

Circular Motion

JEE Main Exercise

1. (c)

$$\theta = \left(\frac{\omega + \omega_0}{2} \right) t$$

$$\Rightarrow \theta = \left(\frac{20+0}{2} \right) (5) = 50 \text{ rad}$$

$$\text{Number of revolutions} = \frac{\theta}{2\pi} = \frac{50}{2\pi} = \frac{25}{\pi}$$

2. (d)

$$\omega = \frac{2\pi}{T}$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$

$$\because T_1 = T_2 \Rightarrow \frac{\omega_1}{\omega_2} = 1$$

3. (a)

$$a_T = \alpha$$

$$\Rightarrow r\alpha = \alpha$$

$$\Rightarrow r = 1 \text{ m}$$

So, diameter = 2m

4. (c)

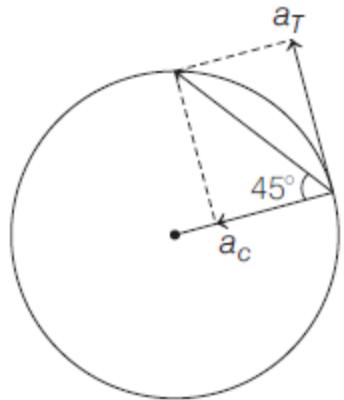
$$\text{Centripetal acceleration } (a_c) = \frac{v^2}{R} = \frac{(30)^2}{500} = 1.8 \text{ m/s}^2$$

$$\text{Tangential acceleration } (a_T) = \frac{dv}{dt} = 2 \text{ m/s}^2$$

$$\begin{aligned} \text{Resultant acceleration} &= \sqrt{a_c^2 + a_T^2} \\ &= \sqrt{(1.8)^2 + 2^2} \\ &= \sqrt{7.24} = 2.7 \text{ m/s}^2 \end{aligned}$$

5. (b)

$$\tan 45^\circ = \frac{a_T}{a_c} = \frac{9}{(v^2/4)}$$



$$\Rightarrow v^2 = 36$$

$$\Rightarrow v = 6 \text{ m/s}$$

and $v = u + at$

$$\Rightarrow 6 = 0 + 9t$$

$$\Rightarrow t = \frac{2}{3} \text{ s}$$

6. (b)

$$a_c = k^2 r t^2 \Rightarrow \frac{v^2}{r} = k^2 r t^2$$

$$\Rightarrow v = krt$$

$$\Rightarrow a_T = \frac{dv}{dt} = kr$$

$$P = Fv \cos 0^\circ = (ma_T)v \cos 0^\circ$$

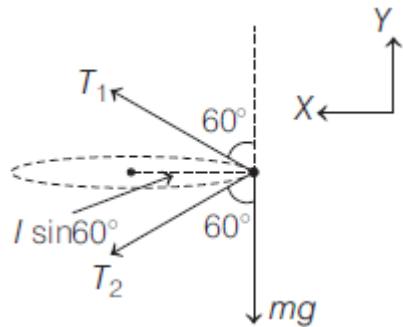
$$= mkrkrt = mk^2 r^2 t$$

7. (b)

$$v = \omega r \text{ and } a = \omega^2 r$$

If radius becomes half, then both v and a will become half of the original values.

8. (b)



$$\sum F_x = ma$$

$$\Rightarrow T_1 \sin 60^\circ + T_2 \sin 60^\circ = m\omega^2 (l \sin 60^\circ) \quad \dots(i)$$

$$\sum F_y = 0$$

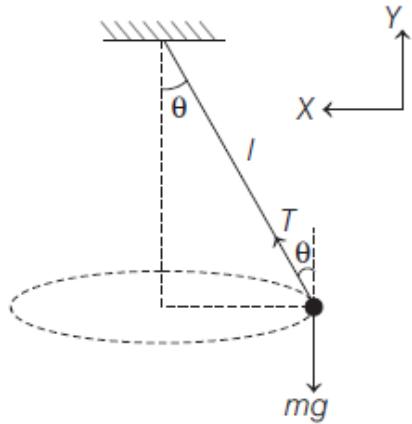
$$\Rightarrow T_1 \cos 60^\circ = T_2 \cos 60^\circ + mg \quad \dots(\text{ii})$$

Solving Eqs. (i) and (ii), we get

$$\omega = 14 \text{ rad/s}$$

9. (c)

$$\sum F_y = 0$$



$$\Rightarrow T \cos \theta = mg \quad \dots(\text{i})$$

$$\sum F_x = ma_x$$

$$T \sin \theta = m\omega^2 (l \sin \theta) \quad \dots(\text{ii})$$

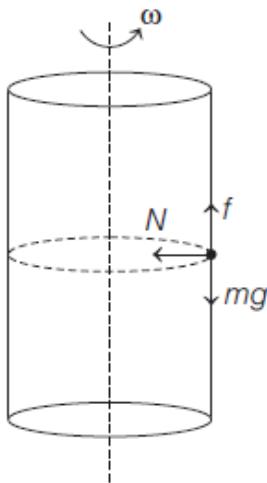
From Eqs. (i) and (ii), we get

$$\omega = \sqrt{\frac{g}{l \cos \theta}}$$

$$\Rightarrow T = \frac{2\pi}{2\omega} = 2\pi \sqrt{\frac{l \cos \theta}{g}}$$

10. (a)

$$\sum F_y = 0 \Rightarrow f = mg$$



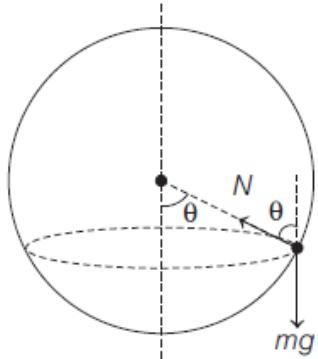
$$\sum F_x = 0 \Rightarrow N = m\omega^2 r$$

$$\text{and } f \leq f_L$$

$$\Rightarrow mg \leq \mu(m\omega^2 r)$$

$$\Rightarrow \omega \geq \sqrt{\frac{g}{\mu r}}$$

11. (b)



$$\sum F_y = 0 \Rightarrow N \cos \theta = mg \quad \dots(i)$$

$$\sum F_x = ma_x \Rightarrow N \sin \theta = m\omega^2 \left(\frac{a}{2} \right) \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\omega = \left(\frac{2g}{\sqrt{3a}} \right)^{1/2}$$

12. (c)

$$v = \sqrt{rg \tan \theta} = \sqrt{(10\sqrt{3})(10) \tan 30^\circ} \\ = 10 \text{ m/s} = 36 \text{ km/h}$$

13. (b)

$$\mu_{\min} = \frac{v^2}{gr} = \frac{\left(72 \times \frac{5}{18}\right)^2}{10(200)} = 0.2$$

14. (a)

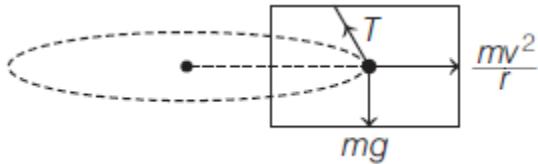
$$v_{\max} = \sqrt{\frac{gr(\tan \theta + \mu)}{1 - \mu \tan \theta}} = \sqrt{\frac{10 \times 20 \left(\frac{3}{4} + \frac{1}{4} \right)}{1 - \frac{3}{4} \times \frac{1}{4}}} \\ = 10 \sqrt{\frac{32}{13}} \text{ m/s}$$

15. (b)

$$\tan \theta = \frac{a}{g} = \frac{v^2/r}{g}$$

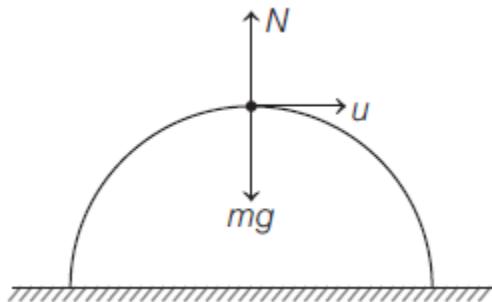
$$\Rightarrow \tan \theta = \frac{v^2}{rg} = \frac{10^2}{10(10)} = 1$$

$$\Rightarrow \theta = 45^\circ$$



16. (a)

$$mg - N = \frac{mu^2}{R}$$



For losing contact, $N = 0$

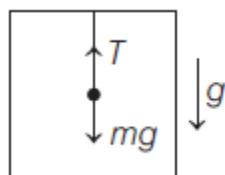
$$\Rightarrow mg = \frac{mu^2}{R}$$

$$\Rightarrow u = \sqrt{gR}$$

17. (d)

In free fall, acceleration of car is g .

For pendulum,



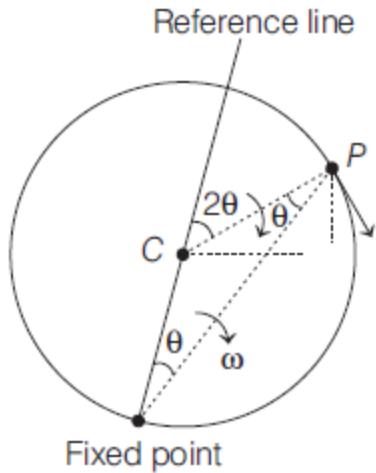
$$\sum F = ma$$

$$\Rightarrow mg - T = m(g)$$

$$\Rightarrow T = 0$$

18. (4)

$$\frac{d\theta}{dt} = 2 \text{ rad/s}$$



Angular velocity of particle w.r.t the centre of circle

$$= \frac{d(2\theta)}{dt}$$

$$= 2 \frac{d\theta}{dt} = 4 \text{ rad/s}$$

$$v = \omega r = 4(1) = 4 \text{ m/s}$$

19. (1)

$$\alpha = \frac{\pi}{4} \text{ rad/s}^2$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\Rightarrow \frac{\pi}{2} = 0 + \frac{1}{2} \times \left(\frac{\pi}{4}\right) t^2$$

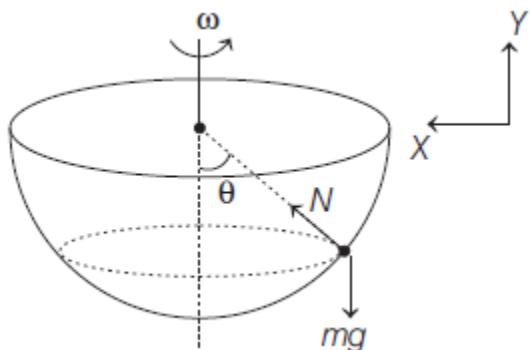
$$\Rightarrow t = 2 \text{ s}$$

$$v_{\text{avg}} = \frac{\Delta S}{\Delta t} = \frac{\sqrt{2}R}{2} = 1 \text{ m/s}$$

20. (100)

$$\sum F_y = 0 \Rightarrow N \cos \theta = mg \quad \dots \dots \text{(i)}$$

$$\sum F_x = ma_x \Rightarrow N \sin \theta = m\omega^2 R \sin \theta \quad \dots \dots \text{(ii)}$$



Solving Eqs. (i) and (ii), we get

$$\omega = \sqrt{\frac{g}{R \cos \theta}}$$

$$\omega = \sqrt{\frac{10}{0.5 \left(\frac{0.3}{0.5} \right)}} = \sqrt{\frac{100}{3}} \text{ rad/s}$$

21. (2)

$$a_c = 2t^2$$

$$\Rightarrow \omega^2 (2) = 2t^2$$

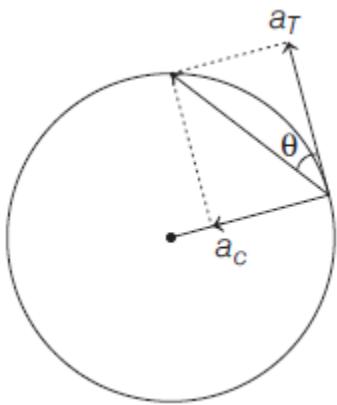
$$\Rightarrow \omega = t$$

$$\Rightarrow \int d\theta = \int_0^2 t dt$$

$$\Rightarrow \theta = \left[\frac{t^2}{2} \right]_0^2 = 2 \text{ rad}$$

22. (2)

$$\tan \theta = \frac{a_c}{a_T} = \frac{\omega^2 R}{R \alpha}$$



$$\tan \theta = \frac{(\alpha t)^2}{\alpha} = \alpha t^2$$

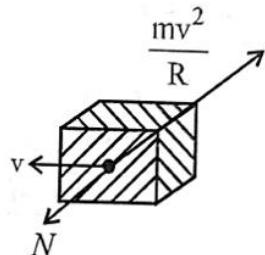
$$= 2 \left(\frac{1}{2} \right)^2 = \frac{1}{2}$$

1. (a)

For observer on block,

$$N = \frac{mv^2}{r} \Rightarrow N \propto v^2$$

So, curve is parabola, symmetric about N -axis



2. (b)

$$\Delta V = |\vec{V} - \vec{u}| = \sqrt{v^2 + u^2}$$

$[\because \vec{V} \pm \vec{u}]$

$$\text{Now, } K_1 + P_1 = K_2 + P_2$$

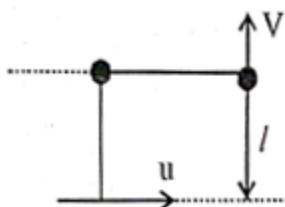
$$\Rightarrow \frac{1}{2}mu^2 + 0 = \frac{1}{2}mV^2 + mg l$$

$$\Rightarrow V^2 = u^2 - 2gl$$

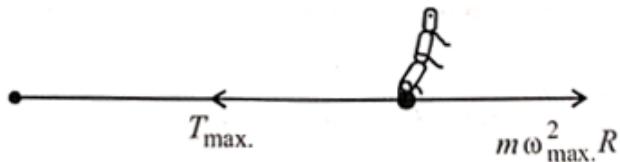
$$\Rightarrow V = \sqrt{u^2 - 2gl}$$

$$\text{So, } \Delta V = \sqrt{V^2 + u^2} = \sqrt{u^2 - 2gl + u^2}$$

$$\sqrt{2(u^2 - gl)}$$



3. (c)



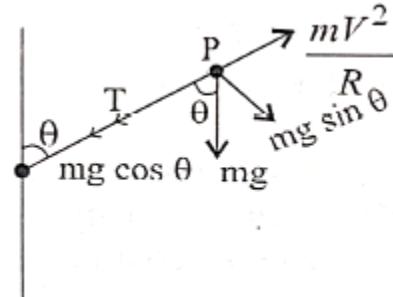
$$T_{\max} = m\omega_{\max}^2 R$$

$$80 = 0.1 \times \left(\frac{k}{30}\right)^2 \times 2 \quad \left[\because \omega = \frac{k}{\pi} \times \frac{2\pi}{60} = \frac{k}{30} \right]$$

$$\Rightarrow k^2 = \frac{30^2 \times 80}{2 \times 0.1} \Rightarrow k^2 = 3600000 \Rightarrow k = 600$$

4. (b)

Let us take a general point 'P'



At P

$$T + mg \cos \theta = \frac{mv^2}{R} \Rightarrow T = \frac{mv^2}{R} - mg \cos \theta$$

So, T will be minimum when, $mg \cos \theta$ is maximum

i.e., when $\cos \theta$ is maximum

i.e., when $\theta = 0^\circ$ and θ is zero when string is at highest point.

5.

(a)

From figure,

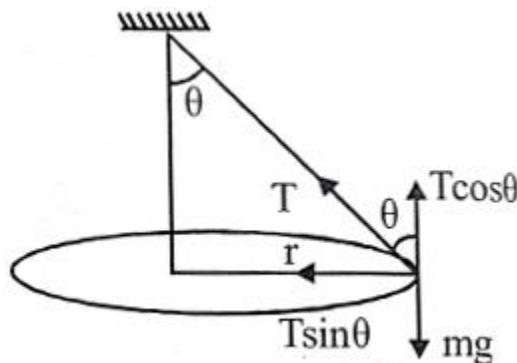
$$\sin \theta = \frac{r}{L} = \frac{L}{L\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

$$T \sin \theta = \frac{mv^2}{r} \quad \dots \text{(i)}$$

$$T \cos \theta = mg \quad \dots \text{(ii)}$$

From (i) and (ii)

$$\tan \theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg}$$



6.

(b)

Given, mass of the block, $m = 200 \text{ g} = 200 \times 10^{-3} \text{ Kg}$

Radius of the circular groove, $r = 20 \text{ cm}$

Time taken to complete one round, $T = 40 \text{ s}$

Here, normal force will provide the necessary centripetal force.

$$N = m\omega^2 r = 200 \times 10^{-3} \times \left(\frac{2\pi}{40} \right)^2 \times 0.2 = 9.859 \times 10^{-4} \text{ N.}$$

7.

(c)

For statement-1

The maximum speed by which cyclist can take a turn on a circular path

$$\Rightarrow v \leq \sqrt{\mu rg} \leq \sqrt{0.2 \times 2 \times 9.8} \Rightarrow v \leq \sqrt{3.92}$$

$$\text{Speed of cyclist, } 7 \text{ kmh}^{-1} = 7 \times \frac{5}{18} = 1.94 \text{ m/s}$$

The maximum safe speed on a banked frictional road

$$\begin{aligned} v_{\text{allowable}} &= \sqrt{r \cdot g \frac{(\mu + \tan \theta)}{1 - \mu \tan \theta}} \\ \Rightarrow v &= \sqrt{\frac{2 \times 9.8 (0.2 + \tan 45^\circ)}{1 - 0.2 \times \tan 45^\circ}} \sqrt{\frac{2 \times 9.8 \times 1.2}{0.8}} \\ &= 5.42 \text{ m/s} \end{aligned}$$

Speed of cyclist, $18.5 \text{ kmh}^{-1} = 5.13 \text{ m/s}$

So, both the statements are true.

8. (d)

$$\text{Using, } \mu mg = \frac{mv^2}{r} = mr\omega^2$$

$$\omega = 2\pi n = 2\pi \times 3.5 = 7\pi \text{ rad/sec}$$

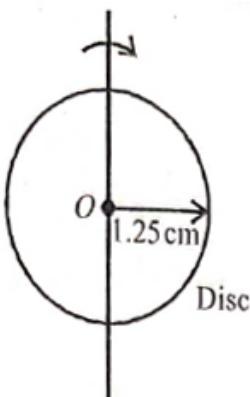
$$\text{Radius, } r = 1.25 \text{ cm} = 1.25 \times 10^{-2} \text{ m}$$

Coefficient of friction, $\mu = ?$

$$\mu mg = \frac{m(r\omega)^2}{r} (\because v = r\omega)$$

$$\mu mg = mr\omega^2$$

$$\Rightarrow \mu = \frac{r\omega^2}{g} = \frac{1.25 \times 10^{-2} \times \left(7 \times \frac{22}{7}\right)^2}{10} \\ = \frac{1.25 \times 10^{-2} \times 22^2}{10} = 0.6$$



9. (d)

$$\text{Given, } \theta = 45^\circ, r = 0.4 \text{ m}, g = 10 \text{ m/s}^2$$

$$T \sin \theta = \frac{mv^2}{r} \quad \dots \text{(i)}$$

$$T \cos \theta = mg \quad \dots \text{(ii)}$$

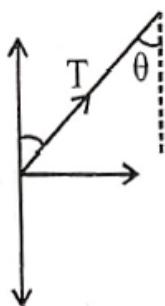
From equation (i) & (ii) we have,

$$\tan \theta = \frac{v^2}{rg}$$

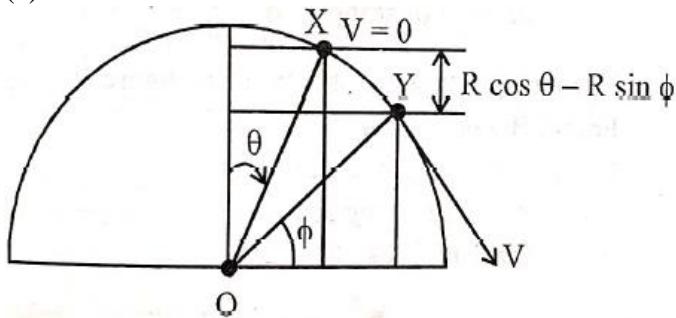
$$v^2 = rg \quad \therefore \theta = 45^\circ$$

Hence, speed of the pendulum in its circular path,

$$v = \sqrt{rg} = \sqrt{0.4 \times 10} = 2 \text{ m/s}$$



10. (c)



$$K.E._x + P.E._x = K.E._y + P.E._y$$

$$\Rightarrow \frac{1}{2}mV^2 = mgR(\cos \theta - \sin \phi)$$

$$\Rightarrow V^2 = 2gR(\cos \theta - \sin \phi)$$

$$\text{At } Y, \frac{mv^2}{R} = mg \sin \phi \quad [\because N_y = 0]$$

$$\Rightarrow 2mg(\cos\theta - \sin\phi) = mg \sin\phi$$

$$\Rightarrow \sin\phi = \frac{2}{3} \cos\theta$$

11. (a)

$$\text{Initially, } k \times 1 = m\omega^2 R \quad \dots (\text{i})$$

$$\text{Finally, } k \times 5 = m(2\omega)^2 (R + 4) \quad \dots (\text{ii})$$

From (i) & (ii), we get : $R = 16$

So, original length = $(16 - 1)$ cm = 15 cm.

EXERCISE - 1

1. (D)

ω = rate of change of angle

$$\therefore \frac{\omega_1}{\omega_2} = 1 \text{ as they both complete } 2\pi \text{ angle in same time.}$$

2. (C)

$$mg - N = \frac{mv^2}{r}$$

$$\Rightarrow N = mg - \frac{mv^2}{r}$$

Since $r_A < r_B$

$$\Rightarrow N_A < N_B$$

3. (A)

Force is always perpendicular to displacement

$$\therefore \text{work done} = 0$$

4. (B)

$$a_{\text{resultant}} = \sqrt{a_c^2 + a_{\text{tang.}}^2}$$

$$= \sqrt{\left(\frac{30^2}{500}\right)^2 + 2^2} = \frac{\sqrt{181}}{5} = 2.7 \text{ m/s}^2$$

5. (A)

$$f = mg$$

$$\Rightarrow \mu N \geq mg$$

$$\Rightarrow \mu m r \omega^2 \geq mg$$

$$\Rightarrow \omega \geq \sqrt{\frac{g}{\mu r}}$$

6. (C)

If the coin just slips at a distance of $4r$ from centre

$$\Rightarrow \mu mg = m4r\omega^2 \dots (1)$$

If angular velocity is doubled

$$\mu mg = mR(2\omega)^2 \dots (2)$$

From (1) and (2)

$$\Rightarrow R = r$$

7. (D)

$$T = mr\omega_0^2 \dots (1)$$

$$2T = mr\omega^2 \dots (2)$$

$$\Rightarrow \omega = \sqrt{2}\omega_0 = \sqrt{2} \times 5 \text{ rpm}$$

8. (C)

Since force is always perpendicular to velocity particle moves in circle.
It's speed is constant and velocity variable.

9. (C)

$$\mu mg = \frac{mv_{\max}^2}{r}$$

$$v_{\max} = \sqrt{\mu gr} = \sqrt{0.3 \times 10 \times 300}$$
$$= 30 \text{ m/s} = 108 \text{ km/hr}$$

10. (C)

$$F_{\text{net}} = ma_{\text{rad.}} = \frac{mv^2}{r}$$

11. (B)

$$T = mr\omega^2 = 0.2 \times 0.5 \times 4^2 = 1.6 \text{ N}$$

12. (C)

Centripetal force is provided by friction

13. (A)

$$N_A - mg = \frac{mv^2}{r}$$

$$N_A = mg + \frac{mv^2}{r_A}, \quad N_B = mg - \frac{mv^2}{r_B}$$

$$N_C = mg + \frac{mv^2}{r_c}$$

14. (A)

Real forces are mg and T only

15. (A)

Bead starts slipping, when

$$\mu N = mL\omega^2$$

$$\mu mL\alpha = mL\omega^2$$

$$\mu\alpha = (0 + \alpha t)^2$$

$$\Rightarrow t = \sqrt{\frac{\mu}{\alpha}}$$

EXERCISE - 2

1. (A, C)

$$\text{Time to fall} = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5}{10}} = 1 \text{ s}$$

Distance covered in y direction = $v \times t$

$$= 3 \times 1$$

$$= 3 \text{ m}$$

Since there is no velocity along x-direction x is always 2m.

2. (A, B)

For no wear and tear friction is zero

$$\Rightarrow \frac{v^2}{Rg} = \tan 15^\circ$$

$$\Rightarrow v = \sqrt{Rg \tan 15^\circ} = 28.1 \text{ m/s}$$

$$v_{\max} = \sqrt{\frac{Rg(\mu + \tan \theta)}{1 - \mu \tan \theta}} = 38.1 \text{ m/s}$$

3. (B, C)

$$\vec{v} = \frac{d\vec{r}}{dt}; \therefore \vec{v} \parallel d\vec{r} \text{ (i.e. along the tangent)}$$

$$\vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{t} = 0$$

4. (A, B)

$$\frac{ds}{dt} = v = K\sqrt{s}$$

$$\Rightarrow \int_0^s \frac{ds}{\sqrt{s}} = \int_0^t K dt$$

$$2\sqrt{s} = Kt$$

$$s = \frac{K^2 t^2}{4}$$

$$v = \frac{ds}{dt} = \frac{K^2 t}{2}$$

5. (A, B, C)

$$T = m\ell\omega^2, v = \ell\omega, F_{\text{vert}} = 0$$

6. (A, B, D)

$$\omega = \frac{v}{R} = \text{constant}, \theta = \omega t$$

$$F_y = -F \sin \theta = -F \sin \omega t$$

$$= -\frac{mv^2}{R} \sin \omega t$$

$$V_r = -v \sin \theta = -v \sin \omega t$$

$$x\text{-coordinate} = R \cos \omega t$$

7. **(B, D)**

$$\text{If } \mu = 0.1, f_{\max} = 0.1 \times 0.5 \times 10 = 0.5 \text{ N}$$

$$\text{Req. centripetal force} = mr\omega^2 = 0.5 \times 1 \times 0.5^2 = 0.125 \text{ N}$$

$$\therefore f = \frac{1}{8} \text{ N, Tension} = \text{zero}$$

$$\text{If } \mu = \frac{1}{20}, f_{\max} = \frac{1}{20} \times 0.5 \times 10 = 0.25 \text{ N}$$

$$\therefore f = \frac{1}{8} \text{ N, Tension} = \text{zero}$$

$$\text{If } \mu = \frac{1}{40}, f_{\max} = \frac{1}{40} \times 0.5 \times 10 = 0.125 \text{ N}$$

$$\therefore f = \frac{1}{8} \text{ N, Tension} = 0$$

EXERCISE - 3

1. **(A)-R; (B)-P; (C)-S; (D)-Q**

$$\frac{v^2}{r} = K^2 rt^2 \text{ (given)}$$

(a) Centripetal force = $mK^2 rt^2$

(b) Tangential force = $m \frac{dv}{dt} = mKr$

(c) Power of centripetal force = $\vec{F}_{\text{centripetal}} \cdot \vec{v} = 0$

(d) Power of tangential force = $\vec{F}_t \cdot \vec{v} = F_t v$
 $= mK^2 r^2 t$

2. **(B)**

Assuming no friction between m_1 and m_2

$$a_1 = R\omega^2 - \frac{T}{m_1}$$

$$a_2 = R\omega^2 - \frac{T}{m_2}$$

$$\therefore a_1 > a_2$$

\therefore Friction on upper block acts towards left and on lower block towards right.

3. **(A)**

Let the required angular velocity be ω

Then

$$T + \mu m_1 g = m_1 R \omega^2 \dots (1)$$

$$T = \mu m_1 g + m_2 R \omega^2 \dots (2)$$

$$\Rightarrow 2 \mu m_1 g = (m_1 - m_2) R \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{2\mu m_1 g}{(m_1 - m_2)R}} = 6.3 \text{ rad/s}$$

4. (B)

$$T = \mu m_1 g + m_2 R \omega^2 \\ = 0.5 \times 2 \times 10 + 1 \times 0.5 \times 40 = 30 \text{ N}$$

5. (B)

$$\tan \theta = \frac{v_{\text{design}}^2}{gR} = \frac{1}{2}; \\ f = m \left(g \sin \theta - \frac{v^2}{R} \cos \theta \right) \\ = 300\sqrt{5} \text{ m/s}$$

6. (A)

$$f = m \left(\frac{v^2}{R} \cos \theta - g \sin \theta \right) \\ = 500\sqrt{5} \text{ m/s}$$

7. (A)

$$\theta = \tan^{-1} \frac{1}{2}$$

8. (C)

$$mg \sin \theta = m \frac{v^2}{L} \quad \dots (1)$$

$$\frac{1}{2} m (\sqrt{3gL})^2 = \frac{1}{2} mv^2 + mgL(1 + \sin \theta) \quad \dots (2)$$

$$\therefore \theta = \sin^{-1} \left(\frac{1}{3} \right); \text{ Also } v^2 = \frac{1}{3} g L$$

9. (C)

$$h_{\max} = L(1 + \sin \theta) + \frac{0^2 - v^2 \cos^2 \theta}{-2g} \\ = \frac{40L}{27}$$

10. (B)

$$\frac{1}{2} m (\sqrt{3gL})^2 = mg h_{\max}$$

$$\Rightarrow h_{\max} = \frac{3L}{2}$$

EXERCISE - 4

1. Here frictional force will provide the required tangential and centripetal force for the circular motion of car.
Force of friction will act along the direction of net acceleration.

$$f = m \sqrt{a^2 + \left(\frac{v^2}{R}\right)^2}$$

Car will skid when $m \sqrt{a^2 + \left(\frac{v^2}{R}\right)^2}$

$$= m \sqrt{a^2 + \frac{a^4 + t^4}{R^2}}$$

$$\Rightarrow t = \left[\frac{(\mu^2 g^2 - a^2) R^2}{a^4} \right]^{1/4}$$

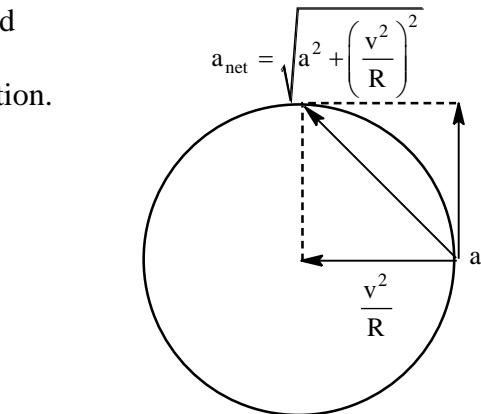
Till this time distance traveled by car is $D = \frac{1}{2} at^2$. Putting value of t we get $D = \frac{R \sqrt{\mu^2 g^2 - a^2}}{2a}$.

2. The insect will slide when $mg \sin \alpha$ becomes equal to limiting friction. At every instant the insect is in equilibrium.

$$\text{So, } N = mg \cos \alpha$$

$$\mu N = mg \sin \alpha$$

$$\Rightarrow \mu mg \cos \alpha = mg \sin \alpha \Rightarrow \mu = \tan \alpha \Rightarrow \alpha = \tan^{-1} \mu = \tan^{-1}(1/3)$$

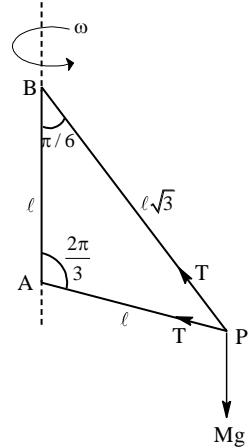
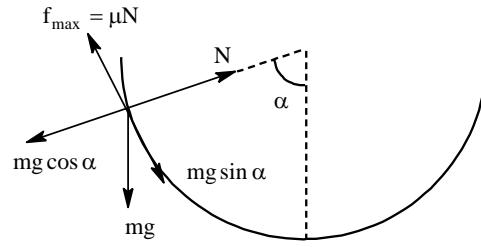


3. Applying Newton's law towards the centre of circle we get
 $N = m\omega^2 R$

Let ω be the minimum angular speed for which man is not falling. At this instant its weight will be balanced by limiting friction acting upwards.

$$\text{i.e., } \mu N = mg \Rightarrow 0.15 \times 70 \omega^2 \times 3 = 70 \times 10$$

$$\Rightarrow \omega = 4.7 \text{ rad/sec.}$$



4. Applying Newton's law along vertical we get

$$T \frac{\cos \pi}{6} + T \frac{\cos \pi}{3} = mg \quad \dots (1)$$

$$\Rightarrow T = \frac{2mg}{\sqrt{3} + 1}$$

Applying Newton's law along horizontal.

$$\text{We get } T \cos \frac{\pi}{3} + T \cos \frac{\pi}{6} = m\omega^2 R = m\omega^2 \frac{\ell\sqrt{3}}{2}$$

$$\Rightarrow \omega^2 = \frac{2g}{\ell\sqrt{3}}$$

5. Particle P and Q will be at same angular position whenever $5\pi t = 2\pi t + \frac{\pi}{2} + 2\pi n$.

$$\Rightarrow t = \frac{1}{6} + \frac{2n}{3} \quad (n = \text{integer}) \quad \dots (1)$$

Similarly, particle P and R will be at same angular position.

Whenever $5\pi t = 3\pi t = \pi + 2\pi m$ ($m = \text{integer}$)

$$\Rightarrow t = \frac{1}{2} + m \quad \dots (2)$$

All three particles will be at same angular position when (1) = (2)

$$\Rightarrow \frac{1}{2} + m = \frac{1}{6} + \frac{2n}{3}$$

$$\Rightarrow 2n = 1 + 3m$$

Smallest integral value of m & n satisfying the above equation is $m = 1, n = 2$.

Putting these values in (1) & (2) we get $t = 1.5$ sec.

So, they will meet for the first time at $t = 1.5$ sec.

6. According to question, $a_c = a_t = \frac{v^2}{R}$

$$\Rightarrow \frac{dv}{dt} = \frac{v^2}{R} \Rightarrow \int_{v_0}^v \frac{dv}{v^2} = \int_0^t \frac{dt}{R}$$

$$\Rightarrow \frac{-1}{v} + \frac{1}{v_0} = \frac{t}{R} \Rightarrow V(t) = \frac{Rv_0}{R - tv_0}$$

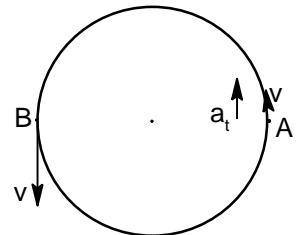
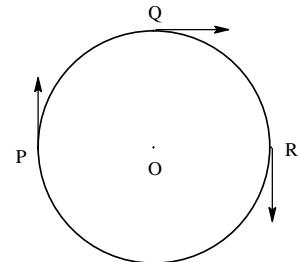
$$\Rightarrow \int_0^{2\pi R} dx = Rv_0 \int_0^T \frac{dt}{R - tv_0} \Rightarrow 2\pi R = \frac{-Rv_0}{v_0} (\ln(R - tv_0))_0^T$$

$$\Rightarrow T = \frac{R}{v_0} (1 - e^{-2\pi})$$

7. Lets assume that the particles meet at time t . Distance traveled by A = distance traveled + πR by B

$$\Rightarrow vt + \frac{1}{2} \times \frac{72v^2 t^2}{25\pi R} = vt + \pi R$$

$$\Rightarrow t = \frac{5\pi R}{6v}$$



$$\text{Angle traced by A} = \frac{\text{distance travelled}}{R} = \frac{11\pi}{6}$$

$$\text{Angular velocity} = \frac{v + at}{R} = \frac{17v}{5R}$$

PYQ : JEE Advanced

1. (C, D)

$\vec{F} \perp \vec{v} \Rightarrow P = 0 \Rightarrow$ kinetic energy = constant; F is constant (given) $\Rightarrow \frac{mv^2}{R} = \text{constant}$
 $\Rightarrow R = \text{constant.}$

2. (A)

Radius of curvature in (a) is minimum

3. (A)

$$mg \sin \alpha = \frac{1}{3} mg \cos \alpha \Rightarrow \cot \alpha = 3$$

4. (C)

$\vec{a} = \vec{a}_{\text{tangential}} + \vec{a}_{\text{normal}}$ and $\vec{a}_{\text{tangential}}$ is downward.

5. $Kx \cos 30^\circ + mg \cos 30^\circ = ma_t$. As $x = \frac{R}{4}$ and $K = \frac{mg}{R}$, $a_t = \frac{5\sqrt{3}}{8}g$;
 $N + Kx \cos 60^\circ = mg \cos 60^\circ \Rightarrow N = \frac{3mg}{8}$