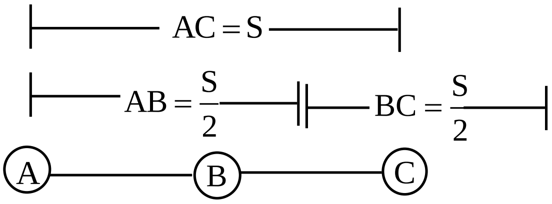
Inchapter Exercise

1.



Given $V_{avg} = 90$

$V_{AB} = 60$

$t_{AB} = \frac{S_{AB}}{V_{AB}}$

$t_{AB} = \frac{S/2}{60} = \frac{S}{120}$

$V_{BC} = ? = V$ let

$t_{BC} = \frac{S_{BC}}{V_{BC}} \rightarrow S/2$

$t_{BC} = \frac{S}{2V}$

$t_{AB} + t_{BC} = t_{AC}$

$\frac{S}{120} + \frac{S}{2V} = t$ ________(1)

Also $V_{avg} = \frac{S}{t} \Rightarrow 90 = \frac{S}{t}$

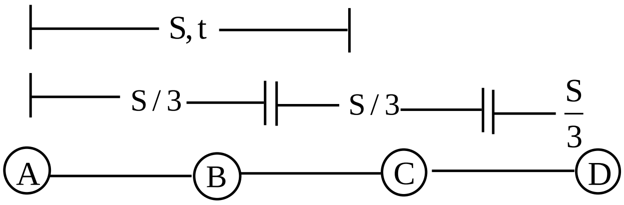
$\Rightarrow t = \frac{S}{90}$ _____(2)

$\Rightarrow t = \frac{S}{90} = \frac{S}{120} + \frac{S}{2V} \Rightarrow \frac{1}{90} = \frac{2V + 120}{240V}$

$\Rightarrow 240V = 180V + 120 \times 90$

$60V = 120 \times 90 \Rightarrow V = 180$

2.



$V_B = u$

$t_{AB} = \frac{S_{AB}}{V_{AB}}$

$$= \frac{S/3}{u}$$

$$t_{AB} = \frac{S}{3u}$$

$$V_{BC} = V$$

$$t_{BC} = \frac{S/3}{V}$$

$$t_{BC} = \frac{S}{3V}$$

$$V_{CD} = w$$

$$t_{CD} = \frac{S/3}{w}$$

$$t_{CD} = \frac{S}{3w}$$

$$t_{AD} = t_{AB} + t_{BC} + t_{CD}$$

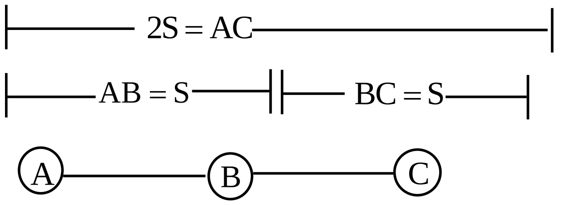
$$\downarrow$$

$$t = \frac{S}{3u} + \frac{S}{3V} + \frac{S}{3w}$$

$$t = \frac{S}{3}\frac{\left[vw + uw + uv\right]}{uvw}$$

$$\text{Avg speed } = \frac{s}{t} = \frac{s}{\frac{s}{3}\left(\frac{vw + uw + uv}{uvw}\right)} = \frac{3uvw}{uv + uw + uv}$$

3.



$$V_{AB} = \frac{30km}{hr}$$

$$t_{AB} = \frac{S}{30}$$

$$V_{BC} = V$$

$$t_{BC} = \frac{S}{V}$$

$$t_{AC} = t_{AB} + t_{BC}$$

$$\downarrow \qquad \downarrow$$

$$10hr = \frac{S}{30} + \frac{S}{V} \qquad \_\_\_\_\_\_\_(1)$$

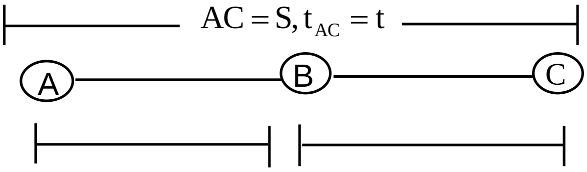
$$V_{avg} = \frac{S_{total}}{t_{total}} \Rightarrow 40 = \frac{2S}{10}$$

$$\left(40km / hr\, given\right) \Rightarrow S = 200 \_\_\_\_\_\_(2)$$

$$\Rightarrow 10 = \frac{200}{30} + \frac{200}{V} \Rightarrow \frac{200}{V} = 10 - \frac{20}{3}$$

$$\Rightarrow \frac{200}{V} = \frac{30-20}{3} \Rightarrow V = \frac{200\times 3}{10} = 60\,\text{km/hr}$$

4. **(a)** half the time is covered with V_1 and other half the time is covered with V_2



$t_{AB} = t/2$
$V_{AB} = V_1$
$S_{AB} = V_{AB}t_{AB}$
$= \frac{V_1 t}{2}$

$t_{BC} = t/2$
$V_{BC} = V_2$
$S_{BC} = V_{BC}t_{BC}$
$= \frac{V_2 t}{2}$

$S_{AC} = S_{AB} + S_{BC}$

$$S = \frac{V_1 t}{2} + \frac{V_2 t}{2} = \left(\frac{V_1+V_2}{2}\right)t$$

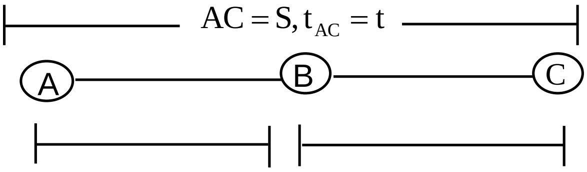
$$\text{Avg. speed} = \frac{S}{t} = \frac{\left(\frac{V_1+V_2}{2}\right)t}{t} = \frac{V_1+V_2}{2}$$

$$t = \frac{S}{2}\left[\frac{V_2+V_1}{V_1V_2}\right]$$

$$\text{Avg.speed} = \frac{S}{t}$$

$$= \frac{S}{\frac{S(V_1+V_2)}{2V_1V_2}} = \frac{2V_1V_2}{V_1+V_2}$$

4. **(b)**

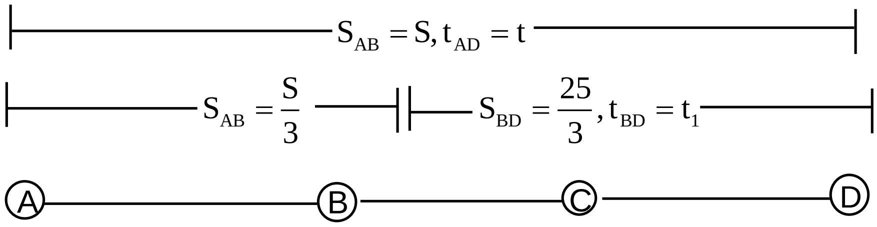


$S_{AB} = S/2$
$V_{AB} = V_1$
$t_{AB} = \frac{S_{AB}}{V_{AB}} = \frac{S/2}{V_1} = \frac{S}{2V_1}$

$S_{BC} = S/2$
$V_{BC} = V_2$
$t_{BC} = \frac{S_{BC}}{V_{BC}} = \frac{S}{2V_2}$

$$t = t_{AC} = t_{AB} + t_{BC} = \frac{S}{2V_1} + \frac{S}{2V_2}$$

4. **(c)**



$$S_{AB} = S/3 \qquad t_{BC} = \frac{t_1}{4} \qquad t_{CD} = \frac{3t_1}{4}$$

$$V_{AB} = V_1 \qquad V_{BC} = V_2 \qquad V_{CD} = V_3$$

$$t_{AB} = \frac{S_{AB}}{V_{AB}} = \frac{S}{3V_1} \qquad S_{BC} = V_{BC}t_{BC} \qquad S_{CD} = V_{CD}t_{CD}$$

$$= \frac{V_2 t_1}{4} \qquad = \frac{3V_3 t_1}{4}$$

Total time $= t = t_{AD} = t_{AB} + t_{BD}$ $\qquad S_{BD} = S_{BC} + S_{CD}$

$$t = \frac{S}{3V_1} + \frac{8S}{3\left(V_2 + 3V_3\right)} \qquad \frac{2S}{3} = \frac{V_2 t_1}{4} + \frac{3V_3 t_1}{4}$$

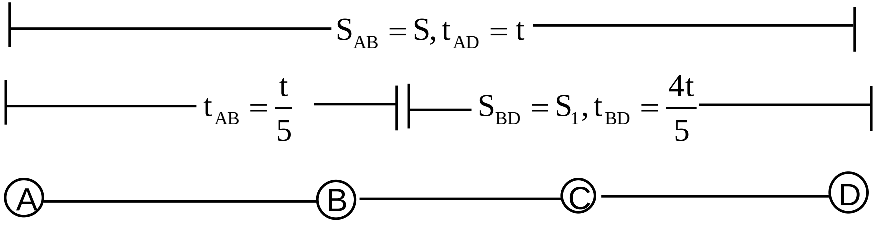
On solving $t_1 = \dfrac{8S}{3\left[V_2 + 3V_3\right]}$

$$t = \frac{S}{3}\left[\frac{\left(V_2 + 3V_3\right) + 8V_1}{V_1\left(V_2 + 3V_3\right)}\right]$$

Avg speed $= \dfrac{S}{t} = \dfrac{S}{\dfrac{S}{3}\left[\dfrac{\left(V_2 + 3V_3\right) + 8V_1}{V_1\left(V_2 + 3V_3\right)}\right]}$

$$= \frac{3V_1\left(V_2 + 3V_3\right)}{V_2 + 3V_3 + 8V_1}$$

4. **(d)**



$$V_{AB} = V_1 \qquad S_{BC} = \frac{3S_1}{4} \qquad S_{CD} = \frac{S_1}{4}$$

$$t_{AB} = t/5 \qquad V_{BC} = V_2 \qquad V_{CD} = V_3$$

$$\Rightarrow S_{AB} = V_{AB}t_{AB} = \frac{V_1 t}{5} \qquad t_{BC} = \frac{S_{BC}}{V_{BC}} = \frac{3S_1}{4V_2} \qquad t_{CD} = \frac{S_{CD}}{V_{CD}} = \frac{S_1}{4V_3}$$

$$S_{Total} = S_{AD} = S_{AB} + S_{BD} \qquad t_{BD} = t_{BC} + t_{CD}$$

$$S = \frac{V_1 t}{5} + \frac{16V_2V_3t}{5\left[3V_3 + V_2\right]} \qquad \frac{4t}{5} = \frac{3S_1}{4V_2} + \frac{S_1}{4V_3}$$

$$S = \frac{t}{5}\frac{\left[V_1\left(3V_3 + V_2\right) + 16V_2V_3\right]}{\left(3V_3 + V_2\right)} \qquad \frac{4t}{5} = \frac{S_1}{4}\left[\frac{3V_3 + V_2}{V_2V_3}\right]$$

$$\Rightarrow S_1 = \frac{16V_2V_3t}{5\left(3V_3 + V_2\right)}$$

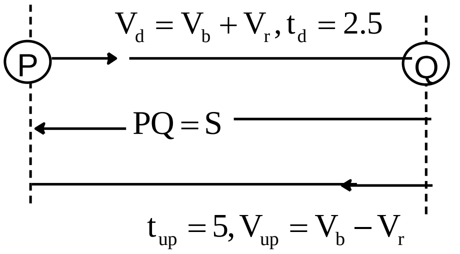
Average speed $= \dfrac{S}{t}$

$$= \frac{\dfrac{t}{5}\dfrac{\left(V_1\left(3V_3 + V_2\right) + 16V_2V_3\right)}{\left(3V_3 + V_2\right)}}{t} = \frac{3V_1V_3 + V_1V_2 + 16V_2V_3}{5\left(3V_3 + V_2\right)}$$

5. Let

$V_{boat} = V_b$ $\qquad V_{downstream} = V_d = V_b + V_r$

$V_{river} = V_r$ $\qquad V_{upstream} = V_{up} = V_b - V_r$



From downstream, $S_d = V_d t_d$

$\Rightarrow S = (V_b + V_r)2.5$ ________(1)

From upstream, $S_{up} = V_{up} t_{up}$

$S = (V_b - V_r)5$ _________(2)

$\Rightarrow S[V_b + V_r]2.5 = (V_b - V_r)5$

$V_b + V_r = 2V_b - 2V_r \Rightarrow V_b = 3V_r$

As, $S = (V_b + V_r)2.5 = (3V_2 + V_r)2.5 \Rightarrow S = 10V_r$ ________(3)

Due to monsoon

$V_r' = 2V_r$

$\Rightarrow V_D' = V_b + V_r' = 3V_r + 2V_r$

$= 5V_r$

From 3

$S_d = V_d' t_d'$

$S = 5V_r t_d \Rightarrow 10V_r = 5V_r t_d'$

$\Rightarrow t_d' = 2$

Last

$V_{up}^i = V_b - V_r' = 3V_r - 2V_r$

$V_{up}' = V_r$

$S_{up} = V_{up}' t_{up}' \Rightarrow S = V_r t_{up}'$

$10V_r = V_r t_{up}'$

$\Rightarrow t_{up}' = 10$

6. $u = 0$

$V = 180\,km/hr = 180 \times \frac{5}{18} = 50m/s$

$t = 25sec$

$a = \frac{V-u}{t} = \frac{50-0}{25} = 2m/s^2$

$S = \frac{V^2 - u^2}{20} = \frac{50^2 - 0^2}{2 \times 2} = 625\,m$

7. $u = 126\frac{km}{hr} = 126 \times \frac{5}{18} = 35\,m/s$

$V = 0, S = 200\,m$

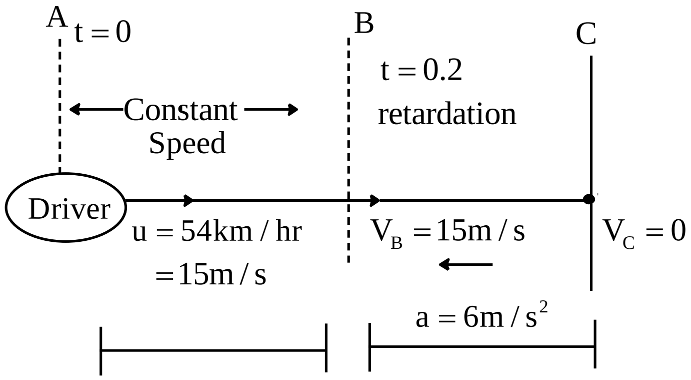
$a = \frac{V^2 - u^2}{2S} = \frac{0^2 - (35)^2}{2 \times 200}$

$a = -\frac{1225}{400} = -3.0625 \leftarrow \text{Acceleration}$

Retardation $= +3.06\,m/s^2$

$t = \frac{V-u}{a} = \frac{0-35}{-3.06}$

8.



$AB = S_{AB} = V_{AB}t$

$= 15 \times 0.2$

$= 3m$

$BC = S_{BC} = \dfrac{V_C^2 - V_A^2}{2}$

$= \dfrac{0^2 - 15^2}{2 \times -6}$

$= \dfrac{225}{12}$

9. $S_{7^{th}} = 20 = u + \frac{a}{2}\left[2(7) - 1\right]$

$20 = u + \frac{13a}{2}$ ________(1)

$S_{9^{th}} = 24 = u + \frac{a}{2}\left[2(9) - 1\right]$

$24 = u + \frac{17a}{2}$ _______(2)

From (1) and (2)

$\Rightarrow a = 2 \Rightarrow u = 7$

$S_{15^{th}} = u + \frac{a}{2}\left[2(15) - 1\right] = 7 + \frac{2}{2}(29) = 36m$

10. $S_{2^{nd}} = 12$ $S_{4^{th}} = 20m$

$S_{2^{nd}} = u + a\left(2 - \frac{1}{2}\right) = 12$

$u + a\left[4 - \frac{1}{2}\right] = 20$

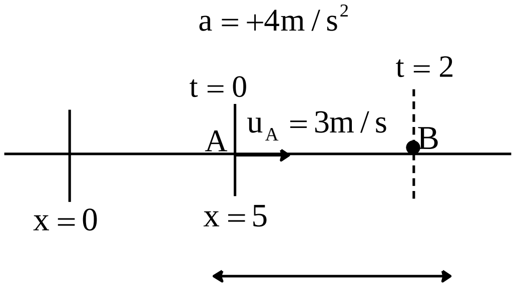
$\therefore a = 4m/s^2,\ u = 6m/s$

Distance covered in 4 sec after 5th second $= S_a - S_5$

$= \left(6 \times 9 + \frac{1}{2} \times 4 \times a^2\right) - \left(6 \times 5 + \frac{1}{2} \times 4 \times 5^2\right)$

$= 216 - 80 = 136m$

11.



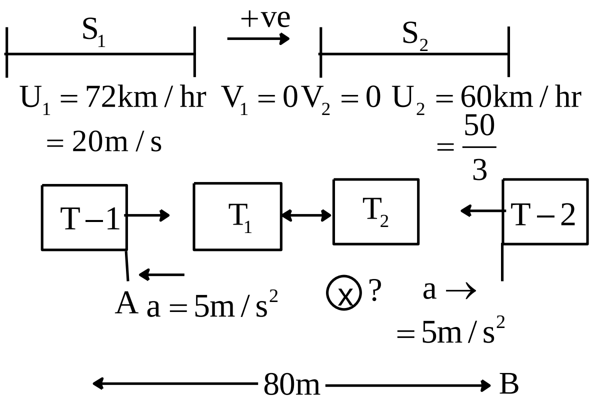
$$S_{AB} = 3(2) + \frac{1}{2}4(2)^2$$

$S_{AB} = 14m$

Position $= 14 + 5 = 19$

$V_{AB} = 3 + 4(2) = 11m/s$

12.



$$|\vec{S}_1| + x + |\vec{S}_2| = 80$$

$$S_1 = \frac{0^2 - (20)^2}{2(-5)} \qquad \Rightarrow 40 + x + \left|\frac{-250}{9}\right| = 80$$

$$= 40m \qquad x = \frac{110}{9}$$

$$S_2 = \frac{0^2 - (50/3)^2}{2(+5)} = \frac{-250}{9}$$

13. (i) $h_{max} = \frac{u^2}{2\rho} = \frac{98^2}{2\times 9.8} = 490m$

(ii) $t_a = \frac{u}{g} = \frac{98}{9.8} = 125^\circ$

(iii) $v^2 = 98^2 - 2\times 9.8\times 196$

$\therefore v = 75.91$

(iv) $v = 98m/s$

(v) $T = \frac{2u}{g} = \frac{2\times 98}{9.8} = 20s$

14. Assume $\uparrow = +ve$

$u = 19.6\ \text{m/s}$

$a = -9.8\ \text{m/s}^2$

$s = -h$

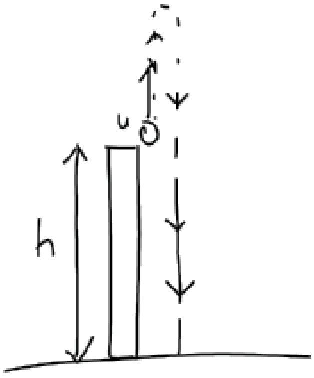
$t = 6\,\text{sec}$

$\Rightarrow\ -h = 19.6\times 6 + \frac{1}{2}(-9.8)\times(6)^2$

$-h = 19.6\times 6 - 9.8\times 18$

$h = 19.6\times 3$

$= 58.8\ \text{m}$



15. $h_{max} = \frac{u^2}{2g}, T = \frac{2u}{g}$ $\quad \therefore T = 4 = \frac{2\times u}{9.8}$

$\therefore u = 19.6 m/s$ $\quad h_{max} = \frac{19.6^2}{2\times 9.8} = 19.6m$

Height of body after 3 S=

$S = 19.6\times 3 - \frac{1}{2}\times 9.8\times 3^2 = 14.7m$

Distance below highest point $= 19.6 - 14.7 = 4.9m$

16. Taking downward positive

Packet is going up with 9.8 m/s at a height of 39.2 m when it was released

$39.2 = -9.8\times t + 4.9t^2$

$\therefore t = 4s \text{ or } t = -2s,$ $\quad \therefore t = 4s$

Also $v = -9.8\times 9.8\times 4 = 29.4 m/s$

17. For $A, S_A = 20t - 4.9t^2$

For B, $S_B = 4.9t^2 + 20t$

$S_A + S_B = 40 = 40t$

$\therefore t = 1s, S_A = 20\times 1 - 4.9\times 1^2 = 151m$

18. $u = 0, g = 9.8 m.s^2$

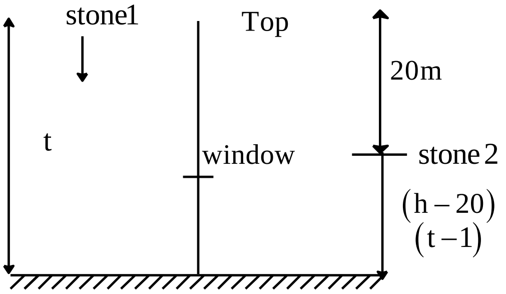
$S_{n^{th}} = u + a\left(n - \frac{1}{2}\right)$

$24.5 = 0 + 9.8\left[n - \frac{1}{2}\right]$

$n = 3$

$S = 0\times 3 + \frac{1}{2}\times 9.8\times 3^2 = 44.1m$

19.



$h = o \times t + \frac{1}{2} \times o \times t^2 \Rightarrow$ for 1st stone

$t = \sqrt{\frac{h}{5}}$

$h - 20 = o \times t + \frac{1}{2} \times o \times \left(t - 1\right)^2$ for 2nd stone

$\therefore h = 20 = 5 \times \left[\sqrt{\frac{h}{5}}^{-1}\right]^2$

$\therefore h = 31.25m$

20. $u = 0, g = 10\,m/s^2,\; s = 45m, t = ?$

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 45}{10}} = 3$

Time taken by sound to reach to of well $= 3.125 - 3 = 0.125s$

Velocity of sound $= \frac{45}{0.125} = 360\,m/s$

21. Safety depends on velocity of reaching ground.

$v = \sqrt{2gh}$

$V_e = \sqrt{2 \times 9.8 \times 2} = \sqrt{9.8 \times 4}$ →Earth

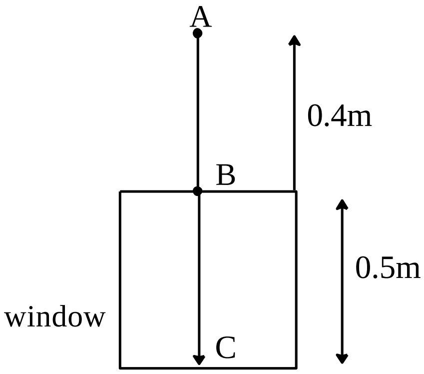
$V_p = \sqrt{2 \times 1.96 \times h}$ →Planet

$V_e = V_p =$ safe velocity

$\therefore \sqrt{9.8 \times 4} = \sqrt{2 \times 1.96 \times h}$

$\therefore h = 10m$

22.



For AB, $0.4 = \frac{1}{2} \times 98 \times t_1^2,\; \therefore t_1 = 0.28s$

For AC, $0.9 = \frac{1}{2} \times 9.8 \times t_2^2 \quad \therefore t_2 = 0.42s$

Time to pass span of window

$= t_2 - t_1 = 0.425 - 0.28s = 0.14s$

23. $y = u_y t + \frac{1}{2} g t^2 = 0 \times 3 + \frac{1}{2} \times 9.8 \times 3^2$

$= 44.1m$

Final vertical velocity $v_y = 0 + 9.8 \times 3 = 29.4 m/s$

Final horizontal velocity $v_x = u$

Resultant velocity makes 45° with horizontal

$t_{on} 45^\circ = \frac{v_y}{v_u}$, $\qquad a = \frac{29.4}{u}$

$\therefore u = 29.4 m/s$

24. $1000 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$

$\therefore t = \sqrt{\frac{1000}{4.9}} = \frac{100}{7} s$

Horizontal velocity of aeroplane

$= 500 km/hr = 500 \times \frac{5}{18} = \frac{1250}{9} m/s$

Distance of missing

Horizontal distance covered when it strikes ground $= \frac{1250}{9} \times \frac{100}{7} = 1984.13m$

25. Horizontal velocity at any instant

$V_x = u = 9.8 m/s$

$V_y = 9.8 = 0 + 9.8t$

$\therefore t = 1s$

26. $\theta = 90 - 30 = 60^\circ$

$u = \frac{u^2 \sin^2 \theta}{2g} = \frac{30^2 \sin^2 60}{2 \times 9.8} = 34.44m$

$T = \frac{2u \sin\theta}{g} = \frac{2 \times 30 \times \sin 60}{9.8} = 5.3s$

$R = \frac{u^2 \sin 2\theta}{g} = \frac{30^2 \sin 120}{9.8} = 79.53m$

27. $R_{max} = \frac{u^2}{g} = 100m$

$u^2 = 100g, u = \sqrt{100g}$

$h_{max} = \frac{u^2}{2g} = \frac{100g}{2g} = 50m$

28. $$H=\frac{u^2\sin^2\theta}{2g}\therefore 25=\frac{40^2\sin^2\theta}{2\times 9.8}$$

$$\sin^2\theta=0.306,\ \therefore \sin\theta=\sqrt{0.306}=0.554$$

$$\cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{0.694}=\sqrt{0.694}=0.833$$

$$R=\frac{u^2\sin^2\theta}{g}=\frac{2u^2\sin\theta\cos\theta}{g}=\frac{2\times 40^2\times 0.554\times 0.833}{9.8}=150.7\text{m}$$

29. $$3000=\frac{u^2\sin 60}{g}$$

$$R_{\max}=\frac{u^2}{g}=\frac{3000\times 2}{\sqrt{3}}=2000\sqrt{3}<5000\text{m}$$

$\therefore 500\text{m}\left(5\text{km}\right)$ target cannot be hit.

30. $$1^2=\frac{u^2\sin^2\theta}{g}=\frac{2u^2\sin\theta\cos\theta}{g}$$

$$4\max=\frac{u^2\sin^2\theta}{2g}$$

Dividing $\frac{H_{\max}}{12}=\frac{\tan\theta}{4}$

$$\therefore \tan\theta=\frac{4u}{12}=\frac{4\times 10}{50}=0.8$$

$$\theta=\tan^{-1}\left(0.8\right)$$

31. Here $H_{\max}=19.6$

$\frac{R}{2}=39.2$ $\therefore R=78.4$

$$\frac{H_{\max}}{R}=\frac{1}{4}\tan\theta$$

$$\tan\theta=\frac{4H_{\max}}{R}=\frac{4\times 19.6}{78.4}=1$$

$$\theta=45^{\circ}\Rightarrow R=\frac{u^2\sin 90^{\circ}}{g}=78.4$$

$$\Rightarrow u=19.6\sqrt{2}\text{m}/\text{s}$$

32. $$H=\frac{(39.2)^2\times\sin^2\theta}{2\times 9.8}$$

$$H+50=\frac{(39.2)^2\times\sin^2}{2\times 9.8}\left(90-\theta\right)=\frac{(39.2)^2\cos^2\theta}{2\times 9.8}$$

Adding $24+50=\frac{(39.2)^2}{2\times 9.8}\left(\sin^2\theta+\cos^2\theta\right)$

$$24+50=\frac{39.2^2}{19.6}=78.4$$

$$\therefore H=14.2\text{m}$$

$$4+50=64.2\text{m}$$

33. $R_1 = \frac{u^2 \sin^2 \theta}{g}, R_2 = \frac{4^2 \sin^2 (90-\theta)}{g}$

$R_2 = \frac{u^2 \sin(180-2\theta)}{g} = \frac{u^2 \sin 2\theta}{g}$

$\therefore R_1 = R_2$

$H_1 = \frac{u^2 \sin^2 \theta}{2g}$ $\quad H_2 = \frac{u^2 \sin^2 (90-\theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$

$H_1 + H_2 = \frac{u^2}{2g}\left(\sin^2 \theta + \cos^2 \theta\right) = \frac{u^2}{2g}$, which is independent of θ.

34. $\theta = 90 - 30 = 60^\circ$

$u_x = u \cos 60 = 19.6$

$\therefore u = \frac{19.6}{\cos 60} = 39.2\text{m/s}$

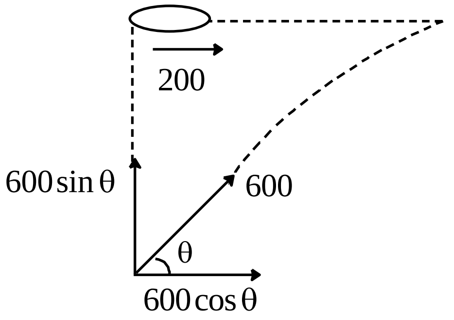
$H_{max} = \frac{u^2 \sin^2 \theta}{2g} = \frac{(39.2)^2}{2 \times 9.8} \times \left(\frac{\sqrt{3}}{2}\right)^2 = 58.8\text{m}$

$R = \frac{u^2 \sin^2 \theta}{g} = \frac{(39.2)^2 \times \sin 120}{9.8}$

$= \frac{(39.2)^2}{9.8} \times \frac{\sqrt{3}}{2} = 135.8\text{m}$

35. Speed of plane $= 720\text{km/hr}$

$= 200\text{m/s}$



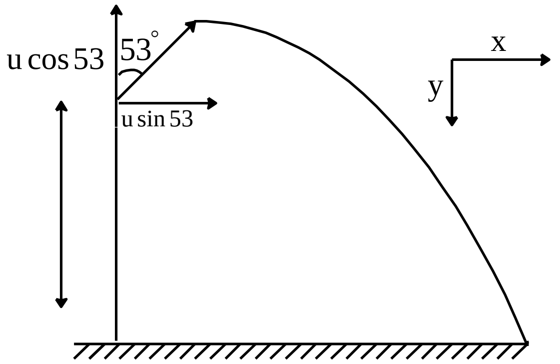
$200 \times t = 600 \cos \theta t$

$\therefore \cos \theta = \frac{1}{3}, \theta = \cos^{-1}\left(\frac{1}{3}\right)$

$h_{max} = \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2\left[1 - \cos^2 \theta\right]}{2 \times 10}$

$= 16000\text{m}$

36.



$800 = -u\cos 53 \times 20 - 5 \times 20^2$

$\therefore u = 100\,m/s$

$$h_{max} = 800 + \frac{100^2 \times \cos^2 53}{2 \times 10} = 980m$$

Horizontal distance travelled by bomb $= u \times t$

$= u\sin 53 \times 20 = 100 \times 0.8 \times 20 = 1600m$

When bomb just strikes ground

$v_y = u\sin 53 = 80m/s$

$v_y = -u\cos 53 + 10 \times 20 = 140$

$v = 80\hat{i} + 140\hat{j}$

37. $Pi = 10^{-3} \times 100\left[20\cos 30\hat{i} + 20\sin 30\hat{j}\right]$

$Pf = 10^{-3} \times 100\left[20\cos 30\hat{i}\right]$

$Pf - pi = 10^{-3} \times 100 \times 20 \times \frac{1}{2}\hat{j} =$

$= 1\hat{j} \Rightarrow 1kgm/s$ magnitude

38. $\overrightarrow{Pf} = m\left[\frac{u}{\sqrt{2}\hat{i}} - \frac{v}{\sqrt{2}\hat{j}}\right]$

$\overrightarrow{Pi} = m\left[\frac{v}{\sqrt{2}}\hat{i} + \frac{v}{\sqrt{2}}\hat{j}\right]$

$\overrightarrow{Pf} - \overrightarrow{Pi} = -mv\sqrt{2}\hat{j} = mv\sqrt{2}$

39. As the projectile returns back to the ground with the same speed so K.E. doesn't change between the point of projection and the point where the body strikes the ground.

40. At highest point, $v_1 = u\cos\theta$ at $y = \frac{h}{2}\max, v_y^2 = u^2\sin^2\theta - 2g\frac{h_{max}}{2}$

$$h_{max} = \frac{u^2\sin^2\theta}{2g}$$

$$\therefore v_y^2 = u^2\sin^2\theta - \frac{2g}{2}\frac{u^2\sin^2\theta}{2g}$$

$$\therefore v_y = \frac{u\sin\theta}{\sqrt{2}}$$

Resultant velocity at $y=\frac{h_{max}}{2}$

$$v_2=\sqrt{{v_x}^2+{v_y}^2}=\sqrt{u^2\cos^2\theta+\frac{u^2\sin^2\theta}{2}}$$

$$\frac{v_1}{v_2}=\sqrt{\frac{2}{5}}=\frac{u\cos\theta}{\sqrt{u^2\cos^2\theta+\frac{u^2\sin^2\theta}{2}}}$$

$$\therefore\frac{2}{5}=\frac{u^2\cos^2\theta}{u^2\cos^2\theta+u^2\sin^2\theta}$$

$\therefore\tan^2\theta=3$ $\qquad\qquad$ $\therefore\tan\theta=\sqrt{3}$

$\theta=60^\circ$

41. $V_{AB}=\vec{V}_\theta-V_B=10-6=4m/s$

$V_{BA}=\vec{V}_B-\vec{V}_A=6-10=-4m/s$

42. $$t=\frac{S_r}{V_r}=\frac{200m}{42\times\frac{5}{18}+\frac{30\times5}{18}}$$

$$=\frac{200m}{20m/s}=10s$$

43. Let x be distance between drives of train A & guard of train B. $u_B-u_A=0$

$\therefore a_r=1,t=50,u_r=0$

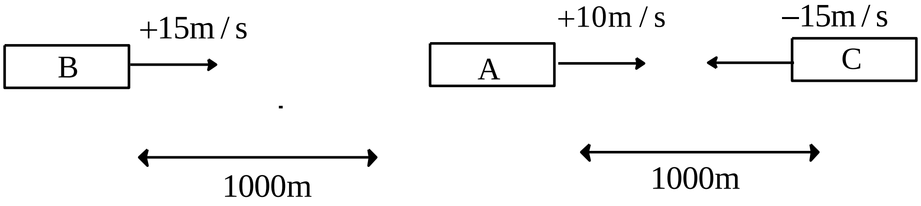
$$x=o\times t+\frac{1}{2}\times1\times{5_o}^2=1250m$$

44. At instant B decides to overtake A,

$V_A=+10m/s$

$V_B=+15m/s$

$V_C=-15m/s$



$V_{CA}=V_C-V_A=-15-10=-25m/s$

Time C require to cross $A=\frac{1000m}{25}\left(\frac{S_r}{V_r}\right)=40s$

In order to avoid accident , B must overtake A in time less than 40 s

$\therefore u_{BA}=4B-4A=15-10=5m/s$

$S_{BA}=100m$

$a_{BA}=a,t=4as$

$$S=ut+\frac{1}{2}at^2$$

$$1000=5\times 40+\frac{1}{2}\times a\times 40^{2}$$

$\therefore a=1m/s^{2}$

45. Let v be full speed of man. He gets into train when his speed become equal to train.

$v=0+at$

$\therefore v=2t$ ______ex 1

Now $v\times t=9+\left(o\times t+\frac{1}{2}\times 2\times t^{2}\right]$ ______ ex 2

$2t^{2}=9+t^{2}$

$\therefore t^{2}=9, t=3s$

$v=3\times 2=6m/s$

$s=6\times 3=18m$

46. $V_{A}=\frac{480}{8}=60\,km/hr$, $V_{B}=\frac{480}{12}=40\,km/hr$

$t=\frac{S_{r}}{V_{r}}=\frac{480}{100}=4.8\,hr$

A will have moved $60\times 4.8=288km$

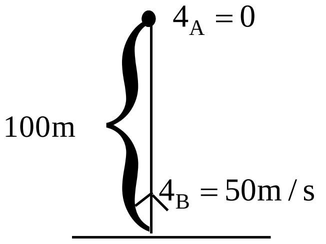
47. $20=\frac{200}{V_{A}-V_{B}},10=\frac{200}{V_{A}+V_{B}}$

$\therefore V_{A}=15m/s, V_{B}=5m/s$

48. (a) $S_{BA}=100, 4_{BA}=50$,

$a_{BA}=0 \therefore S=ut+\frac{1}{2}a+2$

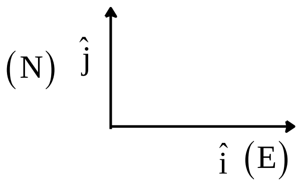
$\therefore 100=50\times t \therefore t=2s$



(b) $S_{BA}=100, 4_{BA}=50-(-25)=75$, $a\,\&\,A=0$

$\therefore 100=75\times t \therefore t=\frac{4}{3}s$

49.



$\vec{V}_{A}=\vec{V}_{B}=\frac{V}{\sqrt{2}}\hat{i}+\frac{V}{\sqrt{2}}\hat{j}(N\varepsilon)$

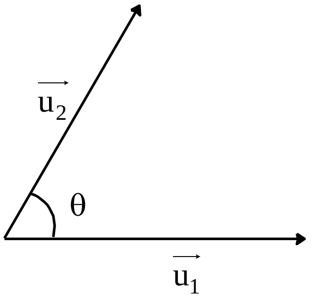
$\vec{V}_B - \vec{V}_C = \frac{-V}{\sqrt{2}}\hat{i} + \frac{V}{\sqrt{2}}\hat{j}\left(N\omega\right)$

$\therefore \vec{V}_A - \vec{V}_C = V\sqrt{2}\hat{j}$

$\therefore \vec{V}_C - \vec{V}_A = -V\sqrt{2}\hat{j}$

$\therefore \vec{V}_{CA}$ is in $-\hat{j}$ i.e. south

50.



$\left|\vec{u}_1\right| = \left|\vec{u}_2\right| = v$

$u_1 = v\hat{i}$ $\qquad$ $u_2 = v\cos\theta\hat{i} + v\sin\theta\hat{j}$

$\overrightarrow{\Delta u} = \vec{u}_2 - \vec{u}_i = \hat{i}\left(v\cos\theta - v\right) + v\sin\theta\hat{j}$

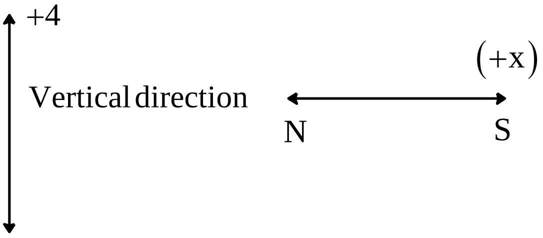
$\left|\overrightarrow{\Delta u}\right| = \sqrt{\left(v\cos\theta - v\right)^2 + v^2\sin^2\theta}$

$= \sqrt{2v^2 - 2v^2\cos\theta}$

$= v\sqrt{2}\sqrt{1-\cos\theta}$

$\left|\Delta\vec{u}\right| = 2v\sin\frac{\theta}{2}$

51.



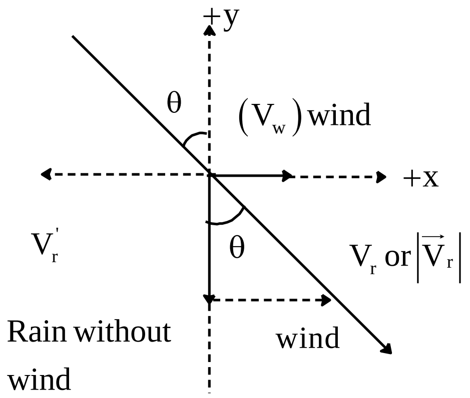
Note-North south or east-west is not vertical direction

Given $\vec{V}_r^{'} = -4\hat{j}, \vec{V}_w = 3\hat{i}$

Last

$\tan\theta = \frac{\left|\vec{V}_{rx}\right|}{\left|\vec{V}_{ry}\right|} = \frac{3}{4}$

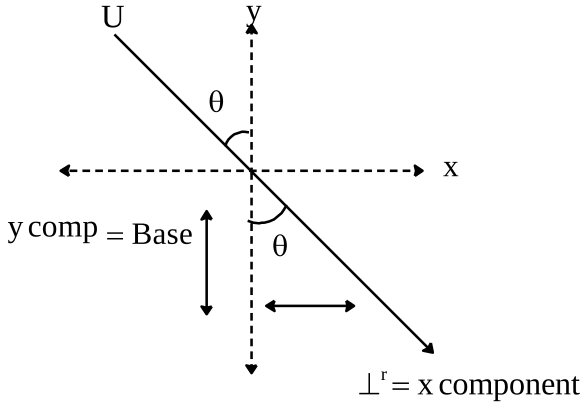
$\Rightarrow \theta = 37^\circ$



Direction of rain ∵ of wind

Using Δ law of vector ⇒ $\vec{V}_r = \vec{V}_w + \vec{V}_r'$

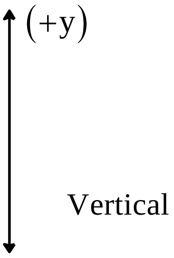
$\vec{V}_r = 3\hat{i} - 4\hat{j}$



4th Quad

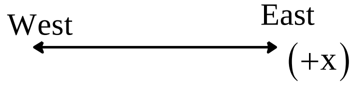
Rain is falling an angle θ w.r.t. vertical

52.



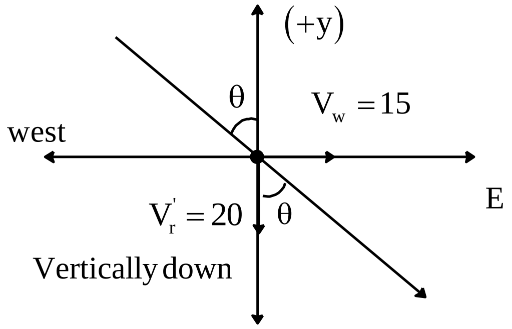
Rain is falling vertically down

$\vec{V}_r' = -20\hat{j}$



Wind and man both are moving west to east with speed 15 m/s and 5 m/s respectively

$\Rightarrow \vec{V}_w = 15\hat{i}$, $\vec{V}_m = 5\hat{i}$



Direction of rain ∵ of wind, speed $= V_r$

$$\Rightarrow \vec{V}_r = \vec{V}_w + \overrightarrow{V_r{}'} = 15\hat{i} - 20\hat{j}$$

Also $\tan\theta = \dfrac{|V_{rx}|}{|\vec{V}_{ry}|} = \dfrac{15}{20} \Rightarrow \theta = 37^\circ$

↑

Rain will fall at an angle θ wrt vertical wrt ground or stationary man

Rain wrt moving man

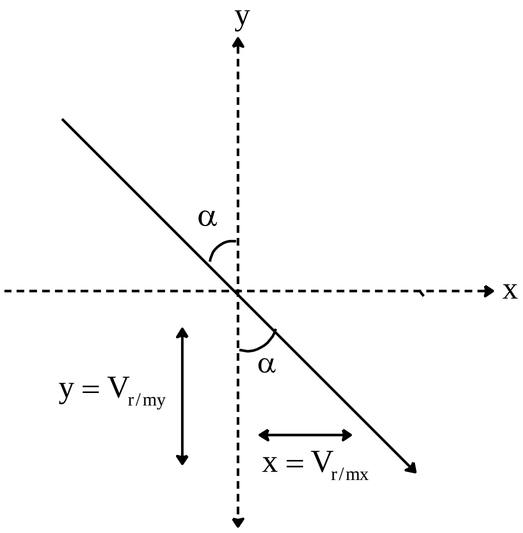
$$\vec{V}_{r/m} = \vec{V}_r - \vec{V}_m = \left(15\hat{i} - 20\hat{j}\right) - \left(5\hat{j}\right)$$

$\vec{V}_{r/m} = 10\hat{i} - 20\hat{j} \leftarrow 4^{th}$ quad

$$\tan\alpha = \frac{\left|\vec{V}_{r/mx}\right|}{\left|\vec{V}_{r/my}\right|}$$

$$\tan\alpha = \frac{10}{20}$$

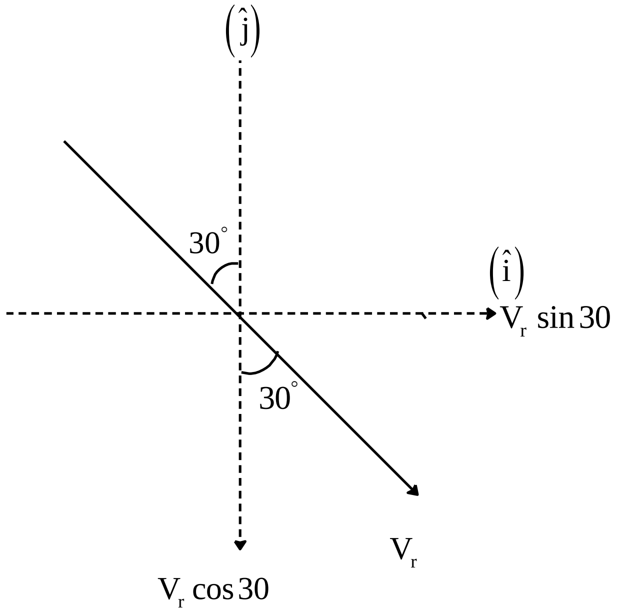
$$\alpha = \tan^{-1}\frac{1}{2}$$



$\vec{V}_{r/m} = 10\hat{i} - 20\hat{j}$

Direction of rain
wrt moving man

53.



$\Rightarrow \vec{V}_r = V_r \sin 30\hat{i} - V_r \cos 30\hat{j}$

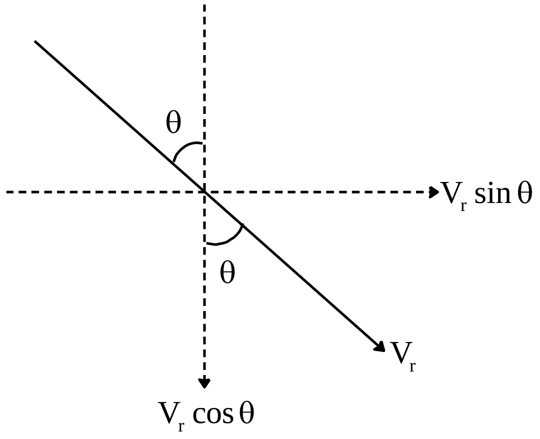
$\vec{V}_m = 10\hat{i}$

$\Rightarrow \vec{V}_{r/m} = \underbrace{(V_r \sin 30 - 10)\hat{i}}_{V_{r/mx}=0} - \underbrace{V_r \cos 30\hat{j}}_{V_{r/my}}$

$\because$ Rain wrt man(moving) falls vertically

$\Rightarrow V_r \sin 30 - 10 \Rightarrow V_r = 20$

54.



$\Rightarrow \vec{V}_r = V_r \sin\theta\hat{i} - V_r \cos\theta\hat{j}$

$\vec{V}_{m_1} = 8\hat{i}$

Hence, $\vec{V}_{r/m_1} = (V_r \sin\theta - 8)\hat{i} - V_r \cos\theta\hat{j}$

$V_{r/m_2x} \quad V_{r/m_2y}$

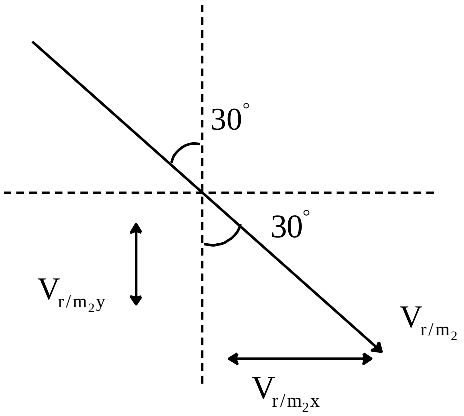
$\vec{V}_{m_2} = 12\hat{i} \Rightarrow \vec{V}_{r/m_2} = [V_r \sin\theta - 12]\hat{i} - V_r \cos\theta\hat{j}$

Given $V_{r/m_1} \Rightarrow$ falls vertically

$\Rightarrow V_{r/m_1x} = V_r \sin\theta - 8 = 0 \Rightarrow V_r \sin\theta = 8$ _______(1)

Given $V_{r/m_2} \Rightarrow$ falls at 30° wrt vertical

$$\Rightarrow \tan 30^\circ = \left|\frac{V_{r/m_2x}}{V_{r/m_2y}}\right|$$



$$\Rightarrow \frac{1}{\sqrt{3}} = \left|\frac{V_r\sin\theta - 12}{V_r\cos\theta}\right| = 8 \qquad \text{From equation (1)}$$

$$\frac{1}{\sqrt{3}} = \left|\frac{-4}{V_r\cos\theta}\right| \Rightarrow V_r\cos\theta = 4\sqrt{3}$$ ______________(2)

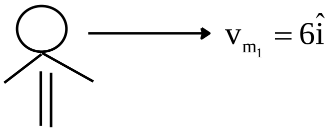
(1) and (2)

$$V_r^2\sin^2\theta + V_r^2\cos^2\theta = \left(8r^2 + \left(4\sqrt{3}\right)\right)^2$$

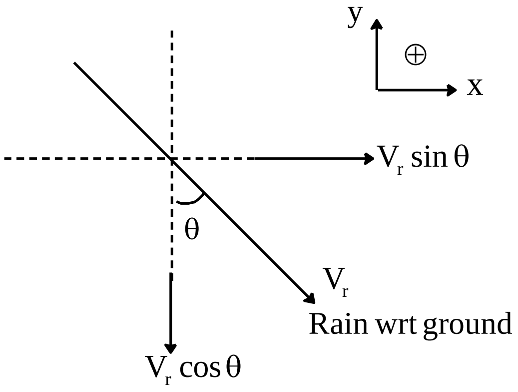
$$V_r^2 = 64 + 48 = 112$$

$$V_r = 4\sqrt{7}$$

55.



Case-I Rain wrt moving m_1 falls vertically



$$\vec{V}_r = V_r\sin\theta\hat{i} - V_r\cos\theta\hat{j}$$

$$\vec{V}m_1 = 6\hat{i}$$

$$\Rightarrow \vec{V}_{r/m_1} = \underbrace{\left(V_r\sin\theta - 6\right)\hat{i}}_{V_{r/m_1x}} - \underbrace{V_r\cos\theta\hat{j}}_{V_{r/m_1y}}$$

It appears to fall vertically

$$\Rightarrow V_{r/m_1x} = 0$$

$$V_r\sin\theta - 6 = 0$$

$$\Rightarrow V_r\sin\theta = 6$$ __________(1)

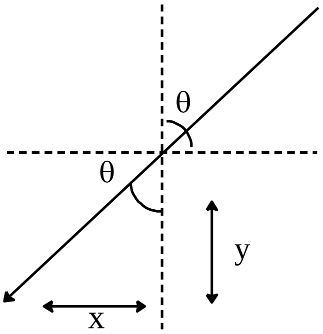
Case-2 Rain wrt moving man

$V_{m_2} = 12\hat{i}$ appears to fall at 30° wrt vertical.

$$\Rightarrow \vec{V}_{r/m_2} = \vec{V}_r - \vec{V}_{m_2}$$

$$= \underbrace{\left(V_r \sin\theta - 12\right)}_{V_{r/m_2x}}\hat{i} - \underbrace{V_r \cos\theta}_{V_{r/m_2y}}\hat{j}$$

$$\Rightarrow \tan\theta = \frac{x}{y} = \frac{\left|V_{r/m_2x}\right|}{\left|V_{r/m_2y}\right|}$$



In our case $\theta = 30^\circ$

$$\Rightarrow \tan 30 = \frac{\left|V_r \sin\theta - 12\right|}{\left|-V_r \cos\theta\right|}$$

$$\downarrow \frac{1}{\sqrt{3}} = \frac{\left|6-12\right|}{V_r \cos\theta}\left(\because V_r \sin\theta = 6\right)$$

$V_r \cos\theta = 6\sqrt{3}$ ____________(2)

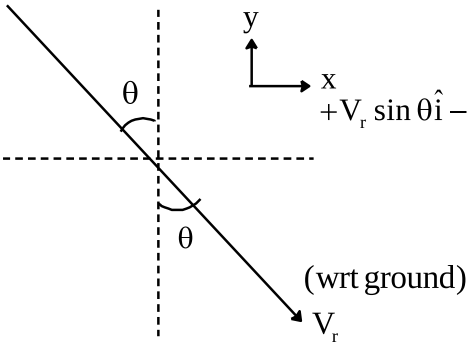
$\Rightarrow$ equation 1 and 2

$$V_r^2 \sin^2\theta + V_r^2 \cos\theta = \left(6\right)^2 + \left(6\sqrt{3}\right)^2$$

$$V_r^2\left(\sin^2\theta + \cos^2\theta\right) = 36 + 108$$

$$V_r^2 = 144 \Rightarrow V_r = 12$$

56.



$+V_r \sin\theta\hat{i} - V_r \cos\theta\hat{j}$

Case-I $\quad V_{m_1} = V\hat{i} \Rightarrow \vec{V}_{r/m_1} = \underbrace{\left[V_r \sin\theta - V\right]}_{V_{r/m_1x}=0} i - V_r \cos\theta\hat{j}$

$V_r \sin\theta - V = 0$

Or $V_r \sin\theta = V$ ________(1)

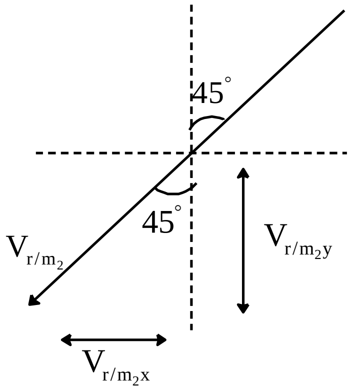
Case-2

$$V_{m_2} = 2V\hat{i} \Rightarrow V_{r/m_2} = \left[V_r \sin\theta - 2V\right]\hat{i} - V_r \cos\theta\hat{j}$$

$\tan 45 = \left|\dfrac{V_r / m_2x}{V_r / m_2y}\right| = V$ ______ from equation (1)

$$I = \left| \frac{V_r \sin\theta - 2V}{V_r \cos\theta} \right|$$

45° with vertical



$$I = \left| \frac{V - 2V}{V_r \cos\theta} \right| \Rightarrow V_r \cos\theta = V \;\underline{\qquad\quad}(2)$$

(1) and (2)

$$V_r^2 \sin^2\theta + V_r^2 \cos^2\theta = V^2 + V^2$$

$$V_r^2 + 2V^2 \Rightarrow V_r = V\sqrt{2}$$

Also $\frac{V_r \sin\theta}{V_r \cos\theta} = \frac{V}{V} \Rightarrow \tan\theta = 1$

$$\Rightarrow \theta = 45^\circ$$

Rain wrt ground

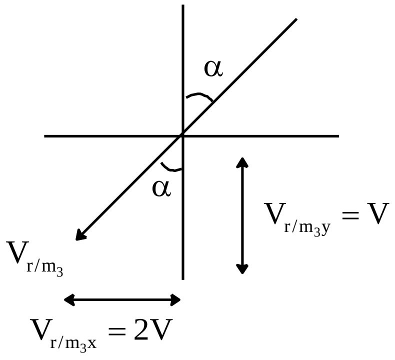
Case-3

$$\vec{V}_{m_3} = 3V\hat{i}, \vec{V}_r = V_r \sin\theta\hat{i} - V_r \cos\theta\hat{j}$$

$$= V\hat{i} - V\hat{j}$$

$$\Rightarrow \vec{V}_{r/m_3} = \underbrace{(V - 3V)\hat{i}}_{(V_{rx} - V_{m3x})\hat{i}} - \underbrace{V\hat{j}}_{(V_{ry} - V_{m3y})\hat{j}} = -2V\hat{i} - V\hat{j}$$

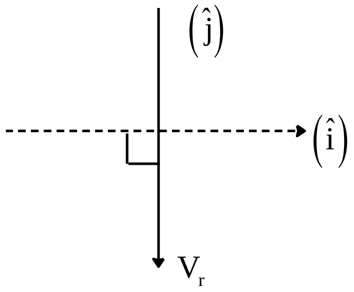
$$\underbrace{V_{r/m_3} = -2V\hat{i} - V\hat{j}}_{3^{rd}\text{ Quad}}$$



$$\tan\alpha = \frac{2V}{V} \Rightarrow \tan\alpha = 2$$

$$\alpha = \tan^{-1}(2)$$

57.



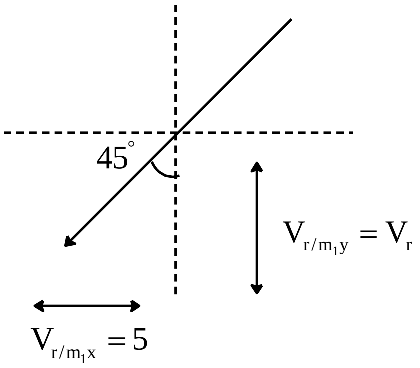
$\vec{V}_r = -V_r\hat{j}$

Case-I

$\vec{V}_{m_1} = 5\hat{i} \Rightarrow \vec{V}_{r/m_1} = \underbrace{-5\hat{i} - V_r\hat{j}}_{3^{rd}}$

$\tan 45 = \dfrac{5}{V_r}$

$V_r = 5$ ____________(1)

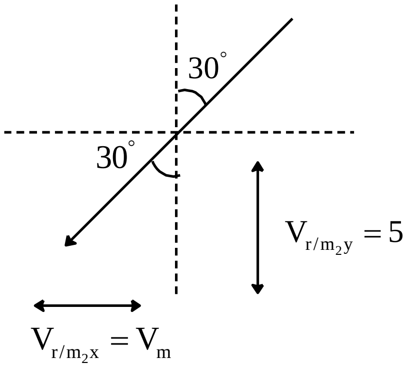


Case-2

$V_{m_2} = V_m\hat{i}$, As $\vec{V}_r = -V_r\hat{j}$

$= -5\hat{j}$

$\Rightarrow V_{r/m_2} = \underbrace{-V_m\hat{i} - 5\hat{j}}_{3rd}$



$\tan 30 = \dfrac{V_m}{5} \Rightarrow V_m = \dfrac{5}{\sqrt{3}}$

JEE-Main Exercise

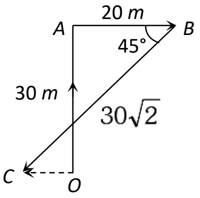
1. (a)

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad \therefore r = \sqrt{x^2 + y^2 + z^2}$$

$$r = \sqrt{6^2 + 8^2 + 10^2} = 10\sqrt{2}\text{ m}$$

2. (c)

From figure, $\overrightarrow{OA} = 0\vec{i} + 30\vec{j}$, $\overrightarrow{AB} = 20\vec{i} + 0\vec{j}$



$$\overrightarrow{BC} = -30\sqrt{2}\cos 45^o\vec{i} - 30\sqrt{2}\sin 45^o\vec{j} = -30\vec{i} - 30\vec{j}$$

$\therefore$ Net displacement, $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} \qquad = -10\vec{i} + 0\vec{j}$

$$|\overrightarrow{OC}| = 10\text{ m.}$$

3. (b)

Total time of motion is 2 *min* 20 *sec* = 140 *sec*.

As time period of circular motion is 40 *sec* so in 140 *sec*. athlete will complete 3.5 revolution *i.e.,* He will be at diametrically opposite point *i.e.,* Displacement = 2*R*.

4. (1204)

An aeroplane flies 400 m north and 300 m south so the net displacement is 100 m towards north.

Then it flies 1200 m upward so $r = \sqrt{(100)^2 + (1200)^2}$

$$= 1204\text{m}$$

The option should be 1204 m, because this value mislead one into thinking that net displacement is in upward direction only.

5. (b)

Distance average speed $= \dfrac{2v_1v_2}{v_1 + v_2} = \dfrac{2 \times 2.5 \times 4}{2.5 + 4}$

$$= \frac{200}{65} = \frac{40}{13}\text{km / hr}$$

6. (d)

Average speed $= \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{x}{t_1 + t_2}$

$$= \frac{x}{\dfrac{x/3}{v_1} + \dfrac{2x/3}{v_2}} = \frac{1}{\dfrac{1}{3 \times 20} + \dfrac{2}{3 \times 60}} = 36\text{ km / hr}$$

7. (c)

Total distance to be covered for crossing the bridge

= length of train + length of bridge

$= 150\text{m} + 850\text{m} = 1000\text{m}$

$$\text{Time } = \frac{\text{Distance}}{\text{Velocity}} = \frac{1000}{45 \times \frac{5}{18}} = 80\text{ sec}$$

8. (c)

Displacement of the particle will be zero because it comes back to its starting point

$$\text{Average speed } = \frac{\text{Total distance}}{\text{Total time}} = \frac{30\text{m}}{10\text{ sec}} = 3\text{ m/s}$$

9. (d)

$$\text{Velocity of particle} = \frac{\text{Total diplacement}}{\text{Total time}}$$

$$= \frac{\text{Diameter of circle}}{5} = \frac{2 \times 10}{5} = 4\text{ m/s}$$

10. (d)

A man walks from his home to market with a speed of 5 km/h.$\text{Distance} = 2.5\text{ km}$ and time $= \frac{d}{v} = \frac{2.5}{5} = \frac{1}{2}\text{hr}$. and he returns back with speed of 7.5 km/h in rest of time of 10 *minutes*.

$$\text{Distance } = 7.5 \times \frac{10}{60} = 1.25\text{ km}$$

$$\text{So, Average speed} = \frac{\text{Total dis tan ce}}{\text{Total time}}$$

$$= \frac{(2.5 + 1.25)\text{km}}{(40/60)\text{hr}} = \frac{45}{8}\text{km/hr}.$$

11. (c)

From given figure, it is clear that the net displacement is zero. So average velocity will be zero.

12. (d)

Length of train = 100 m

$$\text{Velocity of train} = 45\text{ km/hr} = 45 \times \frac{5}{18} = 12.5\text{ m/s}$$

Length of bridge = 1 km = 1000 m

$\therefore$ Total length covered by train = 1100 m

$$\text{Time taken by train to cross the bridge } = \frac{1100}{12.5} = 88\text{ sec}$$

13. (60)

$$\text{Time average speed } = \frac{v_1 + v_2}{2} = \frac{80 + 40}{2} = 60\text{km/hr}.$$

14. (53.33)

Distance travelled by train in first 1 *hour* is 60 *km* and distance in next 1/2 *hour* is 20 *km.*

So Average speed $=\frac{\text{Total distance}}{\text{Total time}}=\frac{60+20}{3/2}$

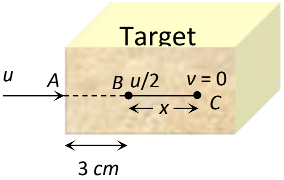
$=53.33\,\text{km}/\text{hour}$

15. (4)

Time average velocity $=\frac{v_1+v_2+v_3}{3}=\frac{3+4+5}{3}=4\text{m}/\text{s}$

16. (1.0)

Let initial velocity of the bullet = *u*



After penetrating 3 *cm* its velocity becomes $\frac{u}{2}$

From $v^2=u^2-2as$

$\left(\frac{u}{2}\right)^2=u^2-2a\,(3)$

$\Rightarrow\ 6a=\frac{3u^2}{4}\ \Rightarrow\ a=\frac{u^2}{8}$

Let further it will penetrate through distance *x* and stops at point *C*.

For distance *BC*, $v=0, u=u/2, s=x, a=u^2/8$

17. (a)

$S=kt^3\ \therefore\ a=\frac{d^2S}{dt^2}=6kt$ *i.e.* $a\propto t$

18. (b)

Let u_1, u_2, u_3 and u_4 be velocities at time $t=0, t_1, (t_1+t_2)$ and $(t_1+t_2+t_3)$ respectively and acceleration is *a* then $v_1=\frac{u_1+u_2}{2}$, $v_2=\frac{u_2+u_3}{2}$ and $v_3=\frac{u_3+u_4}{2}$

Also $u_2=u_1+at_1$, $u_3=u_1+a(t_1+t_2)$

and $u_4=u_1+a(t_1+t_2+t_3)$

By solving, we get $\frac{v_1-v_2}{v_2-v_3}=\frac{(t_1+t_2)}{(t_2+t_3)}$

19. (d)

$v=u+at=10+2\times4=18\,\text{m}/\text{sec}$

20. (c)

If particle starts from rest and moves with constant acceleration then in successive equal interval of time the ratio of distance covered by it will be

$1:3:5:7\ .....(2n-1)$

i.e. ratio of *x* and *y* will be $1:3$ *i.e.* $\frac{x}{y}=\frac{1}{3}\Rightarrow\ y=3x$

21. (a)

$$S_n = u + \frac{a}{2}[2n - 1]$$

$$S_{5^{th}} = 7 + \frac{4}{2}[2 \times 5 - 1] = 7 + 18 = 25m.$$

22. (1)

Distance travelled in 4 *sec*

$$24 = 4u + \frac{1}{2}a \times 16 \qquad \ldots(i)$$

Distance travelled in total 8 *sec*

$$88 = 8u + \frac{1}{2}a \times 64 \qquad \ldots(ii)$$

After solving (i) and (ii), we get $u = 1$ *m/s*.

23. (180)

$S \propto u^2$. If *u* becomes 3 times then *S* will become 9 times *i.e.* $9 \times 20 = 180m$

24. (d)

Total distance $= 130 + 120 = 250\,m$

Relative velocity $= 30 - (-20) = 50\,m/s$

Hence $t = 250/50 = 5s$

25. (b)

Relative velocity of bird *w.r.t* train $= 25 + 5 = 30\,m/s$

time taken by the bird to cross the train $t = \frac{210}{30} = 7\,sec$

26. (d)

Relative velocity

$= 10 + 5 = 15\,m/sec$

$$\therefore \quad t = \frac{150}{15} = 10\,sec$$

27. (a)

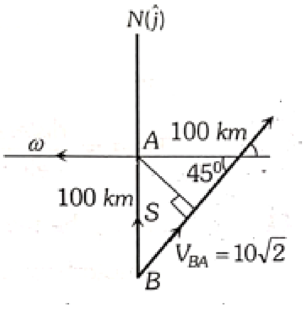
$V_A = 10(-\hat{i})$

$V_B = 10(\hat{j})$

$V_{BA} = 10\hat{j} + 10\hat{i}$

Time for shortest distance

$$= \frac{100/\sqrt{2}}{10\sqrt{2}} = 5.$$



28. (b)

Velocity at the time of striking the floor,

$u=\sqrt{2gh_1}=\sqrt{2\times 9.8\times 10}=14 m/s$

Velocity with which it rebounds.

$v=\sqrt{2gh_2}=\sqrt{2\times 9.8\times 2.5}=7\ m/s$

$\therefore$ Change in velocity $\Delta v=7-(-14)=21 m/s$

$\therefore$ Acceleration $=\frac{\Delta v}{\Delta t}=\frac{21}{0.01}=2100\ m/s^2$ (upwards)

29. (c)

$h=\frac{1}{2}gt^2 \Rightarrow t=\sqrt{2h/g}$

$t_a=\sqrt{\frac{2a}{g}}$ and $t_b=\sqrt{\frac{2b}{g}}\Rightarrow \frac{t_a}{t_b}=\sqrt{\frac{a}{b}}$

30. (a)

Time taken by first stone to reach the water surface from the bridge be *t*, then

$h=ut+\frac{1}{2}gt^2 \Rightarrow 44.1=0\times t+\frac{1}{2}\times 9.8t^2$

$t=\sqrt{\frac{2\times 44.1}{9.8}}=3\,sec$

Second stone is thrown 1 *sec* later and both strikes simultaneously.

This means that the time left for second stone $=3-1=2\,sec$

Hence $44.1=u\times 2+\frac{1}{2}9.8(2)^2$

$\Rightarrow 44.1-19.6=2u\Rightarrow u=12.25\,m/s$

31. (d)

The separation between the two bodies, two seconds after the release of second body

$=\frac{1}{2}\times 9.8[(3)^2-(2)^2]=24.5\,m$

32. (a)

$h=\frac{1}{2}gt^2=\frac{1}{2}\times 10\times(4)^2=80\,m$

33. (c)

Force down the plane $=mg\sin\theta$

$\therefore$ Acceleration down the plane $=g\sin\theta$

Since $l=0+\frac{1}{2}g\sin\theta t^2$

$\therefore\quad t^2=\frac{2l}{g\sin\theta}=\frac{2h}{g\sin^2\theta}\Rightarrow t=\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$

34. (b)

$$v^2 = u^2 + 2gh \Rightarrow (3u)^2 = (-u)^2 + 2gh \Rightarrow h = \frac{4u^2}{g}$$

35. (a)

Height travelled by ball (with balloon) in 2 *sec*

$$h_1 = \frac{1}{2} a\, t^2 = \frac{1}{2} \times 4.9 \times 2^2 = 9.8\, m$$

Velocity of the balloon after 2 sec

$$v = a\, t = 4.9 \times 2 = 9.8\, m/s$$

Now if the ball is released from the balloon then it acquire same velocity in upward direction.

Let it move up to maximum height h_2

$$v^2 = u^2 - 2gh_2 \Rightarrow 0 = (9.8)^2 - 2 \times (9.8) \times h_2 \;\therefore\; h_2 = 4.9m$$

Greatest height above the ground reached by the ball $= h_1 + h_2 = 9.8 + 4.9 = 14.7\, m$

36. (b)

Let h distance is covered in n *sec*

$$\Rightarrow h = \frac{1}{2} gn^2 \qquad \ldots\text{(i)}$$

Distance covered in $n^{th}\, sec = \frac{1}{2} g(2n-1)$

$$\Rightarrow \frac{9h}{25} = \frac{g}{2}(2n-1) \qquad \ldots\text{(ii)}$$

From (i) and (ii), $h = 122.5\, m$

37. (a)

Horizontal velocity of dropped packet $= u$

Vertical velocity $= \sqrt{2gh}$

$\therefore$ Resultant velocity at earth $= \sqrt{u^2 + 2gh}$

38. (a)

$S_n = u + \frac{g}{2}(2n-1)$; when $u = 0$, $S_1 : S_2 : S_3 = 1 : 3 : 5$

39. (b)

It has lesser initial upward velocity.

40. (b)

$$S = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} at^2$$

Hence, $t \propto \sqrt{S}$ *i.e.*, if S becomes one-fourth then t will become half *i.e.*, 2 *sec*

41. (a)

Distance between the balls = Distance travelled by first ball in 3 seconds – Distance travelled by second ball in 2 seconds = $\frac{1}{2}g\,(3)^2 - \frac{1}{2}g\,(2)^2 = 45 - 20 = 25\,m$

42. (a)

When the stone is released from the balloon. Its height $h = \frac{1}{2}at^2 = \frac{1}{2}\times 1.25 \times (8)^2 = 40\,m$ and velocity $v = at = 1.25 \times 8 = 10\,m/s$

Time taken by the stone to reach the ground

$$t = \frac{v}{g}\left[1 + \sqrt{1 + \frac{2gh}{v^2}}\right] = \frac{10}{10}\left[1 + \sqrt{1 + \frac{2\times 10 \times 40}{(10)^2}}\right] = 4\,sec$$

43. (b)

$$h_n = \frac{g}{2}(2n-1) \Rightarrow h_{5^{th}} = \frac{10}{2}(2\times 5 - 1) = 45\,m.$$

44. (6)

Time of flight $= \frac{2u}{g} = \frac{2\times 96}{32} = 6\,sec$

45. (4)

Total distance $= \frac{1}{2}gt^2 = \frac{25}{2}g$

Distance moved in 3 *sec* $= \frac{9}{2}g$

Remaining distance $= \frac{16}{2}g$

If *t* is the time taken by the stone to reach the ground for the remaining distance then

$$\Rightarrow \frac{16}{2}g = \frac{1}{2}gt^2 \Rightarrow t = 4\,\text{sec}$$

46. (30)

If *u* is the initial velocity then distance covered by it in 2 *sec*

$$S = ut + \frac{1}{2}at^2 = u\times 2 + \frac{1}{2}\times 10 \times 4 = 2u + 20 \qquad \ldots\text{(i)}$$

Now distance covered by it in 3rd *sec*

$$S_{3^{rd}} = u + \frac{g}{2}\left(2\times 3 - 1\right)10 = u + 25 \qquad \ldots\text{(ii)}$$

From(i) and (ii), $2u + 20 = u + 25 \Rightarrow u = 5$

$\therefore \quad S = 2\times 5 + 20 = 30\,m$

47. (65)

Speed of stone in a vertically upward direction is 20*m/s*.

So for vertical downward motion we will consider $u = -20\,m/s$

$$v^2 = u^2 + 2gh = (-20)^2 + 2\times 9.8 \times 200 = 4320\,m/s \qquad \therefore \quad v \simeq 65\,m/s.$$

48. (80)

Mass does not affect on maximum height.

$H=\frac{u^2}{2g}\Rightarrow H\propto u^2$,

So if velocity is doubled then height will become four times. *i.e.* $H=20\times 4=80m$

49. (5)

The distance traveled in last second.

$S_{\text{Last}}=u+\frac{g}{2}(2t-1)=\frac{1}{2}\times 9.8(2t-1)=4.9(2t-1)$

and distance traveled in first three second,

$S_{\text{Three}}=0+\frac{1}{2}\times 9.8\times 9=44.1\,m$

According to problem $S_{\text{Last}}=S_{\text{Three}}$

$\Rightarrow 4.9(2t-1)=44.1\Rightarrow 2t-1=9\Rightarrow t=5\,sec.$

50. (200)

$H_{\max}\propto u^2$, It body projected with double velocity then maximum height will become four times *i.e.* 200 *m.*

51. (c)

Due to constant velocity along horizontal and vertical downward force of gravity stone will hit the ground following parabolic path.

52. (c)

Because horizontal velocity is same for coin and the observer.

So relative horizontal displacement will be zero.

53. (c)

Horizontal displacement of the bomb

AB = Horizontal velocity × time available

$AB=u\times\sqrt{\frac{2h}{g}}=600\times\frac{5}{18}\times\sqrt{\frac{2\times 1960}{9.8}}=3.33\,Km.$

54. (b)

$S=u\times\sqrt{\frac{2h}{g}}=100\times\sqrt{\frac{2\times 490}{9.8}}=1000m=1km$

55. (c)

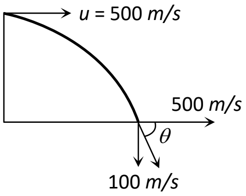
56. (b)

The horizontal distance covered by bomb,

$BC=\upsilon_H\times\sqrt{\frac{2h}{g}}=150\sqrt{\frac{2\times 80}{10}}=660m$

57. (a)

Horizontal component of velocity v_x= 500 *m*/*s* and vertical components of velocity while striking the ground.



$v_y = 0 + 10 \times 10 = 100 m/s$

∴ Angle with which it strikes the ground.

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{100}{500}\right) = \tan^{-1}\left(\frac{1}{5}\right)$$

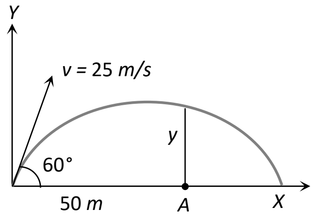
58. (a)

Horizontal component of velocity

$v_x = 25\cos 60° = 12.5 m/s$

Vertical component of velocity

$v_y = 25\sin 60° = 12.5\sqrt{3}\, m/s$



Time to cover 50 *m* distance $t = \frac{50}{12.5} = 4\,\text{sec}$

The vertical height *y* is given by

$$y = v_y t - \frac{1}{2}gt^2 = 12.5\sqrt{3} \times 4 - \frac{1}{2} \times 9.8 \times 16 = 8.2 m$$

59. (a)

For vertical upward motion $h = ut - \frac{1}{2}gt^2$

$$5 = (25\sin\theta) \times 2 - \frac{1}{2} \times 10 \times (2)^2$$

$\Rightarrow$ 25 = 50 sin $\theta \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

60. (d)

Acceleration through out the projectile motion remains constant and equal to *g*.

61. (c)

Time of flight $= \frac{2u\sin\theta}{g} = \frac{2 \times 50 \times \sin 30}{10} = 5s$

62. (c)

Became vertical downward displacement of both (barrel and bullet) will be equal.

63. (b)

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(500)^2 \times \sin 30°}{10} = 12.5 \times 10^3 m$$

64. (a)

$$T=\frac{2u\sin\theta}{g}\Rightarrow u=\frac{T\times g}{2\sin\theta}=\frac{2\times 9.8}{2\times\sin 30}=19.6\,m/s$$

65. (a)

$H=\frac{u^2\sin^2\theta}{2g}$ and $T=\frac{2u\sin\theta}{g}$

So $\frac{H}{T^2}=\frac{u^2\sin^2\theta/2g}{4u^2\sin^2\theta/g^2}=\frac{g}{8}=\frac{5}{4}$

66. (a)

For complementary angles range is same.

67. (d)

$R=4H\cot\theta$, if $R=4\sqrt{3}H$ then $\cot\theta=\sqrt{3}\Rightarrow\theta=30°$

68. (c)

$$H_{\max}=\frac{u^2\sin^2\theta}{2g}$$

According to problem $\frac{u_1{}^2\sin^2 45°}{2g}=\frac{u_2{}^2\sin^2 60°}{2g}$

$$\Rightarrow\frac{u_1{}^2}{u_2{}^2}=\frac{\sin^2 60°}{\sin^2 45°}\Rightarrow\frac{u_1}{u_2}=\frac{\sqrt{3}/2}{1/\sqrt{2}}=\sqrt{\frac{3}{2}}.$$

69. (45)

$R=4H\cot\theta$, if $R=4H$ then $\cot\theta=1\Rightarrow\theta=45°$

70. (200)

$R=\frac{u^2\sin 2\theta}{g}=R\propto u^2$. So if the speed of projection doubled, the range will becomes four times,

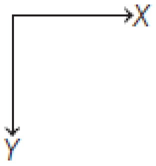
i.e., $4\times 50=200m$

71. (5)

$$\frac{2u\sin\theta}{g}=2\sec\Rightarrow u\sin\theta=10$$

$$\therefore\ H=\frac{u^2\sin^2\theta}{2g}=\frac{100}{2g}=5m$$

72. $10\sqrt{5}$ m/s



$\mathbf{v}_{\text{person}}=10\,\hat{\text{i}}$

$\mathbf{v}_{\text{rain, person}}=20\,\hat{\text{j}}$

$\mathbf{v}_{\text{rain, person}}=\mathbf{v}_{\text{rain}}-\mathbf{v}_{\text{person}}$

$\Rightarrow \quad 20\hat{\mathbf{j}} = \mathbf{v}_{\text{rain}} - 10\,\hat{\mathbf{i}}$

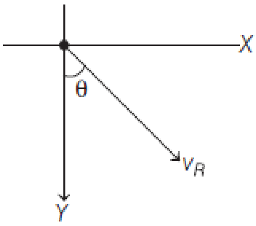
$\Rightarrow \quad \mathbf{v}_{\text{rain}} = 10\hat{\mathbf{i}} + 20\hat{\mathbf{j}}$

$\quad |\mathbf{v}_{\text{rain}}| = 10\sqrt{5}\,\text{m/s}$

73. $2\sqrt{2}$ km/h, 45° with vertically away from the man.

$\mathbf{v}_M = (2\text{ km/h})\,\hat{\mathbf{i}}$

$\mathbf{v}_R = v_R \sin\theta\hat{\mathbf{i}} + v_R\cos\theta\hat{\mathbf{j}}$



$\mathbf{v}_{RM} = \mathbf{v}_R - \mathbf{v}_M$

$\quad = (v_R\sin\theta - 2)\,\hat{\mathbf{i}} + v_R\cos\theta\,\hat{\mathbf{j}}$

Since, rail appears to fall vertically,

$v_R\sin\theta - 2 = 0$

$\Rightarrow \quad \mathbf{v}_R\sin\theta = 2$

$\mathbf{v}_M = 4\hat{\mathbf{i}}$

$\mathbf{v}_R = 2\hat{\mathbf{i}} + v_R\cos\theta\,\hat{\mathbf{j}}$

$\mathbf{v}_{RM} = -2\,\hat{\mathbf{i}} + v_R\cos\theta\,\hat{\mathbf{j}}$

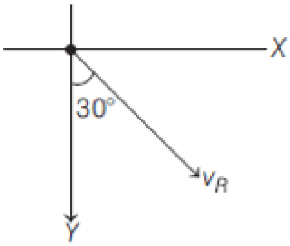
$$\tan\phi = \frac{v_R\cos\theta}{2} = 1$$

$\Rightarrow \quad v_R\cos\theta = 2$

$\mathbf{v}_R = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

$\quad = 2\sqrt{2}$ km/h, at 45° with vertical

74. 20 km/h



$\mathbf{v}_R = v_R\sin 30°\hat{\mathbf{i}} + v_R\cos 30°\hat{\mathbf{j}}$

$\mathbf{v}_M = 10\,\hat{\mathbf{i}}$

$\mathbf{v}_{RM} = \mathbf{v}_R - \mathbf{v}_M$

$\quad = (\mathbf{v}_R\sin 30° - 10)\,\hat{\mathbf{i}} + \mathbf{v}_R\cos 30°$

Since, rain appears to fall vertically

$v_R\sin 30° - 10 = 0$

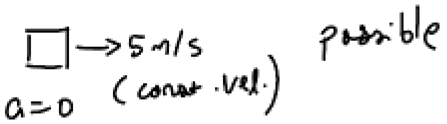
$\Rightarrow \quad v_R = 20\,\text{km/h}$

JEE-Advanced Exercise

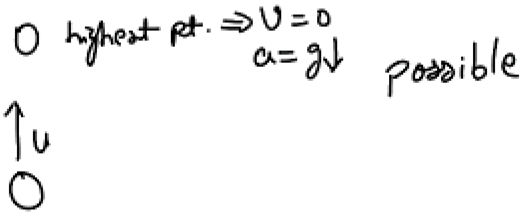
EXERCISE - 1

1. (C)

(a) Eg:



(b)

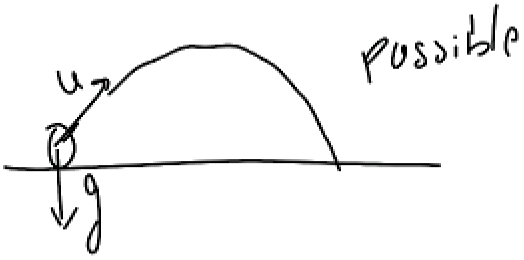


(c) speed is magnitude of velocity

$\therefore$ speed = 0 $\Rightarrow \vec{v} = \vec{0}$

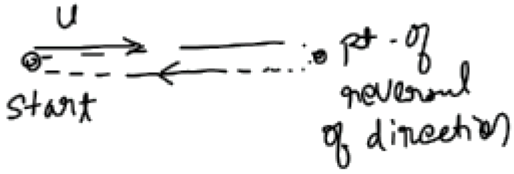
So, (c) is incorrect.

(d) Eg:



2 (C)

$\because$ direction reverses, speed must be zero at some in start.



3 (D)

(a) avg. velocity $= \dfrac{\text{total disp.}}{\text{total time}}$

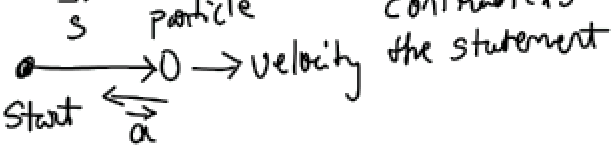
If average velocity = 0

$\Rightarrow$ total displacement = 0

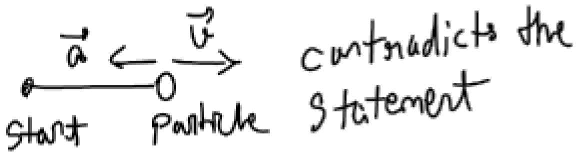
$\Rightarrow$ particle is a starting point

$\Rightarrow$ $\upsilon = 0$ at point of direction reversal

(b) Eg.



(c) Eg.



So correct answer is (D).

4. (C)

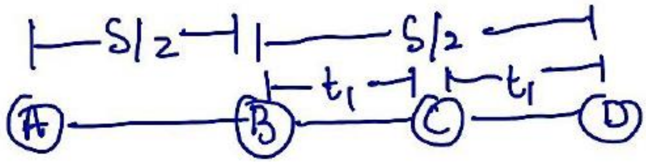
(a) average speed = 0 $\Rightarrow$ distance = 0
$\Rightarrow$ particle is at rest in that time interval
Average velocity = 0 $\Rightarrow$ $S=0$

(b) $\Rightarrow$ speed = 0 at inward of direction reversal

(c) $|\vec{v}|=\text{speed}$ (correct)

(d) $|\text{avg. velocity}|=\dfrac{|\vec{S}|}{\text{time}}$, average speed = $\dfrac{\text{distance}}{\text{time}}$
Both are different.

5. (D)



$V_{AB}=3m/s \quad V_{BC}=4.8m/s \quad V_{CD}=7.5$

$t_{AB}=\dfrac{S_{AB}}{V_{AB}} \qquad \underbrace{S_{BC}=4.5t_1 \qquad S_{CD}=7.5t}$

$\Rightarrow t_{AB}=\dfrac{S/2}{3} \qquad S_{BC}+S_{CD}=S_{BD}$

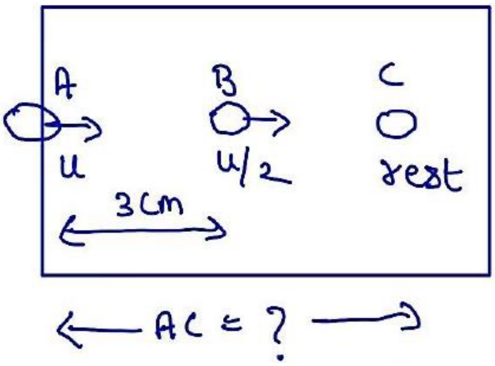
$4.5t_1+7.5t_1=\dfrac{5}{2}$

$\underbrace{t_{AB}=\dfrac{S}{6} \qquad 12t_1=\dfrac{S}{2}\Rightarrow t_1=\dfrac{S}{24}}$

$$Vavg=\frac{S}{\underset{total\,t_{AB}+2t_1}{\cancel{t}}}=\frac{5}{\frac{\cancel{5}}{6}+2\left(\frac{\cancel{5}}{\underset{12}{\cancel{24}}}\right)}$$

= 4m/s

6. (C)



For motion AB

$$V_B^2 - U_A^2 = 2aS_{AB}$$

$$\left(\frac{u}{2}\right)^2 - (u)^2 = 2a(3)$$

$$-\frac{3u^2}{4} = 6a$$

$$\Rightarrow a = -\frac{u^2}{8}$$

For Motion AC

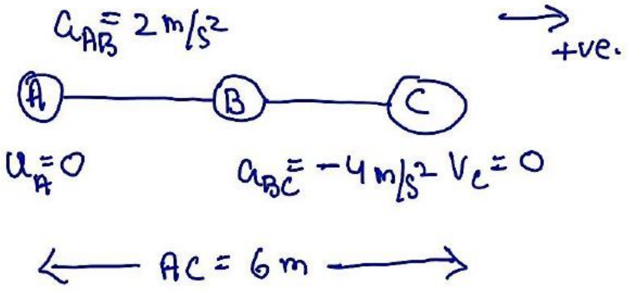
$$\underset{\downarrow}{V_C^2} - U_A^2 = 2\underset{\downarrow}{Q}S_{AC}$$

$$O^2 - u^2 = 2\left[-\frac{u^2}{8}\right]S$$

$$-u^2 = -\frac{u^2}{4}S_{AC}$$

$$S_{AC} = 4cm$$

7. (C)



$$S_{AB} = \frac{v_B^2 - U_A^2}{2Q_{AB}} = \frac{V_B^2 - O^2}{2\times 2} = \frac{V_B^2}{4}$$

$$S_{BC} = \frac{V_C^2 - V_B^2}{2a_{BC}} = \frac{O2 - V_B^2}{2[-4)} = \frac{V_B^2}{8}$$

Also $S_{AB} + S_{BC} = 6 \Rightarrow \frac{V^2}{4} + \frac{V^2}{8} = 6$

$$\Rightarrow V_B^2\frac{(2+1)}{8} = 6 \Rightarrow V_B = 4m/s$$

$$t_{AB} = \frac{V_B - u_A}{a_{AB}} = \frac{4-0}{2} = 2,\ t_{BC} = \frac{V_C - U_B}{a_{BC}} = \frac{0-4}{-4} = 1$$

Ans. = 2 + 1 = 2sec

8. (C)

Rigid Oby $\boxed{C \quad B \quad A} \rightarrow V$ Velo. Of all pts on rigid obj. will be equal at an instance

$\leftarrow L \rightarrow (Pole)$

$\boxed{R \quad M \quad F} \rightarrow U_F = X = U_m = U_R$

$\xleftrightarrow{L/2} \boxed{R \quad M \quad F} \rightarrow V_R = y = V_F = V_m$

$\boxed{R \quad m \quad F} \rightarrow V_m = ?$

*For Front end

$$\left.\begin{aligned} U_F &= X \\ V_F &= y \\ S_F &= L \end{aligned}\right\}$$

$$\Rightarrow a = \frac{V_F^2 - U_F^2}{2SF}$$

$$= \frac{y^2 - x^2}{2L}$$

* For middle end

$$u_m = x, V_m = ?, S_m = \frac{L}{2}$$

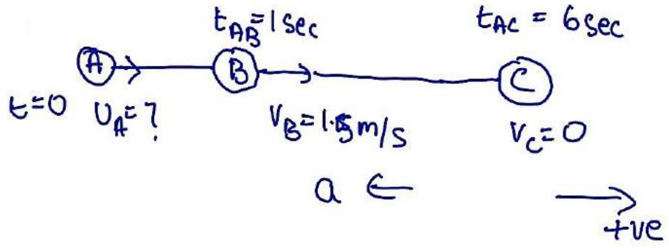
$$a_m = a_{train} = \frac{y^2 - x^2}{2L}$$

As $v_m^2 - u_m^2 = 2amSm$

$$V_m^2 - X^2 = 2\left(\frac{y^2 - x^2}{2L}\right) \times \frac{L}{2}$$

$$V_m = \sqrt{\frac{x^2 + y^2}{2}}$$

9. (C)



*BC motion

$V_C = 0, U_B = 1.5, t_{BC} = 5\text{sec}$

$$aBC = \frac{V_C - U_B}{t_{BC}} = \frac{0 - 1.5}{5} = -0.3m/s^2$$

* AB motion

$V_B = U_A + a t_{AB} \Rightarrow 1.5 = U_A + (0.3)(1)$

$\Rightarrow U_A = 1.8m/s$

*AC motion

$$SAC = \frac{V_C^2 - U_A^2}{2a} = \frac{O^2 - (1.8)^2}{2(-0.3)} = \frac{-1.8 \times 1.8}{-0.6}$$

Ans.: $S_{AC} = 5.4m = \text{distance}$

10. (C)

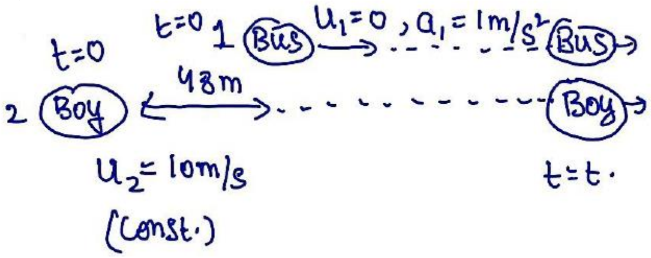
$\vec{v} = \vec{u} + \vec{a}t$

$\vec{V} = (i - 2j + 10k) + (i + j - 2k) \times 2$

$\vec{V} = 3i - 2j + 6k$

$\vec{V} = |\vec{V}| = \frac{3i - 2j + 6k}{\sqrt{49}} = \frac{1}{7}(3\hat{i} - 2\hat{j} + 6\hat{k})$

11. (A)



$S_2 = 4 + S_1$

$10t = 48 + 0(t) + \frac{1}{2}(1)(t)^2$

$\Rightarrow \frac{t^2}{2} - 10t + 48 = 0 \Rightarrow t^2 - 20t + 96 = 0$

$t^2 - 12t - 8t + 96 = 0 \Rightarrow (t-12)(t-8) = 0$

$\Rightarrow \boxed{t = 8}\ or\ \boxed{t = 12}$ Ans

We are getting two times 8 and 12 and both are correct but ans is 8 sec. As in 8 sec boy crosses bus and at this instance $V_b = 8m/s < V_{boy} = 10m$. Hence boy moves ahead but as bus is accelerating hence it will again cross the boy as 12 sec.

12. (B)

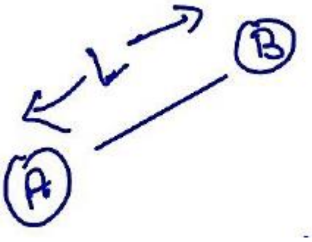
Case 1 Escalator moving

$V_e = 1$

$t_{AB} = \frac{L}{V_e} \Rightarrow l = \frac{L}{V_e}$

$\Rightarrow V_e = L \qquad \ldots\ldots(1)$

Case -2 Man moving

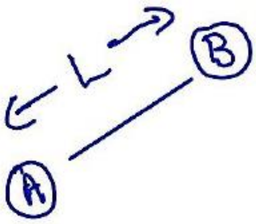


$V_{man} = V_m$

$t_{AB} = \frac{L}{V_m} = 3$

$\Rightarrow V_m = L/3 \qquad (2)$

Case – 3 Both moving



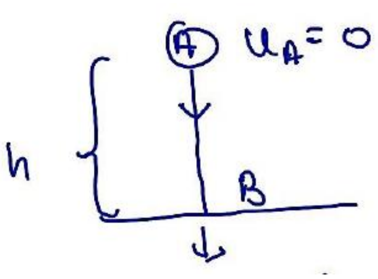
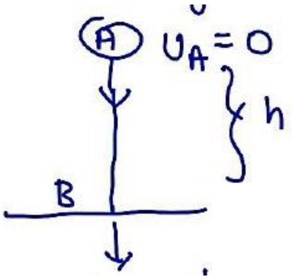
$$V' = V_m + V_E$$

$$V' = L + \frac{L}{3} = \frac{4L}{3}$$

$$tAB = \frac{L}{V'} = \frac{L}{\frac{4}{3}L} = \frac{3}{4}$$

$$t_{AB} = \frac{3}{4} \times 60 = 45\,\text{sec}$$

13. (B)

Planet – 1	Planet -2
$a_1 = 8g \downarrow$	$a_2 = 2g \downarrow$
	
$V_1 = V + V'$	$V_2 = V'$
$t_1 = t' + t$	$t_2 = t'$

$$h = \frac{V_1^2 - U_1^2}{2a_1} = \frac{V_2^2 - U_2^2}{2a_2}$$

$$\frac{(V+V')^2 - (0)^2}{4} = \frac{(V')^2 - 0^2}{2(2g)}$$

$$\Rightarrow \frac{V+V'}{2} = V' \Rightarrow V' = V$$

For 2 $V_2 = U_2 + a_2 t \Rightarrow v'(=V) = 0 + 2gt'$

$\Rightarrow V = 2gt' \Rightarrow t'V/2g$

For (i)

$$V_1 = u_1 + a_1 t_1$$

$$V' + V = 0 + 8g(t' - t)$$

$$V + V = 8g\left(\frac{V}{2g} - t\right)$$

$$\cancel{2}V = 4\cancel{8}g\left(\frac{V}{2g} - t\right)$$

$$\frac{V}{4g} = \frac{V}{2g} - t \Rightarrow t = \frac{V}{2g} - \frac{V}{4g}$$

$$\Rightarrow t = \frac{V}{4g} \Rightarrow V = 4gt$$

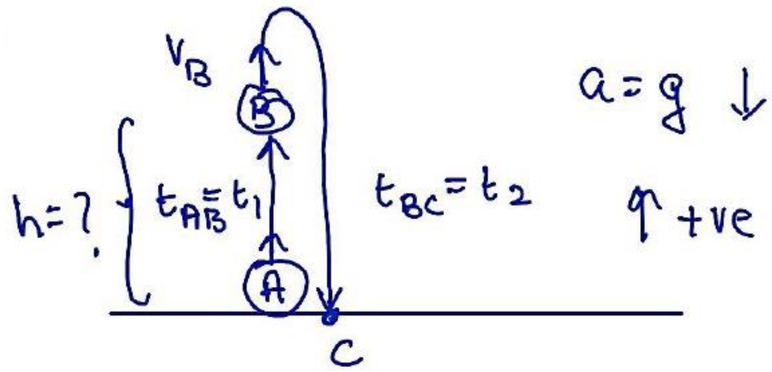
14 (A)

$$S_1 = 5\times4+\frac{1}{2}\times10\times16=100$$

$$S_2 = v\times2+\frac{1}{2}\times10\times4=2v+20$$

$S_1 = S_2$ $\therefore V = 40m/s$

15. (D)



Motion AB

$$\underset{\downarrow}{S} = Vt-\frac{1}{2}at^2$$

$(th) = V_Bt_1+\frac{1}{2}(-g)t_1^2 \Rightarrow h = V_Bt_1+\frac{gt_1^2}{2}$ ....(i)

*Motion BC $\Rightarrow S = ut+\frac{1}{2}at^2 - h = V_Bt_2-\frac{gt_2^2}{2}$ ....(ii)

(i) + (ii) $\Rightarrow h - A = V_Bt_1+\frac{gt_1^2}{2}+V_Bt_2-\frac{gt_2^2}{2}$

$$V_B(t_1+t_2)=\frac{g}{2}(t_2^2-t_1^2)=\frac{g}{2}(t_2+t_1)(t_2-t_1)$$

$\Rightarrow VB = \frac{g(t_2-t_1)}{2} \Rightarrow$ substitute to get h

16 (C)

$h_{max} = 5 = \frac{u^2}{2g}$, is $u = 10m/s$

$\therefore$ No of balls thrown 1 minute will be 60.

17. (C)

$$t_1 = \frac{2u\sin\theta}{g}, t_2 = \frac{2u\sin(90-\theta)}{g}$$

$$t_1t_2 = \frac{2}{g}\times\frac{u^2\sin2\theta}{g} = \frac{2}{g}\times R$$

$\therefore t_1t_2 \,\&\, R$

18. (D)

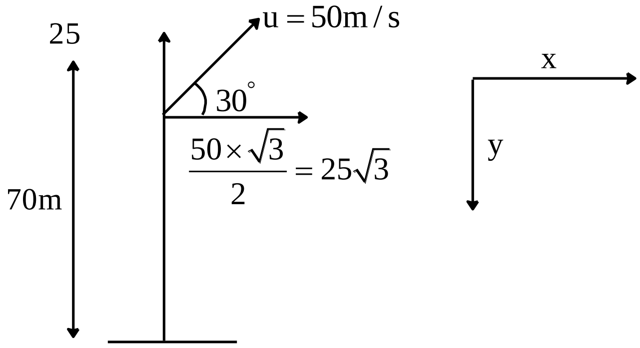
R_1	R_2
$\theta = \pi/3$	$\theta_2 = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$
$h_{1_{\max}} = y_1 = \frac{u^2 \sin^2 \pi/3}{2g}$	$h_{z_{\max}} = \frac{u^2 \sin^2 \pi/6}{2g}$
$y_1 = \frac{u^2}{2g} \times \frac{3}{4}$	$h_{Z\max} = \frac{u^2}{2g} \times \frac{1}{4}$
$\Rightarrow \frac{u^2}{2g} = \frac{4y_1}{3}$	$= \frac{4y_1}{3} \times \frac{1}{4} = \frac{y_1}{3}$

19. (C)

$$y_1 = \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin^2 60}{2g} = \frac{3}{4}\frac{u^2}{2g}$$

$$= \frac{3u^2}{8g}$$

$$\frac{y_2}{y_1} = \frac{1}{3}$$



$$70 = -25t + 5t^2$$

$$\therefore t = 7s$$

20. (A)

$R \to h \text{ and } h'$

$$h = \frac{u^2 \sin^2 \theta}{2g}, h' = \frac{u^2 \cos^2 \theta}{2g}$$

$$R = \frac{u^2 2\sin\theta\cos\theta}{g} \Rightarrow \left[\frac{g}{2u^2}\right] R = \sin\theta \times \cos\theta$$

$$hh' = \frac{u^u(\sin\theta\cos\theta)}{4g^2}$$

$$\sin\theta\cos\theta = \sqrt{\left[\frac{4g^2}{u^4}\right] hh'} = \frac{2g}{u^2}\sqrt{hh'}$$

$$\Rightarrow \sin\theta\cos\theta = \frac{g}{2u^2} R = \frac{2g}{u^2}\sqrt{hh'}$$

$$R = 4\sqrt{hh'}$$

21. (D)

From fig

$$* \underset{\downarrow}{S_{AX}} = \underset{\downarrow}{S_{BX}}$$

$$V_A T = (V_B \cos 30)t \Rightarrow V_A = \frac{V_B\sqrt{3}}{2}$$

A is projected horizontally

$S_{By} = S_{Cy}$ (C is projected vertically)

$$V_B \sin 30t - \frac{gt^2}{2} = V_C t = \frac{gt^2}{2}$$

$$\Rightarrow V_c = V_B / 2$$

$$\Rightarrow V_A : V_B : V_C = \frac{\sqrt{3}V_B}{2} : V_B : \frac{V_B}{2}$$

$$= \sqrt{3} : 2 : 1$$

22. (B)

From fig.

$S_X\ man = S_X truck$

$$ur = \underset{\underset{0}{\downarrow}}{U_t t} + \frac{1}{2}at^2$$

$$Ut = \frac{1}{2}at^2 \Rightarrow u = \frac{at}{2}$$

For time we'll use y axis

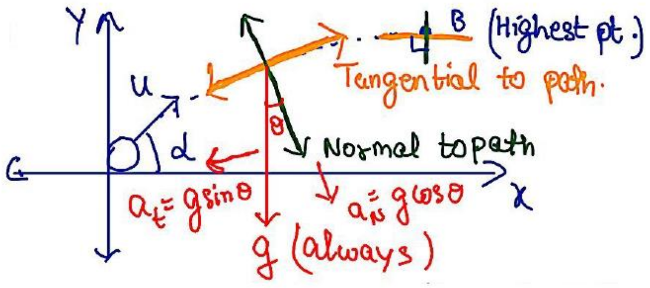
$U_{yman} = 0 \quad S_y = -h\ a_y = -g$

As $S_y = uyt + \frac{1}{2}ayt^2$

$$-h = 0 - \frac{g}{2}t^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

As $u = \frac{at}{2} = \frac{a}{2}\sqrt{\frac{2h}{g}} = a\sqrt{\frac{h}{2g}}$

23. (D)



(A) Throughout the motion $a_x = 0$ and $a_y = g \downarrow \Rightarrow a = g \downarrow$

At all points

* we have taken θ wrt Normal. Hence, at highest pt $\theta = 0^0 \Rightarrow \theta \downarrow$ses during upward motion

* As $\left|\overrightarrow{a_N}\right| = g\cos\theta$ hence as $\theta \downarrow$ ses $\cos\theta \uparrow ses \Rightarrow |\overrightarrow{a_N}|\ also \uparrow$ ses.

* Similarly $\left|\overrightarrow{a_t}\right| = g\sin\theta$ hence $a_t \downarrow ses$

24. (A)

$$\text{time} = \sqrt{\frac{2h}{g}} \text{ for both}$$

25. (D)

From fig

$$h_A \; = \; h_B \; = \; h_C$$

$$\frac{U_A^2 \sin^2 \theta_A}{2g} = \frac{u_B^2 \sin^2 \theta_B}{2g} = \frac{u_C^2 \sin^2 \theta_C}{2g}$$

$$\Rightarrow U_A \sin \theta_A \; = \; u_B \sin \theta_B \; = u_C \sin \theta_C$$

$$u_{yA} \qquad = u_{yB} \qquad = u_{yC}$$

i.e. same initial speed along y axis

$$\Rightarrow T_A = T_B = T_C \qquad \left[\because T = \frac{2u \sin \theta}{g} = \frac{2uy}{g} \right]$$

From fig $R_A > R_B < R_C$

$$\Rightarrow \frac{2u_{XA}u_{yA}}{g} < \frac{2u_{XB}u_{YB}}{g} < \frac{2u_{XC}u_{yC}}{g}$$

$$\Rightarrow u_{XA} < U_{XB} < U_{XC} \qquad \left[\because u_y \; are \; equal \right]$$

Also from fig

$$\theta_A > \theta_B > \theta_C$$

And as $U_A \sin \theta_A = U_B \sin \theta_B = U_C \sin \theta_C$

$$\Rightarrow U_A < U_B < U_C$$

26. (C)

$$\text{H} = \frac{\text{V}^2 \sin^2 60}{2\text{g}} = \frac{\text{V}^2 3}{8\text{g}}, \text{t} = \frac{\text{V} \sin 60}{\text{g}} = \frac{\text{V}\sqrt{3}}{2\text{g}}.$$

For A $\text{H} = \text{v}_\text{e} \text{t} - \frac{1}{2} \text{g}^{+\text{e}}$

$$\therefore \frac{\text{V}}{\text{v}} = \frac{2}{\sqrt{3}}$$

27. (D)

$$R_{\max} = \frac{u 2 \sin}{g} (2 \times 45^0) = \frac{u^2}{g} = ?$$

$$R = \frac{u^2 2 \sin \theta \cos \theta}{g}, \; H = \frac{u^2 \sin^2 \theta}{2g}$$

$$R^2 = \left[\frac{u^2 2 \sin \theta \cos \theta}{g} \right]^2 = \frac{u^4 \times 4 \sin^2 \theta \cos^2 \theta}{g^2}$$

$$\frac{R^2}{H} = \frac{4u^{\overset{2}{\cancel{4}}} \cancel{\sin^2 \theta} \cos^2 \theta}{g^2} \times \frac{2 \cancel{g}}{\cancel{u^2} \cancel{\sin^2 \theta}}$$

$$\Rightarrow \frac{R^2}{H} = \frac{8u^2 \cos^2 \theta}{g}$$

$$\Rightarrow \frac{u^2\cos^2\theta}{g} = \frac{R^2}{8H}\Bigg\}$$

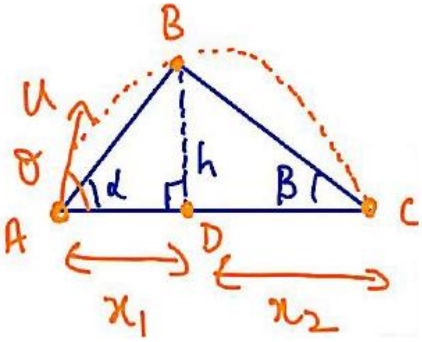
$$\text{As } H = \frac{u^2\sin^2\theta}{2g} \Rightarrow \frac{u^2\sin^2\theta}{g} = 2H$$

$$2H + \frac{R^2}{8H} = \frac{u2\cos 2\theta}{g} + \frac{u^2\sin^2\theta}{g}$$

$$2H + \frac{R^2}{8H} = \frac{u^2}{g}\,(\underbrace{\sin^2\theta + \cos^2\theta}_{1})$$

$$2H + \frac{R^2}{8H} = R_{\max}$$

28. (A)



* From ΔADB

$$\tan\alpha = \frac{BD = H}{AD = x_1}$$

$$\Rightarrow x_1 = \frac{h}{\tan\alpha}$$

*From ΔBDC

$$\tan\beta = \frac{BD = h}{CD = x_2}$$

$$\Rightarrow x_2 = \frac{h}{\tan\beta}$$

Also $R = x_1 + x_2$

From Equation of trajectory (for Pt B)

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

$$h = x_1 + \tan\theta\left[1 - \frac{x_1}{R}\right] \quad R = x_1 + x_2$$

$$\frac{h}{\tan\theta} = x_1\left[1 - \frac{x_1}{x_1 + x_2}\right]$$

$$\frac{h}{\tan\theta} = x1\left[\frac{x_1 + x_2 - x_1}{x_1 + x_2}\right]$$

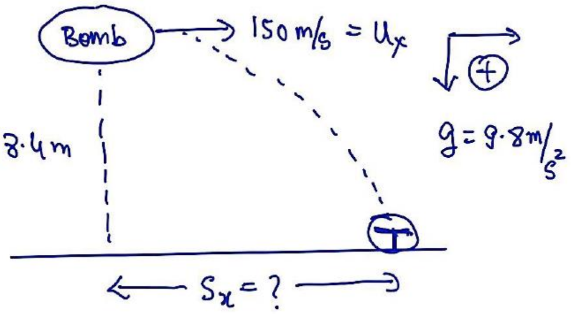
$$\frac{h}{\tan\theta} = \frac{x_1 x_2}{x_1 + x_2}$$

$$\Rightarrow \frac{h}{\tan\theta} = \frac{\dfrac{h}{\tan\alpha}\times\dfrac{h}{\tan\beta}}{\dfrac{h}{\tan\alpha}+\dfrac{h}{\tan\beta}}$$

$$\frac{\cancel{h}}{\tan\theta} = \frac{\cancel{h}^{\overset{1}{\cancel{2}}}}{\cancel{(\tan\alpha\tan\beta)}\ (\cancel{h})\left(\dfrac{\tan\beta+\tan\alpha}{\cancel{\tan\alpha\tan\beta}}\right)}$$

$$\Rightarrow \tan\alpha+\tan\beta=\tan\theta$$

29. (C)



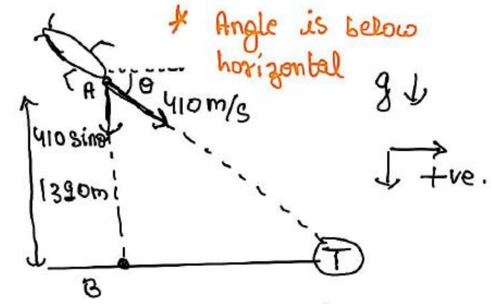
As $S_y = uyt + \frac{1}{2}ayt^2$

$$78.4 = 0(t) + \left(\frac{9.8}{2}\right)t^2$$

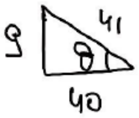
$$t^2 = \frac{\cancel{784}^{8}\times 2}{\cancel{98}} \Rightarrow t = 4\,\text{sec}$$

$$S_x = u\times t = 150\times 4 = 600m$$

30. (A)



$\tan\theta = \frac{9}{40} \Rightarrow$



$$S_y = +1390m,\ ay = 10m/s^2$$

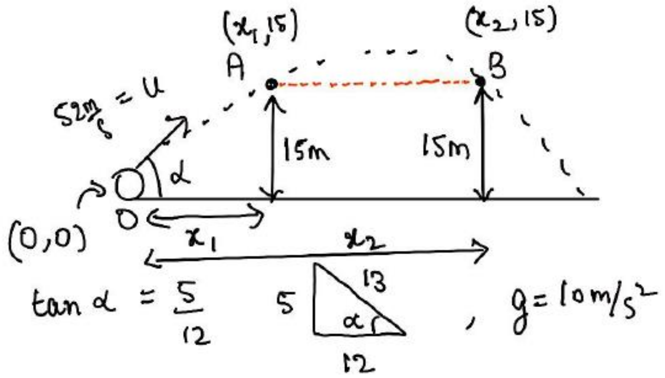
$$u_y = 410\sin\theta = 410\times\frac{9}{41} = 90m/s$$

As $S_y = uyt + \frac{1}{2}ayt^2$

$$1390 = 90t + \frac{10t^2}{2}$$

Solve for t

31. (B)



* Equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{24^2\cos^2\theta}$$

$$15 = x\left(\frac{5}{12}\right) - \frac{10x^2}{2(52)^2\left(\frac{12}{13}\right)^2}$$

Solving this we' ll get two values of x. One will be x_1 and other will be x_2 . Also $x_2 > x_1$

As $S_X = U_X t$

$$\Rightarrow x_1 = u\cos\alpha\left(t_{OA}\right)$$

$$t_{OA} = \frac{x_1}{U\cos\alpha}$$

Similarly $x_2 = u\cos\alpha(t_{OB})$

$$t_{OB} = \frac{x_2}{u\cos\alpha}$$

Ans = t = $t_{OB} - t_{OA}$

Method – 2 $uy = u\sin\alpha = 52\times\frac{5}{13} = \frac{20}{5}m$

$5y = +15$

ay =-10

As $5y = uyt + \frac{1}{2}ayt^2$

$$15 = 20t - \frac{10t^2}{2}$$

$$5t^2 - 20t + 15 = 0$$

$$5(t^2 - 4t + 3) = 0$$

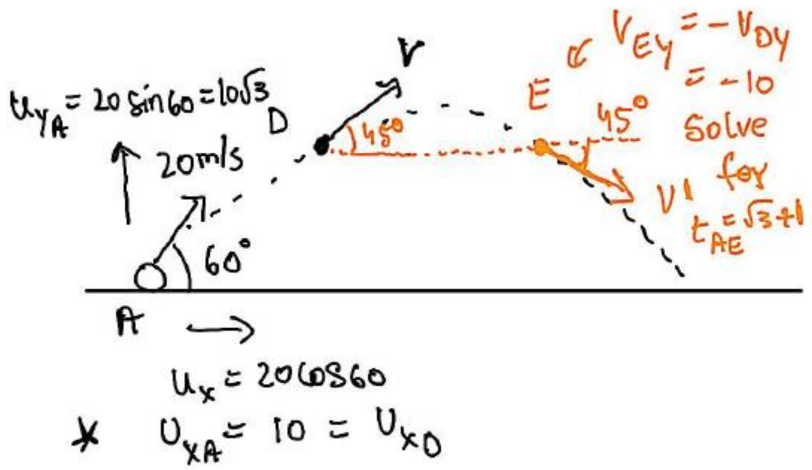
$$(t^2 - 3t - t + 3) = 0$$

$$(t-3)(t-1) = 0$$

$$\Rightarrow \underset{\substack{\uparrow \\ t_{OA}}}{t = 1} \quad and \quad \underset{\substack{\uparrow \\ t_{OB}}}{t = 3}$$

$$\Rightarrow Ans = t = t_{OB} - t_{OA} = 3 - 1 = 2$$

32. (D)



At pt. D$\Rightarrow$ $\tan 45 = \frac{V_{DY}}{V_{DX}}$

$$1 = \frac{V_{DY}}{10} \Rightarrow V_{DY} = 10$$

Also $V_{DY} = U_{AY} + a_y t_{AD}$

$$10 = 10\sqrt{3} - 10t_{AD}$$

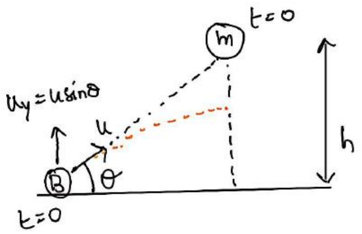
$$10t_{AD} = 10\left(\sqrt{3}-1\right)$$

$$t_{AD} = \sqrt{3} - 1$$

33. (B)

$$\frac{H}{T} = \frac{\frac{u^2\sin^2\theta}{2g}}{\frac{2u\sin\theta}{g}} = \frac{u\sin\theta}{4}$$

34. (A)



* For Bullet

$$Sy = uyt + \frac{1}{2}ayt^2$$

$$S_{Y_B} = (u\sin\theta)t - \frac{gt^2}{2}$$

$\because$ of g, bullet is lowered by height $\frac{gt^2}{2}$ else without g it will achieve height = $(u\sin\theta)t$

* Also as monkey free falls hence

$S_{ym} = 0(t) - \frac{gt^2}{2} = -\frac{gt^2}{2}$ ie monkey unknowingly lower its ht. by $\frac{gt^2}{2}$ $\because$ of fall

Hence bullet will hit the monkey as both hit is lowered by $\frac{gt^2}{2} \because of \;\; g$

35. (B)

$R_1 = R_2 \rightarrow \theta_1 + \theta_2 = 90$

Or $\theta_2 = 90 - \theta_1$ $\qquad \theta_2$

$$T_1 = \frac{2u\sin\theta_1}{g},\ T_2 = \frac{24\sin(90-\theta_1)}{g}\ T_2 = \frac{2u\cos\theta_1}{g}$$

As $R = \frac{u^2 2\sin\theta_1\cos\theta_1}{g}$

$$\Rightarrow T_1T_2 = \frac{2u\sin\theta_1}{g} \times \frac{2u\cos\theta_1}{g}$$

$$T_1T_2 = \left(\frac{2}{g}\right)\underbrace{\left(\frac{u^2 2\sin\theta_1\cos\theta_1}{g}\right)}$$

$$\Rightarrow T_1T_2 \propto R$$

36. (A)

$$R_{\max} = \frac{u^2\sin(2\times 45)}{g} = \frac{u}{g} = d$$

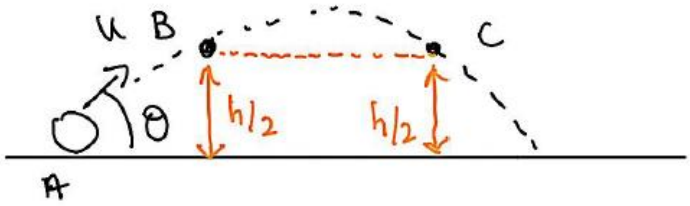
$$\Rightarrow u^2 = gd$$

$$h = \frac{v^2 - u^2}{2a} = \frac{0^2 - gd}{2(-g)}$$

$h = d/2$ Ans

*For $H_{\max}$ player should throw ball vertically.

37. (B)



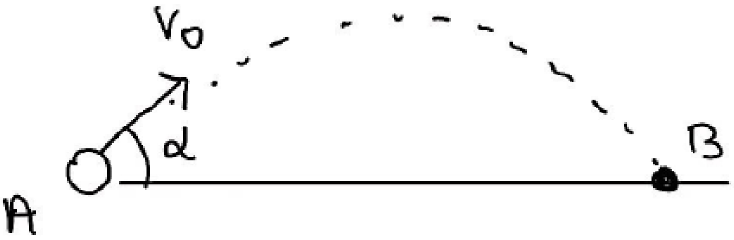
$* BC \rightarrow Sy = 0$

$S_X = U_x t_{BC}$

$$V_{avg_{BC}} = \frac{S}{t_{BC}} = \frac{SX = U_x t_{BC}}{t_{BC}}$$

$$V_{avg_{BC}} = U_x = u\cos\theta$$

38. (A)



$$t_{AB} = \frac{2V_0\sin\alpha}{g}$$

$$S_{ballX} = S_{boyx}$$

Range = $Vb \times t = \frac{2V_0\sin\theta}{g}$

$$\frac{\overset{1}{\cancel{2}}V_0\sin\alpha\cos\alpha}{\cancel{g}} = V_b \times \frac{2V_0\sin\alpha}{g}$$

$$V_b = V_0\cos\alpha$$

39. (D)

$$R_{\max} = R = u^2\ \frac{\sin(2\times45)}{g}$$

$$\Rightarrow u^2 = Rg$$

Or $u = \sqrt{Rg}$

Now we want Range to $\frac{R}{2}$

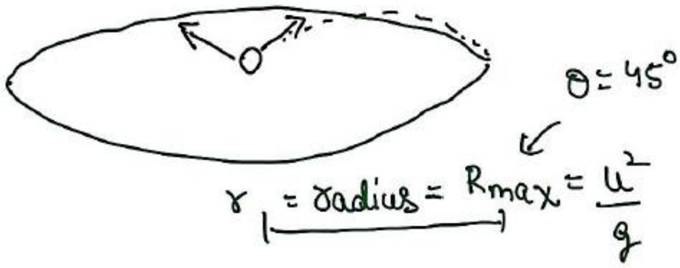
$$\Rightarrow \frac{R}{2} = \frac{u2\sin2\alpha}{g} = \frac{\cancel{Rg}\sin2\alpha}{\cancel{g}}$$

$$\Rightarrow \frac{1}{2} = \sin2\alpha$$

$$\Rightarrow 2\alpha = 30^0\ or\ 150^0\quad [\because \alpha \le 90]$$

$$\alpha = 75^0$$

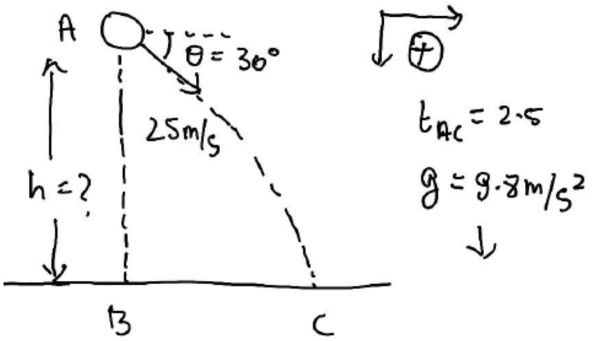
40. (B)



area $_{\max} = \pi r_{\max}^2 = \pi[R_{\max}]^2$

$$= \frac{\pi u^4}{g^2}$$

41. (A)



$$Sy = uyt + \frac{1}{2}ayt^2$$

$$= 25\sin 30(2.5) + \frac{(9.8)(2.5)^2}{2}$$

$$= \frac{25 \times 2.5}{2} + \frac{9.8(2.5)^2}{2}$$

42. (B)

$$t = \sqrt{\frac{2 \times 440}{9.8}} = 10$$

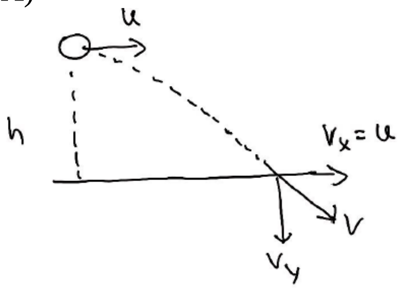
$$h = 200 \times 10 = 2000m$$

43. (A)

$$t = \sqrt{\frac{2 \times 1960}{9.8}} = 20s$$

$$h = \frac{500}{3} \times 20 = 3.33km$$

44. (A)



$$V = \sqrt{V_x^2 + V_y^2}$$

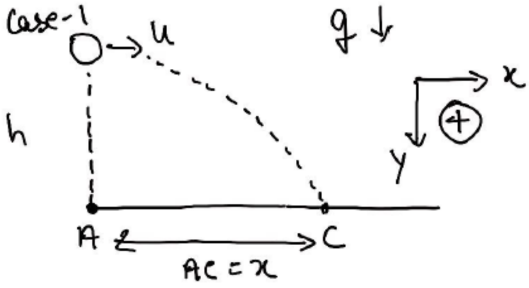
Also $V_y^2 - u_y^2 = 2aysy$

$$V_y^2 - 0 = 2(-g)(-h) = 2gh$$

$$V_y^z = 2gh$$

$$\Rightarrow V = \sqrt{u^2 + 2gh}$$

45. (B)

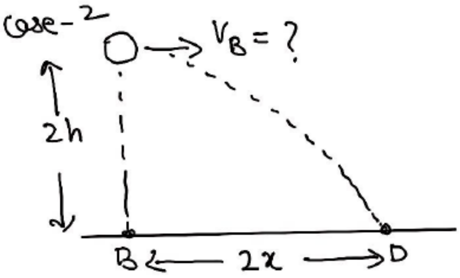


From y axis

$$h = 0(t) + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

From x axis

$$S_x = x = U_x t \Rightarrow x = ut = u\sqrt{\frac{2h}{g}}$$



From y axis

$$2h = 0(t1) + \frac{1}{2}gt_1^2$$

$$\Rightarrow t_1 = \sqrt{\frac{4h}{g}}$$

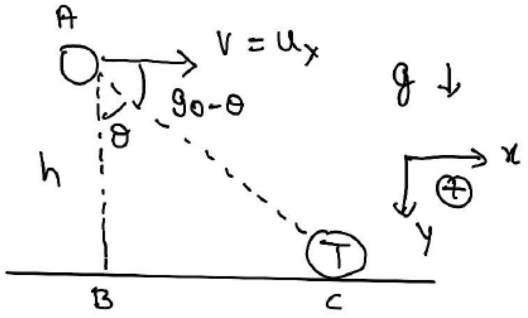
From x axis $\Rightarrow S_x = V_x t_1$

$$2x = V_B\sqrt{\frac{4h}{g}}\ u\sqrt{\frac{2h}{g}}$$

$$\Rightarrow 2\left(u\frac{\sqrt{2h}}{g}\right) = V_B\sqrt{\frac{4h}{g}}$$

$$V_B = \frac{2u}{\sqrt{2}} = u\sqrt{2}$$

46. (B)



From y axis

$$h = 0(t) + \frac{gt^2}{2} \Rightarrow t = \sqrt{\frac{2h}{g}}$$

From x axis

$$S_x = BC = U_x t = Vt = V\sqrt{\frac{2h}{g}}$$

Also from ΔABC

$$\tan\theta = \frac{BC}{AB} \Rightarrow \tan\theta = \frac{V\sqrt{\frac{2h}{g}}}{h}$$

$$\Rightarrow \tan\theta = v\sqrt{\frac{2}{gh}}$$

EXERCISE - 2

1. (A,B)

$$o^2 = 10^2 - 2\times5\times s$$

$$\therefore s = 10\text{m}$$

$$s = 10\times3 - \frac{1}{2}\times5\times9 = 30 - 22.5 = 7.5$$

2. (C, D)

(A) $\upsilon = 0 \Rightarrow a$ can be non-zero

(B) average velocity = 0 $\Rightarrow$ displacement = 0

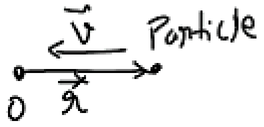
But average speed = $\frac{\text{total distance}}{\text{time}}$

Which can be non-zero.

(C) If $\vec{v}$ and $\vec{a}$ are of opposite signs,

$|\vec{v}|$ decrease $\Rightarrow$ object slows down

(D)

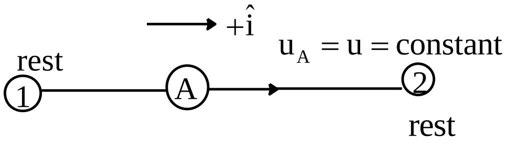


$\vec{r}$ and $\vec{v}$ are opposite sign

$\Rightarrow$ particle moves towards 0

(C) and (D) are correct.

3. (A, B, C)



$$\vec{\upsilon}_1 = 0 = \vec{u}_2$$

$$\vec{\upsilon}_A = u\,\hat{i}$$

$$\vec{\upsilon}_{1/A} = 0 - u\hat{i} = -u\,\hat{i}$$

$$\vec{\upsilon}_{2/A} = 0 - \upsilon\hat{i} = -u\,\hat{i}$$

$\Rightarrow$1 wrt A and 2 wrt A both appears to move along same direction positive x axis

4. (C, D)

Speed increases if $\vec{v}$ and $\vec{a}$ are of same sign and decreases if they are of opposite signs.
Hence (C) and (D) are correct.

5. (A, C)

(A) If speed changes $\Rightarrow$ $|\vec{v}|$ will change

(B), (C) If $\vec{v}$ changes, speed may remain constant as change in $\vec{v}$ may be only due to change in its direction
$\Rightarrow a \neq 0$

(D) $\Rightarrow$ change in speed $\Rightarrow$ velocity will change.

6. (A, C)

$\vec{a}$ and $\vec{v}$ can be either zero as non-zero at any instant of time irrespective of other.
So (A), (C) are correct.

7. (B, D)

(A) If speed changes $\Rightarrow \vec{v}$ charges

(B) $\vec{v}$ can be variable due to changing direction

(C) If $\vec{v} =$ constant $\Rightarrow \vec{a} = \vec{0}$

(D) If $|\vec{v}| =$constant, still direction of $\vec{v}$ can change $\Rightarrow \vec{a} \neq \vec{0}$

8. (B, D)

$$S = u + a\left(n - \frac{1}{2}\right) 25 = u + a\left(5 - \frac{1}{2}\right)$$

$$33 = u + a\left(7 - \frac{1}{2}\right)$$

$$\therefore u = 7 m/s,\ a = 4 m/s^2$$

9. (A, B, C, D)

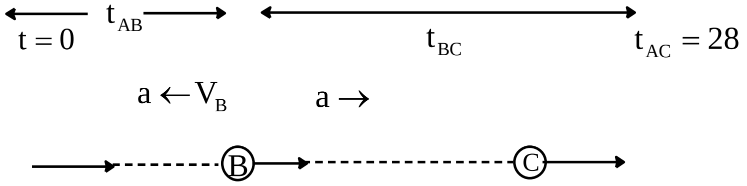
$$u = 4 m/s^2,\ a = 2 m/s^2$$

$$S_{5th} = 4 + 2\left(5 - \frac{1}{2}\right) = 13 m$$

$$50 = 4 + 2 \times t \therefore t = 23 s$$

$$S = 4 \times 6 + \frac{1}{2} \times 2 \times 6^2 = 60 m$$

10. (A,C)



$$\upsilon_A = 54 \frac{km}{hr} = 15 m/s$$

$$V_C = 54 \frac{km}{hr} = 15 m/s$$

$\vdash$ AC = 280m $\dashv$

$$t_{AB} = t_{BC}$$

$$\downarrow \quad \downarrow$$

(1) $\left(\frac{V_B - 15}{-Q}\right)\left(\frac{15 - V_B}{+a}\right)$

$\Rightarrow t_{AB} = t_{BC}$

As $t_{AB} + t_{BC} = t_{AC} \Rightarrow 2t_{AB} = 28$

$t_{AB} = t_{BC} = 14\,\text{sec}$

(2) Also $S_B = S_{BC}$

$\because S_{AB} = ut_{AB} + \frac{1}{2}at_{AB}^2$

$S_{AB} = 15(14) - \frac{a(14)^2}{2}$

And $S_{BC} = VT_{BC} - \frac{1}{2}at_{BC}^2$

$15(14) - \frac{1}{2}(a)(14)^2$

$\Rightarrow S_{AB} = S_{BC}$ and as $S_{AB} + S_{BC} = S_{AC}$

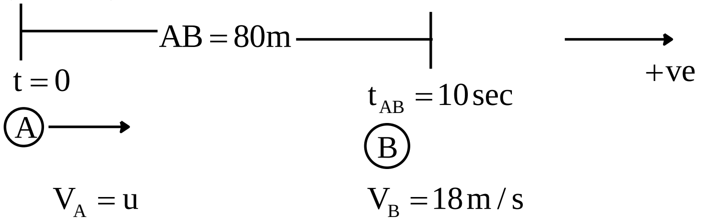
$\Rightarrow 2S_{AB} = 280 \Rightarrow S_{AB} = 140\text{m}$

In motion AB

$\upsilon_A = 15, S_A = +140$

$\upsilon_A = 14 \Rightarrow$ Solve to get a and V_B

11. (A, B, C)



$a \rightarrow$

As, $S = Vt - \frac{1}{2}at^2$

$80 = 18(10) - \frac{a(10)^2}{2}$

$\frac{100a}{2} = 180 - 80 = 100$

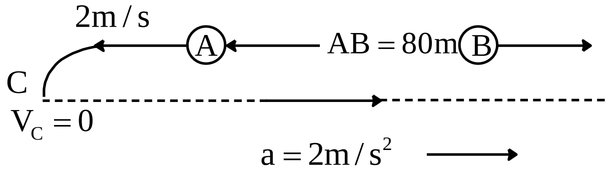
$\Rightarrow a = 2\text{m/s}^2$

Also $V = u + at \Rightarrow 18 = u + 2(10)$

$\Rightarrow u = -2\text{m/s}$

$\Rightarrow \upsilon_A \rightarrow x$

Hence figure



$$S_{AC}=\frac{V_C^2-\upsilon_A^2}{2(2)}=\frac{0^2-(-2)^2}{2(2)}=\frac{-4}{4}=-1$$

Dist $=2|S_{AC}|AC+CA+AB=2(-1)+80$

$=82m$

12. (A, B, C)

$u_x=6, u_y=8$

Max. height $=\frac{4y^2}{2g}=\frac{64}{20}=3.2m$

T= Time of flight $=\frac{2u_y}{g}=\frac{2\times 8}{10}=1.6s$

Range $=4k\times t=6\times 1.6=9.6m$

13. (A, B, C, D)

$$R=\frac{u^2\sin^2\theta}{g}$$

$$180\sqrt{3}=\frac{60\times 66\times \sin^2\theta}{10}$$

$$\sin^2\theta=\frac{\sqrt{3}}{2};\theta=60\,\text{or}\,30^\circ$$

$T=\frac{2u\sin\theta}{g}=6s$ or 10.4 s

Max range $=\frac{u^2}{g}=\frac{60\times 60}{10}=360m$

$\therefore$ Distance to be moved $=360-180\sqrt{3}=48.6m$

14. (A, B, C, D)

$2400=u\times 0.6\times 5+15\times 25$

$\therefore u=666.67ft's$

$R=4\cos 37\times t=2667\,ft$

$v_x=667.67\times\cos 37=534\,ft's$

$v_y=666.67\times\sin 27+32\times 5=560$

EXERCISE - 3

Comprehension

1. (C)

Assume ↓ = +ve

For A and B :

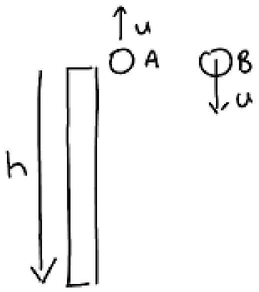
$s=+h \qquad u_A=-u$

$a=+g \qquad u_B=u$

$v^2=u^2+2gh$

$v=$ same for both

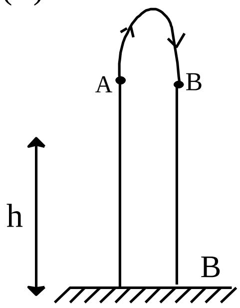
So, (C) is correct.



2. (A)

A will take more time $t_A > t_B$

3. (B)



$t_{AB} = 2$

$t_{AA} = 4 = \frac{2u}{g} \therefore u = 20$

$\therefore h = 20 \times 2 + 5 \times 4 = 60$

4. (C)

If $t_A = 6\,\text{sec}, t_B = 2\,\text{sec}$

$\Rightarrow$ For A : $h = (-u) \times 6 + 5 \times (6)^2$

For B : $h = u \times 2 + 5 \times (2)^2$

$\Rightarrow$ $-6u + 180 = 24 + 20$

$\Rightarrow$ $8u = 160 \Rightarrow u = 20\ \text{m/s}$

5. (B)

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 60}{10}} = \sqrt{12} = 2\sqrt{3} = 3.46$$

Match List – I

$A - p, q, B - p, r, s, C - p, r, D - p, r, s$

Match List - II

$A = q, r, B - r, C - q, r, D - q, r$

EXERCISE - 4

1. **(10s, 30s)**

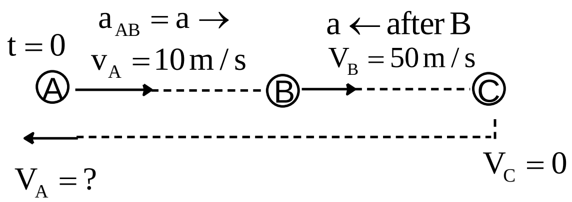
$1500 = 200 \times t - \frac{1}{2} \times 10 \times t^2$

$1500 = 200t - 5t^2$

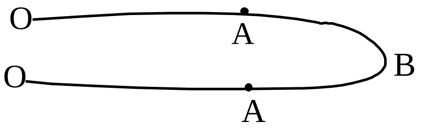
$t^2 - 40t + 300 = 0$

$\therefore t = 10s, t = 30s$

2. **(70 m/s)**



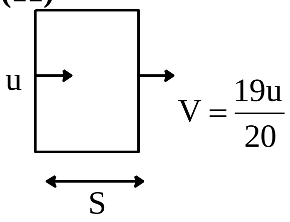
$$50^2 = 10^2 + 29s \text{ (0 to A)}$$
$$1200 = as$$
$$\therefore a = \frac{1200}{s} \qquad \therefore a = \frac{1200}{s}$$



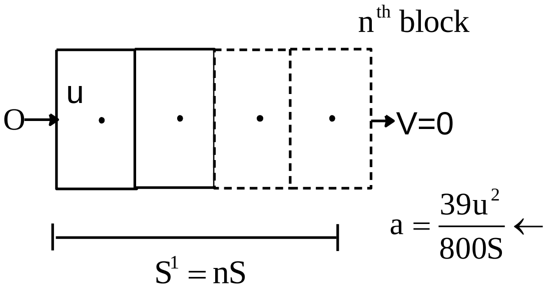
Positive A to B to A to A

$$v^2 = 50^\circ - 2\times\frac{1200}{s}\times(-5)$$
$$v^2 = 2500 + 2400 = 4900$$
$$v = 70m/s$$

3. **(11)**



$$a = \frac{v^2 - u^2}{2S} = \frac{\left(\frac{19u}{20}\right)^2 - u^2}{2S} = \frac{-39u^2}{800S}$$



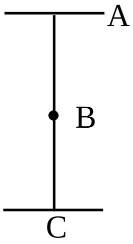
$$S^1 = \frac{v^2 - u^2}{2a} \Rightarrow nS = \frac{0^2 - u^2}{2\left(-\frac{39u^2}{800S}\right)}$$
$$\Rightarrow nS\times 2\left(\frac{+39u^2}{800S}\right) = +u^2$$
$$n = \frac{800}{78} = 10\ldots\ldots$$
$$\Rightarrow \text{Ans. } = 11$$

4. **(26m/s), (10 m/s)**

Avg. speed $= \frac{13}{0.5} = 26\text{m}/\text{s}$

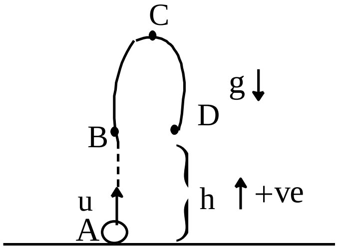
Avg. velocity $= \frac{5}{0.5} = 10\text{m}/\text{s}$

5. **(2984 m)**



$$\text{As} = \frac{1}{2}\times 10\times 10^2 = 500\text{m}$$
$$V_B = 0 + 10\times 10^2 = 100\text{m}/\text{s}$$
$$V_C^2 = V_B^2 - 2\times 8\times \text{BC}$$
$$64 = 10000 - 16\times \text{BC}$$
$$\text{BC} = 2484\text{m}$$
$$\text{AB} + \text{BC} = 500 + 2484$$
$$= 2984\text{m}$$

6.



$$t_{AB} = t_1, \qquad t_{AD} = t_2$$

BD motion

$$S_{BD} = 0, t_{BD} = t_2 - t_1$$
$$a_{BD} = -g$$

As $S = ut + \frac{1}{2}at^2$

$$0 = V_B\left(t_2 - t_1\right) + \frac{-g\left(t_2 - t_1\right)^2}{2}$$
$$\frac{g\left(t_2 - t_1\right)^2}{2} = V_B\left(t_2 - t_1\right)$$
$$V_B = \frac{g\left(t_2 - t_1\right)}{2}$$

AB motion

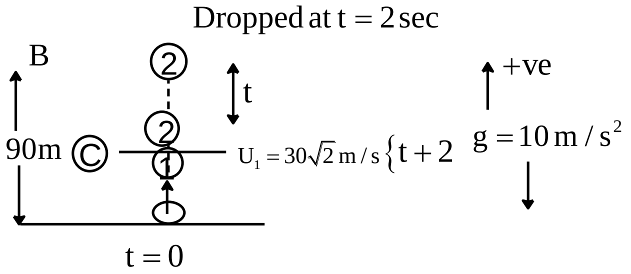
$$V_B = \frac{g\left(t_2 - t_1\right)}{2}, t_{AB} = t_1, a = -g$$

As $V_B = U_A + at_{AB}$

$$\frac{g\left(t_2-t_1\right)}{2}=U_A-g\left(t_1\right)$$

$$\Rightarrow U_A=\frac{g\left(t_2+t_1\right)}{2}$$

7. **312 s, 6.3 m from tower top**



$\left|\vec{S}_{y_1}\right|+\left|\vec{S}_{y_2}\right|=90$ ________(1)

$$S_{y_1}=30\sqrt{2}\left(t+2\right)-\frac{10\left(t+2\right)^2}{2}$$

$\uparrow$ +ve

$S_{y_1}=30\sqrt{2}\left(t+2\right)-5\left(t+2\right)^2$ ________(2)

$$S_{y_2}=O\left(t\right)-\frac{10\left(t\right)^2}{2}=-5t^2$$

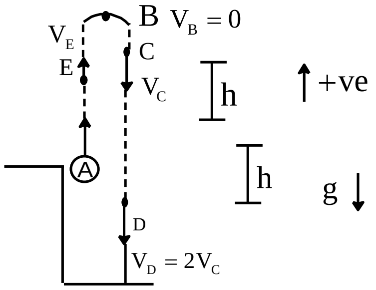
$\downarrow$ –ve

$\Rightarrow\left|\vec{S}_{y_2}\right|=5t^2$ ________(3)

Hence, $30\sqrt{2}\left(t+2\right)-5\left(t+2\right)^2+5t^2=90m$

Solve for t, Sy_1 and Sy_2

8.



Motion CD

$S_{CD}=-2h$

$a=-g$

$V_D^2-u_C^2=2aS$

$\left(-2V_C\right)^2-\left(-V_C\right)^2=2\left(-g\right)\left(-2h\right)$

$2V_C{}^2-V_C{}^2=4gh$

$$V_C^2=\frac{4gh}{3}$$

Motion BC

$$u_B=0,V_C=\sqrt{\frac{4gh}{3}}$$

$a=-g$

$$V_C^2 - U_B^2 = 2as$$

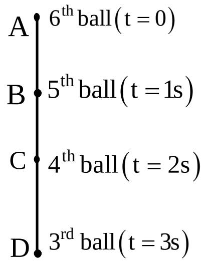
$$\frac{4gh}{3} - 0^2 = 2(-g)S_{BC}$$

$$S_{BC} = \frac{4gh}{3\times -2g} = \frac{-2h}{3}$$

$$\Rightarrow AB = AC + BC$$

$$= h + \frac{2h}{3} = \frac{5h}{3}$$

9. **(45m, 20m, 5m)**



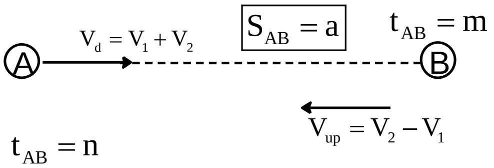
$$5^{th}\text{ball} \Rightarrow \frac{1}{2}\times 10\times 1^2 = 5m$$

$$4^{th}\text{ ball} \Rightarrow \frac{1}{2}\times 10\times 2^2 = 20m$$

$$3^{rd}\text{ball} \Rightarrow \frac{1}{2}\times 10\times 3^2 = 45m$$

10.

$$V_r = V_1, V_S = V_2$$



$t_{AB} = n$

A to B

$$S_{AB} = V_d t_D \Rightarrow a = (V_1 + V_2)m$$

$$\Rightarrow V_1 + V_2 = \frac{a}{m} \qquad \_\_\_\_\_\_\_(1)$$

B to A $\Rightarrow S_{BA} = V_{up}t_{up} \Rightarrow a = (V_2 - V_1)n$

$$\Rightarrow (V_2 - V_1) = \frac{a}{n} \qquad \_\_\_\_\_\_\_(2)$$

Solve for V_1 and V_2

Alternative:

$$\frac{a}{2}\left(\frac{1}{m} - \frac{1}{n}\right), \frac{a}{2}\left(\frac{1}{m} + \frac{1}{n}\right)$$

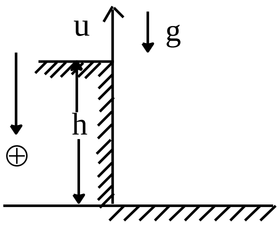
V_{rg} = Velocity of current

V_{sr} = Velocity of steamer relative to current.

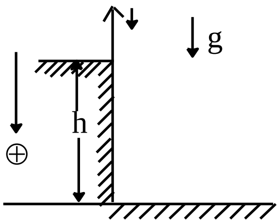
$$n = \frac{a}{V_{sr} + V_{rg}}, n = \frac{a}{V_{sr} - V_{rg}}$$

11. **u = 25 m/sec, h = 180 m**

$S = ut + \frac{1}{2}at^2$



$+h = -u(g) + \frac{1}{2}g(g)^2$ ________(1)



$+h = +u(4) + \frac{1}{2}g(4)^2$ __________(2)

Subtracting

$\Rightarrow 0 = -13u + \frac{1}{2}g(81-16)$

$\Rightarrow 13u = \frac{g}{2}(65)$

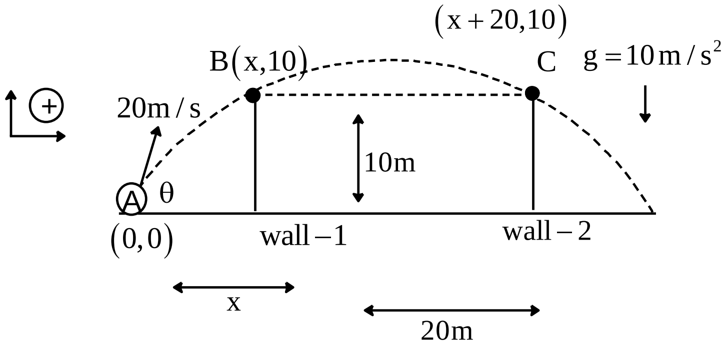
$\Rightarrow u = \frac{5g}{2} = 25m/sec$

$h = -9(25) + \frac{1}{2}10(81)$

$h = -225 + 405$

$h = 180m$

12. **2 sec**



$t_{BC} = ?$

Co-ordinates of pts

If $A = (0,0) \Rightarrow B = (x,10), C = (x+20,10)$

Also from equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

For pt B

$10 = x\tan\theta - \frac{gx^2}{2(20)^2\cos^2\theta}$ ______(1)

For pt $C \Rightarrow 10 = (x+20)\tan\theta - \frac{g(x+20)^2}{2(20)^2\cos^2\theta}$ ______(2)

Using (1) and (2)
We can find θ and x. Once we know θ tjem $U_x = u\cos\theta$

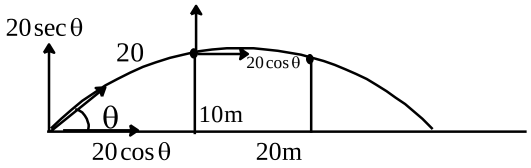
$S_{BCx} = U_x\, t_{BC}$

$\downarrow \qquad \downarrow$

$20 = [20\cos\theta](t_{BC})$

$$t_{BC} = \frac{1}{\cos\theta}$$

Alternative:



$T = \frac{2v_y}{g} \qquad\qquad 2\cos\theta T = 20 \Rightarrow \cos\theta = \frac{1}{T}$

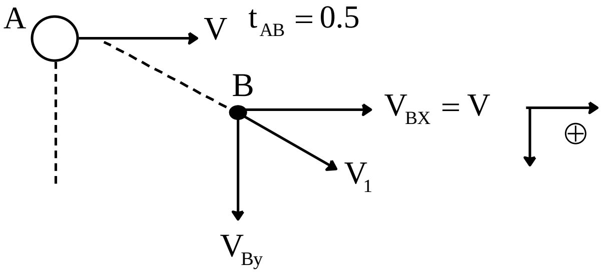
$V_y^2 = (20\sin\theta)^2 - 2g(10)$

$\left(\frac{gT}{2}\right)^2 = 400\left(1-\frac{1}{T^2}\right) - 200$

$25T^2 = 400\left(1-\frac{1}{T^2}\right) - 200$

On solving
$\Rightarrow T = 2\,\text{sec}$

13. **4.5 m/s**



Given $|\vec{V}_1| = 1.5V$

$\sqrt{V_{Bx}^2 + V_{By}^2} = 1.5V$

$V^2 + V_{By}^2 = (1.5V)^2 = 2.25V^2$

$V_{By}^2 = 1.25V^2 \Rightarrow V_{By} = V\sqrt{1.25}$

Also $V_{By} = U_{Ay} + a_{yt} = O + 10(0.5)$

$V\sqrt{1.25} = 5 \Rightarrow V^2 = \frac{25}{1.25}$

$V = \sqrt{20}$

Alternative

After 0.5 s $v_x = 4\hat{i}$

$v_y = (0 + 10\times 0.5)\hat{j}$

$v_y = 5\hat{j}$

$|\vec{v}| = \sqrt{4^2 + 25} = 1.5u$

$\therefore 4^2 + 25 = 2.25u^2$

$\therefore 4.5 m/s$

14. $\boldsymbol{\theta = \tan^{-1} 1.7}$

$t = \sqrt{\dfrac{2 \times 1960}{9.8}} = 20s$

$R = 600 \times \dfrac{5}{18} \times 20 = \dfrac{10000}{3}$

$\tan\theta = \dfrac{R}{h} = \dfrac{\frac{10000}{3}}{1960} = 1.7$

15. **20 s, 2000 m**

$t = \sqrt{\dfrac{2 \times 1960}{9.8}} = 20s$

$R = 100 \times 20 = 2000m$

16. $h_1 = \dfrac{4^2 \sin^2 60}{2g} = \dfrac{3u^2}{8g}$

$h_2 = \dfrac{u^2 \sin^2 30}{2g} = \dfrac{u^2}{8g}$

$h_1 = 3h_2$

$R_1 = R_2$ (as angles are complementary)

17. $\dfrac{1}{\sqrt{3}}, 1:1$

$\dfrac{4_1^2 \sin^2 60}{2g} = \dfrac{4_2^2 \sin^2 30}{2g}$

$\therefore \dfrac{u_1}{u_2} = \dfrac{\sin 30}{\sin 40} = \dfrac{1}{\sqrt{3}}$

For some range, as angles are complementary $\dfrac{u_1}{u_2} = \dfrac{1}{1}$

Alternative:

1st part $\theta_1 = 60^\circ, \theta_2 = 30^\circ$

$h_1 = h_2$

$\dfrac{u_1^2 \sin^2 60}{2g} = \dfrac{u_2^2 \sin^2 30}{2g}$

$u_1^2 \times \dfrac{3}{4} = u_2^2 \times \dfrac{1}{4} \Rightarrow \dfrac{u_1}{u_2} = \dfrac{1}{\sqrt{3}}$

2nd part

$R_1 = R_2$

$$\frac{u_1^2 \sin(2\times 60)}{g} = \frac{u_2^2 \sin(2\times 30)}{g}$$

$$\Rightarrow \frac{u_1}{u_2} = 1$$

18. **3.8 m**

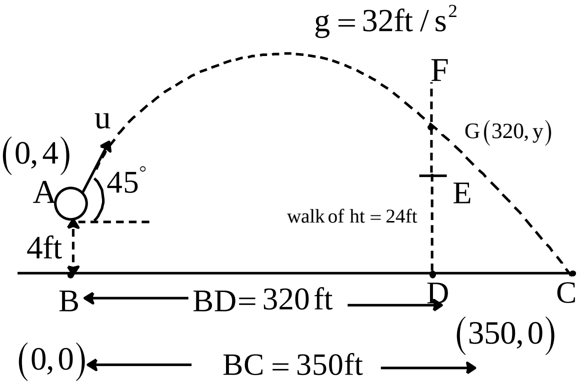
$$t = \sqrt{\frac{2\times 8}{10}} = \sqrt{1.6}$$

$$S_t = 15\sqrt{1.6}$$

$$s_b = 12\sqrt{1.6}$$

$$S = S_t - S_b = 3\sqrt{1.6} = 3.8\text{m}$$

Alternative:



If at 320 ft dist. Away ht of ball from ground is greater 24 ft then it will clear the fence.

Method-1

Equation of trajectory at ptG

$$y = 320 - \frac{32(320)^2}{u^2} \quad \underline{\qquad\quad}(1)$$

Equation of trajectory at C

$$0 = 350\tan 45 - \frac{32(350)^2}{2u^2(\cos^2 45)}$$

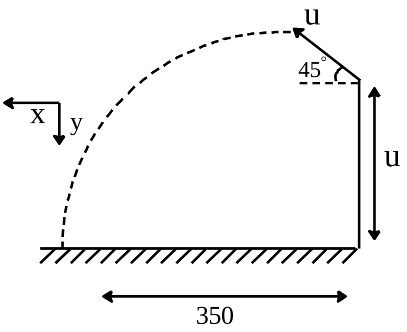
$$\frac{32(350)^2}{u^2} = 350 \Rightarrow u^2 = 32\times 350 \quad \underline{\qquad\quad}(2)$$

Using (2) in equation (1)

$$y = 320 - \frac{32\times(320)^2}{32\times(350)} = 320 - 292.57$$

$$y = 27.42$$

As $y = 27.42 > 24 \Rightarrow$ ball will clear the fence



$$4 = --350\tan 45 + \frac{32\times 350^2}{2\times 4^2\cos^2 45}$$ Equation of trajectory

$\therefore 4^2 = 11073$

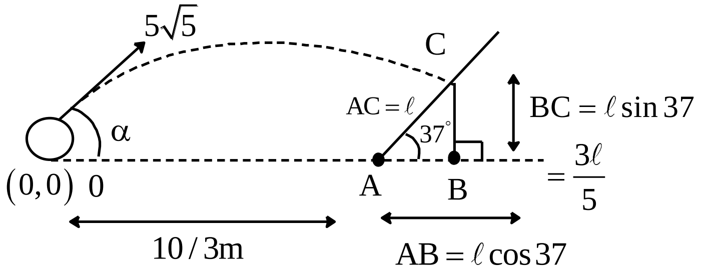
Now check y at $x = 320$.

$$y = -320\tan 4J + \frac{32\times 320^2}{24^2\times\cos^2 45}$$

$\therefore y = -24$. That means of is 24 m above origin or 28 m from ground.

So it will clear 24 feet high fence.

19. **5, 1.25**



$$\because \tan\alpha = 0.5 \quad = \frac{4\ell}{5}$$

$$= \frac{1}{2}$$

$\Rightarrow$ co-ordinates of pt. C

$$\left[\underbrace{\frac{10}{3} + \frac{4\ell}{5}}_{x}, \underbrace{\frac{3\ell}{5}}_{y}\right]$$

Use equation of trajectory at c

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\frac{3\ell}{5} = \left(\frac{10}{3} + \frac{4\ell}{5}\right)\left(\frac{1}{2}\right) - \frac{10\left(\frac{10}{3} + \frac{4\ell}{5}\right)}{2\left(5\sqrt{5}\right)^2\left(\frac{2}{\sqrt{5}}\right)^2}$$

Solve for ℓ

Ans = $x, y = \left(\frac{10}{3} + \frac{4\ell}{5}, \frac{3\ell}{5}\right)$

Alternative:

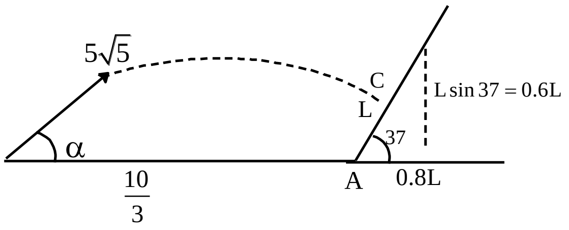
$$y = x\tan\alpha - \frac{gk^2}{2u^2\cos^2\alpha}$$

$$0.6L = \left(\frac{10}{3} + 0.8L\right)\times 0.5 - 10\times\left[\frac{10}{3} + 0.8L\right]^2$$

$$= 2\times 125\times 0.89^2$$

$$L = \frac{6.25}{3}$$

$\therefore (x, y) = 5, 1.25$



20. **5**

Advanced Booklet Solution

Kinematics - II

Inchapter Exercise

1. $x = 4t^2 – 15t + 25$

$$v = \frac{du}{dt} = 8t - 15$$

$$9 = -\frac{dv}{dt} = 8$$

At t = 0, = 25m, v = -15m/s, a = $8m/s^2$

v = 0 means 8t – 15 = 0 $\therefore t = \frac{15}{8} = 1.875s$

Acceleration does not depend on time. So, it is constant

2. $v = 2t^2 + 5$

$t_1 = 2s$, $v_1 = 12$ cm/s, $t_2 = 4s$, $v_2 = 37$ cm/s

Change in velocity = $v_2 – v_1$= 37 – 13 = 24 cm/s

(ii) Avg. acceleration = $\frac{v_2 - v_1}{t_2 - t_1} = \frac{37-13}{4-2} = 12cm/s^2$

(iii) Instantaneous acceleration

$$A_{ar} = \frac{dv}{dt} = \frac{d}{dt}(2t^2+5) = 4t$$

t = 4s, a = 16 cm/s^2

3. $t = \sqrt{x} + 3, \sqrt{x} = t - 3$

$x = (t – 3)^2 = t^2 – 6t + 9$

$$v = \frac{du}{dt} = 2t - 6$$

v =0 $\Rightarrow$ 2t – 6 = 0, $\therefore$ t = 3s

$x = (t – 3)^2$, at t = 3s, x = 0

4. $x = 2 – 5t + 6t^2$

$$v = \frac{du}{dt} = -5 + 12t$$

5. $$v = \frac{du}{dt} = 8 + 14t$$

As = 14

At t = 23, v = 8+14 x 2 = 36 m/s

A = 14

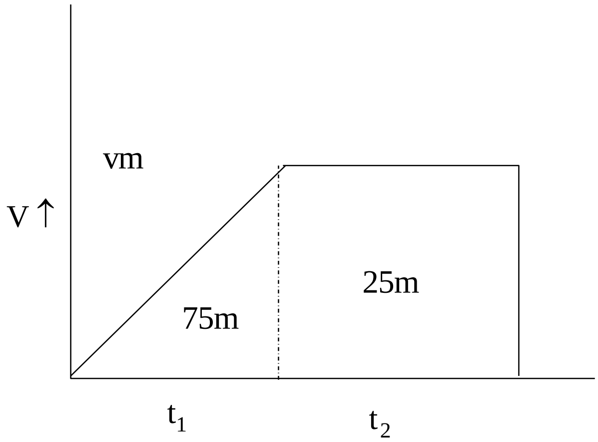
6. $$v = \frac{du}{dt} = 180 + 100t, a = 100$$

At t = 0, v = 180,

At t = 4s, v = 580

At t = 4s , a = 100

7.



$a = \frac{v_m}{t_1} = 1m/s^2$

$vm = t_1$

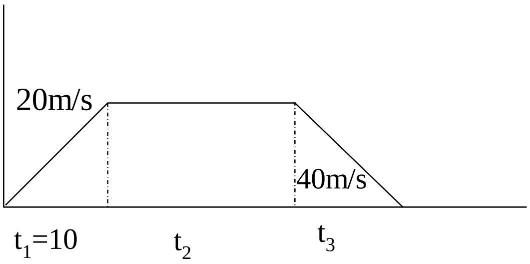
$\frac{1}{2}vmt_2 = 75$

$vmt_1 = 150 \therefore v_m = t_1 = \sqrt{150} = 5\sqrt{6}$

Now, for 1st 50m a = 1m/s^2

$5a = \frac{1}{2}\times 1\times t^2$ t = 100

8.



$\frac{1}{2}\times 20\times t_1 = 40$

$t_3 = 4$

Now ans under graph

$\frac{(2t^2 + 14)}{2}\times 20 = 640$

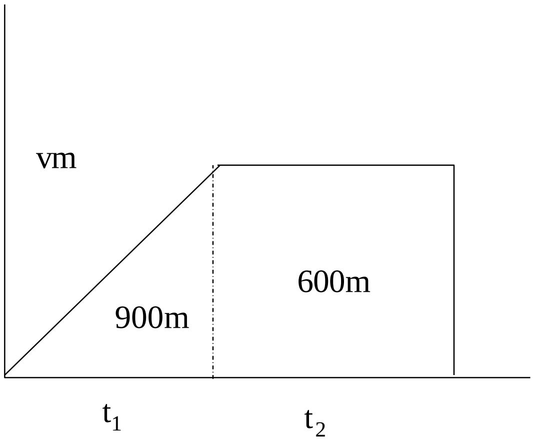
$\therefore t_2 = 25$

Total time = 10 + 25 + 4 = 39s

Accel $\frac{20}{10} = 2m/s^2$

$= \frac{20}{4} = 5m/s^2$

9.

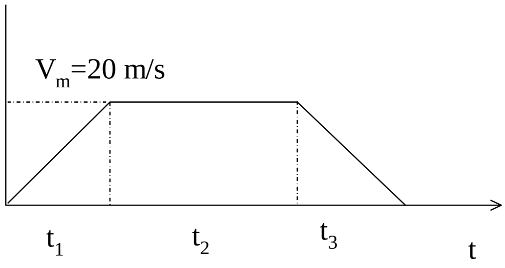


$2 = \frac{Vm}{t_1}, \frac{1}{2}Vmt_1 = 900$

$\therefore t_1 = 30s$, V_m =60m/s vmt_2 $\therefore t_2 = 10s, a = \frac{Vm}{t_1} = 2m$

Now, at centre of track S = 750 and motion : uniformly

10.



$0.1 = \frac{20}{t_1} \therefore t_1 = 200s$

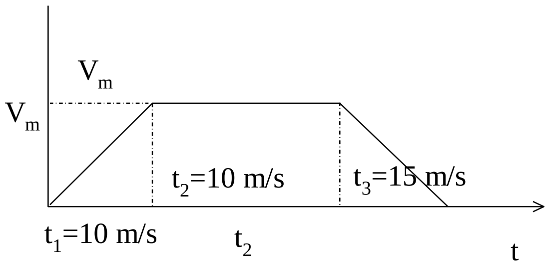
$0.2 = \frac{Vm}{t1}, t3 = \frac{20}{0.2} = 100s$

Now 5000 = $\left[\frac{2t_2 + 200 + 100}{2}\right] \times 20$

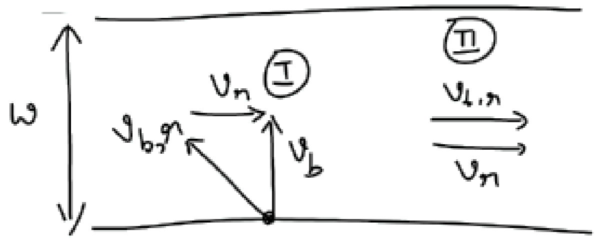
$t_2 = 1000$

$\therefore t_1 + t_2 + t_3 = 400s$

11.



12.



(I) $(\upsilon_b)t_1 = \omega$

$\sqrt{(\upsilon_{h,r})^2 (\upsilon_r)^2}\, t_1 = \omega$

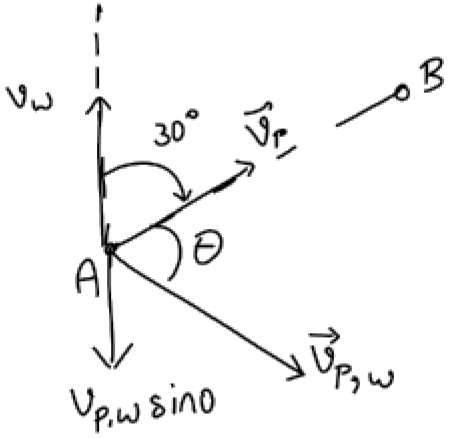
$\sqrt{\upsilon^2 - u^2}\, t_1 = \omega$ **….. (1)**

(II) $(\upsilon_r + \upsilon_{h,r})t_2 = \omega$

$(\upsilon + u)t_2 = \omega$ ...(2)

(1) / (2) $\Rightarrow \dfrac{t_1}{t_2} = \sqrt{\dfrac{\upsilon + u}{v - u}}$

13.



$v_{p,\omega\sin\theta} = \upsilon_\omega$

$\Rightarrow \sin\theta = \dfrac{1}{15}$

14. $\omega = 400\,\text{m}$

$\upsilon_r = 2\,\text{m/s}$

$\upsilon_{b,r} = 10\,\text{m/s}$

(a) $t_{\min} = \dfrac{\omega}{\upsilon_{b,r}} = 40\,s$

(b) drift $= (\upsilon_r)t = 80\,\text{m}$

15. (a) $t = \frac{\omega}{\upsilon_{b,\,r\sin\theta}} = \frac{0.5\,\text{km}}{3km/hr\,\sin\theta} = \frac{10}{\sin\theta}\,\text{mins}$

(b) $t_{\min} = 10\,\text{mins}$

16. $\omega = 0.5\,\text{km}$

$v_r = 5\,\text{km/hr}$

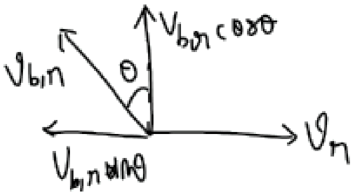
$\upsilon_{b,r} = 3\,\text{km/hr}$

For min drift $\Rightarrow$ $\sin\theta = \frac{v_{b,r}}{v_n} = \frac{3}{5} \Rightarrow \theta = 37°$

$\because\ v_r > \upsilon_{b,r}$

Zero drift not possible.

Min. drift = $\left(\upsilon_r - \upsilon_{b,r}\sin\theta\right)_{\frac{\omega}{V_{b,r}\cos\theta}} = \frac{2}{3}\text{km}$



JEE-Main Exercise

1. (c)

Acceleration $= \frac{d^2x}{dt^2} = 2a_2$

2. (d)

Velocity along *X*-axis $\text{v}_\text{x} = \frac{\text{dx}}{\text{dt}} = 2\text{at}$

Velocity along *Y*-axis $\text{v}_\text{y} = \frac{\text{dy}}{\text{dt}} = 2\text{bt}$

Magnitude of velocity of the particle,

$\text{v} = \sqrt{\text{v}_\text{x}^2 + \text{v}_\text{y}^2} = 2\text{t}\sqrt{\text{a}^2 + \text{b}^2}$

3. (c)

$\text{v} = (180 - 16\text{x})^{1/2}$

As $\text{a} = \frac{\text{dv}}{\text{dt}} = \frac{\text{dv}}{\text{dx}}.\frac{\text{dx}}{\text{dt}}$

$$\therefore a=\frac{1}{2}(180-16x)^{-1/2}\times(-16)\left(\frac{dx}{dt}\right)$$

$$=-8\,(180-16x)^{-1/2}\times v$$

$$=-8\,(180-16x)^{-1/2}\times(180-16x)^{1/2}=-8\,m/s^2$$

4. (c)

Acceleration $a=\frac{dv}{dt}=0.1\times 2t=0.2t$

Which is time dependent *i.e.* non-uniform acceleration.

5. (c)

$v_x=\frac{dx}{dt}=\frac{d}{dt}(3t^2-6t)=6t-6$. At $t=1, v_x=0$

$v_y=\frac{dy}{dt}=\frac{d}{dt}(t^2-2t)=2t-2$. At $t=1, v_y=0$

Hence $v=\sqrt{v_x^2+v_y^2}=0$

6. (c)

$y=a+bt+ct^2-dt^4$

$\therefore v=\frac{dy}{dt}=b+2ct-4dt^3$ and $a=\frac{dv}{dt}=2c-12dt^2$

Hence, at $t=0$, $v_{\text{initial}}=b$ and $a_{\text{initial}}=2c$.

7. (b)

$$a=\sqrt{a_x^2+a_y^2}=\left[\left(\frac{d^2x}{dt^2}\right)^2+\left(\frac{d^2y}{dt^2}\right)^2\right]^{\frac{1}{2}}$$

Here $\frac{d^2y}{dt^2}=0$. Hence $a=\frac{d^2x}{dt^2}=8m/s^2$

8. (c)

$$\frac{dx}{dt}=2at-3bt^2\Rightarrow\frac{d^2x}{dt^2}=2a-6bt=0\Rightarrow t=\frac{a}{3b}$$

9. (b)

$\text{Time}=\frac{\text{Distance}}{\text{Average speed}}=\frac{3.06}{0.34}=9\text{ sec}$

$\text{Acceleration}=\frac{\text{Change in velocity}}{\text{Time}}=\frac{0.18}{9}=0.02\,m/s^2$

10. (d)

$s = 3t^3 + 7t^2 + 14t + 8$ m

$a = \frac{d^2s}{dt^2} = 18t + 14$ at $t = 1 \sec \Rightarrow a = 32 m/s^2$

11. (22)

$v = 4t^3 - 2t$ (given) $\therefore\ a = \frac{dv}{dt} = 12t^2 - 2$

and $x = \int_0^t v\, dt = \int_0^t (4t^3 - 2t)\, dt = t^4 - t^2$

When particle is at 2*m* from the origin $t^4 - t^2 = 2$

$\Rightarrow\ t^4 - t^2 - 2 = 0\ \ (t^2 - 2)(t^2 + 1) = 0\ \Rightarrow\ t = \sqrt{2}$ sec

Acceleration at $t = \sqrt{2}$ sec given by,

$a = 12t^2 - 2 = 12 \times 2 - 2 = 22\ m/s^2$

12. (18)

$v = u + \int adt = u + \int (3t^2 + 2t + 2)dt$

$= u + \frac{3t^3}{3} + \frac{2t^2}{2} + 2t = u + t^3 + t^2 + 2t$

$= 2+8+4+4 = 18\ m/s$ (As $t = 2$ *sec*)

13. (0)

$3t = \sqrt{3x} + 6 \Rightarrow 3x = (3t - 6)^2$

$\Rightarrow x = 3t^2 - 12t + 12$

$v = \frac{dx}{dt} = 6t - 12$, for $v = 0, t = 2 \sec$

$x = 3(2)^2 - 12 \times 2 + 12 = 0$

14. (45)

$a = 6t + 4$

$\Rightarrow\ \frac{dv}{dt} = 6t + 4\ \Rightarrow\ \int_0^v dv = \int_0^v (6t + 4)\, dt$

$\Rightarrow\ [v]_0^v = \left[3t^2 + 4t\right]_0^t$

$\Rightarrow\ v = 0 = 3t^2 + 4t$

$\Rightarrow\ v = 3t^2 + 4t$

$v = \frac{dx}{dt} = 3t^2 + 4t$

$\Rightarrow\ \int_0^x dx = \int_0^t \left(3t^2 + 4t\right) dt$

$\Rightarrow\ [x]_0^x = \left(t^3 + 2t^2\right)_0^t$

$\Rightarrow\ x-0=t^3+2t^2$

$\Rightarrow\ x=t^3+2t^2$

At $\ t=3, x=45$

15. (4)

$$\frac{dv}{dt}=-\frac{1}{30}v^2$$

$$\Rightarrow\ \int_5^v v^{-2}\,dv=-\frac{1}{30}\int_0^t dt$$

$$\Rightarrow\ \left[-\frac{1}{v}\right]_5^v=-\frac{1}{30}[t]_0^t$$

$$\Rightarrow\ \frac{1}{v}-\frac{1}{5}=\frac{t}{30}$$

$$\Rightarrow\ \frac{1}{v}=\frac{t}{30}+\frac{1}{5}$$

$$\Rightarrow\ v=\frac{30}{t+6}$$

When $v=3\ \Rightarrow\ \dfrac{30}{t+6}=3$

$t+6=10$

$t=4\text{s}$

16. (3)

$$s=\sqrt{at^2+bt+c}$$

$$v=\frac{ds}{dt}=\frac{2at+b}{\sqrt{at^2+bt+c}}$$

Acceleration $=\dfrac{dv}{dt}$

$$\Rightarrow \text{Acceleration } =\frac{\left(\sqrt{at^2+bt+c}\right)(2a)-\dfrac{(2at+b)(2at+b)}{2\sqrt{at^2+bt+c}}}{\left(\sqrt{at^2+bt+c}\right)^2}$$

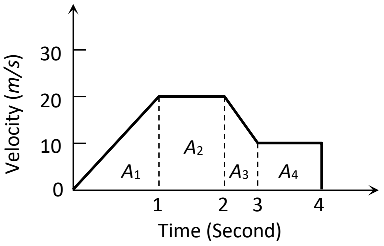
$$\Rightarrow \text{Acceleration } =\frac{4a\left(at^2+bt+c\right)-(2at+b)^2}{2\left(at^2+bt+c\right)^{3/2}}$$

$$\Rightarrow \text{Acceleration } =\frac{4ac-b^2}{\left(\sqrt{at^2+bt+c}\right)^3}=\left(4ac-b^2\right)s^{-3}$$

Acceleration $\propto s^{-3}$

17. (b)

Distance = Area under $v-t$ graph $=A_1+A_2+A_3+A_4$



$$=\frac{1}{2}\times 1\times 20+(20\times 1)+\frac{1}{2}(20+10)\times 1+(10\times 1)$$

$$=10+20+15+10=55\,\text{m}$$

18. (d)

In the positive region the velocity decreases linearly (during rise) and in the negative region velocity increases linearly (during fall) and the direction is opposite to each other during rise and fall, hence fall is shown in the negative region.

19. (a)

Distance = Area covered between graph and displacement axis $=\frac{1}{2}(30+10)10=200$ meter

20. (d)

Because acceleration due to gravity is constant so the slope of line will be constant *i.e.* velocity time curve for a body projected vertically upwards is straight line.

21. (c)

$v^2=u^2+2aS$, If $u=0$ then $v^2\propto S$

i.e. graph should be parabola symmetric to displacement axis.

22. (a)

This graph shows uniform motion because line having a constant slope.

23. (c)

From acceleration time graph, acceleration is constant for first part of motion so, for this part velocity of body increases uniformly with time and as a = 0 then the velocity becomes constant. Then again increased because of constant acceleration.

24. (c)

From given $a-t$ graph it is clear that acceleration is increasing at constant rate

$$\therefore\ \frac{da}{dt}=k\ \text{(constant)}\Rightarrow a=kt\ \text{(by integration)}$$

$$\Rightarrow \frac{dv}{dt}=kt\ \Rightarrow dv=ktdt$$

$$\Rightarrow \int dv=k\int tdt\Rightarrow v=\frac{kt^2}{2}$$

i.e. v is dependent on time parabolically and parabola is symmetric about v-axis.
and suddenly acceleration becomes zero. *i.e.* velocity becomes constant.
Hence (c) is most probable graph.

25. (c)

In first instant you will apply $\upsilon = \tan\theta$ and say, $\upsilon = \tan 30^o = \frac{1}{\sqrt{3}}$ *m*/*s*.

But it is wrong because formula $\upsilon = \tan\theta$ is valid when angle is measured with time axis.

Here angle is taken from displacement axis. So angle from time axis $= 90^o - 30^o = 60^o$

Now $\upsilon = \tan 60^o = \sqrt{3}$

26. (b)

Between time interval 20 *sec* to 40 *sec*, there is non-zero acceleration and retardation. Hence distance travelled during this interval

= Area between time interval 20 *sec* to 40 *sec*

$= \frac{1}{2} \times 20 \times 3 + 20 \times 1$ = 30 + 20 = 50 *m*.

27. (0)

Since total displacement is zero, hence average velocity is also zero.

28. (B)

$$\sqrt{v_{MR}^2 - v_M^2} = \frac{1\text{km}}{\frac{1}{4}\text{h}}$$

$$\Rightarrow \sqrt{5^2 - v_R^2} = 4$$

$$\Rightarrow v_R = 3\,\text{km/h}$$

29. (B)

$$v_{MR}\cos\theta = v_R$$

$$\frac{T_1}{T_2} = \frac{\frac{d}{v_{MR}}}{\frac{d}{(v_{MR})\sin\theta}} = \sin\theta$$

30. (C)

$$t = \frac{d}{v_{MR}\sin\theta}$$

$$\Rightarrow v_{MR}\sin\theta = \frac{60}{6} = 10\,\text{m/s}$$

$$\tan\phi = \frac{v_{MR}\sin\theta}{v_R + v_{MR}\cos\theta}$$

$$\Rightarrow \tan 45^o = \frac{10}{5 + v_{MR}\cos\theta}$$

$$\Rightarrow\ v_{MR}\cos\theta = 5$$

$$v_{MR} = \sqrt{10^2+5^2} = 5\sqrt{5}\ \text{m/s}$$

31. (B)

$$v_{MR} = (4.5\ \text{km/h})\,\hat{\mathbf{j}}$$

$$\mathbf{v}_R = (6\,\text{km/h})\,\hat{\mathbf{i}}$$

$$\mathbf{v}_M = \mathbf{v}_{MR} + \mathbf{v}_R = \left(4.5\hat{\mathbf{j}} + 6\hat{\mathbf{i}}\right)\text{km/h}$$

$$v_M = \left(\sqrt{(4.5)^2+6^2}\right)\text{km/h} = \frac{15}{2}\,\text{km/h}$$

$$= \frac{15}{2}\times\frac{5}{18}\,\text{m/s}$$

32. (B)

$$V_{MR} = 5\,\text{m/s}$$

$$V_R = 4\,\text{m/s}$$

$$t = \frac{d}{\sqrt{v_{MR}^2 - V_R^2}} = \frac{480}{3} = 160\ \text{s}$$

33. (d)

For the round trip he should cross perpendicular to the river

$\therefore$ Time for trip to that side $= \frac{1km}{4km/hr} = 0.25hr$

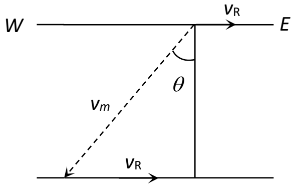
To come back, again he take 0.25 *hr* to cross the river.

Total time is 30 min, he goes to the other bank and come back at the same point.

34. (c)

For shortest possible path man should swim with an angle (90 + θ) with downstream.

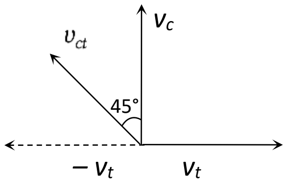
From the fig, $\sin\theta = \frac{v_r}{v_m} = \frac{5}{10} = \frac{1}{2} \Rightarrow \therefore\ \theta = 30°$



35. (b)

$$\overrightarrow{v_{ct}} = \overrightarrow{v_c} - \overrightarrow{v_t}$$

$$\overrightarrow{v_{ct}} = \overrightarrow{v_c} + (-\overrightarrow{v_t})$$



Velocity of car *w.r.t.* train (v_{ct}) is towards West – North

JEE-Advanced Exercise

EXERCISE - 1

1. (C)

Method -1 (Graph)

$acc^n = 4m/s^2 \leftarrow slope 1$

Retardation = $4m/s^2 \leftarrow slope - 3$

Disp = 20m

Slope $= \tan\theta = \frac{\perp^r}{Base}$

Slope – 1 $= \frac{\perp^r = V}{t_1} = 4 \Rightarrow t_1 = \frac{v}{4}$

Slope – 2 $= \frac{\perp^r = V}{t_2} = 4 \Rightarrow t^2 = \frac{V}{4}$

From graph

$t_1 + t' + t_2 = 6$

$t' = 6 - 2t_1 \quad (\because t_1 = t_2)$

As $t_1 = \frac{v}{4} \Rightarrow v = 4t_1$

As Area of V – t graph = disp

$\frac{1}{2}(6 - 2t_1 + 6)4t_1 = 20$

$\frac{1}{2}(6 - 2t1 + 6)4t_1 = 20$

$48t_1 - 8t_1^2 = 40$

$8t^2 - 48t_1 + 40 = 0$

$8[t_1^2 - 6t_1 + 5] = 0$

$(t_1^2 - 5t_1 - t_1 + 5) = 0$

$t_1(t_1 - 5) - 1(t_1 - 5) = 0$

$\Rightarrow t_1 = 5 \; or \; t_1 = 1$

$* t_1 = 5$ not possible

$\because t^1 = 6 - 2t_1 = -4 \; and$

Time can't be –ve

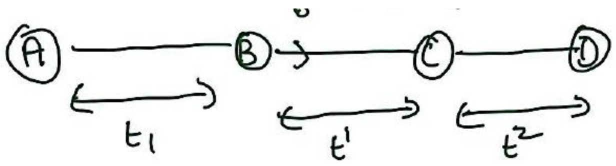
$\Rightarrow t_1 = 1$

And as $t_1 = 6 - 2t_1 = 6 - 2(1)$

$\Rightarrow t' = 4\sec$

Method -2

$u = 0 \;\; a_{AB} = 4m/s \; V_B = const = V_c \;\; V_D = 0$



$t_1 + t' + t_2 = 6\sec$

AB +BC+CCD=20m

From $AB\ t_1 = \frac{V_B - 0}{4} = V_B{\diagup}_4$

From CD = $t2 = \frac{0 - V_B}{-4} = V_B / 4$

$\Rightarrow t_1 = t_2$

2. (C)

$x = (at^2 + 26t + c)^{1/2}$

$V = \frac{dx}{dt} = \frac{1}{2}(at^2 + 2bt + c)^{\frac{1}{2}-1}[2at - 2b]$

$V = \frac{2(at+b)}{(at^2 + 2bt + c)^{1/2}} = \frac{at+b}{(at2 + 2bt + c)^{1/2}}$

Acce^n = $\frac{dv}{dt}$ (Division rule)

$$= \frac{(at^2 + 2bt + 'c)(a) - \frac{(at-b)(at+b)}{(at^2 + 2bt + c)^{1/2}}}{[(at^2 + 2bt + c)^{1/2}]^2}$$

$$= \frac{(at^2 + 2bt + c)a - (at+b)^2}{(at^2 + 2bt + c)^{3/2}}$$

$$= \frac{a^2t^2 + 2abt + ac - a^2t^2 - b^2 - 2abt}{[[at^2 + 2bt + c]^{1/2}]^3}$$

$acc^n = \frac{ac - b^2}{(x)^3} \because x = [at^2 + 2bt + c]^{1/2}$

$acc^n \propto \frac{1}{x^3} \left(\because ac - b^2 = const\right)$

3. (D)

$S^2 = t \Rightarrow S = (t)^{1/2}$

$V = \frac{ds}{dt} = \frac{1}{2}(t)^{-1/2}$

$a = \frac{dv}{dt} = \frac{1}{2}\left[\frac{-1}{2}(t)^{-3/2}\right]$

$a = \frac{-1}{4}\left[(t)^{-1/2}\right]^3$

As $V = \frac{1}{2}(t)^{-1/2} \Rightarrow t^{-1/2} = 2V$

$a = -\frac{1}{4}(2v)^3 = -2V^3$

$a \propto v^3$ and a is –ve

4. (A)

$a = \frac{\lambda}{x^2} = \frac{vdv}{dx}$

$\lambda \int \frac{dx}{x^2} = \int vdv$

$$\lambda\left[\frac{(x)^{-2+1}}{-2+1}\right]=\left[\frac{V^{2}}{2}\right]$$

$$-\lambda\left[\frac{1}{x}\right]_{\mu}^{2\mu}=\left[\frac{V^{2}}{2}\right]_{u=0}^{v}$$

$$-\lambda\underbrace{\left[\frac{1}{2\mu}-\frac{1}{\mu}\right]}=\frac{V^{2}-O^{2}}{2}$$

$$-\lambda\times\frac{-1}{2\mu}=\frac{V^{2}}{2}$$

$$\frac{\lambda}{2\mu}=\frac{v^{2}}{2}\Rightarrow V=\sqrt{\frac{\lambda}{\mu}}$$

5. (D)

$$V_{avg}=\frac{S}{t}$$

Give $v=\mu\sqrt{x}$

$$\frac{dx}{dt}=\mu\sqrt{x}\Rightarrow\int\frac{dx}{\sqrt{x}}=\mu\int dt$$

$$\left[\frac{(x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]=\mu\left[\frac{t^{0+1}}{0+1}\right]$$

$$\left[2\sqrt{x}\right]_{0}^{S}=\left[\mu t\right]_{0}^{t}$$

$$2\left(\sqrt{5}-0\right)=\mu(t-0)$$

$$t=\frac{2\sqrt{5}}{\mu}$$

$$V_{avg}=\frac{S}{t}=\frac{S}{\frac{2\sqrt{S}}{\mu}}=\frac{\mu}{2}\sqrt{S}$$

6. (B)

$$t=ax^{2}+bx$$

$$\frac{dt}{dx}=2ax+b$$

$$\Rightarrow\frac{1}{2ax+b}=\frac{dx}{dt}=v(velo)$$

$$\Rightarrow v=\frac{1}{2ax+b}$$

As, Acc^{n} = $v\frac{dv}{dx}$

$$V=\frac{1}{2ax+b}=(2ax+b)$$

$$\frac{dv}{dx}=\left[-1(2ax+b)^{-2}\right][2a(1)+0]$$

$$\frac{dv}{dx} = \frac{-2a}{(2ax+b)^2}$$

$$\Rightarrow \frac{Vdv}{dx} = \frac{1}{(2ax+b)} \times \frac{-2a}{(2ax+b)^2}$$

$$acc^n = \frac{1}{(2ax+b)^3} \times -2a$$

$$acc^n = (V^3) \times -2a \qquad \left[\because V = \frac{1}{2ax+b}\right]$$

$$acc^n = -2aV^3$$

Retarda^n = $2aV^3$

7. (A)

$$a = 5 + \delta = \frac{VdV}{ds}$$

$$\Rightarrow (5+s)ds = Vdv$$

$$5\int S0ds + \int sds = \int Vdv$$

$$5[S]_0^s + \left[\frac{S^2}{2}\right]_0^s = \left[\frac{V^2}{2}\right]_5^V$$

$$5(S-0) + \frac{S^2 - O^2}{2} = \frac{V^2 - 25}{2}$$

$$\frac{10S + S^2}{2} = \frac{V^2 - 25}{2}$$

$$V^2 = S^2 + 10S + 25$$

$$V^2 = [S+5]^2$$

$$\Rightarrow V = S + 5$$

8. (B)

$$x = -4t^2 + 8t + 2$$

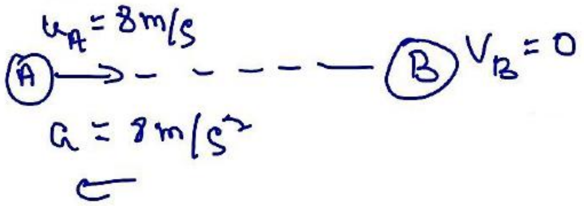
$$\frac{dx}{dt} = v = -8t + 8$$

$$\frac{dV}{dt} = a = -8 = const$$

* u = initial velocity = v at t = 0

U = -8(0)+(8)=8 $(\because V = 8t + 3)$

$$u = 8m/s$$



$$t_{AB} = \frac{V_B - U_A}{a}$$

$$t_{AB} = \frac{0.8}{-8} = 1\text{sec}$$

$\Rightarrow$dist = 2 $2S_{AB}$

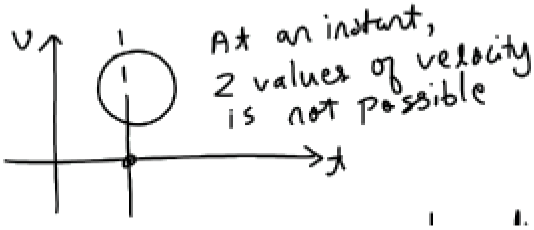
$$S_{AB}=\frac{V_B^2-U_A^2}{2a}=\frac{0^2-(8)^2}{2(-8)}=\frac{-64}{-16}$$

$S_{AB}=4$

Dis = 8m

9. (D)

(A)

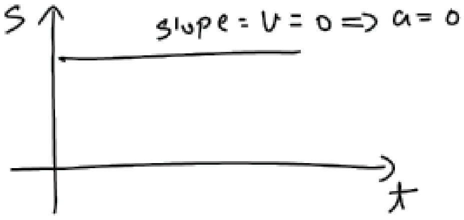


(B) area under v-t graph = displacement

(C) slope of v-t $\neq \frac{da}{dt}$, slope of v-t = a

10. (C)

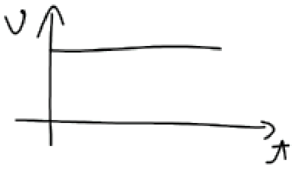
$S-t$ graph = straight line

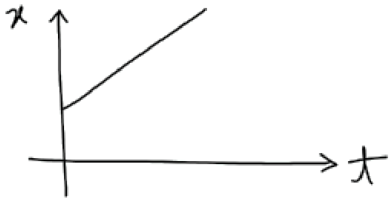


11. (A)

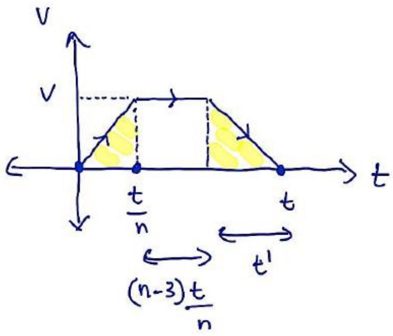
Uniform velocity

v-t graph





12. (B)



$$t^1+\frac{t}{n}+(n-3)\frac{t}{n}=t$$

$$t'=t-\frac{t}{n}[1+n-3]=\frac{nt-t(n-2)}{n}$$

$$t'=\frac{2t}{n}$$

Avg. Speed = $\frac{Total\ dist}{total\ time}\to Area$

$$V_{avg}=\frac{\frac{1}{2}\left[(n-3)\frac{t}{n}+t\right]V}{t}$$

$$=\frac{1}{2}\frac{\cancel{t}v\left(\frac{n-3+n}{n}\right)}{\cancel{t}}$$

$$=\frac{(2n-3)v}{2n}$$

13. (C)

Slope of Q at the end is greater as compared to P.

14. (D)

$$a=v\frac{dv}{ds}$$

$$a=(KS+C)K$$

$$a=k2(s)+(CK)$$

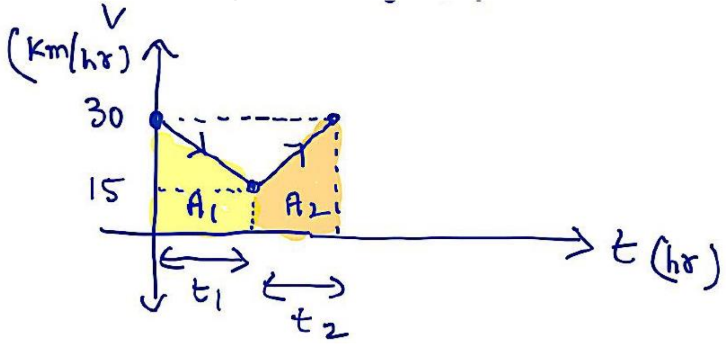
$y=mx+c' \Rightarrow$ st line

15. (A)

Average velocity $=\frac{\text{Area under graph}}{\text{time}}$

16. (D)

Method – 1 (with graph)



During retardation it covers 1km

$$\Rightarrow A_1=1km=\frac{1}{2}(30+15)t_1$$

$$t_1=\frac{2}{45}hrs$$

* During accn it convers $\frac{1}{2}km$

$$\Rightarrow A_2 = \frac{1}{2} km = \frac{1}{2}[30+15]t_2$$

$$t_2 = \frac{1}{45} hr$$

$$\Rightarrow \text{total time} = t_1 + t_2 = \frac{2}{45} + \frac{1}{45} = \frac{3}{45}$$

$$t = \frac{1}{15} hrs = \frac{60}{15} \min s = 4 \min s$$

* Without repair work train will move with (const. velo)
= 30 km/hr for 1.5 km

$$t' = \frac{1.5}{30} hr = \frac{15}{300} = \frac{1}{20} hrs$$

$$\Rightarrow t' = \frac{1}{20} \times 60 \min s = 3 \min s$$

Ans – time lost = $t - t' = 4 - 3 = 1\min$

17. (C)

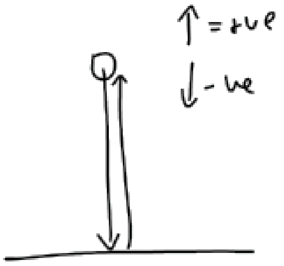
From fig. x = - sint

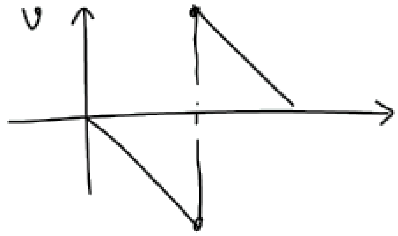
Graph is of $-\sin\theta$

$$\Rightarrow V = \frac{dx}{dt} = \frac{d(-\sin t)}{dt} = -\cos t$$

(C) option

18. (A)





19. (B)

$$\frac{V^2 - u^2}{2} = \text{Area of a – s graph}$$

$$\frac{v^2 - (0.8)^2}{2} = A_1 + A_2 + A_3$$

$$\frac{V^2 - 0.64}{2} = (0.16) + \frac{1}{2}(0.2 + 0.4)0.4 + 0.12$$

$$V^2 - 0.64 = (0.16 + 0.12 + 0.12)2$$

$$V^2 = 1.44 \Rightarrow V = 1.2 m/s$$

20. (A)

B/w PQ →slope = -ve ⇒velo. = -ve

At Q → slope = 0 ⇒ $V_Q = 0$

B/w QR →slope = +ve and ↑sing

⇒velo = *+ve and* ↑sing

B / w Rs →slope = +ve and ↓sing

⇒velo = +ve and ↓sing

At S → slope = 0 → $V_S = 0$

B/w ST → slope = -ve ⇒velo. = -ve

Conclusion

1. Initially velo. ↑ses then ↓*ses*

$\Rightarrow a_i = +ve$ and $a_f = -ve$

Also graph is not a st line implies velo. Is not constant at all implies acc$^n \neq 0$ at all

Hence (b) and (c) options eliminated

(A) $a = b - ct$ (D) $a = b + ct$ } given a and c are +ve constants

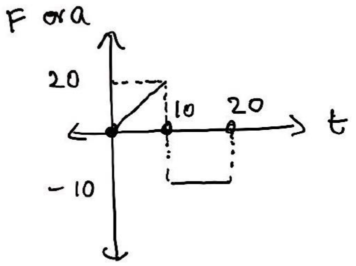
In this accn cant be –ve

In option acc^n will become –ve afer sometime.

21. (B)

$F = ma \leftarrow acc^n$

$a = \frac{F}{m} = 1mg \Rightarrow a = F$



For $V_{\max}$ → a must be +ve ie we'll consider graph till 10 sec.

Area = V – u

$\frac{1}{2} \times 10 \times 20 = V - 10 \Rightarrow V = 110 m/s$

22. (B)

$V = k_1\hat{i} + k_2 x\hat{j}$

$V_x \; V_y$

* $Vx = \frac{dx}{dt} = k_1 \Rightarrow \int dx = k_1 \int dt$

$[x]_0^x = k1[t]_0^t \Rightarrow x = k_1 t$

$\Rightarrow t = \frac{x}{k_1}$

* $V_y = k_2 x = k_2(k_1 t) \quad (\because x = k_1 t)$

$\frac{dy}{dt} = k_1 k_2 t \Rightarrow \int dy = k_1 k_2 \int t dt$

$[y]_0^y - k_1 k_2 \left[\frac{t^2}{2}\right]_0^t$

$$y=\left(\frac{k_1k_2}{2}\right)t^2$$

$$y=\frac{k_1k_2}{2}\left(\frac{x}{k_1}\right)^2 \quad \left(\because t=\frac{x}{k_1}\right)$$

$$y=\frac{k_2x^2}{2k_1}$$

23. (C)

$$\vec{v}_{A,\,B}=\left(8+\frac{v_B}{\sqrt{2}}\right)\hat{i}+\frac{v_B}{\sqrt{2}}\hat{j}$$

$$\vec{S}_{A,\,B}=6\hat{i}+4\hat{j}$$

$$\vec{a}_{A,\,B}=0$$

$$\vec{S}_{A,\,B}=\left(\vec{v}_{A,B}\right)t$$

24. (C)

$$y=ax-bx^2$$

$$=ax\left(1-\frac{bx}{a}\right)$$

$$y=x\tan\theta\left(1-\frac{x}{R}\right)$$

$$\tan\theta=a$$

$$R=\frac{a}{b}$$

EXERCISE - 2

1. (A, B, C, D)

$$x=-\beta t^3+\alpha t$$

$$V=\frac{dx}{dt}=-3\beta t^2+2\alpha t$$

$$a=\frac{dV}{dt}=-6\beta t+2\alpha$$

(A) Find time for $S=0$

$\Rightarrow x=-\beta t^3+\alpha t^2=0$

Solve and check

(B) Find time for velocity =0

ie $V=-3\beta t^2+2\alpha t=0$ solve and check

(C) u is velocity at $t=0$

As, $V=-3\beta t^2+2\alpha t$

At $t=0, v=0 \Rightarrow u=0$

As, $a=-6\beta t+2\alpha$

At $t=0 \Rightarrow a=-6\beta(0)+2\alpha=2\alpha$

$\Rightarrow \text{at}\, u=0, a\neq 0$

(D) Find time for $F_{net}=0$

ie time for $a = 0$

as $a = -6\beta t + 2\alpha = 0$

$$\Rightarrow t = \frac{\alpha}{3\beta}$$

2. (A, D)

(A) $x = t^3 - 3t^2 - gt + 5$

At $t = 0, x = +5$

$\Rightarrow$particle has some initial x co-ordinate $\left(x = 5\right)$ at $t = 0$

$$v = \frac{dx}{dt} = 3t^2 - 6t - 9$$

At $t = 0, u = 3\left(0\right)^2 - 6\left(0\right) - 9$

$\Rightarrow u = -9m/s$

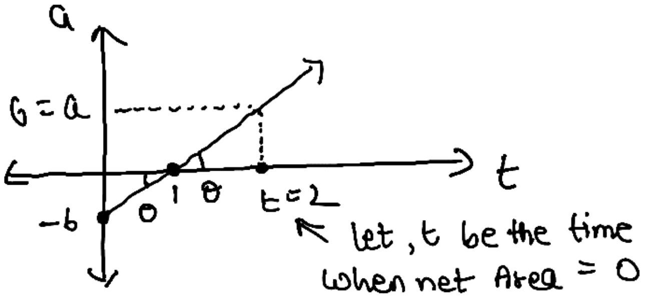
$a = \frac{dv}{dt} = 6t - 6 \leftarrow$non uniform acceleration

$\Rightarrow$ as t $\uparrow$ses acceleration $\uparrow$ses

At $t = 0, a_0 = 6\left(0\right) - 6 = -6m/s^2$

Lets find time when acceleration =0

i.e at, t=1,a=0



Area of $a - t$ graph $= V - u$

(a) Velocity must be positive i.e. $V - u = \text{Area}_{net} > 0$

$A_{net} = 0$

$\downarrow$

$A_{lower} + A_{upper} = 0$

Δ Δ

$\Rightarrow A_{upper} = \left|A_{lower\,\Delta}\right| = \frac{1}{2} \times 6 \times 1 = 3$

Using straight line property

$$\tan\theta = \frac{a}{t-1} = \frac{6}{1} \Rightarrow a = 6t - 6$$

$$\Rightarrow A_{upper\Delta} = \frac{1}{2}\left(t-1\right)\left(a\right) = \frac{1}{2}\left(t-1\right)\left(6t-6\right)$$

$A_{lower\Delta} = 3$

$$\Rightarrow \frac{\left(t-1\right)\left(6t-6\right)}{2} = 3 \Rightarrow 6\left(t-1\right)\left(t-1\right) = 6$$

$$\left(t-1\right)^2 = 1$$

$t - 1 = \sqrt{1} = 1$

$\Rightarrow t = 2 \Rightarrow a = 6t - 6 = 6m/s^2$

3. (C)

Average speed $= \frac{\text{total distance}}{\text{total time}}$

Dist = Area of v-t graph

$= \frac{\pi R^2}{2} = \frac{\pi(1)^2}{2} = \frac{\pi}{2}$

Avg. speed $= \frac{\pi/2}{2} = \frac{\pi}{4}\text{m/s}$

4. (A, B, C)

$u_1 = 40\text{m/s}\,(\text{constant})$

$u_2 = 0, a_2 = 4\text{m/s}^2$

$\vec{u}_{1/2} = 40 - 0 = 40\,\text{m/s}$

$\vec{a}_{1/2} = 0 - 4 = -4\text{m/s}^2$

$\vec{S}_{1/2} = \vec{u}_{1/2}t + \frac{1}{2}\vec{a}_{1/2}t^2$

$\vec{S}_{1/2} = 40t - \frac{4t^2}{2} = 40t - 2t^2$

Use maxima – minima concept to find $S_{1/2}$ max.

For options (b) and (c)

For option (A), $\vec{S}_{1/2} = 0$

$\Rightarrow 40t - 2t^2 = 0$

$\Rightarrow t = 20\,\text{sec}$

5. (A, B, C)

$V = \beta(x)^{1/3} \Rightarrow \frac{dv}{dx} = \frac{\beta}{3}(x)^{\frac{1}{3}-1}$

$\Rightarrow \frac{dv}{dx} = \frac{\beta}{3(x)^{2/3}}$

As, $a = V\frac{dx}{dx} = \left[\beta(x)^{1/3}\right]\left[\frac{\beta}{3(x)^{2/3}}\right]$

$a = \frac{\beta^2}{3(x)^{1/3}} \leftarrow a$ is variable

Hence (a) option is correct

Mean velocity $= V_{avg} = \frac{\text{dist}}{\text{time}}$

As, $V = \beta(x)^{1/3}$

$\frac{dx}{dt} = \beta(x)^{1/3} \Rightarrow \frac{dx}{(x)^{1/3}} = \beta dt$

$\Rightarrow \int (x)^{-1/3}dx = \beta\int dt$

$\left[\frac{(x)^{2/3}}{2/3}\right] = [\beta t]$

$\left[\frac{3}{2}(x)^{2/3}\right]_0^S = \beta[t]_0^T$

$$\frac{3}{2}(S)^{2/3} = \beta T$$

$$(S)^{2/3} = \frac{2\beta T}{3}$$

$$\Rightarrow S = \left[\frac{2\beta T}{3}\right]^{3/2}$$

$$V_{avg} = \frac{S}{T} = \frac{\left[\frac{2\beta T}{3}\right]^{3/2}}{T}$$

$$= \left(\frac{2\beta}{3}\right)^{3/2} (T)^{1/2}$$

For C option

As $\left[\frac{3}{2}(x)^{2/3}\right]_0^{x_{0t}} = \beta[t]_0^t$

$$t = \frac{3}{2\beta}(x_0)^{2/3}$$

$$V_{avg} = \frac{x_0}{t} \leftarrow \text{disp}$$

$$V_{avg} = \frac{x_0}{\frac{3}{2\beta}(x_0)^{2/3}} = \frac{2\beta}{3}(x_0)^{1/3}$$

6. (A, B, C)

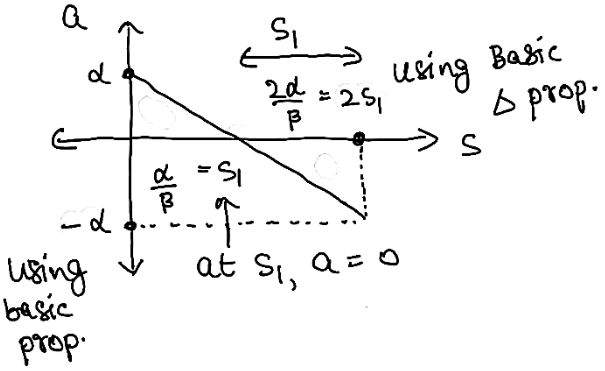
Dist. yet to be covered

$$a = \alpha - \beta S = -\beta S + \alpha$$

$$a = -\beta(s) + \alpha$$

$$\downarrow \quad \downarrow$$

$$y = m x + \alpha$$



$$\Rightarrow a = -\beta S_1 + \alpha = 0$$

$$\Rightarrow S_1 = \alpha / \beta$$

As area of $a - s$ graph $= \frac{V^2 - u^2}{2}$

and $u = 0$ ∵ starts from stat^n A.

also $V = 0$ ∵ it comes to halt at B.

$\Rightarrow$ Net Area $= 0$ when $V = 0$

Solving this $S = \frac{2\alpha}{\beta}$ from graph)

Velocity $\uparrow$ ses till acceleration ≥ 0

$\Rightarrow$ from figure V_{max} is at $S_1 = \alpha/\beta$

As $\frac{V^2 - u^2}{2} =$ Area of $a - s$ graph

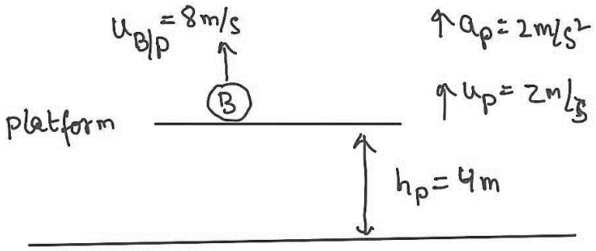
$$\frac{V_{max}^2 - 0^2}{2} = \frac{1}{2}(S_1)(\alpha)$$

$$V_{max}^2 = \left(\frac{\alpha}{\beta}\right)\alpha$$

$$\Rightarrow V_{max} = \frac{\alpha}{\sqrt{\beta}}$$

After this dis. Till station B is again S_1 which is $= \frac{\alpha}{\beta}$

7. (A, B, C, D)



$\vec{u}_P = +2m/s, \vec{u}_{B/P} = +8m/s$

$\vec{u}_B - \vec{u}_P = 8$

$\Rightarrow \vec{u}_B = 8 + \vec{u}_P = 10m/s$

$\vec{a}_P = +2m/s^2, \vec{a}_B = -10\,m/s^2$

$\Rightarrow \vec{a}_{B/P} = (-10) - (2) = -12m/s^2$

$$\vec{S}_{B/P} = \vec{u}_{B/P}t + \frac{1}{2}a_{B/P}t^2$$

$$0 = 8t - \frac{12}{2}t^2 = 8t - 6t^2$$

(means ball reaches back to platform) $\Rightarrow t = \frac{4}{3}$sec

In time $t = \frac{4}{3}, S_P = U_p t + \frac{1}{2}a_P t^2$

$$S_P = 2\left(\frac{4}{3}\right) + \frac{1}{2}(2)\left(\frac{4}{3}\right)^2 = \frac{8}{3} + \frac{16}{9}$$

$S_P = \frac{40}{9} \Rightarrow$ ht.wrt $= S_P + 4$

ground

$$h = \frac{40}{9} + 4 = \frac{76}{9}m$$

At max. ht $V_B = 0$

As, $V_B^2 - V_B^2 = 2aS$

B B

$$0^2 - (10)^2 = 2(-10)S_B$$

$$S_B = 5m$$

$\Rightarrow$ht. of ball wrt ground $= S_B + 4$ =9m

$$\vec{S}_{B/P} = \vec{U}_{B/P}t + \frac{1}{2}\vec{a}_{B/P}t^2$$

$$\vec{S}_{B/P} = 8t - \frac{12}{2}t^2 = 8t - 6t^2$$

Maximize (use concept of maxima-minima)

Alternate to find $\vec{S}_{B/P}$ max

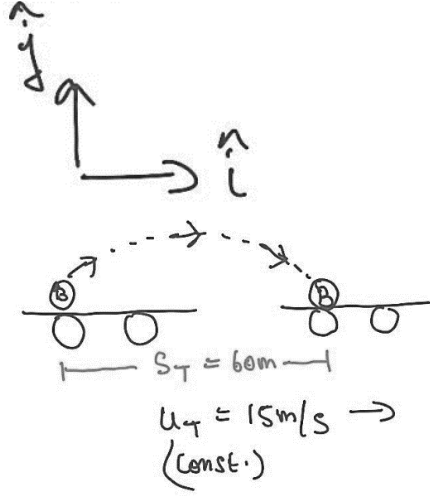
$\vec{S}_{B/P}$ will be maximum when $\vec{V}_{B/P} = 0$

Also $V_{B/P}^2 - U_{B/P}^2 = 2a_{B/P}S_{B/P}$

$$0^2 - (8)^2 = 2(-12)S_{B/P}$$

$$S_{B/P} = \frac{64}{24} = \frac{8}{3}\text{m}$$

8. (A, B, C, D)



$u_{Bx} = u_T = 15\text{m/s}$

$u_{By} = u = ?$

$S_{Bx} = S_T = 60\text{m}$

$$\Rightarrow t = \frac{S_{Bx}}{u_x} = \frac{60}{15} = 4\sec$$

$$t_{up} = \frac{t}{2} = 2\sec$$

In y axis

$V_y = u_y + a_y t$

$0 = u - 10(2) \Rightarrow u = 20$

(at ht. pt)

$\Rightarrow u_{By} = 20\,\text{m/s}$ and $u_{Bx} = 15\,\text{m/s}$

$\Rightarrow \vec{u}_B = 15\hat{i} + 20\hat{j}$

$\vec{u}_T = 15\hat{i}$

Hence $\vec{u}_{B/T} = \vec{u}_B - \vec{V}_T = 20\hat{j}$

(A) option

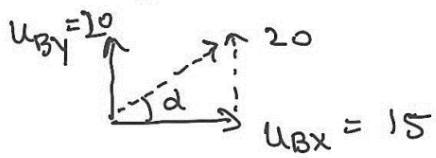
$\because \vec{u}_{B/T} = 20\,\hat{j}$

$\Rightarrow$Ball wrt truck moves along y axis

$\Rightarrow$ (B) option

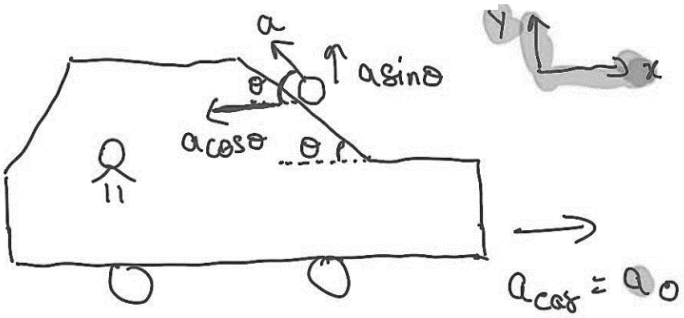
$\vec{u}_B = 15\hat{i} + 20\hat{j}$

$$\Rightarrow \left|\vec{u}_B\right| = \sqrt{15^2 + 20^2} = 25$$



$$\tan\Delta = \frac{20}{15} = \frac{4}{3} \Rightarrow \alpha = 53^\circ$$

9. (B, C)

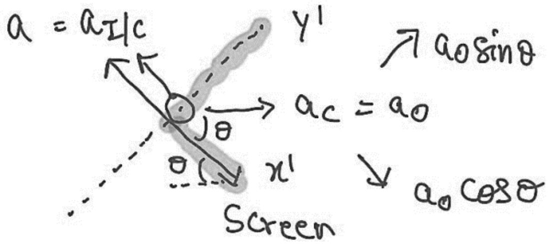


$$\vec{a}_{I/C} = -a\cos\theta\hat{i} + a\sin\theta\hat{j}$$

$$\vec{a}_c = a_0\hat{i}$$

As $\vec{a}_I = \vec{a}_{I/c} + \vec{a}_c$

$$\vec{a}_I = \underbrace{(a_0 - a\cos\theta)}_{a_{Ix}}\hat{i} + \underbrace{a\sin\theta\hat{j}}_{a_{Iy}}$$



Direction

$\perp^r$ to car

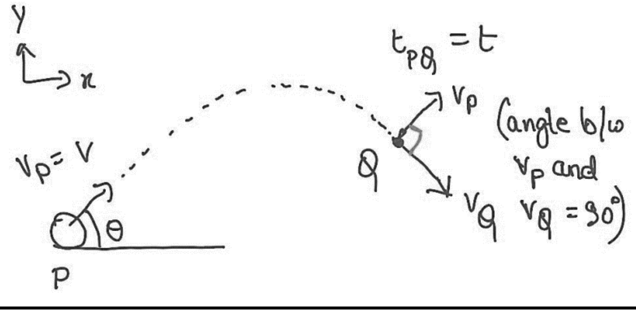
$$\Rightarrow \vec{a}_{I/C} = -a\hat{i}$$

$$\vec{a}_c = a_0\cos\theta\hat{i} + a_0\sin\theta\hat{j}$$

$$\Rightarrow \vec{a}_I = \vec{a}_{I/c} + \vec{a}_c = (a_0\cos\theta - a)\hat{i} \;\; + \underset{\uparrow}{a_0\sin\theta\hat{j}}$$

$a_I \perp^r$ to screen

10. (B, C)



$$\vec{V}_P = V\cos\hat{i} + V\sin\theta\hat{j}$$

$$\vec{V}_Q = \underbrace{V\cos\theta\hat{i}}_{(\because V_{Qx}=V_{Px})} + \underbrace{(V\sin\theta - gt)\hat{j}}_{(\because V_{Qx}=u_{py}+a_{yt\,PQ})}$$

As , $\vec{V}_P \perp^r \vec{V}_Q$

$$\Rightarrow \vec{V}_P.\vec{V}_Q = 0$$

$$\underbrace{\vec{V}_P.\vec{V}_Q}_{0} = V^2\cos^2\theta + V^2\sin^2\theta - V\sin\theta gt$$

$$0 = V^2 - V\sin\theta gt \ (\because \sin^2\theta + \cos^2\theta = 1)$$

$$V\sin\theta gt = V^2$$

$$\Rightarrow t = \frac{V}{g\sin\theta} = \frac{V\cos ec\theta}{g}$$

(C) Option

$$\vec{V}_A = V\cos\theta\hat{i} + \left[V\sin\theta - g\left(\frac{V}{g\sin\theta}\right)\right]\hat{j}$$

$$= V\cos\theta\hat{i} + \frac{V(\sin^2\theta - 1)}{\sin\theta}\hat{j}$$

$$\left|\vec{V}_Q\right| = \sqrt{(V\cos\theta)^2 + \left[\frac{V(\sin^2\theta - 1)}{\sin\theta}\right]^2}$$

11. (B, D)

$$H_{\max} = \text{same}$$

$$\Rightarrow V_y = \text{same} \ \Rightarrow T = \text{same}$$

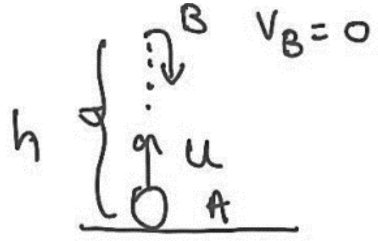
$$R_2 > R_1$$

$$R = V_x T$$

$$(u_x)_2 > (u_x)_1 \text{ and } (v_y)_2 = (v_y)_1$$

$$\Rightarrow v_2 > v_1$$

12. (A, B, C, D)



As $V_B^2 - u_A^2 = 2as$

$$0^2 - u^2 = 2(-g)h$$

$$a = \sqrt{2gh}$$

(A) $R_{\max} = \dfrac{u^2\sin(2(45^\circ))}{g} = \dfrac{u^2}{g} = \dfrac{2gh}{g} = 2h$

(B) $R = nh$

$$\frac{u^2\sin 2\theta}{g} = \frac{n u^2\sin^2\theta}{2g}$$

$$\Rightarrow 2\sin\theta\cos\theta = \frac{n\sin^2\theta}{2}$$

$$\frac{4}{n} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

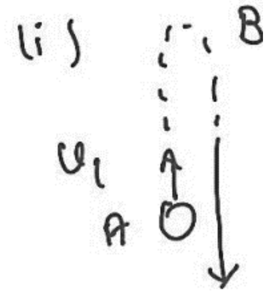
$$\theta = \tan^{-1}\left(\frac{4}{n}\right)$$

(C) $T = \frac{2u\sin\theta}{g}, R = \frac{u^2 2\sin\theta\cos\theta}{g}$

$$gT^2 = g\frac{4u^2\sin^2\theta}{g^2} = \frac{4u^2\sin^2\theta}{g}$$

$$2R\tan\theta = 2\left[\frac{u^2 2\sin\theta\cos\theta}{g}\right]\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$= \frac{4u^2\sin^2\theta}{g}$$

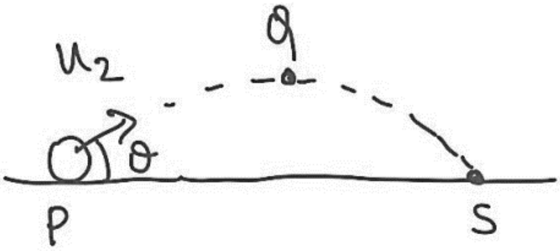


(d) (i)

$$t_{AB} = \frac{V_B - V_A}{a} = \frac{0-u_1}{-g} = \frac{u_1}{g}$$

$$T = 2t_{AB} = \frac{2V_1}{g}$$

$$S_{AB} = \frac{V_B^2 - V_A^2}{2a} = \frac{0-V_1^2}{2(-g)} = \frac{u_1^2}{2g}$$



(ii)

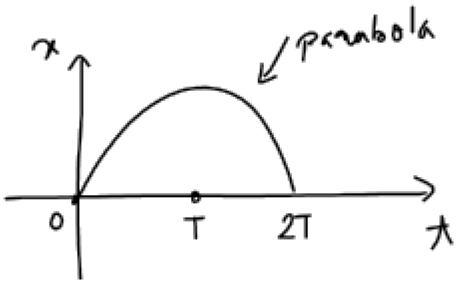
$$t_{PS} = \frac{2u^2\sin\theta}{g} = T = \frac{2u_1}{g}$$

$$\Rightarrow u_2\sin\theta = u_1$$

$$S_{PQ} = H = \frac{u_2^2\sin^2\theta}{2g} = \frac{u_1^2}{2g} = S_{AB}$$

EXERCISE - 3

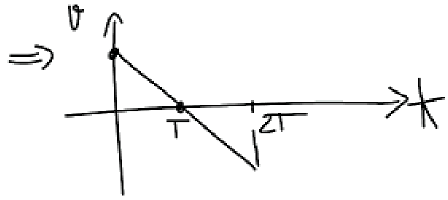
1. (B) 2. (D) 3. (A) 4. (C)

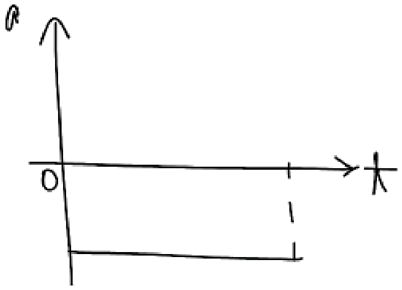


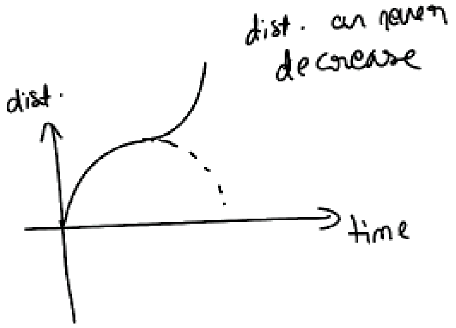
From 0 to $T \Rightarrow$ slope = +ve and |slope| is decreasing

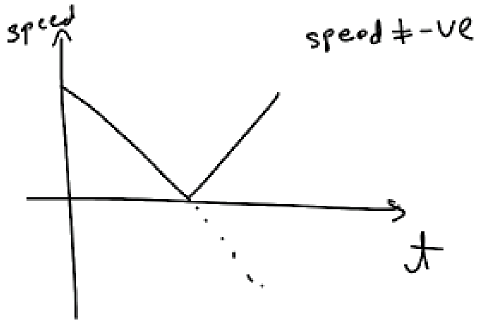
From T to $2T \Rightarrow$ slope = –ve and |slope| is increasing

Also slope = velocity

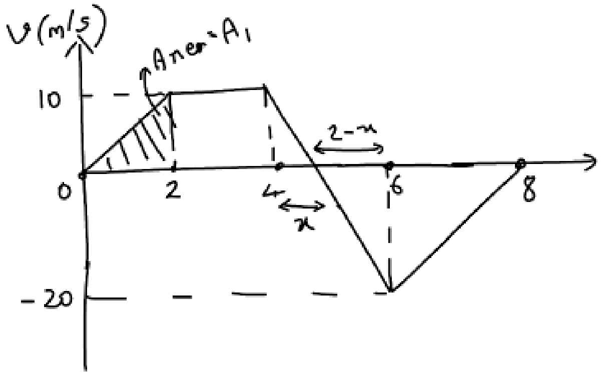








5. (B)



$$\frac{x}{10}=\frac{2-x}{20} \Rightarrow 2-x=2x$$

$$x=\frac{2}{3}=0.67$$

6. (C)

$\left|\frac{\Delta\vec{\upsilon}}{\Delta t}\right| =$ Slope magnitude man for 4 to 6 sec

7. (A)

$x_0 = -15\,\text{m}$

S for t = 2 secs

= Area (A) = $x_f - (-15)$

$x_f = \frac{1}{2} \times 2 \times 10 + (-15)$

$= -5\,\text{m}$

8. (A)

maximum displacement = area from t = 0 to t = 4.67 sec.

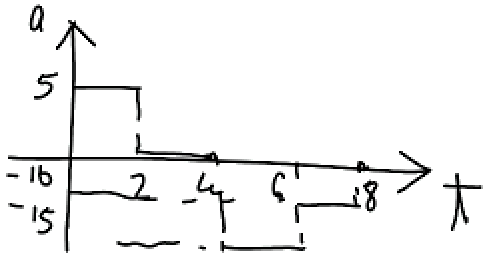
= 33.33 m

9. (A)

total distance = $\sum(\text{magnitude of area under graph})$

= 66.66 m

10. (B)



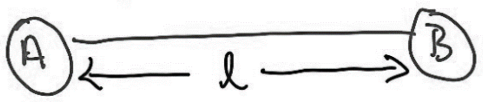
11. (C)

Slope of *s*-*t* graph = velocity

12. (B)

$V_\omega \rightarrow u$

$V_{A|\omega} \rightarrow V$



A to B = down stram

$\text{V}_\text{A} = \text{u} + \text{v}$

$\Rightarrow \text{t}_\text{AB} = \frac{\ell}{\text{u}+\text{v}}$

B to A = upstream

$\text{V}_\text{A} = \text{V} - \text{u}$

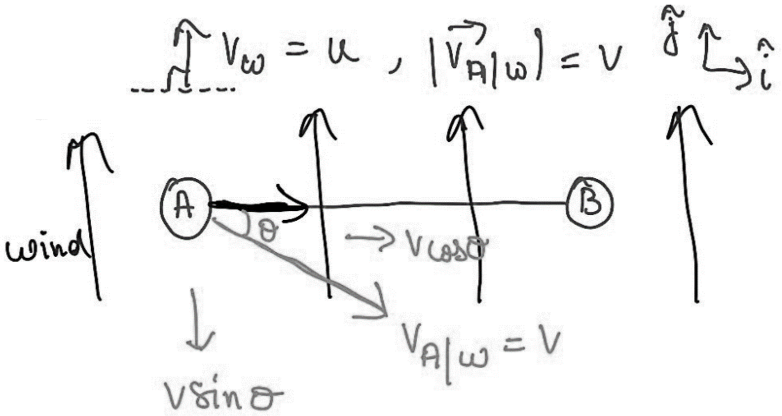
$\text{t}_\text{BA} = \frac{\ell}{\text{V}-\text{u}}$

$\text{T} = \text{t}_\text{AB} + \text{t}_\text{BA} = \frac{\ell}{\text{V}+\text{u}} + \frac{\ell}{\text{V}-\text{u}}$

$$= \frac{\ell\left(\left(V-u\right)+\left(V+u\right)\right)}{V^2-u^2}$$

$$T = \frac{2\ell x}{V^2\left(1-\frac{u^2}{V^2}\right)} = \frac{2\ell}{V\left(1-\frac{u^2}{V^2}\right)}$$

13. (C)



$\vec{V}_{\omega} = u\hat{j}$

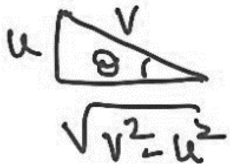
$\vec{A}_{A/\omega} = V\cos\theta\hat{i} - V\sin\theta\hat{j}$

$\vec{V}_A = \vec{V}_{A/\omega} + \vec{V}_{\omega} = \left(V\cos\theta\right)\hat{i} + \left(u - V\sin\theta\right)\hat{j}$

Moves A to B ie only along x axis

$\Rightarrow V_{Ay} = u - V\sin\theta = 0$

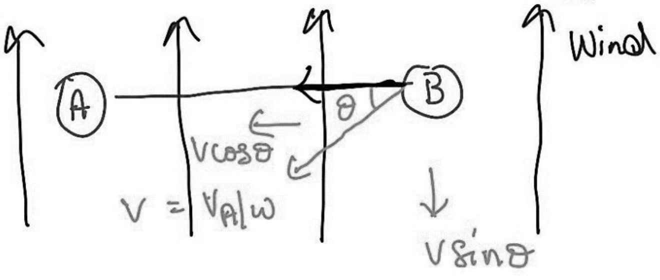
$\Rightarrow \sin\theta = \frac{u}{v}$



$\vec{V}_A = V\cos\theta\hat{i} = V\frac{\sqrt{V^2-u^2}}{V}\hat{i}$

$\vec{V}_A = \sqrt{V^2-u^2}$

$t_{AB} = \frac{\ell}{V_A} = \frac{\ell}{\sqrt{V^2-u^2}}$



Similarly, $V\sin\theta = u$

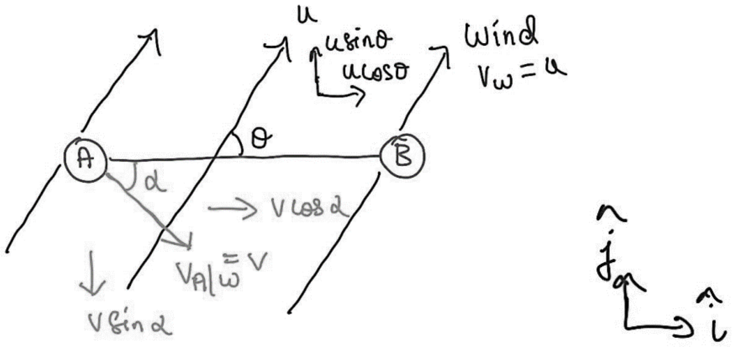
$\because$ it wants to move B to A

$\Rightarrow t_{BA} = \frac{\ell}{V_A} = V\cos\theta = \frac{\ell}{\sqrt{V^2-u^2}}$

$$\Rightarrow T = t_{AB} + t_{BA}$$

$$= \frac{2\ell}{\sqrt{V^2 - u^2}}$$

14. (A)



$$\vec{V}_{A/\omega} = V\cos\alpha\,\hat{i} - V\sin\alpha\,\hat{j}$$

$$\vec{V}_{\omega} = u\cos\theta\hat{i} + u\sin\theta\hat{j}$$

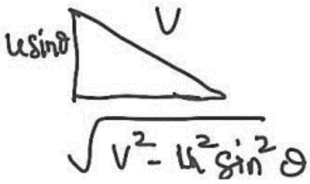
$$\vec{V}_{A} = \vec{V}_{\omega} + \vec{V}_{A/\omega}$$

$$= [V\cos\alpha + u\cos\theta]\hat{i} + \underbrace{[u\sin\theta - V\sin\alpha]}_{V_{Ay}\text{ it must}}\hat{j} = 0$$

As Aeroplane wants to fly along x axis only

$$\Rightarrow u\sin\theta - V\sin\alpha = 0$$

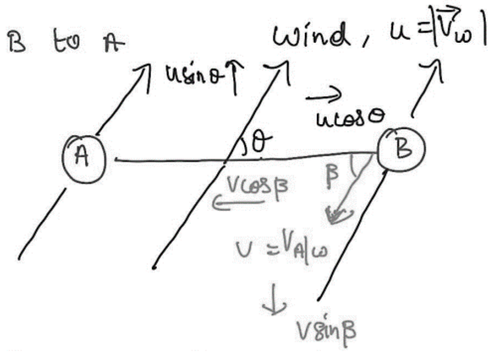
$$\Rightarrow \sin\alpha = \frac{u\sin\theta}{V}$$



$$\vec{V}_{A} = V_{Ax}\hat{i} = \left[V\left(\frac{\sqrt{V^2 - u^2\sin^2\theta}}{V}\right) + u\cos\theta\right]\hat{i}$$

$$\vec{V}_{A} = \left(\sqrt{V^2 - u^2\sin^2\theta} + u\cos\theta\right)\hat{i}\cos\alpha$$

$$t_{AB} = \frac{\ell}{V_{Ax}} = \frac{\ell}{\sqrt{V^2 - u^2\sin^2\theta} + u\cos\theta}$$



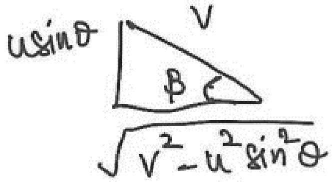
$$\vec{V}_{A/\omega} = -V\cos\beta\hat{i} - V\sin\beta\hat{j}$$

$$\vec{V}_{\omega} = u\cos\theta\hat{i} + u\sin\theta\hat{j}$$

$$\vec{V}_{A} = \vec{V}_{A/\omega} + \vec{V}_{\omega}$$

$$\vec{V}_A = \left(u\cos\theta - V\cos\beta\right)\hat{i} + \underbrace{\left[u\sin\theta - V\sin\beta\right]}_{V_{Ay}=0}\hat{j}$$

$$\Rightarrow \sin\beta = \frac{u\sin\theta}{V}$$



$$\vec{V}_A = \left[u\cos\theta - V\overset{\cos\theta}{\left[\frac{\sqrt{V^2 - u^2\sin^2\theta}}{V}\right]}\right]\hat{i}$$

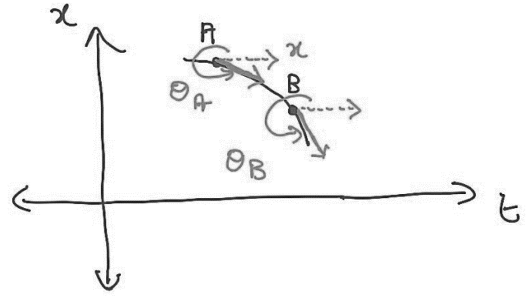
$$\vec{V}_A = u\cos\theta - \sqrt{V^2 - u^2\sin^2\theta}$$

$$t_{BA} = \frac{\ell}{V_A} = \frac{\ell}{u\cos\theta} - \sqrt{V^2 - u^2\sin^2\theta}$$

$$T = t_{AB} + t_{BA} = \frac{\ell}{4\cos\theta\sqrt{V^2 - u^2\sin^2\theta}} + \frac{\ell}{u\cos\theta} - \sqrt{V^2 - u^2\sin^2\theta}$$

<u>Match List</u> – 1 : A – p: B – p: C – q: D – R

<u>Match List</u> – 2 : A – q, s: B – p, r: C – p, r: D – q, s



(B)

From Figure $\theta_A > \theta_B$

And both θ_A and θ_B lies between 270° and 360°

As we move from 360 to 270° $\tan\theta$ becomes more positive

$\Rightarrow V_B$ is move positive then V_A

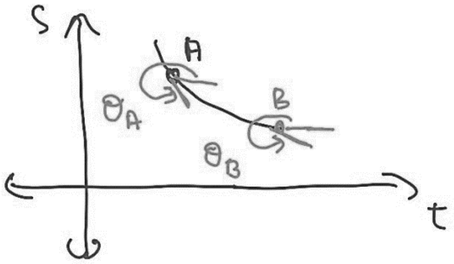
$\Rightarrow$From A to B speed ↑ ses

And acceleration is in direction of velocity is obj is accelerating

↓

Accelerating when velocity and acceleration are in same direction

Decelerating when velocity and acceleration are opposite in direction.



(D)

Both θ_A and θ_B lies between 270 and 360°

Also $\theta_B > \theta_A$

We know $\tan 330^\circ = -\frac{1}{\sqrt{3}}$

$\tan 315^\circ = -1$

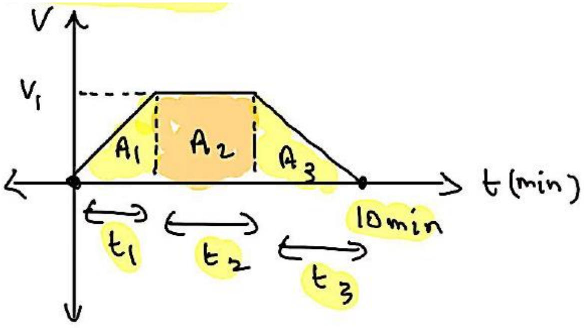
$\Rightarrow \tan\theta_B$ is less positive than $\tan\theta_A$

$\Rightarrow$ V_B is less positive than V_A and $|V_B|$ will be less than $|V_A|$

$\Rightarrow$ speed ↓ ses

EXERCISE - 4

1.



$t_1 + t_2 + t_3 = 10 \min s = \frac{10}{60} hrs = \frac{1}{6} hr$ ……(1)

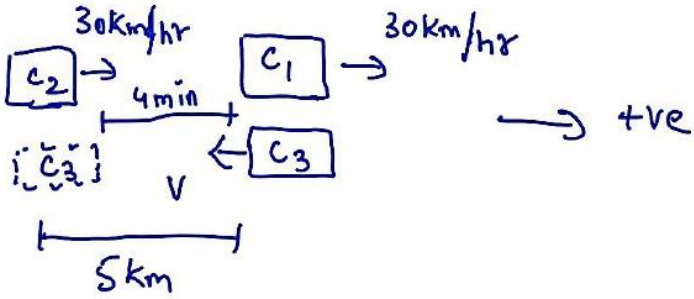
$A_1 = 1km = \frac{1}{2} v_1 t_1 \Rightarrow v_1 t_1 = 2$ ……(2)

$A_2 = 3 = v_1 t_2$ ……(3)

$A_3 = 2 = \frac{1}{2} v_1 t_3 \Rightarrow v_1 t_3 = 4$

(2) + (3) +(4) $\Rightarrow \underbrace{V_1(t_1 + t_2 + t_3)}_{V_1\left(\frac{1}{6}\right)} = 2 + 3 + 4 = g \Rightarrow v_1 = 54$

2.



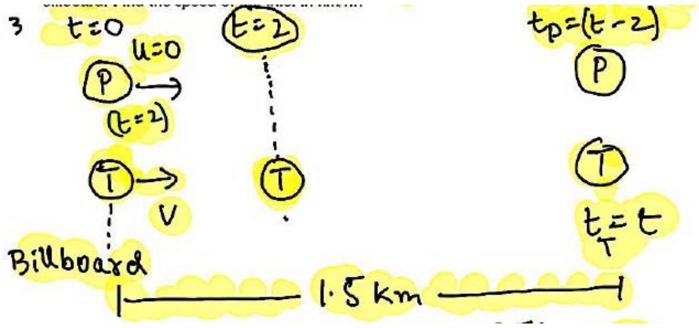
$\overrightarrow{V_{3/2}} = \overrightarrow{V_3} - \overrightarrow{V_2} = -V - 30 = -(V + 30)$

$\vec{S}_{3/2} = -5km$

$t = \frac{S_{3/2}}{V_{3/2}} = \frac{\cancel{4}hr}{\cancel{60}15} = \frac{\delta}{V + 30}$

V + 30 =75$\Rightarrow$V = 45 km/hr

3.



* From fig

$S_T - V_T t_T$

$1.5km = Vt$ ….(1)

* Policeman

U = 0

$V = 150\frac{km}{hr} = 150 \times \frac{5}{18} m/s$

$t = 12\sec$

$a_P = \frac{V-u}{t} = \frac{750/18-0}{12}$

$a_p = \frac{\cancel{750}250}{18\times\cancel{2}4} = \frac{250}{72}$

$S_P = ut + \frac{1}{2}at^2 = 0 + \frac{250}{\cancel{2}\times\cancel{72}}\overset{2}{\cancel{(12)}}$

$S_p = 250m$

$S'_p = V(t^1) \leftarrow 750/18m/s$

$t^1 = (t-2)-(12)$

$t' = t-14$

$\Rightarrow s'_P = \left(\frac{750}{18}\right)(t-14)$

$S_{P_{net}} = S_P + S_P^{'} = 250 + \frac{750}{18}(t-14)_m$

$1.5km = 1500m = 250 + \frac{750}{18}(t-14)$

$\overset{5}{\cancel{1250}} = \frac{\overset{3}{\cancel{750}}}{\underset{6}{\cancel{18}}}(t-14)$

$30 = t-14 \Rightarrow t = 44\sec$

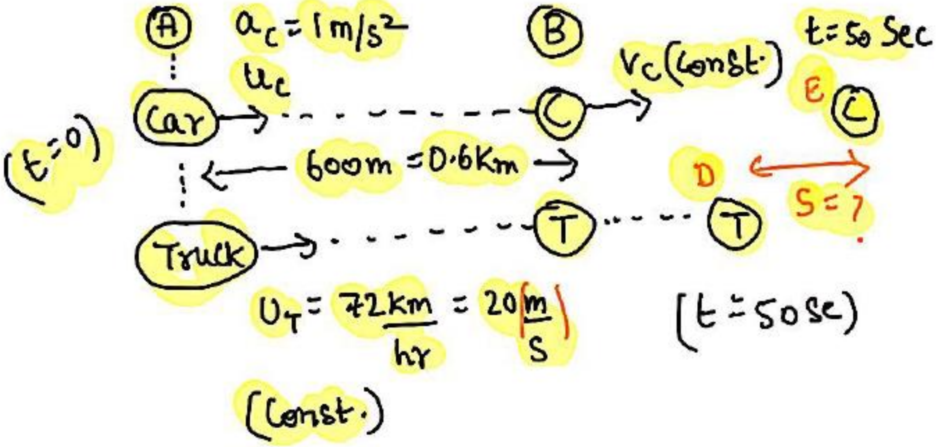
From eqn (i) $1.5km = vt$

$1500m = V(44\sec)$

$V = \frac{1500}{44}\frac{m}{s} = \frac{\overset{\cancel{300}150}{\cancel{1500}}}{\cancel{44}\,\cancel{22}\,11} \times \frac{\overset{9}{\cancel{18}}}{\cancel{5}}$

$V = \frac{1350}{11}\frac{km}{hr}$

4.



For truck A to B

$S_T = 600m, U_T = 20m/s$ $t_T = \frac{600}{20} = 30\sec$

* For car A to B

$S_C = 600m,\ a_c = 1m/s^2$

$t_C = 30\sec$

As $S = Vt - \frac{1}{2}at^2$

$$600 = V_C(30) - \frac{1}{2}(1)(30)^2$$

$$30V_C = 600 + \frac{900}{2}$$

$$V_C = \frac{1050}{30} = 35m/s$$

As $t_{AB} = 30\sec$

$\Rightarrow t = 50 - 30 = 20\sec$

$U_{rel} = (35-20) = 15m/s$

$U_{car} - U_T$

$a_{rel} = 0$ (after B)

$\delta_{rel} = U_{rel} \times t$

$= 15 \times 20 = 300m$

5. $12x = 4t^3 - 15t^2 + 12t + 6$

$$\Rightarrow x = \frac{t^3}{3} - \frac{5}{4}t^2 + t + \frac{1}{2}$$

Disp. Distance in 2 sec=?

At t = 0 $x = \frac{1}{2} = 0.5???$

$V = \frac{dx}{dt} = \frac{3t^2}{3} - \frac{5}{4}(2t) + 1$ (at t = 0)

$$\Rightarrow u = (0)^2 - \frac{5}{2}(0) + 1 = 1m/s$$

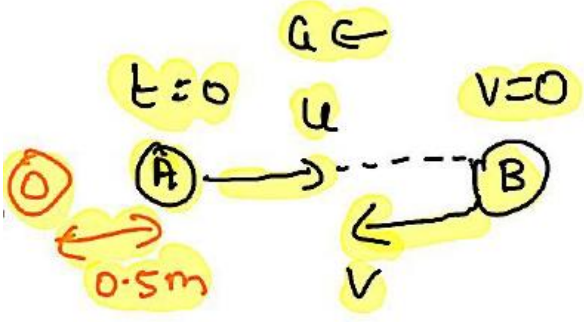
* $a = \frac{dv}{dt} = 2t - \frac{5}{2} \Rightarrow a = 2t - \frac{5}{2}$

$\Rightarrow acc^n$ at t = 0, $a_0 = 2(0) - \frac{5}{2}$

$\Rightarrow a0 = \frac{-5}{2} m/s^2$

At t = 0, y = 1m/s, $a = -\frac{5}{2} m/s^2$

i.e acc^n is oppo. to u hence it will make particle return after some time.
Fig, will be like pt of consideration



$\Rightarrow dist \neq disp$

Also $a = 2t - \frac{5}{2} \neq$ const. Hence Equation of motion cant be applied

1. Lets find t when v =0

As $V = t^2 - \frac{5}{2}t + 1$

$\Rightarrow t^2 - \frac{5}{2}t + 1 = 0$

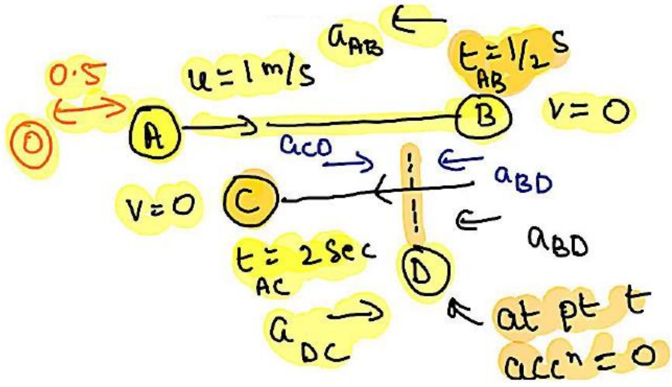
$\frac{2t^2 - 5t + 2}{2} = 0 \Rightarrow 2t^2 - 4t - t + 2 = 0$

$2t(t-2) - 1(t-2) = 0$

$(t-2)(2t-1) = 0$

$\Rightarrow t = \frac{1}{2} \; or \; t = 2$

Hence fig will



$\because$ it changes its direction at D

As $a = 2t - \frac{5}{2}$

$a_0 = 0 = 2t - \frac{5}{2} \Rightarrow t_{AD} = \frac{5}{4}\sec$

Dist = AB + BC $\left(2 - \frac{1}{2}\right)$

$t_{AB} = \frac{1}{2}\sec \quad t_{BC} = \frac{3}{2}\sec$

Disp will be wrt pt o and not wrt pt A

$$S = x = \frac{t^3}{3} - \frac{5}{4}t^2 + t + \frac{1}{2}$$

$$S_{OB} = \frac{1}{3}\left(\frac{1}{2}\right)^3 - \frac{5}{4}\left(\frac{1}{2}\right)^2 + \frac{1}{2} + \frac{1}{2}$$

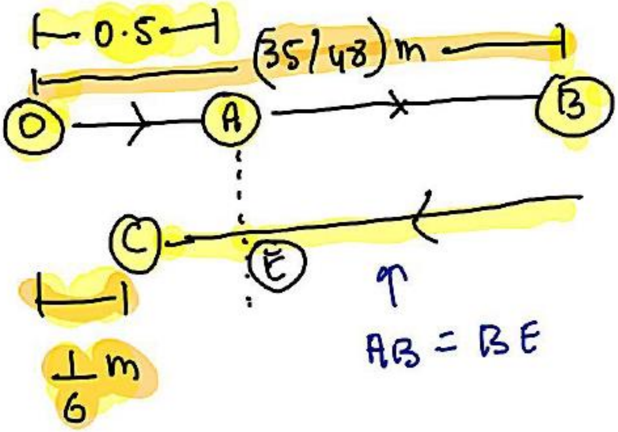
$$= \frac{1}{3} \times \frac{1}{8} - \frac{5}{4} \times \frac{1}{4} + 1$$

$$S_{OB} = \frac{2-15+48}{48} = \frac{35}{48}m$$

Disp. Fro 2 sec = S_{OC}

$$S_{OC} = \frac{1}{3}(2)^3 - \frac{5}{4}(2)^2 + 2 + \frac{1}{2}$$

$$= \frac{32-60+24+6}{12} = \frac{1}{16}m$$



Dist = AB +BC (AB = OB – OA)

$$AB = \frac{35}{48} - 0.5 = \frac{35-24}{48} = \frac{11}{48}m$$

$$EC = OA - OC = 0.5 - \frac{1}{6} = \frac{3-1}{6} = \frac{1}{3}m$$

$$\Rightarrow BC = EC + BE = \frac{1}{3} + \frac{11}{48} = \frac{16+11}{48} = \frac{27}{48}$$

$$\Rightarrow \text{dist} = \frac{27}{48} + \frac{11}{48} = \frac{38}{48} = \frac{19}{24}$$

6. $v = ut^3 - 2t + 1$

$$\frac{ds}{dt} = ut^3 - 2t + 1$$

$$\int ds = u\int t^3 dt - 2\int t dt + \int dt$$

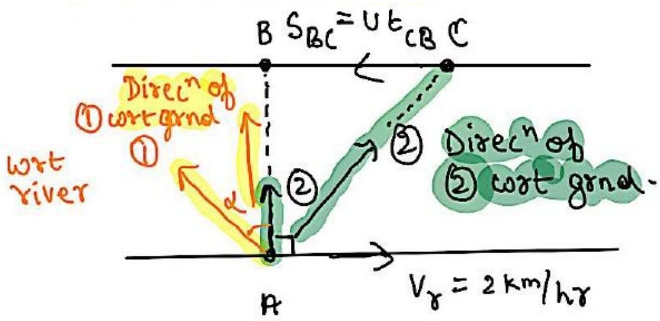
$$[S]_0^{S_{3^{rd}}} = \left[\frac{\cancel{4}t4}{\cancel{4}}\right]_2^3 - \left[\frac{\cancel{2}t^2}{\cancel{2}}\right]_2^3 + [t]_2^3$$

$$S_{3^{rd}} = (81-16) - (9-4) + (3-2)$$

$$= 65 - 5 + 1 = 61cm$$

$$V_{avg} = \frac{S_{3^{rd}}}{t} = \frac{61}{1} = 61cm/s$$

7.



$$\left|\vec{V}_{1/r}\right| = \left|\vec{V}_{2/r}\right| = 2.5\frac{km}{hr}$$

Let width of river = d

$$\vec{V}_{m/r} = 2.5\sin d\hat{i} + 2.5\cos\alpha\hat{j}$$

$$* t_1 = t_2, \vec{v}_r = 2i$$

$$\text{* For 1} \Rightarrow \vec{V}_m = \vec{V}_{m/r} + \vec{V}_r$$

$$= (2.5\sin\alpha + 2)\hat{i} + 2.5\cos d\hat{j}$$

$$\Rightarrow V_{mx} = 0 \Rightarrow -2.5\alpha + 2 = 0$$

$$\Rightarrow \sin\alpha = 2/2.5 = 4/5 \Rightarrow \alpha = 53^0$$

$$t_1 = \frac{width}{V_{my}} = \frac{d}{2.5\cos(\alpha = 53^0)}$$

$$t_1 = \frac{d}{2.5\times\frac{3}{5}} \Rightarrow t_1 = \frac{d}{1.5}$$

* For 2

$$\vec{V}_{m/r} = 2.5j, \vec{V}_r = 2\hat{i}$$

$$\Rightarrow \vec{V}_m = \vec{V}_{m/r} + \vec{V}_r = 2\hat{i} + 2.5\hat{j}$$

$$t_{AC} = \frac{S_y}{V_{my}} = \frac{d}{2.5}$$

$$S_{x_{\max}} = V_{m_x} t_{AC} = 2\times\frac{d}{2.5} = \frac{4d}{5}$$

BC = 4d/5

As $t_1 = t_2$

$$\frac{d}{1.5} = t_{AC} + t_{CB}$$

$$\frac{d}{1.5} = \frac{d}{2.5} + t_{CB}$$

$$t_{CB} = d\left(\frac{1}{1.5} - \frac{1}{2.5}\right) = d\left(\frac{5-3}{7.5}\right)$$

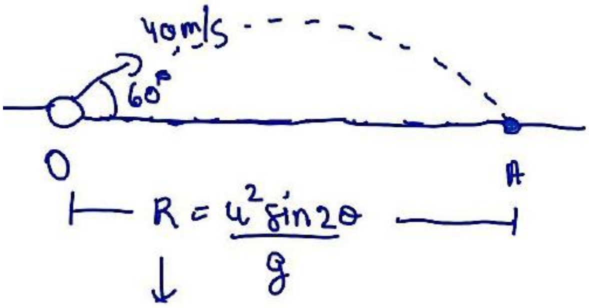
$$t_{CB} = \frac{2d}{7.5}$$

$$BC = \frac{4d}{5} = u\,t_{CD}$$

$$\Rightarrow \frac{\cancel{4d}^{\,2}}{\cancel{5}} = u\frac{\cancel{2d}}{\cancel{7.5}1.5} \Rightarrow u = 3m/s$$

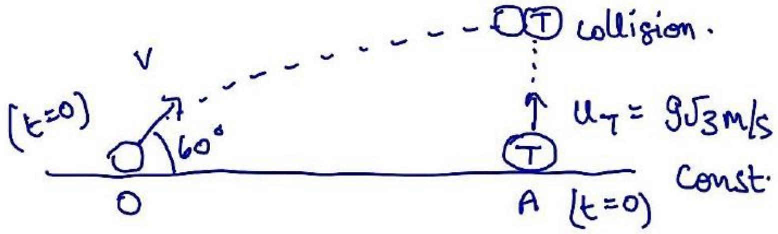
8.

1.



$$OA=\frac{40\times 40\sin(2\times 60)}{10}=160\times\frac{\sqrt{3}}{2}=80\sqrt{3}$$

2.



* For collision

\# $S_{y_{shell}}=S_{yTarget}$

$$uyt+\frac{1}{2}ayt^2=(uy_T)t\ \leftarrow\text{const. velo}$$

$$V\sin 60t-\frac{gt^2}{2}=9\sqrt{3}t \qquad \text{….(1)}$$

\# Also $S_{X_{shell}}=OA$ (in time t)

$$u_x t=80\sqrt{3}$$

$$V\cos 60t=80\sqrt{3}$$

$$\frac{V}{2}t=80\sqrt{3}\Rightarrow V=\frac{160\sqrt{3}}{t} \qquad \text{…..(2)}$$

Using (2) in (1)

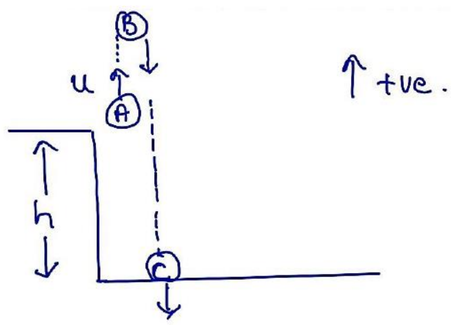
$$\left[\frac{160\sqrt{3}}{t}\right]\sin 60t-\frac{gt^2}{2}=9\sqrt{3}t$$

$$\overset{80}{\cancel{160}}\sqrt{3}\times\frac{\sqrt{3}}{\cancel{2}}-\frac{10t^2}{2}=9\sqrt{3}t$$

$$240-5t^2=9\sqrt{3}t\Rightarrow 5t^2+9\sqrt{3}t\times 240=0$$

$$t=\frac{-9\sqrt{3}\pm\sqrt{243+4800}}{10},t=\frac{\sqrt{5043}-9\sqrt{3}}{10}$$

9.



$$V_{avg} = \frac{S}{t}$$

$$5 = \frac{S}{5}$$

$$\Rightarrow S = 25$$

i.e. H=25m

$$S_y = ut + \frac{1}{2}at^2$$

$$-25 = u\times 5 + \frac{(-\overset{5}{\cancel{10}})(5)^2}{\cancel{2}}$$

5u = 125 – 25 = 100

$$u = 2m/s$$

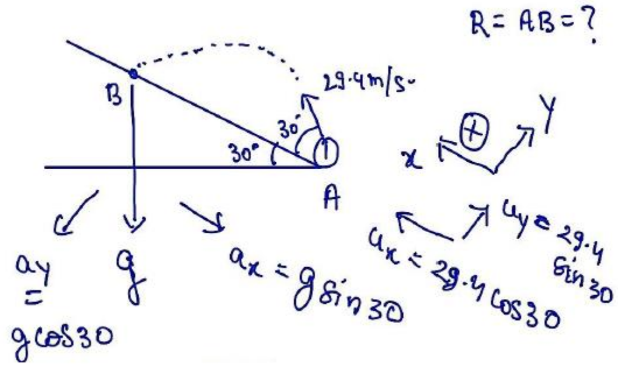
$$S_{AB} = \frac{V_B^2 - U_A^2}{2a} = \frac{0^2 - 2O^2}{2(-10)} = 20$$

Dist. AC = AB +BA +AC

= 20 + 20 + 25 = 65

Speed$_{avg}$ $= \frac{65}{5} = 13m/s$

10.



Find time using y axis

$$T = \frac{2u\sin\theta}{g\cos\alpha} = \frac{2(25.4)\sin 30}{(9.8)\cos 30}$$

$$T = 2\sqrt{3}$$

* $R = S_x = u_x T + \frac{1}{2}a_x T^2$

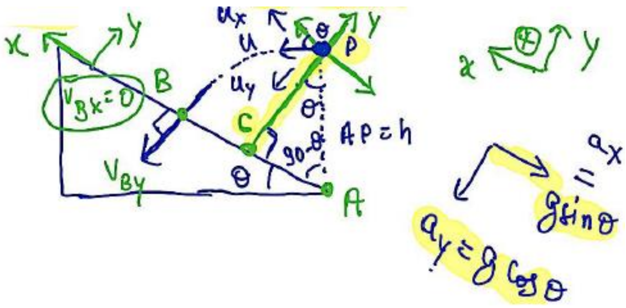
$$R = (29.4\cos)2\sqrt{3} - \frac{g\sin 30}{2}\left(2\sqrt{3}\right)^2$$

$$R = 29.4\times\frac{\sqrt{3}}{\cancel{2}}\times\cancel{2}\sqrt{3}$$

$$-\frac{3.8}{\cancel{2}}\times\frac{1}{\cancel{2}}\times\cancel{4}\times 3$$

R = 88.2 – 29.4
= 58.8m

11.



1. From fig

$$u_x = u\cos\theta$$

$$u_y = -u\sin\theta$$

$$a_x = -g\sin\theta$$

$$a_y = -g\cos\theta$$

2. $V_x = 0$

3. Using $\times$axis

$$T_{PB} = \frac{Vx - ux}{ax}$$

$$T_{PB} = \frac{0 - u\cos\theta}{-g\sin\theta}$$

$$T_{PB} = \frac{u\cos\theta}{g\sin\theta}$$

4. Also from fig.

$$S_y = PC = h\cos\theta$$

From ΔAPC It will be –ve

Also $S_y = u_y t + \frac{1}{2}a_y t^2$ $(-h\cos\theta)$

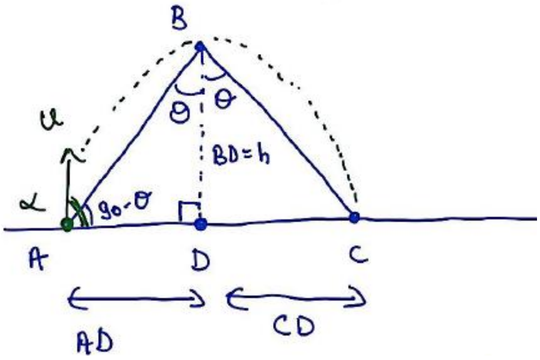
$$+h\cos\theta = +u\sin\theta\frac{u\cos\theta}{g\sin\theta} + \frac{g\cos\theta}{2}\left[\frac{u\cos\theta}{g\sin\theta}\right]^2$$

$$h\cos\theta = \frac{u^2\cos\theta}{g} + \frac{g\cos\theta}{2}\frac{u^2\cos^2\theta}{g^2\sin^2\theta}$$

$$h = \frac{u^2}{g}\left[1 + \frac{\cot 2\theta}{2}\right]$$

$$u = \sqrt{\frac{2gh}{2+\cot^2\theta}}$$

12.



From $\Delta ADB,\ \tan\theta = \dfrac{AD}{BD = h}$

$\Rightarrow AB = h\tan\theta$

Similarly, CD = $h\tan\theta$

$\Rightarrow AC = AD + CD = 2h\tan\theta$

* From Projectile motion

$$h = \frac{u^2\sin^2\alpha}{2g},\ Rang(R) = \frac{u^2\sin 2\alpha}{g}$$

Also from fig,

Rang (R) = AC

$$\frac{u^2\sin 2\alpha}{g} = 2h\tan\theta$$

$$\frac{\cancel{u}^2\,2\sin\alpha\cos\alpha}{\cancel{g}} = 8\frac{\cancel{u}^2\sin^2\alpha}{2\cancel{g}}\tan\theta$$

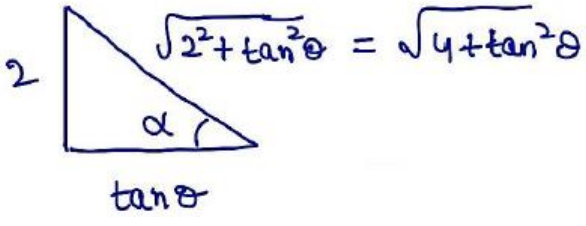
$$2\times\frac{\cos\alpha}{\sin\alpha} = \tan\theta$$

$\Rightarrow \tan\theta = 2\cot\alpha \leftarrow$ Hence proved

As $h = \dfrac{u^2\sin^2\alpha}{2g}$

And as $2\cot\alpha = \tan\theta$

$\Rightarrow \cot\theta = \dfrac{\tan\theta}{2}$

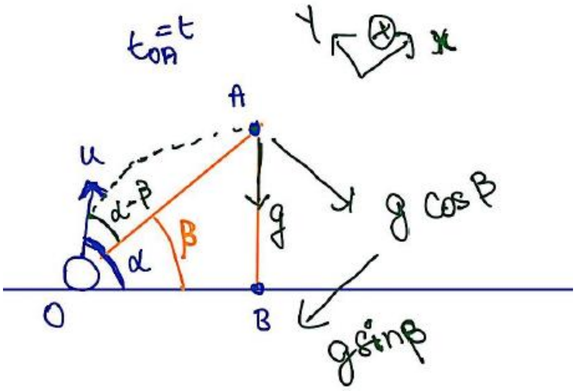


As $h = \dfrac{u^2\sin^2\alpha}{2g} = \dfrac{u^2}{2g}\left[\dfrac{2}{\sqrt{4+\tan^2\theta}}\right]^2$

$$\cancel{2}gh = u^2\left(\frac{\cancel{4}2}{4+\tan^2\theta}\right)$$

$u^2 = \dfrac{gh(4+\tan^2\theta)}{2} \Rightarrow$ Hence proved

13.



- OA becomes incline plane, inclined at angle β wrt horizontal
- Also, let OA be X axis
- $u_x = u\cos(\alpha-\beta)$
  $uy = u\sin(\alpha-\beta)$
- Angle of projection wrt incline $= \alpha - \beta$

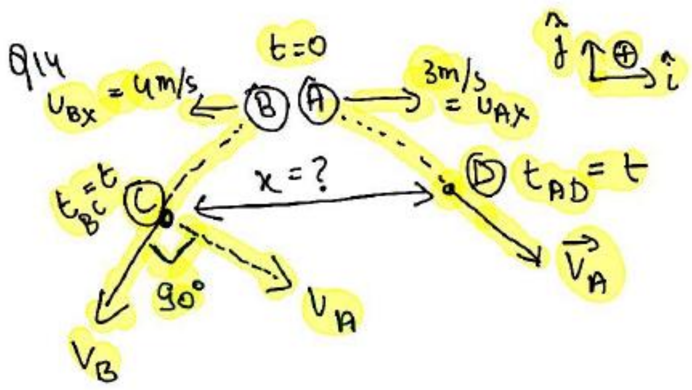
From y axis

$$T = t_{OA} = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

$$t = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

$$\Rightarrow u = \frac{gt\cos\beta}{2u\sin(\alpha-\beta)}$$

14.



*** UY for both = 0 hence their S_y will be equal at all instance $\left[\because S_y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow Sy = -\frac{gt^2}{2}\right]$

$\Rightarrow CD = x =$ Horizontal dist. Below them

$$x = |-4t\hat{i}| + |3t\hat{i}|$$

Dist = 3t + 4t = 7t

For particle A

$\vec{V}_A = V_{AX}\hat{i} + V_{AY}\hat{j}$ at time

$= 3\hat{i} + (U_{Ay} + a_y t)\hat{j}$

$= 3\hat{i} + (0 + 10t)\hat{j}$

$\vec{V}_A = 3\hat{i} - 10t\hat{j}$

For Particle B

$\overrightarrow{V_B}=V_{Bx}\hat{i}+V_{By}\hat{j}$ at time t

$=-4\hat{i}+(u_{By}+a_yt)\hat{j}$

$=-4\hat{i}+(0-10t)\hat{j}$

$\overrightarrow{V}_B=-4\hat{i}-10t\hat{j}$

As $\overrightarrow{V_A}$ is $\perp^r$ *to* $\overrightarrow{V_B}$

$\Rightarrow\overrightarrow{V_A}.\overrightarrow{V_B}=0$

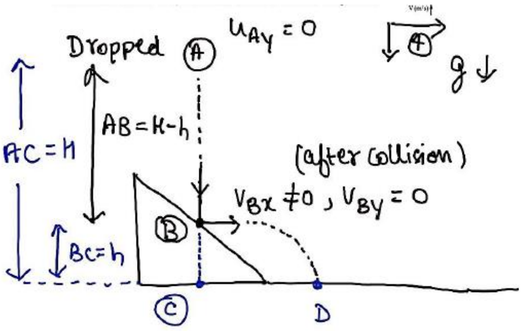
$\left(3\hat{i}+10t\hat{j}\right).(-4\hat{i}-10t\hat{j})=0$

$-12+10t^2=0$

$t^2=\frac{\cancel{12}\,3}{\cancel{100}\,25}\Rightarrow t=\frac{\sqrt{3}}{5}$

$\Rightarrow Dist=7t=\frac{7\sqrt{3}}{5}$

15.



* AB motion

$u_{Ay}=0, S_{ABY}=+(H-h)$

$a_y=g$

As $S=ut+\frac{1}{2}at^2$

$H-h=0+\frac{g}{2}t_{AB}^2$

$\Rightarrow t_{AB}=\sqrt{\frac{2(H-h)}{g}}$

* BD motion

Again,, $u_{By}=0, a_y=g, S_y=h$

As $\delta=ut+\frac{1}{2}at^2$

$h=0+\frac{g}{2}t_{BD}^2\Rightarrow t_{BD}=\sqrt{\frac{2h}{g}}$

$\Rightarrow$Total time $(t_{AD})=T=t_{AB}+t_{BD}$

$T=\sqrt{\frac{2(H-h)}{g}}+\sqrt{\frac{2h}{g}}$

Maximize

Use concept of maxima

$T=\sqrt{\frac{2}{g}}\left[\sqrt{H-h}+\sqrt{h}\right]$

Step - 1 $\frac{dT}{dh}=\sqrt{\frac{2}{g}}\left[\frac{1}{2}(H-h)^{-1/2}(0-1)+\frac{1}{2}(h)\right]^{-1/2}$

$\frac{dT}{dh}=\sqrt{\frac{2}{g}}\left[\frac{-1}{2\sqrt{H-h}}+\frac{1}{2\sqrt{h}}\right]$

Step -2 $\frac{dT}{dh}=0$

$\Rightarrow\sqrt{\frac{2}{g}}\left[\frac{-1}{2\sqrt{H-h}}+\frac{1}{2\sqrt{h}}\right]=0$

$\Rightarrow\frac{-1}{2\sqrt{H-h}}+\frac{1}{2\sqrt{h}}=0$

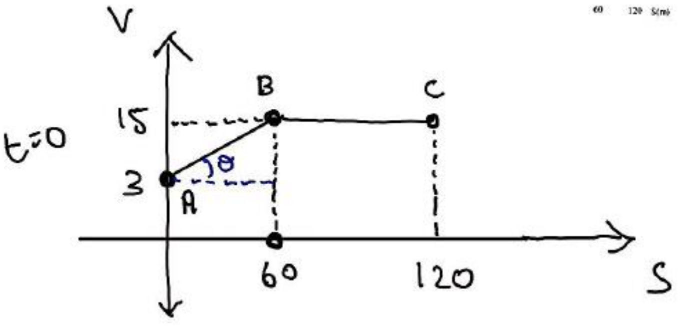
$\frac{1}{\cancel{2}\sqrt{h}}=\frac{1}{\cancel{2}\sqrt{H-h}}$

Squaring both sides

$\frac{1}{h}=\frac{1}{H-h}\Rightarrow H-h=h$

H = 2h or $\frac{h}{H}=\frac{1}{2}$

16.



As $a=v\frac{dv}{ds}$

1. For AB-

*St line with +ve slope

Slope $=\frac{dv}{ds}=\frac{15-3}{60}=\frac{1}{5}$

* Eq. of AB $\Rightarrow y=mx+C$

$V=\frac{1}{5}(S)+3$

$\Rightarrow V=\frac{S}{5}+3$

As $acc^{n}(a)=v\frac{dv}{ds}$

$(a)_{AB}=\left(\frac{S}{5}+3\right)\left(\frac{1}{5}\right)$

$\Rightarrow a_{AB}=\frac{S}{25}+\frac{3}{5}$

Again it’s a st. line for a – s graph with slope $=\frac{1}{25}$ and intercept = $\frac{3}{5}$

2. For BC

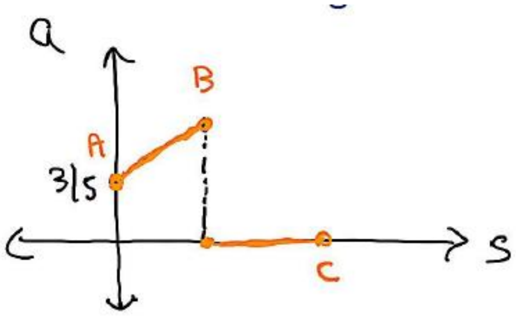
* St line with slope $= \frac{dv}{ds} = 0$

As, accn (a) = $v\frac{dv}{ds}$

$\Rightarrow a_{BC} = 0$

$\Rightarrow$ Acc^n vs disp graph

$a_{AB} = \frac{S}{25} + \frac{3}{5}, a_{BC} = 0$



Calculate of time

$t = t_{AB} + t_{BC}$

(i) $t_{AB} = ?$

$V = \frac{S}{5} + 3 = \frac{S+15}{5}$

$\frac{ds}{dt} = \frac{S+15}{5} \Rightarrow \int \frac{ds}{S+15} = \frac{1}{5}\int dt$

Let P = S + 15

$\frac{dp}{ds} = 1 \Rightarrow ds = dp$

$\Rightarrow \int \frac{ds}{S+15} = \int \frac{dp}{P} = LnP = Ln(S+15)$

As, $\int \frac{ds}{S+15} = \frac{1}{5}\int dt$

$[Ln(S+15)]_0^{60} = \frac{1}{5}[t]_0^{t_{AB}}$

$Ln(60+15) - Ln(0+15) = \frac{(t_{AB} - 0)}{5}$

$Ln\left(\frac{75}{15}\right) = \frac{t_{AB}}{5}$

$Ln5 = \frac{t_{AB}}{5}$

$1.6 = \frac{t_{AB}}{5}$

$t_{AB} = 8$

(ii) $t_{BC} = ?$

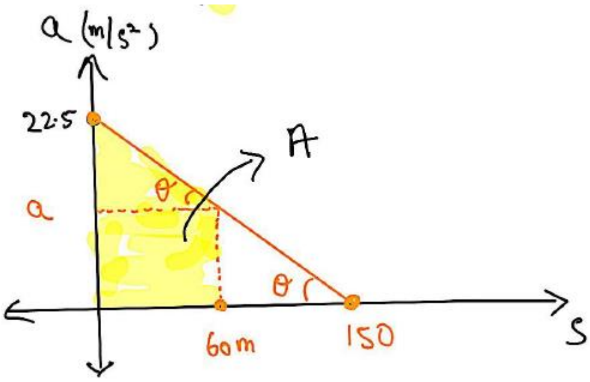
$S_{BC} = V_{BC}\, t_{BC}$ ($\because$ of uniform velocity)

(120 – 60) = 15 t_{BC}

$t_{BC} = 4$

$\Rightarrow t = t_{AB} + t_{BC} = 8 + 4 = 12\sec$

17.



We know $\frac{v^2-u^2}{2}=\int ads=$ Area of a –s graph

* We have to find velo. at 60 m hence area should be calculated till 60 m.

Area = $\frac{1}{2}[a+22.5]60$

Also from fig.

$$\tan\theta=\frac{\overset{9}{\cancel{22.5}}}{\underset{2.5}{\cancel{150}}}=\frac{22.5-a}{\cancel{60}}$$

$\Rightarrow a=13.5$

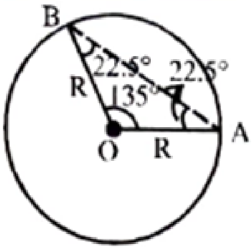
Hence Area = $\frac{1}{\cancel{2}}(13.5+22.5)\,\overset{30}{\cancel{60}}$

$$\frac{V^2-(u-0)^2}{2}=36\times 30=1080$$

$$V=\sqrt{2160}\simeq 46.47m/s$$

PYQ : JEE-Main

1. (b)



From ΔAOB

$$\frac{AB}{\sin 135}=\frac{OB}{\sin 22.5}$$

$$AB=\frac{\sin 135}{\sin 22.5}OB$$

$$=\frac{\sin(135)}{\sin(22.5)}\frac{\text{arc}(AB)}{\frac{3\pi}{4}}=\frac{\sin(45)}{\sin(22.5)}\times\frac{60\times 4}{3\pi}=47\text{m}$$

2. (b)

Given, $\nu=b\sqrt{x}$ or $\frac{dx}{dt}=b\,x^{1/2}$

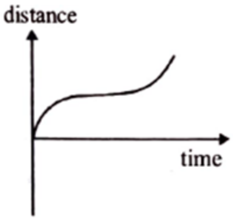
Or $\int_0^x x^{-1/2}dx=\int_0^t bdt\Rightarrow\frac{x^{1/2}}{1/2}=bt\Rightarrow x=\frac{b^2t^2}{4}$

Differentiating w.r.t. time, we get

$$\frac{dx}{dt}=\frac{b^2 2t}{4}\text{ or }\nu=\frac{b^2\tau}{2}\qquad(t=\tau)$$

3. (b)

Graphs in option (c) position-time and option (a) velocity- position are corresponding to velocity-time graph option (d) and its distance-time graph is as given below. Hence distance-time graph option (b) is incorrect.



4. (b)

We have, $\vec{r}=3t\hat{i}+5t^3\hat{j}+7k$

So, $\frac{d^2\vec{r}}{dt^2}=30\,t\hat{j}$

At $t=1\Rightarrow\boxed{\frac{d^2\vec{r}}{dt^2}=30\hat{j}}\Rightarrow\vec{a}=30\hat{j}$

5. (a)

At $t = 20\text{sec}$

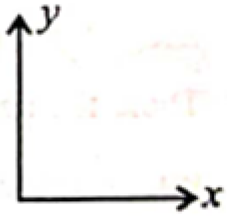
$V_H = at = 5 \times 20 = 100 m/s$

At $t = 1\text{sec}$

[$\because$ After falling there is not acceleration in horizontal direction]

$V_v = (0 + g \times 1) = 10 \times 1 = 10 m/s$

So, $\vec{V} = 100\hat{i} - 10\hat{j}$



6. (c)

Time of fall of packet

$t = \sqrt{\frac{2h}{g}}, x = \sqrt{\frac{2h}{g}} v$

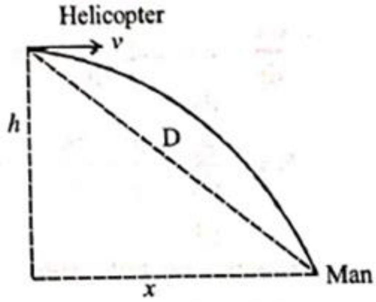
Horizontal range $(x) = \text{time} \times \text{horizontal component of velocity} (\nu)$

$\therefore$ Required distance

$D = \sqrt{x^2 + h^2}$

$= \sqrt{\left(\sqrt{\frac{2h}{g}} v\right)^2 + h^2}$

Or, $D = \sqrt{\frac{2hv^2}{g} + h^2}$



7. (b)

Given: $\vec{u} = 5\hat{j} m/s$

Acceleration, $\vec{a} = 10\hat{i} + 4\hat{j}$ and final coordinate $(20, y_0)$ in time t.

$S_x = u_x t + \frac{1}{2} a_x t^2$ $[\because u_x = 0]$

$\Rightarrow 20 = 0 + \frac{1}{2} \times 10 \times t^2 \Rightarrow t = 2s$

$$S_y = u_y \times t + \frac{1}{2} a_y t^2$$

$$y_0 = 5 \times 2 + \frac{1}{2} 4 \times 2^2 = 18m$$

8. (d)

Given, Position vector,

$\vec{r} = \cos\omega t\hat{i} + \sin\omega t\,\hat{j}$

Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = \omega\left(-\sin\omega t\hat{i} + \cos\omega t\,\hat{j}\right)$

Acceleration,

$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2\left(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}\right)$

$\vec{a} = -\omega^2\vec{r}$

$\therefore \vec{a}$ is antiparallel to $\vec{r}$

Also $\vec{v}.\vec{r} = 0 \therefore \vec{v} \perp \vec{r}$

Thus, the particle is performing uniform circular motion.

9. (d)

$\vec{r} = 15t^2\hat{i} + \left(4 - 20t^2\right)\hat{j}$

$\vec{v}\frac{d\vec{r}}{dt} = 30\,t\hat{i} - 40t\,\hat{j}$

Acceleration , $\vec{a} = \frac{d\vec{v}}{dt} = 30\hat{i} - 40\hat{j}$

$\therefore a = \sqrt{30^2 + 40^2} = 50$

10. (c)

From given equation,

$\vec{V} = K\left(y\hat{i} + x\hat{j}\right)$

$\frac{dx}{dt} = Ky$ and $\frac{dy}{dt} = Kx$

Now $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y} = \frac{dy}{dx} \Rightarrow ydy = xdx$

Integrating both side

$y^2 = x^2 + c$

11. (d)

Equation of trajectory of projectile

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta} = x\tan 45^\circ - \frac{1}{2}\frac{10x^2}{u^2\cos^2 45^\circ}$$

$$\Rightarrow y = x - \frac{10x^2}{2u^2\left(\frac{1}{2}\right)} \Rightarrow 10 = 20 - \frac{(10)(100)}{u^2} \Rightarrow u = 20$$

Time period, $T = \frac{2u\sin\theta}{g} = \frac{(2)(20)}{\sqrt{2}(10)} = 2\sqrt{2}$

$\vec{v} = u\cos\theta\hat{i} + (\mu\sin\theta - gt)\hat{j} = \left[10\sqrt{2}\hat{i} + \left(10\sqrt{2} - 10\sqrt{2}\right)\right]\hat{j}$

Momentum $\vec{P} = M\vec{v} = 100\sqrt{2}\hat{i} + \left(100\sqrt{2} - 200\right)\hat{j}$

12. (c)

Projection speed is v.

$R_1 = \frac{v^2\sin\left(90^\circ\right)}{g}; R_2 = \frac{v^2\sin\left(60^\circ\right)}{g}$

$\therefore \frac{R_1}{R_2} = \frac{2}{\sqrt{3}}$

13. (d)

As range and maximum height are equal

$R = H$

$\Rightarrow \frac{2v_x - v_y}{g} = \frac{v_y^2}{2g},$

$\Rightarrow v_x = \frac{v_y}{4} \Rightarrow u\cos\theta = \frac{u\sin\theta}{4} \Rightarrow \tan\theta = 4$

14. (c)

As, $H_{max} = \frac{u^2\sin^2\theta}{2g} \Rightarrow u^2 \propto \frac{1}{\sin^2}\theta$

$\Rightarrow u \propto \frac{1}{\sin\theta} \Rightarrow \frac{\sin\theta_2}{\sin\theta_1} = \frac{u_1}{u_2}$

$\Rightarrow \sin\theta_1 = \frac{u_2}{u_1} \times \sin\theta_2 = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{2}}$

So, $\theta_1 = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$

15. (a)

$R = \frac{u^2\sin 2\theta}{g}$

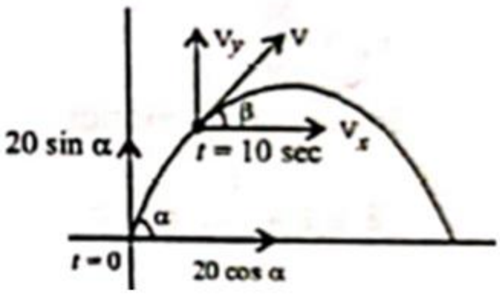
For, $R = R_{max}$

$\sin 2\theta = 1 \Rightarrow 2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}, \text{So } R_{max} = \frac{u^2}{g} = 100\text{m}$

$$\text{So, } H=\frac{u^2\sin^2\left(\frac{\pi}{4}\right)}{2g}=\frac{u^2\times\frac{1}{2}}{2g}=\frac{u^2}{4g}=\frac{100}{4}=25\text{m}$$

16. (b)



$$v_x=20\cos\alpha\left[\because a_x=0\right]$$
$$v_y=20\sin\alpha-g\times t\left[\because a_y=-g\right]$$
$$=20\sin\alpha-10\times10$$
$$=20\sin\alpha-100$$
$$\text{So, } \tan\beta=\frac{v_y}{v_x}=\frac{20\sin\alpha-100}{20\cos\alpha}=\tan\alpha-5\sec\alpha$$

17. (a)

We know that if range is same for two angle of projection, then these angles must be complementary. Let first angle of projection be $'\theta'$ then second will be $(90-\theta)$

$$\therefore h_1=\frac{u^2\sin^2\theta}{2g}\text{and}\,h_2=\frac{u^2\sin^2(90-\theta)}{2g}=\frac{u^2\cos^2\theta}{2g}$$
$$\therefore h_1h_2=\frac{u^2\sin^2\theta}{2g},\frac{u^2\cos^2\theta}{2g}\text{, So, reason is correct}$$
$$\Rightarrow\sqrt{h_1h_2}=\frac{u^2\sin\theta\cos\theta}{2g}\Rightarrow4\sqrt{h_1h_2}=\frac{4u^2\sin\theta\cos\theta}{2g}$$
$$\Rightarrow4\sqrt{h_1h_2}=\frac{u^2(2\sin\theta\cos\theta)}{g}\Rightarrow4\sqrt{h_1h_2}=\frac{u^2\sin^2\theta}{g}=R$$

So, assertion is correct

18. (d)

At maximum height inclination with horizontal be comes zero

$$\text{So, } t=\frac{u\sin\theta}{g}\Rightarrow u=\frac{gt}{\sin\theta}\qquad\text{.....(i)}$$
$$\text{Now, } R=u\cos\theta\times T\Rightarrow R=u\cos\theta\times2t$$
$$\Rightarrow\cos\theta=\frac{R}{2ut}\Rightarrow\cos\theta=\frac{R\sin\theta}{\sin\theta}\qquad\text{[From (i)]}$$
$$\Rightarrow\tan\theta=\frac{2gt^2}{R}\Rightarrow\tan\theta=\frac{20t^2}{R}\Rightarrow\cot\theta=\frac{R}{20t^2}$$

19. (b)

For projectile motion,

Range $R = \frac{u^2 \sin 2\theta}{g}$

As range is same for angle of projection θ and $90 - \theta$. So, range is same for 42° and 48°.

Height in projectile motion, $H = \frac{u^2 \sin^2 \theta}{2g} \Rightarrow H \propto \sin^2 \theta$

So H is higher for 48° than H for 42°

20. (d)

Given: $y = \alpha x - \beta x^2$

For maximum height, maximum value of y

$$\frac{dy}{dx} = 0 \Rightarrow \frac{d\left(\alpha x - \beta x^2\right)}{dx} = 0 \Rightarrow \alpha - 2\beta x = 0$$

$$\therefore x = \frac{\alpha}{2\beta}$$

$$\therefore y = \alpha\left(\frac{\alpha}{2\beta}\right) - \beta\left(\frac{\alpha^2}{4\beta^2}\right) \Rightarrow \left(\frac{\alpha^2}{2\beta}\right) - \left(\frac{\alpha^2}{4\beta}\right) \Rightarrow \frac{\alpha^2}{4\beta}$$

So, $H_{max} = \frac{\alpha^2}{4\beta}$

Hence option (D) is correct answer.

21. (a)

Using principal of conservation of linear momentum for horizontal motion, we have

$$2m\nu_x = mu + mu\cos 60^\circ$$

$$\nu_x = \frac{3u}{4}$$

For vertical motion

$$h = 0 + \frac{1}{2}gT^2 \Rightarrow T = \sqrt{\frac{2h}{g}}$$

Let R is the horizontal distance travelled by the body.

$R = \nu_x T + \frac{1}{2}(0)(T)^2$ (For horizontal motion)

$$R = \nu_x T = \frac{3u}{4} \times \sqrt{\frac{2h}{g}} \Rightarrow R = \frac{3\sqrt{3}u^2}{8g}$$

22. (c)

Given, $y = 2x - 9x^2$

On comparing with,

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta},$$

We have,

$\tan\theta = 2 \text{ or } \cos\theta = \frac{1}{\sqrt{5}}$

And $\frac{g}{2u^2\cos^2\theta} = 9$ or $\frac{10}{2u^2\left(1/\sqrt{5}\right)^2} = 9$

$\therefore u = 5/3 m/s$

23. (d)

R will be same for θ and $90^\circ - \theta$

Time of flights:

$t_1 = \frac{2u\sin\theta}{g}$ and

$t_2 = \frac{2u\sin\left(90^\circ - \theta\right)}{g} = \frac{2u\cos\theta}{g}$

Now, $t_1 t_2 = \left(\frac{2u\sin\theta}{g}\right)\left(\frac{2u\cos\theta}{g}\right) = \frac{2}{g}\left(\frac{u^2\sin 2\theta}{g}\right) = \frac{2R}{g}$

24. (a)

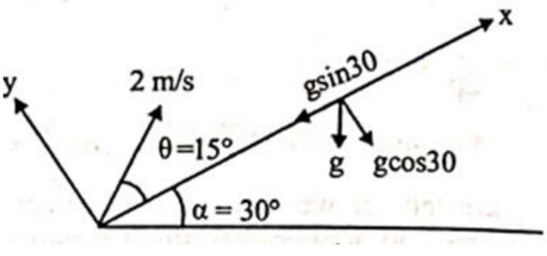
On an inclined plane, time of flight (T) is given by

$T = \frac{2u\sin\theta}{g\cos\alpha}$

Substituting the values, we get

$T = \frac{(2)\left(2\sin 15^\circ\right)}{g\cos 30^\circ} = \frac{4\sin 15^\circ}{10\cos 30^\circ}$

Distance, $S = \left(2\cos 15^\circ\right)T - \frac{1}{2}g\sin 30^\circ (T)^2$



$= \left(2\cos 15^\circ\right)\frac{4}{10}\frac{\sin 15^\circ}{10\cos 30^\circ} - \left(\frac{1}{2} \times 10\sin 30^\circ\right)\frac{16\sin^2 15^\circ}{100\cos^2 30^\circ}$

$= \frac{16\sqrt{3} - 16}{60} \simeq 0.1952 \simeq 20cm$

25. (a)

As we know, range $R = \frac{u^2\sin 2\theta}{g}$

And , area $A = \pi R^2$

$\therefore A \propto R^2$ or , $A \propto u^4$

$$\therefore \frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \left[\frac{1}{2}\right]^4 = \frac{1}{16}$$

26. (b)

Given, $t = \sqrt{x} + 4$

$$\Rightarrow x = (t-4)^2 = t^2 - 8t + 16$$

$$\Rightarrow \frac{dx}{dt} = 2t - 8 \Rightarrow \left.\frac{dx}{dt}\right|_{t=4} = 2\times 4 - 8 = 0$$

27. (c)

Let initial velocity of bullet $= u$

Final velocity of bullet $= \frac{u}{3}$

Distance travelled by bullet, $s = 4\text{cm}$

Using, $v^2 - u^2 = 2as$

$$\Rightarrow \left(\frac{u}{3}\right)^2 = u^2 - 2a(4) \Rightarrow a = \frac{8u^2}{9(8)} = \frac{u^2}{9}$$

Using $v^2 - u^2 = 2as$ again

$$0 = v^2 - 2a(4+x)$$

$$\Rightarrow v^2 = 2\left(\frac{u^2}{9}\right)(4+x) \Rightarrow 4.5 = 4 + x \Rightarrow x = 0.5$$

28. (c)

In ‘t’ sec, $S = 10\,\text{m}$

So, $10 = \frac{1}{2}at^2$ …….(i)

In next, ‘t’ sec, suppose body travel S’ distance

Then, $10 + S' = \frac{1}{2}a(2t)^2$ …….(ii)

Dividing (ii) by (i), we get

$$\Rightarrow \frac{10+S'}{10} = 4 \Rightarrow 10 + S' = 40 \;\Rightarrow S' = 30\text{m}$$

29. (d)

For bus P

$$x_p(t) = \alpha t + \beta t^2$$

$$V_p(t) = \alpha + 2\beta t \qquad \left[\therefore V_p = \frac{dx_p}{dt}\right]$$

For bus Q

$$x_q(t) = \int t - t^2$$

$$V_q(t) = f - 2t\left[\because V_q = \frac{dx_q}{dt}\right]$$

As, $V_p(t) = V_q(t) \Rightarrow \alpha + 2\beta t = f - 2t$

$$\Rightarrow \alpha - f = -2\beta t - 2t \Rightarrow f - \alpha = 2\beta t + 2t$$

$$\Rightarrow t = \frac{f - \alpha}{(2\beta + 2)}$$

30. (b)

We have given,

$$v = \alpha t + \beta t^2$$

$$\Rightarrow \frac{ds}{st} = \alpha t + \beta t^2 \Rightarrow \int_{s_1}^{s_2} ds = \int_1^2 \left(\alpha t + \beta t^2\right) dt$$

$$\Rightarrow s_2 - s_1 = \left[\frac{\alpha t^2}{2} + \frac{\beta t^3}{3}\right]_1^2$$

As particle is moving in a straight line,

$\therefore$ Distance = Displacement

$$\therefore \text{Distance} = \left[\frac{\alpha[4-1]}{2} + \frac{\beta[8-1]}{3}\right] = \frac{3\alpha}{2} + \frac{7\beta}{3}$$

31. (c)

For time interval t_1

$$\nu_1 = u_1 + a_1 t$$

$$\Rightarrow \nu_1 = at_1 \qquad [\because u_1 = 0]$$

For time interval t_2

$$\nu_2 = u_2 + (-a_2 t_2) \text{ or, } 0 = \nu_1 - a_2 t_2 \Rightarrow \nu_1 = a_2 t_2$$

$$\therefore a_1 t_1 = a_2 t_2 \text{ or, } \frac{t_1}{t_2} = \frac{a_2}{a_1}$$

32. (b)

$$x = at + bt^2 - ct^3$$

Velocity, $\nu = \frac{dx}{dt} = \frac{d}{dt}\left(at + bt^2 + ct^3\right)$

$$= a + 2bt - 3ct^2$$

Acceleration, $\frac{d\nu}{dt} = \frac{d}{dt}\left(a + 2bt - 3ct^2\right)$

Or $0 = 2b - 3c \times 2t \qquad \therefore t = \left(\frac{b}{3c}\right)$

and $\nu = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2 = \left(a + \frac{b^2}{3c}\right)$

33. (d)

For constant acceleration, there is straight line parallel to t-axis on $\vec{a} - t$

34. (d)

Position of the particle,

$S =$ area under graph (time t=0 to 5s)

$= \frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1 = 9\text{m}$

35. (b)

Here, $s = \frac{\nu^2}{2a} \Rightarrow s \propto \nu^2$

$\Rightarrow s_2 = s_1\left(\frac{\nu_2}{\nu_1}\right)^2 \Rightarrow s_2 = 40\left(\frac{80}{40}\right)^2 \Rightarrow s_2 = 160\text{m}$

36. (c)

Using equation, $a = \frac{\nu - u}{t}$ and $S = ut + \frac{1}{2}at^2$

Distance travelled by car in 15 sec $= \frac{1}{2}\frac{(45)}{15}(15)^2$

$= \frac{675}{2}\text{m}$

Distance travelled by scooter in 15 seconds $= 30 \times 15 = 450$

($\because$ distance = speed × time)

Difference between distance travelled by car and scooter in 15 sec, $450 - 337.5 = 112.5\text{m}$

Let car catches scooter in time t;

$\frac{675}{2} + 45(t - 15) = 30t \Rightarrow 337.5 + 45t - 675 = 30t \Rightarrow 15t = 337.5$

$\Rightarrow t = 22.5\text{sec}$

37. (a)

Let the car turn of the highway at a distance 'x' from the point M. So, $RM = x$

And if speed of car in field is ν, then time taken by the car to cover the distance $QR = QM - x$ on the highway.

$t_1 = \frac{QM - x}{2\nu}$ .....(i)

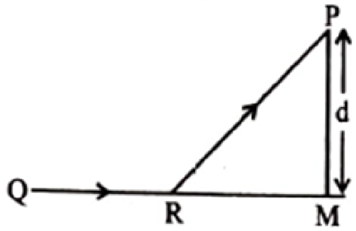
Time taken to travel the distance 'RP' in the field

$t_2 = \frac{\sqrt{d^2 + x^2}}{\nu}$ .....(ii)

Total time elapsed to move the car from Q to P

$$t = t_1 + t_2 = \frac{QM - x}{2\nu} + \frac{\sqrt{d^2 + x^2}}{\nu}$$

For ‘t’to be minimum $\frac{dt}{dx} = 0$



$$\frac{1}{\nu}\left[-\frac{1}{2} + \frac{x}{\sqrt{d^2 + x^2}}\right] = 0$$

Or $x = \frac{d}{\sqrt{2^2 - 1}} = \frac{d}{\sqrt{3}}$

38. (c)

Let L be the length of escalator . Speed of man wrt escalator $= \frac{L}{t_1} = V_1$

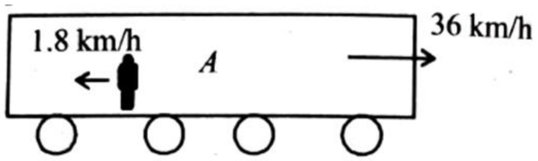
Speed of escalator $= \frac{L}{t_1} = V_2$

Time taken when escalator is moving and man is also walking on it

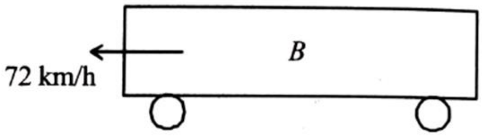
$$= t = \frac{L}{v_1 + v_2} \Rightarrow \frac{1}{t} = \frac{v_1 + v_2}{L} = \frac{1}{t_1} + \frac{1}{t_2} \Rightarrow t = \frac{t_1 t_2}{t_1 + t_2}$$

39. (a)

According to question, train A and B are running on parallel tracks in the opposite direction.



$V_A = 36\,km/h = 10\,m/s$



$V_B = -72\,km/h = -20\,m/s; V_{MA} = -1.8 km/h = -0.5\,m/s$

$V_{man,B} = V_{max,A} + V_{A,B}$

$= V_{man,A} + V_A - V_B = -0.5 + 10 - (-20)$

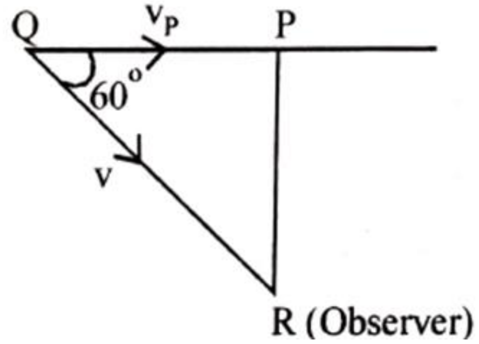
$= -0.5 + 30 = 29.5 m/s$

40. (a)

$$t_1 = \frac{(60+120)/1000\,\text{km}}{80-30}$$

$$t_2 = \frac{(60+120)/1000\,\text{km}}{80+30} \therefore \frac{t_1}{t_2} = \frac{11}{5}$$

41. (d)

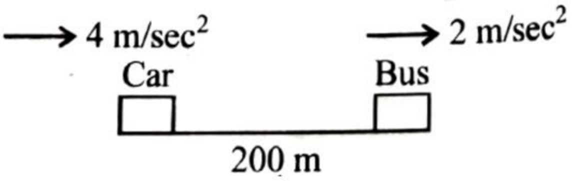


Distance, $PQ = v_P \times t$ (Distance = speed $\times$time)

Distance, $QR = v.t$

$$\cos 60^\circ = \frac{PQ}{QR}; \frac{1}{2} = \frac{v_p \times t}{v.t} \Rightarrow v_p = \frac{v}{2}$$

42. (c)



Given, $u_C = u_B = 0, a_C = 4m/s^2, a_B = 2m/s^2$

Hence relative acceleration , $a_{CB} = 2m/\sec^2$

Now, we know, $s = ut + \frac{1}{2}at^2 \Rightarrow 200 = \frac{1}{2} \times 2t^2$ $[\because u = 0]$

Hence, the car will catch up with the bust after time

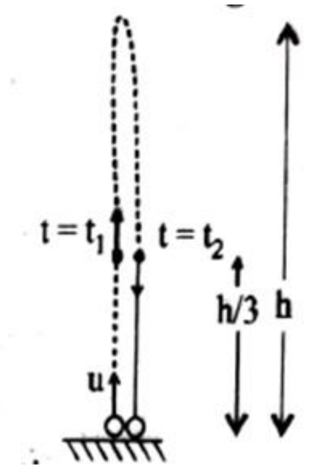
$t = 10\sqrt{2}$ second

43. (b)

Maximum height $= h = \frac{u^2}{2g}$ [$\because$ At maximum height $v = 0$. So $0^2 = u^2 - 2gh$

$\Rightarrow u = \sqrt{2gh}$

As, $s = ut + \frac{1}{2}at^2$



$\Rightarrow u = \sqrt{2gh}$

As, $s = ut + \frac{1}{2}at^2$

$\Rightarrow \frac{h}{3} = t\sqrt{2gh} + \frac{1}{2}(-g)t^2$

$\Rightarrow \frac{gt^2}{2} - t\sqrt{2gh} + \frac{h}{3} = 0$

Let t_1 and t_2 be two roots of above equation.

Then, $\frac{t_2}{t_1} = \frac{\sqrt{2gh} + \sqrt{2gh - 4\times\frac{g}{2}\times\frac{h}{3}}}{\sqrt{2gh} - \sqrt{2gh - 4\times\frac{g}{2}\times\frac{h}{3}}}$

$= \frac{\sqrt{2gh} + \sqrt{\frac{4gh}{3}}}{\sqrt{2gh} - \sqrt{\frac{4gh}{3}}} = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

44. (d)

From equation of motion, $s = ut + \frac{1}{2}gt^2$

For first $\frac{h}{2}, \frac{h}{2} = \frac{1}{2}gt_1^2$ ….(i)

For total height h,

$h = \frac{1}{2}g(t_1 + t_2)^2$ …..(ii)

Divide equation (ii) by (i) we have $\frac{1}{2} = \frac{t_1^2}{(t_1+t_2)^2}$

$\frac{1}{\sqrt{2}} = \frac{t_1}{t_1+t_2}; 1 + \frac{t_2}{t_1} = \sqrt{2}$

$\frac{t_1}{t_2} = \frac{1}{\sqrt{2}-1} \Rightarrow t_2 = (\sqrt{2}-1)t_1$

45. (d)

Time taken by all to reach highest point, $t = \frac{v}{g}$

Frequency of throw, $n = \frac{1}{t} = \frac{g}{v}$

$\Rightarrow v = \frac{g}{n}$ …..(i)

The maximum height the balls can reach,

$$H_{max} = \frac{v^2}{2g} = \frac{\left(\frac{g}{n}\right)^2}{2g} = \frac{g}{2n^2} \text{ from equation (i)}$$

46. (a)

From $h = ut + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

Time taken by mango, $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times19.6}{9.8}} = 2s$

Distance $x = vt = \frac{5}{2}\times 2 = 5m$

47. (a)

At $t = 0$, $u = 100m/s$ downwards

For 0 to 10 sect

$v = u - gt = -100 - 10\times10 = -200m/s$

So the curve will be straight line with -100 as intercept

for 10 to 20 sec, v=0, so option (A) is correct

48. (d)

Let us suppose both ball meat at $t = t$ sec at high h below.

For 'A', $h = \frac{1}{2}gt^2 \Rightarrow -80 = \frac{-1}{2}\times10\times t^2$

$\Rightarrow t^2 = 16 \Rightarrow t = 4sec$

For ball ‘B’

$h = u(t-2) - \frac{1}{2}g(t-2)^2 \Rightarrow -80 = -2u - \frac{1}{2}g\times2^2$

$\Rightarrow -80 = -2u - 20 \Rightarrow 2u = 60$ $\therefore$ u=30m/s

49. (c)

50. (d)

Using $H = \frac{1}{2}gt^2 \Rightarrow 2H/g = t^2$

$$\Rightarrow \frac{9.8\times 2}{9.8} = t^2 \Rightarrow t = \sqrt{2}s$$

For first drop, t=0. For second drop, $t = \Delta t$. For third drop , $t = 2\Delta t$

Let h be the distance travelled by second drop when third drop begins to fall.

$$\therefore h = \frac{1}{2}g\left(\sqrt{2} - \Delta t\right)^2 \Rightarrow 0 = \frac{1}{2}g\left(\sqrt{2} - \Delta t\right)^2 \Rightarrow \Delta t = \frac{1}{\sqrt{2}}$$

$$\therefore h = \frac{1}{2}g\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}\times 9.8\times\frac{1}{2} = \frac{9.8}{4} = 2.45\,m$$

Height of second drop from floor

$$H - h = 9.8 - 2.45 = 7.35mm$$

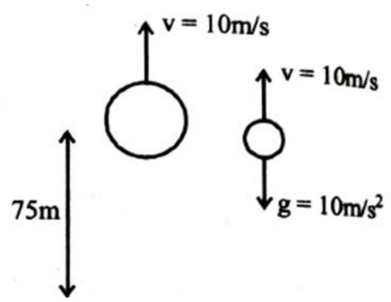
51. (c)

Given

Initial velocity of hot air balloon, $u = 10\,m/s$

For stone

Using,



$$h = ut + \frac{1}{2}gt^2$$

$$\Rightarrow 75 = -10t + \frac{1}{2}gt^2$$

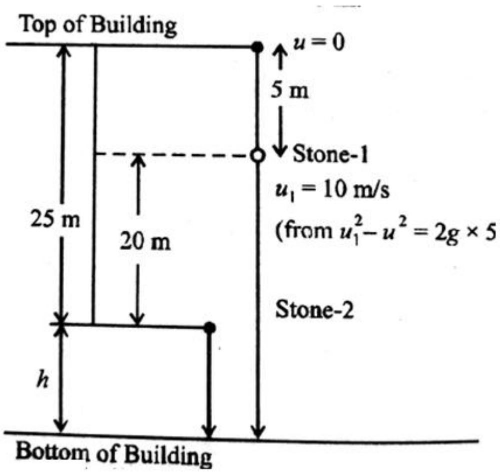
$$\Rightarrow 75 = -10t + 5t^2$$

$$\Rightarrow t^2 - 2t - 15 = 0 \Rightarrow t = 5sec$$

Height of balloon

$$H = vt + 75 \Rightarrow H = 10\times 5 + 75 = 125m$$

52. (b)



For stone $-1, 20+h=10t+\frac{1}{2}gt^2$ ……(i)

And for stone -2, $h=\frac{1}{2}gt^2$ ……(ii)

Putting value of h from eq.(ii) in eq. (i)

$20+\frac{1}{2}gt^2=10t+\frac{1}{2}gt^2 \therefore t=2s$

Therefore, $h=\frac{1}{2}gt^2=\frac{1}{2}\times10\times2^2=20m$

$\therefore$ Height of the building $=h+25=20+25=45m$

53. (c)

For upward motion of helicopter,

$v^2=u^2+2gh \Rightarrow v^2=0+2gh \Rightarrow v=\sqrt{2gh}$

Now, packet will start moving under gravity

Let 't' be the time taken by the food packet to reach the ground.

$s=ut+\frac{1}{2}at^2$

$\Rightarrow -h=\sqrt{2gh}t-\frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2-\sqrt{2gh}t-h=0$

$$\text{or}, t=\frac{\sqrt{2gh}\pm\sqrt{2gh+4\times\frac{g}{2}\times h}}{2\times\frac{g}{2}} \Rightarrow t=\sqrt{\frac{2gh}{g}}\left(1+\sqrt{2}\right)$$

$$\Rightarrow t=\sqrt{\frac{2h}{g}}\left(1+\sqrt{2}\right) \Rightarrow t=3.4\sqrt{\frac{h}{g}}$$

54. (a)

For uniformly accelerated or deceleration motion $v^2=u^2\pm2gh$ equation is quadratic hence $v-h$ graph will be a parabola.

Initial velocity downwards so negative after collision, it reverses its direction with smaller value hence velocity is upwards

Further, $v'^2=2g\times\left(\frac{d}{2}\right)=gd$;

$\therefore\left(\frac{v}{v'}\right)=\sqrt{2}$ or $v=v'\sqrt{2} \Rightarrow v'=\frac{v}{\sqrt{2}}$

As the direction is reversed and speed is decreased so graph(a) correctly depicts these conditions.

55. (3)

$$V_{av}=\frac{S_{total}}{T_{total}}=\frac{200+200+\frac{2\pi\times200}{4}}{238}$$

$$=\frac{400+100\pi}{238}=\frac{714}{238}=3\text{m}/\text{s}$$

56. (18)

$$\nu_{\text{avg}}=\frac{l_{\text{total}}}{t_{\text{total}}}=\frac{l/3+l/3+l/3}{\frac{l}{3\nu_1}+\frac{l}{3\nu_2}+\frac{l}{3\nu_3}}$$

$$=\frac{l}{\frac{l}{3}\left(\frac{1}{\nu_1}+\frac{1}{\nu_2}+\frac{1}{\nu_3}\right)}=\frac{1}{\frac{1}{3}\left(\frac{1}{11}+\frac{1}{22}+\frac{1}{33}\right)}$$

$$=\frac{33}{1+\frac{1}{2}+\frac{1}{3}}=\frac{198}{6+3+2}=\frac{198}{11}=18\text{m}/\text{s}$$

57. (580)

For particle 'A'	For particle 'B'
$X_A=-3t^2+8t+10$	$Y_B=5-8t^3$
$\vec{\nu}_A=(8-6t)\hat{i}$	$\vec{\nu}_B=-24t^2\hat{j}$
$\vec{a}_A=-6\hat{i}$	$\vec{a}_B=-48t\hat{j}$

At $t=1\text{sec}$

$\vec{\nu}_A=(8-6t)\hat{i}=2\hat{i}$ and $\vec{\nu}_B=-24\hat{j}$

$\therefore \vec{\nu}_{B/A}=-\vec{\nu}_A+\vec{\nu}_B=-2\hat{i}-24\hat{j}$

$\therefore$ speed of B w.r.t. A, $\sqrt{\nu}=\sqrt{2^2+24^2}$

$=\sqrt{4+576}=\sqrt{580}$

$\therefore \nu=580(\text{m}/\text{s})$

58. (15 or 75)

We know that range $(R)=\frac{u^2\sin 2\theta}{g}$

So, $R_{\max}=\frac{u^2\sin 2\left(45^\circ\right)}{g}=\frac{u^2}{g}\Rightarrow\frac{R}{2}=\frac{u^2}{2g}=\frac{u^2\sin 2\theta}{g}$

$\Rightarrow \sin 2\theta=\frac{1}{2}\Rightarrow 2\theta=30^\circ,150^\circ\Rightarrow\theta=15^\circ,75^\circ$

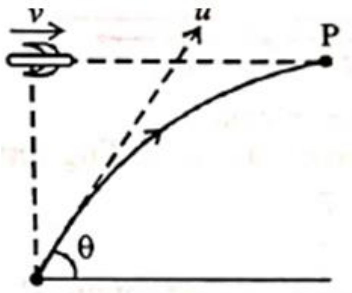
59. (1)

Here, time of flight is same

So, vertical component of velocity is same

Therefore $H_{\max}$ is same.

60. (60)



If fighter, jet and bullet have same horizontal component of velocity , then it will strike after some time as shown above because horizontal displacement of both of them in given time interval will always be same.

i.e. $\nu = u\cos\theta \Rightarrow 200 = 400\cos\theta$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

61. (20)

Let $\vec{\nu}_1 = a\hat{i} + a\hat{j}\left[\because \theta = 45^\circ \Rightarrow u_x = u_y\right]$

Then, $\vec{V}_f = a\hat{i} + \left(a - 2\times10\right)\hat{j}$

$= a\hat{i} + \left(a - 20\right)\hat{j}$

As, $\left|\vec{V}_f\right| = 20$

$\Rightarrow a^2 + \left(a-20\right)^2 = 400$

$\Rightarrow a^2 + a^2 + 400 - 40a = 400 \Rightarrow 2a^2 - 40a = 0$

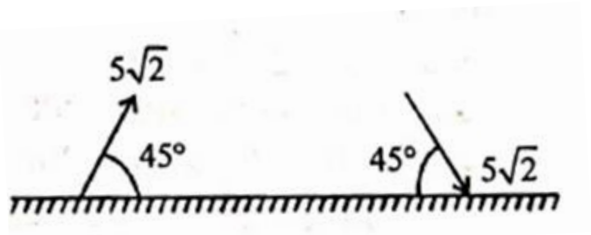
$\Rightarrow a^2 = 20a = 0 \Rightarrow a\left(a-20\right) = 0$

$\Rightarrow a = 20$. So, $V_i = \sqrt{20^2 + 20^2} = 20\sqrt{2}$

$$\therefore h_{max} = \frac{V_i^2 \sin^2 45^\circ}{2g} = \frac{800\times\frac{1}{2}}{2\times10} = 20m$$

62. (5)

Change in momentum,



$\Delta\vec{P} = \vec{P}_B - \vec{P}_A$

$\Delta\vec{P} = mV\cos\theta\hat{i} - mV\sin\theta\hat{j}$

$= m\left(5\sqrt{2}\cos45^\circ\hat{i} - 5\sqrt{2}\sin45^\circ\right)$

$$= -m\left(5\sqrt{2}\times\frac{1}{\sqrt{2}}\hat{i} + 5\sqrt{2}\times\frac{1}{\sqrt{2}}\hat{j}\right) = -2m\times5\sqrt{2}\times\frac{1}{\sqrt{2}}$$

$\left|\Delta\vec{P}\right| = 2\times\left(5\times10^{-3}\right)\times(5) = 5\times10^{-2}\,\text{kg ms}^{-1}$

$\therefore x = 5$

63. (3)

If speed becomes 'n' times by keeping 'a' constant, then stopping distance becomes n^2 times

Here, $V \rightarrow V/3$

So, $S \rightarrow \frac{S}{9}$ i.e. $\frac{27}{9}$m i.e. 3m

64. (100)

$\frac{dv}{ds} = 5;\ \text{As}\ a = \frac{dV}{dt} = \frac{dV}{ds}\times\frac{ds}{dt} = v\frac{dV}{ds}$

So, $a = v\frac{dv}{ds} = 20\times5 = 100\text{m}/\text{sec}^2$

65. (1)

As $v^2 = u^2 + 2ax$

At point 'A'

$40 = u^2 + 2a.10$ ….(i)

At point 'B'

$60 = u^2 + 2a.20$ …..(ii)

Substracting (ii) from (i), we get

$a = 1\text{m}/\text{s}^2$

66. (12)

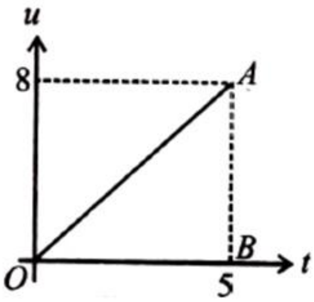
Acceleration equals to $a = v\frac{dv}{dx}$

$a = \frac{vdv}{dx}\ \&\ v = \sqrt{5000+24x}$

$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{5000+24x}}\times24 = \frac{12}{\sqrt{5000+24x}}$

$\Rightarrow a = v\frac{dv}{dx} = \sqrt{5000+24x}\times\frac{12}{\sqrt{5000+24x}} = 12\text{m}/\text{s}^2$

67. (20)



Distance travelled = Area of speed-time graph

$$=\frac{1}{2}\times5\times8=20\text{m}$$

68. (3)

Distance X varies with time t as $x^2=at^2+2bt+c$

$$\Rightarrow 2x\frac{dx}{dt}=2at+2b$$

$$\Rightarrow x\frac{dx}{dt}=at+b\Rightarrow\frac{dx}{dt}=\frac{(at+b)}{x}\Rightarrow x\frac{d^2x}{dt^2}+\left(\frac{dx}{dt}\right)^2=a$$

$$\Rightarrow\frac{d^2x}{dt^2}=\frac{a-\left(\frac{dx}{dt}\right)^2}{x}=\frac{a-\left(\frac{at+b}{x}\right)^2}{x}$$

$$=\frac{ax^2-(at+b)^2}{x^3}=\frac{ac-b^2}{x^3}$$

$\Rightarrow a\propto x^{-3}$ Hence, n=3

69. (392)

$$t_a=\frac{u}{g}=\frac{19.6}{9.8}=2s\begin{bmatrix}\because V=u-gt\\ O=u-gt\\ \Rightarrow t=u/g\end{bmatrix}$$

$$t_d=(6-2)s=\sqrt{\frac{2h_{max}}{g}}$$

$$h_{max}=\frac{16\times9.8}{2}=\frac{392}{5}$$

70. (5)

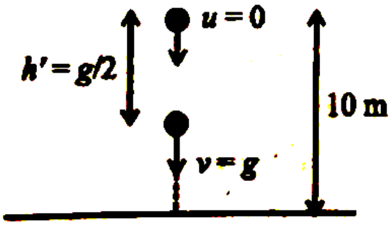
$$v^2=u^2+2gh'$$

$$g^2=0^2+2gh'$$

$$h'=\frac{g}{2}$$

So, req. height

$$=10-h'=10-\frac{g}{2}$$



$$=10-\frac{10}{2}=5\text{m}$$

71. (6)

Let the two ball meet at $t = t\,\text{sec}$

Then, first ball get 't' sec and second ball get $(t-2)$ sec and they will meet at same height

So, $h_1 = h_2$

$$\Rightarrow 50t - \frac{1}{2}gt^2 = 50(t-2) - \frac{1}{2}g(t-2)^2$$

$$\Rightarrow 0 = -100 + \frac{1}{2}g\left[t^2 - (t-2)^2\right] \Rightarrow 20 = 4t - 4$$

$$\Rightarrow 24 = 4t \Rightarrow t = 6\,\text{sec}$$

72. (3)

Taking upward direction as positive, we get

$$h = u \times 6 - \frac{1}{2}g.6^2 \rightarrow \text{Case I}$$

$$h = -u \times 1.5 - \frac{1}{2}g \times 1.5^2 \rightarrow \text{Case II}$$

$$h = -\frac{1}{2}gt^2 \rightarrow \text{Case III}$$

So, $6u - \frac{1}{2}g.6^2 = -1.5u - \frac{1}{2}g \times 1.5^2$ [from I and II]

$$7.5u = \frac{1}{2}g\left(6^2 - 1.5^2\right); u = \frac{g(33.75)}{2 \times 7.5} \Rightarrow u = \frac{45}{2}\text{m/s}$$

Again, $-\frac{1}{2}gt^2 = 6u - \frac{1}{2}g.6^2$ [from I and III]

$$t^2 = \frac{-2}{g}\left(6u - \frac{1}{2}g \times 36\right)$$

$$t^2 = \frac{-2}{10}\left(6 \times \frac{45}{2} - \frac{1}{2} \times 10 \times 36\right)$$

$$\Rightarrow t^2 = -\frac{2}{10}(135 - 180) \Rightarrow t^2 = \frac{+2 \times 45}{10} \Rightarrow t^2 = 9 \Rightarrow t = 3s$$

73. (8)

Let the ball takes time to reach the ground

Using, $S = ut + \frac{1}{2}gt^2 \Rightarrow S = 0 \times t + \frac{1}{2}gt^2$

$$\Rightarrow 200 = gt^2 \qquad [\because 2S = 100m]$$

$$\Rightarrow t = \sqrt{\frac{200}{g}} \qquad \ldots\ldots(i)$$

In last $\frac{1}{2}$s body travels a distance of 19 m, so in $\left(t - \frac{1}{2}\right)$ distance travelled $= 81$

Now, $\frac{1}{2}g\left(t - \frac{1}{2}\right)^2 = 81 \Rightarrow g\left(t - \frac{1}{2}\right)^2 = 81 \times 2$

$$\Rightarrow\left(t-\frac{1}{2}\right)=\sqrt{\frac{81\times 2}{g}}\ \therefore\ \frac{1}{2}=\frac{1}{\sqrt{g}}\left(\sqrt{200}-\sqrt{81\times 2}\right)$$

Using (i)

$$\Rightarrow\sqrt{g}=2\left(10\sqrt{2}-9\sqrt{2}\right)\Rightarrow\sqrt{g}=2\sqrt{2}$$

$$\therefore g=8\,m/s^2$$

PYQ : JEE-Advanced

1. (A)

For a body thrown vertically upwards acceleration remains constant $(a=-g)$ and velocity at anytime t is given by $V=u-gt$. During rise velocity decreases linearly and during fall velocity increases linearly and direction is opposite to each other.

Hence graph (a) correctly depicts velocity versus time.

2. (B)

$y_1=10t-5t^2; y_2=40t-5t^2$

For $y_1=-240m, t=8s$

$\therefore y_2-y_1=30t$ for $t\le 8s$. For $t>8s$,

$y_2-y_1=240-40t-\frac{1}{2}gt^2$

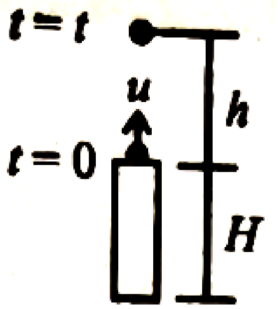
3. (C)

Let t be time to reach maximum height. Then $'nt'$ is total time of flight

So, $h=ut-\frac{1}{2}gt^2$

And $H+h=\frac{1}{2}g(nt-t)^2$

$\Rightarrow H+ut-\frac{1}{2}gt^2=\frac{1}{2}gt^2(n-1)^2$



$\Rightarrow H+ut-\frac{1}{2}gt^2=\frac{1}{2}gt^2n^2+\frac{1}{2}gt^2-gnt^2$

$\Rightarrow H+ut-gt^2=gt^2\left(\frac{n^2}{2}-n\right)$

$\Rightarrow H+\frac{u^2}{g}-\frac{gu^2}{g^2}=\frac{gu^2}{2g^2}(n^2-2n)$

$\Rightarrow H=\frac{u^2}{2g}n(n-2)$

$\Rightarrow 2gH=u^2n(n-2)$

4. (B)

5. (A)

6. (B)

7. (ABCD)

According to questions, equation

$$y = \frac{x^2}{2}$$

At $t = 0, \left.\begin{matrix} x = 0, y = 0 \\ u = 1 \end{matrix}\right\}$ given

$$y = \frac{x^2}{2}$$

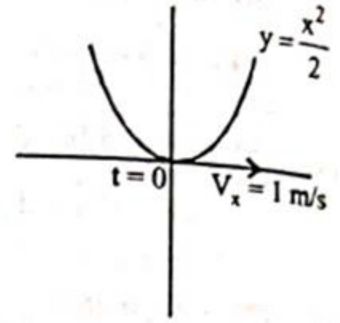
$$\frac{dy}{dt} = \frac{1}{2}.2x\frac{dx}{dt} = x\frac{dx}{dt} \Rightarrow v_y = xv_x$$

Now differenciate wrt time

$$\frac{d^2y}{dt^2} = x\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2$$

$$a_y = \frac{dx}{dt}.V_x + xa_x$$

$$a_y = v_x^2 + xa_x$$



(a) If $a_x = 1$ and particle is at origin $(x = 0, y = 0)$

$a_y = v_x^2$

$a_y = 1^2 = 1 m/s^2$

(b) $a_x = 0$

$a_y = v_x^2 + xa_x \Rightarrow a_y = {v_x}^2$

If $a_x = 0, v_x = \text{constant} = 1 \Rightarrow a_y = 1^2 = 1$

(c) At $t = 0, x = 0, v_y = xv_x$

Speed =1; $v_y = 0 \Rightarrow v_x = 1$

(d) $a_x = 0$ implies that at $t = 1s$

$a_y = {v_x}^2 + xa_x \Rightarrow v_y = xv_x \Rightarrow a_y = {v_x}^2$

If $a_x = 0 \Rightarrow V_x = \text{Constant initially } (v_x = 1) \Rightarrow a_y = 1^2 = 1$

At $t = 1\text{sec}$

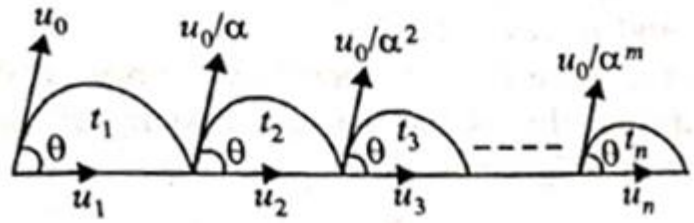
$$v_y = 0 + a_y \times t = 1\times1 = 1$$

$$\tan\theta = \frac{v_y}{v_x} = x\left(\theta \to \text{angle with } x - \text{axis}\right)$$

$$\therefore \tan\theta = \frac{v_y}{v_x} = \frac{1}{1} \Rightarrow \theta = 45^\circ$$

8. (4)

Let $u_1, u_2, u_3, \ldots\ldots$ be the horizontal velocity of the projectiles and $t_1, t_2, t_3, \ldots..$ be the time taken as shown in figure



Average velocity $= \dfrac{\text{total displacement}}{\text{total time taken}}$

Time of flight $T = \dfrac{2u\sin\theta}{g}$

For given value of θ, value of T will change with the value of u.

$\therefore$ Total time taken $= t_1 + t_2 + t_3 + \ldots\ldots$

$$= t_1 + \frac{t_1}{\alpha} + \frac{t_1}{\alpha^2} + \ldots\ldots = \frac{t_1}{1-\frac{1}{\alpha}} = \frac{4\alpha}{(\alpha-1)}$$

Total displacement $= u_1t_1 + u_2t_2 + u_3t_3 + \ldots\ldots$

$$= u_1t_1 + \frac{u_1}{\alpha}.\frac{t_1}{\alpha} + \frac{u_1}{\alpha^2}.\frac{t_1}{\alpha^2} + \ldots\ldots. = \frac{u_1t_1}{1-\frac{1}{\alpha^2}} = \frac{u_1t_1\alpha^2}{\left(\alpha^2-1\right)}$$

$$\therefore \text{Average velocity} = \frac{\text{total displacement}}{\text{total time}}$$

$$= \frac{u_1t_1\alpha^2}{\left(\alpha^2-1\right)} \times \frac{(\alpha-1)}{t_1\alpha} = \frac{u_1\alpha}{\alpha+1}$$

According to questions, average velocity $= 0.8V_1$

Or $0.8V_1 = \dfrac{u_1\alpha}{\alpha+1}$ ……(i)

$V_1 = \dfrac{u_1 \times t_1}{t_1} = u_1$ ……(ii)

From equation (i) and (ii)

$\alpha = 0.8\alpha + 0.8$ or $\alpha - 0.8\alpha = 0.8$

Or $0.2\alpha = 0.8$ or $\alpha = \dfrac{0.8}{0.2} = 4$

9. (0.95)

After entering in new region, time taken by projectile to reach ground is given as

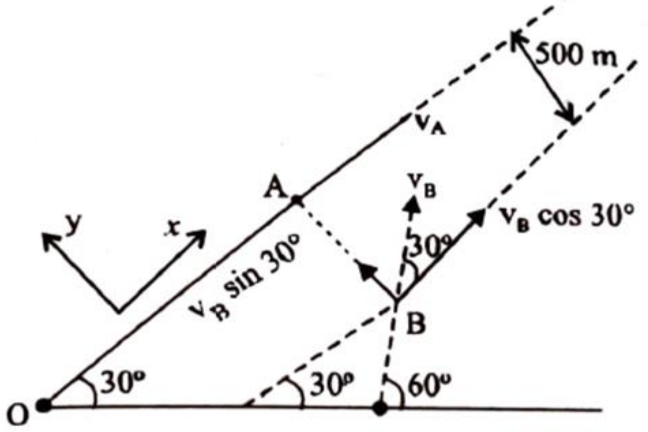
$$t = \sqrt{\frac{2h}{g_{eff}}} = \sqrt{\frac{2 \times 0.81 \times u^2 \sin^2 \theta}{g \times 2g}} = 0.9 \frac{u \sin \theta}{g}$$

So, horizontal displacement done by projectile in new region is given as

$$x = 0.9 \times \frac{u \sin \theta}{g} \times u \cos \theta = 0.9 \left(\frac{d}{2} \right)$$

Now, new range $= \frac{d}{2} + 0.9 \frac{d}{2} = 0.95d$

10. (5)



As 'A' see the motion of 'B' ⊥ r to $\vec{V}_A$.

So, $v_A = v_B \cos 30^\circ$ …….(i)

Now, $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

$= v_B \cos 30\hat{i} + v_B \sin 30\hat{j} - v_A \hat{i}$

$= v_B \sin 30\hat{j}$ [From (i)]

$= \frac{v_A}{\cos 30^\circ} \sin 30^\circ \hat{j}$ [From (i)]

$= v_A \tan 30^\circ \hat{j} = \frac{v_A}{\sqrt{3}} = 100 m/s$

So, $t_0 = \frac{500}{|v_{BA}|} = \frac{500}{100} = 5 sec$

11. (2 or 8)

12. (a) $\left(\frac{\alpha \beta t}{\alpha + \beta} \right)$ (b) $\left(\frac{\alpha \beta t^2}{2(\alpha + \beta)} \right)$

13. displacement = 0

14. 1:1

15. T

16. (a) $\theta = 45^0$ (b) 2 m/s

17. (i) 1 s . ; (ii) $(5\sqrt{3}, 5)$

18. (a) $\frac{u^2 \sin 2\alpha}{g \cos\theta}$, (b) $\frac{u \cos(\alpha + \theta)}{\cos\theta}$

19. u = 7.29 m/s, t = 1 s.