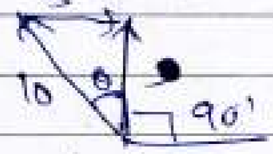
(14)



$V_r = 5 \text{ m/min}$

$V_m = 10 \text{ m/min}$

for shortest possible path he will swim at angle θ with perpendicular line of flow of river



$\sin\theta = 5/10 = 1/2$

$\theta = \pi/3$

$\Rightarrow 90° + 30 = \underline{120°}$ with downstream.

(15)

$0 = (-v_2 + u_1)^2 - 2ad$

$\Rightarrow d = \dfrac{(-v_2 + u_1)^2}{2a}$

$d = (-v_2 + u_1)t - \frac{1}{2}at^2$

$\frac{1}{2}at^2 - (-v_2 + u_1)t + d = 0$

$t = \dfrac{(-v_2 + u_1) \pm \sqrt{(-v_2+u_1)^2 - 4 \times \frac{a}{2} \times \frac{(-v_2+u_1)^2}{2a}}}{2a/2}$

$t = \dfrac{(-v_2 + u_1)}{2a/2} = \dfrac{(-v_2 + u_1)}{a} = \dfrac{(u_1 - v_2)}{a}$

Motion Under Gravity

(1) first 5 sec h_1 $\quad h_1 = \frac{1}{2}g(5)^2$

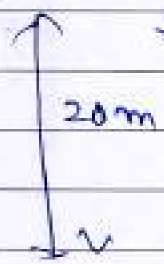
next 5 sec h_2 $\quad h_2 = \frac{1}{2}g(10)^2 - \frac{1}{2}g(5)^2$

and next 5 sec h_3 $\quad h_3 = \frac{1}{2}g(15)^2 - \frac{1}{2}g(10)^2$

$h_1 = \frac{25g}{2}$; $h_2 = \frac{75g}{2}$; $h_3 = \frac{125g}{2}$

$h_1 = h_2/3 = h_3/5$

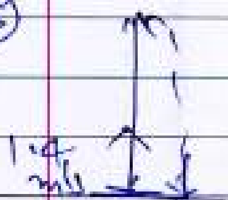
(2)



$v = \sqrt{2gh}$

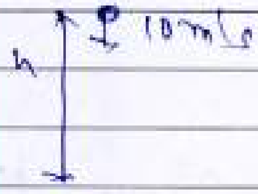
$= \sqrt{2\times10\times20} = 20\,m/s$

(3)



total displacement = zero

(4)



distance in 1st sec

$= 10\times1 + \frac{1}{2}\times10\times1 = 15\,m$

distance in 2 sec. $= 10\times2 + \frac{1}{2}\times10\times(2)^2 = 40\,m$

distance in 3 sec $= 3\times10 + \frac{1}{2}\times10\times(3)^2$

$= 75\,m$

so distance covered in 2nd sec $= 40 - 15 = 25\,m$

distance covered in 3rd sec $= 75 - 40 = 35\,m$

ratio $= \frac{35}{25} = 7/5$

(5) $\downarrow u = 0$ freely from rest

distance in 1st 3 sec

$$= \frac{1}{2} \times \overset{5}{\cancel{10}} \times (3)^2 = \underline{45\,m}$$

if Total time $= T$ $\Rightarrow$ distance $= \frac{1}{2} g T^2$

then in last sec.

distance $\Rightarrow \frac{1}{2} g T^2 - \frac{1}{2} g (T-1)^2 = 45$

$$\Rightarrow \cancel{\frac{1}{2} g T^2} - \cancel{\frac{1}{2} g T^2} - \frac{1}{2} g + \frac{2gT}{2} = 45$$

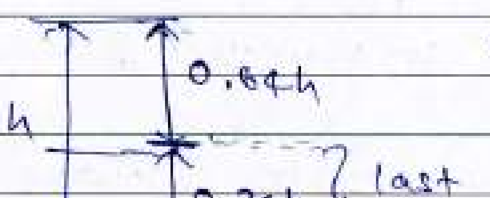
$$\Rightarrow \cancel{\frac{gT}{2} = \frac{2 \times 45}{g} \Rightarrow T = \frac{4 \times 45}{g^2}}$$

$$\Rightarrow \frac{2gT}{2} = 45 + 5 = 50$$

$$\Rightarrow T = \frac{2 \times 50}{2} / g = \underline{10/2 \text{ sec}} = 5$$

$$T = \underline{5 \text{ sec}}$$

(6)



$v^2 = u^2 + 2as$

$v^2 = 2 \times .64 \times 10h = 12.8h$

Total time $= \sqrt{\frac{2h}{g}}$

time to covered $0.64h$ height

$$= \sqrt{\frac{2 \times .64h}{g}}$$

distance in last sec

$$0.36h = \left(\sqrt{\frac{2h}{g}}\right)^2 \times \frac{1}{2} g - \frac{1}{2} \times \left(\sqrt{\frac{2 \times .64h}{g}}\right)^2 g$$

$0.36h = \left(\sqrt{2(.64h)g}\right)1 + \frac{1}{2}g(1)^2$

$\Rightarrow 0.36h = \sqrt{(12.8)h} + 5$

$\Rightarrow (0.36h - 5)^2 = \left(\sqrt{12.8h}\right)^2$

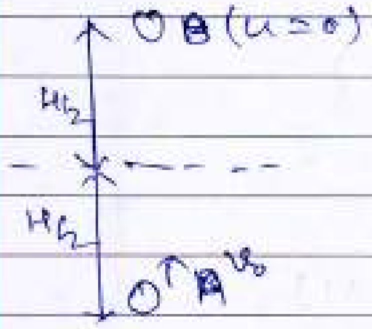
$\Rightarrow (0.36)^2h^2 + 25 - 3.6h = 12.8h$

$\Rightarrow (0.36)^2h^2 - 16.4h + 25 = 0$

$$h = \frac{16.4 \pm \sqrt{268.96 - (100)(.36)^2}}{2\times(.36)^2}$$

$h = 124.8m \approx \underline{125m}$

(7)



time taken by body (B) to cover ~~ped~~ $H/2$ distance

$= \sqrt{\frac{2\times H/2}{g}} = \sqrt{H/g} = T$

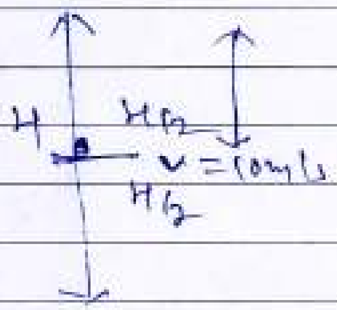
for body (A)

$H/2 = v_0\left(\sqrt{H/g}\right) - \frac{1}{2}\times 10\times\left(\sqrt{H/g}\right)^2$

$\Rightarrow \frac{H}{2} = v_0\left(\sqrt{H/g}\right) - \frac{5H}{g\cdot 2}$

$\Rightarrow H = v_0\sqrt{H/g} \Rightarrow v_0 = \sqrt{gH}$

(8)



$H_{max} = \frac{u^2}{2g}$ at half of H_{max}

$\frac{H_{max}}{2} = \frac{u^2}{4g}$

$v^2 = u^2 + 2as$

$(10)^2 = 2\times 10\times H/2 \Rightarrow H = \underline{10m}$

(9)

$v = \sqrt{2 \times 9.8 \times 50} = \sqrt{980}\ m/s$

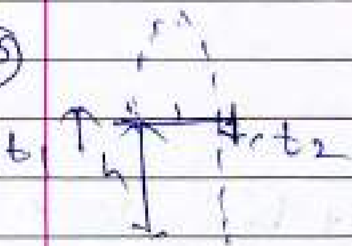
$v^2 = u^2 + 2as$

$a = -2\ m/s^2$ $\quad (3)^2 = (\sqrt{980})^2 - 2 \times 2 \times S$

$S = \frac{971}{4} = 242.75\ m$

$\therefore$ total height $= 50 + 242.75 \approx 293\ m$

(10)



$h = ut_1 - \frac{1}{2} g t_1^2$

$h = ut - \frac{1}{2} g t^2$

$$\boxed{\frac{1}{2} g t^2 - ut + h = 0}$$

$t_1, t_2 = \frac{+u \pm \sqrt{u^2 - 4 \times \frac{g}{2} \times h}}{g}$

$t_1 = \frac{u - \sqrt{u^2 - 2gh}}{g}$; $t_2 = \frac{u + \sqrt{u^2 - 2gh}}{g}$

$t_1 + t_2 = 2u/g \Rightarrow u = \frac{g}{2}(t_1 + t_2)$

(11) time of flight $\geq$ 4 sec

$\frac{2u}{g} \geq 4$

$u \geq 2g \geq 19.6\ m/s$

(12)

(12) 2kg, 72m

4kg in motion under gravity acceleration will be g and its same for all body.

$$\left.\begin{aligned} \text{momentum} &= mv \\ K.E &= \tfrac{1}{2}mv^2 \\ P.E &= mgh \end{aligned}\right\} \text{depends on mass of the body}$$

(13) $\uparrow v$, h

$$-h = vt - \tfrac{1}{2}gt^2$$

$$\tfrac{g}{2}t^2 - vt - h = 0$$

$$t = \frac{+v \pm \sqrt{v^2 + 2gh}}{g}$$

$$= \frac{v}{g} \pm \frac{\sqrt{v^2+2gh}}{g} = \frac{v}{g}\left[1 + \sqrt{1 + \frac{2gh}{v^2}}\right]$$

(14) h

$$t = \sqrt{2h/g} \text{ sec.}$$

at $t/2$ $\quad h' = \tfrac{1}{2}g(t/2)^2 = \tfrac{1}{2}g(t^2/4) = h/4$

So $3h/4$ from the ground.

(15) $\uparrow u$, H, $\downarrow 3u$

$$v^2 = u^2 + 2as$$

$$(3u)^2 = u^2 + 2 \times g \times H$$

$$H = \frac{8u^2}{2g} = 4u^2/g \text{ m}$$

(16) $u = 0$, H

40 m in last 2 sec.

total time $= \sqrt{2H/g}$

$$40 = \frac{1}{2} g T^2 - \frac{1}{2} g (T-2)^2$$

$$\Rightarrow \frac{40 \times 2}{g} = T^2 - (T^2 - 4T + 4)$$

$$\Rightarrow 8 = \cancel{T^2} - \cancel{T^2} + 4T - 4$$

$T = \underline{3 \text{ sec}}$ so $H = \frac{1}{2} \times 10 \times (3)^2$

$= \underline{45 \text{ m}}$

(17) 200 m; 2 sec h_1; 2 sec h_2; 2 sec h_3

$$h_1 = \frac{1}{2} g (2)^2 = 2g$$

$$h_2 = \frac{1}{2} g (4)^2 - \frac{1}{2} g (2)^2 = 6g$$

$$h_3 = \frac{1}{2} g (6)^2 - \frac{1}{2} g (4)^2 = 10g$$

$$h_1 : h_2 : h_3 = 1 : 3 : 5$$

(18) [illegible] m/s

① drop; 400 m; H'; ② $\uparrow$ 50 m/s

$u_{rel} = 50$ m/s

$H_{rel} = 400$ m/s

$T = H/u = \underline{8 \text{ sec}}$

$$H' = 50 \times 8 - \frac{1}{2} \times 10 \times (8)^2 = 80 \text{ m}$$

(19) h; $2h$

$$\left.\begin{aligned} t_1 &= \sqrt{2gh} \\ t_2 &= \sqrt{2g(2h)} \end{aligned}\right\} \quad t_1 : t_2 = 1 : \sqrt{2}$$

(20) (A) (B) Suppose acceleration at planet B is a then at planet A is 9a

$t_A = \sqrt{\frac{2h}{9a}}$ $\quad t_B = \sqrt{\frac{2h'}{a}}$

$t_A \neq t_B \Rightarrow \cancel{\sqrt{\frac{2h}{9a}} = \sqrt{\frac{2h'}{a}}}$

$\cancel{\sqrt{h'} = \sqrt{h}/3 = \sqrt{2}/3m \Rightarrow h' = 2/9m}$

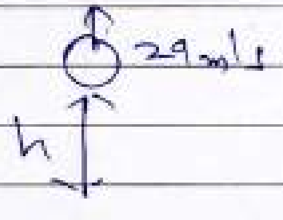
$u_A = \sqrt{2\times(9a)\times h}$
$u_B = \sqrt{2a(h')}$

$\left.\right\}$ $u_A = u_B$

$h' = 9h$ $\underline{= 18m}$

(21) $H = \frac{u^2}{2g} = \frac{15\times15}{2\times10} = \underline{11.25m}$

(22) initial speed of stone $= 29 m/s$ upward



$(t = 10 sec)$

$-h = (29)(10) - \frac{1}{2}\times10\times(10)^2$

$h = \underline{200m}$

(23) In first 3 sec distance $= \frac{1}{2}g(3)^2 = 45m$
in last sec

$45 = \frac{1}{2}gT^2 - \frac{1}{2}g(T-1)^2$

where T is the total time

$\frac{45\times2}{g} = T^2 - (T^2 - 2T + 1)$

$T = \underline{5 sec.}$

(24) $\uparrow a$ net

$\downarrow g$ relative acceleration $= a + g$

(25) distance travelled in 4 sec.

$h_1 = u(4) - \frac{1}{2} g (4)^2$

$= (4u - 8g)$ —(1)

distance in 5 sec

$h_2 = u(5) - \frac{1}{2} g (5)^2$ —(2)

distance in 6 sec

$h_3 = u(6) - \frac{1}{2} g (6)^2$ —(3)

so distance in fifth sec $= h_2 - h_1$

in sixth sec $= h_3 - h_2$

$(h_2 - h_1) = (h_3 - h_2)$

$\boxed{h_1 + h_3 = 2h_2}$ from

eq (1), (2), (3)

$(4u - 8g) + (6u - 18g) = 2(5u - 25g/2)$

$= 10u - 25g$

$\Rightarrow u = 49 m/s^2$

(26) $H_1 = 50 = \frac{u^2}{2g}$ | $H_2 = \frac{(2u)^2}{2g} = \frac{4u^2}{2g}$

$H_2 = 4H_1 = 200 m$

(27) $m_2 = 2m_1$

$u_1 = u_2 =$
$v^2 = u^2 + 2as$

$v = \underline{\sqrt{u^2 + 2ah}}$

m, u — h

velocity and acceleration are independent of mass of body.

(28) $h = \frac{v_0^2}{2g}$ $\quad h' = 3h = \frac{v'^2}{2g}$

$v' = \underline{\sqrt{3}\, v_0}$

(29) $t = 0$ (2), (1), 1 sec later, h

speed of 1st body after 1 sec $= \underline{g}$

Initial Relative velocity $= g$

Separation

$= g(2) + \cancel{\frac{1}{2}(a_{rel})(2)^2}$

$= 2g = \underline{19.6\,m}$

$\boxed{a_{rel} = 0}$

(30) h_1, t_1 ; $h_2\ t_2$

$t_1 = \sqrt{\frac{2h_1}{g}}$

$t_1 + t_2 = \sqrt{\frac{2(h_1 + h_2)}{g}}$

$\frac{t_1 + t_2}{t_1} = \sqrt{\frac{2(h_1 + h_2)}{g}} \times \sqrt{\frac{g}{2h_1}}$

$\Rightarrow 1 + t_2/t_1 = \sqrt{\frac{h_1 + h_2}{h_1}}$

$\Rightarrow 1 + t_2/t_1 = \sqrt{1 + h_2/h_1}$

$t_1/t_2 = \sqrt{h_1/h_2}$

(31) $H/2$, $(10\,m/s)$, $H/2$, u

$$(10)^2 = u^2 + 2(-g)(H/2)$$

$$u^2 = \sqrt{100 + gH} \quad \text{--- (1)}$$

$$H = \frac{u^2}{2g} \Rightarrow u^2 = \sqrt{2gH} \quad \text{--- (2)}$$

$$gH = 100 \Rightarrow H = \underline{10\,m}$$

(32) The speed of particle due to gravity or in motion in effect of gravity does not depends on the mass.

$$\underline{u_1 = u_2 = u_3 = \sqrt{2gh}}$$

(33) Same as Q.(4)

distance in 1 sec $= \frac{1}{2} \times 10 \times 1 + 10 \times 1 = 15\,m$

—"— 2^{nd} sec $= (2 \times 10 + \frac{1}{2} \times 10 \times 2^2) - (15) = 25\,m$

—"— —"— 3^{rd} sec $= (10 \times 3 + \frac{1}{2} \times 10 \times 3^2) - 40 = 35\,m$

ratio $= 25/35 = 5/7$

(34) $h = \frac{1}{2} g t^2$ — (1)

for upwards $\quad -h = ut_1 - \frac{1}{2} g t_1^2$ — (2)

for downwards $\quad h = ut_2 + \frac{1}{2} g t_2^2$ — (3)

(2) + (3) $\quad 0 = u(t_1 + t_2) + \frac{1}{2} g (t_1 + t_2)(t_1 + t_2)$

$t_1 + t_2 = 0 \quad$ or $\quad u + \frac{1}{2} g (t_1 + t_2) = 0$ — (4)

Q(3) - (2) = $2h = u(t_2 - t_1) + \frac{1}{2}g(t_2^2 + t_1^2)$

$gt^2 = u(t_2 - t_1) + \frac{1}{2}g(t_2^2 + t_1^2)$ —(5)

from eq^n (4)

$u = -\frac{1}{2}g(-t_1 + t_2) = \frac{g}{2}(t_2 + t_1)$

Put in eq(5)

$gt^2 = -\frac{g}{2}(t_2^2 + t_1^2 - 2t_1t_2) + \frac{1}{2}g(t_1^2 + t_2^2)$

$\boxed{t = \sqrt{t_1 t_2}}$

(35)

time of flight of balls > 2 (2 sec)

$\frac{2u}{g} > 4$ sec.

$\boxed{u > 2g}$

(36) $T = u/g$ for ascent motion

$s = uT - \frac{1}{2}gT^2$

distance in last t sec.

$s' = (uT - \frac{1}{2}gT^2) - (u(T-t) - \frac{1}{2}g(T-t)^2)$

$= \cancel{uT} - \cancel{\frac{1}{2}gT^2} - \cancel{uT} + ut + \cancel{\frac{1}{2}gT^2} + \frac{1}{2}gt^2 + (-gTt)$

$= ut - \frac{1}{2}gt^2 + gt(-u/g) = \frac{1}{2}gt^2$

$s' = \frac{1}{2}gt^2$

Graphical Questions

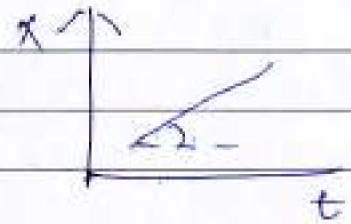
(1) The area under $v-t$ graph = displacement.

$$x = \int v\,dt$$

(2) $x-t$ graph for $\boxed{a = 0}$

$$\frac{d^2x}{dt^2} = 0$$

$$\therefore \frac{dx}{dt} = \text{const.}$$

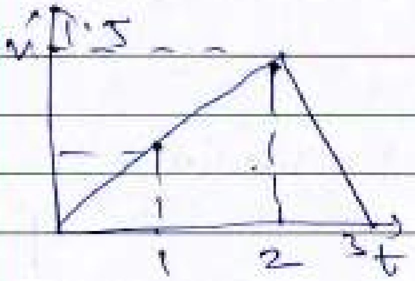


(3) $\theta_A = 30^\circ$ & $\theta_B = 45^\circ$

$v_A = \tan\theta_A$ and $v_B = \tan\theta_B$
$= 1/\sqrt{3}$ $\qquad = 1$

$v_A / v_B = \underline{1/\sqrt{3}}$

(4)



distance = Area under $v-t$ graph.

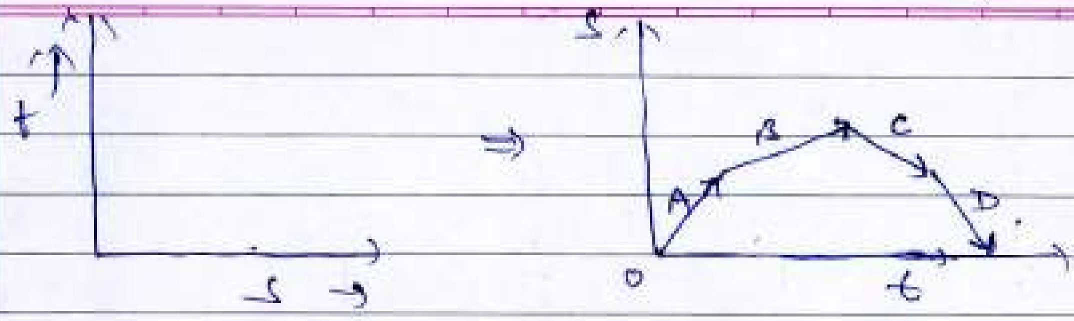
$= \frac{1}{2} \times 3 \times 1.5$

$= 2.25 = 9/4\,m$

(5) average speed

$$= \frac{\text{total distance}}{\text{total time}}$$

(6)



$$v = \frac{ds}{dt}$$

$$\text{avg } v = \frac{\Delta s}{\Delta t} = 0$$

(7)

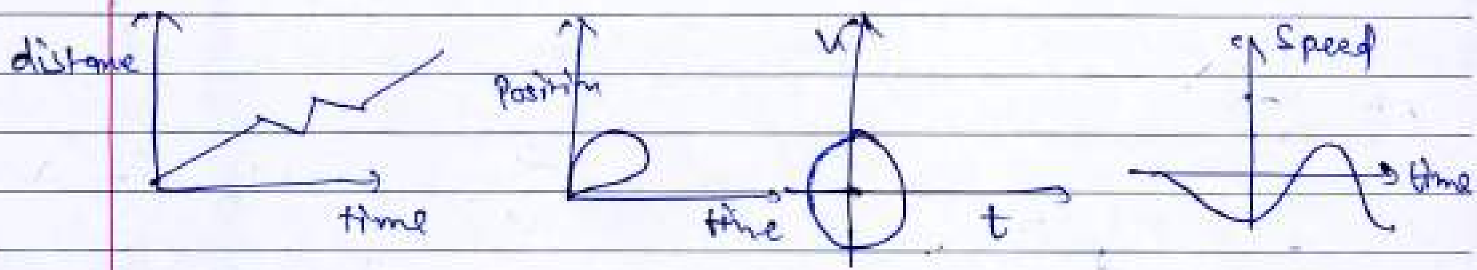
$$v_A = \frac{dx_A}{dt} = \tan\theta_A = \tan 30^\circ = 1/\sqrt{3}$$

$$v_B = \frac{dx_B}{dt} = \tan\theta_B = \tan 60^\circ = \sqrt{3}$$

$$v_A / v_B = 1:3$$

(8)

motion in 1-D



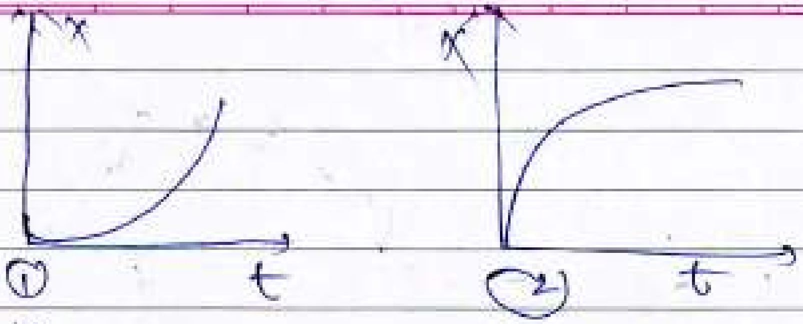
distance will increase w.r.t time always

v-t is a straight line for const acceleration
$\boxed{\text{speed} \geq 0}$ always

(9)

$x = \int v\,dt$ = displacement

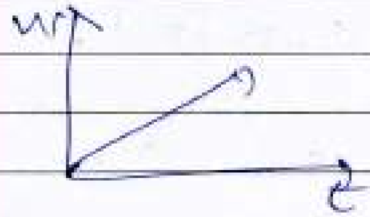
(10)



$\frac{d^2x}{dt^2} > 0$ and $\frac{d^2x}{dt^2} < 0$

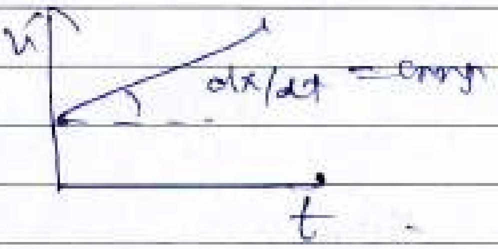
(11) uniform a

$\frac{du}{dt} = a = \text{const.} \neq 0$

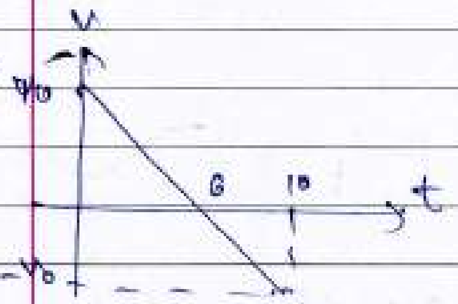


(12) for uniform acceleration

$du/dt = \text{const.}$



(13)



$\frac{du}{dt} = \text{const} = -a$

always

u is zero at t = 6 sec

(14)

total displacement = Area

$= 20\times1 + \frac{1}{2}\times20\times4 + 20\times1$

$= 80\,m$

but when $du/dt = 0$

$= \frac{1}{2}\times20\times4 = 40\,m$

(15) v-t graph for projectile motion

$$\frac{dv}{dt} = -g \quad \text{(always)}$$

$$\boxed{v = -gt + c}$$ straight line

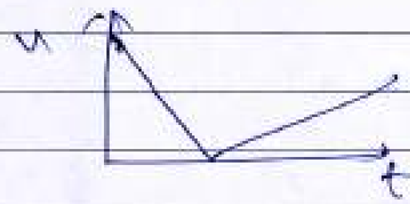
(16) uniform acceleration and instantaneous velocity both are in same direction

$$\frac{dv}{dt} > 0$$

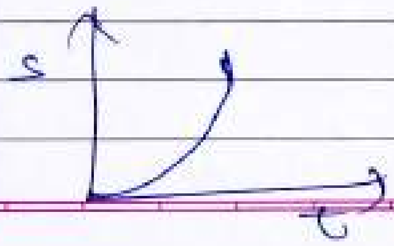
$$ds/dt = v > 0$$

(17) $v = \int a \cdot dt$ = change in velocity.
= Area under a-t graph

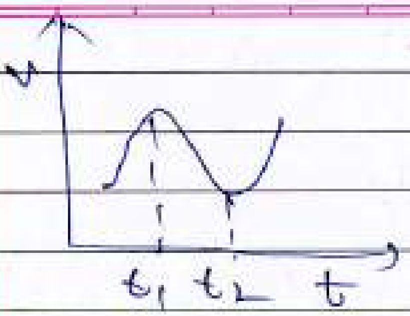
(18) dv/dt = const $\Rightarrow$ $v = mt + c$
and v is zero at highest point



(19) uniform acceleration $\frac{d^2x}{dt^2} > 0$



(20)

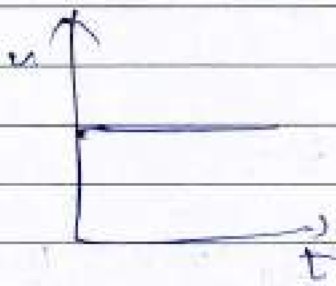


t will increase always

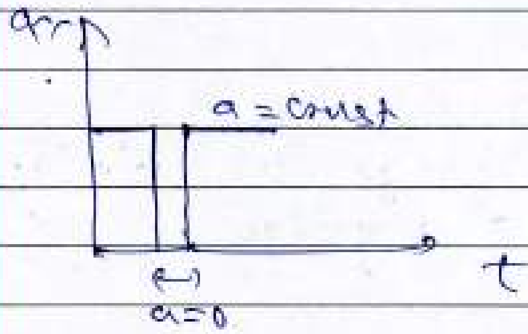
$\boxed{t_2 > t_1}$ always

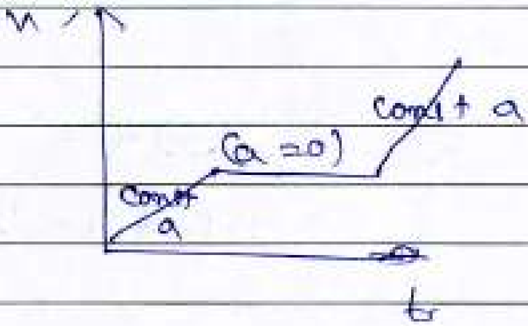
(21) uniform motion $(a = 0)$

$$\frac{dv}{dt} = \cancel{\text{const}} = 0$$



(22)



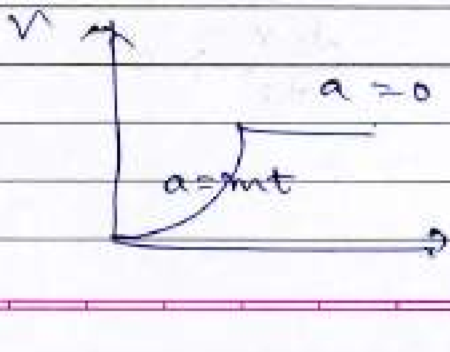


(23)

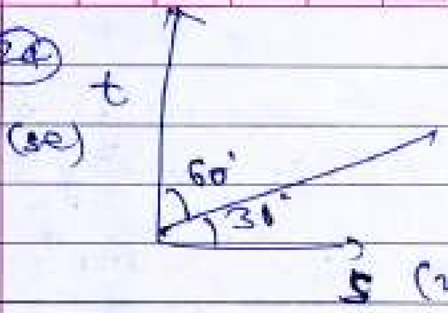
$$a = mt$$

$$\int dv = \int mt\, dt$$

$$v = \underline{\frac{mt^2}{2} + c}$$



(24)

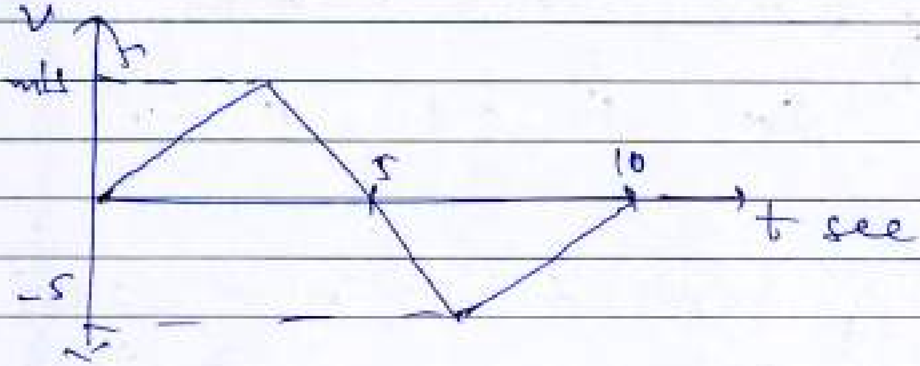


$v = ds/dt = \tan 30°$

$= \sqrt{3}\ m/s$

$\theta \to$ angle with t-axis

(25)



avg velocity = $\dfrac{\text{total displacement}}{\text{total time}}$

total displacement

= Area under v-t graph

$= \frac{1}{2} \times 5 \times 5 - \frac{1}{2} \times 5 \times 5$

$= 0$

$\therefore$ average velocity $= \underline{0}$ Ans

— x —

Uniform Circular Motion

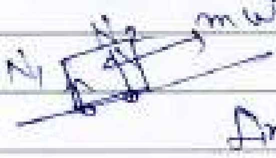
① 

m, R

Acceleration, velocity and Momenta all are vector quantity.

but

$KE = \frac{1}{2}mv^2 = \underline{\frac{1}{2}m\omega^2R^2}$ const.

② 

$m\omega^2R$ during overturns

first $N_1 = 0$ then $\underline{N_2 = 0}$

③ 

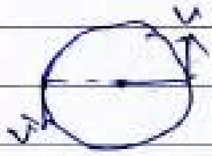
2 rps = f $\therefore \omega = 2\pi f$

$= 4\pi$

$a_c = \omega^2 R = (16\pi^2)(1/4)$

$= 4\pi^2 \ m/s^2$

④ Centripetal force is required force to keep a body in uniform circular motion.

⑤ 

$P_i = mv$ $\quad \Delta P = 2mv$

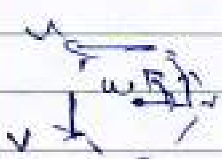
$P_f = -mv$

$KE = \frac{1}{2}mv^2 = $ const.

⑥ uniform circular motion $a_c = 0$

so force (centripetal) and displacement ~~are~~ are perpendicular to each other.

$W = \int \vec{F} \cdot d\vec{s} = 0$

⑦ 

v is tangent.

acceleration (a_c) points to the centre of circle.

(8) For uniform motion

$$a_t = 0 \Rightarrow \frac{dv}{dt} = 0$$

and $a_r \neq 0$

(9) $r_2 = 2r_1$

$$f_{1c} = f_{2c}$$

$$\frac{v_1^2}{R_1} = \frac{v_2^2}{R_2}$$

$$\Rightarrow v_1/v_2 = \sqrt{R_1/R_2} = 1/\sqrt{2}$$

(10) distance travelled in one complete revolution

$$= 2\pi R$$

$$= 200\pi$$

time $= \left(\frac{200\pi}{31.4}\right)$

avg speed $= \left(\frac{200\pi}{200\pi}\right)31.4 = 31.4 \text{ m/s}$

(11) $r = 0.1 \text{ m}$

$m = 1 \text{ kg}$

$f = 3 \text{ r/sec}$

$\omega = 2\pi f = 6\pi$

$$T = \frac{mv^2}{R} = m\omega^2 R$$

$$= 1 \times (6\pi)^2 (0.1)$$

$$= 3.6\pi^2$$

$v = \omega R = (6\pi)(.1)$

$= \underline{0.6\pi}$; $a_c = \omega^2 R = 35.5 \text{ m/s}^2$

(12) $a_c = \frac{v^2}{R} = \frac{(400)(400)}{160} = 1000 \text{ m/s}^2$

$= 1 \text{ km/s}^2$

(13) $\mu \cancel{m} g = \cancel{m} v^2 / R$

$v^2 = (\mu g R) = 0.5 \times 40 \times 10 = 200$ | take $g = 9.8\, m/s^2$

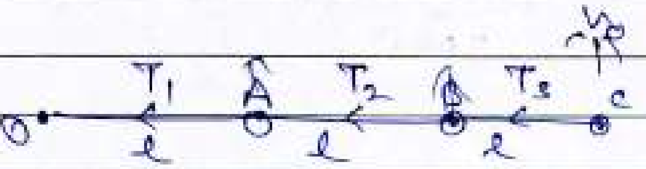
$v = \sqrt{200} \approx 14.14\, m/s$

(14) $\dfrac{f_{c1}}{f_{c2}} = \dfrac{m_1 \omega_1^2 R_1}{m_2 \omega_1^2 R_2}$ | $\omega_1 = \omega_2$ because $T_1 = T_2$

$= \dfrac{\cancel{m_1} R_1}{\cancel{m_2} R_2} = R_1 / R_2$ | $\omega = 2\pi / T$, $m_1 = m_2$

(16) In uniform circular motion momentum will change because momentum is a vector quantity.

(17)



$T_3 = \dfrac{m v_0^2}{3l}$

$T_2 - T_3 = \dfrac{m v_B^2}{2l}$

$T_2 = \dfrac{5}{6}\left(\dfrac{m v_0^2}{l}\right)$

$T_1 - T_2 = \dfrac{m v_A^2}{l}$

$T_1 = \dfrac{m v_0^2}{6l}$

$T_3 = m \omega^2 R_C$

$T_2 - T_3 = m \omega^2 R_B$

$T_1 - T_2 = m \omega^2 R_A$

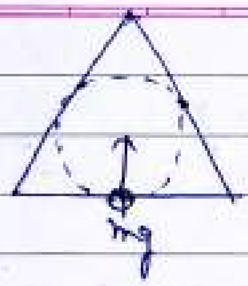
$R_C = 3l$

$R_B = 2l$

$R_A = l$

$\therefore T_1 : T_2 : T_3 = 3 : 5 : 6$

(18)



$mg = \frac{mv^2}{r} = N$

$v^2 = rg$

$\Rightarrow r = v^2/g = \frac{0.5 \times .5}{10}$

$r = 2.5$ cm

(19) angular velocity of ~~p~~ earth

$\omega = 2\pi/T = 2\pi/24\times3600 = \frac{2\pi}{86400}$ rad/sec

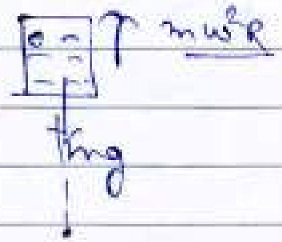
(20) ω of second's hand

$= 2\pi/60 = \pi/30 = 0.1047$ rad/sec

$v = \omega R = \pi/30 \times .03 = 0.00314$ m/s.

NON - Uniform Circular Motion :-

(1) $mg = m\omega^2 R$



$\omega = \sqrt{g/R} = 1.58$

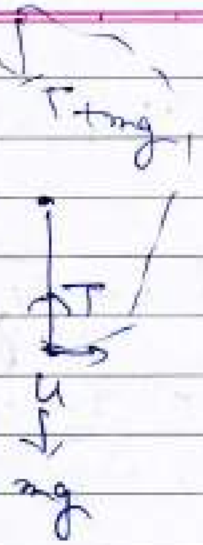
$T = 2\pi/1.58 \approx 4$ sec

(2) $a_t = \frac{dv}{dt} = 2\ m/s^2$

$a_c = v^2/R = \frac{36\times36}{\text{[illegible]}} = 9/5\ m/s^2$

$a = \sqrt{a_t^2 + a_c^2} = \sqrt{4 + (9/5)^2} \approx 2.7\ m/s^2$

(3)



$$T + mg = \frac{mv^2}{l} \quad \text{---(1)}$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = -mg(2l)$$

$$v^2 = u^2 + (-4gl)$$

$$u^2 = v^2 + 4gl \quad \text{---(2)}$$

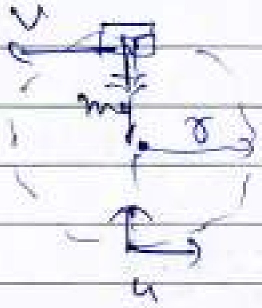
from eq (1)

$T = 0$ for Just slack

$$v^2 = gl$$

$$u = \sqrt{5gl}$$

(4)



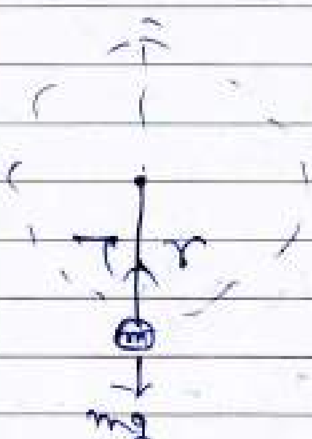
$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = -mg(2r)$$

but at highest Point centripital force will be mg

$$mg = mv^2/r$$

$$v = \sqrt{gr}$$

(5)



$f = n$ rpm

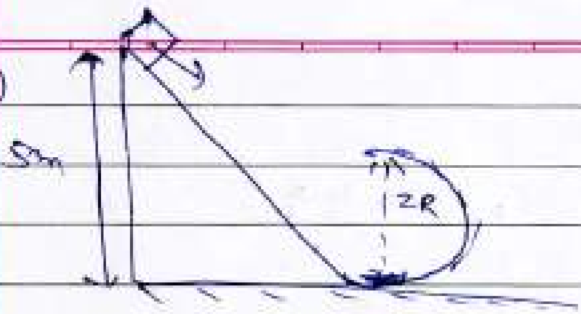
$$\omega = 2\pi f$$

$$= \frac{(2\pi)}{60} n = (2\pi n)/60$$

$$T - mg = m\omega^2 r$$

$$T = mg + m\omega^2 r = m\{g + n^2\pi^2 r/900\}$$

(6)



to complete circle

v_{min} at lower point $\Rightarrow \sqrt{5gr}$

$$u \geq \sqrt{5gr}$$

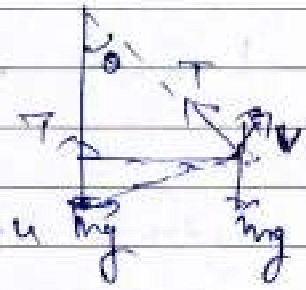
$$\sqrt{2gh} \geq \sqrt{5gr}$$

$$\Rightarrow \sqrt{2 \times g \times 5} \geq \sqrt{5gr}$$

$$\boxed{r = 2m}$$

(7) T_1 at $\theta = 30°$

T_2 at $\theta = 60°$



$$T - mg = \frac{mu^2}{l} \quad \text{—(1)}$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = -mgl(1-\cos\theta)$$

$$\Rightarrow v^2 - u^2 = -2gl(1-\cos\theta) \quad \text{—(2)}$$

$$T\cancel{\cos\theta} = mg\cos\theta \Rightarrow T = mg(\cos\theta) \quad \text{—(3)}$$

from eq (1) & (3)

$$\frac{mg(1+\cos\theta)}{\cancel{\cos\theta}} = \frac{mu^2}{l} \Rightarrow u^2 = \frac{gl(1+\cos\theta)}{\cancel{\cos\theta}}$$

$$\Rightarrow V^2 = u^2 - 2gl(1-\cos\theta)$$

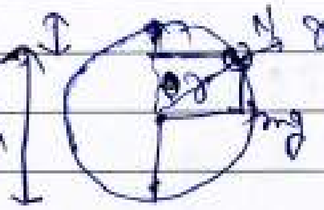
$$= -gl + gl\cos\theta - 2gl + 2gl\cos\theta$$

$$= 3gl\cos\theta - 3gl = 3gl(\cos\theta - 1)$$

$$T = mg\cos\theta$$

$$T_1 = \frac{mg\sqrt{3}}{2} \quad \& \quad T_2 = \frac{mg}{2}$$

$$T_1 > T_2$$

(8)  $r = 21m$ $\quad r(1-\cos\theta)$

$$mg\cos\theta - N = \frac{mv^2}{r} \quad -(1)$$

$$\frac{1}{2}mv^2 = mgr(1-\cos\theta)$$

$N = 0$ when particle will leave the sphere

$$\left.\begin{aligned} mg\cos\theta &= mv^2/r \\ mgr(1-\cos\theta) &= \tfrac{1}{2}mv^2 \end{aligned}\right\}$$

$$2mg(1-\cos\theta) = mg\cos\theta$$

$$\boxed{\cos\theta = 2/3}$$

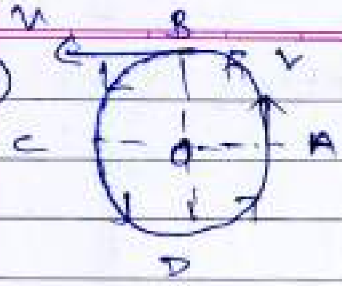
$$h = r + r\cos\theta$$
$$= 21 + 21\times 2/3$$
$$= 35\,m$$

(9) $x = \alpha t^3$ | $y = \beta t^3$

$v_x = 3\alpha t^2$ | $v_y = 3\beta t^2$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

(10) 

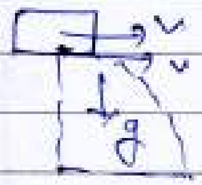
Change in velocity but there is no change in speed.

so $\Delta\vec{v} = v_f - v_i = -v\hat{i} - v\hat{j}$

$|\Delta\vec{v}| = \underline{v\sqrt{2}}$

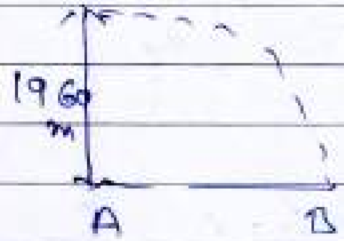
:: <u>Horizontal Projectile Motion</u> :-

(1)



Parabolic Path

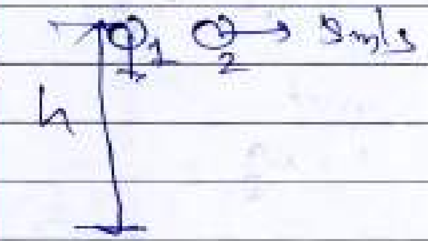
(2)



$AB = \sqrt{\frac{2\times1960}{9.8}} \times 600\times 5/18$

$= \underline{3.33\,Km}$

(3)



$t_1 = t_2 = \sqrt{\frac{2h}{g}}$

(4) if angle b/w a & v is not 0° or 180° then the path of particle will be a parabola.

(5) $x = \sqrt{\frac{2\times 80}{10}} \times 150 = \underline{600\,m}$

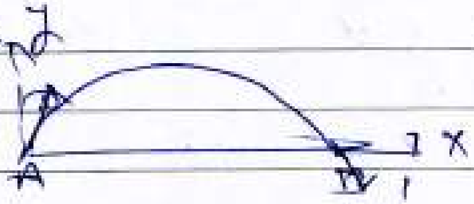
(6) $v_x = 500\,m/s$ $\quad v_y = 10\times10 = 100$

$\tan\theta = \frac{v_y}{v_x} = 1/5$

Oblique Projectile Motion

(1) $\vec{v}_A = 2\hat{i} + 3\hat{j}$

$\vec{v}_B = 2\hat{i} - 3\hat{j}$



$\boxed{v_{Ax} = v_{Bx}}$ $\boxed{v_{Ay} = -v_{By}}$

(2)

$K = \frac{1}{2} m v^2$

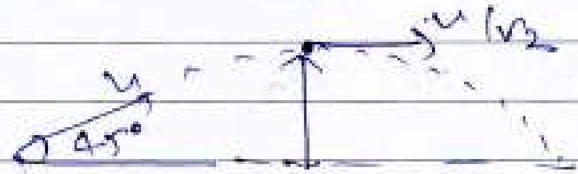
$v^2 = u_x^2 + u_y^2$

$v^2 = 2u_x^2$

at highest Point $K' = \frac{1}{2} m u_x^2$

$K' = \underline{K/2} = E/2$

(3)

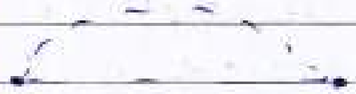


$L = m v_\perp r$

$L = m \frac{v}{\sqrt{2}} H_{max}$

$H_{max} = \frac{v^2 \sin^2 45}{2g} = \frac{m v^3}{4\sqrt{2} g}$

(4)



$T = 2\text{sec} = \frac{2u_y}{g}$

$u_y = g$

$H_{max} = \frac{u_y^2}{2g} = g/2 = \underline{5m}$

(5) In Projectile motion

velocity at $H_{max} = u_x = u\cos\theta$

(6) R will be same for (θ) & $(90-\theta)$

$t_1 = \dfrac{2\sin\theta}{g}$; $t_2 = \dfrac{2\sin(90-\theta)}{g} = \dfrac{2\cos\theta}{g}$

$R = \dfrac{2u\sin\theta\cos\theta}{g}$

$\boxed{t_1 t_2 \propto R}$

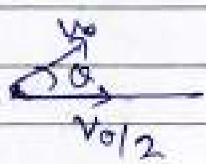
(7) $R_{max} = 100m = \dfrac{u^2}{g} \Rightarrow u^2 = 100g$

$H_{max} = \dfrac{u^2}{2g} = 50m$

(8) Time of flight

$T = \dfrac{2U_{vertical}}{g}$

(9)



~~to~~ to catch the ball

$v_0\cos\theta = v_0/2$

$\cos\theta = 1/2 \quad \therefore \theta = 60^\circ$

(10) $H = \dfrac{u^2\sin^2\theta}{2g}$; $T = \dfrac{2u\sin\theta}{g}$

$\Rightarrow u\sin\theta = \frac{Tg}{2}$

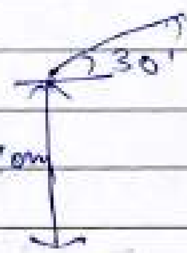
$\sqrt{H} = \frac{u\sin\theta}{\sqrt{2g}} \Rightarrow T = 2\sqrt{\frac{2H}{g}}$

(11) $R = 4\sqrt{3}H$ $\boxed{\frac{H}{R} = \frac{\tan\theta}{4}}$

$\tan\theta = 1/\sqrt{3}$

$\therefore \theta = \underline{30^\circ}$

(12) $u = 50\,m/s$
$\theta = \underline{30^\circ}$



$-70 = (50\sin 30)t - \frac{1}{2}gt^2$

$70 = -25t + 5t^2$

$\Rightarrow t^2 - 5t - 14 = 0 \Rightarrow t = \underline{7\,sec}$

(13) $\theta_1 = 45^\circ$, $H_1 = \frac{u^2\sin^2 45^\circ}{2g} = \frac{u_1^2}{4g}$

$\theta_2 = 60^\circ$, $H_2 = \frac{u^2\sin^2 60^\circ}{2g} = \frac{3u_2^2}{(4g)2}$

$H_1 = H_2 \Rightarrow u_1/u_2 = \sqrt{3}/\sqrt{2} = \sqrt{3/2}$

(14) at highest point a and v are perpendicular to each other.

(19) $R_{max} = 400m$ $H/R = \tan\theta/4$

$\theta = 45$

$H = 2R/4 = 200m$

$R_{max} = \frac{u^2 \sin 90'}{g}$; $H_{max} = \frac{u^2 \sin^2 45}{2g} = \frac{u^2}{4g}$

for $\theta = 45'$

but Here for H_{max} $\theta = 90'$

$H_{max} = \frac{u^2 \sin^2 90}{2g} = \frac{u^2}{2g} = \frac{400}{2} = 200m$