EXERCISE 2 (A)

61. Compare with $Ax^2 + 2Hxy + By^2 + 2ax + 2fy + C = 0$

A = 1, B = 1, H = 0, G = g, F = f, C = 1

$\therefore \Delta = 0$

$$\Rightarrow \begin{vmatrix} 1 & 0 & g \\ 0 & 1 & f \\ g & f & 1 \end{vmatrix} = 0$$

$$\Rightarrow \left(1 - f^2\right) + g(-g) = 0$$

$$f^2 + g^2 = 1$$

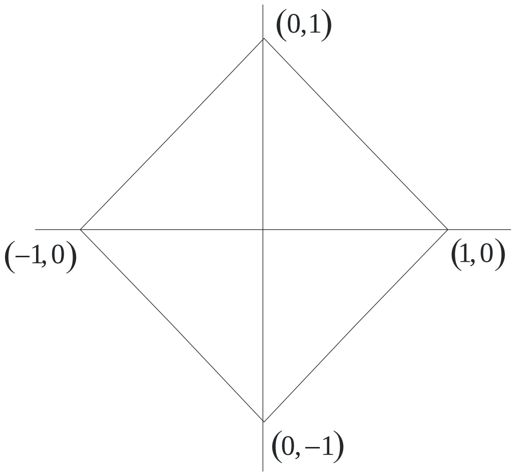
62. $A = \lambda,\ B = 2,\ H = -5/2,\ G = 5/2,\ f = -7/2,\ C = 3$

Put $\Delta = 0$

$$\Rightarrow \begin{vmatrix} \lambda & -5/2 & 5/2 \\ -5/2 & 2 & -7/2 \\ 5/2 & -7/2 & 3 \end{vmatrix} = 0$$

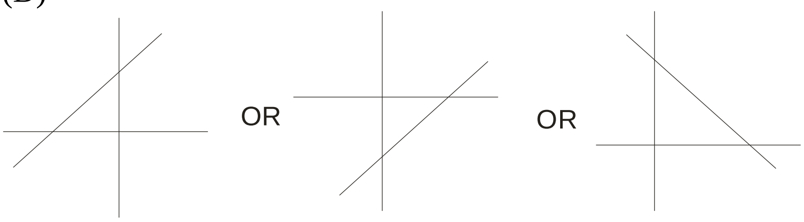
Solve to get $\lambda = 2$

63. Area enclosed by $|x-1| + |y-3| = 1$ is same as enclosed by $|x| + |y| = 1$ (shift of origin)



$\therefore \quad A = 2$

64. (D)



Here

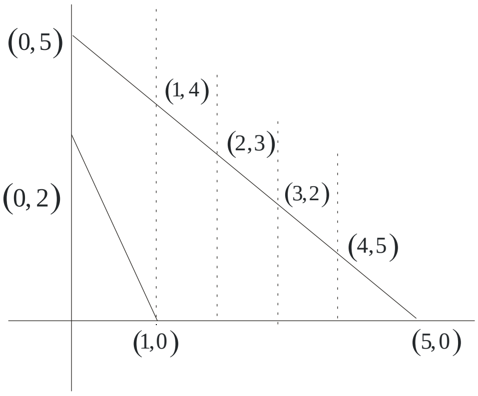
$\left(\frac{-c}{a}\right) < 0$	$\frac{-c}{a} > 0$	$\frac{-c}{a} > 0$
$\& \frac{-c}{b} > 0$	$\frac{-c}{b} < 0$	$\& \frac{-c}{b} > 0$

65. Equation of pair of bisection is $h\left(x^2 - y^2\right) = xy/(a-b)$

$\therefore$ of live pair is coordinate Axes $\Rightarrow h = 0$

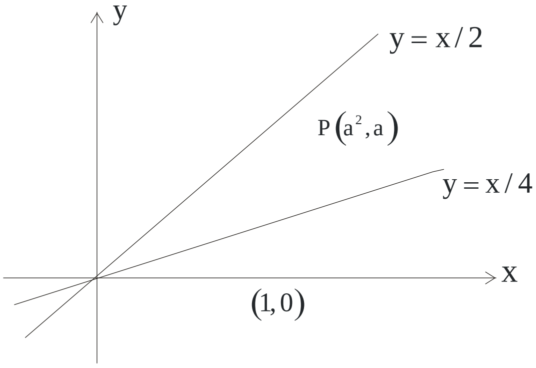
So that equation is $xq = 0$ as $x = 0, y = 0$

66.



Put x = 1,   3 pH here
Put n = 2,   2 pH
Put n = 3,   1 pt here
$\therefore$ Total 6 gourd point

67. Plot lines



Let $L_1 = 2y - x, \quad L_2 = 4y - x$

$\therefore$ pt (1, 0) gives -lve sign to both lines

$\therefore$ P must give + lve to L_1 4$\in$ lve to L_2

$\therefore$ $2a - a^2 < 0$ & $at(0, 4)$

$(-\infty, 0) \cup (3, \infty)$

A (2,4)

68. Let the live by y – mx – c = 0

$\therefore$ adol is algebraic distance

$$\frac{1-2m-c}{\sqrt{Hn2}} + \frac{2-3m-c}{\sqrt{Hn2}} + \frac{7+4m-c}{\sqrt{Hn2}}$$

10 – m – 3c = 0

$$\frac{10}{3} = m + c$$

$\therefore$ pares through (1, 10/3)

69. $\frac{ds}{dx} : ax + hg + g = 0$ if n eqn. put y = 0

$\frac{dg}{dy} ; hx + by + f = 0 \qquad \frac{-g}{a} = \frac{-f}{h}$

Hg = gf

70. Let P^2 (x, y)

$\therefore$ Alfa f_1 P^2 is (y, x)

Alfa f_2 is becomes $\left(y+3x, x_1\right)$

Alfa f_3 is becomes $\left(\frac{y_1+2x_1}{2}, \frac{y_1+4x_1}{2}\right)$

P^2 (A) becomes (0,0)

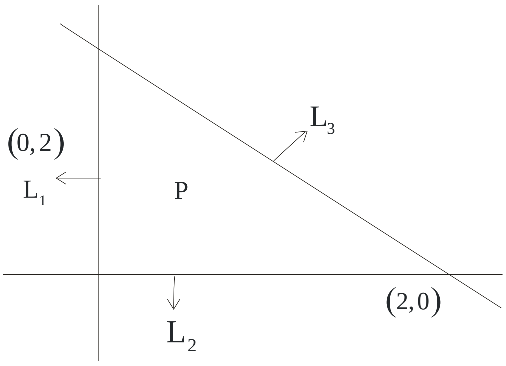
P^2 (B) (4,0) becomes (4, 8)

P^2 (C)(4,2) becomes (5, 9)

P^2 (D) (0,2) becomes (1, 1)

Now pb form 1Lgm

71.



For P to be insides it must give signs (+) $\in$ lve & $\in$ lve w.r.t lives L_1, L_2, L_3 respectively

$\therefore$ $a > 0,\ a^2 > 0$ & at $a^2 - 2 < 0$

$a > 0$ & at $(-2,1)$

$\Rightarrow$ at $(0,1)$

72. (B)

here $L_1 : x\cos\alpha + y\sin\theta = p$

$\Delta\ L_2 = x\sin\alpha - y\cos\alpha = 0$

$ax \perp r$ & $ax + by \perp p$ is @ $\frac{\pi}{4}$ with $L_1 \Rightarrow ax + by$ $cp = 0$ is angle

_______ of $L_1 \& L_2$

= $\left(x\cos\alpha + y\sin\alpha = p\right) = 1\left(x\sin\alpha - y\cos\alpha\right)$

Take (+lve & y_n)

$x\left(\cos\alpha - \sin\alpha\right) + y\left(\sin\alpha + \cos\alpha\right) - p = 0$

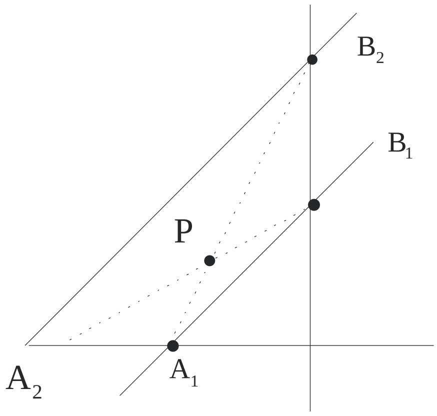
Compare with

$ax + by + p = 0$

$$\frac{a}{\cos\alpha - \sin\alpha} = \frac{b}{\sin\alpha + \cos\alpha} = \frac{-p}{p}$$

$a^2 + b^2 = 2$

73.



$A_1 A_2 B_1 B_2$
Co cyclic
$= m_1 m_2 = 1$ or $m^2 = 1$
$\Rightarrow m = 1 \quad m \leftarrow R^{-1}$
$A_1\left(\frac{-C_1}{m}, 0\right), A_2\left(\frac{-C_2}{m}, 0\right)$
$B_1(0, C_1) \quad B_2(0, C_2)$ let p be (h, k)

$\therefore$ A, P & B_2 collinear

$$\frac{k}{h+C_1} = \frac{C_2}{C_1}$$

$$\rightarrow \frac{k}{C_2} = \frac{H}{C_1} + 1$$

&

A_2 P & B_1 collinear

$$\frac{k}{h+C_2} = \frac{C_1}{C_2}$$

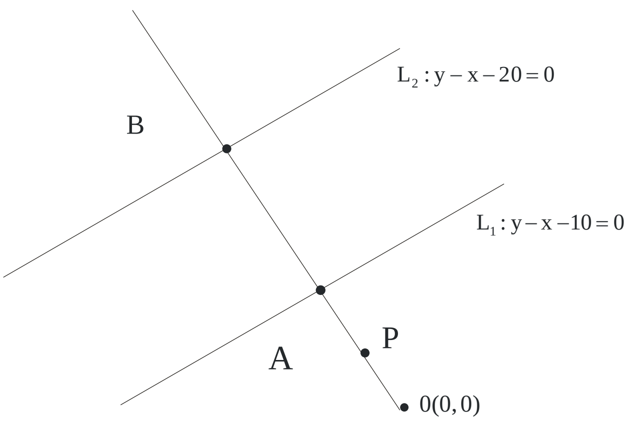
$$\frac{k}{C_1} = \frac{H}{C_2} + 1$$

Subtract to get

$$k\left(\frac{1}{C_2} = \frac{1}{C_1}\right) = h\left(\frac{1}{C_1} = \frac{1}{C_2}\right)$$

$k = -h$

74.



Let $p(h, k)$ Now, parametric eqn. of live PAB can be taken as
$x = r\cos\theta, \quad y = r\sin\theta$

Let OP = r_1 $\therefore$ $h = r_1\cos\theta, \quad k = r\sin\theta$

OA = r_2 $A(r_2\cos\theta, r_2\sin\theta)$

OB = r_3 $B(r_3\cos\theta, r_3\sin\theta)$

Now put pts on L_1 & L_2
$r_2(\sin\theta - \cos\theta) = 10$ $r_3(\sin\theta - \cos\theta) = 20$

$r_2 \frac{10}{\sin\theta - \cos\theta}$ $\qquad$ $r_3 = \frac{20}{\sin\theta - \cos\theta}$

π is given the

$\frac{2}{r_1} = \frac{1}{r_2} + \frac{1}{r_3}$ $\qquad$ $\frac{40}{3} = r_1 \sin\theta - r_2 \cos\theta$

$\frac{2}{r_1} = \frac{3}{20}(\sin\theta - \cos\theta)$ $\qquad$ $\frac{40}{1} = y - x$

75. From 74 put

$r_1^2 = r_2 r_3$

$r_1^2 = \frac{200}{(\sin\theta - \cos\theta)^2}$

$x - (r_1 \sin\theta - r_1 \cos\theta)^2 = 200$

$(x - y)^2 = 200$

76. From 74

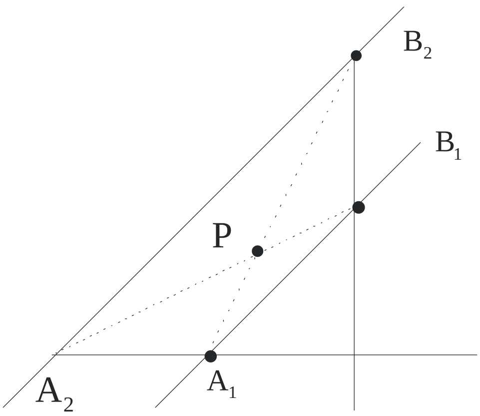
$\frac{1}{r_1^2} = \frac{1}{r_2^2} + \frac{1}{r_3^2}$

$\frac{1}{r_1^2} = \frac{(\sin\theta - \cos\theta)^2}{400}$

$80 = (r_1 \sin\theta = r_1 \cos\theta)^2$

$(x - y)^2 = 80$

77.



$A_1\left(\frac{-C_1}{2}, 0\right)$ $\qquad$ $B_1(0, C_1)$

$A_2\left(\frac{-C_2}{2}, 0\right)$ $\qquad$ $B_2(0, C_2)$

P(h, k) is collinear with A_1 B_2

$\frac{k - C_2}{h} = \frac{2C_2}{C_1}$ or $\frac{k}{C_2} - 1 = \frac{2h}{C_1}$

Also p(h, k) collinear with A_2 & B_2

$\Rightarrow \quad \frac{k}{C_1} - 1 = \frac{2h}{C_2}$

$\therefore$ Subtract

$$k\left(\frac{1}{C_2}-\frac{1}{C_1}\right)=2h\left(\frac{1}{C_1}-\frac{1}{C_2}\right)$$

$$k+2h=0$$

78. Let live be y = mx – c = 0

Homogenize live with curve

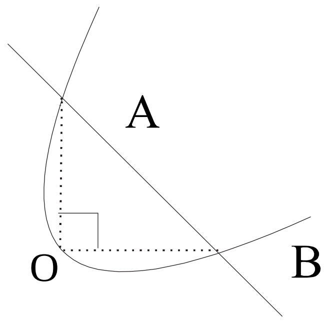
$$3x^2-y^2+\left(4g-2n\right)\left(\frac{y-mx}{c}\right)=0$$

Sin OA & OB

$$\therefore \text{cosec } x^2+\text{cosec } y^2=0$$

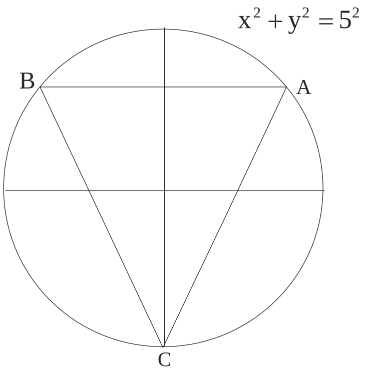
$$\left(3c+2m\right)+\left(-c+4\right)=0$$

$m+c=2$ $\quad$ p^2 is (1, -2)



79. (D)

A(3, 4) $\quad B\left(\cos\theta,\ 5\sin\theta\right)$ $\quad C\left(5\sin\theta,\ -5\cos\theta\right)$



Now, 5(0,0) & centride is $C_1\left(\frac{3+5\cos\theta+5\sin\theta}{3},\frac{4+5\sin\theta-5\cos\theta}{3}\right)$

$$\therefore\ H=\left(3+5\cos\theta+5\sin\theta\right),\left(4+5\sin\theta-5\cos\theta\right)$$

$$h-3=5\cos\theta+5\sin\theta$$

$$k-4=5\sin\theta=5\cos\theta$$

$x+y-7=10\sin\theta$ also $x-y+1=10\cos\theta$

$$\left(x+y-7\right)^2+\left(x-y+1\right)^2=100$$

80.

By parametric live $\qquad$ Also

$k=0+4\sin\theta$ $\qquad$ $h=0+3\cos\left(\theta-90\right)$

$k=4\sin\theta$ $\qquad$ $h=3\sin\theta$

$$\therefore\frac{K}{4}=\frac{H}{3}$$

Paper – 2(A)

18. (A, C)

Let P be (a_1, -39)

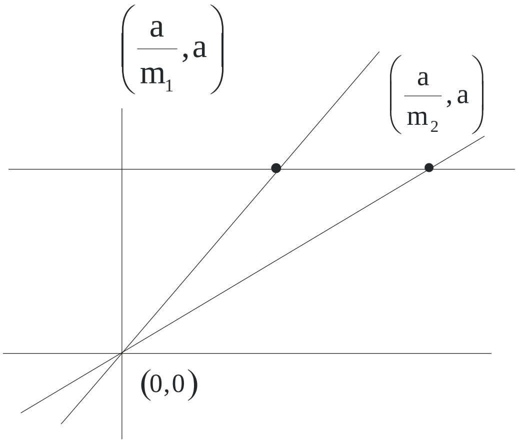
(3, 4) given (- lve sign with 3x – 4y – 8 = 0

$\therefore (a_1 - 3a)$ must give (+lve sign with line

$3a + 12a - 8 > 0$

$a > \frac{8}{15}$

19. (A, C, D)



$m_1 + m_2 = a$

$m_1 m_2 = -(a+1)$

$\therefore A = \frac{1}{2}\left|\frac{a^2}{m_1} - \frac{a^2}{m_2}\right| = \frac{a^2}{2}\frac{|m_1 - m_2|}{m_1 m_2}$

$A = \frac{a^2}{2}\frac{\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{|m_1m_2|} = \frac{a^2}{2}\frac{\left(\sqrt{a^2 + 4(a+1)}\right)}{|a+1|}$

$A = \frac{a^2}{2}\left|\frac{a+2}{a+1}\right|$

20. (A,B, C, D)

If they intersect @ 4 concylic points

$\therefore m_1 m_2 = 1$

$\left(\frac{a}{b}\right)\left(\frac{d}{c}\right) = 1$

$ac = bd$

Now, $(a-1)^2 = (b-d)^2 \Rightarrow (a-c) = 7(b-d)$

Now subtract lives

$x\left(\frac{1}{a} - \frac{1}{c}\right) + y\left(\frac{1}{b} - \frac{1}{d}\right) = 0$

$x\left(\frac{c-a}{ac}\right) + y\left(\frac{b-d}{bd}\right) = 0$

$\Rightarrow \quad x \pm y = 0$

21. (A, C)

Let line be y = mx + c

Put (1,0) $\Rightarrow$ c = - m

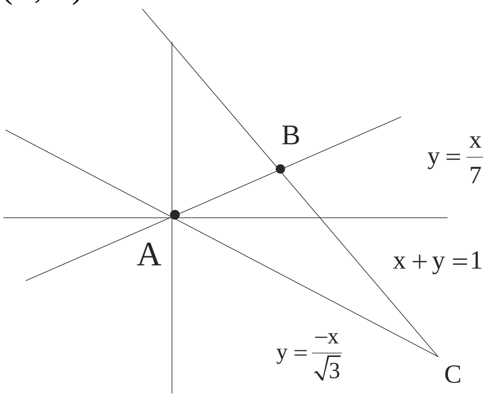
$\therefore \quad y = m(x-1)$ or $mx - y - 1 = 0$

$\therefore$ distance = $\dfrac{\sqrt{3}}{2}$

$$\frac{1}{\sqrt{Hm_2}} = \frac{\sqrt{3}}{2} \Rightarrow \frac{4}{3} = m^2 + 1$$

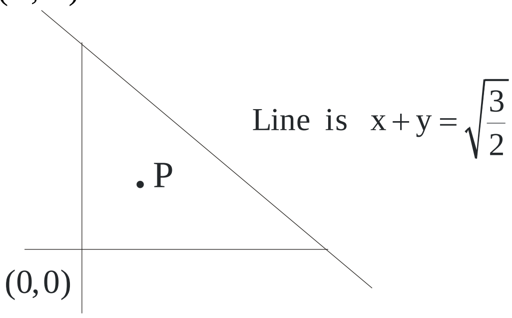
$$m = \pm\frac{1}{\sqrt{3}}$$

22. (B, C)



The Δ is obtuse $\Rightarrow$ interior are In centre & centroid

23. (A, B)



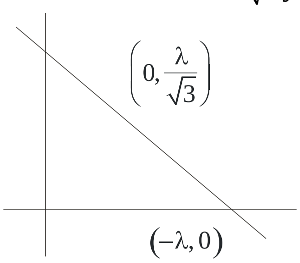
If $P(\sin\theta, \cos\theta)$ inside the Δ

$$\sin\theta > 0 \ \& \cos\theta > 0 \quad \& \quad \sin\theta + \cos\theta < \frac{\sqrt{3}}{\sqrt{2}}$$

$$\Rightarrow \quad q \in 1^{st} \text{ quadrant } \& \ \sin\left(\theta + \frac{\pi}{4}\right) < \sin\frac{\pi}{3}$$

24. (B,D)

Let the line be $x - \sqrt{3y} + \lambda = 0$



$\therefore$ length of intercept

$$\Rightarrow \quad \lambda^2 + \frac{\lambda^2}{3} = 100$$

$$\lambda = y5\sqrt{3}$$

25. (C, D)

$$\begin{vmatrix} 1 & 1 & -1 \\ m-1 & m^2-7 & -5 \\ m-2 & 2m-5 & 0 \end{vmatrix} = 0$$

By property

0

$$\begin{vmatrix} 0 & 0 & -1 \\ -m^2+m+6 & m^2-12 & -5 \\ -m+3 & 2m-5 & 0 \end{vmatrix} = 0$$

$\Rightarrow \quad (2m-5)(-m^2+m+6)+(m-3)(m^2-12)=0$

$\Rightarrow \quad (m-3)\left[-(2m-5)(m12)+(m^2-12)\right]=0$

m = 3 or $m^2-m+2=0$ discard

fro m = 3 lives are parallel

26. (B, C)

Let live have slope ‘m’

$= \frac{1}{2} = \left|\frac{m+2}{1-2m}\right|$

$\frac{m+2}{1-2m} = \frac{1}{2}$ or $\frac{-1}{2}$

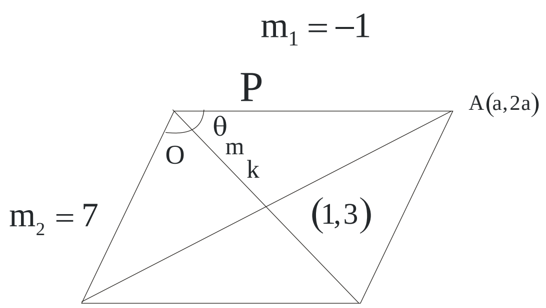
$2m+4=1-2m$ or $2m+2=-1+2m$

4m = - 3 $\Rightarrow m=\infty$

$m=\frac{-3}{4}$

$\therefore$ lives G.M. $\frac{y-3}{x-2}=\frac{-3}{4}$ or ∞

27.



Now diagonal bisects the angle

$\therefore$ equality $\tan\theta$

$\frac{m+1}{1-m}=\frac{7-m}{1+7m}$

$\Rightarrow \quad m=\frac{1}{3}$ or 3

Diagonal are

$$\frac{y-3}{x-1}=\frac{1}{3} \text{ or } -3$$

Put (a, 2a) on there live

$$A=\frac{8}{5} \quad \text{or} \quad \frac{6}{5}$$

28. (A,B,C,D)

Given $\frac{m_1}{m_2}=\frac{9}{2}$

& $\frac{m_1-m_2}{1+m_1m_2}=\pm\frac{7}{9}$

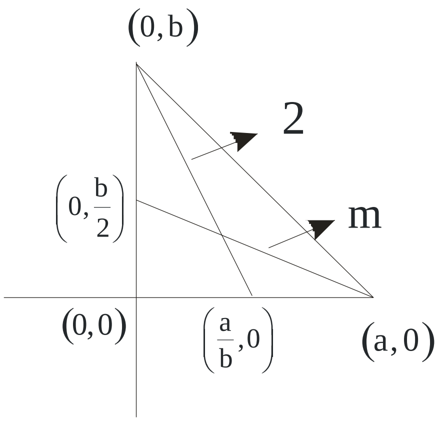
$$\frac{7m_2}{2+gm_2}=\frac{7}{9} \quad \text{or} \quad \frac{-7}{9}$$

$9m_2=2+9{m_2}^2$ or $-9m_2=2+9m_2^2$

$9m_2^2-9m_2^2+2=0$ $\qquad$ $9m_2^2+9m_2+2=0$

$m_2=\frac{2}{3}\text{or}\frac{1}{3}$ or $m_2=\frac{-2}{3}\text{or}\frac{-1}{3}$

29. Consider (0,b)



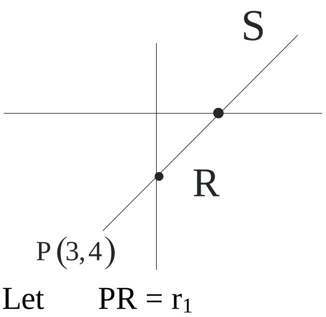
Here $\frac{-2b}{a}=2$ $\qquad$ Or $\frac{-b}{2a}=2$

$\Rightarrow\frac{b}{a}=-1$ $\qquad$ $\Rightarrow\frac{-b}{a}=4$

$m=\frac{-b}{2a}=\frac{1}{2}$ $\qquad$ & $m=\frac{-2b}{a}$

= + 8

30. (A, B, C, D)



Let $\quad PR = r_1$

$\qquad PS = r_2$

$\therefore \quad R(3+r,\cos\theta,\ 4+r,\sin\theta)$

$\Rightarrow \quad 3 + r, \cos\theta = 6$

$r_1 = 3\sec\theta$

The $S(3 + r_2, \cos\theta,\ 4 + r_2, \sin\theta)$

$\Rightarrow \quad 4 + r_2 \sin\theta = 8$

$R = 4\operatorname{cosec}\theta$

31. (A, B, & C)

Put $\frac{y}{x} = m$

$1 + m - m^2 = m^3$

Or $m^3 + m^2 = m + 1$

$(m+1)m^2 = (m+1)$

$m = -1 \quad$ or $\quad m^2 = 1$

m = -1 $\quad$ $m = \pm 1$

32. $x^2 + mxy - 2y^2 + 3y - 1 = 0$

A = 1, B = - 2, G = 0, $f = \frac{3}{2}$ C = -1 $H = \frac{M}{2}$

$$\Delta = 0 \quad \begin{vmatrix} 1 & \frac{m}{2} & 0 \\ \frac{m}{2} & -2 & \frac{3}{2} \\ 0 & \frac{3}{2} & -1 \end{vmatrix} = 0$$

$1\left(2 - \frac{9}{4}\right)\frac{-m}{2}\left[\frac{-m}{2}\right] = 0$

$\frac{m^2}{4} - \frac{1}{4} = 0 \Rightarrow m = \pm 1$

Find intersection Ph

33. (A, B, D)

$\Rightarrow$ Angle between lives must be $180^o - 2\alpha$

$\Rightarrow |\tan 2\alpha| = \left|\frac{2\sqrt{h^2 - 1}}{2}\right|$

$\tan^2 2\alpha = h^2 - 1$

$h = |8a^2\alpha|$

34. (A, B, C, D)

Use condition of both roots common

$3\left(\frac{y}{x}\right)^2 + p\left(\frac{y}{x}\right) + 2 = 0$ has roots m & m_1

$-3\left(\frac{y}{x}\right)^2 + 9\left(\frac{y}{n}\right) + 2 = 0$ has roots m & m_2

$m + m_2 = \frac{-p}{3}, mm_1 = \frac{2}{3}$ & $m + m_2 = \frac{9}{3}$

Also $m_1 m_2 = 1$ $\qquad$ $mm_2 = \frac{-2}{3}$

From here $m^2 = \frac{4}{9}$

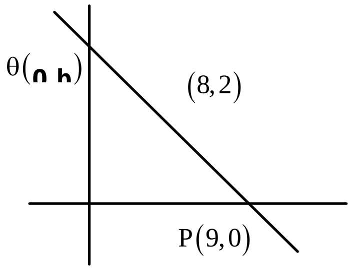
$7m = \frac{2}{3}$ $\qquad$ OR $\qquad$ $7m = \frac{-2}{3}$

$7m = \frac{2}{3}$ $\qquad$ $m_1 = -1$ & $m_2 = 1$

$m_1 = 1$ & $m_2 = -1$ $\qquad$ $p = 5, \quad q = +1$

$P = -5$ $\qquad$ $q = -1$

23.



a, b (+) ve

also $\frac{9}{a}+\frac{2}{b}=1$

Weighted A.M $\geq$ Weighted H.M

$$\frac{\left(2\sqrt{2}\right)\left(\frac{2\sqrt{2}}{a}\right)+\left(\sqrt{2}\right)\left(\frac{\sqrt{2}}{b}\right)}{2\sqrt{2}+\sqrt{2}}$$

$$\frac{1}{3\sqrt{2}}\geq\frac{3\sqrt{2}}{a+b}$$

$a+b\geq 18$

M, n 18

24. From Q. 23

Use A.M $\geq$ G.M

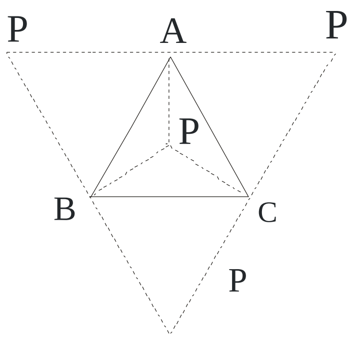
$$\frac{\frac{8}{a}+\frac{1}{b}}{2}\geq\sqrt{\frac{16}{ab}}$$

$$\frac{1}{4}\geq\frac{16}{4b}$$

$ab\geq 64$

$A\geq 32$

25. 4 position possible



Here P such that P,A, B, & C form is any order vertices taken & 4H position is centriod

26. given ax $x^2-(y-2)^2=0$

$X+y-2=0$ or $x-y+2=0$

Bisection Are

$$\frac{x+y-2}{\cancel{\sqrt{2}}}=1\frac{(x-y+2)}{\cancel{\sqrt{2}}}$$

i.e. $\cancel{x}+y-2=\cancel{x}-y+2$ $\quad\quad x+\cancel{y}-2=-x+\cancel{y}-2$

$y=2$ $\quad\quad x=0$

3 lives x + y = 3, y = 2 x = 0

3 vertices (0,2) (0,3)(1,2) $A = \frac{1}{2}$

27. Given $2x + 3y = 6$

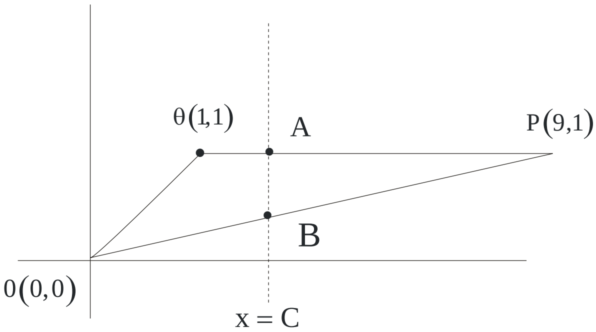
Use equality – Inequality

$(2x + 3y) \le \sqrt{2^2 + 3^2}\sqrt{x^2 + y^2}$

$\Rightarrow \quad \frac{6}{\sqrt{13}} \le \sqrt{x^2 + y^2}$

$m = \frac{6}{\sqrt{13}}$

28.



A is (C , 1)

B is $\left(C_1, \frac{C}{3}\right)$

$\therefore$ Are of Δ ABP should be 2

Area of ΔOPQ

$\therefore \frac{1}{2}\left|\left(\frac{C-9}{3}\right)^2\right| = 2$

Discard C = 15 or C = 3

Take C = 3

29. $a(2x + y - 3) + b(x + 3y + 1) = 0$

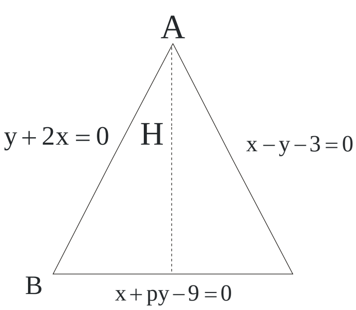
This live passes through intersection of $\begin{matrix} 2x + y - 3 = 0 \\ x + 3y + 1 = 0 \end{matrix}$ i.e. $(2, -1)$

It also satisfies live mx + 2y + 6 = 0

2m – 2 + 6 = 0

m = - 2

30.



Altitudes from A is $(y + 2n) + \lambda(x - y - 3) = 0$

It slope should be P Also H satisfies it

$$-\frac{(\lambda+2)}{1-\lambda}=p \qquad 7+\lambda(-7)$$

$$\lambda=\frac{7}{4}$$

$\Rightarrow$ P is 5

Similarly get q = 45

$$\frac{p+q}{10}=5$$