Inequations & Equations

EXERCISE - 1 [A]

1. **(c)**

$$\Rightarrow \sqrt{\log_{0.5}^{2} 4} = \sqrt{[\log_{10} 4]^2} = \sqrt{\left[\log_{\left(\frac{1}{2}\right)} 4\right]^2}$$

$$= \sqrt{\left[\log_{\left(\frac{1}{2}\right)} \left(\frac{1}{2}\right)^{-2}\right]^2} = \sqrt{\left(-2 \times \log_{\frac{1}{2}} \frac{1}{2}\right)^2}$$

$$= \sqrt{(-2 \times 1)} = \sqrt{4}$$

$= 2$ …… (as a square root of a value can't be negative)

2. **(b)**

$$\Rightarrow \log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8 \log_8 9$$

By formula $\log_a b = \dfrac{\log_c b}{\log_c a}$

Given value is $= \left(\dfrac{\log 4}{\log 3}\right)\left(\dfrac{\log 5}{\log 4}\right)\left(\dfrac{\log 6}{\log 5}\right)\left(\dfrac{\log 7}{\log 6}\right)\left(\dfrac{\log 8}{\log 7}\right)\left(\dfrac{\log 9}{\log 8}\right)$

$$= \frac{\log 9}{\log 3} = \log_3 9$$

$= 2$

3. **(c)**

$$\log_7 \log_7 \left(\sqrt{7\sqrt{7\sqrt{7}}}\right)$$

$$= \log_7 \log_7 \left(7^{1/2}.7^{1/7}.7^{1/8}\right)$$

$$= \log_7 \log_7 7^{7/8} = \log_7 (7/8)$$

$$= 1 - 3\log_7 2$$

4. **(c)**

$$\Rightarrow A = \log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2$$

$$\Rightarrow \log_2 \log_2 \log_4 (4)^2 + \log_{2^{\frac{1}{2}}} (2)$$

$\Rightarrow \log_2 \log_2 4 + 2 \times \frac{1}{\left(\frac{1}{2}\right)} \log_2 2$

$\Rightarrow \log_2 2 + 4 = 1 + 4$

$\Rightarrow 5$

5. **(d)**

$\Rightarrow \log_{10} x = y(\text{given})$

$\Rightarrow \log_{1000} x^2 = 2\log_{10^3} x$

$\Rightarrow 2 \times \frac{1}{3} \log_{10} x$ (by formula $\log_{a^k} b = \frac{1}{k} \log_a b$)

6. **(c)**

$\Rightarrow a^{mn} = a^{m^n}$

Take log both side

$\Rightarrow \log_a a^{mn} = \log_a a^{m^n}$

$\Rightarrow mn = m^n$

$\Rightarrow m^{n-1} = n$

$\Rightarrow m = \left(n^{\frac{1}{n-1}}\right)$

7. **(b)**

$\Rightarrow \frac{(\log x - \log y)(\log x^2 + \log y^2)}{(\log x^2 - \log y^2)(\log x + \log y)}$

$\Rightarrow \frac{(\log x - \log y)2(\log x + \log y)}{2(\log x - \log y)x(\log x + \log y)}$

$= 1$

8. **(c)**

$\Rightarrow a > 0, b > 0, c > 0$

$\Rightarrow \log\left(a^a b^b c^c\right) + \log\left(\frac{1}{abc}\right)$

$\Rightarrow \log\left(\frac{a^a b^b c^c}{abc}\right)$

$\Rightarrow \log\left(a^{a-1} b^{b-1} c^{c-1}\right)$

9. **(a)**

$\log_4\left(\frac{4}{4}\right) - 2\log_4\left(4(-2)^4\right) = \log_4 1 - 2\log_4\left(4^3\right)$

$= 0 - 2 \times 3 = -6$

10. **(c)**

$\Rightarrow \log_x x.\log_5 k = \log_x 5$; given $k \neq 1, k > 0$

$\Rightarrow \frac{\log x}{\log k}.\frac{\log k}{\log 5} = \frac{\log 5}{\log x}$

$\Rightarrow (\log_5 x) = (\log_x 5)$

$\Rightarrow x = 5$ is the only possible solution

11. **(c)**

$\Rightarrow \log_5 a.\log_a x = 2$

$\Rightarrow \frac{\log a}{\log 5}, \frac{\log x}{\log a} = 2$

$\Rightarrow \log_5 x = 2$

$\Rightarrow x = 5^2$

$= 25$

12. **(b)**

$\Rightarrow \log(x+1) + \log(x-1) = \log 3$

$\Rightarrow \log(x+1)(x-1) = \log 3$

$\Rightarrow x^2 - 1 = 3$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

But $x+1 > 0$ & $x-1 > 0$

$\Rightarrow x > -1 \& x > 1$

So, $x \in (1, \infty)$ is our feasible region

Only $x = 2$ lies in the feasible region.

13. **(b)**

$\Rightarrow \log_{10}(2x^2 + 7x + 16) = 1$

$\Rightarrow 2x^2 + 7x + 16 = 10^1$

$\Rightarrow 2x^2 + 7x + 6 = 0$

$\Rightarrow 2x^2 + 4x + 3x + 6 = 0$

$\Rightarrow (2x+3)(x+2) = 0$

$\Rightarrow x = -\frac{3}{2}.-2$

14. **(b)**

$\Rightarrow \log_{10}[\log_{10}(\log_{10} x)] = 0$

$\Rightarrow \log_{10}(\log_{10} x) = 10^0 = 1$

$\Rightarrow \log_{10} x = 10^1 = 10$

$\Rightarrow x = 10^0$

15. **(d)**

$\Rightarrow \log_{16} x + \log_4 x + \log_2 x = 14$

$\Rightarrow \frac{1}{4}\log_2 x + \frac{1}{2}\log_2 x + \log_2 x = 14 \quad \left(\text{by} \log_{a^{k}} b = \frac{1}{k}\log_a b\right)$

$\Rightarrow \frac{7}{4}\log_2 x = 14$

$\Rightarrow \log_2 x = 8$

$\Rightarrow x = 2^8 = 256$

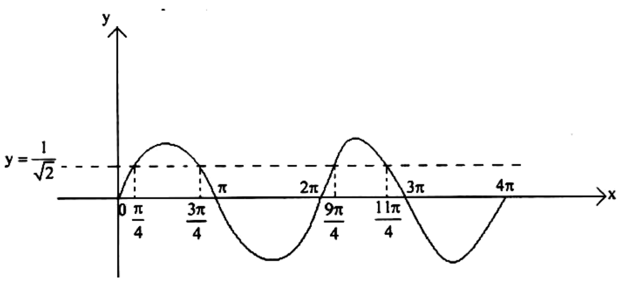
16. **(a)**

$\Rightarrow \log_{\left(\frac{1}{\sqrt{2}}\right)} \sin x > 0; x \in [0, 4\pi]$

As base $\frac{1}{\sqrt{2}}$ lies between 0 to 1satisfy given inequality, $0 < \sin x < 1$

$\Rightarrow x \in (0, \pi) \cup (2\pi, 3\pi)$

As we can see in this interval



We get $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$ as integral

Multiples of $\frac{\pi}{4}$

17. **(b)**

$\Rightarrow \log_{\frac{1}{2}} \left(x^2 - 6x + 12\right) \geq -2$

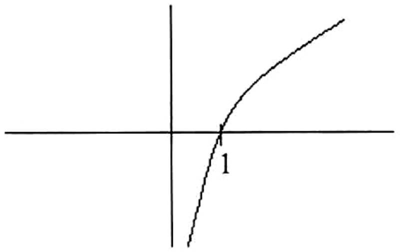
$\Rightarrow \log_{2^{-1}} \left(x^2 - 6x + 12\right) \geq -2$

$\Rightarrow -1 \times \log_2 \left(x^2 - 6x + 12\right) \geq -2$

$\Rightarrow \log_2 \left(x^2 - 6x + 12\right) \leq 2$

$\Rightarrow \log_2 \left(x^2 - 6x + 12\right) - \log_2 4 \leq 0$

$\Rightarrow \log_2 \left(\frac{x^2 - 6x + 12}{4}\right) \leq 0$



$\Rightarrow 0 < \frac{x^2 - 6x + 12}{4} \le 1$

Case-1

$\Rightarrow 0 < \frac{x^2 - 6x + 12}{4}$

$\Rightarrow x^2 - 6x + 12 > 0$

$\Rightarrow x \in R$ …… (1) as discriminant if quadratic expression $x^2 - 6x + 12$ is less than zero.

Descriminant $D = (-6)^2 - 4(12)(1)$

$\Rightarrow D = -12$

Case-2

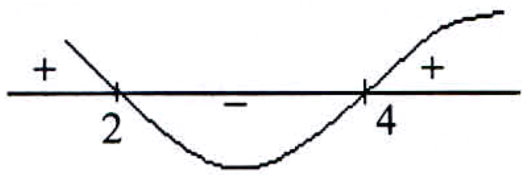
$\Rightarrow \frac{x^2 - 6x + 12}{4} \le 4$

$\Rightarrow x^2 - 6x + 12 \le 4$

$\Rightarrow x^2 - 6x + 8 \le 0$

$\Rightarrow (x-4)(x-2) \le 0$

$\Rightarrow x \in [2, 4]$ ……..(ii)



By taking intersection of (i) & (ii) we get $x \in [2, 4]$

18. **(b)**

$\Rightarrow 2^{\log_{\sqrt{2}}(x-1)} > x + 5$ Here $x - 1 >; x > 1$ ……(1)

$\Rightarrow 2^{\log_{(2)^{\frac{1}{2}}}(x-1)} > x + 5$

$\Rightarrow 2^{2\log_2(x-1)} > x + 5$ (or by formula $\log_{a^k} b = \frac{1}{k} \log \frac{b}{a}$)

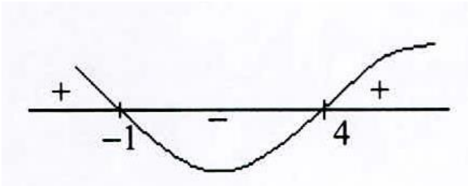
$\Rightarrow 2^{\log_2(x-1)^2} > x + 5$

$\Rightarrow (x-1)^2 > x + 5$

$\Rightarrow x^2 + 1 - 2x > x + 5$

$\Rightarrow x^2 - 3x - 4 > 0$

$\Rightarrow (x-4)(x+1) > 0$



So we get $x \in (-\infty, -1) \cup (4, \infty)$ ……….(ii)

By taking intersection of (i) & (ii)

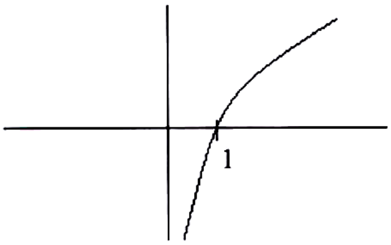
$\Rightarrow x \in (4, \infty)$

19. **(c)**

$\Rightarrow \log_{10}\left(x^2 - 2x - 2\right) \le 0$

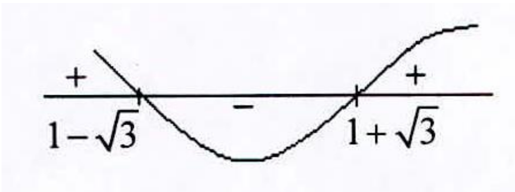
As base is greater than 1 so to hold the inequality true

$\Rightarrow 0 < x^2 - 2x - 2 \le 1$



So, $0 < x^2 - 2x - 2$ and $x^2 - 2x - 2 \le 1$

Case-I



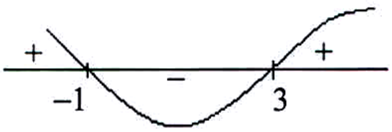
$\Rightarrow x^2 - 2x - 2 > 0$

$\Rightarrow \left[x - \left(1+\sqrt{3}\right)\right]\left[x - \left(1-\sqrt{3}\right)\right] > 0$

So, $x \in \left(-\infty, 1-\sqrt{3}\right) \cup \left(1+\sqrt{3}, \infty\right)$ …….(i)

Case-2

$\Rightarrow x^2 - 2x - 2 \le 1$



$\Rightarrow x^2 - 2x - 3 \le 0$

$\Rightarrow (x-3)(x+1) \le 0$

So we get $x \in [-1, 3]$

By taking intersection of (i) & (ii) we get,

$\Rightarrow x \in \left[-1, 1-\sqrt{3}\right) \cup \left(1+\sqrt{3}, 3\right]$

20. **(a)**

$$\Rightarrow \log_{0.2}\frac{x+2}{x}\le 1$$

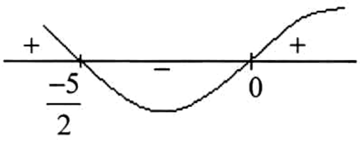
As base of log is less than 1 so hold the inequality true

$$\Rightarrow \frac{x+2}{x}\ge 0.2$$

$$\Rightarrow \frac{x+2}{x}-0.2\ge 0$$

$$\Rightarrow \frac{x+2-0.2x}{x}\ge 0$$

$$\Rightarrow \frac{0.8x+2}{x}\ge 0$$



So, $x\in\left(-\infty,\frac{-5}{2}\right]\cup[0,\infty)$

$$\Rightarrow a^{m\log_a n}\Rightarrow a^{\log_a n^m}\Rightarrow n^m$$

21. **(d)**

$$\Rightarrow \log_{\frac{1}{2}}\left(x^2-1\right)>0$$

As base of log is less than 1

So, $\log_{\frac{1}{2}}\left(x^2-1\right)>0$

$$\Rightarrow 0<x^2-1<1$$

$$\Rightarrow 1<x^2<2$$

$$\Rightarrow x^2>1 \text{ and } x^2<2$$

$$\Rightarrow x\in(-\infty,-1)\cup(1,\infty)\ \&\ x\in\left(-\sqrt{2},\sqrt{2}\right)\Rightarrow x\in\left(-\sqrt{2},-1\right)\cup\left(1,\sqrt{2}\right)$$

22. **(a)**

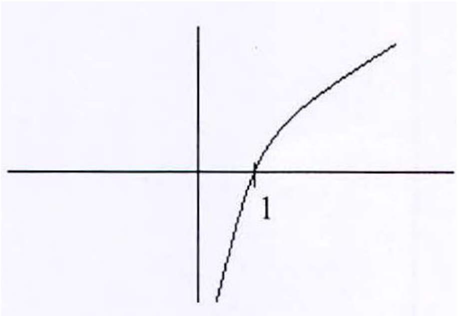
$$\Rightarrow \log_{\frac{\sqrt{3}}{2}}\left(x^2-3x+2\right)\ge 2$$

$$\Rightarrow \log_{\frac{\sqrt{3}}{2}}\left(x^2-3x+2\right)\ge \log_{\frac{\sqrt{3}}{2}}\left(\frac{\sqrt{3}}{2}\right)^2$$

$$\Rightarrow \log_{\frac{\sqrt{3}}{2}}\left(\frac{x^2+3x+2}{\left(\frac{3}{4}\right)}\right)\ge 0$$

$\because$ base is less than 1 so

$$\Rightarrow 0 < \left(\frac{x^2 - 3x + 2}{\frac{3}{4}} \right) \le 1$$



$$\Rightarrow 0 < x^2 - 3x + 2 \ \& \left(x^2 - 3x + 2 \right) \le \frac{3}{4}$$

$$\Rightarrow (x-2)(x-1) > 0 \ \& \ x^2 - 3x + \frac{5}{4} \le 0$$

$$\Rightarrow x \in (-\infty, 1) \cup (2, \infty) \qquad \text{.......(1)} \ \& \ x^2 - \frac{5}{2}x - \frac{1}{2}x + \frac{5}{4} \le 0$$

$$\& \left(x - \frac{5}{2} \right)\left(x - \frac{1}{2} \right) \le 0$$

$$\& x \in \left(\frac{1}{2}, \frac{5}{2} \right) \qquad \text{.......(2)}$$

Taking intersection of (i) & (ii)

$$\Rightarrow x \in \left[\frac{1}{2}, 1 \right) \cup \left(2, \frac{5}{2} \right]$$

23. **(a)**

$$\log_{1/3}\left(x^2 + x + 1 \right) + 1 < 0$$

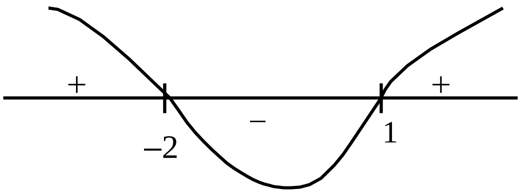
$$\log_{1/3}\left(x^2 + x + 1 \right) < -1$$

$$x^2 + x + 1 > \left(\frac{1}{3} \right)^{-1}$$

$$x^2 + x + 1 > 3$$

$$x^2 + x - 2 > 0$$

$$(x+2)(x-1) > 0$$



$$x \in (-\infty, -2) \cup (1, \infty)$$

24. **(b)**

$$\Rightarrow \left(x^5 \right)^{\frac{1}{3}} \left(16x^3 \right)^{\frac{2}{3}} \left(\frac{1}{4} x^{\frac{4}{9}} \right)^{-\frac{3}{2}} . (4)^{\frac{1}{6}}$$

$\Rightarrow x^{\frac{5}{3}}.\left(4^2\right)^{\frac{2}{2}}.\left(x^3\right)^{\frac{2}{2}}.\left(4^{-1}\right)^{\frac{-3}{2}}.\left(x^{\frac{4}{9}}\right)^{\frac{-3}{2}}$

$\Rightarrow x^{\frac{5}{3}} \times 4^{\frac{4}{3}} \times x^2 \times 4^{\frac{3}{2}} \times x^{\frac{4}{9}\times\frac{-3}{2}}$

$x^{\frac{5}{3}-\frac{2}{3}+2}.4^{\left(\frac{4}{3}+\frac{3}{2}+\frac{1}{6}\right)}$

$\Rightarrow 4^3.x^3$

25. **(c)**

$\Rightarrow \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}.\left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}.\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}$

$\Rightarrow x^{\left(\frac{1}{c}-\frac{1}{b}\right)} \times x^{\left(\frac{1}{a}-\frac{1}{c}\right)} \times x^{\left(\frac{1}{b}-\frac{1}{a}\right)}$

$\Rightarrow x^{\frac{1}{c}-\frac{1}{b}+\frac{1}{a}-\frac{1}{c}+\frac{1}{b}-\frac{1}{a}}$

$\Rightarrow x^0 = 1$

26. **(c)**

$\Rightarrow a^m.a^n = a^{mn}$

$\Rightarrow a^{m+n} = a^{nm}$

$\Rightarrow m + n = mn$ ………(1)

Then $m(n-2) + n(m-2) = ?$

$\Rightarrow 2mn - 2m - 2n$

$\Rightarrow 2(m+n) - 2(m+n)$

27. **(b)**

$\Rightarrow \frac{2^{m+3} \times 3^{2m-2n} \times 5^{m+3+n} \times 6^{n+1}}{6^{m+1} \times 10^{n+3} \times 15^m}$

$\Rightarrow \frac{\left(2^{m+3}\right)\left(3^{2m-2}\right)\left(5^{m+n+3}\right) \times 2^{n+1} \times 3^{n+1}}{\left(2^{m+1}3^{m+1}\right)\left(2^{n+3}5^{n+3}\right)\left(3^m\right)\left(5^m\right)}$

$\Rightarrow 2^{m+3+n+1-(m+1)}3^{2m-n+n+1-(m+1)-m}5^{m+n+3-(n+3)-m}$

$\Rightarrow 2^0 3^0 5^0$

$\Rightarrow 1$

28. **(c)**

$a^{mn} = a^{m^n}$

$\Rightarrow mn = m^n$

$\Rightarrow n = m^{n-1}$

$\Rightarrow m = n^{\frac{1}{n-1}}$

29. **(d)**

$$\Rightarrow \frac{\left(2^{n+1}\right)^m \left(2^{2n}\right)2^n}{\left(2^{m+1}\right)^m 2^{2m}} = 1$$

$$\Rightarrow \frac{2^{nm+m} \times 2^{2n+n}}{2^{mn+m}2^{2m}} = 1$$

$$\Rightarrow 2^{(nm+m+2n+n)-(mn+n-2m)} = 1$$

$$\Rightarrow 2^{2n-m} = 1$$

$$\Rightarrow 2n - m = 0$$

$$\Rightarrow m = 2n$$

30. **(c)**

$$\Rightarrow 5^{x-1} + 5(0.2)^{x-2} = 26$$

$$\Rightarrow 5^{x-1} + 5\left(\frac{1}{5}\right)^{x-2} = 26$$

$$\Rightarrow 5^{x-1} + 5^{1-x+2} = 26$$

$$\Rightarrow 5^{x-1} + 5^{3-x} = 26$$

At $x = 1, 3$ above equation satisfy

31. **(c)**

$$\Rightarrow 2^x - 2^{x-1} = 4$$

$$\Rightarrow 2^x - \frac{2^x}{2} = 4$$

$$\Rightarrow 2.2^x - 2^x = 8$$

$$\Rightarrow 2^x = 8$$

$$\Rightarrow x = 3$$

So, $x^x = 3^x = 27$

32. **(c)**

$$\Rightarrow (25)^{x-2} = (125)^{2x-4}$$

$$\Rightarrow \left(5^2\right)^{x-2} = \left(5^3\right)^{2x-4}$$

$$\Rightarrow 5^{2x-4} = 5^{6x-12}$$

$$\Rightarrow 5^{6x-12-(2x-4)} = 1$$

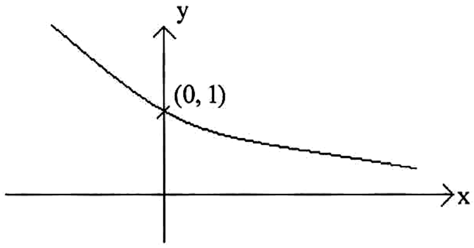
$$\Rightarrow 5^{4x-8} = 1 = 5^{\circ}$$

So, $4x - 8 = 0$

$$\Rightarrow x = 2$$

33. **(a)**

$\Rightarrow a^{x^2-x} \geq a^2; 0<9<1$



$\Rightarrow \dfrac{a^{x^2}-x}{a^2} \geq 1$

$\Rightarrow a^{x^2-x-2} \geq 1$

$\Rightarrow x^2-x-2 \leq 0$

$\Rightarrow (x-2)(x+1) \leq 0$

$\Rightarrow x \in [-1, 2]$

34. **(a)**

$4^{-x+0.5} - 7.2^{-x} < 4$

$\dfrac{4^{0.5}}{4^x} - \dfrac{7}{2^x} < 4$

Let $\dfrac{1}{2^x} = k$

$\therefore 2.k^2 - 7k < 4$

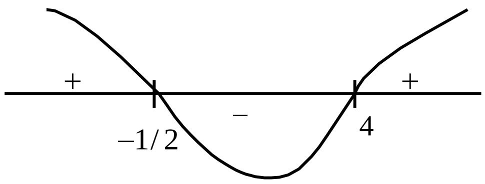
$2k^2 - 7k - 4 < 0$

$2k^2 - 7k - 4 < 0$

$2k^2 - 8k + k - 4 < 0$

$2k(k-4) + (k-4) < 0$

$(2k+1)(k-4) < 0$



$k \in \left(-\dfrac{1}{2}, 4\right)$

As $k = \dfrac{1}{2^x}$ it can only be $+ve$

$\therefore 0 < \dfrac{1}{2^x} < 4$

$\Rightarrow x \in (-2, \infty)$

35. **(d)**

$\Rightarrow f(x) = 5 - |x-3|$

$\Rightarrow f(x) = 5 - (x-3); x \in [3, \infty)$

$\Rightarrow 5 + (x-3); x \in (-\infty, 3)$

$\Rightarrow f(x) = 8 - x; x \in [3, \infty)$

$= 2 + x; x \in (\infty, 3)$

So, greatest value of function occur at $x = 3$

So $f(3) = 8 - 3 = 5$

36. **(b)**

Let $f(x) = |x-4| + 2$

As we know that $|x| \geq 0$ for every $x \in R$

$\therefore |x-4| \geq 0$

The minimum value of function is attained when $|x-4| = 0$

Hence, minimum value of $f(x) = 0 + 2 = 2$

Alternate Method

$f(x) = |x-4| + 2$

There is one critical point i.e. $x = 4$

$f(4) = |4-4| + 2$

$= 0 + 2$

$= 2$

Hence, 2 is the minimum value of $f(x)$.

37. **(b)**

Given equation is $x^2 - 3|x| + 2 = 0$

We can write this as

$|x|^2 - 3|x| + 2 = 0$

$\Rightarrow |x|^2 - |x| - 2|x| + 2 = 0$

$\Rightarrow |x|(|x|-1) - 2(|x|-1) = 0$

$\Rightarrow (|x|-1)(|x|-2) = 0$

This is possible if, at least one of the two factors is zero, i.e.

$|x| - 1 = 0$ or $|x| - 2 = 0$

$\Rightarrow |x| = 1$ or $|x| = 2$

$\Rightarrow x = \pm 1$ or $x = \pm 2$

Clearly, we can see that there is four distinct value of *x*.

38. **(b)**

Given $\left|x^2-12x+32\right|+\left|x^2-9x+20\right|=0$.

Every modulus function is a non-negative function and if two non-negative functions add up to get zero then individual function itself equal to zero simultaneously.

$x^2-12x+32=0$ for $x=$ 4 or 8

$x^2-9x+20=0$ for $x=4$ or 5

Both the equations are zero at $x=4$

So, $x=4$ is the only solution for this equation.

39. **(a)**

Since the modulus function '||' always returns a positive value or 0, its not possible to have –2 as the value of modulus of any expression.

Hence, $|x+2|=-2$ has **no solution**.

40. **(a)**

$\Rightarrow x^2-|x|-6=0$

Case 1: $x \ge 0$

$\Rightarrow x^2-x-6=0$

$\Rightarrow x=3,-2$

But $x \ge 0$ so $x=3$ is the only root.

Case 2: $x<0$

$\Rightarrow x^2-(-x)-6=0$

$\Rightarrow x^2+x-6=0$

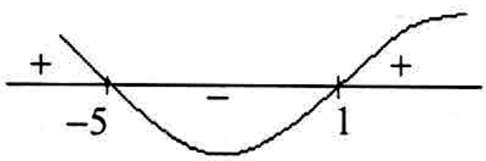
$\Rightarrow x=3,-2$

But $x<0$ so $x=-3$ is the solution.

So multiplication $=3(-3)=-9$

41. **(a)**

$\Rightarrow |x-1|+|x+5|=6$



This is special case as $(x-1)-(x+5)=-6$

So the given expression will hold true if $(x-1)(x+5)\le 0$

$\Rightarrow x\in[-5,1]$

42. **(b)**

$|x-3|+|x+5|=7x$

$2x+2=7x \quad x\ge 3$

$-(x-3)+(x+5)=7x \quad -5<x<3$

$-(x-3)-(x+5)=7x \quad x\le -5$

43. (c)

Let $2x+3>5$ and $2x+3<-5$ {by the property of modulus}

So, first take $2x+3>5$

$\Rightarrow 2x+3-3>5-3$

$\Rightarrow 2x>2$

$\Rightarrow x>1$

Hence, $x\in(1,\infty)$

Now take $2x+3<-5$

$\Rightarrow 2x+3-3<-5-3$

$\Rightarrow 2x<-8$

$\Rightarrow x<-4$

Hence, $x\in(-\infty,-4)$

$\therefore x\in(-\infty,-4)\cup(1,\infty)$

44. (c)

$\Rightarrow |4-3x|\le\frac{1}{2}$

Case 1: $4-3x\ge 0\Rightarrow x\le\frac{4}{3}$ …….(i)

So $4-3x\le\frac{1}{2}$

$\Rightarrow -3x\le-\frac{7}{2}$

$\Rightarrow x\ge\frac{7}{6}$

By intersection of (i) & (ii) $x\in\left[\frac{7}{6},\frac{4}{3}\right]$ ……..(A)

Case 2: $4-3x\le 0\Rightarrow x>\frac{4}{3}$ ………(iii)

$\Rightarrow$ So $-\left(4-3x\right)\le\frac{1}{2}$

$\Rightarrow -4+3x\le\frac{1}{2}$

$\Rightarrow 3x\le\frac{9}{2}$

$\Rightarrow x\le\frac{3}{2}$ ………(iv)

Taking intersection of (iii) & (iv)

$\Rightarrow x\in\left(\frac{4}{3},\frac{3}{2}\right]$ ……….(B)

So union of A & B is the solution of the given inequality $x\in\left[\frac{7}{6},\frac{3}{2}\right]$ ……..(C)

45. **(c)**

$\Rightarrow \frac{|x|-1}{|x|+2} > 0$

So, $|x| > -2$ or $|x| > 1$

$\because |x| > -2$ holds true for $x \in R$

Now, $|x| > 1$

$\Rightarrow x \in (-\infty, -1) \cup (1, \infty)$

46. **(d)**

$||x|-1| < |1-x|$

Case I $x \geq 1$

$x-1 < -(1-x)$

$x-1 < -(1-x)$

$-1 < -1$

No solution

Case II $0 \leq x \leq 1$

$1-x < 1-x$

No solution

Case III $-1 \leq x < 0$

$|-x-1| < 1-x$

$1+x < 1-x$

$x < 0$

$\Rightarrow x \in [-1, 0)$

Case IV: $x < -1$

$|-x-1| < |1-x|$

$|1+x| < 1-x$

$-(1+x) < 1-x$

$-1-x < 1-x$

$-1 < 1$

True for all x.

$x \in (-\infty, -1)$

$\therefore x \in (-\infty, -1] \cup [-1, 0)$

$\Rightarrow x \in (-\infty, 0)$

47. **(a)**

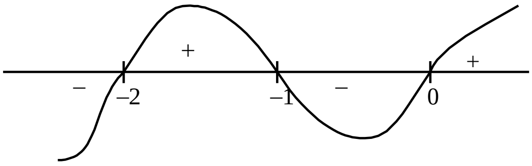
$\left|x+\frac{2}{x}\right|<3$

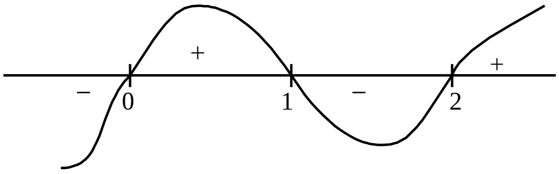
$-3<x+\frac{2}{x}<3$

$x+\frac{2}{x}+3>0$, $x+\frac{2}{x}-3<0$

$\frac{x^2+3x+2}{x}>0$, $\frac{x^2-3x+2}{x}<0$

$\frac{(x+1)(x+2)}{x}>0$, $\frac{(x-1)(x-2)}{x<0}$





$x\in(-2,-1)\cup(0,\infty)$ $x\in(-\infty,0)\cup(1,2)$

$\therefore x\in(-2,-1)\cup(1,2)$

48. **(b)**

$x^2=|x+2|+x>0$

Case I $x\geq-2$

$x^2-(x+2)+x>0$

$x^2-2>0$

$x\in\left(-\infty,-\sqrt{2}\right)\cup\left(\sqrt{2},\infty\right)$

Case II $x\leq-2$

$x^2+(x+2)+x>0$

$x^2+2x+2>0$

$(x+1)^2+1>0$

$x\in R \text{ i.e. } x<-2$

From (1) & (2)

$x\in\left(-\infty,-\sqrt{2}\right)\cup\left(\sqrt{2},\infty\right)$

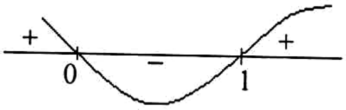
49. **(c)**

$\Rightarrow\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}=\frac{13}{6}$ here $\frac{x}{1-x}\geq0;\frac{x}{1-x}\geq0;\frac{x}{x-1}\leq0$

So, $x\in[0,1)$ is the feasible region for the equation

$\Rightarrow\frac{(x)+(1-x)}{\sqrt{1-x}\sqrt{x6}}=\frac{13}{6}$

$\Rightarrow\frac{1}{\sqrt{x(1-x)}}=\frac{13}{6}$



Taking square both side

$\Rightarrow x(1-x)=\frac{36}{169}$

$\Rightarrow x^2-x+\frac{36}{169}=0$

$\Rightarrow \left(x-\frac{9}{13}\right)\left(x-\frac{4}{13}\right)=0$

$\Rightarrow x=\frac{9}{13},\frac{4}{13}$

Here values lies in the feasible region

So, $x=\frac{9}{13},\frac{4}{13}$

50. **(d)**

$\Rightarrow \sqrt{3y+1}=\sqrt{y-1}$ ……..(1)

$\Rightarrow 3y+1\ge 0\ \&\ y-1\ge 0$

$\Rightarrow y\ge -\frac{1}{3}\ \&\ y\ge 0$

$\Rightarrow y\in[0,\infty)$ is our feasible region

By equation (1), taking square of both side,

$\Rightarrow 3y+1=y-1$

$\Rightarrow 2y=-2$

$\Rightarrow y=-1$; which does not lie in feasible range of y.

So no solution of y.

51. **(a)**

$\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}$

$\Rightarrow (x+1)+(x-1)-2\sqrt{x^2-1}$

$=4x-1$

$\Rightarrow 2x+1-4x=2\sqrt{x^2-1}$

$\Rightarrow 1-2x=2\sqrt{x^2-1}$

$\Rightarrow (1-2x)^2=4\left(x^2-1\right)$

$\Rightarrow 1-4x+4x^2=4x^2-4$

$\Rightarrow 4x=5 \Rightarrow x=\frac{5}{4}$

Putting $x=\frac{5}{4}$ in the original equation,

L.H.S. = 1 & R.H.S. = 2,

Hence, no solution.

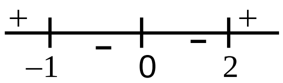
52. **(a)**

$$\Rightarrow \sqrt{1-\left(\frac{x+2}{x^2}\right)} < \frac{2}{3}$$

So, $1-\left(\frac{x+2}{x^2}\right) \geq 0$

$$\Rightarrow \frac{x^2-x-2}{x^2} \geq 0$$

$$\Rightarrow \frac{(x-2)(x+1)}{x^2} \geq 0$$



So $x \in (-\infty,-1] \cup [2,\infty)$ ……(1)

$$\Rightarrow \sqrt{1-\left(\frac{x+2}{x^2}\right)} < \frac{2}{3}$$

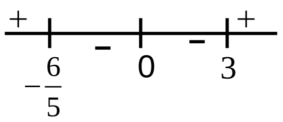
$$\Rightarrow 1-\left(\frac{x+2}{x^2}\right) < \frac{4}{9}$$

$$\Rightarrow \frac{5}{9}-\frac{x+2}{x^2} < 0$$

$$\Rightarrow \frac{5x^2-9x-18}{9x^2} < 0$$

$$\Rightarrow \frac{(5x+6)(x-3)}{x^2} < 0$$

So, $x \in \left(-\frac{6}{5},0\right) \cup (0,3)$ …….(2)



By taking $(1) \cap (2)$

$$\Rightarrow x \in \left(-\frac{6}{5},-1\right] \cup [2,3)$$

53. **(c)**

$$\Rightarrow (x-1)\sqrt{x^2-x-2} \geq 0$$

$\Rightarrow (x^2-x-2) \geq 0$ & $(x-1) \geq 0$

$\Rightarrow (x-2)(x+1) \geq 0$ & $x \geq 1$

$\Rightarrow x \in (-\infty,-1] \cup [2,\infty)$ & $x \in [1,\infty)$

So, $x \in [2,\infty)$

EXERCISE - 1 [B]

1. **(b)**

$\Rightarrow \log(ab) - \log|b|$

We can see that $ab > 0 \Rightarrow a < 0 \& b < 0$ or $a > 0 \& b > 0$

So $\log(ab) - \log|b| = \log|ab| - \log|b|$

$= \log|a|.|b| - \log|b|$

$= \log|a| + \log|b| - \log|b|$

$= \log|a|$

2. **(b)**

$\log_3 M + 9\log_3 N = 3(1 + \log_{0.008} 5)$

$\log_3 MN^9 = 3(\log_{0.008} 5 \times 0.008)$

$\log_3 (MN^9) = 3\log_{0.008} 0.04 = 3 \times \frac{2}{3}$

So $MN^9 = 9$

3. **(b)**

$x = \log_a nc = \frac{\log bc}{\log a}$

$x + 1 = \frac{\log bc}{\log a} + 1$

$\Rightarrow x + 1 = \frac{\log bc + \log x}{\log a}$

$\Rightarrow x + 1 = \frac{\log abc}{\log a}$

Similarly

$y + 1 = \frac{\log abc}{\log b}$ & $z + 1 = \frac{\log abc}{\log c}$

$\therefore \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$

$\Rightarrow \frac{\log a}{\log abc} + \frac{\log b}{\log abc} + \frac{\log c}{\log abc}$

$= \frac{\log abc}{\log abc} = 1$

4. **(c)**

$N = \frac{4^5 + 4^5 + 4^5 + 4^5}{3^5 + 3^5 + 3^5} \cdot \frac{6^5 + 6^5 + 6^5 + 6^5 + 6^5 + 6^5}{2^5 + 2^5}$

$= N = \frac{4 \times 4^5}{3 \times 3^5} \times \frac{6 \times 6^5}{2 \times 2^5}$

$$= \frac{4^6 \times 6^5}{(2\times3)^5} = \frac{4^6 \times 6^5}{6^5}$$

$N = 4^6$

$N = (2^2)^6$

$N = 2^{12}$

$\log_2 N = \log_2 2^{12} = 12$

5. **(c)**

$\log 15 = a \qquad \log 75 = b$

$\log 5 + \log 3 = a \;\Rightarrow\; \log 15 + \log 5 = b \qquad ...(\text{i})$

$\Rightarrow\; \log 3 + 2\log 5 = b$ (from (i))

$\Rightarrow\; -b + 2a = \log 3$

$$\log_{75} 45 = \frac{\log 15 + \log 3}{\log 75} = \frac{a - b + 2a}{b} = \frac{3a - b}{b}$$

6. **(a)**

$\log_a b = 2, \log_b c = 2, \log_3 c = \log_3 a + 3$

$\log_a b = 2 \;\Rightarrow\; b = a^2$

$\log_b c = 2 \;\Rightarrow\; c = b^2 \qquad \Rightarrow\; \log_3 c/a = 3 \Rightarrow c = 27a$

Now $a > 0, b > 0, c > 0, a \neq 1, b \neq 1$

If $b = a^2$ & $c = b^2$

$c = a^4 = 27a = 0 \;\Rightarrow\; a(a^3 - 27) \qquad \Rightarrow$ this gives $a = 0, a = 3$

$a = 3$

$b = a^2 = 9$

$c = b^2 = 81$

7. **(b)**

$$\frac{2}{6\log_4 2000} + \frac{3}{6\log_5 2000}$$

$$= \frac{1}{6}[2\log_{2000} 4 + 3\log_{2000} 5]$$

$$= \frac{1}{6}\left[\log_{2000} 4^2.5^3\right]$$

$$= \frac{1}{6}\log_{2000} 2000 = \frac{1}{6}$$

8. **(b)**

$\log_{10}{}^2 = \beta$

$$\log_{10}\left(\frac{1025}{1024} \times \frac{4}{4}\right) = \alpha$$

$\Rightarrow\; \log_{10} 4100 - \log_{10} 2^{12} = \alpha$

$\Rightarrow\; \log_{10} 4100 = \alpha + 12\beta$

9. (d)

We know that, for any $-1 < r < 1, a + ar + ar^2 + ... + \infty = \frac{a}{1-r}$ therefore

$$\frac{1}{3} + \frac{1}{3^2} + ... + \infty = \frac{1/3}{1-1/3} = \frac{1}{2}$$

Finally we have

$$(0.16)^{\log_{2.5}\left[(1/3)+(1/3^2)+......+\infty\right]} = \left(\frac{2}{5}\right)^{2\log_{2.5}(1/2)} = \left(\frac{2}{5}\right)^{-2\log_{2.5}(2)} = \left(\frac{5}{2}\right)^{\log_{5/2}(4)} = 4$$

10. (d)

$$\Rightarrow 81^{\left(\frac{1}{\log_5 3}\right)} + 27^{(\log_9 36)} + 3^{\frac{4}{\log_7 9}}$$

$$\Rightarrow (81)^{(\log_3 5)} + (3^3)^{\left(\log_{3^2} 36\right)} + 3^4 \log_9 7$$

$$\Rightarrow (3^4)^{\log_3 5} + 3^{3\times\left(\frac{1}{2}\log_3 36\right)} + 3^{4\times\left(\frac{1}{2}\log_3 7\right)}$$

$$\Rightarrow 3^{\log_3 5^4} + 3^{\log_3 (36)^{\frac{3}{2}}} + 3^{\log_3 7^2}$$

$$\Rightarrow 5^4 + (36)^{\frac{3}{2}} + 7^2$$

$$625 + 36 \times 6 + 49 = 890$$

11. (b)

$$A = 12^{300}$$

$$\log_{10} A = 300[\log_{10} 12]$$

$$= 300[0.6010 + 0.4771]$$

$$= 300 \times 1.0781 = 323.43$$

$$\Rightarrow A = 10^{323.43}$$

Hence 324 digits

12. (a)

$$\log_{10} 2 = 0.3010$$

$$\log_5 64 = \frac{\log_{10} 64}{\log_{10} 5} = \frac{6\log_{10} 2}{\log_{10} 10 - \log_{10} 2}$$

$$= \frac{6 \times 0.3010}{1 - 0.3010}$$

$$= \frac{1.8060}{0.6990}$$

$$= \frac{602}{233}$$

13. (a)

$$x = \log_5 (1000) = \log_5 125 + \log_5 8 > 4$$

$$y = \log_{57} (2056) = \log_7 343 + \log_7 6 < 4$$

Hence $x > 4$

14. **(d)**

Note that $x > 0, x \neq 1$.

Using $a^{\log_a x} = a$, we get $(1-x)^2 = 9 \Rightarrow x = 4, -2$

Also $x > 0$, we get $x = 4$.

15. **(b)**

$2^{\log_{10} 3\sqrt{3}} = 3^{k \log_{10} 2}$

$\Rightarrow 2^{\log_{10} 3\sqrt{3}} = 2^{k} \log_{10} 3$

$\Rightarrow \log_{10} 3\sqrt{3} = \log_{10} 3^{k}$

$\Rightarrow k = 3^{3/2}$

$\Rightarrow k = 3/2$

16. **(b)**

$\log_{10}(x-1)^3 - \log_{10}(x-3)^3 = \log_{10} 8$

$\Rightarrow \log_{10}\left(\frac{x-1}{x-3}\right)^3 = \log_{10}(2)^3 \quad \Rightarrow \frac{x-1}{x-3} = 2 \quad \Rightarrow x - 1 = 2x - 6$

$\Rightarrow x = 5$

So, $\log_x 625 = \log_5 (5)^4 = 4$

17. **(d)**

$\log_{x-3} \log_{2x^2-2x+3}(x^2+2x) = 0$

$\Rightarrow \log_{2x^2-2x+3}(x^2+2x) = 1$

$\Rightarrow (x^2) + 2x = 2x^2 - 2x + 3$

$\Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow (x-3)(x-1) = 0$

$\Rightarrow x = 1$ or $x = 3$

Both do not belong to the domain. So, no solution

18. **(c)**

$\log_2 \log x$ is meaningful if $x > 1$

Since $4^{\log_2 \log x} = 2^{2\log_2 \log x} = \left(2^{\log_2 \log x}\right)^2 = (\log x)^2 \qquad \left[a^{\log_a x} = x, a > 0, a \neq 1\right]$

So the given equation reduces to

$2(\log x)^2 - \log x - 1 = 0$

$\Rightarrow \log x = 1, \log x = -\frac{1}{2}$.

But for $x > 1$, $\log x > 0$

So, $\log x = 1$ i.e. $x = e$.

19. **(a)**

The equation is meaningful if $|\sin x| \neq 0, 1$ and $1 + \cos x \neq 0$

So $x \neq k\pi, k = 0, 1, \ldots n, x \neq (2k+1)\frac{\pi}{2}, k = 0, 1, \ldots n-1$.

Now, $\log_{|\sin x|}\left(1+\cos x\right)=2$

$\Leftrightarrow\ 1+\cos x=|\sin x|^2=\sin^2 x=1-\cos^2 x$

$\Rightarrow\ \left(1+\cos x\right)\left(\cos x\right)=0$

$\Leftrightarrow\ \cos x=0$ or $\cos x=-1$

$\Rightarrow\ \cos x=0\ \Rightarrow x=\left(2k+1\right)\frac{\pi}{2}.$

So there is no *x* which satisfies the given equation.

20. (d)

$\Rightarrow \log_2 x+\log_x 2=\frac{10}{3}=\log_2 y+\log_y 2$

$\Rightarrow \log_2 x+\log_x 2=\frac{10}{3}$

$\Rightarrow \log_2 x+\frac{1}{\log_2}x=\frac{10}{3}$

Let's take $\log_2 x=a$

So, $a+\frac{1}{a}=\frac{10}{3}$

$\Rightarrow a^2-\frac{10}{3}a+1=0$

$\Rightarrow \left(a-\frac{9}{3}\right)\left(a-\frac{1}{3}\right)=0$

$\Rightarrow a=\frac{9}{3},\frac{1}{3}$

$\Rightarrow \log_2 x=\frac{9}{3},\frac{1}{3}=3,\frac{1}{3}$

$\Rightarrow x=2^3,2^{\frac{1}{3}}$

$\Rightarrow x=8,2^{\frac{1}{3}}$

Similarly, $y=8,2^{\frac{1}{3}}$

If $x\neq y$ then $x+y=8+2^{\frac{1}{3}}$

21. (b)

$x^{\log_3 x^2}+\left(\log 3x\right)^2-10=\frac{1}{x^2}$

Clearly one solution is $x=1$

OR

$\log_3 x^2+\left(\log_3 x\right)^2-10=-2$

$2\log_3 x+\left(\log_3 x\right)^2=8$

Let $\log_3 x=k$

$\therefore 2k+k^2-8=0$

$k^2+2k-8=0$

$(k+4)(k-2)=0$

$\Rightarrow \log_3 x = 2 \text{ or } \log_3 x = -4$

$\Rightarrow x = 0 \text{ or } x = \frac{1}{81}$

$x \in \left\{1, 9, \frac{1}{81}\right\}$

22. **(c)**

$\log_{1/2} x \ge \log_{1/3} x$

$-\log_2 x \ge -\log_3 x$

$\log_2 x \le \log_3 x$

$\frac{\log x}{\log 2} \le \frac{\log x}{\log 3}$

$\log^x \left[\frac{1}{\log 2} - \frac{1}{\log 3}\right] \le 0$

$\log x \le 0 \left\{\because \frac{1}{\log 2} - \frac{1}{\log 3} > 0\right\}$

$\Rightarrow x \in (0, 1]$

23. **(b)**

$\Rightarrow \log_4\left(\frac{x+1}{x+2}\right) > 0$

$\Rightarrow \frac{x+1}{x+2} > 0$

$\Rightarrow \frac{x+1-x-2}{x+1} > 0$

$\Rightarrow \frac{-1}{x+2} > 0$

$\Rightarrow \frac{1}{x+2} < 0$

$\Rightarrow x \in (-\infty, -2)$

24. **(d)**

$\frac{1}{\log_3\left(2^{2x}-1\right)} > \frac{1}{\log_3\left(2^x+1\right)}$

Domain $x > 0, x \ne \frac{1}{2}$

$\log_3 2^x + 1$ is always positive

Hence, $\log_3\left(2^{2x}-1\right) > 0$

$x > \frac{1}{2}$

And $\log_3\left(2^{2x}-1\right) < \log_3\left(2^x+1\right)$

$\Rightarrow 2^{2x} - 1 < 2^x + 1$

$\Rightarrow t^2 - t - 2 < 0 \qquad (t = 2x)$

$\Rightarrow t \in (-1, 2)$

$\Rightarrow x \in (-\infty, 1)$

Hence solution is $\left(\frac{1}{2}, 1\right)$

25. (b)

For (1) to hold, we must have

$x > 0, x \neq 1$ and $2x^2 + x - 1 > 0$

$\Rightarrow \quad x > 0, x \neq 1$ and $(2x-1)(x+1) > 0$

$\Rightarrow \quad x > \frac{1}{2}, x \neq 1$.

We can write (1) as

$$\log_x\left(\frac{2x^2 + x - 1}{2}\right) > -1 \qquad (2)$$

For $\frac{1}{2} < x < 1$, (2) can be written as

$$\frac{2x^2 + x + 1}{2} < \frac{1}{x}$$

$\Rightarrow \quad 2x^3 + x^2 - x < 2$

$\Rightarrow \quad 2(x^3 - 1) + x(x-1) < 0$

$\Rightarrow \quad (x-1)(2x^2 + 3x + 2) < 0$

$\Rightarrow \quad x < 1 \qquad [\because 2x^2 + 3x + 2 > 0 \ \forall\ x > 0]$

For $x > 1, (2)$ can be written as

$$\frac{2x^2 + x - 1}{2} > \frac{1}{x}$$

$\Rightarrow \quad (x-1)(2x^2 + 3x + 2) > 0$

This is true for each $x > 1$.

Thus, (1) holds for $\frac{1}{2} < x < 1, x > 1$.

26. (d)

The left hand side of the inequality is defined for x's which satisfy the following.

$1 - x > 0, x - 2 > 0$, $1 - x \neq 1$. Obviously there is no single value for which these inequalities are satisfied. Thus the set of its solutions is empty.

27. (c)

$\left\{x : \log_{1/3}\left(\log_4\left(x^2 - 5\right)\right) > 0\right\}$

$= \left\{x : 0 < \log_4\left(x^2 - 5\right) < 1\right\}$

$= \left\{x : 1 < x^2 - 5 < 4\right\}$

$= \left\{x : 6 < x^2 < 9\right\}$

$=\left(-3,-\sqrt{6}\right)\cup\left(\sqrt{6},3\right)$

28. **(b)**

$\left(x+2\right)\left(x+4\right)>0,\ x+2>0$

$\Rightarrow\ x>-2$.

Now (1) can be written as

$\log_3\left(x+2\right)\left(x+4\right)-\log_3\left(x+2\right)<\dfrac{\left(\log 7\right)/2}{\left(\log 3\right)/2}$

$\Rightarrow\ \log_3\left(x+4\right)<\log_3 7$

$\Rightarrow\ x+4<7$ or $x<3$.

29. **(d)**

$\Rightarrow x^{x\sqrt[3]{x}}=\left(x.\sqrt[3]{x}\right)^{x}$; Here $x\neq 0$

$\Rightarrow x^{x+\frac{1}{3}}=\left(x^{1+\frac{1}{3}}\right)^{x}$

$\Rightarrow x^{x^{\frac{4}{3}}}=x^{\frac{4}{3}x}$

Take log both side

$\Rightarrow x^{\frac{4}{3}}\log_x x=\dfrac{4}{3}x\log_x x$

$\Rightarrow x^{\frac{4}{3}}=\dfrac{4}{2}x$

$\Rightarrow x^{\frac{1}{3}}=\dfrac{4}{3}x$

$\Rightarrow x^{\frac{1}{3}}=\dfrac{4}{3}$

$\Rightarrow x=\left(\dfrac{4}{3}\right)^{3}=\dfrac{64}{27}$

30. **(b)**

$\Rightarrow\sqrt{25-5^{x}}=\sqrt{4^{x}-16}$ ………(1)

Here $25-5^{x}\geq 0$ & $4^{x}-16\geq 0$

$\Rightarrow 5^{x}\leq 25$ & $4^{x}\geq 4^{2}$

$\Rightarrow 5^{x}\leq 5^{2}$ & $x\geq 2$

$\Rightarrow x\in\left(-\infty,2\right]$ & $x\in\left[2,\infty\right)$

So only feasible region for given equation is $x=2$

For $x=2$, gives equation is satisfied

So no of solutions $=1$

31. **(c)**

$\Rightarrow 4^{x^2+2} - 9.2^{x^2+2} + 8 = 0$

$\Rightarrow 2^{2x^2+4} - 9.2^2 2^{x^2+8} = 0$

$\Rightarrow 16.2^{2x^2} - 36.2^{x^2} + 8 = 0$

$\Rightarrow 4.2^{2x^2} - 9.2x^2 + 2 = 0$

Put $2^{x^2} = a$

so, $4a^2 - 9a + 2 = 0$

$\Rightarrow (4a-1)(a-2) = 0$

$\Rightarrow a = \frac{1}{4}, 2$

$\Rightarrow 2^{x^2} = \frac{1}{4}, 2$

$\Rightarrow x^2 = -2, 1$

$\Rightarrow x^2 = -2$ is not possible; $x^2 = 1$

$\Rightarrow x = \pm 1$

32. **(b)**

Let $2^{11x} = t$, given equation reduces to $\frac{t^3}{4} + 4t = 2t^2 + 1$

$\Rightarrow \quad t^3 - 8t^2 + 16t - 4 = 0 \quad \Rightarrow t_1 \cdot t_2 \cdot t_3 = 4$

$\Rightarrow \quad 2^{11x_1} \cdot 2^{11x_2} \cdot 2^{11x_3 = 4} \Rightarrow 2^{11(x_1+x_2+x_3)} = 2^2$

$\Rightarrow \quad 11(x_1 + x_2 + x_3) = 2$

$\therefore \quad x_1 + x_2 + x_3 = \frac{2}{11}$

33. **(a)**

$\Rightarrow \frac{2^{x-1}}{2^{x+1}+1} < 2$

We can cross multiply $(2^{x+1}+1)$ as $2^{x+1}+1 > 0$ for $x \in R$

$\Rightarrow 2^{x-1} - 1 < 2(2^{x+1}+1)$

$\Rightarrow \frac{2^x}{2} - 1 < 4.2^x + 4$

$\Rightarrow \frac{7}{2} 2^x > -5$

$\Rightarrow 2^x > \frac{-10}{7}$

This is true for $x \in R$

34. **(c)**

$\Rightarrow \sqrt{2^{2x}-7} < 2^x - 1$ ……..(1)

Here $2^{2x}-7 \ge 0$

$\Rightarrow 2^{2x} \ge 7$

$\Rightarrow 2x \ge \log_2 7$

$\Rightarrow x \ge \frac{1}{2}\log_2 7$ ……..(2) (feasible region)

From feasible region it is clear that $2^x - 1 > 0$

So by taking square of (1)

$\Rightarrow 2^{2x}-7 < \left(2^x-1\right)^2$

$\Rightarrow 2^{2x}-7 < 2^{2x}+1-2.2^x$

$\Rightarrow 2.2^x < 8$

$\Rightarrow 2^x < 4$

$\Rightarrow x < 2$ ………(2)

By taking intersection of (1) & (2)

We get $x \in \left[\frac{1}{2}\log_2 7, 2\right)$

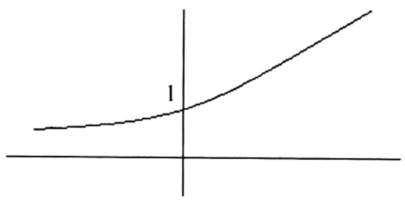
$\Rightarrow x \in \left[\log_4 7, 2\right)$

35. **(c)**

$\Rightarrow 5^{x+2} > \left(\frac{1}{25}\right)^{\frac{1}{x}}$

$\Rightarrow 5^{x+2} > \frac{1}{5^{\frac{2}{x}}}$

$\Rightarrow 5^{x+2}.5^{\frac{2}{x}} > 1$



$\Rightarrow 5^{x+\frac{2}{x}+2} > 5^{\circ}$

So $x + \frac{2}{x} + 2 > 0$

$\Rightarrow \frac{x^2+2x+2}{x} > 0$

Numerator is always > 0

So $\frac{1}{x} > 0$

$\Rightarrow x \in (0, \infty)$

36. **(c)**

$\Rightarrow 49^x + 7^{x+1} - 98 < 0$

$\Rightarrow 7^{2x} + 7.7^x - 98 < 0$

$\Rightarrow 7^x = a; a^2 + 7a - 78 < 0$

$\Rightarrow (a+14)(a-7) < 0$ $\quad (\because a + 14 = 7^x + 14 > 0 \text{ for } x \in R)$

$\Rightarrow 7^x < 7$

So, $x < 1$

37. **(b)**

$|x-p| + |x-15| + |x-p-15| = (x-p) - (x-15) - (x-P-15) = 30 - x$

Minimum = 15

38. **(d)**

$2^x + 2^{|x|} \ge 2\sqrt{2}$

Case I $x \ge 0$

$2^x + 2^x \ge 2\sqrt{2}$

$2^x \ge \sqrt{2}$

$\Rightarrow x \ge \frac{1}{2}$

Case II $x < 0$

$2^x + \frac{1}{2^x} \ge 2\sqrt{2}$

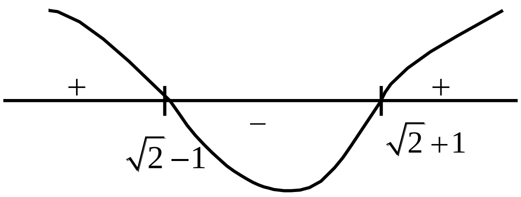
Let $2^x = k$ $\quad k + \frac{1}{k} \ge 2\sqrt{2}$

$k^2 - 2\sqrt{2}k + 1 \ge 1$

$k = \frac{2\sqrt{2} \pm \sqrt{8-4}}{2} = \frac{2\sqrt{2} \pm 2}{2}$

$K = \sqrt{2} \pm 1$

$\left(k - \left(\sqrt{2}+1\right)\right)\left(k - \sqrt{2} - 1\right) \ge 0$



$-\infty < 2^x \le \sqrt{2} - 1 \,\&\, \sqrt{2} + 1 \le 2^x < \infty$

$\Rightarrow 0 < 2^x \le \sqrt{2} - 1 \,\&\, \sqrt{2} + 1 \le 2^x < \infty$

$x \in \left(-\infty, \log_2\left(\sqrt{2}-1\right)\right] \,\&\, \left[\log_2\left(\sqrt{2}+1\right), \infty\right)$

From case I & II

$x \in \left(-\infty, \log_2\left(\sqrt{2}-1\right)\right] \cup \left[\frac{1}{2}, \infty\right)$

39. **(d)**

$\Rightarrow |3^x - 1| > |3^x - 9|$

Take square both side

$\Rightarrow (3^x - 1)^2 - (3^x - 9)^2 > 0$

$\Rightarrow [(3^x - 1) + (3^x - 9)][3^x - 1 - (3^x - 9)] > 0$

$\Rightarrow [2.3^x - 10][8] > 0$

$\Rightarrow 3^x - 5 > 0 \Rightarrow 3^x > 5$

$x > \log_3 5$

40. **(a)**

$\Rightarrow |x^3 - 1| \ge 1 - x$

Case I: $x^3 - 1 \ge 0$

$\Rightarrow x \ge 1$

So $(x^3 - 1) \ge 1 - x$

$\Rightarrow x^3 + x - 2 \ge 0$

$\Rightarrow (x - 1)(x^2 + x + 2) \ge 0$

$\Rightarrow x \in [1, \infty)$ ……..(1)

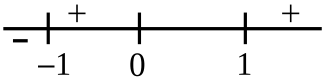
Case 2: $x^3 - 1 < 0$

$\Rightarrow x^3 < 1 \Rightarrow x < 1$

So, $-(x^3 - 1) \ge 1 - x$

$\Rightarrow -x^3 + x \ge 0$

$\Rightarrow x^3 - x \ge 0$



$\Rightarrow x(x - 1)(x + 1) \le 0$

$\Rightarrow x \in (-\infty, 1] \cup [0, 1]$ ……….(2)

So take union of (1) & (2)

$\Rightarrow x \in (-\infty, -1] \cup [0, \infty)$

41. **(b)**

$$\Rightarrow \frac{|x + 2| - x}{x} < 2$$

Case I: $x + 2 \ge 0$

$\Rightarrow x > -2$ ……..(i)

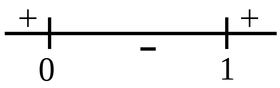
$$\Rightarrow \frac{(x + 2) - x}{x} < 2$$

$$\Rightarrow \frac{2}{x} - 2 < 0$$

$$\Rightarrow \frac{1-x}{x} < 0$$

$$\Rightarrow \frac{x-1}{x} < 0$$

$$\Rightarrow x \in (-\infty, 0) \cup (1, \infty)$$



Taking intersection of (i) & (ii)

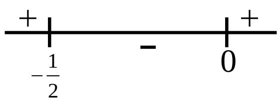
$\Rightarrow x \in [-2, 0) \cup (1, \infty)$ …….(iii)

Case 2: $\Rightarrow x + 2 < 0 \Rightarrow x \in (-\infty, -2)$ ……..(A)

$$\Rightarrow \frac{-(x+2)-x}{x} < 2$$

$$\Rightarrow \frac{-2x-2-2x}{x} < 0$$

$$\Rightarrow \frac{2x-1}{x} > 0$$



$\Rightarrow x \in \left(-\infty, -\frac{1}{2}\right) \cup (0, \infty)$ …….(B)

Taking intersection of A & B

$\Rightarrow x \in (-\infty, -2)$ ………(C)

So by taking union of (iii) & (c)

$$\Rightarrow x \in (-\infty, 0) \cup (1, \infty)$$

42. **(a)**

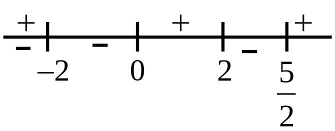
$$\Rightarrow \frac{x^2-5x+4}{x^2-4} \leq 1$$

$$\Rightarrow -1 \leq \frac{x^2-5x+4}{x^2-4} \leq 1$$

Case 1: $\frac{x^2-5x+4}{x^2-4} \geq -1$

$$\Rightarrow \frac{x^2-5x+4+x^2-4}{x^2-4}$$

$$\Rightarrow \frac{x(2x-5)}{(x-2)(x+2)} \geq 0$$



$\Rightarrow x \in (-\infty, -2) \cup [0, 2) \cup \left[\frac{5}{2}, \infty\right)$ ……..(1)

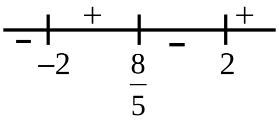
Case 2: $\frac{x^2-5x+4}{x^2-4} \le 1$

$\Rightarrow \frac{x^2-5x+4-x^{2+4}}{x^2-4} \le 0$

$\Rightarrow \frac{8-5x}{(x-2)(x+2)} \le 0$

$\Rightarrow \frac{(5x-8)}{(x-2)(x+2)} \ge 0$

$\Rightarrow x \in \left(-2, \frac{8}{5}\right] \cup (2, \infty)$



By taking intersection of (1) & (2)

$\Rightarrow x \in \left[0, \frac{8}{5}\right] \cup \left[\frac{5}{2}, \infty\right)$

43. **(c)**

$\Rightarrow \frac{x^2-|x|-12}{x-2} \ge 2x$

Case 1: $x \ge 0$

$\Rightarrow \frac{x^2-x-12}{x-3} \ge 2x$

$\Rightarrow \frac{x^2-x-12}{x-3} - 2x \ge 0$

$\Rightarrow \frac{x^2-x-12}{x-3} - 2x \ge 0$

$\Rightarrow \frac{x^2-x-12-2x^2+6x}{x-3} \ge 0$

$\Rightarrow \frac{-x^2+5x-12}{x-3} \ge 0$

$\Rightarrow \frac{x^2-5x+12}{x-3} \le 0$

as $x^2-5x+12$ is always greater than zero for $x \in R$

so, $\frac{1}{x-3} \le .0$

$\Rightarrow x \in (-\infty, 3)$

Case 2: $x < 0$

$\Rightarrow \frac{x^2+x-12}{x-3} - 2x \ge 0$

$\Rightarrow \frac{x^2+x-12-2x^2+6x}{x-3} \ge 0$

$$\Rightarrow \frac{-x^2+7x-12}{x-3} \geq 0$$

$$\Rightarrow \frac{x^2-7x+12}{x-3} \leq 0$$

$$\Rightarrow \frac{(x-4)(x-3)}{(x-3)} \leq 0$$

$$\Rightarrow x-4 \leq 0$$

$$\Rightarrow x \in (-\infty, 4]$$

But for this case $x < 0$

So we get

$$\Rightarrow x \in (-\infty, 0)$$

Take union of $(1)\,\&\,(2)$

$$\Rightarrow x \in (-\infty, 3)$$

44. (c)

$$\Rightarrow \left|\frac{x^2-3x-1}{x^2+x+1}\right| < 3$$

$$\Rightarrow -3\frac{x^3-3x-1}{x^2+x+1} < 3$$

Case 1: $\frac{x^2-3x-1}{x^2+x+1} > -3$

As (x^2+x+1) is always greater than zero

So $x^2-3x-1 > -3(x^2+x+1)$

$$\Rightarrow 4x^2+2 > 0$$

$\Rightarrow x \in R$ …….(1)

Case 2: $\frac{x^2-3x+1}{x^2+x+1} < 3$

$$\Rightarrow x^2-3x-1 < 3x^2+3x+3$$

$$\Rightarrow 2x^2+6x+4 > 0$$

$$\Rightarrow x^2+3x+2 => 0$$

$$\Rightarrow (x+1)(x+2) > 0$$

$\Rightarrow x \in (-\infty, -2) \cup (-1, \infty)$ …….(2)

Take intersection of $(1)\,\&\,(2)$

$$\Rightarrow x \in (-\infty, -2) \cup (-1, \infty)$$

45. (d)

$$\Rightarrow \frac{|x+3|+x}{x+2} > 1$$

$$\Rightarrow \frac{|x+3|+x}{x+2} - 1 > 0$$

$\Rightarrow \dfrac{|x+3|+x-x-2}{x+1>0}>0$

$\Rightarrow \dfrac{|x+3|-2}{x+2}>0$

Case 1: $x+3 \ge x \ge -3$

$\Rightarrow \dfrac{x+3-2}{x+2}>0$

$\Rightarrow \dfrac{x+1}{x+2}>0$

$\Rightarrow x \in (-\infty,-2)\cup(-1,-\infty)$

But for this case $x \ge -3$

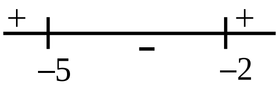
So we get $x \in [-3,-2)\cup(-1,\infty)$

Case 2: $x+3<0$

$\Rightarrow x<-3$

So, $\dfrac{-(x+3)}{x+2}>0$

$\Rightarrow \dfrac{x+5}{x+2}<0$



$\Rightarrow x \in (-5,-2)$

As $x<-3$ so $x \in (-5,-3)$ ……(2)

Take union of $(1)\&(2)$

$\Rightarrow x \in (-5,-2)\cup(-1,\infty)$

So least integral value of $x=-4$

46. **(c)**

$\Rightarrow \dfrac{|x+2|-|x|}{\sqrt{4-x^2}} \ge 0$ here $4-x^3>0$

$\Rightarrow x^3<4 \Rightarrow x \in \left(-\infty,\sqrt[3]{4}\right)$ ………(1)

As $(4-x^3)$ is greater than zero

$\Rightarrow |x+2|-|x| \ge 0$

$\Rightarrow |x+2| \ge |x|$

$\Rightarrow (x+2)^2 \ge x^2$

$\Rightarrow (x+2-x)(x+2+x) \ge 0$

$\Rightarrow 2(2x+2) \ge 0$

$\Rightarrow x \geq -1$ ………(2)

So $(1) \cap (2)$

$\Rightarrow x \in \left[-1, \sqrt[3]{4}\right)$

47. **(b)**

$\Rightarrow \dfrac{x^2 - 5x + 6}{|x| + 7} < 0$

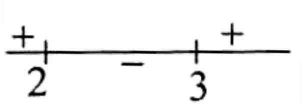
$\Rightarrow \dfrac{(x-3)(x-2)}{|x|+7} < 0$

As $7 + |x| \geq 7$

So we can cross multiply $7 + |x|$

$\Rightarrow (x-3)(x-2) < 0$

$\Rightarrow x \in (2, 3)$



48. **(d)**

$\Rightarrow \left|\dfrac{2x-1}{x-1}\right| > 2$

$\Rightarrow \dfrac{2x-1}{x-1} < -2$ or $\dfrac{2x-1}{x-1} > 2$

$\Rightarrow \dfrac{2x-1+2x-2}{x-1} < 0$ or $\dfrac{2x-1-2x+2}{x-1} > 0$

$\Rightarrow \dfrac{4x-3}{x-1}$ or $\dfrac{1}{x-1} > 0$

$\Rightarrow x \in \left(\dfrac{3}{4}, 1\right)$ 0r $x > 1$

So, $x \in \left(\dfrac{3}{4}, \infty\right) - \{1\}$

49. **(a)**

$\Rightarrow \dfrac{1}{|x|-3} < \dfrac{1}{2}$

Case 1: $x \geq 0$

$\Rightarrow \dfrac{1}{x-3} - \dfrac{1}{2} < 0$

$\Rightarrow \dfrac{5-x}{2(x-3)} < 0$

$\Rightarrow \dfrac{x-5}{(x-3)} > 0$

$\Rightarrow x \in (-\infty, 3) \cup (5, \infty)$

But $x \geq 0$

So, $x \in [0.3) \cup (5, \infty)$ ……..(1)

Case 2: $x < 0$

$\Rightarrow \frac{1}{-x-3} - \frac{1}{2} < 0$

$\Rightarrow \frac{1}{x+3} + \frac{1}{2} > 0$

$\Rightarrow \frac{5+x}{x+3} > 0$

$\Rightarrow x \in (-\infty, -5) \cup (-3, \infty)$

But $x < 0$

So, $x \in (-\infty, -5) \cap (-3, 0)$ ……..(2)

So (1) $\cup$(2)

$\Rightarrow x \in (-\infty, -5) \cup (-3, \infty)$

So least positive integer value $= 1$

50. **(d)**

$|x-1| + |x-2| + |x-3| \geq 6$

Case I : $x \geq 3$

$3x - 6 \geq 6 \Rightarrow x \geq 4$

Case II : $2 < x < 3$

$x \geq 6$ (Not possible)

Case III : $1 \leq x \leq 2$

$4 - x \geq 6$

$\Rightarrow$ $x \leq -2$ (Not possible)

Case IV : $x < 1$

$6 - 3x \geq 6$

$x \leq 0$

$x \in (-\infty, 0] \cup [4, \infty)$

51. **(d)**

$\Rightarrow \sqrt{3y+1} = \sqrt{y-1}$ ……..(1)

$\Rightarrow 3y + 1 \geq 0 \;\&\; y - 1 \geq 0$

$\Rightarrow y \geq -\frac{1}{3} \;\&\; y \geq 0$

$\Rightarrow y \in [0, \infty)$ is our feasible region

By equation (1), taking square of both side,

$\Rightarrow 3y + 1 = y - 1$

$\Rightarrow 2y = -2$

$\Rightarrow y = -1$; which does not lie in feasible range of y.

So no solution of y.

52. **(d)**

$\sqrt{4x+1}+\sqrt{7-x}=0$

As square root is always positive so given equation is feasible only if

$\Rightarrow 4x+1=0$ & $7-x=0$

$\Rightarrow x=-\frac{1}{4}$ & $x=7$

So no common solution.

53. **(a)**

$\sqrt{x^2-4x+3}+\sqrt{x^2-9}$

$=\sqrt{4x-14x+6}$

$\Rightarrow \sqrt{(x-1)(x-3)}+\sqrt{(x-3)(x+3)}$

$=\sqrt{2(2x-1)(x-3)}$

$\Rightarrow x=3$ or $\sqrt{x-1}+\sqrt{x+3}$

$=\sqrt{2(2x-1)}$

$\Rightarrow (x-1)+(x+3)+2\sqrt{(x-1)(x+3)}$

$=2(2x-1)$

$\Rightarrow 2\sqrt{(x-1)(x+3)}=2x-4$

$\Rightarrow \sqrt{(x-1)(x+3)}=x-2$

$\Rightarrow (x-1)(x+3)=(x-2)^2$

$\Rightarrow x^2+2x-3=x^2-4x+4$

$\Rightarrow 6x=-7$

$x=\frac{-7}{6}$

($\because$ $x^2-9<9,$ hence $\sqrt{x^2-9}$ is not defined)

54. **(b)**

We have, $\sqrt{x}+\sqrt{x-\sqrt{(1-x)}}=1$

$\Rightarrow \sqrt{x-\sqrt{1-x}}=1-\sqrt{x}$

On squaring both sides, we get

$x-\sqrt{1-x}=1+x-2\sqrt{x}$

$\Rightarrow -\sqrt{1-x}=1-2\sqrt{x}$

Again, squaring on both sides, we get

$1-x=1+4x-4\sqrt{x}$

$4\sqrt{x}=5x$

$\Rightarrow \sqrt{x}=\frac{4}{5}$ [on squaring both sides]

$\Rightarrow \; x = \frac{16}{25}$

Hence, the number of real solutions is 1.

55. (b)

$\Rightarrow \sqrt{4-\sqrt{1-x}} - \sqrt{2-x} > 0$

Here $1-x \ge 0$

$\Rightarrow 2-x > 0$

$\Rightarrow x \le 1$ ……..(i)

$\Rightarrow 4-\sqrt{1-x} \ge 0$

$\Rightarrow 4 \ge \sqrt{1-x}$

$\Rightarrow 16 \ge 1-x$

$\Rightarrow x > -15$ ………(ii)

So, $(i)\cap(ii)$

$\Rightarrow x \in [-15,1]$ …….. (iii) feasible region

Take intersection of $(iv)\,\&\,(iii)$

$\Rightarrow x \in [-2,1]$

Now

$\Rightarrow \sqrt{4-\sqrt{1-x}} > \sqrt{2-x}$

Take square both side

$\Rightarrow 4-\sqrt{1-x} > 2-x$

$\Rightarrow 2+x > \sqrt{1-x}$

Here $2+x \ge 0 \Rightarrow x \ge -2$ ……..(iv)

So, $2+x > \sqrt{1-x}$

Take square $4+x^2+4x > 1-x$

$\Rightarrow x^2+5x+3 > 0$

$\Rightarrow \left(x-\left(\frac{-5+\sqrt{13}}{2}\right)\right)\left(x-\left(\frac{-5-\sqrt{13}}{2}\right)\right)$

$\Rightarrow x \in \left(-\infty, \frac{-5-\sqrt{13}}{2}\right) \cup \left(\frac{\sqrt{13}-5}{2}, \infty\right)$ ……..(2)

$(1)\cap(2)$

$\Rightarrow x \in \left(\frac{\sqrt{13}-5}{2}, 1\right)$

56. (a)

$\Rightarrow \sqrt{4-x^2} + \frac{|x|}{x} \ge 0$

Here $4-x^2 \ge 0$

$\Rightarrow x^2 \le 4$

$\Rightarrow x \in [-2, 2]$ ……..(i)

Case 1: $x > 0$

$\Rightarrow \sqrt{4-x^2} + \frac{x}{x} \geq 0$ $\quad (x \neq 0)$

$\Rightarrow \sqrt{4-x^2} + 1 \geq 0$

$\Rightarrow \sqrt{4-x^2} \geq -1$ $\quad$ true for $x \in R$

Case 2: $x < 0$

$\Rightarrow \sqrt{4-x^2} + \frac{(-x)}{x} \geq 0$

$\Rightarrow \sqrt{4-x^2} - 1 \geq 0$

$\Rightarrow \sqrt{4-x^2} \geq 1$

Take square $4 - x^2 \geq 1 \Rightarrow x^2 \leq 3$

$\Rightarrow x \in \left[-\sqrt{3}, \sqrt{3}\right]$

But $x < 0$

So $x \in \left[-\sqrt{3}, 0\right)$

So case (1) $\cup$ case (2)

$\Rightarrow x \in R$ $\quad$ ……..(iii)

But our feasible region is $x \in [-2, 2]$

So greatest integral $x = 2$

57. **(b)**

$\Rightarrow \left(x^2 - 1\right)\sqrt{x^2 - x - 2} \geq 0$

$\Rightarrow \left(x^2 - 1\right) \geq 0$ $\quad$ & $x^2 - x - 2 \geq 0$ xh2

$\Rightarrow x \in (-\infty, -1] \cup [1, \infty)$ ……(1) $\quad$ & $(x-2)(x+1) \geq 0$

$x \in (-\infty, -1] \cup [2, \infty)$ $\quad$ ……(2)

Take $(1) \cap (2)$

So, $x \in (-\infty, -1] \cup [2, \infty)$

Least positive integer $= 2$

58. **(c)**

$\sqrt{-x^2 + 10x - 16} < x - 2$

For L.H.S. to be defined

$-x^2 - 10x - 16 \geq 0$

$\Rightarrow \ x^2 - 10x + 16 \leq 0$

$\Rightarrow \ (x-2)(x-8) \leq 0$

$\Rightarrow \ x \in [2, 8]$ $\quad$ ...(1)

Now squring, $(x > 2)$

$-x^2 + 10x - 16 < (x-2)^2$

$= x^2 - 4x + 4$

$\Rightarrow\ 2x^2 - 14x + 20 > 0$

$\Rightarrow\ x^2 - 7x + 10 > 0$

$\Rightarrow\ (x-2)(x-5) > 0$

$\Rightarrow\ x \notin [2, 5] \qquad \ldots(2)$

Possible values $\rightarrow$ 6, 7, 8

1. (1)

$$\frac{1}{\frac{1}{\log_2 x}+\frac{1}{\log_3 x}+......+\frac{1}{\log_{2000} x}}$$

$$\Rightarrow \frac{1}{\log_x 2+\log_x 3+......+\log_x 2000}$$

$$\Rightarrow \frac{1}{\log_x (2.3.......2000)}$$

$$\Rightarrow \frac{1}{\log_x \left(\prod_{n=1}^{2000} n\right)}$$

$$\Rightarrow \frac{1}{\log_x x}=1$$

2. (2)

$$9^{\log_3(\log_2 x)}=\log_2 x-(\log_2 x)^2+1$$

$$(\log_2 x)^2=\log_2 x-(\log_2 x)^2+1$$

Let $\log_2 x=t$

$$2t^2-t-1=0$$

$$2t^2-2t+t-1=0$$

$t=\frac{-1}{2}$ $\quad$ $t=1$

$\log_2 x=\frac{-1}{2}$ $\quad$ $\log_2 x=1$

$x=2$

$x=\frac{1}{\sqrt{2}}$ (not possible)

3. (4)

$$\log_3\left(\frac{4}{3}.\frac{5}{4}...........\frac{243}{242}\right)=\log_3\left(\frac{243}{3}\right)=4.$$

4. (2)

$$\left(5^{\log_5 7}+1\right)^{1/3}=(8)^{1/3}=2$$

5. (4)

Let $t=\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}...}}}$

$$\Rightarrow t^2-4=-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}...}}$$

$$= -\frac{1}{3\sqrt{2}}t$$

$$\Rightarrow \quad t^2 + \frac{1}{3\sqrt{2}}t - 4 = 0$$

$$\Rightarrow \quad t = \frac{8}{3\sqrt{2}}, \frac{-9}{3\sqrt{2}}$$

As $t > 0, t = \frac{8}{3\sqrt{2}}$

Therefore, the value of $6 + \log_{(3/2)}\left[\frac{1}{3\sqrt{2}}\left(\frac{8}{3\sqrt{2}}\right)\right]$

$$= 6 + \log_{(3/2)}\left(\frac{2}{3}\right)^2 = 6 - 2 = 4$$

6. **(9)**

$$N = 6^{\log_{10} 40} \cdot 6^{2\log_{10} 5} = 6^{\log_{10} 1000} = 6^3 = 216$$

7. **(47)**

$$\log_5\left(\frac{a+b}{3}\right) = \frac{\log_5 a + \log_5 b}{2}$$

$$\Rightarrow \quad \log_5\left(\frac{a+b}{3}\right)^2 = \log_5(ab)$$

$$\Rightarrow \quad (a+b)^2 = 9ab \Rightarrow a^2 - 7ab + b^2 = 0$$

$$a^4 + b^4 + 2a^2b^2 = 49a^2b^2$$

$$\Rightarrow \quad \frac{a^4 + b^2}{a^2b^2} = 47$$

8. **(0)**

$$\log_{10}\sqrt{1+x} + 3\log_{10}\sqrt{1-x} = 2 + \log_{10}\sqrt{1-x} + \log_{10}\sqrt{1+x}$$

$$\Rightarrow \quad \log_{10}\sqrt{1-x} = 1$$

$$\sqrt{1-x} = 10 \Rightarrow x = -99 \text{ (not possible)}$$

9. **(2)**

$$\log_b n = 2$$

$$\log_n(2b) = \log_n 2 + \log_n b = 2$$

$$\log_n 2 + \frac{1}{2} = 2$$

$$\log_n 2 = \frac{3}{2} \Rightarrow n = 2^{2/3}$$

If $\log_b n = 2 \Rightarrow b = n^{1/2} = 2^{1/3}$

$$n \cdot b = 2^{2/3} \cdot 2^{1/3} = 2$$

10. **(9)**

$$\log_y x + \frac{1}{\log_y x} = 2$$

$\Rightarrow \log_y x = 1 \Rightarrow x = y$

$x^2 + y = 12$

$\Rightarrow x^2 + x - 12 = 0$

$\Rightarrow (x+4)(x-3) = 0$

$\Rightarrow x = -4$ **or** $x = 3$

But $x > 0$, then $x = 3$

$xy = 9$

11. **(1)**

$\left(\log_2 4 + \log_2\left(4^x + 1\right)\right)\log_2\left(4^x + 1\right) = 3$

Let $\log_2\left(4^x + 1\right) = t$

$t^2 + 2t - 3 = 0 \Rightarrow t = -3$ or 1

$\log_2\left(4^x + 1\right) = 1 \Rightarrow 4^x = 1 \Rightarrow x = 0$

12. **(5)**

$\log_{3^{1/4}}\left(\log_{3\sqrt{5}} x\right) = 4 \Rightarrow \log_3^{\left(\log_{3\sqrt{5}} x\right)} = 1$

$\Rightarrow \log_5 x = 1 \Rightarrow x = 5$

13. **(5)**

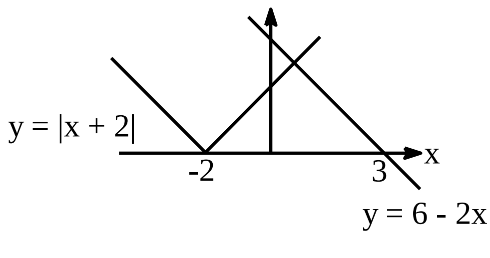
$x^2 - 10x + 16 < 0$

$\Rightarrow (x-3)(x-8) < 0$

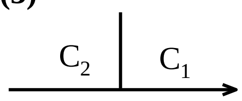
$\Rightarrow x \in (2, 8)$

Integers $\rightarrow$ 3, 4, 5, 6, 7

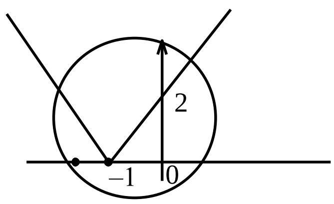
14. **(1)**



15. **(3)**



x = –1



C_1: If $x \geq 1$ … (a)

Then, 2 (x + 1) > x + 4

x > 2 … (b)

(a) n (b)

$x \in (2, \infty)$ (C – 1)

C_2: If $x < -1$ … (a)
Then, $-2(x + 1) > x + 4$
$\Rightarrow x < -3$ … (b)
(a) n (b)
$x \in (-\infty, -3) \cup (2, \infty)$
$C-1 \cup C-2$
So required x = 3

16. **(4)**
$-5 < x^2 - 4x < 5$
(1) (2)
(1) $x^2 - 4x + 5 > 0 \Rightarrow x \in R\ (\therefore D < 0)$ …(1)
(2) $x^2 - 4x - 5 < 0 \Rightarrow (x-5)(x+1) < 0 \Rightarrow -1 < x < 5$ …(2)
(1) n (2)
$x \in (-1, 5)$
So, m = 0, n = 4 $\Rightarrow$ (n – m) = 4

17. **(4)**
$|x^2 + x| - 5 < 0$
$\Rightarrow |x^2 + x| < 5$
$\Rightarrow |x^2 + x|^2 < 5^2$
$\Rightarrow (x^2 + x)^2 - 5^2 < 0$
$\Rightarrow (x^2 + x - 5)(x^2 + x + 5) < 0$
$\Rightarrow x^2 + x - 5 < 0$
$x \in (\alpha, \beta)$ where α, β are the roots of $x^2 + x - 5$

18. **(7)**
Domain $x \geq 3$
So $x \in (3, 10]$
So number of integer n = 7

19. **(2)**
$0 \leq x^2 + 2x - 3 < 1$
(1) (2)
(1) $(x+3)(x-1) \geq 0 \Rightarrow x \in (-\infty, 3] \cup [1, \infty)$ … (1)
(2) $x^2 + 2x - 4 < 0 \Rightarrow x = -1 \pm \sqrt{5} -1-\sqrt{5} < x < -1+\sqrt{5}$ …(2)
(1) n (2)
$x \in (-1-\sqrt{5}, -3] \cup [1, \sqrt{5}-1)$
So integer $x = \{-3, 1\}$

20. **(9)**

$$\frac{\sqrt{2x^2+15x-17}}{10-x} \ge 0$$

$$2x^2+15x-17 \ge 0$$

$$2x^2+17x-2x-17 \ge 0$$

$$x(2x+17)-(2x+17) \ge 0$$

$$(x-1)(2x+17) \ge 0$$

$$x \in (-\infty,-8.5] \cup [1,\infty)$$

Also x < 10

$\therefore$ No. of integers positive are

{1, 2, 3, 4, 5, 6, 7, 8, 9}

No. of positive integers are 9.

1. (b)

For $x < -2, |x+2| = -(x+2)$

$\therefore x^2 - |x+2| + x > 0$

$\Rightarrow x^2 + x + 2 + x > 0 \Rightarrow (x+1)^2 + 1 > 0$,

Which is valid $\forall x \in R$

But $x < -2, \therefore x \in (-\infty, -2)$ …….(i)

For $x \geq 2, |x+2| = x+2$

$\therefore x^2 - |x+2| + x > 0 \Rightarrow x^2 - x - 2 + x > 0$

$\Rightarrow x^2 > 2 \Rightarrow x > \sqrt{2}$ or $x < -\sqrt{2}$

i.e., $x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

But $x \geq -2 \Rightarrow x \in [-2, -\sqrt{2}] \cup (\sqrt{2}, \infty)$

From (i) and (ii),

$x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

But $x \geq -2 \Rightarrow x \in [-2, -\sqrt{2}] \cup (\sqrt{2}, \infty)$

From (i) and (ii)

$x \in (-\infty, -2) \cup [-2, -\sqrt{2}] \cup (\sqrt{2}, \infty)$

$\Rightarrow x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

2. (b)

$$\because |\sqrt{x} - 3| = \begin{cases} \sqrt{x} - 3; & x \geq 9 \\ 3 - \sqrt{x} & x < 9 \end{cases}$$

Case-I: $x \in [0, 9)$

$2(3 - \sqrt{x}) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 8\sqrt{x} + 12 = 0 \Rightarrow \sqrt{x} = 6, 2 \Rightarrow x = 36, 4$

Since $x \in [0, 9); \therefore x = 4$

Case-II: $x \in [9, \infty)$

$2(\sqrt{x} - 3) + x - 6\sqrt{x} + 6 = 0 \Rightarrow x - 4\sqrt{x} = 0 \Rightarrow x = 16, 0$

Since $x \in [9, \infty); \therefore x = 16$

Hence, $x = 4 \,\&\, 16$

3. (d)

Given inequality is,

$2^{\sqrt{\sin^2 x - 2\sin x + 5}} \leq 2^{2\sin^2 y}$

$\Rightarrow \sqrt{\sin^2 x - 2\sin x + 5} \leq 2\sin^2 y$

$\Rightarrow \sqrt{(\sin x - 1)^2 + 4} \leq 2\sin^2 y$

It is true if $\sin x = 1$ and $|\sin y| = 1$

Therefore, $\sin x = |\sin y|$

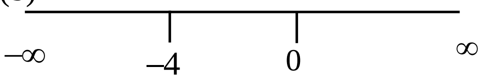
4. (b)

$A = \{x : x \in (-2,2)\}; N = \{x : x \in (-\infty,-1] \cup [5,\infty)\}$

$A \cap B = \{x : x \in (-2,-1]\}; A \cup B = \{x : x \in (-\infty,2) \cup [5,\infty)\}$

$A - B = \{x : x \in (-1,2)\}; B - A = \{x : x \in (-\infty,-2] \cup [5,\infty)\}$

5. (b)



Case I: $x \in (-\infty,-4)$

$(x+3)(x+4) = 6 \Rightarrow x^2 + 7x + 6 = 0 \Rightarrow x = -6$

Case II: $x \in (-4,0)$

$(x+3)(x+4) = -6 \Rightarrow x^2 + 7x + 18 = 0 \Rightarrow$ No real roots

Case III: $x \in (0,\infty)$

$(x-3)(x+4) = 6 \Rightarrow x^2 + x - 18 = 0 \Rightarrow x = \dfrac{\sqrt{73}-1}{2}$

So, the given set contains only 2 elements.

6. (b)

For, S_1 we have

$\Rightarrow \dfrac{(x+2)(x^2+3x+5)}{x^2-3x+2} \le 0 \Rightarrow x \in (-\infty,-2] \cup (1,2)$

For S_2, we have $= 3^x(3^x-3) - 3^2(3^x-3) \le 0$

For $S_2 x \in [1,2] \Rightarrow (-\infty,-2] \cup [1,2]$

7. (d)

$\because S = \{-6,-5,-4,3\}$

Where $-5,-4,3$ Satisfy $T \therefore S \cap T = \{-5,-4,3\}$

8. (256)

$A = \{x \in R : |x-2| > 1\} = (-\infty,1) \cup (3,\infty)$

$B = \left\{x \in R : \sqrt{x^2-3} > 1\right\} = (-\infty,-2) \cup (2,\infty)$

$C = \{x \in R : |x-4| \ge 2\} = (-\infty,2] \cup [6,\infty)$

So, $A \cap B \cap C = (-\infty,-2) \cup [6,\infty)$

$Z \cap (A \cap B \cap C)^C = \{-2,-1,0,1,2,3,4,5\}$

Hence no. of its subsets $= 2^8 = 256$