

EXERCISE 2 (A)

61. Compare with $Ax^2 + 2Hxy + By^2 + 2ax + 2fy + C = 0$

$$A = 1, B = 1, H = 0, G = g, F = f, C = 1$$

$$\therefore \Delta = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & g \\ 0 & 1 & f \\ g & f & 1 \end{vmatrix} = 0$$

$$\Rightarrow (1-f^2) + g(-g) = 0$$

$$f^2 + g^2 = 1$$

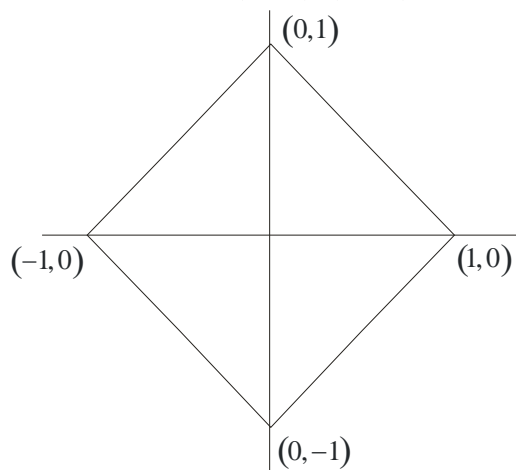
62. $A = \lambda, B = 2, H = -5/2, G = 5/2, f = -7/2, C = 3$

Put $\Delta = 0$

$$\Rightarrow \begin{vmatrix} \lambda & -5/2 & 5/2 \\ -5/2 & 2 & -7/2 \\ 5/2 & -7/2 & 3 \end{vmatrix} = 0$$

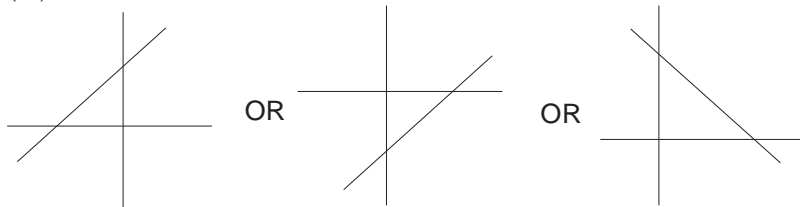
Solve to get $\lambda = 2$

63. Area enclosed by $|x-1| + |y-3| = 1$ is same as enclosed by $|x| + |y| = 1$ (shift of origin)



$$\therefore A = 2$$

64. (D)



Here

$$\left(\frac{-c}{a}\right) < 0$$

$$\& \frac{-c}{b} > 0$$

$$\frac{-c}{a} > 0$$

$$\frac{-c}{b} < 0$$

$$\frac{-c}{a} > 0$$

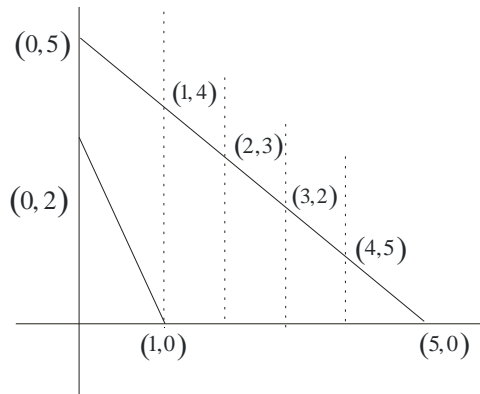
$$\& \frac{-c}{b} > 0$$

65. Equation of pair of bisection is $h(x^2 - y^2) = xy / (a - b)$

\therefore of live pair is coordinate Axes $\Rightarrow h = 0$

So that equation is $xq = 0$ as $x = 0, y = 0$

66.



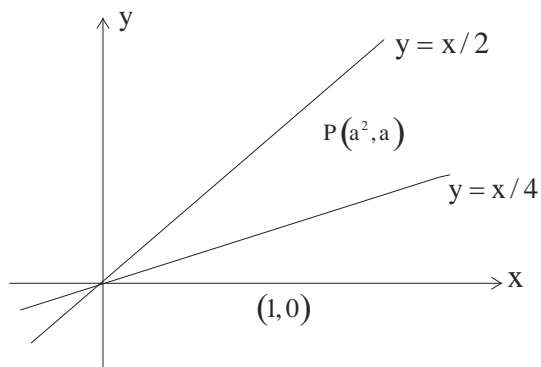
Put $x = 1$, 3 pH here

Put $n = 2$, 2 pH

Put $n = 3$, 1 pt here

\therefore Total 6 gourd point

67. Plot lines



Let $L_1 = 2y - x$, $L_2 = 4y - x$

\therefore pt $(1, 0)$ gives -lve sign to both lines

\therefore P must give + lve to L_1 & - lve to L_2

$\therefore 2a - a^2 < 0$ & at $(0, 4)$

$(-\infty, 0) \cup (3, \infty)$

A $(2, 4)$

68. Let the line be $y - mx - c = 0$

\therefore dist is algebraic distance

$$\frac{1-2m-c}{\sqrt{Hn^2}} + \frac{2-3m-c}{\sqrt{Hn^2}} + \frac{7+4m-c}{\sqrt{Hn^2}}$$

$$10 - m - 3c = 0$$

$$\frac{10}{3} = m + c$$

\therefore passes through $(1, 10/3)$

69. $\frac{ds}{dx} : ax + hg + g = 0$ if n eqn. put $y = 0$

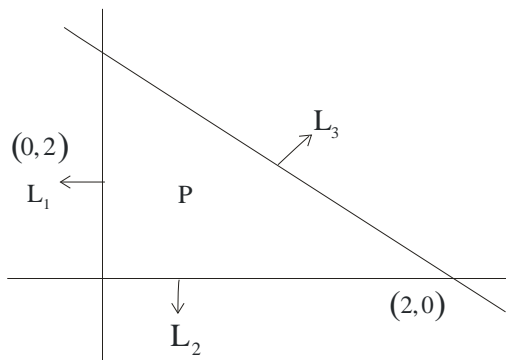
$$\frac{dg}{dy} ; hx + by + f = 0 \quad \frac{-g}{a} = \frac{-f}{h}$$

$$Hg = gf$$

70. Let $P^2(x, y)$
 \therefore Alfa f_1 P^2 is (y, x)
 Alfa f_2 is becomes $(y + 3x, x_1)$
 Alfa f_3 is becomes $\left(\frac{y_1 + 2x_1}{2}, \frac{y_1 + 4x_1}{2}\right)$

$P^2(A)$ becomes $(0, 0)$
 $P^2(B)$ $(4, 0)$ becomes $(4, 8)$
 $P^2(C)$ $(4, 2)$ becomes $(5, 9)$
 $P^2(D)$ $(0, 2)$ becomes $(1, 1)$
 Now pb form 1Lgm

71.



For P to be insides it must give signs (+) \in lve & \in lve w.r.t lives L_1, L_2, L_3 respectively

$$\therefore a > 0, a^2 > 0 \text{ \& \text{ at } a^2 - 2 < 0}$$

$$a > 0 \text{ \& \text{ at } (-2, 1)}$$

$$\Rightarrow \text{ at } (0, 1)$$

72. (B)

$$\text{here } L_1 : x \cos \alpha + y \sin \theta = p$$

$$\Delta L_2 = x \sin \alpha - y \cos \alpha = 0$$

$$ax \perp r \text{ \& \text{ } ax + by \perp p \text{ is @ } \frac{\pi}{4} \text{ with } L_1 \Rightarrow ax + by \quad cp = 0 \text{ is angle}$$

_____ of L_1 & L_2

$$= (x \cos \alpha + y \sin \alpha = p) = 1(x \sin \alpha - y \cos \alpha)$$

Take (+lve & y_n)

$$x(\cos \alpha - \sin \alpha) + y(\sin \alpha + \cos \alpha) - p = 0$$

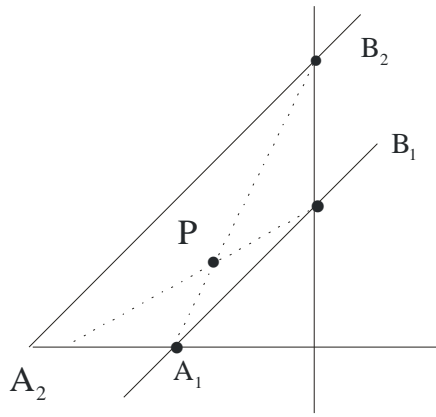
Compare with

$$ax + by + p = 0$$

$$\frac{a}{\cos \alpha - \sin \alpha} = \frac{b}{\sin \alpha + \cos \alpha} = \frac{-p}{p}$$

$$a^2 + b^2 = 2$$

73.



$A_1 A_2 B_1 B_2$

Co cyclic

$$= m_1 m_2 = 1 \text{ or } m^2 = 1$$

$$\Rightarrow m = 1 \quad m \leftarrow \mathbb{R}^{-1}$$

$$A_1 \left(\frac{-C_1}{m}, 0 \right), A_2 \left(\frac{-C_2}{m}, 0 \right)$$

$$B_1(0, C_1) \quad B_2(0, C_2) \quad \text{let } p \text{ be } (h, k)$$

$\therefore A, P \& B_2$ collinear

$A_2, P \& B_1$ collinear

$$\frac{k}{h+C_1} = \frac{C_2}{C_1}$$

$$\frac{k}{h+C_2} = \frac{C_1}{C_2}$$

$$\rightarrow \frac{k}{C_2} = \frac{h}{C_1} + 1$$

&

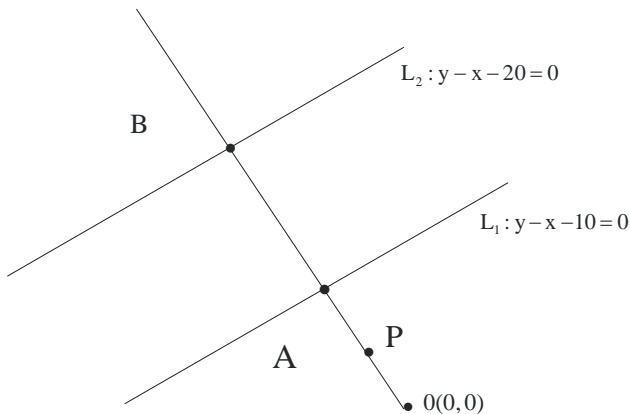
$$\frac{k}{C_1} = \frac{h}{C_2} + 1$$

Subtract to get

$$k \left(\frac{1}{C_2} - \frac{1}{C_1} \right) = h \left(\frac{1}{C_1} - \frac{1}{C_2} \right)$$

$$k = -h$$

74.



Let $p(h, k)$ Now, parametric eqn. of line PAB can be taken as

$$x = r \cos \theta, \quad y = r \sin \theta$$

$$\text{Let } OP = r_1$$

$$\therefore h = r_1 \cos \theta, \quad k = r_1 \sin \theta$$

$$OA = r_2$$

$$A(r_2 \cos \theta, r_2 \sin \theta)$$

$$OB = r_3$$

$$B(r_3 \cos \theta, r_3 \sin \theta)$$

Now put pts on L_1 & L_2

$$r_2 (\sin \theta - \cos \theta) = 10$$

$$r_3 (\sin \theta - \cos \theta) = 20$$

$$r_2 \frac{10}{\sin \theta - \cos \theta}$$

π is given the

$$\frac{2}{r_1} = \frac{1}{r_2} + \frac{1}{r_3}$$

$$\frac{2}{r_1} = \frac{3}{20}(\sin \theta - \cos \theta)$$

$$r_3 = \frac{20}{\sin \theta - \cos \theta}$$

$$\frac{40}{3} = r_1 \sin \theta - r_2 \cos \theta$$

$$\frac{40}{1} = y - x$$

75. From 74 put

$$r_1^2 = r_2 r_3$$

$$r_1^2 = \frac{200}{(\sin \theta - \cos \theta)^2}$$

$$x - (r_1 \sin \theta - r_1 \cos \theta)^2 = 200$$

$$(x - y)^2 = 200$$

76. From 74

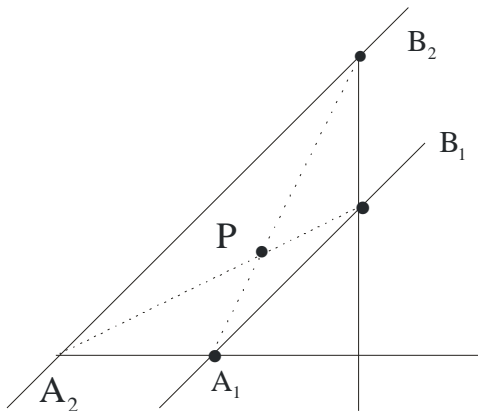
$$\frac{1}{r_1^2} = \frac{1}{r_2^2} + \frac{1}{r_3^2}$$

$$\frac{1}{r_1^2} = \frac{(\sin \theta - \cos \theta)^2}{400}$$

$$80 = (r_1 \sin \theta - r_1 \cos \theta)^2$$

$$(x - y)^2 = 80$$

77.



$$A_1 \left(\frac{-C_1}{2}, 0 \right) \quad B_1 (0, C_1)$$

$$A_2 \left(\frac{-C_2}{2}, 0 \right) \quad B_2 (0, C_2)$$

$P(h, k)$ is collinear with A_1 B_2

$$\frac{k - C_2}{h} = \frac{2C_2}{C_1} \quad \text{or} \quad \frac{k}{C_2} - 1 = \frac{2h}{C_1}$$

Also $p(h, k)$ collinear with A_2 & B_2

$$\Rightarrow \frac{k}{C_1} - 1 = \frac{2h}{C_2}$$

∴ Subtract

$$k \left(\frac{1}{c_2} - \frac{1}{c_1} \right) = 2h \left(\frac{1}{c_1} - \frac{1}{c_2} \right)$$

$$k + 2h = 0$$

78. Let line be $y = mx - c = 0$
Homogenize line with curve

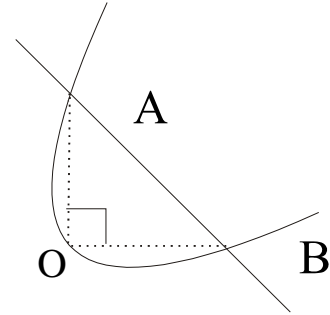
$$3x^2 - y^2 + (4g - 2n) \left(\frac{y - mx}{c} \right) = 0$$

Sin OA & OB

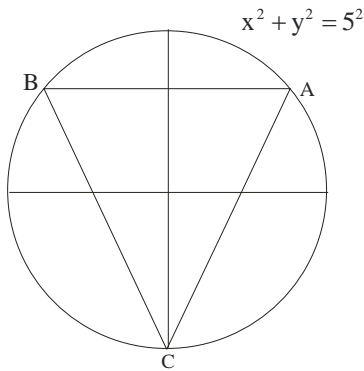
$$\therefore \operatorname{cosec} x^2 + \operatorname{cosec} y^2 = 0$$

$$(3c + 2m) + (-c + 4) = 0$$

$$m + c = 2 \quad p^2 \text{ is } (1, -2)$$



79. (D)
A(3, 4) B(cos θ, 5 sin θ) C(5 sin θ, -5 cos θ)



Now, $(0,0)$ & centroid is $C_1 \left(\frac{3 + 5 \cos \theta + 5 \sin \theta}{3}, \frac{4 + 5 \sin \theta - 5 \cos \theta}{3} \right)$

$$\therefore H = (3 + 5 \cos \theta + 5 \sin \theta), (4 + 5 \sin \theta - 5 \cos \theta)$$

$$h - 3 = 5 \cos \theta + 5 \sin \theta$$

$$k - 4 = 5 \sin \theta - 5 \cos \theta$$

$$x + y - 7 = 10 \sin \theta \text{ also } x - y + 1 = 10 \cos \theta$$

$$(x + y - 7)^2 + (x - y + 1)^2 = 100$$

- 80.

By parametric line

$$k = 0 + 4 \sin \theta$$

$$k = 4 \sin \theta$$

$$\therefore \frac{K}{4} = \frac{H}{3}$$

Also

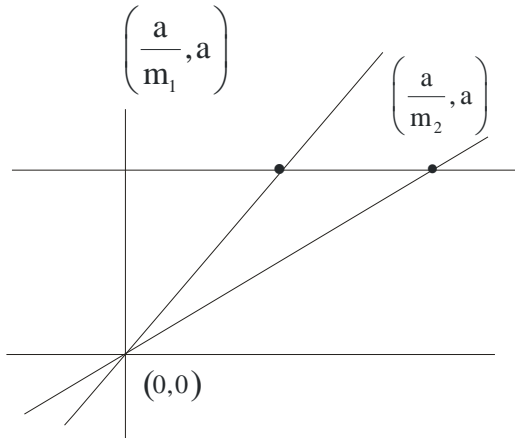
$$h = 0 + 3 \cos(\theta - 90)$$

$$h = 3 \sin \theta$$

Paper – 2(A)

18. (A, C)
 Let P be $(a_1, -39)$
 $(3, 4)$ given (- lve sign with $3x - 4y - 8 = 0$)
 $\therefore (a_1 - 3a)$ must give (+lve sign with line)
 $3a + 12a - 8 > 0$
 $a > \frac{8}{15}$

19. (A, C, D)



$$m_1 + m_2 = a$$

$$m_1 m_2 = -(a + 1)$$

$$\therefore A = \frac{1}{2} \left| \frac{a^2}{m_1} - \frac{a^2}{m_2} \right| = \frac{a^2}{2} \frac{|m_1 - m_2|}{m_1 m_2}$$

$$A = \frac{a^2}{2} \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{|m_1 m_2|} = \frac{a^2}{2} \frac{\left(\sqrt{a^2 + 4(a + 1)} \right)}{|a + 1|}$$

$$A = \frac{a^2}{2} \frac{|a + 2|}{|a + 1|}$$

20. (A, B, C, D)
 If they intersect @ 4 concyclic points

$$\therefore m_1 m_2 = 1$$

$$\left(\frac{a}{b} \right) \left(\frac{d}{c} \right) = 1$$

$$ac = bd$$

$$\text{Now, } (a - 1)^2 = (b - d)^2 \Rightarrow (a - c) = 7(b - d)$$

Now subtract lines

$$x \left(\frac{1}{a} - \frac{1}{c} \right) + y \left(\frac{1}{b} - \frac{1}{d} \right) = 0$$

$$x \left(\frac{c - a}{ac} \right) + y \left(\frac{b - d}{bd} \right) = 0$$

$$\Rightarrow x \pm y = 0$$

21. (A, C)
 Let line be $y = mx + c$

Put $(1,0) \Rightarrow c = -m$

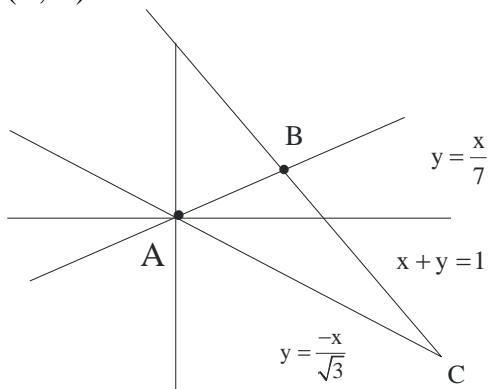
$$\therefore y = m(x-1) \text{ or } mx - y - 1 = 0$$

$$\therefore \text{distance} = \frac{\sqrt{3}}{2}$$

$$\frac{1}{\sqrt{Hm_2}} = \frac{\sqrt{3}}{2} \Rightarrow \frac{4}{3} = m^2 + 1$$

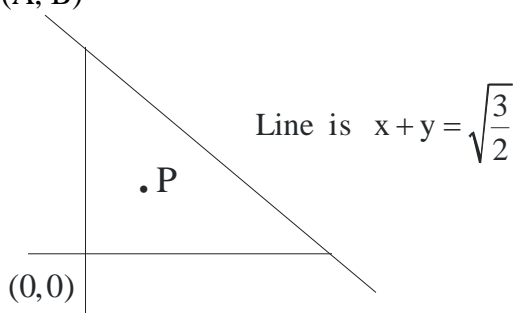
$$m = \pm \frac{1}{\sqrt{3}}$$

22. (B, C)



The Δ is obtuse \Rightarrow interior are In centre & centroid

23. (A, B)



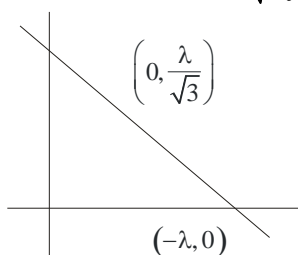
If $P(\sin \theta, \cos \theta)$ inside the Δ

$$\sin \theta > 0 \text{ \& } \cos \theta > 0 \text{ \& } \sin \theta + \cos \theta < \frac{\sqrt{3}}{\sqrt{2}}$$

$$\Rightarrow q \in 1^{\text{st}} \text{ quadrant \& } \sin\left(\theta + \frac{\pi}{4}\right) < \sin \frac{\pi}{3}$$

24. (B,D)

Let the line be $x - \sqrt{3}y + \lambda = 0$



\therefore length of intercept

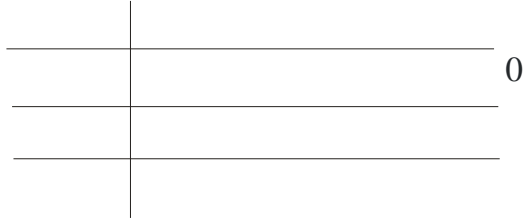
$$\Rightarrow \lambda^2 + \frac{\lambda^2}{3} = 100$$

$$\lambda = 5\sqrt{3}$$

25. (C, D)

$$\begin{vmatrix} 1 & 1 & -1 \\ m-1 & m^2-7 & -5 \\ m-2 & 2m-5 & 0 \end{vmatrix} = 0$$

By property



$$\begin{vmatrix} 0 & 0 & -1 \\ -m^2+m+6 & m^2-12 & -5 \\ -m+3 & 2m-5 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (2m-5)(-m^2+m+6) + (m-3)(m^2-12) = 0$$

$$\Rightarrow (m-3)[-(2m-5)(m-12) + (m^2-12)] = 0$$

$m = 3$ or $m^2 - m + 2 = 0$ discard
 for $m = 3$ lines are parallel

26. (B, C)

Let lines have slope 'm'

$$= \frac{1}{2} = \left| \frac{m+2}{1-2m} \right|$$

$$\frac{m+2}{1-2m} = \frac{1}{2} \quad \text{or} \quad \frac{-1}{2}$$

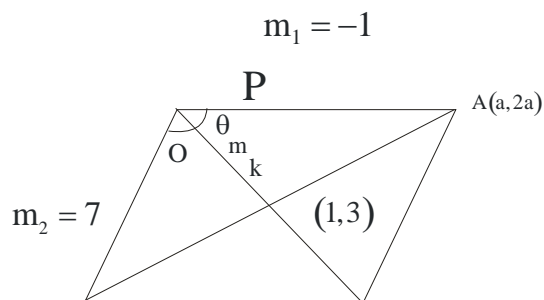
$$2m+4 = 1-2m \quad \text{or} \quad 2m+2 = -1+2m$$

$$4m = -3 \quad \Rightarrow m = \infty$$

$$m = \frac{-3}{4}$$

$$\therefore \text{lines G.M. } \frac{y-3}{x-2} = \frac{-3}{4} \text{ or } \infty$$

27.



Now diagonal bisects the angle

\therefore equality $\tan \theta$

$$\frac{m+1}{1-m} = \frac{7-m}{1+7m}$$

$$\Rightarrow m = \frac{1}{3} \quad \text{or} \quad 3$$

Diagonal are

$$\frac{y-3}{x-1} = \frac{1}{3} \text{ or } -3$$

Put (a, 2a) on there live

$$A = \frac{8}{5} \text{ or } \frac{6}{5}$$

28. (A,B,C,D)

$$\text{Given } \frac{m_1}{m_2} = \frac{9}{2}$$

$$\& \frac{m_1 - m_2}{1 + m_1 m_2} = \pm \frac{7}{9}$$

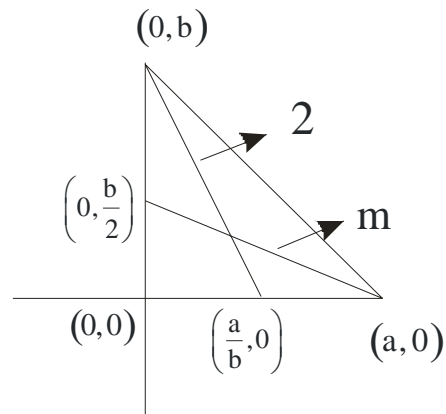
$$\frac{7m_2}{2 + gm_2} = \frac{7}{9} \text{ or } \frac{-7}{9}$$

$$9m_2 = 2 + 9m_2^2 \quad \text{or} \quad -9m_2 = 2 + 9m_2^2$$

$$9m_2^2 - 9m_2^2 + 2 = 0 \quad 9m_2^2 + 9m_2 + 2 = 0$$

$$m_2 = \frac{2}{3} \text{ or } \frac{1}{3} \quad \text{or} \quad m_2 = \frac{-2}{3} \text{ or } \frac{-1}{3}$$

29. Consider (0,b)



$$\text{Here } \frac{-2b}{a} = 2$$

$$\Rightarrow \frac{b}{a} = -1$$

$$m = \frac{-b}{2a} = \frac{1}{2}$$

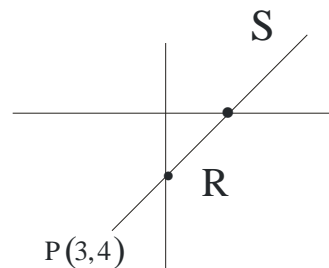
$$\text{Or } \frac{-b}{2a} = 2$$

$$\Rightarrow \frac{-b}{a} = 4$$

$$\& m = \frac{-2b}{a}$$

$$= +8$$

30. (A, B, C, D)



$$\text{Let } PR = r_1$$

$$PS = r_2$$

$$\therefore R(3+r, \cos \theta, 4+r, \sin \theta)$$

$$\Rightarrow 3 + r \cos \theta = 6$$

$$r_1 = 3 \sec \theta$$

The S(3 + r₂, cos θ, 4 + r₂, sin θ)

$$\Rightarrow 4 + r_2 \sin \theta = 8$$

$$R = 4 \operatorname{cosec} \theta$$

31. (A, B, & C)

Put $\frac{y}{x} = m$

$$1 + m - m^2 = m^3$$

$$\text{Or } m^3 + m^2 = m + 1$$

$$\cancel{(m+1)}m^2 = \cancel{(m+1)}$$

$$m = -1 \quad \text{or} \quad m^2 = 1$$

$$m = -1 \quad \quad \quad m = \pm 1$$

32. $x^2 + mxy - 2y^2 + 3y - 1 = 0$

$$A = 1, \quad B = -2, \quad G = 0, \quad f = \frac{3}{2} \quad C = -1 \quad H = \frac{M}{2}$$

$$\Delta = 0 \quad \begin{vmatrix} 1 & \frac{m}{2} & 0 \\ \frac{m}{2} & -2 & \frac{3}{2} \\ 0 & \frac{3}{2} & -1 \end{vmatrix} = 0$$

$$1 \left(2 - \frac{9}{4} \right) - \frac{m}{2} \left[\frac{-m}{2} \right] = 0$$

$$\frac{m^2}{4} - \frac{1}{4} = 0 \Rightarrow m = \pm 1$$

Find intersection Ph

33. (A, B, D)

\Rightarrow Angle between lines must be $180^\circ - 2\alpha$

$$\Rightarrow |\tan 2\alpha| = \left| \frac{2\sqrt{h^2 - 1}}{2} \right|$$

$$\tan^2 2\alpha = h^2 - 1$$

$$h = \sqrt{8a^2\alpha}$$

34. (A, B, C, D)

Use condition of both roots common

$$3\left(\frac{y}{x}\right)^2 + p\left(\frac{y}{x}\right) + 2 = 0 \quad \text{has roots } m \text{ \& } m_1$$

$$-3\left(\frac{y}{x}\right)^2 + 9\left(\frac{y}{n}\right) + 2 = 0 \quad \text{has roots } m \text{ \& } m_2$$

$$m + m_2 = \frac{-p}{3}, \quad mm_1 = \frac{2}{3} \quad \& \quad m + m_2 = \frac{9}{3}$$

$$\text{Also } m_1 m_2 = 1$$

$$m m_2 = \frac{-2}{3}$$

$$\text{From here } m^2 = \frac{4}{9}$$

$$7m = \frac{2}{3}$$

OR

$$7m = \frac{-2}{3}$$

$$7m = \frac{2}{3}$$

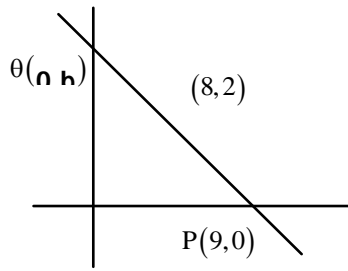
$$m_1 = -1 \quad \& \quad m_2 = 1$$

$$m_1 = 1 \quad \& \quad m_2 = -1$$

$$p = 5, \quad q = +1$$

$$P = -5 \quad q = -1$$

23.



a, b (+) ve

$$\text{also } \frac{9}{a} + \frac{2}{b} = 1$$

Weighted A.M \geq Weighted H.M

$$\frac{(2\sqrt{2})\left(\frac{2\sqrt{2}}{a}\right) + (\sqrt{2})\left(\frac{\sqrt{2}}{b}\right)}{2\sqrt{2} + \sqrt{2}}$$

$$\frac{1}{3\sqrt{2}} \geq \frac{3\sqrt{2}}{a+b}$$

$$a+b \geq 18$$

M, n 18

24. From Q. 23

Use A.M \geq G.M

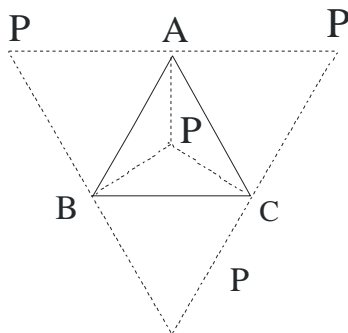
$$\frac{\frac{8}{a} + \frac{1}{b}}{2} \geq \sqrt{\frac{16}{ab}}$$

$$\frac{1}{4} \geq \frac{16}{4b}$$

$$ab \geq 64$$

$$A \geq 32$$

25. 4 position possible



Here P such that P, A, B, & C form is any order vertices taken & 4H position is centroid

26. given ax $x^2 - (y-2)^2 = 0$
 $X + y - 2 = 0$ or $x - y + 2 = 0$

Bisection Are

$$\frac{x+y-2}{\sqrt{2}} = 1 \frac{(x-y+2)}{\sqrt{2}}$$

i.e. $x+y-2 = x-y+2$
 $y = 2$

$$x+y-2 = -x+y-2$$

$$x = 0$$

3 lines $x+y=3$, $y=2$, $x=0$

3 vertices (0,2) (0,3) (1,2) $A = \frac{1}{2}$

27. Given $2x + 3y = 6$

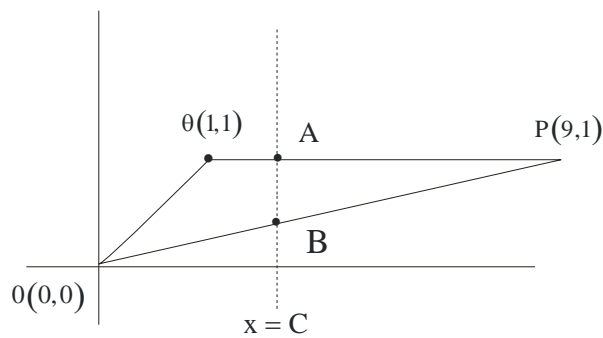
Use equality – Inequality

$$(2x + 3y) \leq \sqrt{2^2 + 3^2} \sqrt{x^2 + y^2}$$

$$\Rightarrow \frac{6}{\sqrt{13}} \leq \sqrt{x^2 + y^2}$$

$$m = \frac{6}{\sqrt{13}}$$

28.



A is (C, 1)

B is $(C, \frac{C}{3})$

\therefore Area of ΔABP should be 2

Area of ΔOPQ

$$\therefore \frac{1}{2} \left| \left(\frac{C-9}{3} \right)^2 \right| = 2$$

Discard $C = 15$ or $C = 3$

Take $C = 3$

29. $a(2x + y - 3) + b(x + 3y + 1) = 0$

This line passes through intersection of $2x + y - 3 = 0$ and $x + 3y + 1 = 0$

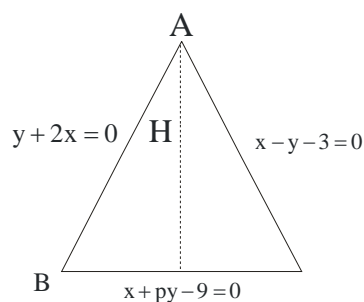
i.e. (2, -1)

It also satisfies line $mx + 2y + 6 = 0$

$$2m - 2 + 6 = 0$$

$$m = -2$$

30.



Altitudes from A is $(y + 2x) + \lambda(x - y - 3) = 0$

Its slope should be $\frac{1}{2}$ Also H satisfies it

$$-\frac{(\lambda+2)}{1-\lambda} = p$$

$$7 + \lambda(-7)$$

$$\lambda = \frac{7}{4}$$

\Rightarrow P is 5

Similarly get q = 45

$$\frac{p+q}{10} = 5$$