ACE OF PACE OBJECTIVE SECTION
(SOLUTION)

1. **(A)**

$2^{17}-2^{13}=2^{13}\left(2^{4}-1\right)$

$=2^{13}\times 15$

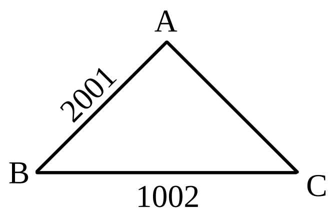
Clearly divisible by 15 $\Rightarrow$ reminder 0.

2. **(B)**

Earth complete 360^{o} in 24 hours i.e. 1440 mins

$\therefore$ 130^{o} in 520 mins.

3. **(C)**



$AC>2001-1002=999$

$AC<2001+1002=3003$

$\therefore$AC takes values 1000 to 3002 i.e., 2003 values

$\therefore$the number of triangles is 2003.

The answer is (c)

4. **(B)**

$LCM\left(a,b\right)=2\times 2\times 2\times 3=24$

$LCM\left(b,c\right)=2\times 2\times 3\times 5=60$

$LCM\left(c,a\right)=2\times 2\times 2\times 5=40$

$\therefore LCM\left(a,b,c\right)=2\times 2\times 2\times 3\times 5=120$

5. **(B)**

Let $x=\sqrt{6+\sqrt{6+\sqrt{6}}}$........squaring both

$x^{2}=6+\sqrt{6+\sqrt{6+\sqrt{6}}}$ ……$\Rightarrow x^{2}=6+x$

$\Rightarrow x^{2}-x-6=0\Rightarrow\left(x-3\right)\left(x+2\right)=0\Rightarrow x=-2,3$

As – 2 is neglected so $x=3$

6. **(A)**

The number of logs in

1^{st} row = 20

2^{nd} row = 19

3^{rd} row = 18 obviously, the numbers 20, 19, 18, ……., are in A. P., such that $a=20$

$d = 19 - 20 = -1$

Let the numbers of rows be n.

$\therefore S_n = 200$

Now, $S_n = \frac{n}{2}\left[2(20)+(n-1)\times(-1)\right] \Rightarrow 200 = \frac{n}{2}\left[40-(n-1)\right]$

$\Rightarrow 2\times 200 = n\times 40 - n(n-1)$

$\Rightarrow 400 = 40n - n^2 + n \Rightarrow n^2 - 41n + 400 = 0$

$\Rightarrow n^2 - 16n - 25n + 400 = 0$

$\Rightarrow n(n-16) - 25(n-16) = 0$

$\Rightarrow (n-16)(n-25) = 0$

Either

$n-16=0 \Rightarrow n=16$ or $n-25=0 \Rightarrow n=25$

$T_n = 0 \Rightarrow a+(n-1)d = 0 \Rightarrow 20+(n-1)\times(-1) = 0$

$\Rightarrow n-1 = 20 \Rightarrow n = 21$ i.e., 21^{st} term becomes 0

$\therefore n = 25$ is not required

$\therefore$Number of rows $=16$

Now, $T_{16} = a+(16-1)d = 20+15\times(-1) = 20-15 = 5$

$\therefore$Number of logs in the 16^{th} (top) row is 5.

7. **(A)**

$\angle A = 180^0 - (70^0 + 50^0) = 60^0$

$\frac{BD}{DC} = \frac{AB}{AC}$ means AD is the bisector of $\angle A$

$\therefore \angle BAD = \frac{1}{2}\times 60^0 = 30^0$

8. **(B)**

In ΔBAC and ΔADC, we have

$\angle BAC = \angle ADC = 90^0$

And $\angle ACB = \angle DCA = \angle C$

$\therefore \Delta BAC \sim \Delta ADC$ [AA similarity]

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DAC)} = \frac{BC^2}{AC^2} = \frac{(13)^2}{(5)^2} = \frac{169}{25}$

9. **(C)**

In figure $\angle DAC = 180^0 - (25^0 + 130^0) = 25^0$

$\therefore$ AD is the bisector of $\angle BAC$

By angle bisector theorem $\frac{AB}{AC} = \frac{BD}{DC}$

$\Rightarrow \frac{15}{9} = \frac{x}{6} \Rightarrow x = 10\,cm$

10. **(C)**

Let the required ratio be $k:1$

Then, $2=\frac{6k-4\times1}{k+1}\Rightarrow k=\frac{3}{2}$

$\therefore$ The required ratio is $\frac{3}{2}:1$ that is 3 : 2

Also, $y=\frac{3\times3+2\times3}{3+2}=3$

11. **(D)**

The co – ordinates of A are (0, 0) and co – ordinates of B are (a, b). Let the co – ordinates of C be $(x,0)$. Area of $\Delta ABC=20$

$\Rightarrow\frac{1}{2}\left[a(0-0)+x(0-b)\right]=20$

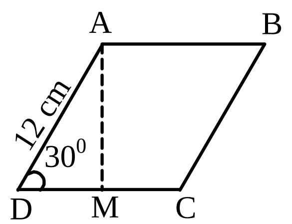
$\Rightarrow|-bx|=40$ $\qquad\Rightarrow x=\frac{40}{b}$

Co – ordinates of C are $\left(\frac{40}{b},0\right)$

12. **(C)**

In $\Delta ADM,\sin30^0=\frac{AM}{AD}$

$\Rightarrow\frac{1}{2}=\frac{AM}{12}\Rightarrow AM=6\,cm$



Area of parallelogram ABCD =CD $\times$AM

$\therefore CD\times AM=60\left[\because \text{Area of } ||^{gm}\ 60\,cm^2,\text{given}\right]$

$\Rightarrow CD\times6=60\Rightarrow CD=10cm$

13. **(B)**

We, have $AR=AP$ and $CR=CQ$

$\therefore OQ=BP$ [radius of the circle]

$AP=AB-PB, AP=8-x$

And $CQ=BC-BQ=6-x$

In $AC=AR+RC=8-x+6-x=14-2x$

Now in ΔABC

$AC^2=AB^2+BC^2$

$(14-2x)^2=(8)^2+(6)^2$

$196-56x+4x^2=64+36$

$\Rightarrow 4x^2-56x+96=0$

$\Rightarrow x^2-14x+24=0$

$(x-12)(x-2)=0$

$x=2, x=12, \text{but } x\neq 12$

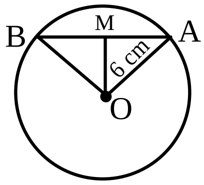
$\therefore x=2\,\text{cm}$

14. (C)

AB is a chord of length $=16\,\text{cm}$

$OM=6\,\text{cm}$

In ΔOMA



$OA^2=OM^2+AM^2$

$\Rightarrow OA^2=6^2+8^2$

$\left[\because AM=\frac{1}{2}AB=\frac{1}{2}\times 16=8\,\text{cm}\right]$

$\Rightarrow OA^2=36+64=100$

$\Rightarrow OA=10\,\text{cm}$

15. (A)

$C_1:C_2=2:3=2\pi r_1:2\pi r_2=2:3$

$\Rightarrow r_1:r_2=2:3$

Now, $r_1^2:r_2^2=\left(\frac{2}{3}\right)^2=4:9$

16. (C)

Given that $r:h=5:7$

$\Rightarrow h=\frac{7r}{5}$

Volume $=550\,\text{cm}^3$

$\Rightarrow \pi r^2h=550 \Rightarrow \frac{22}{7}.r^2.\frac{7r}{5}=550$

$\Rightarrow r^3=125 \Rightarrow r=5\,\text{cm}$

17. (A)

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$= \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} \times \frac{\left(\sqrt{10}-\sqrt{3}\right)}{\left(\sqrt{10}-\sqrt{3}\right)} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} \times \frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}$$

$$= \frac{7\sqrt{3}\left(\sqrt{10}-\sqrt{3}\right)}{7} - 2\sqrt{5}\left(\sqrt{6}-\sqrt{5}\right) - \frac{3\sqrt{2}}{-3}\left(\sqrt{15}-3\sqrt{2}\right)$$

$$= \frac{7\sqrt{30}-21}{7} - 2\sqrt{30} + 10 + \sqrt{30} - 6$$

$$= \sqrt{30} - 3 - \sqrt{30} + 4 = 1$$

18. **(A)**

We make the denominator same in each fraction by taking L. C. M. of 9, 11, 13.

$$\therefore \frac{7}{9} = \frac{7}{9} \times \frac{11\times13}{11\times13} = 1001/\left(9\times11\times13\right)$$

$$\frac{9}{11} = \frac{9}{11} \times \frac{9\times13}{9\times13} = 1053/\left(9\times11\times13\right)$$

$$\frac{11}{13} = \frac{11}{13} \times \frac{9\times11}{9\times11} = 1089/\left(9\times11\times13\right)$$

$$\therefore \frac{11}{13} > \frac{9}{11} > \frac{7}{9}$$

19. **(B)**

Let the angle be x.

Complement of $x = \left(90^0 - x\right)$

Since the difference is 14^0 we have

$$x - \left(90^0 - x\right) = 14^0$$

$$\Rightarrow 2x = 104^0 \Rightarrow x = 52^0$$

20. **(A)**

Given α, β are the roots of $x^2 + bx + c = 0$

$$\Rightarrow \alpha + \beta = \frac{-b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

$\therefore$ equation whose roots are $\frac{-b}{a}$ and $\frac{c}{a}$

Is $x^2 - x\left(\frac{-b}{a} + \frac{c}{a}\right) + \left(\frac{-b}{a}\right)\left(\frac{a}{c}\right) = 0$

$$\Rightarrow x^2 - x\left(\frac{-b+c}{a}\right) + \left(\frac{-bc}{a^2}\right) = 0$$

$$\Rightarrow a^2x^2 + a\left(b-c\right)x - bc = 0$$

$\therefore$ answer is (a)

21. **(D)**

$$\frac{2^{n+4}-2\left(2^{n}\right)}{2\left(2^{n+3}\right)}=\frac{2^{n+4}-2^{n+1}}{2^{n+4}}=1-\frac{2^{n+1}}{2^{n+4}}$$

$$=1-2^{n+1-n-4}=1-2^{-3}=1-\frac{1}{8}=\frac{7}{8}$$

Trick: Put n = 0

22. **(B)**

$25^{x-1}=5^{2x-1}-100\left(\text{given}\right)$

Or, $5^{2(x-1)}=2^{2x-1}-100$

Or, $5^{2x-1}-5^{2x-2}=100$

Only $x=2$ satisfy above equation.

23. **(A)**

Consider $a^2:b^2$

Adding ab on both sides, we get

$a^2+ab:b^2+ab=a\left(a+b\right):b\left(a+b\right)=a:b$

24. **(B)**

In figure, the points on the line are $\left(-1,-2\right),\left(0,0\right),\left(1,2\right)$. By inspection, $y=2x$ is the equation corresponding to this graph, as the y-coordinate in each case is double that of the x –coordinate

25. **(A)**

$$s=\frac{1}{2}\left(a+b+c\right)$$

$$s'=\frac{1}{2}\left(4a+4b+4c\right)=2\left(a+b+c\right)=4s$$

$\Delta=\sqrt{s\left(s-a\right)\left(s-b\right)\left(s-c\right)}$ and

$$\Delta'=\sqrt{s'\left(s'-4a\right)\left(s'-4b\right)\left(s'-4c\right)}$$

$$\Rightarrow\Delta'=\sqrt{4s\left(4s-4a\right)\left(4s-4b\right)\left(4s-4c\right)}$$

$$=16\sqrt{s\left(s-a\right)\left(s-b\right)\left(s-c\right)}=16\Delta$$

Increase in the area of the triangle

$=\Delta'-\Delta=16\Delta-\Delta=15\Delta$

$\therefore$ percentage increase $=\dfrac{15\Delta}{\Delta}\times100=1500\%$

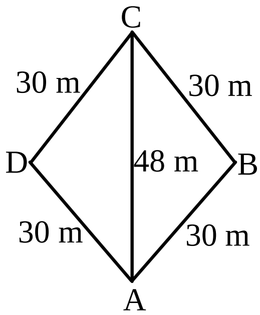
26. **(A)**

For ΔABC, $s=\dfrac{30+30+48}{2}=54m$

$\therefore$ area of $\Delta ABC=\sqrt{s\left(s-a\right)\left(s-b\right)\left(s-c\right)}$

$= \sqrt{54 \times 24 \times 24 \times 6} m^2$

$= 432 \, m^2$



$\therefore$ area of rhombus ABCD

$= 2 \times 432 \, m^2 = 864 \, m^2$

Required area in which

Each cow can graze $= \frac{864}{12} m^2 = 72 \, m^2$

27. (C)

$\frac{3^6}{4^6} \times \frac{4^4}{3^{10}} = \frac{4^4}{3^4} = \left(\frac{4}{3}\right)^{x+2}$

$\therefore x + 2 = 4$

$x = 2$

28. (B)

$P(x) = 5x^2 - 3x + 7$

$P(1) = 5(1)^2 - 3(1) + 7$

$= 5 - 3 + 7$

$= 9$

29. (C)

$x + y + 2 = 0$

$x + y = -2$ ……. (i)

Taking cube both sides

$x^3 + y^3 + 3xy(x + y) = -8$

$x^3 + y^3 + 8 = -3xy(-2)$ from (i)

$x^3 + y^3 + 8 = 6xy$

30. (A)

$$\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{25}{9}\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right] = \left[\left(\frac{3}{2}\right)^4\right]^{-\frac{3}{4}} \times \left[\left\{\left(\frac{5}{3}\right)^2\right\}^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$=\left(\frac{3}{2}\right)^{-3}\times\left[\left(\frac{5}{3}\right)^{-3}\div\left(\frac{5}{2}\right)^{-3}\right]$$

$$=\left(\frac{2}{3}\right)^{3}\times\left[\left(\frac{3}{5}\right)^{3}\div\left(\frac{2}{5}\right)^{3}\right]$$

$$=\frac{8}{27}\times\left[\frac{27}{125}\times\frac{125}{8}\right]$$

$$=\frac{8}{27}\times\frac{27}{8}=1$$

31. **(B)**

Let one angle be α, other will be $180-\alpha$

By using the relation stated

$\alpha-20=\frac{1}{3}(180-\alpha)$

$3\alpha-60=180-\alpha$

$4\alpha-60=180-\alpha$

$\alpha=60$

32. **(B)**

$$\left[\left\{(81)^{-\frac{1}{2}}\right\}^{-\frac{1}{4}}\right]^{2}=\left\{(81)^{\frac{-1}{2}}\right\}$$

$$=(81)^{\frac{1}{4}}=\left(3^{4}\right)^{\frac{1}{4}}=3$$

33. **(D)**

$2^{x}\times4^{x}=(8)^{\frac{1}{3}}\times(32)^{\frac{1}{5}}$

$2^{x}\times\left(2^{2}\right)^{x}=\left(2^{3}\right)^{\frac{1}{3}}\times\left(2^{5}\right)^{\frac{1}{5}}$

$2^{x}.2^{2x}=2^{1}\times2^{1}$

$2^{x+2x}=2^{1+1}$

$2^{3x}=2^{2}$

Comparing powers on both sides, we get

$3x=2$

$x=\frac{2}{3}$

34. **(C)**

If the sum of 3 prime is even, then one of the numbers must be 2.

Let the second number be x. Then as per the given condition,

$x+(x+36)+2=100\Rightarrow x=31$

So, the number are 2, 31, 67

Hence largest number is 67

35. **(B)**

Time taken for entire journey $= \frac{1500}{x}$ hr

New speed when it leave half an hour late than scheduled time $= (x + 250)$ km / h

Time taken $= \frac{1500}{x+250}$ h

So, according to question,

$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{30}{60}$$

$$\Rightarrow 1500\left(\frac{1}{x} - \frac{1}{x+250}\right) = \frac{1}{2} \Rightarrow \frac{x+250-x}{x(250+x)} = \frac{1}{3000}$$

$$\Rightarrow x^2 + 250x = 750000 \Rightarrow x^2 + 250x - 750000 = 0$$

$$\Rightarrow x^2 + 1000x - 750x - 750000 = 0$$

$$\Rightarrow x(x+1000) - 750(x+1000) = 0$$

$$\Rightarrow (x+1000)(x-750) = 0 \Rightarrow x = -1000, 750$$

Speed cannot be negative so we have $x = 750$

$\therefore$Usual speed = 750 km/h