Inchapter Exercise - I - Rahul Rai (RRP) ①

6. $\frac{d}{dx}\left[(4x^2-7x+5).\sec x\right]$

$= \frac{d}{dx}(4x^2-7x+5).\ \sec x + (4x^2-7x+5)\frac{d}{dx}(\sec x)$

$= (8x-7)\sec x + (4x^2-7x+5)\sec x \tan x$

7. $\frac{d}{dx}\left[x^4(5\sin x - 3\cos x)\right]$

$= \frac{d}{dx}(x^4).\ (5\sin x - 3\cos x) + x^4 . \frac{d}{dx}(5\sin x - 3\cos x)$

$= 4x^3(5\sin x - 3\cos x) + x^4(5\cos x + 3\sin x)$

8. $\frac{d}{dx}\frac{\sqrt{x}+1}{\sqrt{x}-1} = \frac{\frac{d}{dx}(\sqrt{x}+1).(\sqrt{x}-1) - (\sqrt{x}+1)\frac{d}{dx}(\sqrt{x}-1)}{(\sqrt{x}-1)^2}$

$= \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1) - \frac{1}{2\sqrt{x}}(\sqrt{x}+1)}{(\sqrt{x}-1)^2}$

$= \frac{1}{2\sqrt{x}} . \frac{(\sqrt{x}-1-\sqrt{x}+1)}{(\sqrt{x}-1)^2} = -\frac{1}{\sqrt{x}(\sqrt{x}-1)^2}$

9. $\frac{d}{dx}\cdot\frac{x\ln x}{e^x} = \frac{\ln x}{e^x}\frac{d}{dx}(x) + x\ln x\frac{d}{dx}(e^{-x}) + \frac{x}{e^x}\frac{d}{dx}\ln(x)$

$= \frac{\ln x}{e^x} - \frac{x\ln x}{e^x} + \frac{1}{e^x}$

$= \frac{1 + \ln x - x\ln x}{e^x}$

10. (i) $y = \frac{x^3}{3} - x$

Maxima or minima exist where $\frac{dy}{dx} = 0$.

$$\frac{dy}{dx} = x^2 - 1 = 0$$

$$(x-1)(x+1) = 0$$

at $x = 1, -1$

~~at x=1, -1~~

(ii) $y = \sin x \qquad x \in (0, 2\pi)$

$$\frac{dy}{dx} = \cos x$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

11. (i) $y = x^3 - 3x + 10$

$\frac{dy}{dx} = 0 \Rightarrow 3x^2 - 3 = 0$ or $3(x-1)(x+1) = 0$

$$x = \pm 1$$

$\frac{d^2y}{dx^2} = 6x \qquad \frac{d^2y}{dx^2} > 0$ at $x = +1$ (minima)

$\frac{d^2y}{dx^2} < 0$ at $x = -1$ (maxima)

Local maxima at $x = -1$, $y = 12$

Local minima at $x = +1$, $y = 8$.

(ii) $y = \frac{x^2}{2} + \frac{1}{x}$. $\frac{dy}{dx} = x - \frac{1}{x^2}$ $\qquad \frac{dy}{dx} = 0 \Rightarrow \frac{x^3 - 1}{x^2} = 0$

or $\underline{x = 1}$.

$$\frac{d^2y}{dx^2} = 1 + \frac{2}{x^3}$$

$\frac{d^2y}{dx^2} > 0$ at $x = 1$. $\qquad$ Local minimum value is $\frac{3}{2}$ at $x = 1$.

Inchapter Exercise II

1. $\int \left(x^5 + \frac{2}{x^2} - \frac{1}{x} - \frac{4}{\sqrt{x}} + 10\right) dx$

$= \int x^5 dx + 2\int x^{-2} dx - \int x^{-1} dx - 4\int x^{-1/2} dx + 10\int dx$

$= \frac{x^6}{6} - \frac{2}{x} - \ln x + 8\sqrt{x} + 10x + C$

2. $\cancel{\int \left(7e^x + 4\sin x - \frac{9}{x^3} + e\right) dx}$

$\cancel{= 7\int e^x dx + 4\int \sin x\, dx - 9\int x^{-3} dx + e\int dx}$

$= \cancel{7e^x +}$

2. $\int \left(7e^x + 4\sin x - \frac{9}{x^3} + e\right) dx$

$= 7\int e^x dx + 4\int \sin x\, dx - 9\int x^{-3} dx + e\int dx$

$= 7e^x - 4\cos x + \frac{9}{2x^2} + ex + C$

3. $\int_1^5 (3+2t)\, dt \quad = \cancel{3\int dt}$

$= \left[3t + t^2\right]_1^5 = (3\times 5 + 5^2) - (3\times 1 + 1^2)$

$= 40 - 4 = 36$

4. $\int_{\pi/4}^{\pi/2} (2\sin x - \cos x)\, dx$

$= \left[-2\cos x - \sin x\right]_{\pi/4}^{\pi/2} = \left[2\cos x + \sin x\right]_{\pi/2}^{\pi/4}$

$= \left(\sqrt{2} + \frac{1}{\sqrt{2}}\right) - 1 = \frac{3}{\sqrt{2}} - 1$

5. $s = 2t + 4t^2$

$v = \frac{ds}{dt} = 2 + 8t$

$a = \frac{dv}{dt} = 8$

t (s)	$v (= 2+8t)$ in m/s	a (in m/s^2)
0	2	8
2	18	8
10	82	8

6. $v = 3t^2$

$a = \frac{dv}{dt} = 6t$

t	a (m/s^2)
0	0
2	12
10	60

$v = \frac{ds}{dt}$

$\frac{ds}{dt} = 3t^2$

$ds = 3t^2\, dt$

$\int_0^s ds = 3\int_0^t t^2\, dt$

$s = t^3$

t (s)	s (m)
0	0
2	8
10	1000

7. $v = u + at$

$\frac{ds}{dt} = v$ or $ds = v\,dt$

or $\int_0^s ds = \int_0^t (u + at)\, dt$

$s = u\int_0^t dt + a\int_0^t t\,dt$

$s = ut + \frac{1}{2}at^2$

8. $v\frac{dv}{ds} = a$ or $\int_u^v v\,dv = \int_0^s a\,ds$

$\frac{v^2 - u^2}{2} = as$

or $v^2 = u^2 + 2as$.

Subjective Questions

1. $y = (x^2 - 3x + 3)(x^2 + 2x - 1)$

$$\frac{dy}{dx} = \left[\frac{d}{dx}(x^2 - 3x + 3)\right]\left[x^2 + 2x - 1\right] + (x^2 - 3x + 3)\frac{d}{dx}(x^2 + 2x - 1)$$

$$= (2x - 3)(x^2 + 2x - 1) + (x^2 - 3x + 3)(2x + 2)$$

$$= 2x^3 + 4x^2 - 2x - 3x^2 - 6x + 3 + 2x^3 - 6x^2 + 6x$$
$$+ 2x^2 - 6x + 6$$

$$= 4x^3 - 3x^2 - 8x + 9$$

2. $y = \dfrac{x+1}{x-1}$; $\dfrac{dy}{dx} = \dfrac{(x-1)\frac{d}{dx}(x+1) - (x+1)\frac{d}{dx}(x-1)}{(x-1)^2}$

$$= \frac{\cancel{x} - 1 - \cancel{x} - 1}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

3. $y = \dfrac{x}{x^2+1}$; $\dfrac{dy}{dx} = \dfrac{(x^2+1)\frac{d}{dx}x - x\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$

$$= \frac{x^2 + 1 - 2x^2}{(x^2+1)^2} = \frac{(1 - x^2)}{(1 + x^2)^2}$$

4. $y = \dfrac{ax+b}{cx+d}$; $\dfrac{dy}{dx} = \dfrac{(cx+d)\frac{d}{dx}(ax+b) - (ax+b)\frac{d}{dx}(cx+d)}{(cx+d)^2}$

$$= \frac{a(cx+d) - c(ax+b)}{(cx+d)^2}$$

$$= \frac{\cancel{acx} + ad - \cancel{acx} - bc}{(cx+d)^2} = \frac{ad - bc}{(cx+d)^2}$$

Que 5. $z = \dfrac{x^2+1}{3(x^2-1)} + (x^2+1)(1-x)$

$$\frac{dz}{dx} = \frac{\left[\frac{d}{dx}(x^2+1)\right](x^2-1) - (x^2+1)\frac{d}{dx}(x^2-1)}{3(x^2-1)^2} + (x^2+1).\frac{d}{dx}(1-x) + (1-x)\frac{d}{dx}(x^2+1)$$

$$= \frac{(x^2-1)2x - (x^2+1).2x}{3(x^2-1)^2} + (x^2+1)(-1) + (1-x).2x$$

$$= \frac{2x(\cancel{x^2}-1-\cancel{x^2}-1)}{3(x^2-1)^2} - x^2 - 1 + 2x - 2x^2$$

$$= \frac{-4x}{3(x^2-1)^2} - 3x^2 + 2x - 1$$

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5. $z = \dfrac{x^2+1}{3(x^2-1)} + (x^2-1)(1-x)$

$$\frac{dz}{dx} = \frac{1}{3}\,\frac{(x^2-1)\frac{d}{dx}(x^2+1) - (x^2+1).\frac{d}{dx}(x^2-1)}{(x^2-1)^2} + (x^2-1)\frac{d}{dx}(1-x) + (1-x)\frac{d}{dx}(x^2-1)$$

$$= \frac{2x(x^2-1) - 2x(x^2+1)}{3(x^2-1)^2} - (x^2-1) + 2x(1-x)$$

$$= \frac{2x(\cancel{x^2}-1-\cancel{x^2}-1)}{3(x^2-1)^2} - x^2 + 1 + 2x - 2x^2$$

$$= -\frac{4x}{3(x^2-1)^2} - 3x^2 + 1 + 2x$$

6. $\frac{d}{dx}\left(\frac{1-x^3}{1+x^3}\right)$

$$= \frac{(1+x^3)\frac{d}{dx}(1-x^3) - (1-x^3)\frac{d}{dx}(1+x^3)}{(1+x^3)^2}$$

$$= \frac{-3x^2(1+x^3) - 3x^2(1-x^3)}{(1+x^3)^2}$$

$$= \frac{-3x^2(1+\cancel{x^3}+1-\cancel{x^3})}{(1+x^3)^2}$$

$$= \frac{-6x^2}{(1+x^3)^2}$$

7. $y = \frac{2}{x^3-1}$

putting $t = x^3-1$, $y = \frac{2}{t}$

$$\boxed{\frac{dy}{dx} = \frac{\quad}{(x^3-1)^2}}$$

$\frac{dt}{dx} = 3x^2$, $\frac{dy}{dt} = \frac{-2}{t^2}$

On applying chain rule,

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx}$$

$$= -\frac{2}{t^2}\cdot 3x^2 = \frac{-6x^2}{(x^3-1)^2}$$

8. $\frac{dy}{dx} = \frac{1}{a^3-3}\cdot\frac{d}{dx}(x^2-x+1)$

$$= \frac{2x-1}{(a^3-3)}$$

9. $\frac{dy}{dx} = \frac{1}{\sqrt{\pi}} \frac{d}{dx}(1-x^3)$

$= \frac{-3x^2}{\pi}$

10. $\frac{d}{dx}(\sin x + \cos x) = \frac{d}{dx}\sin x + \frac{d}{dx}\cos x$

$= \cos x - \sin x$

11. $\frac{dy}{dx} = x \cdot \frac{d}{dx}\sin x + \sin x \cdot \frac{d}{dx} x + \frac{d}{dx}\cos x$

$= x\cos x + \cancel{\sin x} - \cancel{\sin x}$

$= x\cos x$

12. $\frac{d}{dx}(\cos^2 x) = 2\cos x \cdot \frac{d}{dx}(\cos x)$

$= -2\cos x \cdot \sin x.$

13. $y = 3\sin^2 x - \sin^3 x$

$\frac{dy}{dx} = 3 \cdot 2\sin x \cdot \frac{d}{dx}(\sin x) - 3\sin^2 x \cdot \frac{d}{dx}(\sin x)$

$= 6\sin x \cdot \cos x - 3\sin^2 x \cdot \cos x$

$= 3\sin x \cos x(2 - \sin x).$

14. $\frac{d}{dx}(x\ln x) = \ln x \cdot \frac{d}{dx}x + x \cdot \frac{d}{dx}\ln x$

$= \ln x + x \cdot \frac{1}{x} = 1 + \ln x$

15. $y = \ln^2 x$

putting $\ln x = t$; $\frac{dt}{dx} = \frac{1}{x}$

$y = t^2$; $\frac{dy}{dt} = 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$= 2t \cdot \frac{1}{x}$

$= \frac{2\ln x}{x}.$

16. $y = \ln x^2$

Let $t = x^2$, $\frac{dt}{dx} = 2x$

$y = \ln t \qquad \frac{dy}{dt} = \frac{1}{t}.$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{1}{t} \cdot 2x = \frac{2x}{x^2} = \frac{2}{x}$$

17. $\int \sqrt{x}\, dx$

$$= \int x^{1/2} dx = \frac{x^{1+\frac{1}{2}}}{\frac{1}{2}+1} + c = \frac{2}{3} x^{3/2} + C$$

18. $\int \sqrt[m]{x^n}\, dx = \int x^{n/m} dx$

$$= \frac{x^{\frac{n}{m}+1}}{\frac{n}{m}+1} + C = \frac{m}{m+n} \cdot x^{\frac{n+m}{m}} + C.$$

19. $\int \frac{dx}{x^2} = \int x^{-2} dx = \frac{x^{-2+1}}{-2+1} + C$

$$= \frac{x^{-1}}{-1} + C = \frac{-1}{x} + C$$

20. $\int \frac{dx}{2\sqrt{x}} = \frac{1}{2} \int x^{-1/2} dx = \frac{1}{2} \cdot \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$

$$= \sqrt{x} + C$$

Level - I

1. $v = at + \frac{b}{t+c}$.

$[at] = [v]$

$[a]\,T = LT^{-1}$

$[a] = LT^{-2}$

$[t] = [c]$

$\Rightarrow [c] = T$

$\frac{[b]}{[t+c]} = LT^{-1}$

$\frac{[b]}{T} = LT^{-1}$

$[b] = L$

2. $[F] = MLT^{-2}$

$V = (10g)\,(10\,cm)\,(0.1s)^{-2}$

$= 10^{-2}\,kg \times 0.1\,m \times (0.01)^{-2} s^{-2} = 10^{-2} \times 10^{-1} \times 10^{4}\,kgm/s^2$

$= 10\,N$

$= \cancel{10^{-5}\,kg\,m s^{-2}}$

3. $P = P_0\, e^{(-\alpha t^2)}$

$[\alpha t^2] = M^0 L^0 T^0$

$[\alpha]\,T^2 = T^0$

$[\alpha] = T^{-2}$

5. $S_{nth} = u + \frac{a}{2}[2n-1]$

S_{nth} is the displacement in n^{th} second.

so $[S_{nth}] = L$

6. $[F] = MLT^{-2}$

$T^{-2} = F M^{-1} L^{-1}$

$T = F^{-1/2} M^{1/2} L^{1/2}$

7. Intensity $= \dfrac{\text{Power}}{\text{Area}}$

$$[I] = \frac{ML^2T^{-3}}{L^2} =$$

$$= M^1T^{-3}$$

$q = CV$

$C = \dfrac{q}{V}$

8. $F = \dfrac{\alpha}{\text{Density} + \beta^3}$

$[\text{Density}] = [\beta]^3$

$ML^{-3} = [\beta]^3$

$\Rightarrow [\beta] = M^{1/3}L^{-1}$

$[\alpha] = [F][\text{Density}]$

$= MLT^{-2} . ML^{-3}$

$= M^2L^{-2}T^{-2}$

$BVl = V$

$[B] = \dfrac{MLT^{-2}}{LT^{-1} . L}$

$= ML^{-1}T^{-1}$

$iBL = F$

$[B] = MT^{-2}I^{-1}$

$MT^{-2}I^{-1}L^2T^{-1} = V$

$ML^2T^{-3}I^{-1} = [V]$

9. $[P] = ML^{-1}T^{-2}$

14. Impulse = change in momentum.

$\vec{J} = \Delta\vec{p}$

16. $F = Pt^{-1} + \alpha t$

$[Pt^{-1}] = [F]$

$\dfrac{[P]}{T} = MLT^{-2}$

$[P] = MLT^{-1} = [\text{momentum}]$.

18. $[\text{Calorie}] = [\text{Energy}] = ML^2T^{-2}$

$1\,C = 4.2\ \text{Kg m}^2/\text{s}^2$, $1J = 1\text{Kg} \times (1m)^2 \times (1s)^{-2}$

$$1U = \frac{(\alpha\ \text{kg}) \times (\beta\ m)^2}{(\gamma s)^2} = \frac{\alpha\beta^2}{\gamma^2}\frac{\text{Kg m}^2}{s^2} = \frac{\alpha\beta^2}{\gamma^2}\ J$$

or $$1\,J = \frac{\gamma^2}{\alpha\beta^2}\ U$$

So $$1\,C = \frac{4.2\,\gamma^2}{\alpha\beta^2}\ U$$

19. $[\eta] = ML^{-1}T^{-1}$

$1\ \text{poise} = \dfrac{gm}{cm.\ s} \Rightarrow \dfrac{gm}{cm.\ s} = x\,\dfrac{(2\,\text{kg})}{(2m)(4s)}$

or $$\frac{gm}{cm \cdot s} = \frac{x}{4} \cdot \frac{1000\ gm}{100\ cm.\ s}$$

or $$x = 0.4$$

20. We know $E = \dfrac{hc}{\lambda}$ or $h = \dfrac{E\lambda}{c}$

so $\dfrac{h}{e} = \dfrac{E\lambda}{eC}$ or $\left[\dfrac{h}{e}\right] = \dfrac{ML^2T^{-2} \cdot L}{I\,T.\ LT^{-1}} = ML^2T^{-2}I^{-1}$

which is same as dimension of magnetic flux.

21. Since $BLv = V$ — (I) B → magnetic field $E = qV$ — (III)

& $V = \dfrac{q}{C}$ — (II) L → length E → Energy

v → speed

V → potential difference

q → charge

C → capacitance

$$B^2L^2C = (BL)^2 \cdot \frac{q}{V} = \frac{V^2}{v^2} \cdot \frac{q}{V}$$

$$= \frac{Vq}{v^2} = \frac{\text{Energy}}{v^2} \Rightarrow [B^2L^2C] = \frac{ML^2T^{-2}}{L^2T^{-2}} = M$$

23. $y = a\cos(\omega t - kx)$

$[\omega t] = [kx]$ $\quad$ $[\omega]T = [k]L = M^0L^0T^0$

$[k] = L^{-1}$

24. $x(t) = \frac{v_0}{\alpha}(1 - e^{-\alpha t})$

$\Rightarrow \left[\frac{v_0}{\alpha}\right] = L$ & $[\alpha t] = M^0L^0T^0$

$\cancel{LT^{-1} = [\alpha]L}$ & $[\alpha]T = M^0L^0T^0$

$[\alpha] = T^{-1}$

Since, $[\alpha] = T^{-1}$

& $[v_0] = [\alpha][L]$

$[v^0] = LT^{-1}$

25. $A = B + \frac{C}{D+E}$

$\Rightarrow [A] = [B] = \left[\frac{C}{D+E}\right]$ & $[D] = [E]$

So $[A] = M^0LT^{-1}$ & $[D] = \frac{[C]}{[B]} = \frac{M^0LT^0}{M^0LT^{-1}} = T$

$[E] = T.$

26. $\int \frac{dv}{\sqrt{2bv - v^2}} = a^n \sin^{-1}\left(\frac{x}{a} - 1\right)$

$[x] = [a] = L$ & $[a]^n = \frac{[dv]}{[v]} = M^0L^0T^0$

$L^n = M^0L^0T^0$

$\Rightarrow n = 0$

28. $E \propto h^a c^b \lambda^d$

or $E = k h^a c^b \lambda^d$

$[E] = [h]^a [c]^b [\lambda]^d$

$ML^2T^{-2} = [ML^2T^{-1}]^a [LT^{-1}]^b L^d$

$ML^2T^{-2} = M^a L^{2a+b+d} T^{-a-b}$

$\Rightarrow a = 1, \quad 2a+b+d = 2, \quad a+b = 2$

& $b = 1$ & $d = -1$

$E = \frac{hc}{\lambda}$

$[h] = \frac{ML^2T^{-2}.L}{LT^{-1}} = ML^2T^{-1}$

29. $d = 10.1$ cm.

Least count is 0.1 cm

So uncertainty $= \frac{0.1}{10.1} \times 100 = \pm 1\%$

30. $T = 25$ s.

ΔT (error in T) $= 0.2$ s

% error $= \frac{0.2}{25} \times 100 = 0.8\%$

Level-II

1. $K \propto p^a m^b$

$[K] = [p]^a [m]^b$

$ML^2T^{-2} = [MLT^{-1}]^a \; M^b$

$ML^2T^{-2} = M^{a+b} \; L^a \; T^{-a}$

$a+b=1$, $a=2$ so $b=-1$

3. $F \propto \rho^a v^b A^c$

$[F] = [\rho]^a [v]^b [A]^c$

$MLT^{-2} = [ML^{-3}]^a [LT^{-1}]^b [L^2]^c$

$MLT^{-2} = M^a \; L^{-3a+b+2c} \; T^{-b}$

$a=1$, $b=2$, $-3a+b+2c=1$ which means $c=1$

so $F \propto \rho v^2 A$

4. $r = 2.12$ cm

$A = \pi r^2$

$= \pi.(2.12)(2.12)$

$= 14.1 \text{ cm}^2$ [must have same no. of significant digit – Product Rule].

5. $[F] = MLT^{-2}$

$[L] = F M^{-1} T^2$

8. $U = \dfrac{\alpha\sqrt{y}}{y+\beta}$ From here we can conclude that $[y]=[\beta]=L$

and $[U] = \dfrac{[\alpha]\,[y]^{1/2}}{[y]}$

$ML^2T^{-2} = [\alpha]\, L^{-1/2}$ or $[\alpha] = ML^{5/2}T^{-2}$

So $[\alpha.\beta] = [\alpha][\beta] = ML^{5/2}T^{-2}.L = ML^{7/2}T^{-2}$

9. (a) $t = 2\pi\sqrt{\frac{ml^2}{E}}$

$[LHS] = T$; $[RHS] = [2\pi]\left[\frac{ml^2}{E}\right]^{1/2}$

$= \left[\frac{ML^2}{ML^2T^{-2}}\right]^{1/2} = T.$

Since $[LHS] = [RHS]$, it can be dimensionally correct.

(b) $[LHS] = L$, $[RHS] = \frac{\cancel{M}L\cancel{T^{-2}}}{\cancel{M}\cancel{T^{-2}}} = L$ [correct]

(c) $[RHS] = \left[\frac{ML^2}{MLT^{-2}}\right]^{1/2} = L^{1/2}T \neq [LHS]$ [incorrect]

(d) $[LHS] = LT^{-2}$; $[RHS] = \frac{L(LT^{-1})^2}{M^{-1}L^3T^{-2}.M} = \frac{L^3T^{-2}}{L^3T^{-2}} = M^0L^0T^0$

since $[LHS] \neq [RHS]$; incorrect.

11. $\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{d}{dx}x^{-1/2} = \left(\frac{-1}{2}\right)x^{\frac{-1}{2}-1} = \frac{-1}{2}x^{-3/2}$

12. $\frac{d}{dx}(ax+b)^{-2} = (-2).(ax+b)^{-3}.\frac{d}{dx}(ax+b) = -2a(ax+b)^{-3}$

13. $\frac{d}{dx}(x^3 + x^{-3} + 8) = 3x^2 - \frac{3}{x^4}$

14. $\frac{d}{dx}\sin(x^3) = \cos(x^3).\frac{d}{dx}(x^3) = 3x^2\cos(x^3).$

15. $\frac{d}{dx}(4x^3-5)^{1/2} = \frac{1}{2}(4x^3-5)^{-1/2}.\frac{d}{dx}(4x^3-5)$

$= \frac{1}{2}.(4x^3-5)^{-1/2}.(12x^2)$

$= \frac{6x^2}{\sqrt{4x^3-5}}$

16. $\frac{d}{dx}\sin(\ln x) = \cos(\ln x)\cdot\frac{d}{dx}\ln x = \frac{\cos(\ln x)}{x}$

17. $\frac{d}{dx}(2x^2+1)^{1/2} = \frac{1}{2}\cdot(2x^2+1)^{\frac{1}{2}-1}\cdot\frac{d}{dx}(2x^2+1)$

$= \frac{1}{2}\cdot(2x^2+1)^{-1/2}\cdot 4x = 2x(2x^2+1)^{-1/2}$

18. $\frac{d}{dx}e^{\sqrt{2x}} = e^{\sqrt{2x}}\cdot\frac{d}{dx}\sqrt{2x} = e^{\sqrt{2x}}\cdot\sqrt{2}\cdot\frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{2x}}e^{\sqrt{2x}}$

19. $\frac{d}{dx}(x^4 - 2\sin x + 3\cos x)$

$= 4x^3 - 2\cos x - 3\sin x$

20. $\frac{d}{dx}(x^2\sin x\cdot nx) = \frac{d}{dx}(x^2)(\sin x\cdot nx) + x^2\cdot\frac{d}{dx}(\sin x)\cdot nx + x^2\sin x\cdot\frac{d}{dx}(nx)$

$= 2x\sin x\cdot nx + x^2\cdot\cos x\cdot nx + x^2\sin x\cdot n$

21. $\frac{d}{dx}\left(\frac{x^2+1}{x+1}\right) = \frac{(x+1)\frac{d}{dx}(x^2+1)-(x^2+1)\frac{d}{dx}(x+1)}{(x+1)^2} = \frac{2x(x+1)-(x^2+1)}{(x+1)^2}$

$= \frac{2x^2+x-x^2-1}{(x+1)^2} = \frac{x^2+x-1}{(x+1)^2}$

22. $\frac{dV}{dr} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) = \frac{4\pi}{3}\frac{d}{dr}(r^3) = \frac{4\pi}{3}\cdot 3r^2 = 4\pi r^2$

23. $y = \frac{c^2}{x}$ $\quad \frac{dy}{dx} = \frac{-c^2}{x^2} = \frac{-xy}{x^2} = -\frac{y}{x}$ $\quad$ [as $c^2 = xy$]

24. $x = at^2$ $\quad y = 2at$ $\quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$

$\frac{dx}{dt} = 2at$ $\quad \frac{dy}{dt} = 2a$

~~25.~~ $\cancel{\int x^{5/2}\,dx = \frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1}+c = \frac{2}{7}x^{7/2}+c}$

25. $\int x^{1/5}\,dx = \frac{x^{\frac{1}{5}+1}}{\frac{1}{5}+1}+C = \frac{x^{6/5}}{6/5}+C = \frac{5}{6}x^{6/5}+C$

26. $\int \frac{1}{(ax+b)^2}\,dx$ put $ax+b=t$

$dt = a\,dx$ or $dx = \frac{dt}{a}$

$= \frac{1}{a}\int \frac{dt}{t^2} = \frac{-1}{at} = \frac{-1}{a(ax+b)}+C$

27. $\int \sin x.\cos x\,dx$

putting $\sin x = t$; $dt = \cos x\,dx$

$I = \int t\,dt = \frac{t^2}{2}+C = \frac{\sin^2 x}{2}+C =$

or

28. $\int \frac{x\,dx}{x^2+a^2}$ putting $x^2+a^2=t$

$2x\,dx = dt$

$= \int \frac{dt}{2t} = 2\ln|t|+C = 2\ln|x^2+a^2|+C$

29. $\int_{-\pi/2}^{\pi/2} \cos x\,dx = [\sin x]_{-\pi/2}^{\pi/2} = \sin\frac{\pi}{2} - \sin\left(\frac{-\pi}{2}\right)$

$= 1+1=2$

30. $\int_0^{\pi/2} \sqrt{1+\cos x}\,dx = \sqrt{2}\int_0^{\pi/2} \cos\left(\frac{x}{2}\right)dx = 2\sqrt{2}\left[\sin\frac{x}{2}\right]_0^{\pi/2} = 2$

31. $\int \sqrt{x}\, dx - \int x\sqrt{x}\, dx$

$= \int x^{1/2} dx - \int x^{3/2} dx$

$= \frac{2}{3} x^{3/2} - \frac{2}{5} x^{5/2} + C$

32. $\int_0^{\pi/2} \sin x \, dx + \int_0^{\pi/2} \cos x \, dx$

$= \left[\sin x - \cos x\right]_0^{\pi/2} = (1 - 0) - (0 - 1) = 2$

33. $\rho = \frac{m}{V}$ $\qquad \frac{\Delta m}{m} \times 100 = 0.3$ (Given)

$\& \quad \frac{\Delta l}{l} \times 100 = 0.2.$

$\rho = \frac{m}{l^3}$

$\ln \rho = \ln m - 3 \ln l$

$\frac{d\rho}{\rho} = \frac{dm}{m} - 3\frac{dl}{l}$

$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + 3\frac{\Delta l}{l}$ [for maximum error].

$\frac{\Delta \rho}{\rho} \times 100 = \frac{\Delta m}{m} \times 100 + 3\frac{\Delta l}{l} \times 100$

$= 0.3 + 3(0.2) = 0.9$

34. $KE = \frac{1}{2} m v^2$ $\qquad \frac{\Delta V}{V} \times 100 = 40 \Rightarrow \Delta V = 0.4 V$

$\Delta KE = \frac{m}{2}\left[(V + \Delta V)^2 - V^2\right] = \frac{m}{2} V^2 (1.4^2 - 1) = 0.96 \frac{m}{2} V^2$

$\Delta KE = 0.96\ KE$; So $\frac{\Delta KE}{KE} = 0.96$ or $\frac{\Delta KE}{KE} \times 100 = 96$

123. $T_1 = 2.63\ s$

$T_2 = 2.56\ s$

$T_3 = 2.42\ s$

$T_4 = 2.71\ s$

$T_5 = 2.80\ s$

$$T_{mean} = \frac{T_1 + T_2 + T_3 + T_4 + T_5}{5} = 2.62\ s$$

$\Delta T_1 = |T_1 - T_{mean}| = 0.01\ s$

$\Delta T_2 = 0.06\ s$

$\Delta T_3 = 0.20\ s$

$\Delta T_4 = 0.09\ s$

$\Delta T_5 = 0.18\ s$

$$\Delta T_{mean} = \frac{\Delta T_1 + \Delta T_2 + \Delta T_3 + \Delta T_4 + \Delta T_5}{5} = 0.11\ s$$

124. $\rho = \frac{m}{V}$, $V = \pi r^2 l$

$$\rho = \frac{m}{\pi r^2 l}$$

$$\ln \rho = \ln m - \ln\left(\frac{1}{\pi}\right) - 2\ln r - \ln l$$

$$\frac{d\rho}{\rho} = \frac{dm}{m} - 2\frac{dr}{r} - \frac{dl}{l}$$

or $\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 2\frac{\Delta r}{r} + \frac{\Delta l}{l}$ [For maximum error]

$$\frac{\Delta\rho}{\rho}\times 100 = \frac{0.003}{0.3}\times 100 + 2\times\frac{0.005}{0.5}\times 100 + \frac{0.006}{0.6}\times 100$$

$$= 4\ \%$$

or $$-0.1 + \frac{\Delta P}{P} + \frac{\Delta P}{P}(-0.1) = 0$$

$$\frac{\Delta P}{P}(1 - 0.1) = 0.1$$

$$0.9\ \frac{\Delta P}{P} = 0.1$$

or $$\frac{\Delta P}{P} \times 100 = \frac{100}{9} = 11.1\%$$

120. $$KE = \frac{1}{2} m v^2$$

$$\ln(KE) = \ln\left(\frac{1}{2}\right) + \ln m + 2 \ln v$$

$$\frac{d(KE)}{KE} = \frac{dm}{m} + 2\frac{dv}{v}$$

or $$\frac{\Delta KE}{KE} = \frac{\Delta m}{m} + 2\frac{\Delta v}{v}$$

$$\frac{\Delta KE}{KE} \times 100 = \frac{\Delta m}{m} \times 100 + 2\frac{\Delta v}{v} \times 100$$

$$= 2 + 2 \times 3$$

$$= 8\%$$

121. $$V = \frac{4}{3}\pi R^3$$

$$\ln V = \ln\left(\frac{4\pi}{3}\right) + 3 \ln R$$

$$\frac{dV}{V} = 3\frac{dR}{R}$$

$$\frac{\Delta V}{V} = 3\frac{\Delta R}{R}$$

$$\frac{\Delta V}{V} \times 100 = 3\frac{\Delta R}{R} \times 100 = 6\%$$

$\left[\text{Given } \frac{\Delta R}{R} \times 100 = 2\right]$

17. $a^3 = 6a^2$

$a^3 - 6a^2 = 0$

$a^2(a-6) = 0$

$a = 6$

So $V = a^3 = 216\ m^3$.

Dimension

47. $P = \frac{\alpha}{\beta} e^{-\frac{\alpha Z}{k\theta}}$

$[\alpha Z] = [k\theta] = [ML^2T^{-2}]$

$[\alpha].L = ML^2T^{-2}$.

$[\alpha] = MLT^{-2}$

& $[P] = \frac{[\alpha]}{[\beta]}$ or $ML^{-1}T^{-2} = \frac{MLT^{-2}}{[\beta]}$

$\Rightarrow [\beta] = L^2$.

49. $P = \frac{RT}{V-b} e^{-\frac{\alpha V}{RT}}$

$\Rightarrow [\alpha][V] = [R][T]$

We know $[P][V] = [R][T]$

So $[\alpha] = [P]$

50. $V = at^2 + bt + c$

$[v] = [at^2] = [bt] = [c]$

$LT^{-1} = [a]T^2 = [b]T = [c]$

From here, $[a] = LT^{-3}$

$[b] = LT^{-2}$

$[c] = LT^{-1}$

51. We know $[E] = \frac{[G][M]}{[R]} \Rightarrow \frac{[E]}{[G]} = \frac{M}{L}$

$$\frac{E^2L^2}{M^5G^2} = \left[\frac{E}{G}\right]^2 \frac{M^2L^2T^{-2}}{M^5}$$

$$= \frac{M^2}{\cancel{L^2}} \frac{M^2\cancel{L^2}T^{-2}}{M^5} = M^{-1}T^{-2}$$

55. $y = A\sin(Bx + Ct + D)$

$[y] = [A] = L$, $[B][x] = [C][t] = [D] = M^0L^0T^0$

$\Rightarrow [B] = L^{-1}$, $[C] = T^{-1}$, $[D] = M^0L^0T^0$

$[ABCD] = [A][B][C][D] = T^{-1}$

56. $[P]^x[Q]^y[c]^z = (ML^{-1}T^{-2})^x(M^1L^0T^{-3})^y(LT^{-1})^z$

$= M^{x+y}L^{-x+z}T^{-2x-3y-z}$

$x + y = 0$ $\quad y = -x$

$-x + z = 0$ $\quad z = x$

$-2x - 3y - z = 0$ $\quad$ So (x, y, z) is $(x, -x, x)$

$-2x + 3x - x = 0.$ $\quad$ Correct option is (b).

62. $y = At^2 - Bt^3$

$[y] = [A]T^2 = [B]T^3 = L$

$[A] = LT^{-2}, \quad [B] = LT^{-3}$

68. Use $E = h\nu$

69. Impulse is same as change in momentum.

74. Use τ (torque) = Force × distance

75. Use $F = 6\pi \eta r v$

$F \to$ force, $r \to$ radius, $v \to$ speed

$\eta \to$ coefficient of viscosity.

87. Use $Q = i t$

$t \to$ time

$i \to$ current

$Q \to$ charge

96. $[W] = ML^2T^{-2}$

$[W] = [k]L^2$

$[k] = MT^{-2}$

106. Pressure gradient is same as $\frac{P}{l}$

111. $E = h\nu$

$h = E.T$

$[I] = [\text{moment of inertia}] = ML^2$

$\left[\frac{h}{I}\right] = \frac{ML^2T^{-1}}{ML^2} = T^{-1}$ (same as frequency)

112. EMF of a battery is the potential difference across terminals of a battery when there is no current drawn from it.

114. $\rho = 0.625 \ gm/cm^3$

$$= \frac{0.625 \times 10^{-3} \ kg}{(10^{-2} m)^3}$$

$$= \frac{0.625 \times 10^{-3}}{10^{-6}} \frac{kg}{m^3}$$

$$= 0.625 \times 10^{3} \ kg/m^3$$

$$= 625 \ kg/m^3$$

Previous Year's Questions (Error)

115. $v = \sqrt{2gh}$

$p = mv = m\sqrt{2g}\ \sqrt{h}$

$p' = m\sqrt{2g}\ \sqrt{(2h)} = \sqrt{2}\ m\sqrt{2gh} = p\sqrt{2}$

$\Delta p = p' - p = (\sqrt{2}-1)\,p$

$\frac{\Delta p}{p} \times 100 = (1.41-1)\times 100 = 41\ \%$

116. $\lambda = \frac{h}{p}$

$d\lambda = -\frac{h}{p^2}\,dp$

or $\Delta\lambda = -\frac{h}{p^2}\,\Delta p$ or $\Delta\lambda = -\frac{\lambda}{p}\cdot\Delta p$

$\frac{\Delta\lambda}{\lambda}\times 100 = 0.5$ [given].

$\frac{\Delta\lambda}{\lambda} = \frac{\Delta p}{p}$ where Δp is change in momentum & p is initial momentum.

Here momentum is changed by 'p'.
we have to find ~~new~~ initial momentum p'.

$\frac{\Delta\lambda}{\lambda}\times 100 = \frac{p}{p'}\times 100$

$0.5 = \frac{p}{p}\times 100$

$p' = \frac{100p}{0.5} = \underline{200P}$

117. $PV = nRT$

$\ln P + \ln V = \ln(nRT)$

$\frac{dP}{P} + \frac{dV}{V} = 0$ [as T is constant].

$\frac{\Delta P}{P} = -\frac{\Delta V}{V}$

$\frac{\Delta P}{P} \times 100 = -\frac{\Delta V}{V} \times 100$

Given $\frac{\Delta V}{V} \times 100 = -4$.

then $\frac{\Delta P}{P} \times 100 = 4$

so increase is 4%.

119. At constant temperature $P_1V_1 = P_2V_2$

$PV = (P+\Delta P)(V+\Delta V)$

$PV = PV + P\Delta V + V\Delta P + \Delta V . \Delta P$

on dividing both sides by PV, we get

$\cancel{1} = \cancel{1} + \frac{\Delta V}{V} + \frac{\Delta P}{P} + \frac{\Delta V}{V} \cdot \frac{\Delta P}{P}$

or $\frac{\Delta V}{V} + \frac{\Delta P}{P} + \frac{\Delta V}{V} \cdot \frac{\Delta P}{P} = 0$

Given that $\frac{\Delta V}{V} \times 100 = -10$ or $\frac{\Delta V}{V} = -0.1$

$$\boxed{\cancel{\begin{aligned} 0.1 + \frac{\Delta P}{P} + \frac{\Delta P}{P} \times 0.1 &= 0 \\ \frac{\Delta P}{P}(1.1) &= -0.1 \\ \text{or } \frac{\Delta P}{P} \times 100 &= -\frac{100}{11} \end{aligned}}}$$

125. $V = l \times b \times h$

$= 12\,cm \times 6\,cm \times 2.45\,cm$

$= 176.4\,cm^3$

$= 2 \times 10^2\,cm^3$ [ubto 1 significant digit]

130. $X = M^a L^b T^c$

$\ln X = a \ln M + b \ln L + c \ln T$

$$\frac{dX}{X} = a\frac{dM}{M} + b\frac{dL}{L} + c\frac{dT}{T}$$

$$\frac{\Delta X}{X} = a\frac{\Delta M}{M} + b\frac{\Delta L}{L} + c\frac{\Delta T}{T}$$

$$\frac{\Delta X}{X}\times 100 = a\left(\frac{\Delta M}{M}\times 100\right) + b\left(\frac{\Delta L}{L}\times 100\right) + c\left(\frac{\Delta T}{T}\times 100\right)$$

$$= a\alpha + b\beta + c\gamma$$

131. $A = \dfrac{a^2 b^3}{c\sqrt{d}}$ $\quad \ln A = 2\ln a + 3\ln b - \ln c - \frac{1}{2}\ln d$

$$\frac{dA}{A} = 2\frac{da}{a} + 3\frac{db}{b} - \frac{dc}{c} - \frac{1}{2}\frac{dd}{d}$$

$$\cancel{\frac{\Delta A}{A} = 2\frac{\Delta a}{a} + 3\,d}$$

$$\frac{\Delta A}{A} = 2\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta d}{d}$$ [For maximum error]

$$\frac{\Delta A}{A}\times 100 = 2\times 1 + 3\times 3 + 2 + \frac{1}{2}\cdot 2$$

$$= 14.$$