

JEE Main Exercise

1. (c)

$$\theta = \left( \frac{\omega + \omega_0}{2} \right) t$$

$$\Rightarrow \theta = \left( \frac{20 + 0}{2} \right) (5) = 50 \text{ rad}$$

$$\text{Number of revolutions} = \frac{\theta}{2\pi} = \frac{50}{2\pi} = \frac{25}{\pi}$$

2. (d)

$$\omega = \frac{2\pi}{T}$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$

$$\because T_1 = T_2 \Rightarrow \frac{\omega_1}{\omega_2} = 1$$

3. (a)

$$a_T = \alpha$$

$$\Rightarrow r\alpha = \alpha$$

$$\Rightarrow r = 1 \text{ m}$$

So, diameter = 2m

4. (c)

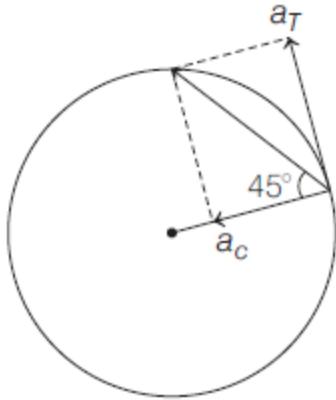
$$\text{Centripetal acceleration } (a_c) = \frac{v^2}{R} = \frac{(30)^2}{500} = 1.8 \text{ m/s}^2$$

$$\text{Tangential acceleration } (a_T) = \frac{dv}{dt} = 2 \text{ m/s}^2$$

$$\begin{aligned} \text{Resultant acceleration} &= \sqrt{a_c^2 + a_T^2} \\ &= \sqrt{(1.8)^2 + 2^2} \\ &= \sqrt{7.24} = 2.7 \text{ m/s}^2 \end{aligned}$$

5. (b)

$$\tan 45^\circ = \frac{a_T}{a_c} = \frac{9}{(v^2/4)}$$



$$\Rightarrow v^2 = 36$$

$$\Rightarrow v = 6 \text{ m/s}$$

and  $v = u + at$

$$\Rightarrow 6 = 0 + 9t$$

$$\Rightarrow t = \frac{2}{3} \text{ s}$$

6. (b)

$$a_c = k^2 r t^2 \Rightarrow \frac{v^2}{r} = k^2 r t^2$$

$$\Rightarrow v = k r t$$

$$\Rightarrow a_T = \frac{dv}{dt} = kr$$

$$P = Fv \cos 0^\circ = (ma_T)v \cos 0^\circ$$

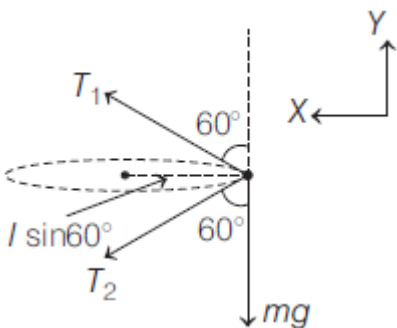
$$= mkrkrt = mk^2 r^2 t$$

7. (b)

$$v = \omega r \text{ and } a = \omega^2 r$$

If radius becomes half, then both  $v$  and  $a$  will become half of the original values.

8. (b)



$$\Sigma F_x = ma$$

$$\Rightarrow T_1 \sin 60^\circ + T_2 \sin 60^\circ = m\omega^2 (l \sin 60^\circ) \quad \dots(i)$$

$$\Sigma F_y = 0$$

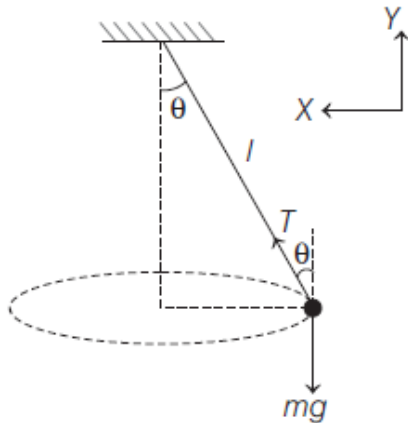
$$\Rightarrow T_1 \cos 60^\circ = T_2 \cos 60^\circ + mg \quad \dots(\text{ii})$$

Solving Eqs. (i) and (ii), we get

$$\omega = 14 \text{ rad/s}$$

9. (c)

$$\Sigma F_y = 0$$



$$\Rightarrow T \cos \theta = mg \quad \dots(\text{i})$$

$$\Sigma F_x = ma_x$$

$$T \sin \theta = m\omega^2 (l \sin \theta) \quad \dots(\text{ii})$$

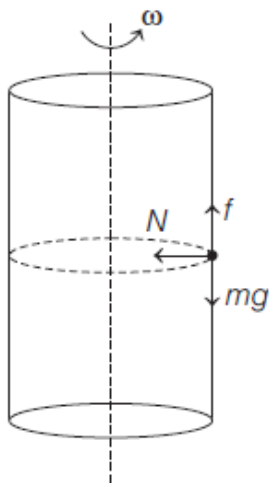
From Eqs. (i) and (ii), we get

$$\omega = \sqrt{\frac{g}{l \cos \theta}}$$

$$\Rightarrow T = \frac{2\pi}{2\omega} = 2\pi \sqrt{\frac{l \cos \theta}{g}}$$

10. (a)

$$\Sigma F_y = 0 \Rightarrow f = mg$$



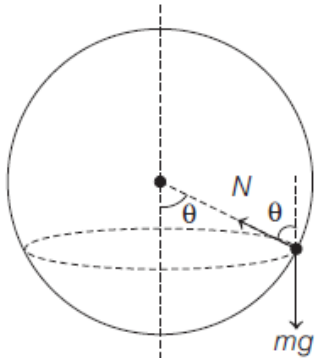
$$\Sigma F_x = 0 \Rightarrow N = m\omega^2 r$$

$$\text{and } f \leq f_L$$

$$\Rightarrow mg \leq \mu(m\omega^2 r)$$

$$\Rightarrow \omega \geq \sqrt{\frac{g}{\mu r}}$$

11. (b)



$$\Sigma F_y = 0 \Rightarrow N \cos \theta = mg \quad \dots(i)$$

$$\Sigma F_x = ma_x \Rightarrow N \sin \theta = m\omega^2 \left(\frac{a}{2}\right) \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\omega = \left(\frac{2g}{\sqrt{3}a}\right)^{1/2}$$

12. (c)

$$v = \sqrt{rg \tan \theta} = \sqrt{(10\sqrt{3})(10) \tan 30^\circ}$$

$$= 10 \text{ m/s} = 36 \text{ km/h}$$

13. (b)

$$\mu_{\min} = \frac{v^2}{gr} = \frac{\left(72 \times \frac{5}{18}\right)^2}{10(200)} = 0.2$$

14. (a)

$$v_{\max} = \sqrt{\frac{gr(\tan \theta + \mu)}{1 - \mu \tan \theta}} = \sqrt{\frac{10 \times 20 \left(\frac{3}{4} + \frac{1}{4}\right)}{1 - \frac{3}{4} \times \frac{1}{4}}}$$

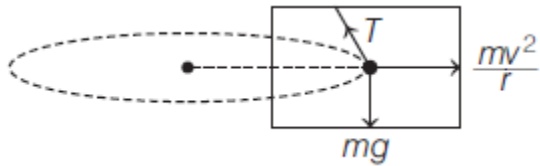
$$= 10\sqrt{\frac{32}{13}} \text{ m/s}$$

15. (b)

$$\tan \theta = \frac{a}{g} = \frac{v^2/r}{g}$$

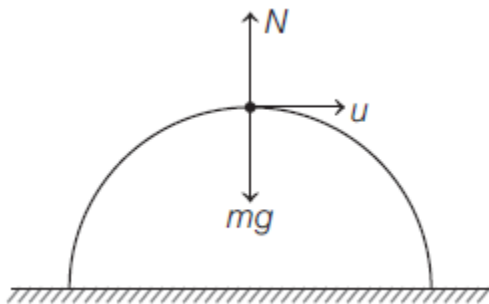
$$\Rightarrow \tan \theta = \frac{v^2}{rg} = \frac{10^2}{10(10)} = 1$$

$$\Rightarrow \theta = 45^\circ$$



16. (a)

$$mg - N = \frac{mu^2}{R}$$



For losing contact,  $N = 0$

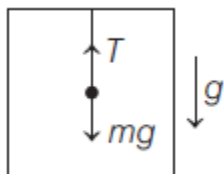
$$\Rightarrow mg = \frac{mu^2}{R}$$

$$\Rightarrow u = \sqrt{gR}$$

17. (d)

In free fall, acceleration of car is  $g$ .

For pendulum,



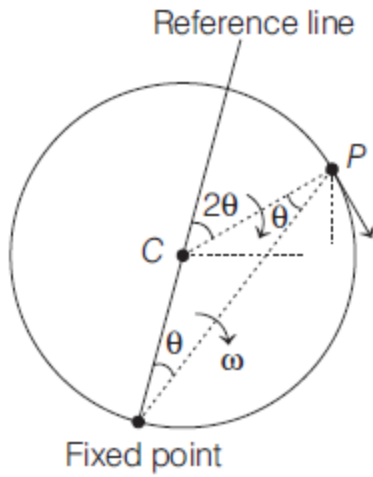
$$\Sigma F = ma$$

$$\Rightarrow mg - T = m(g)$$

$$\Rightarrow T = 0$$

18. (4)

$$\frac{d\theta}{dt} = 2 \text{ rad/s}$$



Angular velocity of particle w.r.t the centre of circle

$$= \frac{d(2\theta)}{dt}$$

$$= 2 \frac{d\theta}{dt} = 4 \text{ rad/s}$$

$$v = \omega r = 4(1) = 4 \text{ m/s}$$

19. (1)

$$\alpha = \frac{\pi}{4} \text{ rad/s}^2$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\Rightarrow \frac{\pi}{2} = 0 + \frac{1}{2} \times \left(\frac{\pi}{4}\right) t^2$$

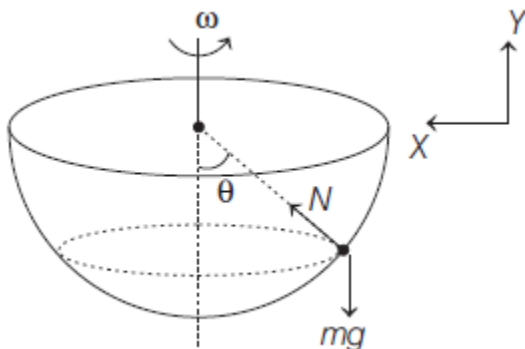
$$\Rightarrow t = 2 \text{ s}$$

$$v_{\text{avg}} = \frac{\Delta S}{\Delta t} = \frac{\sqrt{2}R}{2} = 1 \text{ m/s}$$

20. (100)

$$\Sigma F_y = 0 \Rightarrow N \cos \theta = mg \quad \dots (i)$$

$$\Sigma F_x = ma_x \Rightarrow N \sin \theta = m\omega^2 R \sin \theta \quad \dots (ii)$$



Solving Eqs. (i) and (ii), we get

$$\omega = \sqrt{\frac{g}{R \cos \theta}}$$

$$\omega = \sqrt{\frac{10}{0.5 \left( \frac{0.3}{0.5} \right)}} = \sqrt{\frac{100}{3}} \text{ rad/s}$$

21. (2)

$$a_c = 2t^2$$

$$\Rightarrow \omega^2 (2) = 2t^2$$

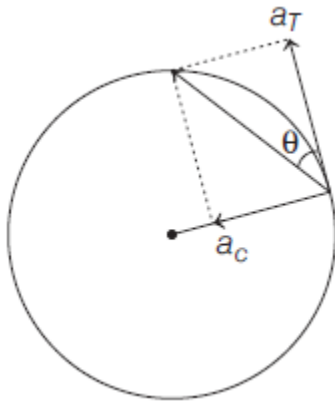
$$\Rightarrow \omega = t$$

$$\Rightarrow \int d\theta = \int_0^2 t dt$$

$$\Rightarrow \theta = \left[ \frac{t^2}{2} \right] = 2 \text{ rad}$$

22. (2)

$$\tan \theta = \frac{a_c}{a_T} = \frac{\omega^2 R}{R\alpha}$$

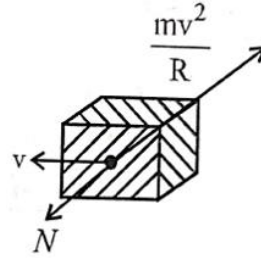


$$\begin{aligned} \tan \theta &= \frac{(\alpha t)^2}{\alpha} = \alpha t^2 \\ &= 2 \left( \frac{1}{2} \right)^2 = \frac{1}{2} \end{aligned}$$

1. (a)  
For observer on block,

$$N = \frac{mv^2}{r} \Rightarrow N \propto v^2$$

So, curve is parabola, symmetric about  $N$ -axis



2. (b)

$$\Delta \vec{V} = |\vec{V} - \vec{u}| = \sqrt{v^2 + u^2}$$

$$[\because \vec{V} \perp \vec{u}]$$

$$\text{Now, } K_1 + P_1 = K_2 + P_2$$

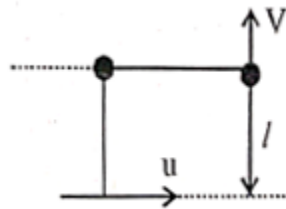
$$\Rightarrow \frac{1}{2} mu^2 + 0 = \frac{1}{2} mV^2 + mg l$$

$$\Rightarrow V^2 = u^2 - 2gl$$

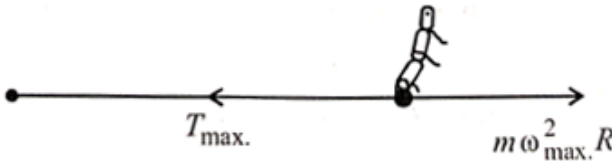
$$\Rightarrow V = \sqrt{u^2 - 2gl}$$

$$\text{So, } \Delta V = \sqrt{V^2 + u^2} = \sqrt{u^2 - 2gl + u^2}$$

$$\sqrt{2(u^2 - gl)}$$



3. (c)



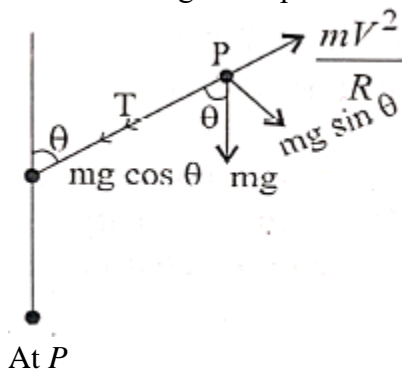
$$T_{\max} = m\omega_{\max}^2 R$$

$$80 = 0.1 \times \left(\frac{k}{30}\right)^2 \times 2 \quad \left[\because \omega = \frac{k}{\pi} \times \frac{2\pi}{60} = \frac{k}{30}\right]$$

$$\Rightarrow k^2 = \frac{30^2 \times 80}{2 \times 0.1} \Rightarrow k^2 = 3600000 \Rightarrow k = 600$$

4. (b)

Let us take a general point 'P'





$$T + mg \cos \theta = \frac{mv^2}{R} \Rightarrow T = \frac{mv^2}{R} - mg \cos \theta$$

So,  $T$  will be minimum when,  $mg \cos \theta$  is maximum

i.e., when  $\cos \theta$  is maximum

i.e., when  $\theta = 0$  and  $\theta$  is zero when string is at highest point.

5. (a)

From figure,

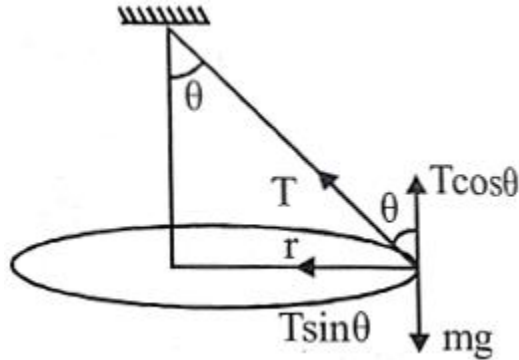
$$\sin \theta = \frac{r}{L} = \frac{L}{L\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

$$T \sin \theta = \frac{mv^2}{r} \quad \dots(i)$$

$$T \cos \theta = mg \quad \dots(ii)$$

From (i) and (ii)

$$\tan \theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg}$$



6. (b)

Given, mass of the block,  $m = 200 \text{ g} = 200 \times 10^{-3} \text{ Kg}$

Radius of the circular groove,  $r = 20 \text{ cm}$

Time taken to complete one round,  $T = 40 \text{ s}$

Here, normal force will provide the necessary centripetal force.

$$N = m\omega^2 r = 200 \times 10^{-3} \times \left(\frac{2\pi}{40}\right)^2 \times 0.2 = 9.859 \times 10^{-4} \text{ N}.$$

7. (c)

For statement-1

The maximum speed by which cyclist can take a turn on a circular path

$$\Rightarrow v \leq \sqrt{\mu rg} \leq \sqrt{0.2 \times 2 \times 9.8} \Rightarrow v \leq \sqrt{3.92}$$

$$\text{Speed of cyclist, } 7 \text{ kmh}^{-1} = 7 \times \frac{5}{18} = 1.94 \text{ m/s}$$

The maximum safe speed on a banked frictional road

$$v_{\text{allowable}} = \sqrt{r \cdot g \frac{(\mu + \tan \theta)}{1 - \mu \tan \theta}}$$

$$\Rightarrow v = \sqrt{\frac{2 \times 9.8 (0.2 + \tan 45^\circ)}{1 - 0.2 \times \tan 45^\circ}} \sqrt{\frac{2 \times 9.8 \times 1.2}{0.8}}$$

$$= 5.42 \text{ m/s}$$

Speed of cyclist,  $18.5 \text{ kmh} = 5.13 \text{ m/s}$

So, both the statement are true.

8. (d)

$$\text{Using, } \mu mg = \frac{mv^2}{r} = mr\omega^2$$

$$\omega = 2\pi n = 2\pi \times 3.5 = 7\pi \text{ rad/sec}$$

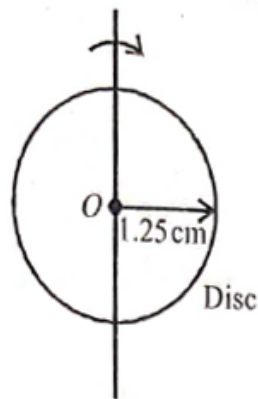
$$\text{Radius, } r = 1.25 \text{ cm} = 1.25 \times 10^{-2} \text{ m}$$

Coefficient of friction,  $\mu = ?$

$$\mu mg = \frac{m(r\omega)^2}{r} \quad (\because v = r\omega)$$

$$\mu mg = mr\omega^2$$

$$\begin{aligned} \Rightarrow \mu &= \frac{r\omega^2}{g} = \frac{1.25 \times 10^{-2} \times \left(7 \times \frac{22}{7}\right)^2}{10} \\ &= \frac{1.25 \times 10^{-2} \times 22^2}{10} = 0.6 \end{aligned}$$



9. (d)

Given,  $\theta = 45^\circ$ ,  $r = 0.4 \text{ m}$ ,  $g = 10 \text{ m/s}^2$

$$T \sin \theta = \frac{mv^2}{r} \quad \dots(i)$$

$$T \cos \theta = mg \quad \dots(ii)$$

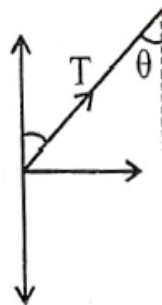
From equation (i) & (ii) we have,

$$\tan \theta = \frac{v^2}{rg}$$

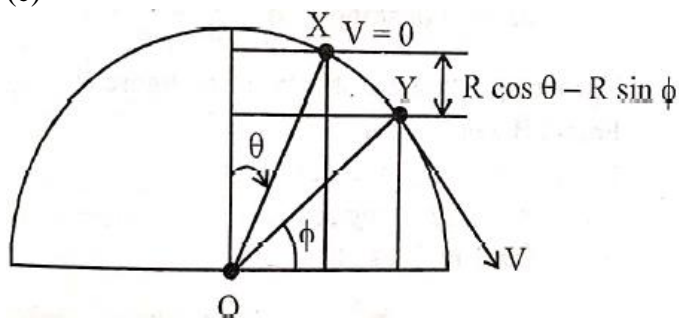
$$v^2 = rg \quad \because \theta = 45^\circ$$

Hence, speed of the pendulum in its circular path,

$$v = \sqrt{rg} = \sqrt{0.4 \times 10} = 2 \text{ m/s}$$



10. (c)



$$K.E._X + P.E._X = K.E._Y + P.E._Y$$

$$\Rightarrow \frac{1}{2} mV^2 = mgR(\cos \theta - \sin \phi)$$

$$\Rightarrow V^2 = 2gR(\cos \theta - \sin \phi)$$

$$\text{At } Y, \frac{mv^2}{R} = mg \sin \phi \quad [\because N_Y = 0]$$

$$\Rightarrow 2mg(\cos\theta - \sin\phi) = mg\sin\phi$$

$$\Rightarrow \sin\phi = \frac{2}{3}\cos\theta$$

11. (a)

$$\text{Initially, } k \times 1 = m\omega^2 R \quad \dots(\text{i})$$

$$\text{Finally, } k \times 5 = m(2\omega)^2 (R + 4) \quad \dots(\text{ii})$$

From (i) & (ii), we get :  $R = 16$

So, original length =  $(16 - 1) \text{ cm} = 15 \text{ cm}$ .