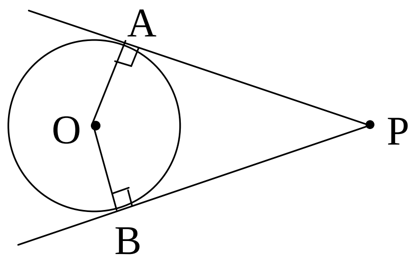
ACE OF PACE SOLUTION

1. **(A)**



$\angle PAO = 90^o$
$\angle PBO = 90^o$
$\therefore \angle PAO + \angle PBO = 180^o$
Hence PAOB is cyclic.

2. **(C)**
Since 1st day is Sunday, 1, 8, 15, 22 & 29 are also Sunday $\therefore$ total 5 Sundays.
Avg. no. of visitors = $\dfrac{510\times5+240\times25}{30}$
$=17\times5+8\times25$
$=285$

3. **(D)**
Let side of cube be $'a'$
Volume of cube $=a^3$
$\therefore$ Radius of sphere $=\dfrac{a}{2}$
Volume of sphere $=\dfrac{4}{3}\pi\left(\dfrac{a}{2}\right)^3=\dfrac{\pi a^3}{6}$
$\dfrac{\text{Vol. of Cube}}{\text{Vol. of Sphere}}=\dfrac{a^3}{\frac{\pi a^3}{6}}=\dfrac{6}{\pi}$

4. **(A)**
$(\alpha+\beta)r=\left(\dfrac{-q}{p}\right)r$
$(\alpha\beta q)=\left(\dfrac{r}{p}\right)q$
$\therefore(\alpha+\beta)r+\alpha\beta q=0$

5. **(C)**
$\dfrac{\cos^2\theta+\tan^2\theta-1}{\sin^2\theta}=\dfrac{\tan^2\theta-\sin^2\theta}{\sin^2\theta}$
$=\sec^2\theta-1=\tan^2\theta$

6. **(A)**

$x^2 - 6x + (5+K) = 0$

$|\alpha - \beta| = \frac{\sqrt{D}}{a} = \sqrt{36 - 4(5+K)} = 2$

$= \sqrt{16 - 4K}$

Hence $K = 3$

7. **(A)**

$8^2 + 2^2 + 3^2 + 4^2 = 93$

$6^2 + 5^2 + 7^2 + 8^2 = 174$

Hence $8^2 + 5^2 + 7^2 + 3^2 = 147$

8. **(B)**

(A) $13 + 7 - 6 \div 2 < 3 \times 4 \Rightarrow 17 < 12$ False

(B) $9 + 5 + 4 = 18 \div 9 + 16 \Rightarrow 18 = 18$ True

(C) $9 - 3 - 2 + 1 > 8 \times 2 \Rightarrow 5 > 16$ False

(D) $28 \div 4 \times 2 < 6 \times 4 \div 2 \Rightarrow 14 < 12$ False

9. **(B)**

Since parabola opens upwards $a > 0$

Product of roots $= \frac{c}{a}$ is $-$ve $\therefore c < 0$

$\frac{-b}{2a} > 0 \Rightarrow \quad b < 0$

10. **(C)**

$\angle POA = 180^o - 112^o = 68^o$

$\angle OAP = 90^o$ (Tangent property)

In ΔOAP

$\angle APO = 180^o - 90^o - 68^o = 22^o$

$\angle ACB = \frac{\angle AOB}{2} = \frac{112}{2} = 56^o$

11. **(D)**

$$\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b} = \frac{\alpha(a\alpha + b) + \beta(a\beta + b)}{(a\beta + b)(a\alpha + b)}$$

$$= \frac{a\left[(\alpha+\beta)^2 - 2\alpha\beta\right] + b(\alpha+\beta)}{a^2\alpha\beta + ab(\alpha+\beta) + b^2}$$

$$= \frac{a\left[\frac{b^2}{a^2} - \frac{2c}{a}\right] + b\left(-\frac{b}{a}\right)}{a^2\left(\frac{c}{a}\right) + ab\left(-\frac{b}{a}\right) + b^2}$$

$$= \frac{b^2 - 2ac - b^2}{a^2c - ab^2 + ab^2} = \frac{-2ac}{a^2c} = -\frac{2}{a}$$

12. **(C)**

Let Rajan's age be x

$(x-8)\frac{6}{5} = x$

$x = 48$

His sister is still 10 yrs younger $x - 10 = 38$ years.

13. **(C)**

Let the rate upstream of the boat = x kmph

And the rate downstream of the boat = y kmph

Distance travelled upstream in 8 hrs 48 min = Distance travelled downstream in 4 hrs.

Since distance = speed × time, we have

$x \times 8.8 = y \times 4 \Rightarrow y = 2.2x$

Speed of boat = $\frac{x+y}{2}$

Speed of water = $\frac{y-x}{2}$

$\therefore$ required ratio = $\frac{x+y}{y-x} = \frac{3.2}{1.2} = \frac{8}{3}$

14. **(A)**

Total surface area to be plastered = $2h(\ell + b) + \ell b$

$= 2 \times 6(25+12) + 25 \times 12$

= 744 m^2

$\therefore$ cost = $0.75 \times 744 =$ Rs. 558

15. **(B)**

For a year to have the same calendar with 2007 ,the total odd days from 2007 should be 0.

Take the year 2014 given in the choice.

Total odd days in the period 2007 – 2013 = 5 normal years + 2 leap years

= 5 x 1 + 2 x 2 = 9 odd days

= 2 odd day *(As we can reduce multiples of 7 from odd days which will not change anything)*

Take the year 2016 given in the choice.

Number of odd days in the period 2007-2015 = 7 normal years + 2 leap years

= 7 x 1 + 2 x 2 = 11 odd days

= 4 odd days

(Even if the odd days were 0, calendar of 2007 will not be same as the calendar of 2016 because 2007 is not a leap year whereas 2016 is a leap year. In fact, you can straight away ignore this choice due to this fact without even bothering to check the odd days)

Take the year 2017 given in the choice.

Number of odd days in the period 2007-2016 = 7 normal years + 3 leap years

= 7 x 1 + 3 x 2 = 13 odd days

= 6 odd days

Take the year 2018 given in the choice.

Number of odd days in the period 2007-2017 = 8 normal years + 3 leap years

= 8 x 1 + 3 x 2 = 14 odd days

= 0 odd day *(As we can reduce multiples of 7 from odd days which will not change anything)*

Also, both 2007 and 2018 are not leap years.

Since total odd days in the period 2007-2017 = 0 and both 2007 and 2018 are of same type, 2018 will have the same calendar as that of 2007

16. **(B)**
x weeks x days = (7 × x) + x = 7x + x = 8x = (7 × x) + x = 7x + x = 8x days

17. **(A)**
Total runs in first 10 overs = $10 \times 3.2 = 32$
Remaining runs = $282 - 32 = 250$
Minimum run rate = $\frac{250}{40} = 6.25$

18. **(B)**
Meal for 200 children = Meal for 120 men
$\Rightarrow$ Meal for 1 child = Meal for 120/200 men
$\Rightarrow$ Meal for 150 children
= Meal for 120×150/200 men = Meal for 90 men
Total meal available = Meal for 120 men
Remaining meal
= Meal for 120 men - Meal for 90 men
= Meal for 30 men

19. **(B)**
Total working hours required = $3 \times 8 \times 2 = 4 \times x \times 1 \Rightarrow x = 12$ hours

20. **(C)**
Let the numbers be 2x, 3x and 4x
LCM of 2x, 3x and 4x = 12x
12x=240 ⇒x=240/12=20
H.C.F of 2x, 3x and 4x =x=20

21. **(B)**
LCM of 5, 6, 7 and 8 = 840
Hence the number can be written in the form (840k + 3) which is divisible by 9.
If k = 1, number = (840 × 1) + 3 = 843 which is not divisible by 9.
If k = 2, number = (840 × 2) + 3 = 1683 which is divisible by 9.
Hence 1683 is the least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder.

22. **(A)**
$P(y) = y^3 + 4y^2 - 3y + 10$
$P(-4) = -64 + 64 + 12 + 10 = 22$

23. **(D)**
Number = 56x + 29 (∵ since the number gives 29 as remainder on dividing by 56)
= (7 × 8 × x) + (3 × 8) + 5
Hence, if the number is divided by 8, we will get 5 as remainder.

24. **(D)**
$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2$
$= 8 + 3\sqrt{7} + 8 - 3\sqrt{7} - 2$
= 14
$\therefore \sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{14}$

25. (B)

$$\frac{1}{1+x^{q-p}+x^{r-p}}+\frac{1}{1+x^{p-q}+x^{r-q}}+\frac{1}{1+x^{q-r}+x^{p-r}}$$

$$=\frac{x^p}{x^p+x^q+x^r}+\frac{x^q}{x^p+x^q+x^r}+\frac{x^r}{x^p+x^2+x^r}$$

= 1

26. (D)

If the hands of the clock are in the same straight line, but not together, they will be 30 minute spaces apart.

At 7'o clock, the hands of the clock are 25 minute spaces apart. Hence the minute hand should gain 5 minute spaces over the hour hand so that the hands will be 30 minute spaces apart.
In 60 minutes, minute hand gains 55 minute spaces over the hour hand.

Hence, to gain 5 minute spaces for the minute hand, time needed

= 60/55 × 5 = 60/11 minutes = $5\frac{5}{11}$ minutes

That means when the time is $5\frac{5}{11}$ minutes past 7, the hands of a clock will be in the same straight line but not together.

27. (D)

Let the commodity P costs 40 paise more than the commodity Q after n years.
Price of the commodity P in 2001 = Rs.4.20
Since the price of the commodity P increases by Rs 0.40 every year,
Price of the commodity P after n years from 2001 = Rs.4.20 + (n × .40)
Price of the commodity Q in 2001 = Rs.6.30
Since the price of the commodity Q increases by Rs 0.15 every year,
price of the commodity Q after n years from 2001 = Rs.6.30 + (n × .15)
Since the commodity P costs Rs. 0.40 more that the commodity Q after n years from 2001,
4.20 + (n × .40) = 6.30 + (n × .15) + 0.40
$\Rightarrow$ (.40n − .15n) = 6.30 − 4.20 + 0.40 = 2.5
$\Rightarrow$ .25n = 2.5
$\Rightarrow$ n =2.5/.25=250/25=10
$\Rightarrow$ Commodity P costs Rs.0.40 more that the commodity Q after 10 years from 2001.
i.e., in 2011

28. (A)

$\log(648)^5$
= 5 log(648)
= 5 log(81 × 8)
= 5[log(81) + log(8)]
=5 $[\log(3^4) + \log(2^3)]$
=5[4log(3) + 3log(2)]
= 5[4 × 0.4771 + 3 × 0.30103]
= 5(1.9084 + 0.90309)
= 5 × 2.81149
≈ 14.05
ie, $\log(648)^5 \approx 14.05$
ie, its characteristic = 14
Hence, number of digits in $(648)^5$ = 14+1 = 15

29. **(C)**

=log4/log3 × log5/log4 × log6/log5 × log7/log6 × log8/log7 × log9/log8×1

= log9/log3 = log3^2/log3 = 2log3/log3 = 2

30. **(C)**

Let x and (12− x) litres of milk be mixed from the first and second container respectively.

Amount of milk in x litres of the the first container =.75x

Amount of water in x litres of the the first container =.25x

Amount of milk in (12 − x) litres of the the second container =.5(12 − x)

Amount of water in (12 − x) litres of the the second container =.5(12 − x)

Ratio of water to milk

=[.25x + .5(12 − x)]:[.75x + .5(12 − x)]= 3:5

⇒(.25x + 6 − .5x)/(.75x + 6 − .5x) = 3/5 ⇒ (6 − .25x)/(.25x + 6) = 3/5 ⇒ 30 − 1.25x = .75x + 18 ⇒ 2x = 12 ⇒ x = 6

Since x = 6,12 – x = 12 – 6 = 6

Hence 6 and 6 litres of milk should mixed from the first and second container respectively.

31. **(D)**

(x^n+1) is divisible by (x+1)when n is odd.

⇒ (67^{67} + 1) is divisible by (67 + 1)

⇒(67^{67} + 1) is divisible by 68

⇒ (67^{67}+1) ÷ 68 gives a remainder of 0

⇒ [(67^{67}+1) + 66] ÷ 68 gives a remainder of 66

⇒ (67^{67} + 67) ÷ 68 gives a remainder of 66

32. **(C)**

1 + 1 + 1 + ⋯ n terms =∑1 = n

$$1+2+3+\cdots+n=\sum n=\frac{n(n+1)}{2}$$

$$\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+.....(\text{up to n terms})$$

$$=(1+1+1+.....\text{up to n terms})-\left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+......\text{up to n terms}\right)$$

$$=n-\frac{1}{n}(1+2+3+.....\text{up to n terms})$$

$$=n-\frac{1}{n}\left[\frac{n(n+1)}{2}\right]$$

$$=n-\frac{(n+1)}{2}$$

$$=\frac{(2n-n-1)}{2}$$

$$=\frac{n-1}{2}$$

33. **(B)**

$A=5,\ A:B:C=1:3:5$

So, $B = 5\times3=15$

$C=5\times5=25$

At constant C , AB = constant

So, $5\times15=$ new value of A $\times$ New value of B

$= A'\times9 \Rightarrow A'=\frac{75}{9}8.33$

34. **(A)**

We need to final quadratic in y, s.t. $y=\frac{1}{ax+b}$

$\therefore x=\frac{1-by}{ay}$

Quadratic becomes. $a\left(\frac{1-by}{ay}\right)^2+b\left(\frac{1-by}{ay}\right)+c=0$

$\left(ay^2+by\left(1-by\right)\right)+\left(1-by\right)^2$

$=\left(cay^2-by+1\right)=0$

$\therefore$ sum of roots = $\frac{b}{ca}$

35. **(B)**

Amount invested by Amar = x

Amount invested by Akbar = x + 5000

Amount invested by Anthony = x + 7000

Total Invested = $3x+12000$

Total Interest @ 12% = $\frac{12}{100}\left(3x+12000\right)$

$\frac{36x}{100}+1440=3240$

X = 5000

So, By Akbar = x + 5000 = 10000

36. **(D)**

$\sin\theta_1+\sin\theta_2+\sin\theta_3=3$

$\sin\theta_2\le1$

$\sin\theta_2\le1 \Rightarrow \sin\theta_1+\sin\theta_2+\sin\theta_3\le3$

$\sin\theta_3\le1$

Hence, it can only be true if $\sin\theta_1=\sin\theta_2=\sin\theta_3=1$

So, $\cos\theta_1=\cos\theta_2=\cos\theta_3=0$

37. **(A)**

Venu has as many sisters as his brothers. Suppose venu has $\times$ sisters,

So he has $\times$ brothers

So, total boys = x + 1 , total girls = x

Karuna has x – 1 sisters & x + 1 brothers

It is given, $x + 1 = 3(x - 1) \Rightarrow x = 2$

Karuna has x – 1 = 1 sister

38. **(D)**

25 minutes = $\frac{25}{60}$ hrs

1 hr = 20 rev. $\frac{25}{60} \Rightarrow \frac{25}{60} \times 20 = \frac{25}{3}$ rev.

$\frac{25}{3}$ rev. $\Rightarrow \frac{25}{3} \times 2\pi$ radius = $\frac{50\pi}{3}$ radius

39. **(A)**

Suppose the person correctly answer x questions, incorrectly answers y question & left 3 questions unattempted.

So, (i) x + y + 3 = 100 (total no of question)

(ii) $2x - y - \frac{3}{2} = 135$

(iii) $2x - \frac{y}{2} - 3 = 133$

$3x(i) + 2 \times (ii) + 2 \times (iii)$

11x = 836

$x = 76 \Rightarrow y + z = 24 (iv)$

(ii) – (iii)

$\Rightarrow 3 - y = 4$ (v)

Solving (iv) & (v)

z = 14

40. **(B)**

1

$2 \times 1 - 1 = 1$

$2 \times 1 + 1 = 3$

$2 \times 3 - 1 = 5$

$2 \times 5 + 1 = 11$

$2 \times 11 - 1 = 21$

$2 \times 21 + 1 = 43$

$2 \times 43 - 1 = 85$

41. **(A)**

The first fortune teller would allow you to have the best prediction of your future. The first fortune teller predicts with only 20% accuracy. So, there is a 80% chance of the first fortune teller being wrong. If you consider the opposite of the first fortune tellers predictions, then you get the prediction of your future with 80% accuracy.

42. **(C)**

Centre of smaller ring has to travel $2\pi(11R)$ distance. In one rotation it travels $2\pi R$

43. **(B)**

Out of the 4 persons, 3 persons told that the mugger was tall. Hence it is make probable the mugger was tall. By the similar method, we can tell that B is probably right.

44. **(A)**

$ax^2+bx+c=0 \rightarrow \alpha,\beta$

$cx^2+bx+a=0 \rightarrow \frac{1}{\alpha},\frac{1}{\beta}$

Sum of roots = $\alpha+\frac{1}{\beta}+\beta+\frac{1}{\alpha}=-\frac{b}{a}+\left(-\frac{b}{c}\right)$

$=\frac{-b(a+c)}{ac}$

Product of roots $=\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$

$=\alpha\beta+1+1+\frac{1}{\alpha\beta}$

$=\frac{c}{a}+\frac{a}{c}+2=\frac{(a+c)^2}{ac}$

$\therefore$ required quadratic = $x^2+\frac{b(a+c)}{ac}x+\frac{(a+c)^2}{ac}=0$

$\frac{1}{ac}\left(acx^2+b(a+c)x+(a+c)^2\right)=0$

45. **(B)**

$7+3=10$ $\quad 8+3=11$ $\quad 9+3=12$

$10-2=8$ $\quad 11-2=9$ $\quad 12-2=10$

46. **(A)**

$\cot 4=\tan 86=m$

$\cot 86=\frac{1}{\tan 86}=\frac{1}{m}=\tan 4$

$\frac{\cot 4-\cot 86}{m+\tan 4}=\frac{m-\frac{1}{m}}{m+\frac{1}{m}}=\frac{m^2-1}{m^2+1}$

47. **(B)**

If the smallest squares in the figure are 1×1, then

No. of 1×1 square = $7\times4=28$

No. of 2×2 square = $6\times3=18$

No. of 3×3 square = $5\times2=10$

No. of 4×4 square = $4\times1=4$
Total squares = 60

48. **(C)**
n^3-n^2

49. **(B)**
$\sec A-\tan A=p$
$\sec A-p=\tan p$
Squaring
$\sec^2 A+p^2-2p\sec A=\tan^2 A$
$=\sec^2 A-1$
$2p\sec A=p^2+1$
$\cos A=\dfrac{1}{\sec A}=\dfrac{2p}{p^2+1}$

50. **(A)**
Age of captain = 26yrs
Age of wicket keeper = 26 + 3 = 29 yrs
Let average age of team = x yrs.
Total age of remaning 9 players = 11x – 26 – 29
= 11x – 55
$\therefore \dfrac{11x-55}{9}=x-1$
$\therefore 2x=46\Rightarrow x=23$ years