CIRCULAR MOTION

EXERCISE – 1

1. **d**

ω = rate of change of angle

$\therefore \quad \frac{\omega_1}{\omega_2} = 1$ as they both complete 2π angle in same time.

2. **c**

$$mg - N = \frac{mv^2}{r}$$

$$\Rightarrow \quad N = mg - \frac{mv^2}{r}$$

Since $r_A < r_B$

$\Rightarrow \quad N_A < N_B$

3. **a**

Force is always perpendicular to displacement

$\therefore$ work done = 0

4. **b**

$$a_{resultant} = \sqrt{a_c^2 + a_{tang.}^2}$$

$$= \sqrt{\left(\frac{30^2}{500}\right)^2 + 2^2} = \frac{\sqrt{181}}{5} = 2.7\ m/s^2$$

5. **a**

f = mg

$\Rightarrow \quad \mu N \geq mg$

$\Rightarrow \quad \mu\, mr\omega^2 \geq mg$

$\Rightarrow \quad \omega \geq \sqrt{\frac{g}{\mu r}}$

6. **c**

If the coin just slips at a distance of 4r from centre

$\Rightarrow \quad \mu\, mg = m4r\omega^2 \qquad \ldots (1)$

If angular velocity is doubled

$\mu\, mg = mR\, (2\omega)^2 \qquad \ldots (2)$

From (1) and (2)

$\Rightarrow \quad R = r$

7. **d**

$T = mr\, \omega_0^2 \qquad \ldots (1)$

$2T = mr\, \omega^2 \qquad \ldots (2)$

$\Rightarrow \quad \omega = \sqrt{2}\omega_0 = \sqrt{2} \times 5$ rpm

8. **c**

Since force is always perpendicular to velocity particle moves in circle. It's speed is constant and velocity variable.

9. **c**

$$\mu mg = \frac{mv_{max}^2}{r}$$

$$v_{max} = \sqrt{\mu gr} = \sqrt{0.3 \times 10 \times 300}$$

$$= 30 m/s = 108 \text{ km/hr}$$

10. **c**

$$F_{net} = ma_{rad.} = \frac{mv^2}{r}$$

11. **b**

$T = mr\omega^2 = 0.2 \times 0.5 \times 4^2 = 1.6N$

12. **c**

Centripetal force is provided by friction

13. **a**

$$N_A - mg = \frac{mv^2}{r}$$

$$N_A = mg + \frac{mv^2}{r_A}, \quad N_B = mg - \frac{mv^2}{r_B}$$

$$N_C = mg + \frac{mv^2}{r_c}$$

14. **a**

Real forces are mg and T only

15. **a**

Bead starts slipping, when

$\mu N = mL\omega^2$

$\mu mL\alpha = mL\omega^2$

$\mu\alpha = (0 + \alpha t)^2$

$\Rightarrow \quad t = \sqrt{\frac{\mu}{\alpha}}$

EXERCISE – 2

1. **(a, c)**

Time to fall = $\sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5}{10}} = 1\,s$

Distance covered in y direction = $v \times t$

= 3×1

= 3m

Since there is no velocity along x-direction x is always 2m.

2. **(a, b)**

For no wear and tear friction is zero

$\Rightarrow \quad \frac{v^2}{Rg} = \tan 15^o$

$\Rightarrow \quad v = \sqrt{Rg \tan 15^o} = 28.1\,m/s$

$$v_{max} = \sqrt{\frac{Rg(\mu + \tan\theta)}{1 - \mu\tan\theta}} = 38.1 m/s$$

3. **(b, c)**

$$\vec{v} = \frac{d\vec{r}}{dt} ;\ \therefore\ \vec{v} \parallel d\vec{r}\ \text{(i.e. along the tangent)}$$

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{t} = 0$$

4. **(a, b)**

$$\frac{dS}{dt} = v = K\sqrt{S}$$

$$\Rightarrow \quad \int_0^S \frac{dS}{\sqrt{S}} = \int_0^t K dt$$

$$2\sqrt{S} = Kt$$

$$S = \frac{K^2t^2}{4}$$

$$v = \frac{dS}{dt} = \frac{K^2t}{2}$$

5. **(a, b, c)**

$T = m\ell\omega^2$, $v = \ell\omega$, $F_{vert} = 0$

6. **(a, b, d)**

$$\omega = \frac{v}{R} = \text{constant},\ \theta = \omega t$$

$F_y = -F \sin\theta = -F \sin\omega t$

$$= -\frac{mv^2}{R}\sin\omega t$$

$V_r = -v\sin\theta = -v \sin \omega t$

x-coordinate = $R \cos \omega t$

7. **(b, d)**

If $\mu = 0.1$, $f_{max} = 0.1 \times 0.5 \times 10 = 0.5N$

Req. centripetal force = $mr\omega^2 = 0.5 \times 1 \times 0.5^2 = .125N$

$$\therefore \quad f = \frac{1}{8}N,\ \text{Tension} = \text{zero}$$

$$\text{If } \mu = \frac{1}{20},\ f_{max} = \frac{1}{20} \times 0.5 \times 10 = 0.25\ N$$

$$\therefore \quad f = \frac{1}{8}N,\ \text{Tension} = \text{zero}$$

$$\text{If } \mu = \frac{1}{40},\ f_{max} = \frac{1}{40} \times 0.5 \times 10 = 0.125\ N$$

$\therefore \quad f = \frac{1}{8}\text{N, Tension} = 0$

EXERCISE – 3

1. **(a)**

$\frac{v^2}{r} = K^2rt^2$ (given)

(a) Centripetal force = mK^2rt^2

(b) Tangential force = $m\frac{dv}{dt} = mKr$

(c) Power of centripetal force $= \vec{F}_{centripetal} \cdot \vec{v} = 0$

(d) Power of tangential force $= \vec{F}_t \cdot \vec{v} = F_t\ v$

$= m.K^2r^2t$

2. **(b)**

Assuming no friction between m_1 and m_2

$a_1 = R\omega^2 - \frac{T}{m_1}$

$a_2 = R\omega^2 - \frac{T}{m_2}$

$\because \quad a_1 > a_2$

$\therefore$ Friction on upper block acts towards left and on lower block towards right.

3. **(a)**

Let the required angular velocity be ω

Then

$T + \mu m_1 g = m_1 R\omega^2 \quad \ldots (1)$

$T = \mu m_1 g + m_2 R\omega^2 \quad \ldots (2)$

$\Rightarrow \quad 2\mu m_1 g = (m_1 - m_2)R\omega^2$

$\Rightarrow \quad \omega = \sqrt{\frac{2\mu m_1 g}{(m_1 - m_2)R}} = 6.3\ \text{rad/s}$

4. **(b)**

$T = \mu m_1 g + m_2 R\omega^2$

$= 0.5 \times 2 \times 10 + 1 \times 0.5 \times 40 = 30\text{N}$

5. **(b)**

$\tan\theta = \frac{v^2_{design}}{gR} = \frac{1}{2};$

$f = m\left(g\sin\theta - \frac{v^2}{R}\cos\theta\right)$

$= 300\sqrt{5}\ \text{m/s}$

6. **(a)**

$$f = m\left(\frac{v^2}{R}\cos\theta - g\sin\theta\right)$$

$$= 500\sqrt{5}\ m/s$$

7. **(a)**

$$\theta = \tan^{-1}\frac{1}{2}$$

8. **(c)**

$$mg\sin\theta = m\frac{v^2}{L} \qquad \ldots (1)$$

$$\frac{1}{2}m(\sqrt{3gL})^2 = \frac{1}{2}mv^2 + mgL(1+\sin\theta) \qquad \ldots (2)$$

$$\therefore \qquad \theta = \sin^{-1}\left(\frac{1}{3}\right); \ \text{Also } v^2 = \frac{1}{3}gL$$

9. **(c)**

$$h_{max} = L(1+\sin\theta) + \frac{0^2 - v^2\cos^2\theta}{-2g}$$

$$= \frac{40L}{27}$$

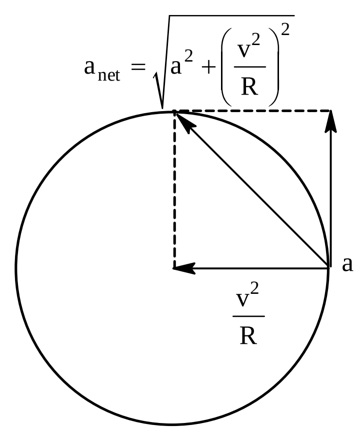
10. **(b)**

$$\frac{1}{2}m(\sqrt{3gL})^2 = mg\,h_{max}$$

$$\Rightarrow \qquad h_{max} = \frac{3L}{2}$$

EXERCISE – 4

1. Here frictional force will provide the required tangential and centripetal force for the circular motion of car.
Force of friction will act along the direction of net acceleration.



$$f = m\sqrt{a^2 + \left(\frac{v^2}{R}\right)^2}$$

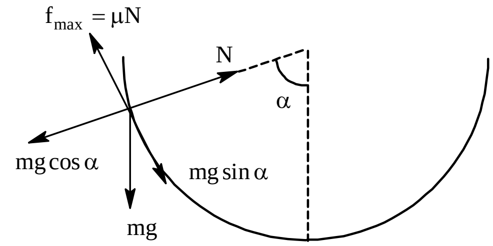
Car will skid when $m\sqrt{a^2 + \left(\frac{v^2}{R}\right)^2}$

$$= m\sqrt{a^2 + \frac{a^4 + t^4}{R^2}}$$

$$\Rightarrow \qquad t = \left[\frac{(\mu^2 g^2 - a^2)R^2}{a^4}\right]^{1/4}$$

Till this time distance traveled by car is $D = \frac{1}{2}at^2$. Putting value of t we get $D = \frac{R\sqrt{\mu^2 g^2 - a^2}}{2a}$.

2. The insect will slide when mg sin α becomes equal to limiting friction. At every instant the insect is in equilibrium.



So, N = mg cosα

μN = mg sin α

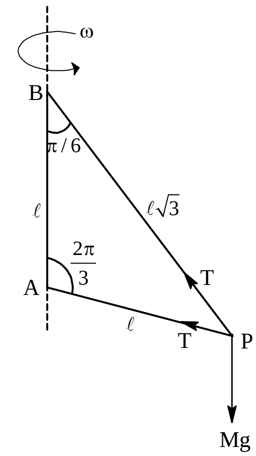
$\Rightarrow \mu$ mg cos α = mg sin α ⇒ μ = tan α ⇒ α = $\tan^{-1}\mu$ = $\tan^{-1}(1/3)$

3. Applying Newton's law towards the centre of circle we get $N = m\omega^2 R$

Let ω be the minimum angular speed for which man is not falling. At this instant its weight will be balanced by limiting friction acting upwards.

i.e, μN = mg ⇒ $0.15 \times 70\ \omega^2 \times 3 = 70 \times 10$

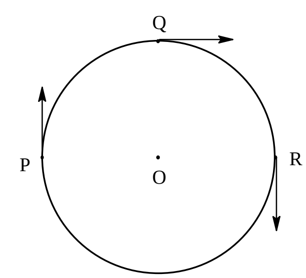
⇒ ω = 4.7 rad/sec.



4. Applying Newton's law along vertical we get

$T\frac{\cos\pi}{6} + T\frac{\cos\pi}{3} = mg$ … (1)

⇒ $T = \frac{2mg}{\sqrt{3}+1}$



Applying Newton's law along horizontal.

We get $T\cos\frac{\pi}{3} + T\cos\frac{\pi}{6} = m\omega^2 R = m\omega^2\frac{\ell\sqrt{3}}{2}$

⇒ $\omega^2 = \frac{2g}{\ell\sqrt{3}}$

5. Particle P and Q will be at same angular position whenever $5\pi t = 2\pi t + \frac{\pi}{2} + 2\pi n$.

⇒ $t = \frac{1}{6} + \frac{2n}{3}$ (n = integer) … (1)

Similarly, particle P and R will be at same angular position.

Whenever 5πt = 3πt = π + 2πm (m = integer)

⇒ $t = \frac{1}{2} + m$ … (2)

All three particles will be at same angular position when (1) = (2)

$\Rightarrow \quad \frac{1}{2}+m=\frac{1}{6}+\frac{2n}{3}$

$\Rightarrow \quad 2n = 1 + 3m$

Smallest integral value of m & n satisfying the above equation is m = 1, n = 2.
Putting these values in (1) & (2) we get t = 1.5 sec.
So, they will meet for the first time at t = 1.5 sec.

6. According to question, $a_c = a_t = \frac{v^2}{R}$

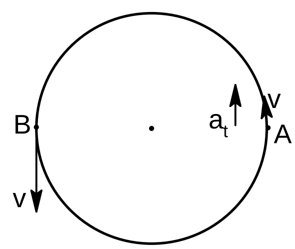
$\Rightarrow \quad \frac{dv}{dt}=\frac{v^2}{R} \Rightarrow \int_{v_0}^{v}\frac{dv}{v^2}=\int_0^t\frac{dt}{R}$

$\Rightarrow \quad \frac{-1}{v}+\frac{1}{v_0}=\frac{t}{R} \Rightarrow V(t)=\frac{Rv_0}{R-tv_0}$

$\Rightarrow \quad \int_0^{2\pi R}dx = Rv_0\int_0^T\frac{dt}{R-tv_0} \Rightarrow 2\pi R=\frac{-Rv_0}{v_0}(\ell n(R-tv_0)_0^T$

$\Rightarrow \quad T=\frac{R}{v_0}(1-e^{-2\pi})$

7. Lets assume that the particles meet at time t. Distance traveled by A = distance traveled + πR by B



$\Rightarrow \quad vt+\frac{1}{2}\times\frac{72v^2t^2}{25\pi R}=vt+\pi R$

$\Rightarrow \quad t=\frac{5\pi R}{6v}$

Angle traced by $A=\frac{\text{distance travelled}}{R}=\frac{11\pi}{6}$

Angular velocity $=\frac{v+at}{R}=\frac{17v}{5R}$

EXERCISE – 5

1. **(c, d)**

$\vec{F}\perp\vec{v} \Rightarrow P = 0 \Rightarrow$ kinetic energy = constant; F is constant (given) $\Rightarrow \frac{mv^2}{R}=$ constant

$\Rightarrow$ R = constant.

2. **(a)**

Radius of curvature in (a) is minimum

3. **(a)**

mg sin α = $\frac{1}{3}$ mg cos $\alpha \Rightarrow$ cot α = 3

4. **(c)**

$\vec{a}=\vec{a}_{\tan gential}+\vec{a}_{normal}$ and $\vec{a}_{\tan gential}$ is downward.

5. $Kx\cos 30° + mg\cos 30° = ma_t$. As $x = \frac{R}{4}$ and $K = \frac{mg}{R}$, $a_t = \frac{5\sqrt{3}}{8}g$;

$N + Kx\cos 60° = mg\cos 60° \Rightarrow N = \frac{3mg}{8}$

WORK POWER & ENERGY

EXERCISE – 1

1. **D**

$$W = \int_{x=x_1}^{x=x_2} F dx = \int_{x=0}^{x=5} (7 - 2x + 3x^2) dx = (7x - x^2 + x^3)\Big|_{x=0}^{x=5} y$$

= 135 J

2. **A**

This is the statement of Work – Kinetic Energy Theorem

3. **B**

Since the force acting on the particle is perpendicular to the displacement everywhere, the work done iz zero

4. **B**

Instantaneous power, P = $\vec{F} \cdot \vec{v} = (10\hat{i} + 10\hat{j} + 20\hat{k}) \cdot (5\hat{i} + 3\hat{j} + 6\hat{k})$

= 140 W

5. **D**

Mass of the hanging part = $\frac{M}{3}$; when the hanging part of the chain is paralled on the table, its centre of mass is raised by $\frac{L}{6}$.

The work done = rise in potential energy = $= \left(\frac{M}{3}\right) g \left(\frac{L}{6}\right) = \frac{MgL}{18}$

6. **B**

$\frac{1}{2} mv^2 = mgR \Rightarrow v = \sqrt{2gR}$

7. **B**

$K_{longer}\, l_{longer} = K_{original}\, l_{original}$

$\Rightarrow \quad K_{longer} = \frac{k\ell}{(2\ell / 3)} = \frac{3}{2} k$

8. **C**

W_F = increase in potential energy = mgL (1 – cos θ)

9. **D**

Mean power of gravity = $\frac{\text{work done by gravity}}{\text{time elapsed}} = 0$

10. **B**

Acceleration, a = –kx

$\Rightarrow$ F = –Kx $\therefore$ loss of KE : gain of potential energy $\propto x^2$.

11. **C**

$$x = \frac{t^3}{3} \Rightarrow v = t^2$$

Now, W = ΔKE = $\frac{1}{2}m(v_f^2 - v_2^2)$ = 16J

12. **C**

KE_{max} = Maximum loss of KE = Mgl (1 – cos θ)

13. **A**

Centre of mass of the rope is lifted by $\frac{h}{2}$ and the back by h. Therefore,

$$W = Mgh + mg\,\frac{h}{2} = \left(M + \frac{m}{2}\right)gh$$

14. **D**

$$\mu x\, mg = m\,v\frac{dv}{dx} \Rightarrow \int_{x=0}^{x} \mu\, xg\, dx = \int_{v=0}^{v} mv\, dv$$

$\Rightarrow$ E $\propto x^2$

15. **D**

$$W = \left(\frac{-3mg}{4}\right)d$$

16. **C**

$$W = \int_A^B \vec{F}\,.\,d\vec{r} = \frac{F\pi R}{2}$$

17. **C**

$$P = \frac{dW}{dt} = \frac{3t^2}{2} \Rightarrow W = 4J \Rightarrow v = 2m/s$$

18. **C**

$W_1 : W_2 : W_3$ = Ratio of corresponding displacements = $1^2 : (2^2 - 1^1) : (3^2 - 2^2)$

= 1 : 3 : 5

19. **C**

$$\frac{dv}{dx} = \frac{-12a}{x^{13}} + \frac{6h}{x^7} = 0 \Rightarrow x = \left(\frac{2a}{b}\right)^{1/6} \Rightarrow U_{min} = \frac{a}{(2a/b)^2} - \frac{b}{(1a/b)} = -\frac{b^2}{4a}$$

$\therefore$ Minimum energy required = $\frac{b^2}{4a}$

20. **D**

$\mu \text{ mg } v_{max} = P \Rightarrow v_{max} = \dfrac{P}{\mu \text{ mg}}$

21. **C**

$W = \dfrac{\mu mg}{1+\mu} = 163.3 \text{ J}$

22. **C**

The KE intercepted $\propto v^3$

23. **D**

$T - mg = m\dfrac{v^2}{\ell} \Rightarrow T = m\left(g + \dfrac{5g\ell}{\ell}\right) = 6mg$

24. **A**

$mg(h + x) = \dfrac{1}{2}kx^2 \Rightarrow 980x^2 - 2 \times 9.8\,(0.4 + x) = 0$

$\Rightarrow \quad 50x^2 - x - 0.4 = 0$

$\Rightarrow \quad (10x - 1)(5x + 0.4) = 0$

$\Rightarrow \quad x = 0.1 \text{ m} = 10 \text{ cm}$

25. **D**

$W = (kx\,\hat{j}).(a\hat{i}) + \int_{y=0}^{y=a} k(y\hat{i} + a\hat{j}).dy\,\hat{j} = ka^2$

EXERCISE – 2

1. **c, d**

Since no work is done by the force speed is constant not velocity. $\vec{a} = \dfrac{v^2}{r}$ along the centre of circle.

2. **b, c**

W.d by all forces = ΔK.E.

$\Rightarrow \quad W_g + W_N = K.E_f - K.E_i$

$\Rightarrow \quad mgh + 0 = \dfrac{1}{2}mv^2 - 0$

$\Rightarrow \quad v = \sqrt{2gh} = v_P = v_Q$

where h is the initial height of both blocks from ground.

3. **b, c**

4. **b, c**

w.d = $\vec{F}.\vec{d}$

$= (-\hat{i} + 2\hat{j} + 3\hat{k}).3\hat{j}$

= 6 J

w.d. = $\vec{F}.\vec{d}$

$= (-\hat{i}+2\hat{j}+3\hat{k}).(3\hat{j}+4\hat{k}) = 18J$

5. **a, b, c**

W.d = Area enclosed by the F-x graph

6. **a, d**

$a = \frac{F_{net}}{m} = \frac{10-0.2\times2\times10}{2} = 3\,m/s^2$

v(t = 4s) = 0 + 3 × 4 = 12 m/s

s (t = 4s) = $\frac{1}{2}\times3\times4^2 = 24$ m

w.d by net force = ΔK.E

$= \frac{1}{2}mv^2 = \frac{1}{2}\times2\times12^2$

= 144 J

w.d. by applied force = 10 × 24 = 240 J

w.d by friction = $\vec{F}.\vec{d}$ = – 4 × 24 = –96 J

7. **b, c**

8. **b, c, d**

9. **a, b, d**

w.d by all forces = ΔK.E.

⇒ Fx – mgx = 40 J

⇒ 20x = 40 J

x = 2 m

w.d.$_{gravity}$ = – mgx = –2 × 10 × 2 = – 40 J

w.d.$_{tension}$ = Fx = 40 × 2 = 80 J

10. **a, d**

Power = $\vec{F}.\vec{v} = Fv = F\times at$ or $F\times\sqrt{2ax}$

Since 'a' and 'F' are constants

Power varies linearly with time and parabolically with displacement

11. **a, c**

Hint: Direction of spring force and displacement are same in (a) & (c)

12. b**, c**

$P_{mg} = \vec{F}.\vec{v}$

$= mg(-\hat{j}).[u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}]$

= – mg (u sin θ – gt)

⇒ P < 0 for $t < \frac{u\sin\theta}{g}$ and P > 0 for $\frac{2u\sin\theta}{g} > t > \frac{u\sin\theta}{g}$

13. **a, c, d**

$\vec{a} = \frac{\vec{F}}{m} = -\frac{\delta U}{m\delta x}\hat{i} - \frac{\delta U}{m\delta y}\hat{j} = -3\hat{i} - 4\hat{j}$

$v(\text{at } x = 0) = \sqrt{u^2 + 2as}$

$= \sqrt{0^2 + 2\times 5\times 10}$

= 10 m/s

$\vec{x} = \vec{x}_0 + \vec{u}t + \frac{1}{2}\vec{a}t^2$

$= 6\hat{i} + 4\hat{j} + 0 + \frac{1}{2}(-3\hat{i} + 4\hat{j})1^2$

$= 4.5\hat{i} + 2\hat{j}$

14. **c, d**

w.d by all force = increase in spring energy

$\Rightarrow \quad Fx_0 + mgx_0 = \frac{1}{2}k(x_0 + \frac{mg}{k})^2 - \frac{1}{2}k\left(\frac{mg}{k}\right)^2$

$\Rightarrow \quad x_0 = \frac{2F}{k}$

15. **b, c, d**

w.d. by $\vec{F}_2 = 15\times\frac{\pi}{2}\times 6 = 45\pi$ J

w.d. by $\vec{F}_3 = 30\times 6 = 180$ J

w.d. by $\vec{F}_1 = \int F_1\cos\left(90 - \frac{\theta}{2}\right) rd\theta$

$\vec{F}_1$ is conservative in nature as it is always directed towards P.

16. **b, d**

At highest point $F_{net} = \frac{mv'^2}{\ell}$

$\Rightarrow \quad 2mg + mg = \frac{mv'^2}{\ell}$

$\Rightarrow \quad v' = \sqrt{3g\ell}$

Conserving energy velocity at lowest point

$v = \sqrt{7g\ell}$

17. **b, d**

w.d. = $\vec{F}\,.\,\vec{d} = (6\hat{i} - 6\hat{j})\,.\,(-3\hat{i} + 4\hat{j})$

= –18 – 24 = – 42 J

Had there be no initial velocity particle must have moved along straight line making an angle of 45° with x-axis.

18. **a, c, d**

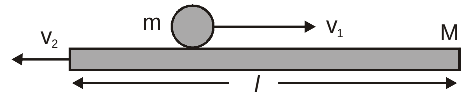
$P = \vec{F}.\vec{v} = +$ ive angle is acute

$= -$ ive angle is obtuse

Area under graph = ΔK.E.

= 20J

19. **(a, c)**



$mv_1 = Mv_2$, where v_1 and v_2 are speeds of mass m and M, as seen from ground. The velocity of m relative to M is $v_{12} = v_1 - (-v_2)$.

Hence, or $t = \frac{1}{V_{12}} = \frac{1}{V_1 + V_2}$ or $v_1 + v_2 = l/t$.

20. **(a, b)**

$\because F = -\frac{dU}{dr} = \frac{2A}{r^3} - \frac{B}{r^2}$ at equilibrium F = or, $r = \frac{2A}{B}$

At infinity U = 0 $r = \frac{2A}{B}$, $U = -\frac{B^2}{4A}$ $\Delta U = \frac{B^2}{4A}$.

21. **(a, c)**

From conservation of linear momentum (1 + 2)v = (6 x 1) + (2 – 3) v = 4m/s (of both the blocks)

From work energy therom i.e., W_{total} = ΔKE on 1kg block, $W_f = \frac{1}{2}\times 1\times(4^2 - 6^2) = -10J$

on 2kg block $W_f = \frac{1}{2}\times 2(4^2 - 3^2) = +7J$. $\therefore$ Net work done by friction is –3J.

22. **(b, d)**

In region OA particle is acccelerated, in region AB particle has uniform velocity while in region BD particle is deceleration., Therefore, work done is positive in region OA , zero in region AB and negative in region BC.

23. **(a, c)**

at B acceleration of block $= \frac{v^2}{R} = \frac{2gR}{R} = 2g$

24. **(a, d)**

EXERCISE – III

1. (A – q)

Work energy theorem – w.d. by all forces is equal to change in K.e.

(B – s)

Negative of work done by conservative force is equal to change in potential energy

(C – r)

$W.d_{ext.} + W_{non\ cons.} = \Delta K.E. - W_{cons.}$

$= \Delta K.E. + \Delta U$

$= \Delta T.M.E$

2. (A – r)

$$\frac{1}{2}mu^2 = mgR + \frac{1}{2}mv_B^2$$

$$\Rightarrow \quad v_B = \sqrt{7gR}$$

(B - q)

$$\frac{1}{2}mu^2 = mg \times 2R + \frac{1}{2}mv_C^2$$

$$v_C = \sqrt{5gR}$$

(C – p)

$$T_B = \frac{mv_B^2}{R} = 7\ mg$$

(D – t)

$$T_C + mg = \frac{mv_C^2}{R}$$

$$T_C = \frac{m5gR}{R} - mg = 4mg$$

3. (A – q)

$$w.d = \int_2^4 kx\,dx = \left[\frac{kx^2}{2}\right]_2^4 = \frac{1}{2}k[4^2 - 2^2] = +ive$$

(B – p)

$$w.d = \int_{-4}^{-2} kx\,dx = \left[\frac{kx^2}{2}\right]_{-4}^{-2} = \frac{1}{2}k[2^2 - 4^2] = -ive$$

(C – r)

$$w.d = \int_{-2}^{2} kx\,dx = \left[\frac{kx^2}{2}\right]_{-2}^{2} = 0$$

4. (A – t), (B – p), (C – s), (D – q)

$$S = \frac{1}{2} \times 2 \times (4)^2 = 16m$$

$w.d_{gravity}$ = – mg × 16 = –1 × 10 × 16 = –160 J

$w.d_{normal\ reaction}$ = N cos θ × S = m(g+a) $\cos^2\theta$ × S = 144J

$w.d_{friction}$ = f × S × sinθ

= m(g+a) $\sin^2\theta$ × S = 48 J

$w.d._{forces}$ = ΔK.E.

$$= \frac{1}{2}m(at)^2 = \frac{1}{2} \times 1 \times (2 \times 4)^2 = 32J$$

5. **c**

work done by both against gravity = mgh

6. **b**

$$\text{Average Power} = \frac{mgh}{t} = \frac{50 \times 10 \times 15}{30} = 250W$$

7. **b**

Chemical energy expended by the physicist ends up increasing the potential energy.

8. **b**

As the physicist falls, gravitational potential gets converted into kinetic energy, increasing his speed. After he hits the cushion, this kinetic energy gets converted into heat.

9. d

$\Delta K.E. = K.E_f - K.E_i = K.E_f - 0$

= w.d. by mg

= mgh′

$= 50 \times 10 \times \frac{15}{3} = 2500J$

10. **a**

11. **c**

12. **a**

From conservation of energy at A and B, we have $\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mgR(1+\sin\theta)$ … (1)

At B the string becomes slack. Therefore

$mg\sin\theta = \frac{mv_B^2}{R}$ … (2)

After passing through B, the ball goes in a projectile

$\Rightarrow \quad v_B \sin\theta t = R\cos\theta$ … (3)

and $-v_B\cos\theta t + \frac{1}{2}gt^2 = R + R\sin\theta$ … (4)

On solving 1, 2, 3 & 4

$\theta = 30°$

$v = \sqrt{\frac{7gR}{2}}$ and $v_B = \sqrt{\frac{gR}{2}}$

13. **c**

$\lambda(\ell - x)v + \lambda hg\,dt - \lambda v^2 dt$

$= \lambda(\ell - (x + dx)](v + dv)$, where λ is mass per unit length

$\Rightarrow \quad hg\,dt = (\ell - x)\,dv$

$\Rightarrow \quad hg\int_{x=0}^{x}\frac{dx}{\ell - x} = \int_{v=0}^{v} v\,dv$

$\Rightarrow \quad \frac{v^2}{2} = hg\ln\frac{\ell}{\ell - x}$

$\therefore \quad v_{\text{at B}} = \sqrt{2gh\ln\frac{\ell}{h}}$

14. **a**

$KE_x = \lambda(\ell - x)gh\ln\frac{\ell}{\ell - x}$

It is maximum, when $\frac{\ell}{\ell - x} = e$

$\therefore \quad KE_{max} = \lambda hg\frac{\ell}{e} = \frac{mgh}{e}$

15. **b**

Heat generated = $\lambda(\ell - h)\, gh - \frac{1}{2}.\lambda h\,.\,2gh\ln\frac{\ell}{h}$

$= \frac{mgh}{\ell}\left[\ell - h - h\ln\frac{\ell}{h}\right]$

16. **b**

17. **a**

18. **b**

19. **c**

$U(x) = 20 + (x - 3)^2$
At x = 0,
T.M.E = U + K.E
= 20 + 9 + 20 = 49 J
At extreme positions, K.E = 0
$\Rightarrow \quad U = 49J$
$\Rightarrow \quad 20 + (x - 3)^2 = 49$
$\Rightarrow \quad x - 3 = \pm\sqrt{29}$
$\Rightarrow \quad x = 3 \pm \sqrt{29}$ i.e., -3.4 and 7.4 m
$K.E._{max}$ = T.M.E – Min. potential energy
= 49 – 20 (at x = 3)
= 29 J
Body is in equilibrium at min. potential energy
i.e., at x = 3

20. W_{mg} is path and rate independent

21. Total energy is conserved when there is no external and no internal non conservative force.

22. Work done for conservative forces are path independent

23. $W_{mg} = mgh$

24. $F.\Delta\vec{r} = \vec{F}.(\vec{r}_2 - \vec{r}_1)$

25. $t = \frac{2v\sin\theta}{g}$ is time of flight and vertical displacement is zero.

EXERCISE – 4

1. $W_F = Fh = 80J$; $W_{weight} = -(mg)h = -40$ J.

2. Tension, $T = \frac{2m_1m_2}{m_1+m_2}g$; acceleration $a = \frac{m_2-m_1}{m_1+m_2}g$

$\therefore \quad W = T \cdot \frac{1}{2}at^2 = \frac{m_1m_2(m_2-m_1)}{(m_1+m_2)^2}g^2t^2$

$= \frac{200}{9}J$

3. $W = \int_{x=1}^{x=2}(2+x)\,dx = 3.5J$

4. $\frac{F}{\sqrt{2}} + N = mg$; $\frac{F}{\sqrt{2}} = \mu N$

$\therefore \quad \frac{F}{\sqrt{2}} = \mu\left(mg - \frac{F}{\sqrt{2}}\right) \Rightarrow \frac{F}{\sqrt{2}} = \frac{\mu}{\mu+1}mg = 3.6$

(a) $W_F = \frac{F}{\sqrt{2}}S = 7.2J$

(b) $W_{friction} = -W_F = -7.2J$

(c) $W_{gravity} = 0$

5. W = Area under the curve $= 10 \times 2 + \frac{1}{2}\times 2\times 10$

$= 30J$

6. W_F = increase in potential energy = $mg\,\ell(1-\cos\theta)$

7. $dW = mg(\mu\cos\theta + \sin\theta)\,ds = mg\,(\mu d\ell + dh)$

$\therefore \quad W = mg(\mu\ell + h)$

8. $W = \Delta KE = -\frac{1}{2}(2)\,20^2 = -400J$

9. $W = \Delta KE = \frac{1}{2}m\alpha^2 v$

10. a) $(2m)g\,x_m = \frac{1}{2}k\,x_m^2 \Rightarrow x_m = \frac{4mg}{k}$

b) $\frac{1}{2}(3m)v^2 + \frac{1}{2}K\left(\frac{x_m}{2}\right)^2 = (2m)\,g\left(\frac{x_m}{2}\right)$

$\Rightarrow v = 2g\sqrt{\frac{m}{3k}}$

c) $2mg - k\frac{x_m}{4} = (3m)\,a \Rightarrow a = \frac{g}{3}$

11. $K\left(\frac{2m_Ag}{K}\right) = mg \Rightarrow m_A = \frac{m}{2}$

12. Let x be the extension of the spring and θ the angle that the spring makes with the vertical at break off.

$$K\ x\cos\theta = mg \Rightarrow 40x\frac{0.4}{0.4+x} = 3.2$$

$\Rightarrow$ x = 0.1m; The … of B = the slide of $A = \sqrt{(0.5)^2-(0.4)^2} = 0.3$ metres = h (say)

$$\frac{1}{2}(2m)v^2 + \frac{1}{2}Kx^2 = mg\ h$$

$\Rightarrow$ v = 1.54 m/s

13. $-\mu\frac{mv^2}{R} = m\frac{dv}{dt} \Rightarrow \int_{v=v_0}^{v}\frac{dv}{v^2} = -\frac{\mu}{R}\int_{t=0}^{t}dt$

$\Rightarrow$ $\frac{1}{v} = \frac{1}{v_0} + \frac{\mu t}{R} \Rightarrow v = v_0\frac{R}{R+\mu v_0 t};\ S = \int_0^t v\,dt = \frac{R}{\mu}\ln\left(1+\frac{\mu v_0}{R}t\right)$

14. $Fb(1^{-\sin\theta}) = 2\times\frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{Fb(1-\sin\theta)}{m}}$, $F_{max} = 2mg$

15.

Conserving mechanical energy:

$$2\times 10\times 1 = 0.5\times 10\times(\sqrt{5}-1) + \frac{1}{2}\times 2\times v^2 + \frac{1}{2}\times 0.5\left(\frac{2v}{\sqrt{5}}\right)^2$$

$\Rightarrow$ v = 3.39 m/s

16. $W = \int_1^2 \vec{F}\cdot d\vec{s} = \int_{(2,3)}^{(4,6)}(3x^2\hat{i}+2y\hat{j})\cdot(dx\hat{i}+dy\,\hat{j}+dz\hat{k})$

$$= \int_{(2,3)}^{(4,6)}(3x^2\,dx + 2y\,dy) = x^3\Big|_2^4 + y^2\Big|_3^6$$

= 83J

17. (i) $W = \int_{0\,(\text{along}OC)}^{c}(xy\,\hat{i} + xy\,\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j})$

$$= \int_0^4 x^2(\hat{i}+\hat{j})\cdot 2dx\,\hat{i} = \int_0^1 2x^2\,dx = \frac{2}{3}\,J$$

(ii) $W = \int_{0\,(\text{along OA})}^{A} xy\,(\hat{i}+\hat{j})\cdot dx\,\hat{i} + \int_{A\,(\text{along AC})}^{C} xy\,(\hat{i}+\hat{j})\cdot dy\,\hat{j}$

$$= 0 + \int_{y=0}^{y=1} y\,dy = \frac{1}{2}\,J$$

(iii) $W = 0 + \int_{k=0}^{k=1} x\,dx = \frac{1}{2}\,J$

18. (a) $\int dmg\,h = mgh$

(b) $\int_{x=0}^{x=\ell}\left(\frac{m}{\ell}dx\right)gx = \frac{1}{2}mg\ell$

(c) $\int_{\theta=0}^{\theta=\ell/R}\left(\frac{m}{\ell}Rd\theta\right)g\,(R\cos\theta)=\frac{mgR^2}{\ell}\sin\frac{\ell}{R}$

19. (a) $u\cos\theta = v$

(b) $m\times 10\times 5=\frac{1}{2}mv^2+\frac{1}{2}m\left(\frac{v}{0.8}\right)^2$

$\Rightarrow v=\frac{40}{\sqrt{41}}\ m/s$

20. $mg\left(\frac{3}{4}d\right)+\frac{1}{2}k\left(\frac{D}{4}\right)^2=\frac{1}{2}mv^2$

$\Rightarrow \quad v=d\sqrt{\frac{3g}{2d}+\frac{k}{16m}}$

21. $mg\,h_{min}=mg\,2r+\frac{1}{2}m\,(\sqrt{gr})^2$

$\Rightarrow \quad h_{min}=\frac{5r}{2};\ mg(5r)-mg\,2r=\frac{1}{2}mv^2$

Now, $F_{resultant}=\frac{mv^2}{r}=6mg$

22. $mg\,(1-\cos\theta)=\frac{mv^2}{\ell}$ … (1)

$\frac{1}{2}mg\ell+mg\ell(1-\cos\theta)=\frac{3}{2}mv^2$ … (2)

From (1) and (2)

$v=\sqrt{\frac{g\ell}{3}};\ \theta=\cos^{-1}\frac{2}{3}$

23. $\frac{1}{2}mv_0^2=mg\ell(1-\cos 60^\circ)$

$\Rightarrow \quad v_0=\sqrt{g\ell}=\sqrt{9.8\times 5}=7\ m/s$

24. $\frac{1}{2}m(\sqrt{5gR})^2-mgR(1+\cos\alpha)=\frac{1}{2}mv^2$ … (1)

Also, $t_{flight}=\frac{2R\sin\alpha}{v\cos\alpha}=\frac{2v\sin\alpha}{g}$ … (2)

From (1) and (2)

$\alpha = 0$ or $\alpha = 60°$

25. $\frac{1}{2}mv^2-mg.2R=\frac{1}{2}mv^2$, where v is velocity at the highest point.

$\Rightarrow \quad v=\sqrt{u^2-4gR}$

Now, $v\,t_{flight}=3R$

$\Rightarrow \quad \sqrt{u^2-4gR}\sqrt{\frac{4R}{g}}=3R$

$\Rightarrow \quad u = \frac{5}{2}\sqrt{gR}$;

$x_{min} = v_{min}\ t_{flight} = \sqrt{gh}\sqrt{\frac{4R}{g}} = 2R$

26. $\int_{x=0}^{x=\pi R} (\lambda dx)\ g\left[r\sin\frac{x}{r} + x\right] + \frac{1}{2}(\pi r\lambda)\ v^2$

$\Rightarrow \quad v = \sqrt{2gr\left(\frac{2}{\pi} + \frac{\pi}{2}\right)}$

27. $mgR\left(\frac{1}{4} + 1 - \cos\theta\right) = \frac{1}{2}mv^2$ … (1)

$mg\cos\theta = m\frac{v^2}{R}$ … (2)

$\Rightarrow \quad \theta = \cos^{-1}\frac{5}{6}$

28. $mg\sqrt{\left(\frac{n+1}{n+3}\right) - 1} - Mg\left(\sqrt{n^1 - 1} - \sqrt{\left(\frac{n+1}{2}\right)^2 - 1}\right) > 0$

$\Rightarrow \quad \frac{m}{M} > 2\sqrt{\frac{n+1}{n+3}} - 1$

29. $(F\,R\sqrt{2} - mgR) = \frac{1}{2}mv^2$

$\Rightarrow \quad v = \sqrt{2R\left(\frac{F\sqrt{2}}{m} - g\right)}$

30. $\frac{1}{2}mv_0^2 = \int_{x=0}^{x=2L} (Gx)(mg)\ dx + \frac{1}{2}k\ L^2$

$\Rightarrow \quad v = \sqrt{4ag + \frac{k}{m}}$

31. Stretch x = 0.4 (sec 30° - 1) = 0.4 $\left(\frac{2}{\sqrt{3}} - 1\right)$; kx sin 30° = μ (mg – kx cos 30°)

$\Rightarrow \quad kx = \frac{\mu mg}{\sin 30^o + \mu\cos 30^o}$

Now, W = ΔU = $\frac{1}{2}kx^2 = 0.09J$

32. a) $mg(1 - \cos\theta) = \frac{1}{2}mv^2$ … (1)

$F + mg\cos\theta = \frac{mv^2}{R}$ … (2)

From (1) and (2)

F = mg (2 – 3 cos θ)

$N = Mg - 2F\cos\theta, = Mg - 2\,mg\,(2 - 3\cos\theta)\cos\theta$

which is minimum when $\theta = \cos^{-1}\frac{1}{3}$

b) $N = 0 \Rightarrow \frac{m}{M} = \frac{3}{2}$

EXERCISE - V

1. **(b)**

The centripetal acceleration

$a_c = k^2rt^2$ or $\frac{v^2}{r} = k^2rt^2$

$\therefore$ $v = krt$

So, tangential acceleration, $a_t = \frac{dv}{dt} = kr$

Work is done by tangential force.

Power $= F_t \cdot v \cdot \cos 0^o$

$= (m\,a_t)\,(krt)$

$= (mkr)\,(krt)$

$= mk^2\,r^2t$

2. **(b)**

The force constant of a spring is inversely proportional to the length of the spring.

Let the original length of spring be L and spring constant is K (given)

Therefore,

$K \times L = \frac{2L}{3} \times K'$ $\Rightarrow K' = \frac{3}{2}K$

3. **(d)**

$dU_{(x)} = -Fdx$

$\therefore$ $U_x = -\int_0^x Fdx$

$= \frac{kx^2}{2} - \frac{ax^4}{4}$

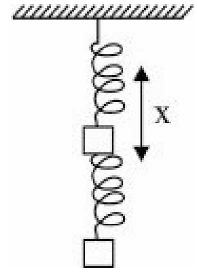
U = 0 at x = 0 and at $x = \sqrt{\frac{2k}{a}}$; $\Rightarrow$ we have potential energy zero twice (out of which one is at origin).

Also, when we put x = 0 in the function,

We get F= 0. But $F = -\frac{dU}{dx}$

$\Rightarrow$ At $x = 0$; $\frac{dU}{dx} = 0$ i.e. the slope of the graph should be zero. These characteristics are represents by (d).

4. **(b)**



Let the maximum extension of the spring be x as shown in the figure. Work is done by the gravitational and the spring force. There is no change in the kinetic energy between the initial and final position of the mass.
From Work-energy theorem;
$W_g + W_s = 0$

Where W_g = work done by gravity
And W_s^g = work done by spring

$$\Rightarrow +Mgx - \frac{1}{2}kx^2 = 0$$

$$\Rightarrow x = \frac{2Mg}{k}$$

5. **(b)**
In a conservative field work done does not depend on the path. The gravitational field is a conservative field.
$\therefore W_1 = W_2 = W_3$

6. **(b)**
We know that

$$\Delta U = -\int_0^x F dx \text{ or } \Delta U = -\int_0^x k\,x dx$$

$$\Rightarrow U_{(x)} - U_{(0)} = -\frac{kx^2}{2}$$

Given $U_{(0)} = 0$

$$U_{(x)} = -\frac{kx^2}{2}$$

7. **(d)**

$v = \sqrt{5gL}$ … (1)

$\left(\frac{v}{2}\right)^2 = v^2 - 2gh$ … (2)

$h = L(1 - \cos\theta)$ … (3)

Solving Eqs. (1), (2) and (3), we get

$$\cos\theta = -\frac{7}{8} \text{ or } \theta = \cos^{-1}\left(-\frac{7}{8}\right) = 151^o$$

8. **(c)**
When the block B is displaced towards wall 1, only spring S_1 is compressed and S_2 is in its natural state. This happens because the other end of S_2 is not attached to the wall but is free. Therefore the energy stored in the system $= \frac{1}{2}k_1x^2$. When the block is released, it will come back to the equilibrium position, gain momentum, overshoot to equilibrium position and move towards wall 2. As this happens, the spring S_1 comes to its natural length and S_2 gets compressed. As there are no frictional forces involved, the P.E. stored in the spring S_1 gets stored as the P.E. of spring S_2 when the block B reaches its extreme position after compressing S_2 by y.

$\therefore \quad \frac{1}{2}k_1x^2 = \frac{1}{2}k_2y^2$

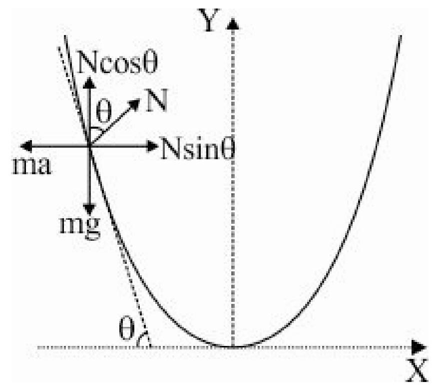
$\frac{1}{2}\times kx^2 = \frac{1}{2}4ky^2$

$x^2 = 4y^2$

$\therefore \quad \frac{y}{x} = \frac{1}{2}$

9. **(b)**

The forces acting on the bead as seen by the observer in the accelerated frame are: (a) N; (b) mg; (c) ma (Pseudo force).



Let θ is the angle which the tangent at P makes with the X-axis. As the bead is in equilibrium with respect to the wire, therefore

N sin θ = ma and N cos θ = mg

$\therefore \quad \tan\theta = \frac{a}{g} \qquad ...(i)$

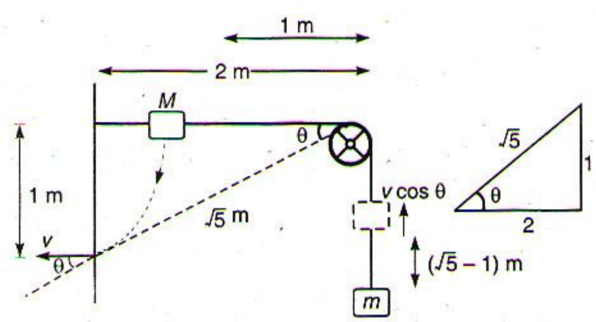
But y = kx^2. Therefore,

$\frac{dy}{dx} = 2kx = \tan\theta \qquad ..(ii)$

From (i) & (ii)

$2kx = \frac{a}{g} \Rightarrow x = \frac{a}{2kg}$

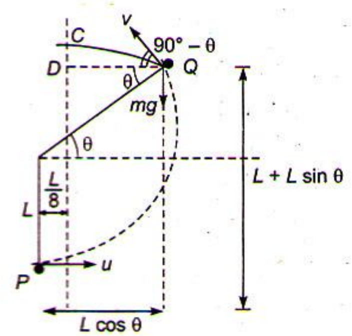
10. Let M strikes with speed v. Then, velocity of m at this instant will be v cos θ or $\frac{2}{\sqrt{5}}v$. Further M will fall a distance of 1 m while m will rise up by $(\sqrt{5}-1)m$. From energy conservation: decrease in potential energy of M = increase in potential energy of m + increase in kinetic energy of both the blocks.



or (2) (9.8) (1) = (0.5) (9.8) $(\sqrt{5}-1)+\frac{1}{2}\times 2\times v^2 + \frac{1}{2}\times 0.5\times\left(\frac{2v}{\sqrt{5}}\right)^2$

Solving this equation, we get v = 3.29 m/s

11. Let the string slacks at point Q as shown in figure. From P to Q path is circular and beyond Q path is parabolic. At point C, velocity of particle becomes horizontal, therefore, QD = half the range of the projectile.



Now, we have following equations

(1) $T_Q = 0$. Therefore, $mg \sin\theta = \dfrac{mv^2}{L}$ … (i)

(2) $v^2 = u^2 - 2gh = u^2 - 2gL(1 + \sin\theta)$ … (ii)

(3) $QD = \dfrac{1}{2}$ (Range)

$$\Rightarrow \left(L\cos\theta - \frac{L}{8}\right) = \frac{v^2 \sin 2(90^\circ - \theta)}{2g} = \frac{v^2 \sin 2\theta}{2g} \quad \text{… (iii)}$$

Eq. (iii) canbe written as

$$\left(\cos\theta - \frac{1}{8}\right) = \left(\frac{v^2}{gL}\right)\sin\cos\theta$$

Substituting value of $\left(\dfrac{v^2}{gL}\right) = \sin\theta$ from eq. (i), we get

$$\left(\cos\theta - \frac{1}{8}\right) = \sin^2\theta - \theta = (1 - \cos^2\theta)\cos\theta$$

or $\cos\theta - \dfrac{1}{8} = \cos\theta - \cos^3\theta$

$\therefore$ $\cos^3\theta = \dfrac{1}{8}$ or $\cos\theta = \dfrac{1}{2}$ or $\theta = 60^\circ$

From Eq. (i) $v^2 = gL \sin\theta = gL \sin 60°$

or $v^2 = \dfrac{\sqrt{3}}{2} gL$

$\therefore$ Substituting this value of v^2 in Eq. (ii)

$u^2 = v^2 + 2gL(1 + \sin\theta)$

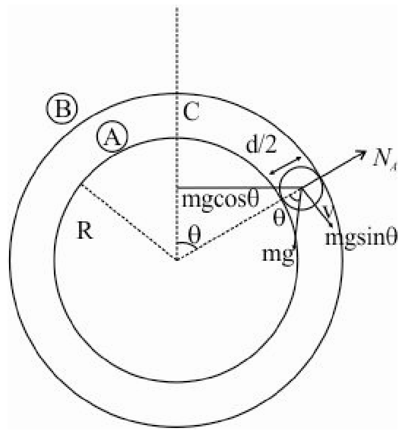
$$= \frac{\sqrt{3}}{2} gL + 2gL\left(1 + \frac{\sqrt{3}}{2}\right)$$

$$= \frac{3\sqrt{3}}{2} gL + 2gL$$

$$= gL\left(2 + \frac{3\sqrt{3}}{2}\right)$$

$$u = \sqrt{gL\left(2 + \frac{3\sqrt{3}}{2}\right)}$$

12. The ball is moving in a circular motion. The necessary centripetal force is provided by ($mg\cos\theta - N$). Therefore,



$$mg\sin\theta - N_A = \frac{mv^2}{\left(R+\frac{d}{2}\right)} \qquad ...(i)$$

According to energy conservation

$$\frac{1}{2}mv^2 = mg\left(R+\frac{d}{2}\right)(1-\cos\theta)...(ii)$$

From (i) and (ii)

$N_A = mg\,(3\cos\theta - 2)$ …(iii)

The above equation shows that as θ increases N_A decreases. At a particular value of θ, N_A will become zero and the ball will lose contact with sphere A. This condition can be found by putting $N_A = 0$ in eq. (iii)

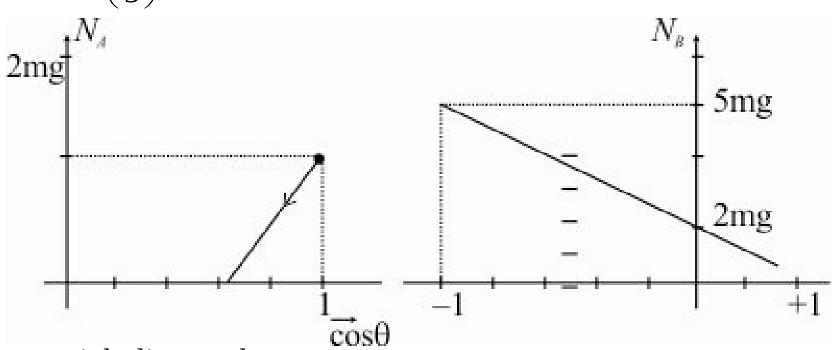
$0 = mg(3\cos\theta - 2)$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

The graph between N_A and cosθ

From equation (iii) when θ = 0, N_A = mg.

When $\therefore \quad \theta = \cos^{-1}\left(\frac{2}{3}\right)$



The graph is a straight line as shown

when $\theta > \cos^{-1}\left(\frac{2}{3}\right)$

$$N_B - (mg\cos\theta) = \frac{mv^2}{R+\frac{d}{2}}$$

$$\Rightarrow \quad N_B + mg\cos\theta = \frac{mv^2}{\left(R+\frac{d}{2}\right)} \qquad ...(iv)$$

Using energy conservation

$$\frac{1}{2}mv^2 = mg\left[\left(R+\frac{d}{2}\right) - \left(R+\frac{d}{2}\right)\cos\theta\right]$$

$$\frac{mv^2}{\left(R+\frac{d}{2}\right)} = 2mg[1-\cos\theta] \qquad ...(v)$$

From (iv) and (v), we get

N_B + mg cosθ = 2mg – 2mg cosθ

N_B = mg(2 – 3 cosθ)

When $\cos\theta = \frac{2}{3}$, $N_B = 0$

When cosθ = –1, N_B = 5 mg

13. Given m = 0.36 kg, M = 0.72 kg.

The figure shows the forces on m and M. When the system is released, let the acceleration be a. Then

T – mg = ma

Mg – T = Ma

$$\therefore \quad a = \frac{(M-m)g}{M+m} = g/3$$

and T = 4mg/3

For block m:

u = 0, a = g/3, t = 1, s=?

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\times\frac{g}{3}\times 1^1 = g/6$$

∴ Work done by the string on m is

$$\vec{T}.\vec{s} = Ts = 4\frac{mg}{3}\times\frac{9}{6} = \frac{4\times 0.36\times 10\times 10}{3\times 6} = 8J$$