PART I : PHYSICS

SECTION 1 (Maximum Marks: 28)

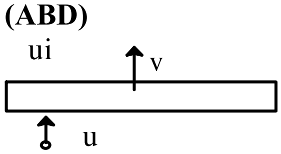
- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories:

Full Marks	: +4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
Partial Marks	: +1	For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened
Zero Marks	: 0	If none of the bubbles is darkened
Negative Marks :		-2 In all other cases

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get -2 marks, as a wrong option is also darkened

1. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true?
(A) The resistive force experienced by the plate is proportional to v
(B) At a later time the external force F balance the resistive force
(C) The plate will continue to move with constant non-zero acceleration, at all times
(D) The pressure difference between the leading and trailing faces of the plate is proportional to uv

1. **(ABD)**



For upper surface change in momentum will be proportional to $(u+v)$

Pressure difference between upper and lower surface $\propto \left[(u+v)^2-(u-v)^2\right] = uv$

2. A human body has a surface area of approximately $1m^2$. The normal body temperature is 10K above the surrounding room temperature T_0. Take the room temperature to be T_0 = 300K. For T_0 = 300K, the value of $\sigma T_0^4 = 460 Wm^{-2}$ (where σ is the Stefan-Boltzmann constant). Which of the following option is/are correct?
(A) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time.
(B) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths
(C) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation
(D) The amount of energy radiated by the body in 1 second is close to 60 Joules.

2. **(ACD or C)**

$Q=\sigma\left[\left(T_0+10\right)^4-T_0^{\ 4}\right]$

$\sigma\left[\left(T_0+10\right)^4-T_0^{\ 4}\right]-\sigma\left[\left(T_0+10\right)^4-\left(T_0-\Delta T\right)^4\right]$

$-\sigma T_0^{\ 4}+\sigma\left(T_0-\Delta T\right)^4$

$\sigma T_0^{\ 4}\left[1-\frac{4\Delta T}{T_0}-1\right]$

$-4\sigma T_0^{\ 3}\Delta T$ (hence option A is correct)

A = 1m^2

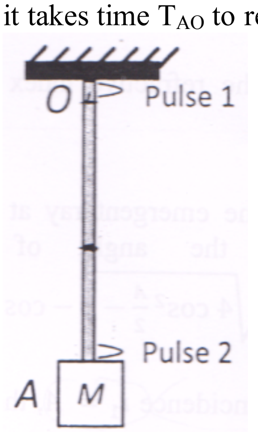
$\sigma_T=10K$ $\qquad U=\sigma A\left(T^4-T_0^4\right)$

$=\sigma A\left[\left(T_0+\Delta T\right)^4-T_0^4\right]$

$U=\sigma.AT_0^3\Delta T$

$U=\frac{4\times 450}{300}\times 10=4\times 15=60w$

3. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A If the wave pulse of wavelength λ_0 is produced at point A(Pulse 2) without distributing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct?



(A) The time $T_{AO}=T_{OA}$
(B) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope
(C) The wavelength of pulse 1 becomes longer when it reaches point A
(D) The velocity of any pulse along the rope is independent of its frequency and wavelength

3. **(AD)**

Velocity at a distance X from the lower end $V=\sqrt{\frac{Mg+\mu X}{\mu}}=\sqrt{\frac{Mg}{\mu}+X}$

4. For an isosceles prism of angle A and refractive indes μ, it is found that the angle of minimum deviation δ_m = A. which of the following options is/are correct?

(A) At minimum deviation the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1=\left(i_1/2\right)$

(B) For this prism the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

(C) For this prism the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2}-1} - \cos A\right]$

(D) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism.

4. **(ACD)**

$S = i_1 + i_2 - A$

For minimum deviation

$i_1 = i_2$ and $r_1 = r_2 = \frac{A}{2}$

Given $S = A$

$2A = 2i_1$

$i_1 = A$ and $r_1 = \frac{A}{2}$

$\therefore r_1 = \frac{i_1}{2}$ option A

$i_2 = 90^o$ $r_2 = C$

$\therefore$ $r_1 = A - C$

$i = \sin^{-1}\left(\mu\sin\left(A - C\right)\right)$

$\sin i_1 = \mu \sin r_1$

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin A}{\sin\frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$$

$\cos\frac{A}{2} = \frac{\mu}{2}$

$\frac{A}{2} = \cos^{-1}\frac{\mu}{2}$

$\therefore$ option B is incorrect

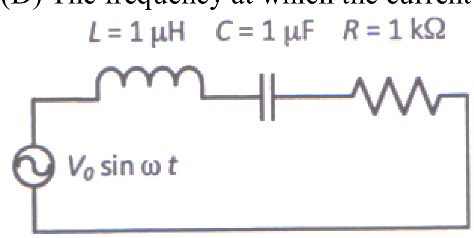
5. In the circuit shown, $L = 1\mu H, C = 1\mu F$ and $R = 1k\Omega$, they are connected in series with and a. c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct?

(A) At $\omega >> 10^6 \text{rad.s}^{-1}$, the circuit behave like a capacitor

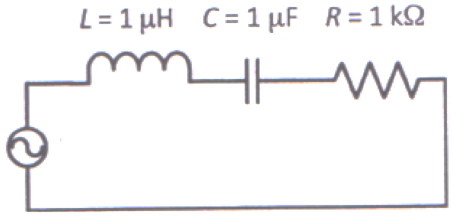
(B) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero

(C) The current will be in phase with the voltage if $\omega = 10^4 \text{rad.s}^{-1}$

(D) The frequency at which the current will be in phase with the voltage is independent of R



5. **(BD)**



(A) $\omega = 10^6$ $x_C = 1$

$x_C = \frac{1}{\omega_C} = 1$ (A – wrong)

(B) $\omega \sim 0$ $x_C \Rightarrow \infty$

So $z \to \infty$

$1 \to 0$

B – correct

(C) If $\omega = 10^4$

$x_L = 10^{-2}$

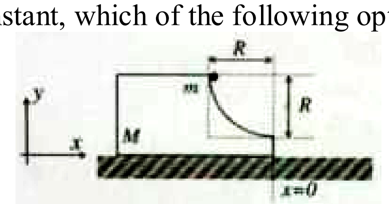
$x_C = 10^2$

$x_C \neq x_L$ so C wrong

(D) If $x_C = x_L$ then current is in phase with voltage

D – correct

6. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are correct?



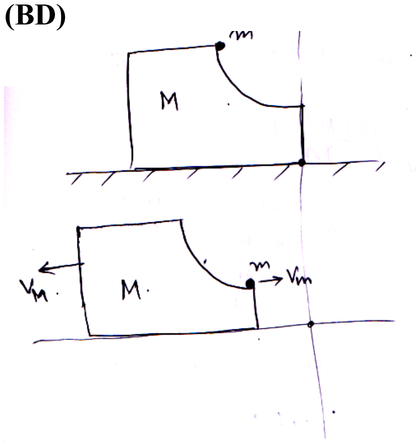
(A) The velocity of the block M is $V = -\frac{m}{M}\sqrt{2gR}$

(B) The x component of displacement of the centre of mass of the block M is : $-\frac{mR}{M+m}$

(C) The position of the point mass is : $x = -\sqrt{2}\frac{mR}{M+m}$

(D) The velocity of the point mass m is: $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$

6. **(BD)**



Let displacement of M be x towards left so displacement of m will be x – R towards left displacement of c.o.m towards left $= \dfrac{m(x-R)+Mx}{M+m} = 0$

$\Rightarrow x = \left(\dfrac{m}{M+m}\right)R$

$\therefore$ displacement of m & M are $\left(\dfrac{M}{M+m}\right)R$ & $\left(\dfrac{m}{M+m}\right)R$ respectively

Let velocity of m be V_m & of M be V_M

CLM $\rightarrow mV_m = MV_M$, COE $mgR = \dfrac{1}{2}mV_m^2 + \dfrac{1}{2}MV_M^2$

Solving the two equations

$V_m = \sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}$ & $V_M = \sqrt{\dfrac{2gR\ m^2}{M(M+m)}}$

$= \dfrac{1}{2}\dfrac{mM}{m+M}.(V_m + V_M)^2 = mgR$

$\Rightarrow V_M + V_m = \sqrt{2gR\dfrac{(M+m)}{M}}$

$\dfrac{V_m}{V_M} = \dfrac{M}{m}$ $\therefore V_m = \dfrac{M}{m+M}\sqrt{\dfrac{2gR(M+m)}{M}}$

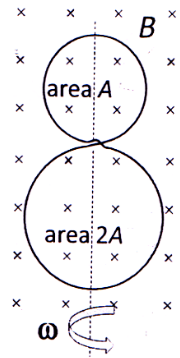
$= \sqrt{\dfrac{2gR(M)}{M+m}}$

$V_M = \dfrac{m}{M+m}\sqrt{\dfrac{2gR(M+m)}{M}}$

$= \sqrt{\dfrac{2gRm^2}{M}}$

7. A circular insulated copper wire loop is twisted to form two loops of area A and 2A as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper.

At t = 0, the loop starts rotating about the common diameter as axis with a constant angular velocity w in the magnetic field. Which of the following options is/are correct?



(A) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone
(B) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper
(C) The emf induced in the loop is proportional to the sum of the areas of the two loops
(D) The net emf induced due to both the loops is proportional to $\cos\omega t$

7. (AB)

E induced $= \frac{d}{dt}\left(-\bar{B}.\bar{A} + \bar{B}\times 2\bar{A}\right)$

$= \frac{d}{dt}\left(\bar{B}.\bar{A}\right) = \frac{d}{dt}\left(B.A\cos\omega t\right)$

$= -BA\omega\sin\omega t$, amplitude $= BA\omega$

E induced A $= \frac{d}{dt}\left(-B.A\right)$

$= +BA\omega\sin\omega t$, amplitude $= BA\omega$

E induced 2A $= \frac{d}{dt}\left(\bar{B}.2\bar{A}\right)$

$= -2BA\omega\sin\omega t$, amplitude $= 2BA\omega$

Also $\frac{d\phi}{dt}$ is maximum when $\sin\omega t = 1$

$\Rightarrow \omega t = \frac{\pi}{2}$

$\Rightarrow \bar{B} \perp \bar{A}$

E induced $\propto$ subtraction of the area $\propto \sin\omega t$

SECTION 2 (Maximum Marks: 15)

- This section contains **FIVE** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from -0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened

  Zero Marks : 0 In all other cases

8. A stationary source emits sound of frequency $f_0 = 492\,\text{Hz}$. The sound is *reflected* by a large car *approaching* the source with a speed of 2 ms^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound at the frequency *it* has received).

8. **(6)**

$f_0 = 492 Hz$

$f' = 492\left(\frac{330+2}{330}\right) = 492\left(\frac{332}{330}\right)$

$f'' = 492\left(\frac{332}{330}\right)\left\{\frac{330}{330-2}\right\}$

$= \frac{492 \times \cancel{332}^{\,83}}{330} \times \frac{\cancel{330}}{\cancel{328}_{\,82}}$

$= 6 \times 83$

$f'' = 498$

$\therefore$ Beat frequency $= 498 - 492$

$= 6$ A

9. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $\frac{V_i}{V_f} = 6.28$, then the *smallest possible* n_f is

9. **(5)**

$P.E = \frac{-27.2}{v^2} eV$

$\frac{V_i}{V_f} = \left(\frac{n_i}{n_f}\right)^2$

$6.25 = \left(\frac{n_i}{n_f}\right)^2$

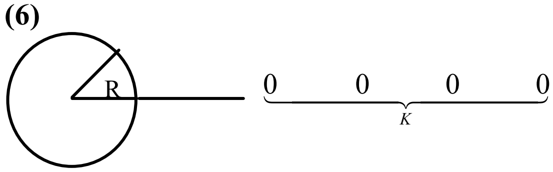
$\therefore \frac{n_f}{n_i} = 2.5$

$n_f = 2.5 \times n_i$

$\therefore n_f = 5 A$

10. A drop of liquid of radius R = 10^{-2}m having surface tension $S = \frac{0.1}{4\pi} Nm^{-1}$ divisible itself into K identical drops. If this process the total change in the surface energy $\Delta U = 10^{-3} J$

10. **(6)**



$R^3 = K.r^3$

$$\therefore r = \frac{R}{K^{1/3}}$$

$$U_i = T.4\pi R^2$$

$$U_f = T.4\pi R^2.K = T.4\pi\left\{\frac{R}{K^{1/3}}\right\}.K$$

$$= 4\pi R^2 T\ K^{1-2/3}$$

$$= 4\pi R^2 T\ K^{1/3}$$

$$\Delta U = U_f - U_i$$

$$= 4\pi R^2 T\left\{K^{1/3} - 1\right\}$$

$$10^{-3} = 4\pi \times 10^{-4} \times \frac{0.1}{4\pi}\left(K^{1/3} - 1\right)$$

$$10^{-3} = 10^{-5}\left(K^{1/3} - 1\right)$$

$$100 = K^{1/3} - 1$$

$$K^{1/3} = 101$$

$$K = (101)^3$$

$$= (100)^3 \approx 10^6$$

11. ^{131}I is an isotope of Iodine that β decays to an isotope of Xenon with a half – life of 8 days. A small amount of a serum labeled with ^{131}I is injected into the blood of a person. The activity of the amount of 131I injected was 2.4×10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of bleed is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in liters is approximately. (You may use $e^x \approx 1 + x$ for $|x| << 1$ and $\ln 2 \approx 0.7$)

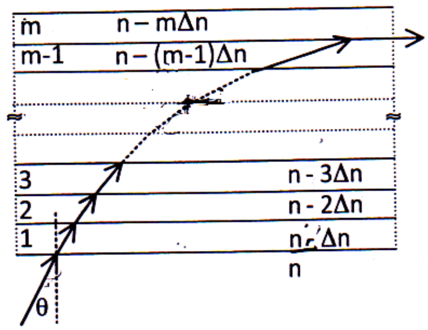
11. **(5)**

$$\frac{115 \times V}{2.5} = 2.A \times 10^5 e^{-\frac{0.7}{8\times 24}} = 11.5$$

$$V = \frac{2.4 \times 10^5 \times 2.5}{11.5} e^{-\frac{0.7\times 11.5}{8\times 24}}$$

= 5 litres

12. A monochromatic light is traveling in a medium of a refractive index n = 1.6. It enters a stack of glass layers from the bottom side at an angle $\theta = 30^0$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the m^{th} slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)^{th}$ and m^{th} slabs from the right side of the stack. What is the value of m?



12. **(8)**

$n\sin\theta = (n - m\Delta n)\sin 90^0$

$1.6 \times \frac{1}{2} = (1.6 - 0.1m)$

0.8 = 1.6 – 0.1 m

0.8 = 0.1 m

SECTION 3 (Maximum Marks: 18)

- This section contains **SIX** questions of matching type
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options [A], [B], [C], and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +3 If only the bubble corresponding to the correct option is darkened

  Zero Marks : 0 If none of the bubbles is darkened

  Negative Marks: –1 In all other cases

Answer Q.13, Q.14 and Q.15 by appropriately matching the information given in the three columns of the following table.

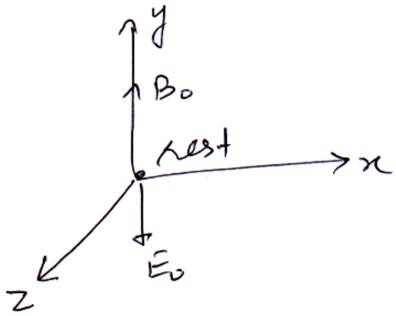
A charged particle (electron or proton) is introduced at the origin ($x = 0$, $y = 0$, $z = 0$) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in columns 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.

Column 1	Column 2	Column 3
(I) Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i) $\vec{E} = E_0\hat{z}$	(P) $\vec{B} = -B_0\hat{x}$
(II) Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii) $\vec{E} = -E_0\hat{y}$	(Q) $\vec{B} = B_0\hat{x}$
(III) Proton with $\vec{v} = 0$	(iii) $\vec{E} = -E_0\hat{x}$	(R) $\vec{B} = B_0\hat{y}$
(IV) Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv) $\vec{E} = E_0\hat{x}$	(S) $\vec{B} = B_0\hat{z}$

13. In which case would the particle move in a straight line along the negative direction of y – axis (i.e., move along $-\hat{y}$)?

[A] (III) (ii) (P) [B] (II) (iii) (Q) [C] (IV) (ii) (S) [D] (III) (ii) (R)

13. (D)

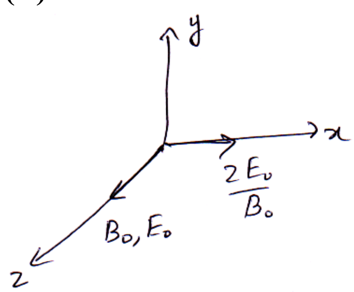


No magnetic force particle will move along –ve y axis

14. In which case will the particle describe a helical path with axis along the positive z direction?

[A] (II) (ii) (R) [B] (III) (iii) (P) [C] (IV) (ii) (R) [D](IV) (i) (S)

14. (D)

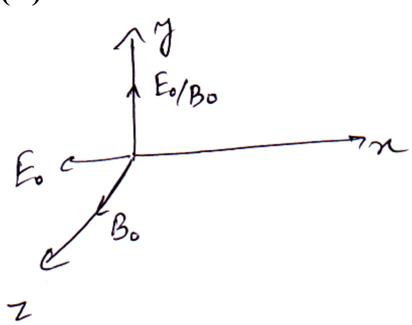


Superposition of circular motion parallel to xy plane + uniformly accelerated motion along z – axis

15. In which case will the particle move in a straight line with *constant* velocity?

[A] (II) (iii) (S) [B] (III) (ii) (R) [C] (IV) (i) (S) [D] (III) (iii) (P)

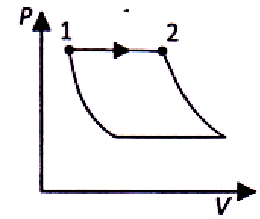
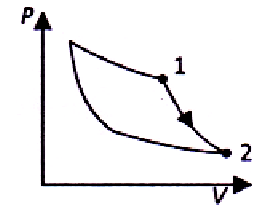
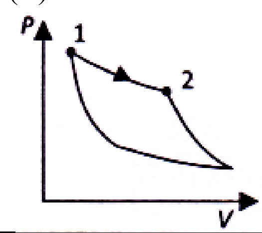
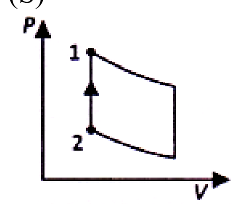
15. (A)



Electric and magnetic force balances each other so move with constant velocity.

Answer Q.16, Q.17 and Q.18 by appropriately matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding $P–V$ diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n.

Column 1	Column 2	Column 3
(I) $W_{1\to 2} = \frac{1}{\gamma - 1}\left(P_2V_2 - P_1V_1\right)$	(i) Isothermal	(P) 
(II) $W_{1\to 2} = -PV_2 + PV_1$	(ii) Isochoric	(Q) 
(III) $W_{1\to 2} = 0$	(iii) Isobaric	(R) 
(IV) $W_{1\to 2} = -nRT\ln\left(\frac{V_2}{V_1}\right)$	(iv) Adiabatic	(S) 

16. Which one of the following options correctly represents a thermodynamics process that is used as a correction in the determination of the speed of sound in an ideal gas?

[A] (I) (ii) (Q) [B] (I) (iv) (Q) [C] (IV) (ii) (R) [D] (III) (iv) (R)

16. (B)

Correction: Propagation of sound is adiabatic and not isothermal

$\Rightarrow work = \frac{P_2V_2 - P_1V_1}{\gamma - 1}$

$(Q) 1 \to 2$

Represents an adiabatic process

$\Rightarrow$ (I) (iv) (Q)

17. Which one of the following options is the correct combinations?

[A] (II) (iv) (P)   [B] (III) (ii) (S)   [C] (II) (iv) (R)   [D] (IV) (ii) (S)

17. **(B)**

Consider Process S:

Area under Pv curve = 0   (III)

$Volume(1 \to 2) = const$

$\Rightarrow$ isochoric   (ii)

$\Rightarrow$ (III) (ii) (S)

18. Which of the following options is the only correct representation of a process in which $\Delta V = \Delta Q - P\Delta V$

[A] (III) (iii) (P)   [B] (II) (iii) (P)   [C] (II) (iii) (S)   [D] (II) (iv) (R)

18. **(B)**

$\Delta U = \Delta Q - P\Delta V$

$w = P\Delta V$ when P = const

$w \neq 0$

$\Rightarrow$ (II) (iii) (P)

PART II : CHEMISTRY

SECTION 1 (Maximum Marks: 28)

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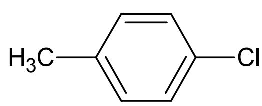
  Zero Marks : 0 If none of the bubbles is darkened

  Negative Marks : -2 In all other cases

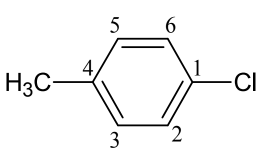
- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get -2 marks, as a wrong option is also darkened

19. The IUPAC name(s) of the following compound is(are)

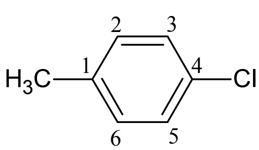


(A) 1-chloro-4-methylbenzene (B) 1-methy-l-4-chlorobenzene
(C) 4-chloroluene (D) 4-methylchlorobenzene

Sol. **(AC)**



1-Chloro-4-methylbenzene



4-Chlorotoluene

20. The correct statement(s) about the oxoacids, $HClO_4$ and HClO, is (are)

(A) The central atom in both $HClO_4$ and HClO is sp^3 hybridized

(B) The conjugate base of $HClO_4$ is weaker base than H_2O

(C) $HClO_4$ is formed in the reaction between Cl_2 and H_2O

(D) $HClO_4$ is more acidic than HClO because of the resonance stabilization of its anion

Sol. **(ABD)**

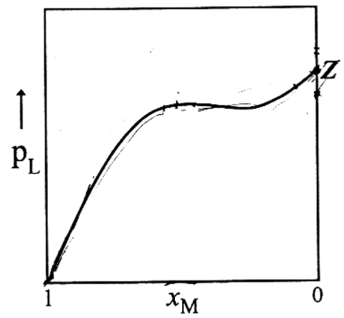
(A) The central atom in both $HClO_4$ and HClO is sp^3 hybridized

(B) The conjugate base of $HClO_4$ is weaker base than H_2O

(C) $Cl_2 + H_2O \rightarrow HCl + HClO$

(D) $HClO_4$ is more acidic than HClO because of the resonance stabilization of its anion

21. For a solution formed by mixing liquids **L** and **M**, the vapour pressure of **L** plotted against the mole fraction of **M** in solution is shown in the following figure. Here x_L and x_M represents mole fractions of **L** and **M**, respectively, in the solution. The correct statement(s) applicable to this system is(are)



(A) The point **Z** represents vapour pressure of pure liquid **M** and Raoult's law is obeyed when $x_L \rightarrow 0$

(B) The point **Z** represents vapour pressure of pure liquid **M** and Raoult's law is obeyed from $x_L = 0$ to $x_L = 1$

(C) Attractive intermolecular interactions between **L-L** in pure liquid **L** and **M-M** in pure liquid **M** are stronger than those between **L-M** when mixed in solution

(D) The point **Z** represents vapour of pure liquid **L** and Raoult's law in obeyed when $x_L \rightarrow 1$

Sol. **(CD)**

$\because$ Solution of L & M shows positive deviation from Raoult's Law

$\therefore$ Interaction between L-L in pure liquid 4

M-M in pure liquid M are stronger than L-M is mixture

At point $Z, x_M = 0$ or $x_L = 1$

$\therefore$ It represents V.P. of pure liquid L

22. Addition of excess aqueous ammonia to a pink coloured aqueous solution of $MCl_2 \cdot 6H_2O$ (**X**) and NH_4Cl gives an octahedral complex **Y** in the presents of air. In aqueous solution complex **Y** behaves as 1:3 electrolyte. The reaction of **X** with excess HCl at room temperature results in the formation of a blue coloured complex **Z**. The calculated spin only magnetic moment of **X** and **Z** is 3.87 B.M., whereas it is zero for complex **Y**.

Among the following options, which statement(s) is(are) correct?

(A) **Z** is tetrahedral complex

(B) The hybridization of the central metal ion in **Y** is d^2sp^3

(C) When **X** and **Z** are in equilibrium at 0^oC, the colour of the solution is pink

(D) Addition of silver nitrate to **Y** gives only two equivalents of silver chloride

Sol. **(ABC)**

$$\underset{(X)}{\left[Co(H_2O)_6\right]^{2+}Cl_2} \xrightarrow{O_2} \underset{(Y)}{\left[Co(NH_3)_6\right]^{3+}Cl_3}$$

$$\downarrow \text{excess HCl}$$

$$\underset{(Z)}{(COCl_4)^{2-}}$$

$X = \left[CO(H_2O)_6\right]Cl_2$

$Y = \left[CO(NH_3)_6\right]Cl_3$

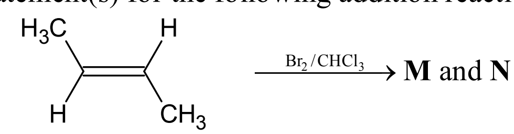
$Z = (COCl_4)^{2-}$

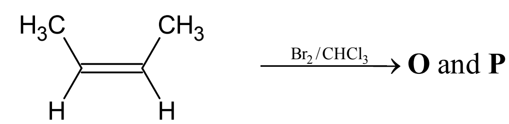
'Z' is tetrahedral complex

'Y' is d^2sp^3

When 'X' and Z are in Eq. at 0^oC the solution is pink

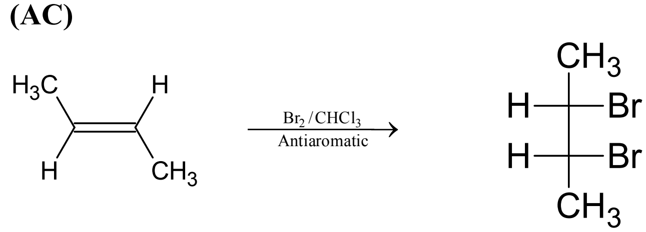
23. The correct statement(s) for the following addition reactions is (are)

(i) 

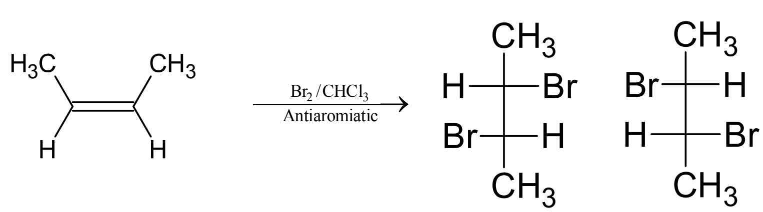
(ii) 

(A) Bromination proceeds through *trans*-addition in both the reactions
(B) **O** and **P** are identical molecules
(C) (**M** and **O**) and (**N** and **P**) are two pairs of diastereomers
(D) (**M** and **O**) and (**N** and **P**) are two pairs of enantiomers

Sol. **(AC)**



M & N $\Rightarrow$ Meso



O & P $\Rightarrow$ Racemic mixture

M & O $\Rightarrow$ Diastereomers

N & P $\Rightarrow$ Diastereomers

24. An ideal gas is expanded from (p_1, V_1, V_2) to (p_2, V_2, V_2) under different conditions. The correct statement(s) among the following is (are)

(A) The work done on the gas is maximum when it is compressed irreversibly from (p_2, V_2) to (p_1, V_1) against constant pressure p_1

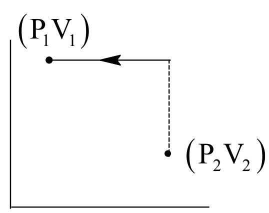
(B) The work done by the gas is less when it is expanded reversible from V_1 to V_2 under adiabatic conditions as compared to that when expanded reversible from V_1 to V_2 under isothermal conditions

(C) If the expansions is carried out freely, it is simultaneously both isothermal as well as adiabatic

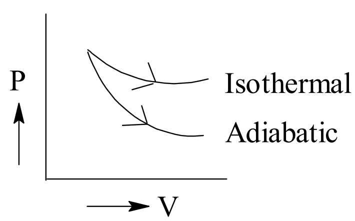
(D) The change in internal energy of the gas is (i) zero, if it is expanded reversible with $T_1 = T_2$, and (ii) positive, if it is expanded reversible under adiabatic conditions with $T_1 \neq T_2$

Sol. **(ABC)**

(A)



(B)



Work done is area under the graph which is more for isothermal & less for adiabatic

(C) $\Delta U = q + w$

For free expansion $w = 0$

For adiabatic $q = 0$

Hence $\Delta U = 0$

25. The colour of the X_2 molecules of group 17 elements changes gradually from yellow to violet down the group. This is due to

(A) the physical state of X_2 at room temperature changes from gas to solid down the group

(B) decrease in π^*-σ^* gas down the group

(C) decrease in HOMO-LUMO gap down the group

(D) decrease in ionization energy down the group

Sol. **(BC)**

Colour of X_2 molecules in GP. NO. 17 changes from yellow to violet down the group. Due to decrease in HOMO-LUMO gap down the group or decrease in π^*-σ^* gap down the group

SECTION 2 (Maximum Marks: 15)

- This section contains **FIVE** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from -0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened

  Zero Marks : 0 In all other cases

26. The conductance of a 0.0015 M aqueous solution of a weak monobasic acid was determined by using a conductivity cell consisting of platinized Pt electrodes. The distance between the electrodes is 120 cm with an area of cross section of 1 cm^2. The conductance of this solution was found to be $5\times10^{-7}S$. The pH of the solution is 4. The value of limiting molar conductivity $\left(\Lambda_m^o\right)$ of this weak monobasic acid in aqueous solution is $Z\times10^2 Scm^{-1}mol^{-1}$. The value of Z is

Sol. **(6)**

$$K = 5\times10^{-7}\times\frac{120}{1}$$

$$= 6\times10^{-5}\text{ S cm}^{-1}$$

$$\Lambda_m = 6\times10^{-5}\times\frac{1000}{1.5\times10^{-3}}$$

$$= 40\text{ s cm}^2\text{ mol}^{-1}$$

$$\left[H^+\right] = 10^{-4} = 1.5\times10^{-3}\alpha$$

$$\alpha = \frac{1}{15}$$

$$\Lambda_m^o = \frac{\Lambda_m}{\alpha} = 600$$

Ans. 6

27. The sum of the number of lone pairs of electrons on each central atom in the following species is

$$\left[TeBr_6\right]^{2-}, \left[BrF_2\right]^+, SNF_3 \text{ and } \left[XeF_3\right]^-$$

(Atomic number: $N = 7, F = 9, S = 16, Br = 35, Te = 52, Xe = 54$)

Sol. **(6)**

$\left[TeBr_6\right]^{2-} - 1$

$\left[BrF_2\right]^+ - 2$

$SNF_3 - 0$

$\left[XeF_3\right]^- - 3$

Ans. 6

28. A crystalline solid of a pure substance has a face-centred cubic structure with a cell edge of 40 pm. If the density of the substance in the crystal is 8 g cm^{-3}, then the number of atoms present in 256 g of the crystal is $N \times 10^{24}$. The value of N is

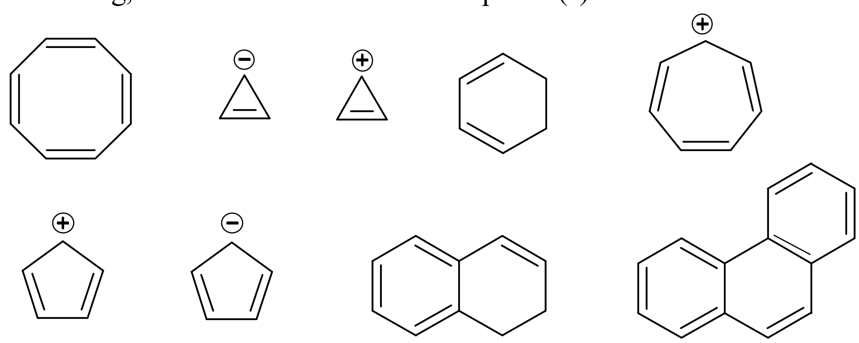
Sol. **(2)**

$$8 = \frac{4 \times \frac{256}{N \times 10^{24}}}{\left(400 \times 10^{-10}\right)^3}$$

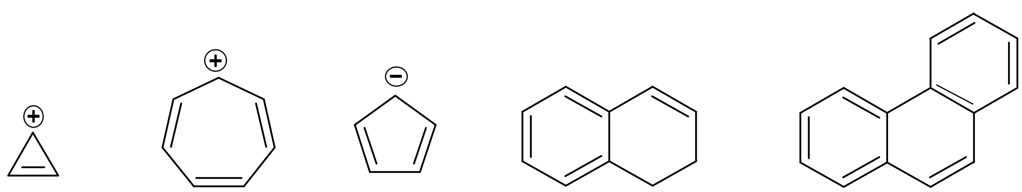
$$8 = \frac{4 \times 256}{64 \times N}$$

$$N = 4$$

29. Among the following, the number of aromatic compound(s) is



Sol. **(5)**



30. Among H, He_2^+, Li_2, Be_2, B_2, C_2, N_2, O_2^- and F_2, the number of diamagnetic species is

(Atomic numbers. H = 1, He = 2, Li = 3, Be = 4, B = 5, C = 6, N = 7, O = 8, F = 9)

Sol. **(6)**

di	H_2	σ_{1s}^2
para	He_2^+	$\sigma_{1s}^2\ \sigma_{1s}^{*1}$
di	Li_2	$\sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2$
di	Be_2	$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}$
para	B_2	$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\left(\pi_{2p_x}^1 = \pi_{2p_y}^1\right)$
di	C_2	$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\left(\pi_{2p_x}^2 = \pi_{2p_y}^2\right)$
di	N_2	$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\left(\pi_{2p_x}^2 = \pi_{2p_y}^2\right)\sigma_{2p}^2$
para	O_2^-	$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\sigma_{2p}^2\left(\pi_{2p_x}^2 = \pi_{2p_y}^2\right)\left(\pi_{2p_x}^2 = \pi_{2p_y}^1\right)$
di	F_2	$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\sigma_{2p_z}^2\left(\pi_{2p_x}^2 = \pi_{2p}^2\right)\left(\pi_{2p_x}^{*2} = \pi_{2p_y}^{*2}\right)$

SECTION 3 (Maximum Marks: 18)

- This section contains **SIX** questions of matching type
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options [A], [B], [C], and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

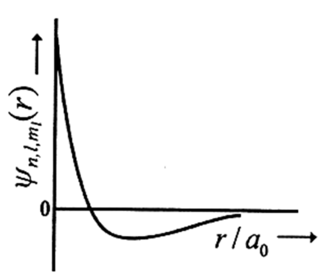
  Full Marks : +3 If only the bubble corresponding to the correct option is darkened

  Zero Marks : 0 If none of the bubbles is darkened

  Negative Marks: –1 In all other cases

Answer Q.31, Q.32 and Q.33 by appropriately matching the information given in the three column of the following table.

The waves function, ψ_{n,l,m_l} is mathematic function whose value depends upon spherical polar coordinates (r, θ, ϕ) of the electron and characterized by the quantum numbers n, l and m_l. Here r is distance from nucleus, θ is colatitudes and ϕ is azimuth. In the mathematic functions given in the table, Z is atomic number and a_0 is Bohr radius.

Column 1	Column 2	Column 3
(I) 1s orbital	(i) $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\left(\frac{Zr}{a_0}\right)}$	(P) 
(II) 2s orbital	(ii) One radial node	(Q) Probability density at nucleus $\propto \frac{1}{a_0^3}$
(III) $2p_z$ orbital	(iii) $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{5}{2}} e^{-\left(\frac{Zr}{2a_0}\right)} \cos\theta$	(R) Probability density is maximum at nucleus
(IV) $3d_z^2$ orbital	(iv) *xy*-plane is a nodal plane	(S) Energy needed to excite electron from $n = 2$ state to $n = 4$ state is $\frac{27}{32}$ times the energy needed to excite electron from $n = 2$ state to $n = 6$ state

31. For hydrogen atom, the only CORRECT combination is
(A) (II) (i) (Q) (B) (I) (iv) (R) (C) (I) (i) (S) (D) (I) (i) (P)

Sol. **(C)**

$\frac{1}{4}-\frac{1}{36}$ $\frac{9-1}{36}=\frac{8}{36}$

$= n-\ell-1$

$\frac{5}{2}$ $n-\ell=5$

$\mu=4$

$\left[\frac{1}{4}-\frac{1}{16}\right]=\frac{4-1}{16}=\frac{3}{16}$

$n-\ell=3$

For Hydrogen is orbital

$\psi \propto \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\left(\frac{Zr}{a_0}\right)}$

Probability density will be max. at nucleus.

32. For He^+ ion, the only **INCORRECT** combination is
(A) (I) (iii) (R) (B) (II) (ii) (Q) (C) (I) (i) (S) (D (I) (i) (R)

Sol. **(A)**

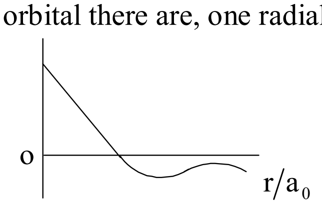
For He^+

33. For the given orbital in Column 1, the only CORRECT combination for any hydrogen-like species is
(A) (IV) (iv) (R) (B) (III) (iii) (P) (C) (II) (ii) (P) (D) (I) (ii) (S)

Sol. **(C)**

For Hydrogen like atom

For 2s orbital there are, one radial node.



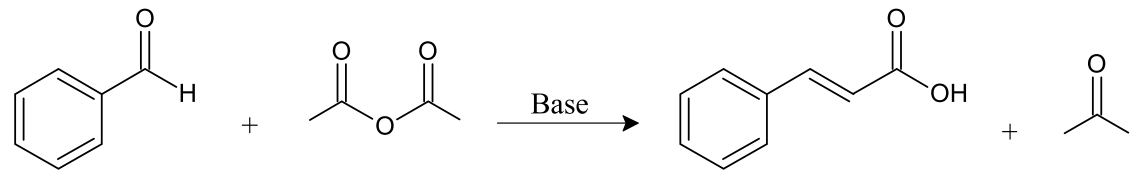
Answer Q.34, Q.35 and Q.36 by appropriately matching the information given in the three column of the following table.

Column1, 2 and 3 contain starting materials, reaction conditions, and type of reactions, respectively.		
Column 1	**Column 2**	**Column 3**
(I) Toluene	(i) $NaOH/Br_2$	(P) Condensation
(II) Acetophenone	(ii) $Br_2/h\nu$	(Q) Carboxylation
(III) Benzaldehyde	(iii) $(CH_3CO)_2O/CH_3COOK$	(R) Substitution
(IV) Phenol	(iv) $NaOH/CO_2$	(S) Haloform

34. The only CORRECT combination that gives two different carboxylic acids is

(A) (II) (iv) (R) (B) (III) (iii) (P) (C) (I) (i) (S) (D) (IV) (iii) (Q)

Sol. **(B)**



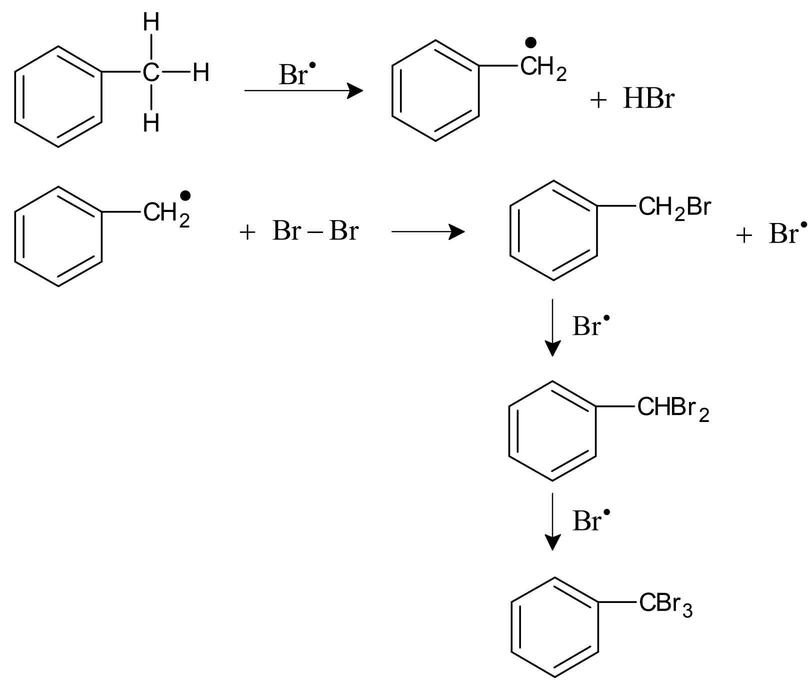
Perkin reaction

35. The only CORRECT combination in which the reaction proceeds through radical mechanism is

(A) (III) (ii) (P) (B) (I) (ii) (R) (C) (IV) (i) (Q) (D) (II) (iii) (R)

Sol. **(B)**

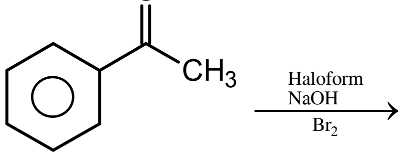
$Br - Br \rightarrow 2Br^{\bullet}$

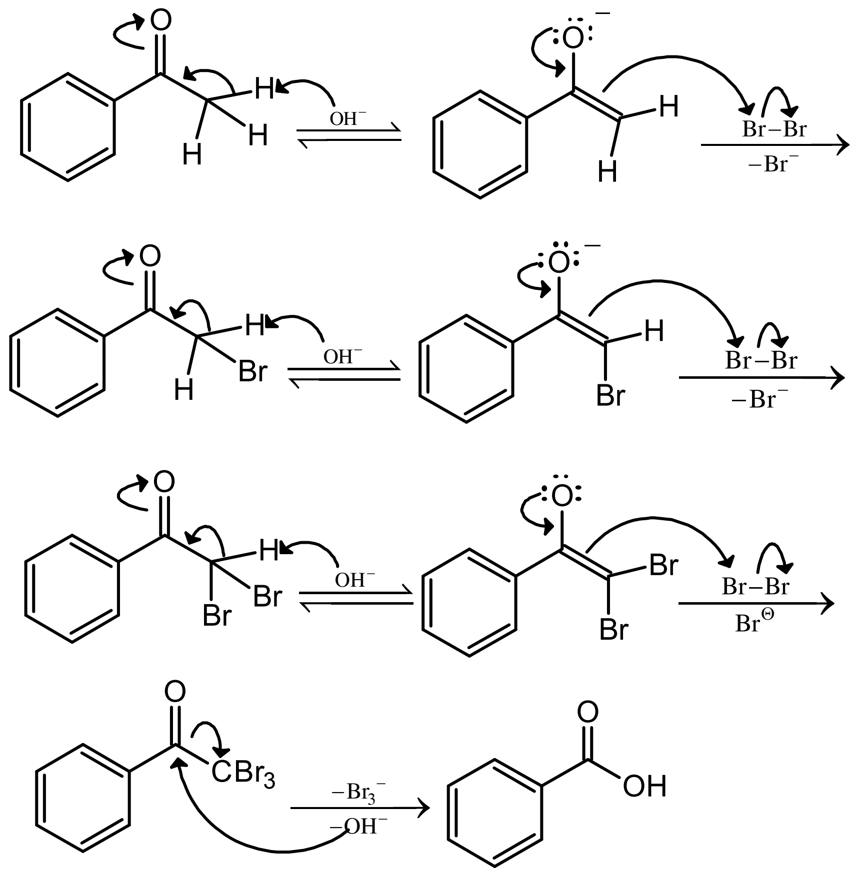


36. For the synthesis of benzoic, the only CORRECT combination is

(A) (I) (iv) (Q) (B) (III) (iv) (R) (C) (IV) (ii) (P) (D) (II) (i) (S)

Sol. **(D)**





O
H
H
H
OH⁻
:Ö:⁻
H
H
Br–Br
–Br⁻
O
H
Br
H
OH⁻
:Ö:⁻
H
Br
Br–Br
–Br⁻
O
H
Br
Br
OH⁻
:Ö:
Br
Br
Br–Br
BrΘ
O
CBr3
–Br3⁻
–OH⁻
O
OH

PART III : MATHEMATICS

SECTION 1 (Maximum Marks: 28)

- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories:

Full Marks	: +4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
Partial Marks	: +1	For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened
Zero Marks	: 0	If none of the bubbles is darkened
Negative Marks :		-2 In all other cases

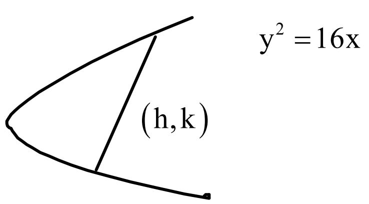
- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get -2 marks, as a wrong option is also darkened

37. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is (are) possible value (s) of p, h and k?

(A) $p = -1, h = 1, k = -3$
(B) $p = 5, h = 4, k = -3$
(C) $p = -2, h = 2, k = -4$
(D) $p = 2, h = 3, k = -4$

Answer: (D)

Sol:



If mid point is (h, k)

Then equations of chord is

$T = S_1$

$yk - 8(x + h) = k^2 - 16h$

$ky - 8x - 8h - k^2 + 16h = 0$

$-8x + ky + 8h - k^2 = 0$

Compare $2x + y - p = 0$

$-4 = \frac{k}{1} = \frac{k^2 - 8h}{p}$

$\Rightarrow k = -4$

$k^2 - 8h = -4p$

$4 - 2h = -p$

$p = 2h - 4$

38. Let a, b, x and y be real number such that $a-b=1$ and $y \neq 0$. If the complex number $z=x+iy$ satisfies $\operatorname{Im}\left(\frac{az+b}{z+1}\right)=y$, then which of the following is (are) possible value (s) of x?

(A) $1-\sqrt{1+y^2}$ (B) $-1+\sqrt{1-y^2}$ (C) $1+\sqrt{1+y^2}$ (D) $-1-\sqrt{1-y^2}$

Answer: (B,D)

Sol: $\frac{(1+b)z+b}{z+1}$

$\frac{z+b(z+1)}{(z+1)}$

$\Rightarrow b+\frac{z}{z+1}$

$\operatorname{Im}\left(b+\frac{z}{z+1}\right)=y$

Hence

$\operatorname{Im}\left(\frac{z}{z+1}\right)=y$

$\operatorname{Im}\left(\frac{z+1-1}{z+1}\right)=y$

$\operatorname{Im}\left(\frac{-1}{z+1}\right)=y$

$-y=-\frac{1}{z+1}=\frac{(x+1)-iy}{(x+1)^2+y^2}$

$x=-1\pm\sqrt{1-y^2}$

39. Let X and Y be two events such that $P(X)=\frac{1}{3}, P(X|Y)=\frac{1}{2}$ and $P(Y|X)=\frac{2}{5}$. Then

(A) $P(X\cup Y)=\frac{2}{5}$ (B) $P(Y)=\frac{4}{15}$ (C) $P(X'|Y)=\frac{1}{2}$ (D) $P(X\cap Y)=\frac{1}{5}$

Answer: (B,C)

Sol: $P(x)=\frac{1}{3}$

$P\left(\frac{x}{y}\right)=\frac{p(x\cap y)}{p(y)}=\frac{1}{2}$

$p\left(\frac{y}{x}\right)=\frac{p(x\cap y)}{p(x)}=\frac{2}{5}$

$\Rightarrow p(x\cap y)=\frac{2}{5}\times p(x)$

$\Rightarrow P(x\cap y)=\frac{2}{5}\times\frac{1}{3}=\frac{2}{15}$

Also, $p(y)=2p(x\cap y)$

$=\frac{4}{15}$

$p(x\cup y)=p(x)+p(y)-p(x\cap y)$

$=\frac{1}{3}+\frac{4}{15}-\frac{2}{15}$

$=\frac{1}{3}+\frac{2}{15}=\frac{7}{15}$

$p\left(\frac{x'}{y}\right)=\frac{p(\bar{x}\cap y)}{p(y)}=\frac{p(y)-p(x\cap y)}{p(y)}$

$=1-\frac{2}{15}\times\frac{15}{4}=\frac{1}{2}$

40. If $2x-y+1=0$ is a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{16}=1$, then which of the following CANNOT be sides of a right angled triangle?

(A) $2a,4,1$ (B) $a,4,1$ (C) $a,4,2$ (D) $2a,8,1$

Answer: (B,C,D)

Sol: $y=2x+1$

$y=mx+\sqrt{a^2m^2-b^2}$

$a^2m^2-b^2=1$

$a^2\times 4=17$

$a=\pm\frac{\sqrt{17}}{2}$

(a) $2a,4,1$ $4a^2=17$

(b) $a,4,1$

(c) $a,4,2$

(d) $2a,8,1$

41. Let $[x]$ be the greatest integer less than or equals to x. Then, at which of the following point (s) the function $f(x)=x\cos(\pi(x+[x]))$ is discontinuous?

(A) $x=2$ (B) $x=0$ (C) $x=1$ (D) $x=-1$

Answer: (A,C,D)

$f(x)x\cos\pi(x+[x])$

Disc

$x=2$ $f(2)=2$

$x=2^+$ $f(2^+)=2$

$x=2^-$ $f(2^-)=2$

Disc

$x=0$ $f(0)=0$

$x = 0^+$ $f(0^+) = 0$

$x = 0^-$ $f(0^-) = 0$

Disc

$x = 1$ $f(1) = 1$

$x = 1^+$ $f(1^+) = 1$

$x = 1^-$ $f(1^-) = -1$

Disc

$x = -1$ $f(-1) = -1$

$x = -1^+$ $f+(-1) = 1$

$x = -1^-$ $f(-1^-) = +1$

Sol: $f(x) = x\cos(\pi(x+[x]))$

$\cos\pi(x+[x])$ is discontinuous at $x \in I$ but $f(x) = x\cos(\pi(x+[x]))$

May be continuous at $x = 0$

$f(0^+) = f(0^-) = f(0) = 0$

$\Rightarrow$ continuous only at $x = 0$

42. Which of the following is (are) NOT the square of a 3×3 matrix with real entries?

(A) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&-1\end{bmatrix}$ (B) $\begin{bmatrix}1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}$ (C) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ (D) $\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}$

Answer: (A,D)

Sol: (A) $|A| = -1$

So if $A = B^2$

And $|B^2| = 1$ this is a contradiction

(B) $\begin{pmatrix}1&0&0\\0&1&1\\0&-2&-1\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&1\\0&-2&-1\end{pmatrix} = \begin{pmatrix}1&0&0\\0&-1&0\\0&0&-1\end{pmatrix}$

Hence (B) is square of a matrix

(C) Note that $I^2 = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

(D) $|A| = -1$

So $A \neq B^2$

43. Let $f : R \to (0,1)$ be a continuous function. Then, which of the following function (s) has (have) the value zero at some point in the interval $(0,1)$?

(A) $f(x) + \int_0^{\frac{\pi}{2}} f(t)\sin t\,dt$ (B) $e^x - \int_0^x f(t)\sin t\,dt$

(C) $x-\int_0^{\frac{\pi}{2}-x} f(t)\cos t\,dt$ (D) $x^9-f(x)$

Answer: (C,D)

Sol: (a) $\therefore \int_0^{\frac{\pi}{2}} f(t)\sin t\,dt =$

$k\int_0^{\frac{\pi}{2}} \sin t\,dt = k(-\cos t)_0^{\frac{\pi}{2}}$ (because $f(x)$ is constants functions)

$= -k(0-1) = k \neq 0$

(b) $e^x - \int_0^x f(t)\sin t\,dt = 0$

$\Rightarrow e^x - f(x)\sin x = 0$

$f(x) = e^x \operatorname{cosec} x$

$f(0) = \rightarrow \infty$ $f\left(\frac{\pi}{2}\right) = e^{\frac{\pi}{2}}$

(e) $x - \int_0^{\frac{\pi}{2}} f(t)\cos t\,dt = 0$

$1 - f\left(\frac{\pi}{2} - x\right)\sin x(-1) = 0$

$\Rightarrow f\left(\frac{\pi}{2} - x\right) = \operatorname{cosec} x$

$f(x) = -\sec x$

$f(0) = -1$

$f\left(\frac{\pi}{2}\right) = \infty$

$y = x^9 - f(x)$

$(0) = -f(0)$

$y(1) = 1 - f(x)$

SECTION 2 (Maximum Marks: 15)

- This section contains **FIVE** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from -0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened

  Zero Marks : 0 In all other cases

44. For a real number α, if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Of linear equations, has infinitely many solutions, then $1+\alpha+\alpha^2=$

Answer: (1)

Sol: $\begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha & \alpha & 1 \end{vmatrix}=0$

$\Rightarrow 1-\alpha^2-\alpha^2+\alpha^4+\alpha^4-\alpha^4=0$

$\alpha^4-2\alpha^2+1=0$

$\left(\alpha^2-1\right)^2=0$

$\alpha=\pm1$

For $\alpha=1$, No solution

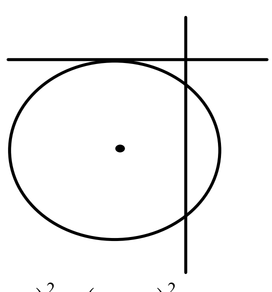
For $\alpha=-1$, infinite solution

$1+\alpha+\alpha^2=1-1+1=1$

45. For how many values of n, the circle $x^2+y^2+2x+4y-p=0$ and the coordinate axes have exactly three common points?

Answer: (2)

Sol:



For $(x+1)^2+(y+2)^2=p+5$

At $p=0$, if passes through origin

$r=2$

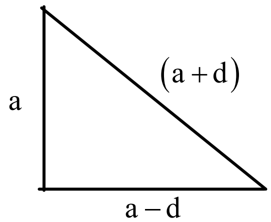
$p+5=4$

$p=-1$

46. The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24 then what is the length of its smallest side?

Answer: (6)

Sol: $(a^2)+(a-d)^2=(a+d)^2$



$\Rightarrow a^2=(a+d)^2-(a-d)^2$

$a^2=4ad$

$\Rightarrow a=4d$

Area $=\frac{1}{2}a(a-d)=24$

$\Rightarrow 4d\times 3d=48$

$d^2=4$

$d=2$

$a=8$

Smallest side $=6$

47. Words of length 10 are formed using the letters A,B,C,D,E,F,G,H,I,J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x}=$

Answer: (5)

Sol: $x=10!$

$y={}^{10}C_1.\frac{10!}{2!}=5\times 10!$

So, $\frac{y}{9x}=\frac{9\times 5\times 10!}{9\times 10!}=5$

48. Let $f:R\to R$ be a differentiable function such that $f(0)=0, f\left(\frac{\pi}{2}\right)=3$ and $f'(0)=1$. If

$$g(x)=\int_x^{\frac{\pi}{2}}\left[f'(t)\operatorname{cosec} t-\cot t\operatorname{cosec} t\, f(t)\right]dt$$

For $x\in\left(0,\frac{\pi}{2}\right]$, then $\lim_{x\to 0} g(x)=$

Answer: (2)

Sol: $f(0)=0, f\left(\frac{\pi}{2}\right)=3$

$f'(0)=1$

$$g(x)=\int_x^{\pi/2}\left[f'(t).\operatorname{cosec}t-\cot t.\operatorname{cosec}t+f(t)\right]dt$$

$$=\int_x^{\pi/2}d\left(f(t).\operatorname{cosec}t\right)$$

$$f(t)\operatorname{cosec}t\Big|_x^{\pi/2}$$

$$f\left(\frac{\pi}{2}\right)-f(x).\operatorname{cosec}x$$

$$\underset{x\to0^+}{\text{Lt}}\,g(x)=\underset{x\to0^+}{\text{Lt}}\left(3-\frac{f(x)}{\sin x}\right)$$

$$=3-\underset{x\to0^+}{\text{Lt}}\frac{f'(x)}{\cos x}$$

$$=3-\frac{1}{1}=3-1=2$$

SECTION 3 (Maximum Marks: 18)

- This section contains **SIX** questions of matching type
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options [A], [B], [C], and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +3 If only the bubble corresponding to the correct option is darkened

  Zero Marks : 0 If none of the bubbles is darkened

  Negative Marks: –1 In all other cases

Answer Q 49, Q 50 and Q 51 by appropriately matching the information given in the three columns of the following table

Column 1,2 and 3 contains conics, equations of tangents to the conics and points of contact, respectively		
Column I	Column 2	Column 3
(I) $x^2+y^2=a^2$	(i) $my=m^2x+a$	(P) $\left(\frac{a}{m^2},\frac{2a}{m}\right)$
(II) $x^2+a^2y^2=a^2$	(ii) $y=mx+a\sqrt{m^2+1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}},\frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2=4ax$	(iii) $y=mx+\sqrt{a^2m^2-1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}},\frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2-a^2y^2=a^2$	(iv) $y=mx+\sqrt{a^2m^2+1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}},\frac{-1}{\sqrt{a^2m^2-1}}\right)$

49. If a tangent to a suitable conic (Column 1) is found to be $y = x + 8$ and its point of contact is $(8,16)$, then which of the following options is the only CORRECT combination?

(A) $(\text{III})(\text{ii})(\text{Q})$ (B) $(\text{III})(\text{i})(\text{P})$ (C) $(\text{II})(\text{iv})(\text{R})$ (D) $(\text{I})(\text{ii})(\text{Q})$

Answer: (B)

Sol: $y = x + 8$

$m = 1, (x_1, y_1) = (8,16)$

For $y^2 = 4ax$

Tangent is

$yy_1 = 2a(x + x_1)$

$m = \frac{2a}{y_1} = \frac{2a}{16} = 1 \Rightarrow a = 8$

For $a = 8$ and $m = 1$

$my = m^2x + a$ gives $y = x + 8$

and $\left(\frac{a}{m^2}, \frac{2a}{m}\right) = (8,16)$

50. For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact $(-1,1)$, then which of the following options is the only CORRECT combination for obtaining its equation?

(A) $(\text{I})(\text{ii})(\text{Q})$ (B) $(\text{I})(\text{i})(\text{P})$ (C) $(\text{II})(\text{ii})(\text{Q})$ (D) $(\text{III})(\text{i})(\text{P})$

Answer: (A)

Sol: For $(-1,1)$ and $a = \sqrt{2}$

Must be $x^2 + y^2 = 2$

Equation of tangent is

$x(-1) + y(1) = 2$

$\Rightarrow y = x + 2 \Rightarrow m = 1$

$\therefore y = x + \sqrt{2}\sqrt{1^2 + 1} \Rightarrow y = x + 2$

Is the tangent

$\therefore (\text{I})(\text{ii})(\text{Q})$

51. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only CORRECT combination?

(A) $(\text{IV})(\text{iii})(\text{S})$ (B) $(\text{II})(\text{iv})(\text{R})$ (C) $(\text{II})(\text{iii})(\text{R})$ (D) $(\text{IV})(\text{iv})(\text{S})$

Answer: (B)

Sol: Point is $(x_1, y_1) = \left(\sqrt{3}, \frac{1}{2}\right)$

and equation of tangent is $3x + 2y = 4$

So, for conic –II i.e. $x^2 + a^2y^2 = a^2$

i.e. $\frac{x^2}{a^2} + \frac{y^2}{1} = 1 \rightarrow$ equation of tangent at $\left(\sqrt{3}, \frac{1}{2}\right)$ will be $\frac{x\sqrt{3}}{a^2} + y.\frac{1}{2} = 1$ ______(1)

and $\left(\sqrt{3},\frac{1}{2}\right)$ should also i.e. on $\frac{x^2}{a^2}+\frac{y^2}{1}=1$ giving us $a^2=4$

Putting $a^2=4$ in (1); we get

$x\sqrt{3}+2y=4$ $\rightarrow$ which is the given equation of tangent

So, conic is $\frac{x^2}{a^2}+\frac{y^2}{1}=1$

Equation of tangent $\Rightarrow y=mx+\sqrt{a^2m^2+1}$

& point of contact $=\left(\frac{-a^2m}{\sqrt{a^2m^2+1}},\frac{1}{\sqrt{a^2m^2+1}}\right)$

Answer Q 52, Q 53 and Q 54 by appropriately matching the information given in the three columns of the following table

Let $f(x)=x+\log_e x-x\log_e x, x\in(0,\infty)$

Column 1 contains information about zeros of $f(x), f'(x)$ and $f''(x)$

Column 2 contains information about the limiting behavior of $f(x), f'(x)$ and $f''(x)$ at infinity.

Colum 3 contains information about increasing/ decreasing nature of $f(x)$ and $f'(x)$

Column I	Column 2	Column 3
(I) $f(x)=0$ for some $x\in(1,e^2)$	(i) $\lim_{x\to\infty} f(x)=0$	(P) f is increasing in $(0,1)$
(II) $f'(x)=0$ for some $x\in(1,e)$	(ii) $\lim_{x\to\infty} f(x)=-\infty$	(Q) f is decreasing in (e,e^2)
(III) $f'(x)=0$ for some $x\in(0,1)$	(iii) $\lim_{x\to\infty} f'(x)=-\infty$	(R) f' is increasing in $(0,1)$
(IV) $f''(x)=0$ for some $x\in(1,e)$	(iv) $\lim_{x\to\infty} f''(x)=0$	(S) f' is decreasing in (e,e^2)

52. Which of the following options is the only CORRECT combination?

(A) (IV)(iv)(S) (B) (II)(ii)(Q) (C) (I)(i)(P) (D) (III)(iii)(R)

Answer: (B)

53. Which of the following options is the only INCORRECT combination?

(A) (II)(iii)(P) (B) (II)(iv)(Q) (C) (I)(iii)(P) (D) (III)(i)(R)

Answer: (D)

54. Which of the following options is the only CORRECT combination?

(A) (II)(iii)(S) (B) (I)(ii)(R) (C) (III)(iv)(P) (D) (IV)(i)(S)

Answer: (A)

Sol 52, 53, 54

$f(x)=x+\ln x-x\ln x$

$=1-(\ln x-1)(x-1)$

$f(1)=1>0$

$f(e^2)=2-e^2<0$

$f(1)|(e^2) < 0 \Rightarrow f|x| = 0$ for some $x \in (1, e^2)$

(I) is correct

$f'(x) = 1 + \frac{1}{x} - (\ln x + 1)$

$= \frac{1}{x} - \ln x$

$f'(1) = 1 > 0$

$f'(e) = \frac{1}{e} - 1 < 0$ (II is correct)

$\lim_{x \to 0^+} f'(x) \to \infty$ So (III is wrong)

$f''(x) = -\frac{1}{x^2} - \frac{1}{x}$

$= \frac{-(x+1)}{x^2}$ roots $x = -1$

No root between $(1, e)$

(IV) is correct

$f'(x) = \frac{1}{x} - \ln x$

For $x \in (0,1)$

$\frac{1}{x} > 0$

$-\ln x > 0$

So

$f'(x) > 0$

(P) correct of increasing in $(0,1)$

For $x > e$

$\frac{1}{x} < \frac{1}{e}$

$\ln x > 1$

So, $f'(x)$ will be negative

$f(x)$ is decreasing in (e, e^2)

(Q) is correct

$f''(x) = -\frac{1}{x^2} - \frac{1}{x} = -\frac{(x+1)}{x^2} < 0$ for $x > -1$

If $x \in (0,1)$

$\therefore$ $f'(x)$ is decreasing in $(0,1)$

(R)is false

(S) is True