

**INEQUATIONS & EQUATION**  
**EXERCISE-1-A**

1. (B)

$$\Rightarrow \log(ab) - \log|b|$$

We can see that  $ab > 0 \Rightarrow a < 0 \& b < 0$  or  $a > 0 \& b > 0$

$$\text{So } \log(ab) - \log|b| = \log|ab| - \log|b|$$

$$= \log|a \cdot |b|| - \log|b|$$

$$= \log|a| + \log|b| - \log|b|$$

$$= \log|a|$$

2. (C)

$$\Rightarrow \sqrt{\log_{0.5}^2 4} = \sqrt{[\log_{10} 4]^2} = \sqrt{\left[\log_{\left(\frac{1}{2}\right)} 4\right]^2}$$

$$= \sqrt{\left[\log_{\left(\frac{1}{2}\right)} \left(\frac{1}{2}\right)^{-2}\right]^2} = \sqrt{\left(-2 \times \log_{\frac{1}{2}} \frac{1}{2}\right)^2}$$

$$= \sqrt{(-2 \times 1)} = \sqrt{4}$$

$$= 2$$

..... (as a square root of a value can't be negative)

3. (B)

$$\Rightarrow \log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8 \log_8 9$$

$$\text{By formula } \log_a b = \frac{\log_c b}{\log_c a}$$

$$\text{Given value is } = \left(\frac{\log 4}{\log 3}\right) \left(\frac{\log 5}{\log 4}\right) \left(\frac{\log 6}{\log 5}\right) \left(\frac{\log 7}{\log 6}\right) \left(\frac{\log 8}{\log 7}\right) \left(\frac{\log 9}{\log 8}\right)$$

$$= \frac{\log 9}{\log 3} = \log_3 9$$

$$= 2$$

4. (C)

$$\log_7 \log_7 \left( \sqrt{7 \sqrt{7 \sqrt{7}}} \right)$$

$$= \log_7 \log_7 \left( 7^{1/2} \cdot 7^{1/7} \cdot 7^{1/8} \right)$$

$$= \log_7 \log_7 7^{7/8} = \log_7 (7/8)$$

$$= 1 - 3 \log_7 2$$

5. (D)

$$\Rightarrow 81^{\left(\frac{1}{\log_5 3}\right)} + 27^{(\log_9 36)} + 3^{\frac{4}{\log_7 9}}$$

$$\Rightarrow (81)^{(\log_3 5)} + (3^3)^{(\log_{3^2} 36)} + 3^4 \log_9 7$$

$$\begin{aligned} &\Rightarrow (3^4)^{\log_3 5} + 3^{3 \times \left(\frac{1}{2} \log_3 36\right)} + 3^{4 \times \left(\frac{1}{2} \log_3 7\right)} \\ &\Rightarrow 3^{\log_3 5^4} + 3^{\log_3 (36)^{\frac{3}{2}}} + 3^{\log_3 7^2} \\ &\Rightarrow 5^4 + (36)^{\frac{3}{2}} + 7^2 \\ &625 + 36 \times 6 + 49 = 890 \end{aligned}$$

6. (C)

$$\begin{aligned} &\Rightarrow \log_x x \cdot \log_5 k = \log_x 5 && ; \text{ given } k \neq 1, k > 0 \\ &\Rightarrow \frac{\log x}{\log k} \cdot \frac{\log k}{\log 5} = \frac{\log 5}{\log x} \\ &\Rightarrow (\log_5 x) = (\log_x 5) \\ &\Rightarrow x = 5 \text{ is the only possible solution} \end{aligned}$$

7. (C)

$$\begin{aligned} &\Rightarrow \log_5 a \cdot \log_a x = 2 \\ &\Rightarrow \frac{\log a}{\log 5} \cdot \frac{\log x}{\log a} = 2 \\ &\Rightarrow \log_5 x = 2 \\ &\Rightarrow x = 5^2 \\ &= 25 \end{aligned}$$

8. (C)

$$\begin{aligned} &\Rightarrow A = \log_2 \log_2 \log_4 256 + 2 \log_{\sqrt{2}} 2 \\ &\Rightarrow \log_2 \log_2 \log_4 (4)^2 + \log_{\frac{1}{2^2}} (2) \\ &\Rightarrow \log_2 \log_2 4 + 2 \times \frac{1}{\left(\frac{1}{2}\right)} \log_2 2 \\ &\Rightarrow \log_2 2 + 4 = 1 + 4 \\ &\Rightarrow 5 \end{aligned}$$

9. (D)

$$\begin{aligned} &\Rightarrow \log_{10} x = y \text{ (given)} \\ &\Rightarrow \log_{1000} x^2 = 2 \log_{10^3} x \\ &\Rightarrow 2 \times \frac{1}{3} \log_{10} x && \text{(by formula } \log_{a^k} b = \frac{1}{k} \log_a b) \end{aligned}$$

10. (A)

$$\begin{aligned} &\frac{(\log a)^2}{\log b / \log c} - 1 + \frac{(\log b)^2}{\log a \log c} - 1 + \frac{(\log c)^2}{\log a \log b} - 1 = 0 \\ &\Rightarrow (\log a)^3 + (\log b)^3 + (\log c)^3 = 3 \log a \log b \log c \end{aligned}$$

$$\begin{aligned} \because a, b, c \text{ are distinct} \\ \Rightarrow \log a + \log b + \log c = 0 \\ \Rightarrow abc = 1 \end{aligned}$$

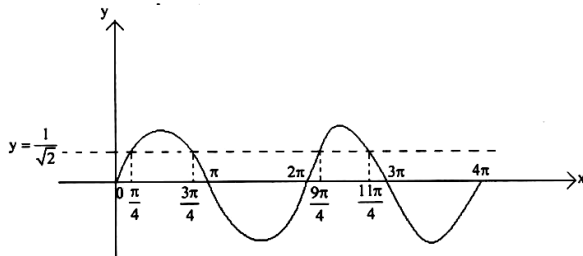
11. (A)

$$\Rightarrow \log_{\left(\frac{1}{\sqrt{2}}\right)} \sin x > 0; x \in [0, 4\pi]$$

As base  $\frac{1}{\sqrt{2}}$  lies between 0 to 1 satisfy given inequality,  $0 < \sin x < 1$

$$\Rightarrow x \in (0, \pi) \cup (2\pi, 3\pi)$$

As we can see in this interval



We get  $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$  as integral

Multiples of  $\frac{\pi}{4}$

12. (B)

$$\Rightarrow \log_{\frac{1}{2}}(x^2 - 6x + 12) \geq -2$$

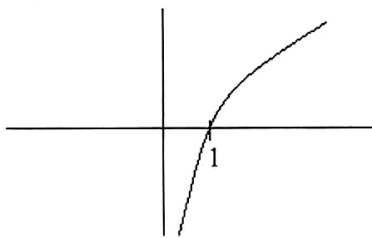
$$\Rightarrow \log_{2^{-1}}(x^2 - 6x + 12) \geq -2$$

$$\Rightarrow -1 \times \log_2(x^2 - 6x + 12) \geq -2$$

$$\Rightarrow \log_2(x^2 - 6x + 12) \leq 2$$

$$\Rightarrow \log_2(x^2 - 6x + 12) - \log_2 4 \leq 0$$

$$\Rightarrow \log_2\left(\frac{x^2 - 6x + 12}{4}\right) \leq 0$$



$$\Rightarrow 0 < \frac{x^2 - 6x + 12}{4} \leq 1$$

**Case-1**

$$\Rightarrow 0 < \frac{x^2 - 6x + 12}{4}$$

$$\Rightarrow x^2 - 6x + 12 > 0$$

$\Rightarrow x \in \mathbb{R} \dots\dots$  (1) as discriminant of quadratic expression  $x^2 - 6x + 12$  is less than zero.

$$\text{Discriminant } D = (-6)^2 - 4(12)(1)$$

$$\Rightarrow D = -12$$

**Case-2**

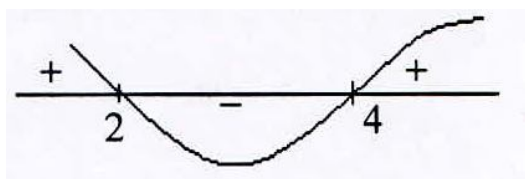
$$\Rightarrow \frac{x^2 - 6x + 12}{4} \leq 4$$

$$\Rightarrow x^2 - 6x + 12 \leq 4$$

$$\Rightarrow x^2 - 6x + 8 \leq 0$$

$$\Rightarrow (x - 4)(x - 2) \leq 0$$

$$\Rightarrow x \in [2, 4] \dots\dots(ii)$$



By taking intersection of (i) & (ii) we get  $x \in [2, 4]$

13. (B)

$$\Rightarrow 2^{\log_{\sqrt{2}}(x-1)} > x + 5 \text{ Here } x - 1 > 1; x > 2 \dots\dots(1)$$

$$\Rightarrow 2^{\log_2 (2)^{\frac{1}{2}(x-1)}} > x + 5$$

$$\Rightarrow 2^{2 \log_2(x-1)} > x + 5 \quad (\text{or by formula } \log_{a^k} b = \frac{1}{k} \log_a b)$$

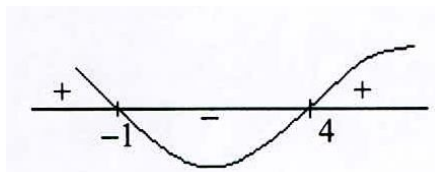
$$\Rightarrow 2^{\log_2(x-1)^2} > x + 5$$

$$\Rightarrow (x - 1)^2 > x + 5$$

$$\Rightarrow x^2 + 1 - 2x > x + 5$$

$$\Rightarrow x^2 - 3x - 4 > 0$$

$$\Rightarrow (x - 4)(x + 1) > 0$$



So we get  $x \in (-\infty, -1) \cup (4, \infty) \dots\dots(ii)$

By taking intersection of (i) & (ii)

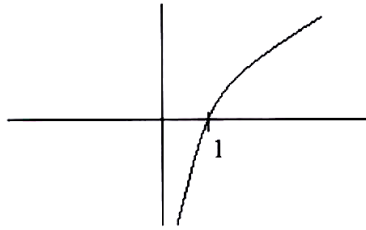
$$\Rightarrow x \in (4, \infty)$$

14. (C)

$$\Rightarrow \log_{10}(x^2 - 2x - 2) \leq 0$$

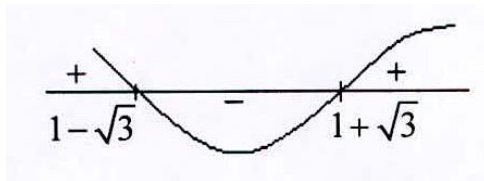
As base is greater than 1 so to hold the inequality true

$$\Rightarrow 0 < x^2 - 2x - 2 \leq 1$$



So,  $0 < x^2 - 2x - 2$  and  $x^2 - 2x - 2 \leq 1$

**Case-I**



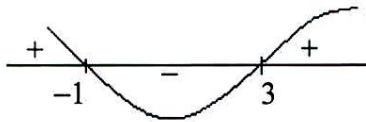
$$\Rightarrow x^2 - 2x - 2 > 0$$

$$\Rightarrow [x - (1 + \sqrt{3})][x - (1 - \sqrt{3})] > 0$$

So,  $x \in (-\infty, 1 - \sqrt{3}) \cup (1 + \sqrt{3}, \infty)$  .....(i)

**Case-2**

$$\Rightarrow x^2 - 2x - 2 \leq 1$$



$$\Rightarrow x^2 - 2x - 3 \leq 0$$

$$\Rightarrow (x - 3)(x + 1) \leq 0$$

So we get  $x \in [-1, 3]$

By taking intersection of (i) & (ii) we get,

$$\Rightarrow x \in [-1, 1 - \sqrt{3}) \cup (1 + \sqrt{3}, 3]$$

15. (A)

$$\Rightarrow \log_{0.2} \frac{x+2}{x} \leq 1$$

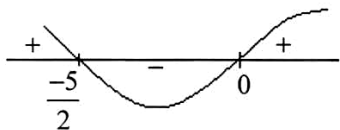
As base of log is less than 1 so hold the inequality true

$$\Rightarrow \frac{x+2}{x} \geq 0.2$$

$$\Rightarrow \frac{x+2}{x} - 0.2 \geq 0$$

$$\Rightarrow \frac{x+2-0.2x}{x} \geq 0$$

$$\Rightarrow \frac{0.8x+2}{x} \geq 0$$



$$\text{So, } x \in \left(-\infty, \frac{-5}{2}\right] \cup [0, \infty)$$

$$\Rightarrow a^{m \log_a n} \Rightarrow a^{\log_a n^m} \Rightarrow n^m$$

16. (C)

$$\Rightarrow a^{mn} = a^{m^n}$$

Take log both side

$$\Rightarrow \log_a a^{mn} = \log_a a^{m^n}$$

$$\Rightarrow mn = m^n$$

$$\Rightarrow m^{n-1} = n$$

$$\Rightarrow m = \left(n^{\frac{1}{n-1}}\right)$$

18. (B)

$$\Rightarrow (x^5)^{\frac{1}{3}} (16x^3)^{\frac{2}{3}} \left(\frac{1}{4}x^9\right)^{\frac{3}{2}} \cdot (4)^{\frac{1}{6}}$$

$$\Rightarrow x^{\frac{5}{3}} \cdot (4^2)^{\frac{2}{3}} \cdot (x^3)^{\frac{2}{3}} \cdot (4^{-1})^{\frac{3}{2}} \cdot \left(x^{\frac{4}{9}}\right)^{\frac{-3}{2}}$$

$$\Rightarrow x^{\frac{5}{3}} \times 4^{\frac{4}{3}} \times x^2 \times 4^{\frac{3}{2}} \times x^{\frac{4}{9} \times \frac{-3}{2}}$$

$$x^{\frac{5}{3} - \frac{2}{3} + 2} \cdot 4^{\left(\frac{4}{3} + \frac{3}{2} + \frac{1}{6}\right)}$$

$$\Rightarrow 4^3 \cdot x^3$$

19. (D)

$$\Rightarrow \frac{(2^{n+1})^m (2^{2n})2^n}{(2^{m+1})^m 2^{2m}} = 1$$

$$\Rightarrow \frac{2^{nm+m} \times 2^{2n+n}}{2^{mn+m} 2^{2m}} = 1$$

$$\Rightarrow 2^{(nm+m+2n+n)-(mn+n-2m)} = 1$$

$$\Rightarrow 2^{2n-m} = 1$$

$$\Rightarrow 2n - m = 0$$

$$\Rightarrow m = 2n$$

20. (D)

$$\Rightarrow x^{x\sqrt[3]{x}} = \left(x \cdot \sqrt[3]{x}\right)^x$$

Here  $x \neq 0$

$$\Rightarrow x^{x+\frac{1}{3}} = \left(x^{1+\frac{1}{3}}\right)^x$$

$$\Rightarrow x^{x^{\frac{4}{3}}} = x^{\frac{4}{3}x}$$

Take log both side

$$\Rightarrow x^{\frac{4}{3}} \log_x x = \frac{4}{3} x \log_x x$$

$$\Rightarrow x^{\frac{4}{3}} = \frac{4}{3} x$$

$$\Rightarrow x^{\frac{1}{3}} = \frac{4}{3} x$$

$$\Rightarrow x^{\frac{1}{3}} = \frac{4}{3}$$

$$\Rightarrow x = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$$

21. (C)

$$\text{If } x = 2^{\frac{1}{3}} - 2^{-\frac{1}{3}}$$

$$\text{Then } 2x^3 + 6x = 2 \left[ 2^{\frac{1}{3}} - 2^{-\frac{1}{3}} \right]^3 + 6 \left[ 2^{\frac{1}{3}} - 2^{-\frac{1}{3}} \right]$$

$$= 2 \left( 2^{\frac{1}{3}} - 2^{-\frac{1}{3}} \right) \left[ \left( 2^{\frac{1}{3}} - 2^{-\frac{1}{3}} \right)^2 + 3 \right]$$

$$= 2 \left( 2^{\frac{1}{3}} - 2^{-\frac{1}{3}} \right) \left[ 2^{\frac{2}{3}} + 2^{-\frac{2}{3}} - 2 + 3 \right]$$

$$= 2 \left[ 2^{\frac{1}{3}} - 2^{-\frac{1}{3}} \right] \left[ 2^{\frac{2}{3}} + 2^{-\frac{2}{3}} + 1 \right]$$

$$= 2 \left[ \left( 2^{\frac{1}{3}} \right)^3 - \left( 2^{-\frac{1}{3}} \right)^3 \right]$$

$$(\because \text{by } a^3 - b^3 = (a - b)(a^2 + b^2 + ab))$$

$$= 2 [2 - 2^{-1}]$$

$$= 3$$

22. (A)

$$\Rightarrow (x)^{x\sqrt{x}} = (x\sqrt{x})^x \quad (\text{here } x \neq 0)$$

$$\Rightarrow x^{x^{\frac{3}{2}}} = \left(x^{\frac{3}{2}}\right)^x$$

$$\Rightarrow x^{x^{\frac{3}{2}}} = x^{\frac{3}{2}x}$$

Take log both sides

$$\Rightarrow x^{\frac{3}{2}} \log_x x = \frac{3}{2} x \log_x x$$

$$\Rightarrow x^{\frac{3}{2}} = \frac{3}{2}x$$

$$\Rightarrow x^{\frac{1}{2}} = \frac{3}{2}$$

$$\Rightarrow x = \frac{9}{4}$$

23. (C)

$$\Rightarrow 5^{x-1} + 5(0.2)^{x-2} = 26$$

$$\Rightarrow 5^{x-1} + 5\left(\frac{1}{5}\right)^{x-2} = 26$$

$$\Rightarrow 5^{x-1} + 5^{1-x+2} = 26$$

$$\Rightarrow 5^{x-1} + 5^{3-x} = 26$$

At  $x = 1, 3$  above equation satisfy

24. (D)

$$\Rightarrow x - \frac{1}{x} = 2$$

$$\Rightarrow x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2$$

$$= \left[\left(x - \frac{1}{x}\right)^2 + 2\right]^2 - 2$$

$$= [2^2 + 2]^2 - 2$$

$$= 36 - 2 = 24$$

25. (C)

$$\Rightarrow \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} \cdot \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}$$

$$\Rightarrow x^{\left(\frac{1}{c} - \frac{1}{b}\right)} \times x^{\left(\frac{1}{a} - \frac{1}{c}\right)} \times x^{\left(\frac{1}{b} - \frac{1}{a}\right)}$$

$$\Rightarrow x^{\frac{1}{c} - \frac{1}{b} + \frac{1}{a} - \frac{1}{c} + \frac{1}{b} - \frac{1}{a}}$$

$$\Rightarrow x^0 = 1$$

26. (C)

$$\Rightarrow a^m \cdot a^n = a^{mn}$$

$$\Rightarrow a^{m+n} = a^{mn}$$

$$\Rightarrow m + n = mn$$

.....(1)

$$\text{Then } m(n-2) + n(m-2) = ?$$

$$\Rightarrow 2mn - 2m - 2n$$

$$\Rightarrow 2(m+n) - 2(m+n)$$



27.

(B)

$$\Rightarrow \log(x+1) + \log(x-1) = \log 3$$

$$\Rightarrow \log(x+1)(x-1) = \log 3$$

$$\Rightarrow x^2 - 1 = 3$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

But  $x+1 > 0$  &  $x-1 > 0$

$$\Rightarrow x > -1 \text{ \& } x > 1$$

So,  $x \in (1, \infty)$  is our feasible region

Only  $x = 2$  lies in the feasible region.

28.

(D)

$$\Rightarrow f(x) = 5 - |x - 3|$$

$$\Rightarrow f(x) = 5 - (x - 3); x \in [3, \infty)$$

$$\Rightarrow 5 + (x - 3); x \in (-\infty, 3)$$

$$\Rightarrow f(x) = 8 - x; x \in [3, \infty)$$

$$= 2 + x; x \in (-\infty, 3)$$

So, greatest value of function occur at  $x = 3$

$$\text{So } f(3) = 8 - 3 = 5$$

29.

(B)

$$\Rightarrow \frac{2^{m+3} \times 3^{2m-2n} \times 5^{m+3+n} \times 6^{n+1}}{6^{m+1} \times 10^{n+3} \times 15^m}$$

$$\Rightarrow \frac{(2^{m+3})(3^{2m-2})(5^{m+n+3}) \times 2^{n+1} \times 3^{n+1}}{(2^{m+1}3^{m+1})(2^{n+3}5^{n+3})(3^m)(5^m)}$$

$$\Rightarrow 2^{m+3+n+1-(m+1)} 3^{2m-n+n+1-(m+1)-m} 5^{m+n+3-(n+3)-m}$$

$$\Rightarrow 2^0 3^0 5^0$$

$$\Rightarrow 1$$

30.

(A)

$$\Rightarrow x^{\frac{2}{3}} + x^{\frac{1}{3}} = 2$$

Take cube both side

$$\Rightarrow x^2 + x + 3x^{\frac{2}{3}}x^{\frac{1}{3}} \left( x^{\frac{2}{3}} + x^{\frac{1}{3}} \right) = 8$$

$$\Rightarrow x^2 + x + 3x(2) = 8$$

$$\Rightarrow x^2 + 7x - 8 = 0$$

$$\Rightarrow (x+8)(x-1) = 0$$

$$\Rightarrow x = 1, -8$$

31.

(D)

$$\text{If } a + b + c = 0$$

$$\Rightarrow \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} \Rightarrow \frac{a^3 + b^3 + c^3}{abc}$$

$$\Rightarrow \frac{\{(a+b)^3 - 3ab(a+b)\} + c^3}{abc} \Rightarrow \frac{\{(-c)^3 - 3ab(-c)\} + c^3}{abc}$$

$$\Rightarrow \frac{3abc}{abc} = 3$$

32.

(C)

$$\Rightarrow 2^x - 2^{x-1} = 4$$

$$\Rightarrow 2^x - \frac{2^x}{2} = 4$$

$$\Rightarrow 2 \cdot 2^x - 2^x = 8$$

$$\Rightarrow 2^x = 8$$

$$\Rightarrow x = 3$$

$$\text{So, } x^x = 3^3 = 27$$

33.

(B)

$$\Rightarrow \frac{(\log x - \log y)(\log x^2 + \log y^2)}{(\log x^2 - \log y^2)(\log x + \log y)}$$

$$\Rightarrow \frac{(\log x - \log y)2(\log x + \log y)}{2(\log x - \log y)x(\log x + \log y)}$$

$$= 1$$

34.

(B)

$$\Rightarrow \log_{10}(2x^2 + 7x + 16) = 1$$

$$\Rightarrow 2x^2 + 7x + 16 = 10^1$$

$$\Rightarrow 2x^2 + 7x + 6 = 0$$

$$\Rightarrow 2x^2 + 4x + 3x + 6 = 0$$

$$\Rightarrow (2x+3)(x+2) = 0$$

$$\Rightarrow x = -\frac{3}{2}, -2$$

35.

(C)

$$\Rightarrow a > 0, b > 0, c > 0$$

$$\Rightarrow \log(a^a b^b c^c) + \log\left(\frac{1}{abc}\right)$$

$$\Rightarrow \log\left(\frac{a^a b^b c^c}{abc}\right)$$

$$\Rightarrow \log(a^{a-1} b^{b-1} c^{c-1})$$

36.

(B)

$$\Rightarrow \log_{10} [\log_{10} (\log_{10} x)] = 0$$

$$\Rightarrow \log_{10} (\log_{10} x) = 10^0 = 1$$

$$\Rightarrow \log_{10} x = 10^1 = 10$$

$$\Rightarrow x = 10^0$$

37.

(C)

$$\Rightarrow (25)^{x-2} = (125)^{2x-4}$$

$$\Rightarrow (5^2)^{x-2} = (5^3)^{2x-4}$$

$$\Rightarrow 5^{2x-4} = 5^{6x-12}$$

$$\Rightarrow 5^{6x-12-(2x-4)} = 1$$

$$\Rightarrow 5^{4x-8} = 1 = 5^0$$

$$\text{So, } 4x - 8 = 0$$

$$\Rightarrow x = 2$$

38.

(C)

$$\Rightarrow \sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = \frac{13}{6} \text{ here } \frac{x}{1-x} \geq 0; \frac{x}{1-x} \geq 0; \frac{x}{x-1} \leq 0$$

So,  $x \in [0, 1)$  is the feasible region for the equation

$$\Rightarrow \frac{(x) + (1-x)}{\sqrt{1-x}\sqrt{x}6} = \frac{13}{6}$$

$$\Rightarrow \frac{1}{\sqrt{x(1-x)}} = \frac{13}{6}$$



Taking square both side

$$\Rightarrow x(1-x) = \frac{36}{169}$$

$$\Rightarrow x^2 - x + \frac{36}{169} = 0$$

$$\Rightarrow \left(x - \frac{9}{13}\right) \left(x - \frac{4}{13}\right) = 0$$

$$\Rightarrow x = \frac{9}{13}, \frac{4}{13}$$

Here values lies in the feasible region

$$\text{So, } x = \frac{9}{13}, \frac{4}{13}$$

39.

(D)

$$\Rightarrow \sqrt{3y+1} = \sqrt{y-1} \quad \dots\dots\dots(1)$$

$$\Rightarrow 3y+1 \geq 0 \text{ \& } y-1 \geq 0$$

$$\Rightarrow y \geq -\frac{1}{3} \text{ \& } y \geq 0$$

$\Rightarrow y \in [0, \infty)$  is our feasible region

By equation (1), taking square of both side,

$$\Rightarrow 3y+1 = y-1$$

$$\Rightarrow 2y = -2$$

$\Rightarrow y = -1$ ; which does not lie in feasible range of  $y$ .

So no solution of  $y$ .

40. (D)

$$\Rightarrow x = 7 + 4\sqrt{3}; x = 4 + 4\sqrt{3} + 3$$

$$\Rightarrow x = (2)^2 + 2 \cdot 2\sqrt{3} + (\sqrt{3})^2$$

$$\Rightarrow x = (2 + \sqrt{3})^2$$

$$\Rightarrow \sqrt{x} = 2 + \sqrt{3} \quad \dots\dots\dots(1)$$

$$\Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2 + \sqrt{3}}$$

$$\Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{2 - \sqrt{3}}{4 - 3}$$

$$\Rightarrow \frac{1}{\sqrt{x}} = 2 - \sqrt{3} \quad \dots\dots\dots(2)$$

$$\Rightarrow \sqrt{x} + \frac{1}{\sqrt{x}} = (2 + \sqrt{3}) + \left( \frac{1}{2 + \sqrt{3}} \right)$$

$$= (2 + \sqrt{3}) + (2 - \sqrt{3})$$

$$= 4$$

41. (A)

$$\Rightarrow x^3 - 3x + 9 = 0$$

Let the roots are  $\alpha, \alpha, \beta$

$$\text{So, } \alpha + \alpha + \beta = 0$$

$$\Rightarrow 2\alpha + \beta = 0 \quad \dots\dots\dots(1)$$

$$\Rightarrow \alpha \cdot \alpha + \alpha \cdot \beta + \beta \cdot \alpha = -3$$

$$\Rightarrow \alpha^2 + 2\alpha\beta = -3 \quad \dots\dots\dots(2)$$

$$\Rightarrow \alpha^2\beta = -q$$

$$\Rightarrow q = -\alpha^2\beta \quad \dots\dots\dots(3)$$

By equations (2), (1)

$$\Rightarrow \alpha^2 + 2\alpha(-2\alpha) = -3$$

$$\Rightarrow -3\alpha^2 = -3$$

$$\Rightarrow \alpha^2 = 1$$

$$\Rightarrow \alpha = \pm 1 \quad \dots\dots\dots(4)$$

$$\begin{aligned} \Rightarrow \because q &= -\alpha^2\beta \\ &= -\alpha^2(-2\alpha) \\ \Rightarrow q &= 2\alpha^3 \\ \Rightarrow q &= \pm 2 \end{aligned}$$

42. (D)

$$\begin{aligned} \Rightarrow \log_{16} x + \log_4 x + \log_2 x &= 14 \\ \Rightarrow \frac{1}{4} \log_2 x + \frac{1}{2} \log_2 x + \log_2 x &= 14 && \left( \text{by } \log_{a^k} b = \frac{1}{k} \log_a b \right) \\ \Rightarrow \frac{7}{4} \log_2 x &= 14 \\ \Rightarrow \log_2 x &= 8 \\ \Rightarrow x &= 2^8 = 256 \end{aligned}$$

43. (C)

$$\Rightarrow |4 - 3x| \leq \frac{1}{2}$$

$$\text{Case 1: } 4 - 3x \geq 0 \Rightarrow x \leq \frac{4}{3} \quad \dots\dots(i)$$

$$\text{So } 4 - 3x \leq \frac{1}{2}$$

$$\Rightarrow -3x \leq -\frac{7}{2}$$

$$\Rightarrow x \geq \frac{7}{6}$$

$$\text{By intersection of (i) \& (ii) } x \in \left[ \frac{7}{6}, \frac{4}{3} \right] \quad \dots\dots(A)$$

$$\text{Case 2: } 4 - 3x \leq 0 \Rightarrow x > \frac{4}{3} \quad \dots\dots(iii)$$

$$\Rightarrow \text{So } -(4 - 3x) \leq \frac{1}{2}$$

$$\Rightarrow -4 + 3x \leq \frac{1}{2}$$

$$\Rightarrow 3x \leq \frac{9}{2}$$

$$\Rightarrow x \leq \frac{3}{2} \quad \dots\dots(iv)$$

Taking intersection of (iii) & (iv)

$$\Rightarrow x \in \left( \frac{4}{3}, \frac{3}{2} \right] \quad \dots\dots(B)$$

$$\text{So union of A \& B is the solution of the given inequality } x \in \left[ \frac{7}{6}, \frac{3}{2} \right] \quad \dots\dots(C)$$

44. (A)  
 $\Rightarrow x^2 - |x| - 6 = 0$

**Case 1:**  $x \geq 0$

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow x = 3, -2$$

But  $x \geq 0$  so  $x = 3$  is the only root.

**Case 2:**  $x < 0$

$$\Rightarrow x^2 - (-x) - 6 = 0$$

$$\Rightarrow x^2 + x - 6 = 0$$

$$\Rightarrow x = 3, -2$$

But  $x < 0$  so  $x = -3$  is the solution.

$$\text{So multiplication} = 3(-3) = -9$$

45. (C)  
 $\Rightarrow \frac{|x|-1}{|x|+2} > 0$



So,  $|x| > -2$  or  $|x| > 1$

$\therefore |x| > -2$  holds true for  $x \in \mathbb{R}$

Now,  $|x| > 1$

$$\Rightarrow x \in (-\infty, -1) \cup (1, \infty)$$

46. (A)  
 $\Rightarrow |x-1| + |x+5| = 6$



This is special case as  $(x-1) - (x+5) = -6$

So the given expression will hold true if  $(x-1)(x+5) \leq 0$

$$\Rightarrow x \in [-5, 1]$$

47. (D)  
 $\Rightarrow \log_{\frac{1}{2}}(x^2 - 1) > 0$

As base of log is less than 1

$$\text{So, } \log_{\frac{1}{2}}(x^2 - 1) > 0$$

$$\Rightarrow 0 < x^2 - 1 < 1$$

$$\Rightarrow 1 < x^2 < 2$$

$$\Rightarrow x^2 > 1 \text{ and } x^2 < 2$$

$$\Rightarrow x \in (-\infty, -1) \cup (1, \infty) \text{ \& } x \in (-\sqrt{2}, \sqrt{2}) \Rightarrow x \in (-\sqrt{2}, -1) \cup (1, \sqrt{2})$$

48. (D)

$$\Rightarrow \log_2 x + \log_x 2 = \frac{10}{3} = \log_2 y + \log_y 2$$

$$\Rightarrow \log_2 x + \log_x 2 = \frac{10}{3}$$

$$\Rightarrow \log_2 x + \frac{1}{\log_2 x} = \frac{10}{3}$$

Let's take  $\log_2 x = a$

$$\text{So, } a + \frac{1}{a} = \frac{10}{3}$$

$$\Rightarrow a^2 - \frac{10}{3}a + 1 = 0$$

$$\Rightarrow \left(a - \frac{9}{3}\right)\left(a - \frac{1}{3}\right) = 0$$

$$\Rightarrow a = \frac{9}{3}, \frac{1}{3}$$

$$\Rightarrow \log_2 x = \frac{9}{3}, \frac{1}{3} = 3, \frac{1}{3}$$

$$\Rightarrow x = 2^3, 2^{\frac{1}{3}}$$

$$\Rightarrow x = 8, 2^{\frac{1}{3}}$$

Similarly,  $y = 8, 2^{\frac{1}{3}}$

If  $x \neq y$  then  $x + y = 8 + 2^{\frac{1}{3}}$

49. (A)

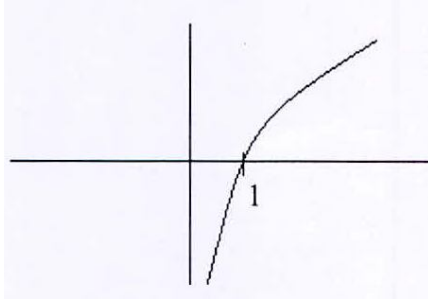
$$\Rightarrow \log_{\frac{\sqrt{3}}{2}}(x^2 - 3x + 2) \geq 2$$

$$\Rightarrow \log_{\frac{\sqrt{3}}{2}}(x^2 - 3x + 2) \geq \log_{\frac{\sqrt{3}}{2}}\left(\frac{\sqrt{3}}{2}\right)^2$$

$$\Rightarrow \log_{\frac{\sqrt{3}}{2}}\left(\frac{x^2 + 3x + 2}{\left(\frac{3}{4}\right)}\right) \geq 0$$

$\because$  base is less than 1 so

$$\Rightarrow 0 < \left(\frac{x^2 - 3x + 2}{\frac{3}{4}}\right) \leq 1$$



$$\Rightarrow 0 < x^2 - 3x + 2 \text{ \& } (x^2 - 3x + 2) \leq \frac{3}{4}$$

$$\Rightarrow (x-2)(x-1) > 0 \text{ \& } x^2 - 3x + \frac{5}{4} \leq 0$$

$$\Rightarrow x \in (-\infty, 1) \cup (2, \infty) \quad \dots\dots(1) \text{ \& } x^2 - \frac{5}{2}x - \frac{1}{2}x + \frac{5}{4} \leq 0$$

$$\text{\& } \left(x - \frac{5}{2}\right) \left(x - \frac{1}{2}\right) \leq 0$$

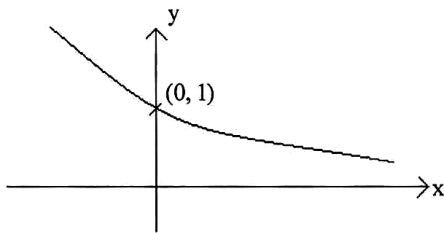
$$\text{\& } x \in \left(\frac{1}{2}, \frac{5}{2}\right) \quad \dots\dots(2)$$

Taking intersection of (i) & (ii)

$$\Rightarrow x \in \left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$$

50. (A)

$$\Rightarrow a^{x^2-x} \geq a^2; 0 < a < 1$$



$$\Rightarrow \frac{a^{x^2-x}}{a^2} \geq 1$$

$$\Rightarrow a^{x^2-x-2} \geq 1$$

$$\Rightarrow x^2 - x - 2 \leq 0$$

$$\Rightarrow (x-2)(x+1) \leq 0$$

$$\Rightarrow x \in [-1, 2]$$

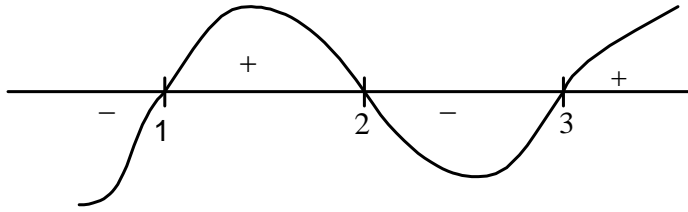
51. (C)

$$(x-1)(x^2 - 5x + 7) < (x-1)$$

$$(x-1)[x^2 - 5x + 6] < 0$$

$$(x-1)(x-2)(x-3) < 0$$





$$x \in (-\infty, 1) \cup (2, 3)$$

52. (D)

$$||x|-1| < |1-x|$$

**Case I**  $x \geq 1$

$$x-1 < -(1-x)$$

$$x-1 < -(1-x)$$

$$-1 < -1$$

No solution

**Case II**  $0 \leq x < 1$

$$1-x < 1-x$$

No solution

**Case III**  $-1 \leq x < 0$

$$|-x-1| < 1-x$$

$$1+x < 1-x$$

$$x < 0$$

$$\Rightarrow x \in [-1, 0)$$

**Case IV:**  $x < -1$

$$|-x-1| < |1-x|$$

$$|1+x| < 1-x$$

$$-(1+x) < 1-x$$

$$-1-x < 1-x$$

$$-1 < 1$$

True for all  $x$ .

$$x \in (-\infty, -1)$$

$$\therefore x \in (-\infty, -1] \cup [-1, 0)$$

$$\Rightarrow x \in (-\infty, 0)$$

53. (A)

$$4^{-x+0.5} - 7 \cdot 2^{-x} < 4$$

$$\frac{4^{0.5}}{4^x} - \frac{7}{2^x} < 4$$

$$\text{Let } \frac{1}{2^x} = k$$

$$\therefore 2 \cdot k^2 - 7k < 4$$

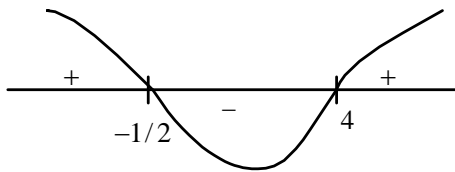
$$2k^2 - 7k - 4 < 0$$

$$2k^2 - 7k - 4 < 0$$

$$2k^2 - 8k + k - 4 < 0$$

$$2k(k-4) + (k-4) < 0$$

$$(2k+1)(k-4) < 0$$



$$k \in \left(-\frac{1}{2}, 4\right)$$

As  $k = \frac{1}{2^x}$  it can only be +ve

$$\therefore 0 < \frac{1}{2^x} < 4$$

$$\Rightarrow x \in (-2, \infty)$$

54. (A)

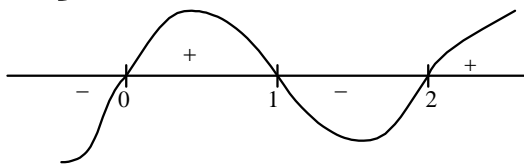
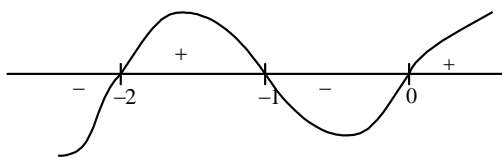
$$\left|x + \frac{2}{x}\right| < 3$$

$$-3 < x + \frac{2}{x} < 3$$

$$x + \frac{2}{x} + 3 > 0$$

$$\frac{x^2 + 3x + 2}{x} > 0$$

$$\frac{(x+1)(x+2)}{x} > 0$$



$$x \in (-2, -1) \cup (0, \infty)$$

$$x + \frac{2}{x} - 3 < 0$$

$$\frac{x^2 - 3x + 2}{x} < 0$$

$$\frac{(x-1)(x-2)}{x} < 0$$

$$x \in (-\infty, 0) \cup (1, 2)$$

$$\therefore x \in (-2, -1) \cup (1, 2)$$

55. (D)

$$2^x + 2^{|x|} \geq 2\sqrt{2}$$

**Case I**  $x \geq 0$

$$2^x + 2^x \geq 2\sqrt{2}$$

$$2^x \geq \sqrt{2}$$

$$\Rightarrow x \geq \frac{1}{2}$$

**Case II**  $x < 0$

$$2^x + \frac{1}{2^x} \geq 2\sqrt{2}$$

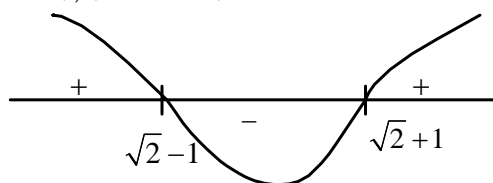
$$\text{Let } 2^x = k \quad k + \frac{1}{k} \geq 2\sqrt{2}$$

$$k^2 - 2\sqrt{2}k + 1 \geq 1$$

$$k = \frac{2\sqrt{2} \pm \sqrt{8-4}}{2} = \frac{2\sqrt{2} \pm 2}{2}$$

$$K = \sqrt{2} \pm 1$$

$$(k - (\sqrt{2} + 1))(k - \sqrt{2} - 1) \geq 0$$



$$-\infty < 2^x \leq \sqrt{2} - 1 \text{ \& } \sqrt{2} + 1 \leq 2^x < \infty$$

$$\Rightarrow 0 < 2^x \leq \sqrt{2} - 1 \text{ \& } \sqrt{2} + 1 \leq 2^x < \infty$$

$$x \in (-\infty, \log_2(\sqrt{2} - 1)] \text{ \& } [\log_2(\sqrt{2} + 1), \infty)$$

From case I & II

$$x \in (-\infty, \log_2(\sqrt{2} - 1)] \cup \left[\frac{1}{2}, \infty\right)$$

56. (A)

$$\log_{1/3}(x^2 + x + 1) + 1 < 0$$

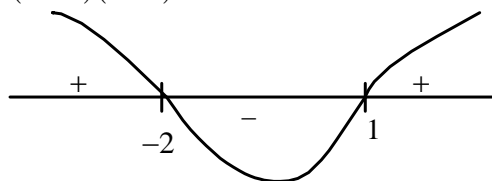
$$\log_{1/3}(x^2 + x + 1) < -1$$

$$x^2 + x + 1 > \left(\frac{1}{3}\right)^{-1}$$

$$x^2 + x + 1 > 3$$

$$x^2 + x - 2 > 0$$

$$(x + 2)(x - 1) > 0$$



$$x \in (-\infty, -2) \cup (1, \infty)$$

57. (B)

$$x^2 = |x + 2| + x > 0$$

**Case I**  $x \geq -2$

$$x^2 - (x+2) + x > 0$$

$$x^2 - 2 > 0$$

$$x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$$

**Case II**  $x \leq -2$

$$x^2 + (x+2) + x > 0$$

$$x^2 + 2x + 2 > 0$$

$$(x+1)^2 + 1 > 0$$

$$x \in \mathbf{R} \text{ i.e. } x < -2$$

From (1) & (2)

$$x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$$

58. (C)

$$\log_{1/2} x \geq \log_{1/3} x$$

$$-\log_2 x \geq -\log_3 x$$

$$\log_2 x \leq \log_3 x$$

$$\frac{\log x}{\log 2} \leq \frac{\log x}{\log 3}$$

$$\log^x \left[ \frac{1}{\log 2} - \frac{1}{\log 3} \right] \leq 0$$

$$\log x \leq 0 \left\{ \because \frac{1}{\log 2} - \frac{1}{\log 3} > 0 \right\}$$

$$\Rightarrow x \in (0, 1]$$

59. (B)

$$x^{\log_3 x^2} + (\log_3 x)^2 - 10 = \frac{1}{x^2}$$

Clearly one solution is  $x = 1$

OR

$$\log_3 x^2 + (\log_3 x)^2 - 10 = -2$$

$$2\log_3 x + (\log_3 x)^2 = 8$$

Let  $\log_3 x = k$

$$\therefore 2k + k^2 - 8 = 0$$

$$k^2 + 2k - 8 = 0$$

$$(k+4)(k-2) = 0$$

$$\Rightarrow \log_3 x = 2 \text{ or } \log_3 x = -4$$

$$\Rightarrow x = 9 \text{ or } x = \frac{1}{81}$$

$$x \in \left\{ 9, \frac{1}{81} \right\}$$

60. (B)

$$x = \log_a nc = \frac{\log bc}{\log a}$$

$$x+1 = \frac{\log bc}{\log a} + 1$$

$$\Rightarrow x+1 = \frac{\log bc + \log x}{\log a}$$

$$\Rightarrow x+1 = \frac{\log abc}{\log a}$$

Similarly

$$y+1 = \frac{\log abc}{\log b} \quad \& \quad z+1 = \frac{\log abc}{\log c}$$

$$\therefore \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$$

$$\Rightarrow \frac{\log a}{\log abc} + \frac{\log b}{\log abc} + \frac{\log c}{\log abc}$$

$$= \frac{\log abc}{\log abc} = 1$$

**INEQUATIONS & EQUATION**  
**EXERCISE 1 (B)**

1. (D)

$$\Rightarrow |3^x - 1| > |3^x - 9|$$

Take square both side

$$\Rightarrow (3^x - 1)^2 - (3^x - 9)^2 > 0$$

$$\Rightarrow [(3^x - 1) + (3^x - 9)][3^x - 1 - (3^x - 9)] > 0$$

$$\Rightarrow [2 \cdot 3^x - 10][8] > 0$$

$$\Rightarrow 3^x - 5 > 0 \Rightarrow 3^x > 5$$

$$x > \log_3 5$$

2. (B)

$$\Rightarrow \sqrt{25 - 5^x} = \sqrt{4^x - 16} \quad \dots\dots\dots(1)$$

$$\text{Here } 25 - 5^x \geq 0 \quad \& \quad 4^x - 16 \geq 0$$

$$\Rightarrow 5^x \leq 25 \quad \& \quad 4^x \geq 4^2$$

$$\Rightarrow 5^x \leq 5^2 \quad \& \quad x \geq 2$$

$$\Rightarrow x \in (-\infty, 2] \quad \& \quad x \in [2, \infty)$$

So only feasible region for given equation is  $x = 2$

For  $x = 2$ , gives equation is satisfied

So no of solutions = 1

3. (D)

$$\sqrt{4x+1} + \sqrt{7-x} = 0$$

As square root is always positive so given equation is feasible only if

$$\Rightarrow 4x+1=0 \quad \& \quad 7-x=0$$

$$\Rightarrow x = -\frac{1}{4} \quad \& \quad x = 7$$

So no common solution.

4. (C)

$$\Rightarrow 4^{x^2+2} - 9 \cdot 2^{x^2+2} + 8 = 0$$

$$\Rightarrow 2^{2x^2+4} - 9 \cdot 2^2 \cdot 2^{x^2+2} = 0$$

$$\Rightarrow 16 \cdot 2^{2x^2} - 36 \cdot 2^{x^2} + 8 = 0$$

$$\Rightarrow 4 \cdot 2^{2x^2} - 9 \cdot 2^{x^2} + 2 = 0$$

Put  $2^{x^2} = a$

$$\text{so, } 4a^2 - 9a + 2 = 0$$

$$\Rightarrow (4a-1)(a-2) = 0$$

$$\Rightarrow a = \frac{1}{4}, 2$$

$$\Rightarrow 2^{x^2} = \frac{1}{4}, 2$$

$$\Rightarrow x^2 = -2, 1$$

$$\Rightarrow x^2 = -2 \text{ is not possible; } x^2 = 1$$

$$\Rightarrow x = \pm 1$$

5. (A)

$$\Rightarrow \sqrt{(x-1)^2} + \sqrt[4]{(2x+1)^4} - \sqrt[3]{\left(x - \frac{1}{2}\right)^3}; x \in (0,1)$$

$$\Rightarrow |x-1| + (2x+1) - \left(x - \frac{1}{2}\right) \quad (\text{as } 1+2x > 0 \text{ for } x \in (0,1))$$

$$\Rightarrow -(x-1) + (2x+1) - \left(x - \frac{1}{2}\right)$$

$$\Rightarrow \frac{5}{2}$$

6. (A)

$$\Rightarrow \frac{2^{x-1}}{2^{x+1} + 1} < 2$$

We can cross multiply  $(2^{x+1} + 1)$  as  $2^{x+1} + 1 > 0$  for  $x \in \mathbb{R}$

$$\Rightarrow 2^{x-1} - 1 < 2(2^{x+1} + 1)$$

$$\Rightarrow \frac{2^x}{2} - 1 < 4 \cdot 2^x + 4$$

$$\Rightarrow \frac{7}{2} 2^x > -5$$

$$\Rightarrow 2^x > \frac{-10}{7}$$

This is true for  $x \in \mathbb{R}$

7. (C)

$$\Rightarrow \sqrt{2^{2x} - 7} < 2^x - 1 \quad \dots\dots(1)$$

Here  $2^{2x} - 7 \geq 0$

$$\Rightarrow 2^{2x} \geq 7$$

$$\Rightarrow 2x \geq \log_2 7$$

$$\Rightarrow x \geq \frac{1}{2} \log_2 7 \quad \dots\dots(2) \text{ (feasible region)}$$

From feasible region it is clear that  $2^x - 1 > 0$

So by taking square of (1)

$$\Rightarrow 2^{2x} - 7 < (2^x - 1)^2$$

$$\Rightarrow 2^{2x} - 7 < 2^{2x} + 1 - 2 \cdot 2^x$$

$$\Rightarrow 2 \cdot 2^x < 8$$

$$\Rightarrow 2^x < 4$$

$$\Rightarrow x < 2 \quad \dots\dots\dots(2)$$

By taking intersection of (1) & (2)

$$\text{We get } x \in \left[ \frac{1}{2} \log_2 7, 2 \right)$$

$$\Rightarrow x \in [\log_4 7, 2)$$

8. (A)

$$\Rightarrow |x^3 - 1| \geq 1 - x$$

Case I:  $x^3 - 1 \geq 0$

$$\Rightarrow x \geq 1$$

So  $(x^3 - 1) \geq 1 - x$

$$\Rightarrow x^3 + x - 2 \geq 0$$

$$\Rightarrow (x-1)(x^2 + x + 2) \geq 0$$

$$\Rightarrow x \in [1, \infty) \quad \dots\dots\dots(1)$$

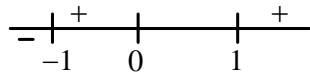
Case 2:  $x^3 - 1 < 0$

$$\Rightarrow x^3 < 1 \Rightarrow x < 1$$

So,  $-(x^3 - 1) \geq 1 - x$

$$\Rightarrow -x^3 + x \geq 0$$

$$\Rightarrow x^3 - x \geq 0$$



$$\Rightarrow x(x-1)(x+1) \leq 0$$

$$\Rightarrow x \in (-\infty, 1] \cup [0, 1] \quad \dots\dots\dots(2)$$

So take union of (1) & (2)

$$\Rightarrow x \in (-\infty, -1] \cup [0, \infty)$$

9. (B)

$$\Rightarrow \frac{|x+2| - x}{x} < 2$$

Case I:  $x + 2 \geq 0$

$$\Rightarrow x > -2 \quad \dots\dots\dots(i)$$

$$\Rightarrow \frac{(x+2) - x}{x} < 2$$

$$\Rightarrow \frac{2}{x} - 2 < 0$$

$$\Rightarrow \frac{1-x}{x} < 0$$

$$\Rightarrow \frac{x-1}{x} < 0$$

$$\Rightarrow x \in (-\infty, 0) \cup (1, \infty)$$



$$\begin{array}{c} + \quad | \quad - \quad | \quad + \\ \hline 0 \quad \quad \quad 1 \end{array}$$

Taking intersection of (i) & (ii)

$$\Rightarrow x \in [-2, 0) \cup (1, \infty) \quad \dots\dots(iii)$$

$$\text{Case 2: } \Rightarrow x + 2 < 0 \Rightarrow x \in (-\infty, -2) \quad \dots\dots(A)$$

$$\Rightarrow \frac{-(x+2) - x}{x} < 2$$

$$\Rightarrow \frac{-2x - 2 - 2x}{x} < 0$$

$$\Rightarrow \frac{2x - 1}{x} > 0$$

$$\begin{array}{c} + \quad | \quad - \quad | \quad + \\ \hline -\frac{1}{2} \quad \quad \quad 0 \end{array}$$

$$\Rightarrow x \in \left(-\infty, -\frac{1}{2}\right) \cup (0, \infty) \quad \dots\dots(B)$$

Taking intersection of A & B

$$\Rightarrow x \in (-\infty, -2) \quad \dots\dots(C)$$

So by taking union of (iii) & (c)

$$\Rightarrow x \in (-\infty, 0) \cup (1, \infty)$$

10. (A)

$$\Rightarrow \frac{x^2 - 5x + 4}{x^2 - 4} \leq 1$$

$$\Rightarrow -1 \leq \frac{x^2 - 5x + 4}{x^2 - 4} \leq 1$$

$$\text{Case 1: } \frac{x^2 - 5x + 4}{x^2 - 4} \geq -1$$

$$\Rightarrow \frac{x^2 - 5x + 4 + x^2 - 4}{x^2 - 4}$$

$$\Rightarrow \frac{x(2x - 5)}{(x - 2)(x + 2)} \geq 0$$

$$\begin{array}{c} + \quad | \quad - \quad | \quad + \quad | \quad - \quad | \quad + \\ \hline -2 \quad \quad 0 \quad \quad 2 \quad \quad \frac{5}{2} \end{array}$$

$$\Rightarrow x \in (-\infty, -2) \cup [0, 2) \cup \left[\frac{5}{2}, \infty\right) \quad \dots\dots(1)$$

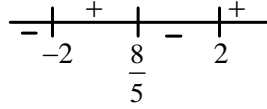
$$\text{Case 2: } \frac{x^2 - 5x + 4}{x^2 - 4} \leq 1$$

$$\Rightarrow \frac{x^2 - 5x + 4 - x^2 + 4}{x^2 - 4} \leq 0$$

$$\Rightarrow \frac{8-5x}{(x-2)(x+2)} \leq 0$$

$$\Rightarrow \frac{(5x-8)}{(x-2)(x+2)} \geq 0$$

$$\Rightarrow x \in \left(-2, \frac{8}{5}\right] \cup (2, \infty)$$



By taking intersection of (1) & (2)

$$\Rightarrow x \in \left[0, \frac{8}{5}\right] \cup \left[\frac{5}{2}, \infty\right)$$

11. (C)

$$\Rightarrow \frac{x^2 - |x| - 12}{x-2} \geq 2x$$

Case 1:  $x \geq 0$

$$\Rightarrow \frac{x^2 - x - 12}{x-3} \geq 2x$$

$$\Rightarrow \frac{x^2 - x - 12}{x-3} - 2x \geq 0$$

$$\Rightarrow \frac{x^2 - x - 12}{x-3} - 2x \geq 0$$

$$\Rightarrow \frac{x^2 - x - 12 - 2x^2 + 6x}{x-3} \geq 0$$

$$\Rightarrow \frac{-x^2 + 5x - 12}{x-3} \geq 0$$

$$\Rightarrow \frac{x^2 - 5x + 12}{x-3} \leq 0$$

as  $x^2 - 5x + 12$  is always greater than zero for  $x \in \mathbb{R}$

so,  $\frac{1}{x-3} \leq 0$

$$\Rightarrow x \in (-\infty, 3)$$

Case 2:  $x < 0$

$$\Rightarrow \frac{x^2 + x - 12}{x-3} - 2x \geq 0$$

$$\Rightarrow \frac{x^2 + x - 12 - 2x^2 + 6x}{x-3} \geq 0$$

$$\Rightarrow \frac{-x^2 + 7x - 12}{x-3} \geq 0$$

$$\Rightarrow \frac{x^2 - 7x + 12}{x-3} \leq 0$$

$$\Rightarrow \frac{(x-4)(x-3)}{(x-3)} \leq 0$$

$$\Rightarrow x-4 \leq 0$$

$$\Rightarrow x \in (-\infty, 4]$$

But for this case  $x < 0$

So we get

$$\Rightarrow x \in (-\infty, 0)$$

Take union of (1) & (2)

$$\Rightarrow x \in (-\infty, 3)$$

12. (C)

$$\Rightarrow \left| \frac{x^2 - 3x - 1}{x^2 + x + 1} \right| < 3$$

$$\Rightarrow -3 \frac{x^2 - 3x - 1}{x^2 + x + 1} < 3$$

$$\text{Case 1: } \frac{x^2 - 3x - 1}{x^2 + x + 1} > -3$$

As  $(x^2 + x + 1)$  is always greater than zero

$$\text{So } x^2 - 3x - 1 > -3(x^2 + x + 1)$$

$$\Rightarrow 4x^2 + 2 > 0$$

$$\Rightarrow x \in \mathbb{R} \quad \dots\dots(1)$$

$$\text{Case 2: } \frac{x^2 - 3x + 1}{x^2 + x + 1} < 3$$

$$\Rightarrow x^2 - 3x - 1 < 3x^2 + 3x + 3$$

$$\Rightarrow 2x^2 + 6x + 4 > 0$$

$$\Rightarrow x^2 + 3x + 2 > 0$$

$$\Rightarrow (x+1)(x+2) > 0$$

$$\Rightarrow x \in (-\infty, -2) \cup (-1, \infty) \quad \dots\dots(2)$$

Take intersection of (1) & (2)

$$\Rightarrow x \in (-\infty, -2) \cup (-1, \infty)$$

13. (D)

$$\Rightarrow \frac{|x+3| + x}{x+2} > 1$$

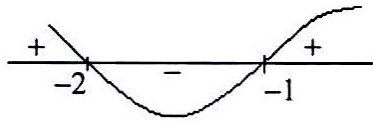
$$\Rightarrow \frac{|x+3| + x}{x+2} - 1 > 0$$

$$\Rightarrow \frac{|x+3| + x - x - 2}{x+1} > 0$$

$$\Rightarrow \frac{|x+3| - 2}{x+1} > 0$$

$$\text{Case 1: } x+3 \geq x \geq -3$$

$$\Rightarrow \frac{x+3-2}{x+2} > 0$$



$$\Rightarrow \frac{x+1}{x+2} > 0$$

$$\Rightarrow x \in (-\infty, -2) \cup (-1, -\infty)$$

But for this case  $x \geq -3$

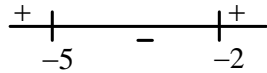
$$\text{So we get } x \in [-3, -2) \cup (-1, \infty)$$

Case 2:  $x+3 < 0$

$$\Rightarrow x < -3$$

$$\text{So, } \frac{-(x+3)}{x+2} > 0$$

$$\Rightarrow \frac{x+5}{x+2} < 0$$



$$\Rightarrow x \in (-5, -2)$$

$$\text{As } x < -3 \text{ so } x \in (-5, -3)$$

.....(2)

Take union of (1) & (2)

$$\Rightarrow x \in (-5, -2) \cup (-1, \infty)$$

So least integral value of  $x = -4$

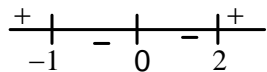
14. (A)

$$\Rightarrow \sqrt{1 - \left(\frac{x+2}{x^2}\right)} < \frac{2}{3}$$

$$\text{So, } 1 - \left(\frac{x+2}{x^2}\right) \geq 0$$

$$\Rightarrow \frac{x^2 - x - 2}{x^2} \geq 0$$

$$\Rightarrow \frac{(x-2)(x+1)}{x^2} \geq 0$$



$$\text{So } x \in (-\infty, -1] \cup [2, \infty)$$

.....(1)

$$\Rightarrow \sqrt{1 - \left(\frac{x+2}{x^2}\right)} < \frac{2}{3}$$

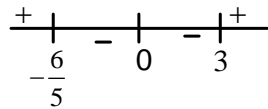
$$\Rightarrow 1 - \left(\frac{x+2}{x^2}\right) < \frac{4}{9}$$

$$\Rightarrow \frac{5}{9} - \frac{x+2}{x^2} < 0$$

$$\Rightarrow \frac{5x^2 - 9x - 18}{9x^2} < 0$$

$$\Rightarrow \frac{(5x+6)(x-3)}{x^2} < 0$$

$$\text{So, } x \in \left(-\frac{6}{5}, 0\right) \cup (0, 3) \quad \dots\dots(2)$$



By taking (1)  $\cap$  (2)

$$\Rightarrow x \in \left(-\frac{6}{5}, -1\right] \cup [2, 3)$$

15.  $\Rightarrow \sqrt{4-\sqrt{1-x}} - \sqrt{2-x} > 0$

Here  $1-x \geq 0$

$$\Rightarrow 2-x > 0$$

$$\Rightarrow x \leq 1 \quad \dots\dots(i)$$

$$\Rightarrow 4 - \sqrt{1-x} \geq 0$$

$$\Rightarrow 4 \geq \sqrt{1-x}$$

$$\Rightarrow 16 \geq 1-x$$

$$\Rightarrow x > -15 \quad \dots\dots(ii)$$

So, (i)  $\cap$  (ii)

$$\Rightarrow x \in [-15, 1] \quad \dots\dots (iii) \text{ feasible region}$$

Take intersection of (iv) & (iii)

$$\Rightarrow x \in [-2, 1]$$

Now

$$\Rightarrow \sqrt{4-\sqrt{1-x}} > \sqrt{2-x}$$

Take square both side

$$\Rightarrow 4 - \sqrt{1-x} > 2-x$$

$$\Rightarrow 2+x > \sqrt{1-x}$$

$$\text{Here } 2+x \geq 0 \Rightarrow x \geq -2 \quad \dots\dots(iv)$$

So,  $2+x > \sqrt{1-x}$

Take square  $4+x^2+4x > 1-x$

$$\Rightarrow x^2+5x+3 > 0$$

$$\Rightarrow \left(x - \left(\frac{-5+\sqrt{13}}{2}\right)\right) \left(x - \left(\frac{-5-\sqrt{13}}{2}\right)\right)$$

$$\Rightarrow x \in \left(-\infty, \frac{-5-\sqrt{13}}{2}\right) \cup \left(\frac{\sqrt{13}-5}{2}, \infty\right) \quad \dots\dots(2)$$

$$(1) \cap (2)$$

$$\Rightarrow x \in \left( \frac{\sqrt{13}-5}{2}, 1 \right)$$

16. (A)

$$\Rightarrow \sqrt{4-x^2} + \frac{|x|}{x} \geq 0$$

Here  $4-x^2 \geq 0$

$$\Rightarrow x^2 \leq 4$$

$$\Rightarrow x \in [-2, 2] \quad \dots\dots(i)$$

Case 1:  $x > 0$

$$\Rightarrow \sqrt{4-x^2} + \frac{x}{x} \geq 0 \quad (x \neq 0)$$

$$\Rightarrow \sqrt{4-x^2} + 1 \geq 0$$

$$\Rightarrow \sqrt{4-x^2} \geq -1 \quad \text{true for } x \in \mathbb{R}$$

Case 2:  $x < 0$

$$\Rightarrow \sqrt{4-x^2} + \frac{(-x)}{x} \geq 0$$

$$\Rightarrow \sqrt{4-x^2} - 1 \geq 0$$

$$\Rightarrow \sqrt{4-x^2} \geq 1$$

Take square  $4-x^2 \geq 1 \Rightarrow x^2 \leq 3$

$$\Rightarrow x \in [-\sqrt{3}, \sqrt{3}]$$

But  $x < 0$

$$\text{So } x \in [-\sqrt{3}, 0)$$

So case (1)  $\cup$  case (2)

$$\Rightarrow x \in \mathbb{R} \quad \dots\dots(iii)$$

But our feasible region is  $x \in [-2, 2]$

So greatest integral  $x = 2$

17. (C)

$$\Rightarrow \frac{|x+2| - |x|}{\sqrt{4-x^2}} \geq 0 \text{ here } 4-x^2 > 0$$

$$\Rightarrow x^3 < 4 \Rightarrow x \in (-\infty, \sqrt[3]{4}) \quad \dots\dots(1)$$

As  $(4-x^3)$  is greater than zero

$$\Rightarrow |x+2| - |x| \geq 0$$

$$\Rightarrow |x+2| \geq |x|$$

$$\Rightarrow (x+2)^2 \geq x^2$$

$$\Rightarrow (x+2-x)(x+2+x) \geq 0$$

$$\Rightarrow 2(2x+2) \geq 0$$

$$\Rightarrow x \geq -1 \quad \dots\dots\dots(2)$$

So  $(1) \cap (2)$

$$\Rightarrow x \in [-1, \sqrt[3]{4})$$

18. (B)

$$\Rightarrow \log_4 \left( \frac{x+1}{x+2} \right) > 0$$

$$\Rightarrow \frac{x+1}{x+2} > 0$$

$$\Rightarrow \frac{x+1-x-2}{x+1} > 0$$

$$\Rightarrow \frac{-1}{x+2} > 0$$

$$\Rightarrow \frac{1}{x+2} < 0$$

$$\Rightarrow x \in (-\infty, -2)$$

19. (A)

$$\Rightarrow x^3 + ax + b = 0$$

$$\Rightarrow \alpha + \beta + \gamma = 0$$

$$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = a$$

$$\Rightarrow \alpha\beta\gamma = -b$$

$$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) + \gamma^3$$

$$= (-\gamma)((\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta) + \gamma^3$$

$$= (-\gamma)(\gamma^2 - 3\alpha\beta) + \gamma^3$$

$$= +3\alpha\beta\gamma = -3b$$

$$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$\Rightarrow \frac{\alpha^3 + \beta^3 + \gamma^3}{\alpha^2 + \beta^2 + \gamma^2} = \frac{-3b}{-2a} = \frac{3b}{2a}$$

20. (A)

$$\Rightarrow f(x) = x^3 + 2x^2 + k$$

If  $(x - 1)$  is divisor of  $f(x)$

Then  $f(1) = 0$

$$\Rightarrow f(1) = 1 + 2 + k = 0$$

$$\Rightarrow k = -3$$

21. (D)

$$\Rightarrow 4x^3 + 20x^2 - 23x + 6 = 0$$

Let's take roots are  $\alpha, \alpha, \beta$

$$\Rightarrow 2\alpha + \beta = \frac{-20}{4} = -5 \quad \dots\dots\dots(1)$$

$$\Rightarrow \alpha^2 + \alpha\beta + \alpha\beta = \frac{-23}{4}$$

$$\Rightarrow \alpha^2\beta = \frac{-6}{4}$$

$$\Rightarrow \alpha^2(-5 - 2\alpha) = -\frac{6}{4}$$

$$\Rightarrow 5\alpha^2 + 2\alpha^3 - \frac{6}{4} = 0$$

$$\Rightarrow \alpha^3 + 20\alpha^3 - 6 = 0$$

Which gives  $\alpha = \frac{1}{2}$

$$\text{So } 2\left(\frac{1}{2}\right) + \beta = -5 \Rightarrow \beta = -6$$

So roots are  $-6, \frac{1}{2}, \frac{1}{2}$

22. (D)

$$\Rightarrow 2x^3 - 5x^2 + 3x - 1 = 0$$

$$\Rightarrow \alpha\beta\gamma = \frac{1}{2}$$

$$\Rightarrow \alpha + \beta + \gamma = \frac{5}{2}$$

$$\Rightarrow \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\beta\alpha} = \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma} = \frac{\left(\frac{5}{2}\right)}{\left(\frac{1}{2}\right)} = 5$$

23. (A)

$$\Rightarrow x^3 - 4x^2 + x + 6 = 0$$

Roots are  $2\alpha, 3\alpha, \beta$

So,  $2\alpha, 3\alpha, \beta = 4$

$$\Rightarrow 5\alpha + \beta = 4 \quad \dots\dots\dots(1)$$

$$\Rightarrow (2\alpha)(3\alpha)(\beta) = -6$$

$$\Rightarrow \alpha^2\beta = -1 \quad \dots\dots\dots(2)$$

By (1) & (2)

$$\Rightarrow \alpha^2(4 - 5\alpha) = -1$$

$$\Rightarrow 5\alpha^3 - 4\alpha^2 - 1 = 0$$

By this we get  $\alpha = 1$

So roots are  $2, 3, -1$

24. (B)

$$\Rightarrow 4x^3 - 12x^2 + 11x + k = 0$$



$$\text{Sum of roots } (a - d + a + a + d) = \frac{12}{4} = 3$$

$$\Rightarrow a = 1$$

As  $a$  is root of equation so it will satisfy if.

$$\Rightarrow 4(a)^3 - 12a^2 + 11a + k = 0$$

$$\Rightarrow 4 - 12 + 11 + k = 0$$

$$\Rightarrow k = 3$$

25. (B)

$$\Rightarrow x^3 - 11x^2 + 37x - 35 = 0$$

If root is  $3 + \sqrt{2}$ , second root has to be  $3 - \sqrt{2}$

Let's take third root  $\alpha$

$$\text{So, } (3 + \sqrt{2}) + (3 - \sqrt{2}) + \alpha = 11$$

$$\Rightarrow \alpha = 5$$

26. (B)

As  $x + 1$  is factor so  $x = -1$  will satisfy the expression

$$\Rightarrow (-1)^4 + (p - 3)(-1)^3 - (3p - 5)(-1)^2 + (2p + q)(-1) + 12 = 0$$

$$\Rightarrow p = 2$$

27. (D)

$$\Rightarrow x^3 - ix + 1 - i = 0$$

Roots will be  $1 + i, 1 - i, \alpha$

$$\Rightarrow 1 + i + 1 - i + \alpha = 0$$

$$\Rightarrow \alpha = -2$$

So equation will roots  $(1 - i)$  &  $-2$  is

$$\Rightarrow x^2 - (1 - i - 2)x + (1 - i)(-2) = 0$$

$$\Rightarrow x^2 + (1 + i)x - 2(1 - i) = 0$$

28. (D)

$$\Rightarrow 2x^4 + 9x^3 + 8x^2 + 9x + 2 = 0$$

29. (A)

$$\Rightarrow x^3 - 1 = 0$$

Roots will be  $\alpha, \beta, \gamma$

$$\Rightarrow \alpha + \beta + \gamma = 0$$

$$\Rightarrow \sum \alpha\beta = 0$$

$$\Rightarrow \alpha\beta\gamma = 1$$

$$\Rightarrow \sum (\alpha + 1) = (\alpha + \beta + \gamma) + 3 = 3$$

$$= \sum (\alpha + 1)(\beta + 1) = (\alpha + 1)(\beta + 1) + (\alpha + 1)(\gamma + 1) + (\beta + 1)(\gamma + 1)$$

$$= \sum \alpha\beta + (\alpha + \beta + 1) + (\alpha + \gamma + 1) + (\beta + \gamma + 1)$$

$$\begin{aligned}
&= 3 \\
&\Rightarrow (\alpha+1)(\beta+1)(\gamma+1) = (\alpha\beta + \alpha + \beta + 1)(\gamma+1) \\
&= \alpha\beta\gamma + \alpha\beta + \alpha\gamma + \alpha + \beta\gamma + \beta + \gamma + 1 = 2 \\
&\text{So equation with } (\alpha+1), (\beta+1), (\gamma+1) \text{ as roots} \\
&= x^3 - 3x^2 + 3x - 2 = 0
\end{aligned}$$

30. (B)

$$\begin{aligned}
&\Rightarrow x^3 + 3x + 2 = 0 \\
&\Rightarrow a + b + c = 0 \\
&\Rightarrow ab + bc + ca = 3 \\
&\Rightarrow abc = -2 \\
&\Rightarrow a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 + ab + bc + ca) = 0 \\
&\Rightarrow a^3 + b^3 + c^3 = 3(-2) = -6
\end{aligned}$$

31. (A)

$$\begin{aligned}
&\Rightarrow x^3 + 1 = 0 \\
&\Rightarrow \alpha + \beta + \gamma = 0 \\
&\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = 0 \\
&\Rightarrow \alpha\beta\gamma = 1 \\
&\Rightarrow (\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha) \\
&\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 0 \\
&\Rightarrow (\alpha^2 + \beta^2 + \gamma^2)^2 = \alpha^4 + \beta^4 + \gamma^4 + 2(\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2) \\
&\Rightarrow 0 = \alpha^4 + \beta^4 + \gamma^4 + 2\left[(\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2(\alpha\beta\cdot\beta\gamma + \beta\gamma\cdot\gamma\alpha + \gamma\alpha\cdot\alpha\beta)\right] \\
&= \alpha^4 + \beta^4 + \gamma^4 + 2\left[0 - 2(\beta + \gamma + \alpha)\right] \quad (\because \alpha\beta\gamma = 1) \\
&\Rightarrow 0 = \alpha^4 + \beta^4 + \gamma^4 + 2\left[0 - 2(0)\right] \\
&\Rightarrow \alpha^4 + \beta^4 + \gamma^4 = 0
\end{aligned}$$

32. (A)

$$\begin{aligned}
&\Rightarrow x^3 + 3x^2 + 2x + 4 = 0 \\
&\Rightarrow \alpha\beta\gamma = -3 \\
&\Rightarrow \sum \alpha\beta = 2 \\
&\Rightarrow \alpha\beta\gamma = -4 \\
&\Rightarrow x^3 - (2\alpha + 2\beta + 2\gamma)x^2 + (\sum \alpha 2\beta)x - (2\alpha \cdot 2\beta \cdot 2\gamma) = 0 \\
&\Rightarrow x^3 + 6x^2 + 8x + 32 = 0
\end{aligned}$$

33. (A)

As  $\alpha, \beta, \gamma$  satisfy cubic equation  $x^3 - 5x^2 + 5x - 3 = 0$

So,  $\alpha, \beta, \gamma$  are roots of this equation

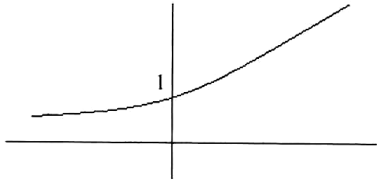
$$\Rightarrow \alpha + \beta + \gamma = 5$$

34. (C)

$$\Rightarrow 5^{x+2} > \left(\frac{1}{25}\right)^{\frac{1}{x}}$$

$$\Rightarrow 5^{x+2} > \frac{1}{5^{\frac{2}{x}}}$$

$$\Rightarrow 5^{x+2} \cdot 5^{\frac{2}{x}} > 1$$



$$\Rightarrow 5^{x+\frac{2}{x}+2} > 5^0$$

So  $x + \frac{2}{x} + 2 > 0$

$$\Rightarrow \frac{x^2 + 2x + 2}{x} > 0$$

Numerator is always  $> 0$

So  $\frac{1}{x} > 0$

$$\Rightarrow x \in (0, \infty)$$

35. (B)

$$\Rightarrow \frac{x^2 - 5x + 6}{|x| + 7} < 0$$

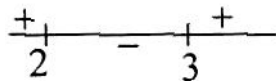
$$\Rightarrow \frac{(x-3)(x-2)}{|x| + 7} < 0$$

As  $7 + |x| \geq 7$

So we can cross multiply  $7 + |x|$

$$\Rightarrow (x-3)(x-2) < 0$$

$$\Rightarrow x \in (2, 3)$$



36. (D)

$$\Rightarrow \left| \frac{2x-1}{x-1} \right| > 2$$

$$\Rightarrow \frac{2x-1}{x-1} < -2$$

$$\text{or } \frac{2x-1}{x-1} > 2$$

$$\Rightarrow \frac{2x-1+2x-2}{x-1} < 0 \quad \text{or} \quad \frac{2x-1-2x+2}{x-1} > 0$$

$$\Rightarrow \frac{4x-3}{x-1} \quad \text{or} \quad \frac{1}{x-1} > 0$$

$$\Rightarrow x \in \left(\frac{3}{4}, 1\right) \quad \text{Or } x > 1$$

So,  $x \in \left(\frac{3}{4}, \infty\right) - \{1\}$

37. (A)

$$\Rightarrow \frac{1}{|x|-3} < \frac{1}{2}$$

Case 1:  $x \geq 0$

$$\Rightarrow \frac{1}{x-3} - \frac{1}{2} < 0$$

$$\Rightarrow \frac{5-x}{2(x-3)} < 0$$

$$\Rightarrow \frac{x-5}{(x-3)} > 0$$

$$\Rightarrow x \in (-\infty, 3) \cup (5, \infty)$$

But  $x \geq 0$

$$\text{So, } x \in [0, 3) \cup (5, \infty) \quad \dots\dots\dots(1)$$

Case 2:  $x < 0$

$$\Rightarrow \frac{1}{-x-3} - \frac{1}{2} < 0$$

$$\Rightarrow \frac{1}{x+3} + \frac{1}{2} > 0$$

$$\Rightarrow \frac{5+x}{x+3} > 0$$

$$\Rightarrow x \in (-\infty, -5) \cup (-3, \infty)$$

But  $x < 0$

$$\text{So, } x \in (-\infty, -5) \cap (-3, 0) \quad \dots\dots\dots(2)$$

So (1)  $\cup$  (2)

$$\Rightarrow x \in (-\infty, -5) \cup (-3, \infty)$$

So least positive integer value = 1

38. (C)

$$\Rightarrow (x-1)\sqrt{x^2-x-2} \geq 0$$

$$\Rightarrow (x^2-x-2) \geq 0 \quad \& \quad (x-1) \geq 0$$

$$\Rightarrow (x-2)(x+1) \geq 0 \quad \& \quad x \geq 1$$

$$\Rightarrow x \in (-\infty, -1] \cup [2, \infty) \quad \& \quad x \in [1, \infty)$$

So,  $x \in [2, \infty)$

39. (B)

$$\Rightarrow (x^2 - 1)\sqrt{x^2 - x - 2} \geq 0$$

$$\Rightarrow (x^2 - 1) \geq 0 \quad \& \quad x^2 - x - 2 \geq 0 \quad \text{.....(1)}$$

$$\Rightarrow x \in (-\infty, -1] \cup [1, \infty) \quad \& \quad (x - 2)(x + 1) \geq 0$$
$$x \in (-\infty, -1] \cup [2, \infty) \quad \text{.....(2)}$$

Take  $(1) \cap (2)$

So,  $x \in (-\infty, -1] \cup [2, \infty)$

Least positive integer = 2

40. (C)

$$\Rightarrow 49^x + 7^{x+1} - 98 < 0$$

$$\Rightarrow 7^{2x} + 7 \cdot 7^x - 98 < 0$$

$$\Rightarrow 7^x = a; a^2 + 7a - 78 < 0$$

$$\Rightarrow (a + 14)(a - 7) < 0 \quad (\because a + 14 = 7^x + 14 > 0 \text{ for } x \in \mathbb{R})$$

$$\Rightarrow 7^x < 7$$

So,  $x < 1$

41. (B)

$$\alpha, \beta, \gamma \rightarrow x^3 + 2x^2 - 3x - 1 = 0$$

The equation whose roots are  $\frac{1}{\alpha^2}, \frac{1}{\beta^2}, \frac{1}{\gamma^2}$

$$x = \frac{1}{\alpha^2}$$

$$\Rightarrow \alpha = \frac{1}{\sqrt{x}}$$

$$\left(\frac{1}{x}\right)^{\frac{3}{2}} + \frac{2}{x} - \frac{2}{x^{\frac{1}{2}}} - 1 = 0$$

$$\Rightarrow 1 + 2\sqrt{x} - 3x - x^{\frac{3}{2}} = 0$$

$$\Rightarrow 1 - 3x = x^{\frac{3}{2}} - 2x^{\frac{1}{2}}$$

$$\Rightarrow 1 + 9x^2 - 6x = x^3 + 4x - 4x^2 \text{ (squaring)}$$

$$\Rightarrow x^3 - 13x^2 + 10x - 1 = 0$$

$$\text{Hence, } \frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4} = S_1^2 - 2S_2$$
$$= 169 - 20 = 149$$

42. (C)

Let roots one diminished by k

$$\text{So, } \alpha - k + \beta - k + \gamma - k = 0$$

$$\Rightarrow (\alpha + \beta + \gamma) - 3k = 0$$

$$\Rightarrow k = \frac{9/2}{3} = \frac{3}{2}$$

43. (A)

$$\alpha + \beta, \beta + \gamma, \gamma + \alpha$$

$$\alpha + \beta + \gamma = p$$

Hence roots of the equations are  $p - \alpha, p - \beta, p - \gamma$

$$x = p - \alpha \Rightarrow \alpha = p - x$$

Equation is

$$(p - x)^3 - p(p - x)^2 + q(p - x) - r = 0$$

$$\Rightarrow x^3 - 3px^2 + 3p^2x - p^3 + p^3 - 2xp^2 + px^2 - qp + qx + r = 0$$

$$\Rightarrow x^3 - 2px^2 + 2(p^2 + q)x + r - pq = 0$$

44. (B)

$$x_1 = \beta\gamma + \frac{1}{\alpha} = \frac{\alpha\beta\gamma + 1}{\alpha} = \frac{3 + 1}{\alpha} = \frac{4}{\alpha}$$

$$\text{Hence, } x_1 = \frac{4}{\alpha} \Rightarrow \alpha = \frac{4}{x_1}$$

Equation

$$\frac{64}{x_1^3} - \frac{2 \cdot 16}{\alpha_1^2} + \frac{5.4}{\alpha_1} - 3 = 0$$

$$\Rightarrow 3x_1^3 - 20x_1^2 + 32x_1 - 64 = 0$$

45. (B)

$$x = \sqrt{3} - \sqrt{2}$$

$$x + \sqrt{2} = \sqrt{3}$$

$$\Rightarrow x^2 + 2\sqrt{2}x + 2 = 3 \quad (\text{roots are } -\sqrt{2} \pm \sqrt{3})$$

$$\Rightarrow x^2 - 1 = -2\sqrt{2}x$$

$$\Rightarrow x^2 - 2x^2 + 1 = 8x^2$$

$$\Rightarrow x^4 - 10x^2 + 1 = 0 \quad (\text{roots are } \pm\sqrt{2} \pm \sqrt{3})$$

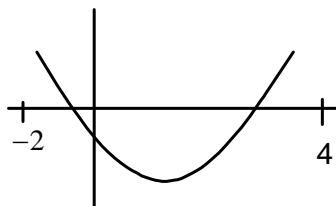
46. (B)

$$(x - m)^2 = 1$$

$$= m - 1, m + 1$$

$$m - 1 > -2$$

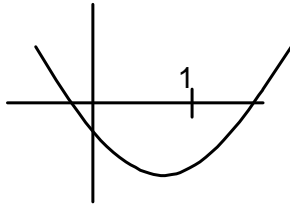
$$\& m + 1 < 4$$



$$m > -1 \quad m < 3$$

$$(-1, 3)$$

47. (A)  
1 lies in between the roots  
So,  $a.f(1) < 0$



$$\Rightarrow (2k+1) \cdot [2k+1-k+k-2] < 0$$

$$(2k+1)(2k-1) < 0$$

$$\begin{array}{c} + \quad | \quad - \quad | \quad + \\ \hline -\frac{1}{2} \quad \quad \quad \frac{1}{2} \end{array}$$

$$k \in \left( -\frac{1}{2}, \frac{1}{2} \right)$$

48. (C)  
 $(r_1 + 2)(r_2 + 2)(r_3 + 2)$   
Equation whose roots are  $(r_1 + 2), (r_2 + 2)$  &  $(r_3 + 2)$  is  
 $(x - 2)^3 - 2(x - 2)^2 + 4(x - 2) + 5074 = 0$   
 $\Rightarrow x^3 - 6x^2 + 12x - 8 - 2x^2 + 8x - 8 + 4x - 8 + 5074 = 0$   
 $\Rightarrow 1x^3 - 8x^2 + 24x + 5050 = 0$   
Product =  $-5050$

49. (A)  
 $P(x) = (x - 2)(4x - 1)(x) + Ax + B$

$$2A + B = 1$$

$$\frac{A}{4} + B = 2$$

$$\frac{7A}{4} = -1 \quad \Rightarrow A = -\frac{4}{7}, B = 1 + \frac{8}{7} = \frac{15}{7}$$

$$\text{Hence, } -\frac{4x+15}{7}$$

50. (D)  
 $\frac{1}{\log_3(2^{2x} - 1)} > \frac{1}{\log_3(2^x + 1)}$

$$\text{Domain } x > 0, x \neq \frac{1}{2}$$

$\log_3 2^x + 1$  is always positive

Hence,  $\log_3(2^{2x} - 1) > 0$

$$x > \frac{1}{2}$$

And  $\log_3(2^{2x} - 1) < \log_3(2^x + 1)$

$$\Rightarrow 2^{2x} - 1 < 2^x + 1$$

$$\Rightarrow t^2 - t - 2 < 0 \quad (t = 2x)$$

$$\Rightarrow t \in (-1, 2)$$

$$\Rightarrow x \in (-\infty, 1)$$

Hence solution is  $\left(\frac{1}{2}, 1\right)$

51. (B)

$$A = 12^{300}$$

$$\log_{10} A = 300[\log_{10} 12]$$

$$= 300[0.6010 + 0.4771]$$

$$= 300 \times 1.0781 = 323.43$$

$$\Rightarrow A = 10^{323.43}$$

Hence 324 digits

52. (A)

$$\log_{10} 2 = 0.3010$$

$$\log_5 64 = \frac{\log_{10} 64}{\log_{10} 5} = \frac{6 \log_{10} 2}{\log_{10} 10 - \log_{10} 2}$$

$$= \frac{6 \times 0.3010}{1 - 0.3010}$$

$$= \frac{1.8060}{0.6990}$$

$$= \frac{602}{233}$$

53. (B)

$$2^{\log_{10} 3\sqrt{3}} = 3^{k \log_{10} 2}$$

$$\Rightarrow 2^{\log_{10} 3\sqrt{3}} = 2^k \log_{10} 3$$

$$\Rightarrow \log_{10} 3\sqrt{3} = \log_{10} 3^k$$

$$\Rightarrow k = 3^{3/2}$$

$$\Rightarrow k = 3/2$$

54. (A)

$$x = \log_5(1000) = \log_5 125 + \log_5 8 > 4$$

$$y = \log_{57}(2056) = \log_7 343 + \log_7 6 < 4$$



Hence  $x > 4$

55. (B)

For compare  $\frac{b}{a}$ . Sum of the roots  $\geq 3$

$$\text{Hence } \frac{-b}{a} \geq 3 \quad \text{and} \quad -\frac{b}{2a} \geq 2$$

$$\Rightarrow \left(\frac{b}{a}\right) \geq 3 \quad \Rightarrow |b| \geq 4|a|$$

$$\Rightarrow |b| \geq 3|a| \quad \text{Hence } |b| \geq 4a$$

56. (A)

$$x = \frac{1+\alpha}{1-\alpha}$$

$$\Rightarrow \alpha = \frac{x-1}{x+1}$$

$$\left(\frac{x-1}{x+1}\right)^3 - \left(\frac{x-1}{x+1}\right) - 1 = 0$$

$$\Rightarrow (x-1)^3 - (x-1)(x+1)^2 - (x+1)^3 = 0$$

$$\Rightarrow -6x^2 - 2 - x^3 - x^2 + x + 1 = 0$$

$$\Rightarrow x^3 + 7x^2 - x + 1 = 0$$

$$\Rightarrow x^3 + 7x^2 - x + 1 = 0$$

57. (D)

$$x = \alpha(\beta+r) = \alpha\beta + \alpha v$$

$$= 3 - \beta r$$

$$= 3 - \frac{\beta r \alpha}{\alpha}$$

$$= 3 - \frac{(-2)}{\alpha} = 3 + \frac{2}{\alpha}$$

$$\Rightarrow x = 3 + \frac{2}{\alpha}$$

$$\Rightarrow \alpha = \frac{2}{x-3}$$

58. (A)

$$ax + by = 1$$

$$px^2 + qy^2 = 1$$

$$\Rightarrow px^2 + q\left(\frac{ax-1}{b}\right)^2 = 1$$

$$\Rightarrow b^2px^2 + qa^2x^2 - 2aqx + q - b^2 = 0$$

Has one root

$$\Rightarrow 4a^2q^2 - 4(b^2q + a^2q)(q - b^2) = 0$$

$$\Rightarrow 4(a^2/q^2 - b^2pq - a^2/q^2 + b^4p + b^2a^2q) = 0$$

$$\Rightarrow b^2(-pq + b^2p + a^2q) = 0$$

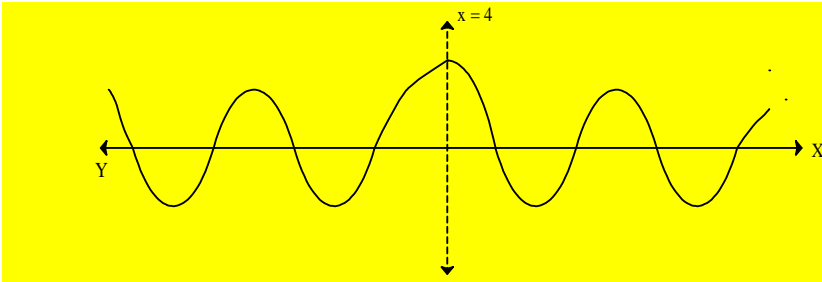
$$\text{Hence } \frac{b^2}{q} + \frac{a^2}{p} = 1$$

59. (C)

Express is symmetrical about  $x = 4$

So, roots will be  $4 \pm \alpha, 4 \pm \beta, 4 \pm \gamma,$

Hence sum  $= 8 \times 4 = 32$



60. (D)

$$x^2 + 2xy + 3y^2 - 6x + 6y \geq k \quad (\text{where } k \text{ is min unvalue})$$

$$\Rightarrow x^2 + (2y - 6)x + 3y^2 + 6y - k \geq 0$$

Should have all real  $x$

$$\text{Hence } 4(y - 3)^2 - 4(3y^2 + 6y - k) \leq 0$$

$$\Rightarrow 4[y^2 + 9 - 6y - 3y^2 - 6y + k] \geq 0$$

$$\Rightarrow [2y^2 + 12y - (9 + k)] \leq (\forall y \in \mathbb{R})$$

$$\text{Hence, } 144 + 8(9 + k) \leq 0$$

$$\Rightarrow k + 9 \leq -18$$

$$\Rightarrow k \leq -27$$

$$\text{Hence } k = -27$$

## EXERCISE – 2 (A)

1. (AC)

$$\begin{aligned} \Rightarrow 18^{4x-3} &= (54\sqrt{2})^{3x-4} \\ \Rightarrow 18^{4x-3} &= (18)^{3x-4} (3\sqrt{2})^{3x-4} \\ \Rightarrow 18^{x+1} &= (3\sqrt{2})^{3x-4} \\ \Rightarrow \left[ (3\sqrt{2})^2 \right]^{x+1} &= (3\sqrt{2})^{3x-4} \\ \Rightarrow (3\sqrt{2})^{2x+2} \times (3\sqrt{2})^{-3x+4} &= 1 \\ \Rightarrow (3\sqrt{2})^{-x+6} &= 1 \end{aligned}$$

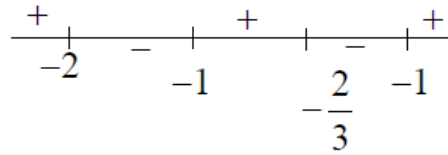
$$\text{So } -x = 6 \Rightarrow x = 6$$

2. (BC)

$$\Rightarrow \frac{2x}{2x^2+5x+2} > \frac{1}{x+1} \Rightarrow \frac{2x}{2x^2+5x+2} - \frac{1}{x+1} > 0$$

$$\Rightarrow \frac{2x^2+2x-2x^2-5x+2}{(2x^2+5x+2)(x+1)} > 0$$

$$\Rightarrow \frac{3x+2}{(2x+1)(x+2)(x+1)} < 0$$



$$\text{So, } x \in (-2, -1) \cup \left(-\frac{2}{3}, -1\right)$$

3. (ABC)

$$\Rightarrow 2576a456b$$

$\Rightarrow b$  has to 0 or 5

Sum of digits has to be multiple of 3, to divide it by 3.

$$\Rightarrow 2 + 5 + 7 + 6 + a + 4 + 5 + 6 + b = 35 + a + b$$

If a take value of 4, then b will also take 5.

If a take value of 4, then b has to be zero.

If take 6, then  $35 + a + b$  can't become multiple of 3, for  $b = 0.5$

4. (ABD)

$$\Rightarrow x^2 - 6x - 5|x-3| - 5 = 0$$

**Case – I :**  $x \geq 3$

$$\Rightarrow x^2 - 6x - 5|x-3| - 5 = 0$$

$$\Rightarrow x^2 - 11x + 10 = 0$$

$$\Rightarrow x = 1, 10$$

$\Rightarrow x = 1$  not a solution for  $x \geq 3$

$$\Rightarrow x = 10$$

**Case – 2:**  $x < 3$

$$\Rightarrow x^2 - 6x + 5x - 15 - 5 = 0$$

$$\Rightarrow x^2 - x - 20 = 0$$

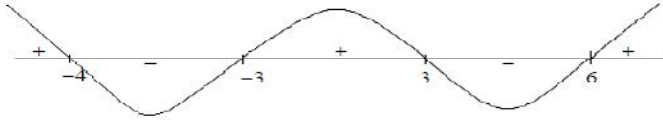
$$\Rightarrow (x-5)(x+4) = 0$$

For  $x < 3$ ,  $x = 1 - 4$

So,  $l = 1, m = 1$

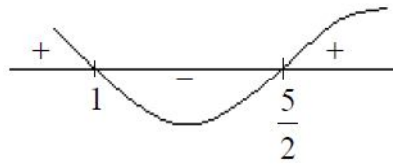
5. (BCD)

$$\begin{aligned} \Rightarrow \frac{2x+3}{x^2+x-12} &\leq \frac{1}{2} \\ \Rightarrow \frac{2(2x+3)-(x^2+x-12)}{2(x^2+x-12)} &\leq 0 \\ \Rightarrow \frac{-x^2+3x+18}{(x+4)(x-3)} &\leq 0 \\ \Rightarrow \frac{(x-6)(x+3)}{(x+4)(x-3)} &\geq 0 \\ \Rightarrow x &\in (-\infty, -4) \cup [-3, 3] \cup [6, \infty) \end{aligned}$$



6. (AD)

$$\begin{aligned} \Rightarrow |x-1| + |5-2x| &= |3x-6| \\ \text{As } (x-1) - (5-2x) &= 3x-6 \\ \text{So, } (x-1)(5-2x) &\leq 0 \\ \Rightarrow (x-1)(2x-5) &\geq 0 \\ \Rightarrow x &\in (-\infty, 1] \cup \left[\frac{5}{2}, \infty\right) \end{aligned}$$



7. (AC)

$$\begin{aligned} \Rightarrow \log_2 3 &> 1 \\ \Rightarrow \log_{12} 5 &< 1 \\ \text{So, } \log_2 3 &> \log_{12} 5 \\ \text{Similarly, } \log_6 5 &< 1 \\ \Rightarrow \log_7 11 &> 1 \\ \Rightarrow \log 82 &> \log_3 3^4 \\ \Rightarrow \log_3 81 &> 4 \\ \Rightarrow \log_2 15 &< \log_2 16 \\ \Rightarrow \log_2 15 &< \log_2 2^4 \\ \Rightarrow \log_2 15 &< 4 \\ \Rightarrow \log_{16} 15 &< 1 \\ \Rightarrow \log_{10} 11 &> 1 \\ \Rightarrow \log_7 6 &< 1 \end{aligned}$$

8. (BC)

$$\begin{aligned} \Rightarrow 2x^2 + 6\sqrt{2}x + 1 &= 0 \\ \Rightarrow x &= \frac{-6\sqrt{2} \pm \sqrt{72-8}}{4} \\ \Rightarrow x &= \frac{-3\sqrt{2}}{2} + 2, \frac{-3\sqrt{2}}{2} - 2 \end{aligned}$$

9. (BD)

$$\Rightarrow \log_{x+1}(x-0.5) = \log_{x-0.5}(x+1)$$

$$\Rightarrow x-5 > 0; x-0.5 > 0; x-0.5 \neq 1$$

$$\Rightarrow x > 0.5; x > 0.5; x \neq 1.5$$

$$\Rightarrow x+1 > 0; x+1 > 0$$

$$\Rightarrow x > -1; x > -1$$

So,  $x \in (0.5, 1.5) \cup (1.5, \infty)$  is feasible reason

$$\Rightarrow \frac{\log(x-0.5)}{\log(x+1)} = \frac{\log(x+1)}{\log(x-0.5)}$$

$$\Rightarrow \log^2(x-0.5) - \log^2(x+1) = 0$$

$$\Rightarrow \log\left(\frac{x-0.5}{x+1}\right) \log\{(x-0.5)(x+1)\} = 0$$

$$\text{So, } \frac{x-0.5}{x+1} = 1 \text{ or } (x-0.5)(x+1) = 1$$

$$\Rightarrow x-0.5 = x+1 \text{ (no solution)}$$

$$\text{Or } x^2 + 0.5x - 1.5 = 0$$

$$\Rightarrow 2x^2 + x - 3 = 0$$

$$\Rightarrow 2x^2 + 3x - 2x - 3 = 0$$

$$\Rightarrow x = 1$$

$$\text{As } x \neq \frac{-3}{2}$$

So  $x = 1$  is the only solution.

10. (CD)

$$\Rightarrow \log \left[ (1)^{\frac{1}{5}} + (32)^{\frac{1}{5}} + (243)^{\frac{1}{5}} \right]$$

$$\Rightarrow \log \{1 + 2 + 3\}$$

$$\Rightarrow \log 6$$

$$\Rightarrow \frac{1}{5} \{ \log 1 + \log 32 + \log 243 \} = \frac{1}{5} \log \{1 \times 32 \times 243\}$$

$$= \log \left\{ (1)^{\frac{1}{5}} (32)^{\frac{1}{5}} (243)^{\frac{1}{5}} \right\}$$

$$= \log 6$$

11. (BCD)

$$\Rightarrow \alpha, \beta, \gamma \text{ are roots of } x^3 - 2x + 3 = 0$$

$$\Rightarrow \alpha + \beta + \gamma = 0$$

$$\Rightarrow \sum \alpha\beta = -2$$

$$\Rightarrow \alpha\beta\gamma = -3$$

Equation with roots  $-\alpha, -\beta, -\gamma$  is  $x^3 - (0)x^2 + (-2)x - (3) = 0$

$$\Rightarrow x^3 - 2x - 3 = 0$$

$$\Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\sum \alpha\beta}{\alpha\beta\gamma} = \frac{-2}{-3} = \frac{2}{3}$$

$$\Rightarrow \sum \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma} = 0$$

$$\Rightarrow \frac{1}{\alpha} \cdot \frac{1}{\beta} \cdot \frac{1}{\gamma} = -\frac{1}{3}$$

So, equation  $x^3 - \left(\frac{2}{3}\right)x^2 + (0)x - \left(-\frac{1}{3}\right) = 0$

$$\Rightarrow 3x^3 - 2x^2 + 1 = 0$$

$$\Rightarrow \sum(\alpha + \beta) = 0$$

$$\Rightarrow \sum(\alpha + \beta)(\beta + \gamma) = (\alpha\beta + \beta\gamma + \gamma\alpha + \beta^2) + (\alpha\beta + \beta\gamma + \gamma\alpha + \alpha^2) + (\alpha\beta + \beta\gamma + \gamma\alpha + \gamma^2)$$

$$= 3(-2) + (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$= 3(-2) + 0 - 2(-2)$$

$$\Rightarrow \sum(\alpha + \beta)(\beta + \alpha) = -2$$

$$\Rightarrow (\alpha\beta + \beta\gamma + \gamma\alpha) = (\alpha\beta + \beta\gamma + \gamma\alpha + \beta^2)(\gamma + \alpha)$$

$$= (-2 + \beta^2)(-\beta)$$

$$= 2\beta - \beta^3$$

$$= 3 \quad (\because \beta^3 - 2\beta + 3 = 0)$$

So equation  $x^3 - 0x^2 + (-2)x - (3) = 0$

$$\Rightarrow x^3 - 2x - 3 = 0$$

$$\Rightarrow \sum \frac{\alpha + \beta}{\gamma + 1} = \frac{-\gamma}{\gamma + 1} - \frac{\alpha}{\alpha + 1} - \frac{\beta}{\beta + 1} = -3 + \left[ \frac{1}{\gamma + 1} + \frac{1}{\alpha + 1} + \frac{1}{\beta + 1} \right]$$

$$= -3 + \left[ \frac{\sum(\alpha + 1)(\beta + 1)}{(\alpha + 1)(\beta + 1)(\gamma + 1)} \right]$$

$$= -3 - \frac{1}{4} = \frac{-13}{4}$$

12.

(B)

$$\Rightarrow S_1 : x^2 + |x| + 1 = 0$$

$$\Rightarrow x^2 + 1 = -|x|$$

No solution

$$\Rightarrow S_2 : x^2 - 5|x| + 6 = 0$$

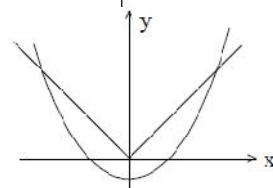
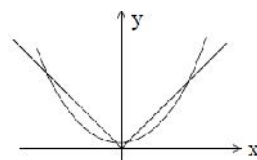
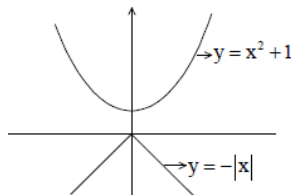
$$\Rightarrow x^2 + 6 = 5|x|$$

$\Rightarrow$  4 solution.

$$\Rightarrow S_3 : x^2 - |x| - 2 = 0$$

$$\Rightarrow x^2 - 2 = |x|$$

$\Rightarrow$  2 solution



13.

(BC)

$$\Rightarrow x^{1 - \log_5 x} = 0.04$$

$$\Rightarrow x > 0$$

$$\Rightarrow x \cdot x^{\log \frac{1}{x}} = 0.04$$

Only  $x = 25, \frac{1}{5}$  satisfy the equation

14.

(AD)

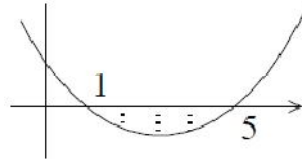
$$\Rightarrow f(x) = x^2 - 6x + 5$$

$$\text{Least value at } x = \frac{6}{2} = 3$$

$$\text{Least value} = (3)^2 - 6(3) + 5 = -4$$

$$\Rightarrow f(2) = (2)^2 - 6(2) + 5 = -3$$

$$\Rightarrow f(4) = (4)^2 - 6(4) + 5 = -3$$



15.

(AB)

$$\Rightarrow 10^{\frac{2}{x}} + 25^{\frac{1}{x}} = \frac{17}{4}(50)^{\frac{1}{x}}$$

$$\Rightarrow 100^{\frac{1}{x}} + 25^{\frac{1}{x}} = \frac{17}{4}(50)^{\frac{1}{x}}$$

$$\Rightarrow 4^{\frac{1}{x}} + 1 = \frac{17}{4}2^{\frac{1}{x}}$$

$$\Rightarrow 2^{\frac{2}{x}} - \frac{17}{4}2^{\frac{1}{x}} + 1 = 0$$

$$\Rightarrow \left(2^{\frac{1}{x}} - 4\right)\left(2^{\frac{1}{x}} - \frac{1}{x}\right) = 0$$

$$\Rightarrow 2^{\frac{1}{x}} = 4, \frac{1}{4}$$

$$\Rightarrow x = \frac{-1}{2}, \frac{1}{2}$$

16.

(BC)

$$\Rightarrow |x^2 + 4x + 3| + 2x + 5 = 0$$

$$\Rightarrow |(x+1)(x+3)| + 2x + 5 = 0$$

**Case 1:**  $x \in (-\infty, -3] \cup [-1, \infty)$

$$\Rightarrow x^2 + 4x + 3 + 2x + 5 = 0$$

$$\Rightarrow x^2 + 6x + 8 = 0$$

$$\Rightarrow x = -4, -2$$

So  $x = -4$

**Case 2:**  $x \in (-3, -1)$

$$\Rightarrow -(x^2 + 4x + 3) + 2x + 5 = 0$$

$$\Rightarrow x^2 + 2x - 2 = 0$$

$$\Rightarrow x = \frac{-2 \pm \sqrt{12}}{2}$$

$$\Rightarrow x = -1 + \sqrt{3}, -1 - \sqrt{3}$$

So,  $x = -1 - \sqrt{3}$



17.

(BC)

$$\Rightarrow 4x^2 + 4x + 12\lambda^2 \geq 47 \quad \forall x \in \mathbf{R}$$

$$\Rightarrow 4(x^2 + x + 3\lambda^2) \geq 47$$

$$\Rightarrow x^2 + x + 3\lambda^2 - \frac{47}{4} \geq 0$$

So,  $8 \leq 0$

$$\Rightarrow 1 - 4 \left( 3\lambda^2 - \frac{47}{4} \right) \leq 0$$

$$\Rightarrow 4 - \lambda^2 \leq 0$$

$$\Rightarrow \lambda^2 \geq 4$$

$$\Rightarrow \lambda \in (-\infty, -2] \cup [2, \infty)$$

18. (ABD)

$$\Rightarrow \frac{1}{2} \leq \log_{0.1} x \leq 2; x > 0$$

$$\Rightarrow \frac{1}{2} \leq -\log_{10} x \leq 2$$

$$\Rightarrow -\frac{1}{2} \geq \log_{10} x \geq -2$$

$$\Rightarrow \log_{10} x \leq -\frac{1}{2} \quad \& \quad \log_2 x \geq -2$$

$$\Rightarrow x \leq (10)^{-\frac{1}{2}} \quad \& \quad x \geq 10^{-2}$$

$$\Rightarrow x \leq \frac{1}{\sqrt{10}} \quad \dots(1) \quad x \geq \frac{1}{100} \quad \dots(2)$$

(1)  $\cap$  (2)

$$\Rightarrow x \in \left[ \frac{1}{100}, \frac{1}{\sqrt{10}} \right]$$

19. (A)

$$\Rightarrow \log_2 (3^{2x+2} + 7) = 2 + \log_2 (3^{x-1} + 1)$$

$$\Rightarrow \log_2 \left( \frac{9 \cdot 3^{2x} + 7}{\frac{3^x}{3} + 1} \right) = 2$$

$$\Rightarrow \frac{3(9 \cdot 3^{2x} + 7)}{3^x + 3} = 4$$

Put  $3^x = a$

$$\Rightarrow 27a^2 + 21 = 4a + 12$$

$$\Rightarrow 21a^2 - 4a + a = 0$$

Which is not possible for values of a.

So no solution.

20. (AB)

$$\Rightarrow \log_{a_1 a_2} a_1 = 4$$

$$\Rightarrow \log a_1 = 4 \left[ \log_{a_1} + \log_{a_2} \right] 3 \log_{a_1} + 4 \log_{a_2} = 0$$



$$\Rightarrow \log_{a_1 a_2} \frac{(a_1)^{\frac{1}{3}}}{(a_2)^{\frac{1}{2}}} \Rightarrow \log_{a_1 a_2} (a_1)^{\frac{1}{3}} - \log_{a_1 a_2} (a_2)^{\frac{1}{2}}$$

$$\Rightarrow \frac{1}{3}[4] - x$$

$$\Rightarrow x = \frac{1}{2} \left[ \frac{\log(a_2)}{\log a_1 a_2} \right] = \frac{1}{2} \left[ \frac{\log a_1 a_2}{\log a_1 a_2} \right]$$

$$= \frac{1}{2} [1 - \log_{a_1 a_2} a_1]$$

$$= \frac{1}{2} [1 - 4] = -\frac{3}{2}$$

$$\text{So, } \log_{a_1 a_2} \frac{(a_1)^{\frac{1}{3}}}{(a_2)^{\frac{1}{2}}} = \frac{4}{3} + \frac{3}{2} = \frac{17}{6}$$

21. (AD)

$$f(x) = x^3 - 3x^2 + 4 = 0$$

clearly  $x = -1$  is a root of given equation.

$$\frac{x^3 - 3x^2 + 4}{x + 1} = (x - 2)^2$$

$x = 2$  is a repeated root &  $x = -1$  is a root

22. (AD)

Given  $a < \alpha, \beta$

(i)  $f(a) > 0$

(ii)  $D > 0$

(iii)  $-\frac{B}{2A} > a$

(i)  $f(a) = a^2 + a + a = a^2 + 2a > 0$



$$a \in (-\infty, -2) \cup (0, \infty) \quad \dots (1)$$

(ii)  $D > 0$

$$1 - 4a > 0$$

$$a < \frac{1}{4} \quad \dots (2)$$

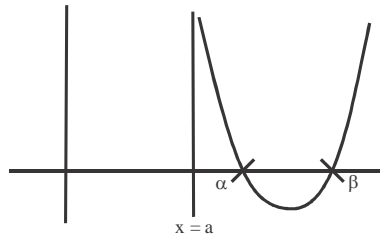
(iii)  $-\frac{B}{2A} > a$

For  $x^2 + x + a$ ,  $A = 1$  &  $B = 1$

$$-\frac{1}{2} > a \quad \dots (3)$$

From (1), (2), (3)

$$a < -2$$



23. (CD)

$$\tan x \geq 3 \cot x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \& x \neq 0$$

{  $\because \cos x > 0$  for given integral }

(i) for  $x > 0$

$$\tan x \geq \frac{3}{\tan x} \quad \{\text{here } \tan x > 0\}$$

$$\tan^2 x \geq 3$$

$$\Rightarrow \tan x \geq \sqrt{3}$$

$$x \in \left[ \frac{\pi}{3}, \frac{\pi}{2} \right]$$

(ii) for  $x < 0$

$$\tan x \geq \frac{3}{\tan x} \quad \{\text{here } \tan x < 0\}$$

$$\tan^2 x \leq 3$$

$$\tan x \leq -\sqrt{3}$$

$$\Rightarrow x \in \left( -\frac{\pi}{3}, 0 \right]$$

24. (AD)

$$\frac{3}{\sqrt{2-x}} - \sqrt{2-x} < 2$$

$$3 - (2-x) < 2\sqrt{2-x}$$

$$1+x < 2\sqrt{2-x}$$

$$\text{Here } 2-x > 0$$

$$\Rightarrow x < 2$$

$$(1+x)^2 < 4(2-x)$$

$$1+x^2+2x < 8-4x$$

$$x^2+6x-7 < 0$$

$$(x+7)(x-1) < 0$$



$$x \in (-7, 1)$$

25. (ABC)

$$xy + 3y^2 - x + 4y - 7 = 0$$

$$\& 2xy + y^2 - 2x - 2y + 1 = 0$$

$$xy + 3y^2 - x + 4y - 7 = 0$$

$$xy - x + 3y^2 - 3 + 4y - 4 = 0$$

$$x(y-1) + 3(y-1)(y+1) + 4(y-1) = 0$$

$$(y-1)[x + 3(y+1) + 4] = 0$$

$$\Rightarrow \text{either } y = 1 \text{ or } x + 3y + 7 = 0 \quad \dots(i)$$

$$2xy + y^2 - 2x - 2y + 1 = 0$$

$$2xy - 2x + y^2 - 2y + 1 = 0$$

$$2x(y-1) + (y-1)^2 = 0$$

$$(y-1)[2x + y - 1] = 0$$

$$\text{Either } y = 1 \text{ or } 2x + y - 1 = 0$$

26. (ABC)

$$kx^2 + kx - (2k+1) > 0$$

If true for at least one real x

$$\begin{aligned} \Rightarrow D > 0 \\ k^2 - 4k \times (-1)(2k+1) > 0 \\ k^2 + 8k^2 + 4k > 0 \\ k(9k+4) > 0 \\ \Rightarrow k \in \left(-\infty, -\frac{4}{9}\right) \cup (0, \infty) \end{aligned}$$

27. (BC)

$$\begin{aligned} \log_x 2 \cdot \log_{2x} 2 &= \log_{4x} 2 \\ \frac{\log 2}{\log x} \cdot \frac{\log 2}{\log 2x} &= \frac{\log 2}{\log 4x} \\ \log 2 (\log 4x) &= \log x \cdot \log 2x \\ \log 2 (2 \log 2 + \log x) &= \log x (\log 2 + \log x) \\ 2(\log 2)^2 + \cancel{\log 2 \cdot \log x} &= \cancel{\log 2 \cdot \log x} + (\log x)^2 \\ \log x &= \pm \sqrt{2} \log 2 \\ x &= 2^{\pm\sqrt{2}} \end{aligned}$$

28. (a)

$$\begin{aligned} \log_{(2x+3)}(6x^2 + 23x + 21) \\ = 4 - \log_{(3x+7)}(4x^2 + 12x + 9) \\ \Rightarrow \log_{(2x+3)}[(2x+3)(3x+7)] \\ = 4 - \log_{(3x+7)}[(2x+3)^2] \\ \Rightarrow 1 + \frac{\log(3x+7)}{\log(2x+3)} = 4 - \frac{2\log(2x+3)}{\log(3x+7)} \\ \text{Let } \frac{\log(3x+7)}{\log(2x+3)} = k \\ \Rightarrow 1+k = 4 - \frac{2}{k} \\ k + \frac{2}{k} - 3 = 0 \\ k^2 - 3k + 2 = 0 \\ (k-1)(k-2) = 0 \\ \frac{\log(3x+7)}{\log(2x+3)} = 1, 2 \\ 3x+7 = 2x+3 \text{ or } 3x+7 = (2x+3)^2 \\ x = -4 \text{ or } 3x+7 = 4x^2 + 9 + 12x \\ 4x^2 + 9x + 2 = 0 \quad 4x^2 + 8x + x + 2 = 0 \\ 4x(x+2) + (x+2) = 0 \\ x = -2 \text{ or } x = -\frac{1}{4} \\ \text{So, } x = -\frac{1}{4}, -2, -4 \\ \text{Also } 2x+3 > 0 \quad \& \quad 3x+7 > 0 \\ x > -\frac{3}{4} \quad \& \quad x > -\frac{7}{4} \end{aligned}$$

$\Rightarrow$  only value of  $x = -\frac{1}{4}$

29.

(BD)

$$2^{\log_{\sqrt{2}}(x-1)} > x + 5$$

$$2^{\log_2(x-1)^2} > x + 5$$

$$(x-1)^2 > x + 5$$

$$x^2 + 1 - 2x > x + 5$$

$$x^2 - 3x - 4 > 0$$

$$(x-4)(x+1) > 0$$

$$x \in (-\infty, -1) \cup (4, \infty)$$

30.

(BCD)

$$\text{Given } \frac{1}{3} < \frac{x^2 - 2x + 4}{x^2 + 2x + 4} < 3$$

$$\Rightarrow \frac{1}{3} < \frac{x^2 + 2x + 4}{x^2 - 2x + 4} < 3$$

$$\frac{9 \cdot 3^{2x} + 6 \cdot 3^x + 4}{9 \cdot 3^{2x} - 6 \cdot 3^x + 4}$$

$$\frac{3^{2x+2} + 2 \cdot 3 \cdot 3^x + 4}{3^{2x+2} - 2 \cdot 3 \cdot 3^x + 4}$$

$$\frac{(3^{x+1})^2 + 2 \cdot (3^{x+1}) + 4}{(3^{x+1})^2 - 2 \cdot (3^{x+1}) + 4}$$

$$\frac{(3^{x+1})^2 + 2 \cdot (3^{x+1}) + 4}{(3^{x+1})^2 - 2 \cdot (3^{x+1}) + 4}$$

$$\text{Let } 3^{x+1} = y$$

$$\Rightarrow \frac{y^2 + 2y + 4}{y^2 - 2y + 4}$$

$$\Rightarrow \frac{y^2 + 2y + 4}{y^2 - 2y + 4}$$

$$\Rightarrow \frac{y^2 + 2y + 4}{y^2 - 2y + 4}$$

It will also lie between (1/3, 3)

31.

(A)

$$\log_2 x + \log_x 2 = \frac{10}{3} = \log_2 y + \log_y 2$$

Also  $x \neq y$

$$\frac{\log x}{\log^2} + \frac{\log 2}{\log x} = \frac{10}{3}$$

Solving we get

$$\frac{\log x}{\log 2} = 3, \frac{1}{3}$$

$$x = 8 \text{ or } x = 2^{\frac{1}{3}}$$

$$\text{If } x = 8 \Rightarrow y = 2^{\frac{1}{3}}$$

$$\& \text{ if } x = 2^{\frac{1}{3}} \Rightarrow y = 8$$

$$\therefore \frac{x}{y} = \frac{2^3}{2^{\frac{1}{3}}} \text{ or } \frac{2^{\frac{1}{3}}}{2^3}$$

$$\frac{x}{y} = 2^{\frac{8}{3}} \text{ or } 2^{-\frac{8}{3}}$$

32. (ACD)  
 $2x^2 - 2(2a+1)x + a(a+1) = 0$   
 (i)  $f(a) < 0$   
 (ii)  $D > 0$   
 (i)  $f(a) = 2a^2 - 2(2a+1)a + a(a+1)$   
 $= 2a^2 - 4a^2 - 2a + a^2 + a$   
 $= -a^2 - a < 0$   
 $= a(a+1) > 0$   
 $a \in (-\infty, -1) \cup (0, \infty)$   
 (ii)  $D > 0$   
 $4(2a+1)^2 - 4 \cdot 2 \cdot a(a+1) > 0$   
 $4a^2 + 1 + 4a - 2a^2 - 2a > 0$   
 $2a^2 + 2a + 1 > 0$   
 $\Rightarrow a \in \mathbb{R}$   
 $\therefore a \in (-\infty, -1) \cup (0, \infty)$
33. (D)  
 $2^x + 2^{|x|} \geq 2\sqrt{2}$   
 Case - I:  $x \geq 0$   
 $2^x + 2^x \geq 2\sqrt{2}$   
 $2^x \geq \sqrt{2}$   
 $\Rightarrow x \geq \frac{1}{2}$   
 Case - II:  $x < 0$   
 $2^x + 2^{-x} \geq 2\sqrt{2}$   
 Let  $2^x = k$   
 $k + \frac{1}{k} \geq 2\sqrt{2}$   
 $k^2 - 2\sqrt{2}k + 1 \geq 0$   
 $(k - (\sqrt{2} + 1))(k - (\sqrt{2} - 1)) \geq 0$   
 $-\infty < 2^x \leq \sqrt{2} - 1 \text{ \& } \sqrt{2} + 1 \leq 2^x < \infty$   
 $\Rightarrow 0 < 2^x < \sqrt{2} - 1 \text{ \& } \sqrt{2} + 1 \leq 2^x < \infty$   
 $x \in (-\infty, \log_2(\sqrt{2} - 1)) \text{ \& } (\log_2(\sqrt{2} + 1), \infty)$   
 From case I & II  
 $x \in (-\infty, \log_2(\sqrt{2} - 1)] \cup [\frac{1}{2}, \infty)$
34. (AC)  
 $(3-x)^4 + (2-x)^4 = (5-2x)^4$   
 $(3-x)^4 + (2-x)^4 = (3-x+2-x)^4$   
 Let  $3-x = a$  &  $2-x = b$   
 $a^4 + b^4 = (a+b)^4$   
 clearly either  $a = 0$  or  $b = 0$   
 $\Rightarrow$  either  $x = 3$  or  $x = 2$   
 Now  
 $a^4 + b^4 = (a^2 + b^2 + 2ab)^2$

$$a^4 + b^4 = a^4 + b^4 + 4a^2 b^2 + 2a^2 b^2 + 4ab^3 + 4a^3 b$$

$$\Rightarrow 4ab^3 + 6a^2 b^2 + 4a^3 b = 0$$

$$2ab [2b^2 + 3ab + 2a^2] = 0$$

Either  $a = 0$  or  $b = 0$

$$\text{Or } 2a^2 + 3ab + 2b^2 = 0$$

$$2\left(\frac{a}{b}\right)^2 + 3\frac{a}{b} + 2 = 0$$

Here  $D < 0$

$\Rightarrow$  two real roots & two non real roots

35. (AB)

$$4^x - (a-3)2^x + a - 4 = 0$$

Roots are non positive

$$x \leq 0$$

$$\Rightarrow 0 < 2^x \leq 1$$

$$(2^x)^2 - (a-3)2^x + a - 4 = 0$$

$$k^2 - (a-3)k + (a-4) = 0$$

$$(i) f(0) > 0$$

$$(ii) f(1) \geq 0$$

$$(iii) D > 0$$

$$(i) f(0) = a - 4 > 0$$

$$a > 4$$

$$(ii) f(1) \geq 0$$

$$1 - (a-3) + a - 4 \geq 0$$

$$(iii) D > 0$$

$$(a-3)^2 - 4(a-4) > 0$$

$$a^2 + 9 - 6a - 4a + 16 > 0$$

$$a^2 - 10a + 25 > 0$$

$$(a-5)^2 > 0$$

$$a \in \mathbb{R}$$

$$(iv) 0 < -\frac{B}{2A} \leq 1$$

$$0 < -\frac{-(a-3)}{2 \times 1} \leq 1$$

$$0 < \frac{a-3}{2} \leq 1$$

$$3 < a \leq 5$$

From (i), (ii), (iii), (iv)

$$a \in (4, 5]$$

36. (BCD)

$$27^x + 2\cos 3y + 8 + 6\cos y = 4 \cdot 3^{x+1} \cdot \cos y$$

$$3^{3x} + 2\cos 3y + 8 + 6\cos y = 4 \cdot 3^{x+1} \cdot \cos y$$

$$(3^x)^3 + 8\cos^3 y - \cancel{6\cos y} + 8 + \cancel{6\cos y} = 3 \cdot 2 \cdot 3^x \cdot 2\cos y$$

$$(3^x)^3 + (2\cos y)^3 + 2^3 = 3 \cdot (3^x) \cdot (2\cos y) \cdot (2)$$

$$3^x + 2 \cos y + 2 = 0$$

$$\text{Or } 3^x = 2 \cos y = 2$$

$$x = \log_3 2, \cos y = 1$$

37. (ABD)

$$x + y = \sqrt{3} \quad \dots(1)$$

$$y + z = \sqrt{5} \quad \dots(2)$$

$$z - x = \sqrt{7} \quad \dots(3)$$

(1) & (2) we get

$$x - z = \sqrt{3} - \sqrt{5}$$

Add in (3) we get

$$0 = \sqrt{3} - \sqrt{5} + \sqrt{7}$$

Not possible

No real values of x, y, z

38. (ABCD)

$$y = \frac{x^2 + 2x - 7}{x^2 + 2x + 7}$$

Range in given D

$$\text{For } x = 1, y = -\frac{2}{5}$$

$$x = -1, y = -\frac{4}{3}$$

$$x = 0, y = -1$$

$$x = 2, y = -1$$

39. (CD)

$$\log_2 x = \log_4 y + \log_4 (4 - x)$$

$$4 - x > 0 \Rightarrow x < 4$$

$$\& x, y > 0$$

$$\log_2 x = \frac{1}{2} \log_2 (y \cdot (4 - x))$$

$$x^2 = (4 - x) \cdot y \quad \dots(1)$$

$$\log_3 (x + y) = \log_3 x - \log_3 y$$

$$x + y > 0 \& x > 0 \& y > 0$$

$$x + y = \frac{x}{y}$$

$$y = \frac{x}{y} - x$$

$$y = \left( \frac{1}{y} - 1 \right)^x \quad \dots (2)$$

$$x = \frac{4}{3}, y = \frac{2}{3}$$

40. (AC)

$$2 \cos^2 x > \frac{3}{2}$$

$$\left(\cos x - \frac{\sqrt{3}}{2}\right)\left(\cos x + \frac{\sqrt{3}}{2}\right) > 0$$

$$-1 \leq \cos x < -\frac{\sqrt{3}}{2} < \cos x \leq 1$$

41. (ABCD)

$$f(x) = x^8 - x^5 + x^2 - x + 1$$

$$f(x) = x^5(x^3 - 1) + x(x - 1) + 1$$

$$f(x) = x(x - 1)[x^4(x^2 + x + 1) + 1] + 1$$

$$x^4(x^2 + x + 1) > 0$$

Also  $x(x - 1) > 0$  for  $n \in (-\infty, 0) \cup (1, \infty)$

Now for  $x \in [0, 1]$

$$x^8 + x^2 + 1 - x^5 - x > 0$$

$$\therefore f(x) > 0 \Rightarrow x \in \mathbb{R}$$

42. (ABC)

$$x^2 + y^2 - xy - x - y + 1 \geq k$$

$$x^2 - (y + 1)x + y^2 - y + 1 = k \geq 0$$

$$D \leq 0 \Rightarrow (y + 1)^2 - 4y^2 + 4y - 4 + 4k \leq 0$$

$$3y^2 - 6y + 3 - 4k \geq 0 \quad (\forall y \in \mathbb{R})$$

$$(D \leq 0) \Rightarrow 36 - 36 + 48k \leq 0$$

Minimum value of expected = 0

$$k \leq 0$$

Have minimum value is 0

In-equation has solution  $x, y \in \mathbb{R}$

$\therefore$  All equations with real values if  $x$  &  $y$  are true.

43. (ABCD)

$$x^2 + (a \log(1 - a^2))x - (a^2 - 1) = 0$$

Roots are opposite in sign

$$\frac{C}{A} < 0$$

$$-(a^2 - 1) < 0$$

$$a \in (-\infty, -1) \cup (1, \infty) \quad \dots(1)$$

As we have  $\tan \log(1 - a^2)$

$$\therefore a^2 < 1 \quad \dots(2)$$

From (1) & (2) no value of  $a$  for which roots are opposite in sign

44. (CD)

$$x^4 - 4x^3 + ax^2 - bx + 1 = 0$$

$$\alpha + \beta + \gamma + \delta = 4$$

$$\alpha, \beta, \gamma, \delta = 1$$

Now by AM  $\geq$  am

$$AM = \frac{\alpha + \beta + \gamma + \delta}{4} = 1$$



$$GM = (\alpha.\beta.\gamma.\delta)^{\frac{1}{4}} = 1$$

Now,  $\therefore AM = GM$

$$\Rightarrow \alpha = \beta = \gamma = \delta = 1$$

Now,  $a = \alpha\beta + \beta\gamma + \gamma\delta + \delta\gamma + \alpha\gamma + \beta\delta$

$$\Rightarrow b = 4$$

45. (BC)

$$\log_{(a)} x \leq \log_a x^2$$

For  $0 < a < 1$

$$x \geq x^2$$

$$x \in (0, 1] \quad \{\text{as } x \neq 0\}$$

For  $1 < a$

$$x \leq x^2$$

$$x(x-1) \geq 0$$

$$x \in [1, \infty)$$

46. (AC)

$$\log_3(x+2) > \log_{x+2} 81$$

$$\frac{\log(x+2)}{\log 3} > \frac{4 \log 3}{\log(x+2)}$$

(i) for  $x > -1$

$$\log(x+2) > 0$$

$$\therefore [\log(x+2)]^2 > 4(\log 3)^2$$

$$(\log(x+2) - 2 \log 3)(\log(x+2) + 2 \log 3) > 0$$

$$\Rightarrow \log(x+2) < -2 \log 3 \text{ \& } \log(x+2) > 2 \log 3$$

$$\& \log(x+2) > 2 \log 3$$

$$x < \frac{1}{9} - 2 \text{ \& } x > 7$$

$$x < \frac{-17}{9}$$

(ii) for  $x < -1$

$$\log(x+2)^2 \leq 4(\log 3)^2$$

$$-2 \log 3 < \log(x+2) < 2 \log 3$$

$$\Rightarrow \frac{-17}{9} < x < 7$$

47. (BC)

$$\frac{1}{\log_4\left(\frac{x+1}{x+2}\right)} \leq \frac{1}{\log_4(x+3)}$$

$$\frac{x+1}{x+2} > 1$$

$$\frac{x+1-x-2}{x+2} > 0$$

$$-\frac{1}{x+2} > 0$$

$$\frac{1}{x+2} < 0$$

$$\Rightarrow x < -2 \quad \dots (1)$$

$$\frac{x+1}{x+2} > 0$$

$$\Rightarrow x > -1 \text{ \& } x < -2 \quad \dots (2)$$

$$\Rightarrow 0 < \frac{x+1}{x+2} < 1$$

For  $x > -1$

Case I take  $x > -1$

$$\frac{1}{\log_4\left(\frac{x+1}{x+2}\right)} \leq \frac{1}{\log_4(x+3)}$$

$$\log_4(x+3) \geq \log_4\left(\frac{x+1}{x+2}\right)$$

$$\left\{ \text{as } \log_4\left(\frac{x+1}{x+2}\right) < 0 \text{ for } x > -1 \right\}$$

$$x+3 \geq \frac{x+1}{x+2}$$

$$x+3 - \left(\frac{x+1}{x+2}\right) \geq 0$$

$$\Rightarrow x > -2$$

{ we have taken  $x > -1$  }

i.e.  $x > -1$

case II:  $-3 < x < -2$

$$x+3 = \frac{x+1}{x+2} \quad \frac{x^2+4x+5}{x+2} \geq 0$$

$$x > -2$$

no solution

48. (ABC)

$$n = x(x+1)(x+2)(x+3)$$

$$= x^4 + 6x^3 + 11x^2 + 6x$$

$$\text{Now } (x^2 + 3x + 1)^2 = x^4 + 6x^3 + 11x^2 + 6x + 1$$

$$\Rightarrow n + 1 \text{ is a perfect square}$$

$n$  is never a perfect square.

Also in four consecutive integers there is a factor of 2, a factor of 4, a factor of 3

$\therefore n$  is always divisible by  $2 \times 4 \times 3 = 24$

49. (BC)

Possible values 10 & 12 from options

50. (C)

$$28x + 30y + 31z = 365$$

One of solution is (1, 4, 7) for (x, y, z)

$$z - 2x = 5$$

**INEQUATION & EQUATION**  
**EXERCISE 2 (B)**

1. (D)

$$\begin{aligned} & \left| |x+1| - 2 \right| = 1 \\ & \Rightarrow |x-1| = 3 \quad \text{or} \quad |x-1| = 1 \\ & \Rightarrow (x-1)^2 = 9 \quad x-1 = \pm 1 \\ & \Rightarrow (x-4)(x+2) = 0 \quad x = 2, 0 \\ & \Rightarrow x = -2, 4 \end{aligned}$$

2. (B)

$$\begin{aligned} & \left| |x-2| - 3 \right| = 4 \\ & \Rightarrow |x-2| = 7 \\ & \Rightarrow (x-2)^2 = 7^2 \\ & \Rightarrow (x-9)(x-5) = 0 \\ & \Rightarrow x = 5, 9 \end{aligned}$$

3. (C)

$$\begin{aligned} & |x-2| = 10, 0 \\ & x = 2, 12, -8 \end{aligned}$$

4. (A)

$$\begin{aligned} & A = 2^{\log_2 4} + 3^{\log_2 4} + 4^{\log_2 2} - 4^{\log_2 3} \\ & \Rightarrow A = 2^2 + 3^{\log_2 4} + 4^1 - 3^{\log_2 4} = 8 \\ & D = (\log_5 49)(\log_7 125) \\ & \Rightarrow D = \frac{\log 49}{\log 5} \times \frac{\log 125}{\log 7} = \frac{2 \log 7}{\log 5} \times \frac{3 \log 5}{\log 7} \\ & \Rightarrow D = 6 \\ & \Rightarrow a = \log_A D = \log_8 6 \\ & \Rightarrow a = \frac{\log_2 6}{3} \\ & \text{Now } \log_6 12 = \log_6 6 + \log_6 2 \\ & \Rightarrow \log_6 12 = 1 + \frac{1}{\log_2 6} \\ & \Rightarrow \log_6 12 = 1 + \frac{1}{3a} = \frac{1+3a}{3a} \end{aligned}$$

5. (A)

$$\begin{aligned} & N = 7^{\log_{49} 900} \\ & \Rightarrow N = 7^{\log_{7^2} 30^2} = 7^{\log_7 30} = 30 \\ & \log_6 12 = \frac{1+ma}{ma} \Rightarrow m = n = 3 \\ & \log_N m = \log_{30} 3 \\ & \log_m N = \log_3 30 \\ & \log_n N = \log_3 30 \\ & \text{Clearly } \log_{30} 3 < \log_3 30 \end{aligned}$$

$$\therefore \log_N m < \log_m N = \log_n N$$

6. (B)

$$\begin{aligned} \log_{\left(\frac{A-N}{10}\right)} |N + A + D + m + n| - \log_5 2 &= \log_{\left(\frac{8-30}{10}\right)} |30 + 8 + 6 + 3 + 3| - \log_5 2 \\ &= \log_5 50 - \log_5 2 = \log_5 25 + \log_5 2 - \log_5 2 \\ &= \log_5 5^2 = 2 \end{aligned}$$

7. (A)

$$\begin{aligned} y = 4 - |4x^2 - 9| &\Rightarrow |4x^2 - 9| = 4 - y \geq 0 \\ &\Rightarrow y \leq 4 \end{aligned}$$

8. (A)

$$\begin{aligned} &\Rightarrow (4x^2 - 9)^2 = 7^2 \\ &\Rightarrow (4x^2 - 16)(4x^2 - 2) = 0 \\ &\Rightarrow x = \pm 2, \pm \frac{1}{\sqrt{2}} \end{aligned}$$

9. (B)

$$\begin{aligned} |Z| + 4 = 4 - |4x^2 - 9| &\Rightarrow |Z| + |4x^2 - 9| = 0 \\ &\Rightarrow 4x^2 - 9 = 0, Z = 0 \\ &\Rightarrow x = \pm \frac{3}{2}, Z = 0 \end{aligned}$$

10. (D)

$$\begin{aligned} y &= \log_x (4-x) + \log_3 (2+x) \\ x &> 0, x \neq 1, 4-x > 0 \& 2+x > 0 \\ x &\in (0,1) \cup (1,4) \end{aligned}$$

11. (A)

$$\begin{aligned} &\Rightarrow \frac{x}{x+3} \geq 1 \Rightarrow \frac{3}{x+3} \leq 0 \\ &\Rightarrow x < -3 \end{aligned}$$

12. (C)

$$\begin{aligned} y &= \log_{\frac{1}{2}} \left( \frac{x+4}{x+1} \right) + \sqrt{x-1} \\ &\Rightarrow \frac{x+4}{x+1} > 0 \& x-1 \geq 0 \\ &\Rightarrow x < -4 \text{ or } x > -1 \& x \geq 1 \\ &\Rightarrow (-\infty, -4) \cup [1, \infty) \end{aligned}$$

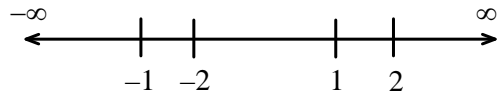
13. (A)

$$\begin{aligned} f(x) &= |x-2| + |x-1| + |x+1| + |x+2| \\ f(-x) &= f(x) \text{ then given functions is symmetric about } y\text{-axis} \end{aligned}$$

14.

(B)

$$|x-1| + |x-2| + |x+1| + |x+2| = 6$$

**Case I:**  $x < -2$ 

$$\therefore \cancel{x} - x + \cancel{x} - x - n - \cancel{x} - n - \cancel{x} = 6$$

$$-4x = 6$$

$$x = -\frac{3}{2}$$

**Case II:**  $-2 \leq x < -1$ 

$$\cancel{x} - n + \cancel{x} - n - \cancel{x} - \cancel{x} + 2 = 6$$

$$-2x = 2 \quad \therefore x = -1$$

**Case III:**  $-1 \leq x < 1$ 

$$1 - \cancel{x} + 2 - \cancel{x} + \cancel{x} + 1 + \cancel{x} + 2 = 6$$

$$6 = 6$$

**Case IV:**  $1 \leq x < 2$ 

$$\cancel{x} + 1 + 2 - \cancel{x} + n + 1 + n + 2 = 6$$

$$2x = 0 \quad \therefore n = 0$$

**Case V:**  $x \geq 2$ 

$$4x = 6 \quad \therefore x = \frac{3}{2}$$

$$\therefore x \in [-1, 1]$$

15.

(C)

$$f(x) = |x-2| + |x-1| + |x+1| + |x+2|$$

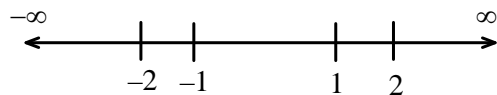
$$f(x)_{\min} = 6$$

$$\therefore k = 3$$

16.

(C)

$$|x-1| + |x-2| + |x+1| + |x+2| \leq 20$$

**Case I:**  $x < -2$ 

$$\cancel{x} - x + \cancel{x} - x - x - \cancel{x} - x - \cancel{x} \leq 20$$

$$-4x \leq 20, \quad x \geq -5$$

$$\therefore x \in [-5, -2]$$

**Case II:**  $-2 \leq x < -1$ 

$$\cancel{x} - x + \cancel{x} - x - \cancel{x} - \cancel{x} + 2 \leq 20$$

$$-2x \leq 16 \quad \therefore x \geq -8$$

$$x \in [-2, -1]$$

**Case III:**  $-1 \leq x < 1$ 

$$1 - \cancel{x} + 2 - \cancel{x} + \cancel{x} + 1 + \cancel{x} + 2 \leq 20$$

$$6 \leq 20$$

**Case IV:**  $1 \leq x < 2$ 

$$\cancel{x} - \cancel{x} + 2 - \cancel{x} + n + \cancel{x} + x + 2 \leq 20$$

$$2x \leq 16 \quad x \leq 8$$

**Case V:**  $x \geq 2$

$$4x \leq 20 \quad x \leq 5$$

$$x \in [-5, 5]$$

17. (D)

$$\text{If } x \in (-\infty, -1), y \in (0, 1)$$

18. (B)

$$\text{If } x \in (-1, 2) \text{ then } y \in (-\infty, 0)$$

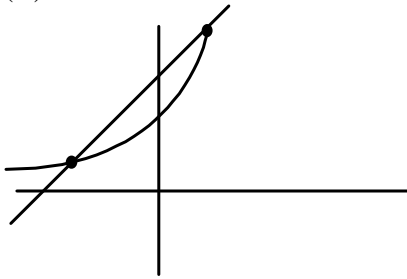
19. (C)

$$\text{If } n \in (2, 4), \text{ then } y \in (-\infty, \infty)$$

20. (A)

$$f(x) = k \text{ has root in } (4, \infty) \text{ then } k \in (1, \infty)$$

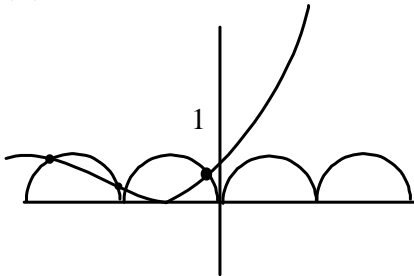
21. (C)



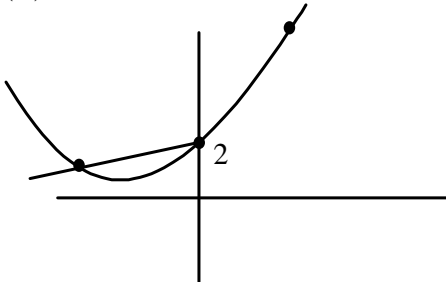
$$2^x + 3^x - 1 = 2x + 3$$

$$2^x + 3^x = 2x + 4$$

22. (B)



23. (C)



$$2^x + 3^x - 1 = 2x^2 - 2$$

$$-2^x - 3$$

$$\Rightarrow 2^x + 3^x = 2(x^2 + x + 1)$$

For 24 – 26

$$\tan\left(\theta + \frac{\pi}{4}\right) = 3 \tan 3\theta$$

$$\frac{1 + t\theta}{1 - t\theta} = \frac{3(3t\theta - t^3\theta)}{(1 - 3\tan^2 \theta)}$$

$$\text{Get } \tan \theta = t \quad (1 + t)(1 - 3t^2) = 3(1 - t)(3t - t^3)$$

$$1 - 3t^2 + t - 3t^3 = 9t - 3t^3 - 9t^2 + 3t^4$$

$$3t^4 - 6t^2 + 8t - 1 = 0$$

24. (D)

$$\sum \tan \alpha = 0$$

25. (A)

$$\pi \tan \alpha = -\frac{1}{3}$$

26. (B)

$$\frac{1}{t\alpha} + \frac{1}{t\beta} + \frac{1}{t\gamma} + \frac{1}{t\delta} = \frac{\sum t\alpha t\beta \tan \gamma}{t\alpha t\beta t\gamma t\delta}$$

$$= \frac{-8/\sqrt{3}}{-1/\sqrt{3}} = 8$$

27. (A)

$$\text{Statement - 1 is correct as } x^4 - 5x^3 + 56|x - 3| = 0$$

$$\text{Statement - 2: } a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1}x + a_n = 0 \text{ where } a_0, a_1, \dots, a_n \in I, a_0 \neq 0, a_n \neq 0$$

$$\Rightarrow (a_0 x^{n-1} + a_1 x^{n-2} + \dots + a_{n-1}) = -\frac{a_n}{x}$$

Clearly for an integral root  $x$  must be divisible by  $x$ .

Hence statement -2 is also correct & is a correct explanation of statement -1

28. (D)

For common root of equations  $x^2 - 4x + 1 = 0$  and  $x^2 - ax + b = 0$ , if  $a = 4$  &  $b = 1$ , then both the roots will be common.

If quadratic equations  $ax^2 + bx + c = 0$  and  $a_1x^2 + b_1x + c_1 = 0$  with rational coefficient have both the roots common, then we must have  $a = a_1k$ ,  $b = b_1k$  and  $c = c_1k$ , for some real number  $k \neq 0$ .

29. (D)

Let the polynomial be  $P(x) = (x - 1)(x - 2)Q(x) + ax + b$

$$P(1) = -1 \Rightarrow a + b = -1$$

$$P(2) = 1 \Rightarrow 2a + b = 1$$

$$\Rightarrow a = 2, b = -3$$

$$\Rightarrow \text{remainder is } 2x - 3$$

Statement -1 is false.

Statement – 2 is correct.

30. (D)

As  $x^2 + 1 \geq 1$  hence  $\log_3(x^2 + 1) \geq 0$ , also  $|\log_4 y^2| \geq 0$

Hence  $\log_3(x^2 + 1) + |\log_4 y^2| = 0 \Rightarrow \log_3(x^2 + 1) = 0$  &  $|\log_4 y^2| = 0$

$\Rightarrow x^2 + 1 = 1$  &  $y^2 = 1$

Statement – 1 is false

Statement – 2 is correct

31. (A)

$\log_{0.1}(\sqrt{14} - \sqrt{13}) = \log_{10}(\sqrt{14} + \sqrt{13})$

Also  $\sqrt{13} - \sqrt{12} < \sqrt{14} + \sqrt{13} \Rightarrow \log_{0.1}(\sqrt{14} - \sqrt{13}) > \log_{10}(\sqrt{13} - \sqrt{12})$

Statement – 1 is true

Statement – 2 is also correct & correct explanation of statement – 1

32. (C)

Statement – 1  $\log_{\sin^2 \theta} b < \log_{\sin^2 \theta} C$

$\Rightarrow b > c$

As  $\log_x b < \log_x c$

If  $x \in (0, 1)$   $b > c$

& if  $x > 1$   $b < c$

True

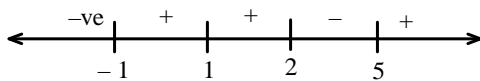
$\Rightarrow$  statement 2 is false. It's not always true.

33. (A)

34. (A)  $\frac{(x-1)^2(x+1)(x-2)}{(x-5)} \geq 0$

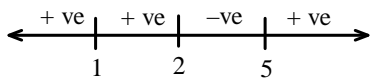
$x = 1, 1, -1, 2$  (roots)

$x = 5$  (pole)



$x \in [-1, 2] \cup (5, \infty)$

(B)  $\frac{|x|(x-5)}{(x-2)} \geq 0$



$x \in (-\infty, 2] \cup [5, \infty)$

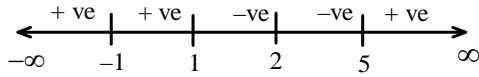
(c)  $\log_{10} x$

If  $x$  is two digit number then  $Q = [1, 2]$

$P = \log_{10} x$ ,  $x$  is four digit number then  $P = [3, 4)$



$$(d) \frac{(x-1)(x+1)(x-2)^2}{(x-5)} \leq 0$$



$$x \in [1, 5]$$

35. (A)  $-(r, s)$

$$\Rightarrow \log_{49} 7 = \frac{1}{2}$$

$$\Rightarrow \log_3 (5 + 8 \log_{49} (5 + 4 \log_{49} 7))$$

$$= \log_3 (5 + 8 \log_{49} 7)$$

$$= \log_3 \left(5 + \frac{8}{2}\right)$$

$$= \log_3 9 = 2$$

$$|k| = 2 \Rightarrow k = \pm 2$$

$$(B) \sqrt{(\log_{2^{-1}} .2^2)^2} = \sqrt{(-2)^2} = |-2| = 2$$

$$(C) \log_x (x^2 - 1) = 0$$

$$\therefore x^2 - 1 = 1$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

But  $x = -\sqrt{2}$  (rejected)

$\therefore$  only one solution

$$(D) \frac{1}{x} = \frac{\sqrt{9 - \sqrt{77}}}{\sqrt{81 - 77}} \quad \therefore \quad \frac{2}{x} = \sqrt{9 - \sqrt{77}}$$

$$x + \frac{2}{x} = \sqrt{9 + \sqrt{77}} + \sqrt{9 - \sqrt{77}}$$

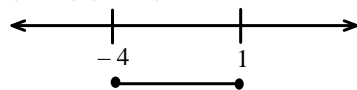
$$\left(x + \frac{2}{x}\right)^2 = 18 + 2 \times 2 = 22$$

$$\frac{1}{11} \left(x + \frac{2}{x}\right)^2 = 2$$

36. (A)  $x^2 + 4x - x - 4 \leq 0$

$$x(x+4) - (x+4) \leq 0$$

$$(x-1)(x+4) \leq 0$$



$$x \in [-4, 1]$$

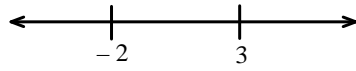
$$(B) x^2 - 2x + 2 > 0$$

$$\therefore x \in R$$

$$(C) \quad x^2 - 3x + 2x - 6 < 0$$

$$x(x-3) + 2(x-3) < 0$$

$$(x+2)(x-3) < 0$$

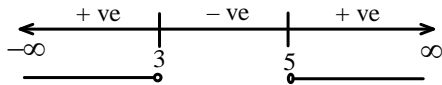


$$x \in (-2, 3)$$

$$(D) \quad x^2 - 3x - 5x + 15 > 0$$

$$x(x-3) - 5(x-3) > 0$$

$$(x-3)(x-5) > 0$$



$$x \in (-\infty, 3) \cup (5, \infty)$$

37. (A) - (QR)

$$1 - x^2 > 0 \quad \therefore x^2 < 1$$

$$2 - x^2 > 2\sqrt{1-x^2} \quad |x| < 1$$

$$4 + x^4 - 4x^2 > 4 - 4x^2$$

$$x^4 > 0 \quad \therefore x \in (-1, 1)$$

(B) - (PQRS)

$$x^8 - x^5 + x^2 - x + 1 > 0 \quad x \in R$$

For  $x < 0$  (time)

For  $x \in (0, 1)$

$$\frac{x^8}{+ve} + \frac{(x^2 - x^5)}{+ve} + \frac{(x)}{+ve} > 0$$

For  $x > 1$

$$\frac{(x^8 - x^5)}{+ve} + \frac{(x^2 - \alpha)}{+ve} + \frac{1}{+ve} > 0$$

(C) - (PQRS)

$$x^{12} - x^9 + x^4 - x + 3 > 0 \quad (\text{same})$$

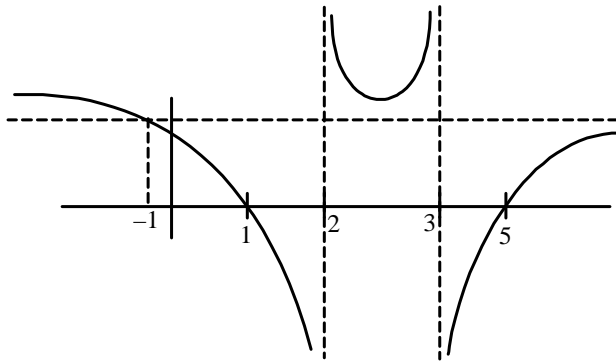
(D) - (PQRS)

$$2x^2 + 1 > \sqrt{4x^2 + 1}$$

$$4x^4 + 4x^2 + 1 > 4x^2 + 1$$

$$x^4 > 0 \quad n \in (-\infty, \infty) - \{0\}$$

38. 
$$\frac{(x-1)(x-5)}{(x-2)(x-3)}$$



(A) – (PRS)

$$-1 < x < 1, \quad f(x) \in (0,1)$$

(B) – (QS)

$$1 < x < 2, \quad f(x) \in (-\infty, 0)$$

(C) – (QS)

$$3 < x < 5, \quad f(x) \in (-\infty, 0)$$

(D) – (PRS)

$$x > 5, \quad f(x) \in (0,1)$$

39. (A) – (R)

$$|x-2| < 5$$

$$-5 < x-2 < 5$$

$$-3 < x < 7$$

(B) – (P)

$$|x-1| + |x-2| < 6$$

Case (i)  $x \geq 2$

$$2x - 3 < 6$$

$$x < 9.5$$

Case (ii)  $1 \leq x < 2$

$$x - 1 - x + 2 < 6$$

$$1 < 6$$

Case (iii)  $x \leq 1$

$$-x + 1 - x + 2 < 6$$

$$3 < 6 + 2x$$

$$x > -3$$

(C) – (S)

$$|4-3x| \leq \frac{1}{2}$$

$$-\frac{1}{2} \leq 4-3x \leq \frac{1}{2}$$

$$-\frac{9}{2} \leq -3x \leq -\frac{7}{2}$$

$$\frac{3}{2} \geq x \geq \frac{7}{6}$$

(D) – (Q)

$$|x^2 - 2| < 7$$

$$\begin{aligned}
-7 &< x^2 - 2 < 7 \\
-5 &< x^2 < 9 \\
0 &< x^2 < 9 \\
-3 &< x < 3
\end{aligned}$$

40. (A) – (QS)

$$x^{\log_{10} x} = 100x$$

$$\Rightarrow (\log_{10} \lambda)^2 = 2(\log_{10} \alpha)$$

$$\Rightarrow \log_{10} x = 2, -1$$

$$x = 100, \frac{1}{10}$$

$$x_1 \cdot x_2 = 10$$

(B) – (QS)

$$\log_2 (9 - 2^x) = 3 - x$$

$$\Rightarrow 9 - 2^x = \frac{8}{2^x}$$

$$\Rightarrow (2^x)^2 - 9 \cdot (2^x) + 8 = 0$$

$$2^x = 1, 2^x = 8$$

$$x = 0, x = 3$$

$$\lambda_1^2 + \alpha_2^2 = 9$$

(C) – (P)

$$\log_{1/8} \log_{1/4} \log_{1/2} (x) = \frac{1}{3}$$

$$\Rightarrow \log_{1/4} \log_{1/2} x = \frac{1}{2}$$

$$\Rightarrow \log_{1/2} x = \left(\frac{1}{4}\right)^{1/2} = \frac{1}{2}$$

$$\Rightarrow x = \left(\frac{1}{2}\right)^{1/2} = \frac{1}{\sqrt{2}}$$

(D) – (QR)

$$\log_b a = 3 \quad \log_b c = -4$$

$$a = b^3, \quad c = \frac{1}{b^4}$$

$$b^{9x} = \frac{1}{b^{4x-4}}$$

$$\Rightarrow b^{9x} = b^{-4x+4}$$

$$9x = -4x + 4$$

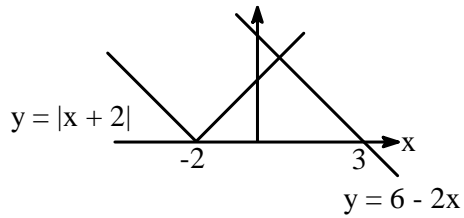
$$\Rightarrow x = \frac{4}{13}$$

$$p + q = 17$$



**INEQUATION & EQUATION  
EXERCISE – 2(C)**

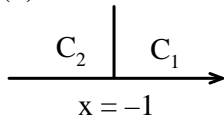
1. (1)



2. (2)

Same as above.

3. (3)



$C_1$ : If  $x \geq 1$  ... (a)

Then,  $2(x + 1) > x + 4$

$x > 2$  ... (b)

(a) n (b)

$x \in (2, \infty)$  (C-1)

$C_2$ : If  $x < -1$  ... (a)

Then,  $-2(x + 1) > x + 4$

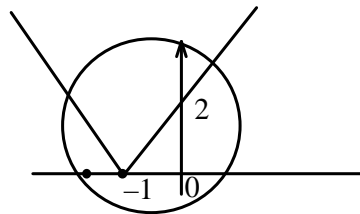
$\Rightarrow x < -3$  ... (b)

(a) n (b)

$x \in (-\infty, -3) \cup (2, \infty)$

$C-1 \cup C-2$

So required  $x = 3$



4. (2)

Same as above take three cases.

5. (5)

$(|x| - 2)(|x| - 3) < 0 \Rightarrow 2 < |x| < 3$

$\Rightarrow x \in (-3, -2) \cup (2, 3)$

So  $(a + b) = 5$

6. (4)

$-5 < x^2 - 4x < 5$

(1) (2)

(1)  $x^2 - 4x + 5 > 0 \Rightarrow x \in \mathbb{R} (\because D < 0)$  ... (1)

(2)  $x^2 - 4x - 5 < 0 \Rightarrow (x - 5)(x + 1) < 0 \Rightarrow -1 < x < 5$  ... (2)

(1) n (2)

$x \in (-1, 5)$

So,  $m = 0, n = 4 \Rightarrow (n - m) = 4$

7. (4)

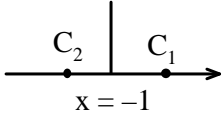
Same as question 6.

8. (7)  
 Domain  $x \geq 3$   
 So  $x \in (3, 10]$   
 So number of integer  $n = 7$
9. (2)  
 $0 \leq x^2 + 2x - 3 < 1$   
 (1) (2)  
 (1)  $(x+3)(x-1) \geq 0 \Rightarrow x \in (-\infty, 3] \cup [1, \infty) \dots (1)$   
 (2)  $x^2 + 2x - 4 < 0 \Rightarrow x = -1 \pm \sqrt{5} - 1 - \sqrt{5} < x < -1 + \sqrt{5} \dots (2)$   
 (1) n (2)  
 $x \in (-1 - \sqrt{5}, -3] \cup [1, \sqrt{5} - 1)$   
 So integer  $x = \{-3, 1\}$
10. (1)  
 $0 \leq \frac{x^2 - x - 2}{x^2} < \frac{4}{3}$  & solve as above
11. (9)  
 $\frac{\sqrt{2x^2 + 15x - 17}}{10 - x} \geq 0$   
 $2x^2 + 15x - 17 \geq 0$   
 $2x^2 + 17x - 2x - 17 \geq 0$   
 $x(2x + 17) - (2x + 17) \geq 0$   
 $(x - 1)(2x + 17) \geq 0$   
 $x \in (-\infty, -8.5] \cup [1, \infty)$   
 Also  $x < 10$   
 $\therefore$  No. of integers positive are  
 $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$   
 No. of positive integers are 9.
12. (6)  
 Domain:  $x \in [1, 9]$  & on solving  $x \in [1, 5]$   
 So  $(m + n) = 6$
13. (5)  
 Domain (a)  $-x^2 + 2x + 2y \geq 0 \Rightarrow x^2 - 2x - 2y \leq 0$   
 $\Rightarrow (x - 6)(x - 4) \leq 0 \Rightarrow -4 \leq x \leq 6 \dots (a)$   
 (b)  $8x - x^2 \geq 0 \Rightarrow x^2 - 8x \leq 0 \Rightarrow x \in [0, 8] \dots (b)$   
 (a) n (b)  $x \in [0, 6] \dots (1)$   
 (b)  $-x^2 + 2x + 2y \geq 8x - x^2 \Rightarrow x \leq y \dots (2)$   
 (1) & (2)  $x \in [0, 4]$   
 So answer is (5)
14. (1)  
 As function is increasing so equation change at most one solution

15. (1)  
Same as Q. 13

16. (1)  
Same as above

17. (2)  
 $|2^{x+1} - 1| + |2^{x+1} + 1| = 2^{|x+1|}; x \in \mathbb{R}$



$\begin{array}{c} C_2 \quad | \quad C_1 \\ \hline x = -1 \end{array}$

$C_1$  : if  $x \geq -1$  ... (a)

$$2 \cdot 2^x - 1 + 2 \cdot 2^x + 1 \Rightarrow 2^x \cdot 2$$

$$\Rightarrow 2^x = 0 \Rightarrow x \in \phi \quad \dots (b)$$

(a) & (b)  $x \in \phi$  ...  $C_1$

$C_2$ : If  $x < -1$

$$1 - 2 \cdot 2^x + 2 \cdot 2^x + 1 = \frac{1}{2} \cdot 2^{-x}$$

$$\Rightarrow 2^{-x} = 4 \Rightarrow x = -2 \quad \dots (b)$$

(a) & (b)  $x = -2$  ...  $C_2$

$$C_1 \cup C_2$$

$$x = -2 \Rightarrow |x| = 2$$

18. (7)  
 $(x - 2) = 0$  or  $1 \Rightarrow x = 2, 3$   
 or  $\log_2 x^3 - 3 \log_x 4 = 3 \Rightarrow 3 \log_2 x - \frac{6}{\log_2 x} = 3$   
 $\Rightarrow \log_2^2 x - \log_2 x - 2 = 0 \Rightarrow (\log_2 x - 2)(\log_2 x + 1) = 0$   
 $\log_2 x = -1, 2 \Rightarrow x = \frac{1}{2}, 4$   
 So sum = 9.5

19. (8)  
 As function is increasing so only one solution  
 $x = (16)^2 = 2^8$

20. (2)  
 $\sqrt{\log_2 x} + \sqrt[3]{\log_2 x} = 2$   
 $x = 2$

21. (0)  
 $(|x| - 3)(|x| + 2) = 0 \Rightarrow |x| = -2, 3 \Rightarrow x = \pm 3$

22. (0)  
 $\log(x - 2)(1 - x) \geq -1$   
 $x \in \phi \Rightarrow$  No solution



23. (0)  
 $(x+1)(x-1) - 2\sqrt{x^2-1} = 4x-1$   
 $\Rightarrow -2\sqrt{x^2-1} = (2x-1)$   
 $\Rightarrow 4(x^2-1) = 4x^2+1-4x \Rightarrow x = \frac{5}{4}$   
 But  $x = \frac{5}{4}$ . Doesn't satisfy so no solution

24. (2)  
 Same as Q. 18

25. (4)  
 $D \geq 0 \Rightarrow 16 - \log_2 n \geq 0 \Rightarrow \log_2 n \leq 2$   
 $(0 < n \leq 4)$

26. (1)  
 $(2x)^2 = x^2 + 75 \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$   
 Checking at  $x = \pm 5$  then only  $x = 5$  possible

27. (2)  
 Same as above

28. (7)  
 (1) Domain: a)  $x > 0, x \neq \frac{1}{2}$  ... (a)  
 b)  $x^2 - 5x + 6 > 0$   
 $\Rightarrow (x-2)(x-3) > 0 \Rightarrow x \in (-\infty, 2) \cup (3, \infty)$  ... (b)

(a) & (b)  $x \in (0, 2) \cup (3, \infty) - \left\{ \frac{1}{2} \right\}$  ... (1)

(2) C-1: If  $x > \frac{1}{2}$  ... (a)

$x^2 - 5x + 6 > 2x$   
 $\Rightarrow x^2 - 7x + 6 > 0 \Rightarrow (x-6)(x-1) > 0$   
 $\Rightarrow x \in (-\infty, 1) \cup (6, \infty)$  ... (b)

(a) & (b)  $x \in (6, \infty)$  ... (C-1)

C-2 if  $x < \frac{1}{2}$  ... (a)

$x^2 - 5x + 6 < 2x$   
 $x \leq (1, 6)$  ... (b)

$n \times x \in \phi$  ... (C-2)

$(C-1) \cup (C-2)$

$x \in (6, \infty)$  ... (2)

(1) n (2)  $x \in (6, \infty)$

29. (2)

$$y^{(6-y)^2+7(6-y)+12} = 1$$

$$\Rightarrow y^{y^2-19y+90} = 1 \Rightarrow (y-10)(y-9) = 1$$

So  $y = 1, 9, 10, -1 \Rightarrow x = 5, -3, -4, 7$

So  $(x, y) \equiv (5, 1)$  or  $(7, -1)$

30. (1)

$$3 + |x + 2| = \sqrt{9 - y^2}$$

So  $x = -2, y = 0$

So  $(-2, 0)$

31. (2)

$$\log(-x) = \sqrt{\log|x|}$$

Domain:  $x < 0$

$$\log^2(-x) = \log(-x)$$

$$\log(-x) = 0, 1$$

$$-x = 1, 10 \Rightarrow x = -1, -10$$

i.e. two solutions

32. (2)

Let  $\log_a^x = t$

$$\frac{2}{t} + \frac{1}{1+t} + \frac{3}{2+t} = 0$$

$$\Rightarrow 2(1+t)(2+t) + t(2+t) + 3t(1+t) = 0$$

$$\Rightarrow 6t^2 + 11t + 2 = 0$$

$$t = \frac{-11 \pm \sqrt{121 - 48}}{12} = \frac{-11 \pm \sqrt{73}}{12}$$

i.e. two values

33. (3)

$$N = \log_3(3^3 \cdot 5) \cdot \log(5 \cdot 3) - \log_3^5 \cdot \log_3(3^4 \cdot 5)$$

$$= (3 + \log_3^5)(1 + \log_3^5) - (\log_3^5)(4 + \log_3^5)$$

$$= 3$$

34. (0)

$$\log(x-3)(x-1)$$

$$\left(\frac{1}{10}\right)(x-3) \geq 1$$

$$\log(x-3)(x+1)$$

$$\Rightarrow 10(x-3) \leq 1$$

Domain:  $x \in (3, \infty) - \{4\}$

$$10^{1+\log_{(x-3)}^{(x-1)}} \leq 1$$

$$\Rightarrow \log_{(x-3)}^{(x-1)} \leq -1$$

Not possible as  $x > 3$  so no solution

35. (2)

Same as Q. 32

36.

(2)

$$\sqrt{(x-1)+4-2\sqrt{4}\sqrt{2-1}} + \sqrt{(x-1)+9-2\sqrt{9}\sqrt{-1}} = 1$$

$$\Rightarrow |\sqrt{x-1}-2| + |\sqrt{x-1}-3| = 1$$

$C_1$ : If  $x \geq 4$  ... (a)

$$(\sqrt{x-1}-2) + (\sqrt{x-1}-3) = 1 \Rightarrow \sqrt{x-1} = 3$$

$$x = 10 \quad \dots (b)$$

(a) n (b)  $x = 10$  ... (C-1)

$C_2$ : If  $x \in (3,4)$  ... (a)

$$(\sqrt{x-1}-2) - (\sqrt{x-1}-3) = 1 \Rightarrow 1 = 1 \Rightarrow x \in \mathbb{R} \quad \dots (b)$$

(a) n (b)  $x \in (3,4)$  ... (C-2)

$C_3$ : If  $x \leq 3$  ... (a)

$$-(\sqrt{x-1}-2) - (\sqrt{x-1}-3) = 1$$

$$\Rightarrow (\sqrt{x-1}-2)(\sqrt{x-1}-3) = 1$$

$$\Rightarrow \sqrt{x-1} = 2 \Rightarrow x = 5 \quad \dots (b)$$

(a) n (b)  $x \in \phi$  ... (C-3)

$$(C-1) \cup (C-2) \cup (C-3)$$

$$x \in (3,4) \cup \{10\}$$

So,  $x = 10$

37.

(0)

$$2x^2(2^x) + 4 \cdot 2^{|x-3|} = 16x^2 \cdot 2^{|x-3|} + \frac{2^x}{2}$$

$$\Rightarrow 4x^2 \cdot (2^x) + 8 \cdot 2^{|x-3|} = 32x^2 \cdot 2^{|x-3|} + 2^x$$

$$\Rightarrow 4x^2(2^x - 8 \cdot 2^{|x-3|}) = (2^x - 8 \cdot 2^{|x-3|})$$

$$\text{So, } 4x^2 = 1 \text{ or } 2^x = 2^{|x-3|+3}$$

$$x = \pm \frac{1}{2} \text{ or } x = |x-3| + 3$$

$$|x-3| = (x-3) \quad x \geq 3$$

So negative integral  $x = \phi$

38.

(0)

Domain:  $x \geq -1$

$$(x+1) + (x+4) + 2\sqrt{(x+1)(x+4)} = (x+2) + (x+3) + 2\sqrt{(x+2)(x+3)}$$

$$\Rightarrow (x+1)(x+4) = (x+2)(x+3)$$

$$\Rightarrow y = 6 \Rightarrow x \in \phi$$

39.

(2)

$$x^{x-y} = y^{x+y}, (\sqrt{x})y = 1$$

$$x^{x-y} = \left[ \left( \frac{1}{x} \right)^{1/2} \right]^{x+y}$$

$$x - y = -\frac{(x+y)}{2}$$

$$\Rightarrow 3x = y$$

$$\Rightarrow (3x) \cdot \sqrt{x} = 1$$

$$\Rightarrow x = \left(\frac{1}{3}\right)^{\frac{2}{3}}$$

Also  $x = y = 1$  is a possible solution

So two pairs possible

40.

(1)

$$2^{x+1} = y^2 + 4 \quad \& \quad 2^{x-1} \leq y$$

$$\Rightarrow 2^x = \frac{y^2 + 4}{2} \quad \& \quad 2^x \leq 2y$$

$$\Rightarrow \frac{y^2 + 4}{2} \leq 2y$$

$$\Rightarrow (y-2)^2 \leq 0$$

$$\Rightarrow y = 2$$

So no. of values is 1

41.

(6)

$$|x| + |y| = 1, k > 0$$

$$xy(x+y) = 0$$

$$\Rightarrow x = 0 \text{ or } y = 0 \text{ or } x + y = 0$$

$$(i) \text{ If } x = 0 \Rightarrow |y| = k$$

$$\Rightarrow y = \pm k$$

$$(ii) \text{ If } y = 0 \Rightarrow |x| = k$$

$$\Rightarrow x = \pm k$$

$$(iii) \text{ If } x + y = 0 \Rightarrow x = -y$$

$$\Rightarrow |x| = |y|$$

$$\Rightarrow |x| + |x| = k$$

$$\Rightarrow x = \pm \frac{k}{2} = y$$

Hence total 6 solutions

42.

(7)

$$x^2 - mx + 2m = 0$$

$$x_1 + x_2 = 1m \quad \dots (1)$$

$$x_1 x_2 = 2m \quad \dots (2)$$

$$\text{Now, } x_1^3 + x_2^3 = x_1^2 + x_2^2$$

$$(x_1 + x_2)^3 - 3x_1 x_2 (x_1 + x_2) = (x_1 + x_2)^2 - 2x_1 x_2$$

$$(m)^3 - 3 \cdot 2m \cdot m = (m)^2 - 2 \cdot 2m$$

$$m^3 - 6m^2 = m^2 - 4m$$

$$m^3 - 7m^2 + 4m = 0$$

$$\Rightarrow m_1 = 0 \quad \& \quad m_2 + m_3 = 7$$

$$\Rightarrow m_1 + m_2 + m_3 = 7$$

43.

(2)

$$|x^2 + 3x| + x^2 - 2 = 0$$

Case – I

$$x^2 + 3x \geq 0$$

$$x(x + 3) \geq 0$$

$$x \in (-\infty, -3) \cup (0, \infty)$$

$$\text{Now, } x^2 + 3x + x^2 - 2 = 0$$

$$2x^2 + 3x - 2 = 0$$

$$x = \frac{1}{2} \text{ \& } x = -2$$

$\therefore x = \frac{1}{2}$  is the solution

Case – II

$$x^2 + 3x < 0$$

$$x(x + 3) < 0$$

$$x \in (-3, 0)$$

$$\text{Now, } -(x^2 + 3x) + x^2 - 2 = 0$$

$$-3x - 2 = 0$$

$$\Rightarrow x = -\frac{2}{3}$$

$\therefore x = -\frac{2}{3}$  is the solution

$\Rightarrow$  Total 2 solutions

44. (1)

$$4^{\frac{x+y}{x}} = 32$$

$$2^{2\left(\frac{x+y}{x}\right)} = 2^5$$

$$\Rightarrow 2\left(\frac{x+y}{x}\right) = 5 \quad \dots(1)$$

$$\log_3(x - y) = 1 - \log_3(x + y)$$

$$\Rightarrow \log_3(x^2 - y^2) = 1$$

$$\Rightarrow x^2 - y^2 = 3$$

$$\Rightarrow x = \pm\sqrt{3 + y^2} \quad \dots (2)$$

Solving (1) & (2) we get

$$x = \pm 2 \text{ \& } y = \pm 1$$

Also  $x - y > 0$  &  $x + y > 0$

Only solution which satisfy is

$$x = 2 \text{ \& } y = 1$$

So one solution

45. (0)

$$x_1 + x_2 - x_3 - x_4 = 1 \quad \dots (1)$$

$$x_1 + 2x_2 + 3x_3 - x_4 = 2 \quad \dots (2)$$

$$3x_1 + 5x_2 + 5x_3 - 3x_4 = 6 \quad \dots (3)$$

Here (1) & (2) gives

$$x_2 + 4x_3 = 1 \quad \dots (4)$$

Also  $2 \times (2) - (3)$  gives

$$x_2 + 4x_3 = 0 \quad \dots (5)$$

$\therefore$  From (4) & (5)

No solution

