INEQUATIONS & EQUATION
EXERCISE-1-A

1. (B)

$\Rightarrow \log(ab) - \log|b|$

We can see that $ab > 0 \Rightarrow a < 0 \& b < 0$ or $a > 0 \& b > 0$

So $\log(ab) - \log|b| = \log|ab| - \log|b|$

$= \log|a|.|b| - \log|b|$

$= \log|a| + \log|b| - \log|b|$

$= \log|a|$

2. (C)

$\Rightarrow \sqrt{\log_{0.5}^2 4} = \sqrt{[\log_{10} 4]^2} = \sqrt{\left[\log_{\left(\frac{1}{2}\right)} 4\right]^2}$

$= \sqrt{\left[\log_{\left(\frac{1}{2}\right)} \left(\frac{1}{2}\right)^{-2}\right]^2} = \sqrt{\left(-2 \times \log_{\frac{1}{2}} \frac{1}{2}\right)^2}$

$= \sqrt{(-2 \times 1)} = \sqrt{4}$

$= 2$ …… (as a square root of a value can't be negative)

3. (B)

$\Rightarrow \log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8 \log_8 9$

By formula $\log_a b = \dfrac{\log_c b}{\log_c a}$

Given value is $= \left(\dfrac{\log 4}{\log 3}\right)\left(\dfrac{\log 5}{\log 4}\right)\left(\dfrac{\log 6}{\log 5}\right)\left(\dfrac{\log 7}{\log 6}\right)\left(\dfrac{\log 8}{\log 7}\right)\left(\dfrac{\log 9}{\log 8}\right)$

$= \dfrac{\log 9}{\log 3} = \log_3 9$

$= 2$

4. (C)

$\log_7 \log_7 \left(\sqrt{7\sqrt{7\sqrt{7}}}\right)$

$= \log_7 \log_7 \left(7^{1/2}.7^{1/7}.7^{1/8}\right)$

$= \log_7 \log_7 7^{7/8} = \log_7 (7/8)$

$= 1 - 3\log_7 2$

5. (D)

$\Rightarrow 81^{\left(\frac{1}{\log_5 3}\right)} + 27^{(\log_9 36)} + 3^{\frac{4}{\log_7 9}}$

$\Rightarrow (81)^{(\log_3 5)} + \left(3^3\right)^{\left(\log_{3^2} 36\right)} + 3^4 \log_9 7$

$$\Rightarrow \left(3^4\right)^{\log_3 5} + 3^{3\times\left(\frac{1}{2}\log_3 36\right)} + 3^{4\times\left(\frac{1}{2}\log_3 7\right)}$$

$$\Rightarrow 3^{\log_3 5^4} + 3^{\log_3 (36)^{\frac{3}{2}}} + 3^{\log_3 7^2}$$

$$\Rightarrow 5^4 + (36)^{\frac{3}{2}} + 7^2$$

$$625 + 36\times 6 + 49 = 890$$

6. (C)

$\Rightarrow \log_x x.\log_5 k = \log_x 5$; given $k \neq 1, k > 0$

$$\Rightarrow \frac{\log x}{\log k}.\frac{\log k}{\log 5} = \frac{\log 5}{\log x}$$

$$\Rightarrow (\log_5 x) = (\log_x 5)$$

$\Rightarrow x = 5$ is the only possible solution

7. (C)

$$\Rightarrow \log_5 a.\log_a x = 2$$

$$\Rightarrow \frac{\log a}{\log 5}, \frac{\log x}{\log a} = 2$$

$$\Rightarrow \log_5 x = 2$$

$$\Rightarrow x = 5^2$$

$$= 25$$

8. (C)

$$\Rightarrow A = \log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2$$

$$\Rightarrow \log_2 \log_2 \log_4 (4)^2 + \log_{2^{\frac{1}{2}}}(2)$$

$$\Rightarrow \log_2 \log_2 4 + 2\times\frac{1}{\left(\frac{1}{2}\right)}\log_2 2$$

$$\Rightarrow \log_2 2 + 4 = 1 + 4$$

$$\Rightarrow 5$$

9. (D)

$$\Rightarrow \log_{10} x = y(\text{given})$$

$$\Rightarrow \log_{1000} x^2 = 2\log_{10^3} x$$

$\Rightarrow 2\times\frac{1}{3}\log_{10} x$ (by formula $\log_{a^{k^b}} = \frac{1}{k}\log_a b$)

10. (A)

$$\frac{(\log a)^2}{\log b / \log c} - 1 + \frac{(\log b)^2}{\log a \log c} - 1 + \frac{(\log c)^2}{\log a \log b} - 1 = 0$$

$$\Rightarrow (\log a)^3 + (\log b)^3 + (\log c)^3 = 3\log a \log b \log c$$

$\because$ a, b, c are distinct

$\Rightarrow \log a + \log b + \log c = 0$

$\Rightarrow abc = 1$

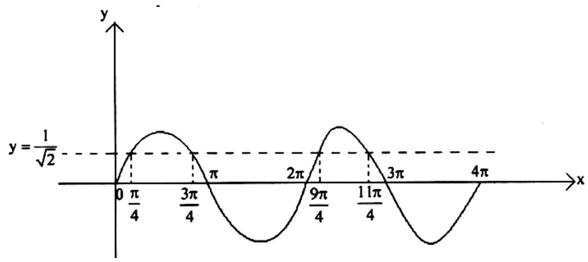
11. (A)

$\Rightarrow \log_{\left(\frac{1}{\sqrt{2}}\right)} \sin x > 0; x \in [0, 4\pi]$

As base $\frac{1}{\sqrt{2}}$ lies between 0 to 1satisfy given inequality, $0 < \sin x < 1$

$\Rightarrow x \in (0, \pi) \cup (2\pi, 3\pi)$

As we can see in this interval



We get $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$ as integral

Multiples of $\frac{\pi}{4}$

12. (B)

$\Rightarrow \log_{\frac{1}{2}} \left(x^2 - 6x + 12\right) \geq -2$

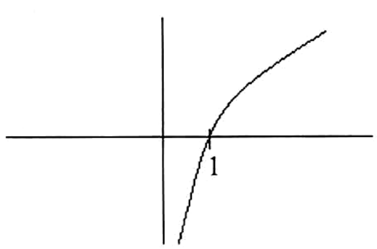
$\Rightarrow \log_{2^{-1}} \left(x^2 - 6x + 12\right) \geq -2$

$\Rightarrow -1 \times \log_2 \left(x^2 - 6x + 12\right) \geq -2$

$\Rightarrow \log_2 \left(x^2 - 6x + 12\right) \leq 2$

$\Rightarrow \log_2 \left(x^2 - 6x + 12\right) - \log_2 4 \leq 0$

$\Rightarrow \log_2 \left(\frac{x^2 - 6x + 12}{4}\right) \leq 0$



$\Rightarrow 0 < \frac{x^2 - 6x + 12}{4} \leq 1$

Case-1

$\Rightarrow 0 < \frac{x^2 - 6x + 12}{4}$

$\Rightarrow x^2 - 6x + 12 > 0$

$\Rightarrow x \in R$ …… (1) as discriminant if quadratic expression $x^2 - 6x + 12$ is less than zero.

Descriminant $D = (-6)^2 - 4(12)(1)$

$\Rightarrow D = -12$

Case-2

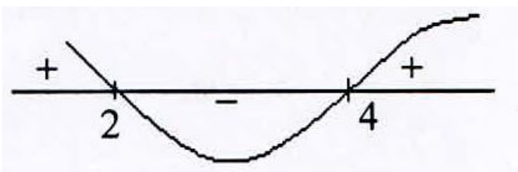
$\Rightarrow \frac{x^2 - 6x + 12}{4} \le 4$

$\Rightarrow x^2 - 6x + 12 \le 4$

$\Rightarrow x^2 - 6x + 8 \le 0$

$\Rightarrow (x-4)(x-2) \le 0$

$\Rightarrow x \in [2, 4]$ ……..(ii)



By taking intersection of (i) & (ii) we get $x \in [2, 4]$

13. (B)

$\Rightarrow 2^{\log_{\sqrt{2}}(x-1)} > x + 5$ Here $x - 1 >; x > 1$ ……(1)

$\Rightarrow 2^{\log (2)^{\frac{1}{2}}(x-1)} > x + 5$

$\Rightarrow 2^{2\log_2(x-1)} > x + 5$ (or by formula $\log_{a^{k^b}} = \frac{1}{k}\log\frac{b}{a}$)

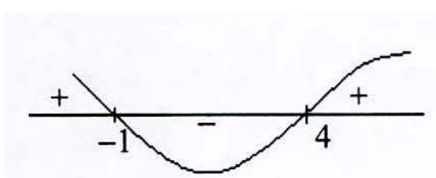
$\Rightarrow 2^{\log_2(x-1)^2} > x + 5$

$\Rightarrow (x-1)^2 > x + 5$

$\Rightarrow x^2 + 1 - 2x > x + 5$

$\Rightarrow x^2 - 3x - 4 > 0$

$\Rightarrow (x-4)(x+1) > 0$



So we get $x \in (-\infty, -1) \cup (4, \infty)$ ……….(ii)

By taking intersection of (i) & (ii)

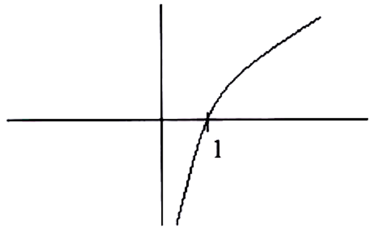
$\Rightarrow x \in (4, \infty)$

14. (C)

$\Rightarrow \log_{10}(x^2 - 2x - 2) \le 0$

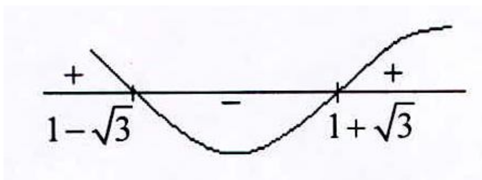
As base is greater than 1 so to hold the inequality true

$\Rightarrow 0 < x^2 - 2x - 2 \le 1$



So, $0 < x^2 - 2x - 2$ and $x^2 - 2x - 2 \le 1$

Case-I



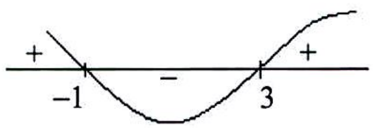
$\Rightarrow x^2 - 2x - 2 > 0$

$\Rightarrow \left[x - \left(1+\sqrt{3}\right)\right]\left[x - \left(1-\sqrt{3}\right)\right] > 0$

So, $x \in \left(-\infty, 1-\sqrt{3}\right) \cup \left(1+\sqrt{3}, \infty\right)$ …….(i)

Case-2

$\Rightarrow x^2 - 2x - 2 \le 1$



$\Rightarrow x^2 - 2x - 3 \le 0$

$\Rightarrow (x-3)(x+1) \le 0$

So we get $x \in [-1, 3]$

By taking intersection of (i) & (ii) we get,

$\Rightarrow x \in \left[-1, 1-\sqrt{3}\right) \cup \left(1+\sqrt{3}, 3\right]$

15. (A)

$\Rightarrow \log_{0.2} \frac{x+2}{x} \le 1$

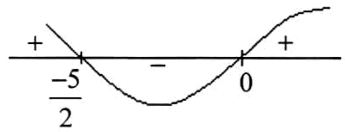
As base of log is less than 1 so hold the inequality true

$\Rightarrow \frac{x+2}{x} \ge 0.2$

$\Rightarrow \frac{x+2}{x} - 0.2 \ge 0$

$\Rightarrow \frac{x+2-0.2x}{x} \ge 0$

$\Rightarrow \frac{0.8x+2}{x} \ge 0$



So, $x \in \left(-\infty, \frac{-5}{2}\right] \cup [0, \infty)$

$\Rightarrow a^{m \log_a n} \Rightarrow a^{\log_a n^m} \Rightarrow n^m$

16. (C)

$\Rightarrow a^{mn} = a^{m^n}$

Take log both side

$\Rightarrow \log_a a^{mn} = \log_a a^{m^n}$

$\Rightarrow mn = m^n$

$\Rightarrow m^{n-1} = n$

$\Rightarrow m = \left(n^{\frac{1}{n-1}}\right)$

18. (B)

$\Rightarrow \left(x^5\right)^{\frac{1}{3}} \left(16x^3\right)^{\frac{2}{3}} \left(\frac{1}{4} x^{\frac{4}{9}}\right)^{-\frac{3}{2}} . (4)^{\frac{1}{6}}$

$\Rightarrow x^{\frac{5}{3}} . \left(4^2\right)^{\frac{2}{2}} . \left(x^3\right)^{\frac{2}{2}} . \left(4^{-1}\right)^{\frac{-3}{2}} . \left(x^{\frac{4}{9}}\right)^{\frac{-3}{2}}$

$\Rightarrow x^{\frac{5}{3}} \times 4^{\frac{4}{3}} \times x^2 \times 4^{\frac{3}{2}} \times x^{\frac{4}{9} \times \frac{-3}{2}}$

$x^{\frac{5}{3} - \frac{2}{3} + 2} . 4^{\left(\frac{4}{3} + \frac{3}{2} + \frac{1}{6}\right)}$

$\Rightarrow 4^3 . x^3$

19. (D)

$\Rightarrow \frac{\left(2^{n+1}\right)^m \left(2^{2n}\right) 2^n}{\left(2^{m+1}\right)^m 2^{2m}} = 1$

$\Rightarrow \frac{2^{nm+m} \times 2^{2n+n}}{2^{mn+m} 2^{2m}} = 1$

$\Rightarrow 2^{(nm+m+2n+n)-(mn+n-2m)} = 1$

$\Rightarrow 2^{2n-m} = 1$

$\Rightarrow 2n - m = 0$

$\Rightarrow m = 2n$

20. (D)

$\Rightarrow x^{x\sqrt[3]{x}} = \left(x.\sqrt[3]{x}\right)^x$ Here $x \neq 0$

$\Rightarrow x^{x+\frac{1}{3}} = \left(x^{1+\frac{1}{3}}\right)^{x}$

$\Rightarrow x^{x^{\frac{4}{3}}} = x^{\frac{4}{3}x}$

Take log both side

$\Rightarrow x^{\frac{4}{3}} \log_x x = \frac{4}{3} x \log_x x$

$\Rightarrow x^{\frac{4}{3}} = \frac{4}{2} x$

$\Rightarrow x^{\frac{1}{3}} = \frac{4}{3} x$

$\Rightarrow x^{\frac{1}{3}} = \frac{4}{3}$

$\Rightarrow x = \left(\frac{4}{3}\right)^{3} = \frac{64}{27}$

21. (C)

If $x = 2^{\frac{1}{3}} - 2^{-\frac{1}{3}}$

Then $2x^3 + 6x = 2\left[2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right]^3 + 6\left[2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right]$

$= 2\left(2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right)\left[\left(2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right)^2 + 3\right]$

$= 2\left(2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right)\left[2^{\frac{2}{3}} + 2^{\frac{-2}{3}} - 2 + 3\right]$

$= 2\left[2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right]\left[2^{\frac{2}{3}} + 2^{\frac{-2}{3}} + 1\right]$

$= 2\left[\left(2^{\frac{1}{3}}\right)^3 - \left(2^{\frac{-1}{3}}\right)^3\right]$ $\left(\because \text{by } a^3 - b^3 = (a-b)(a^2 + b^2 + ab)\right)$

$= 2\left[2 - 2^{-1}\right]$

$= 3$

22. (A)

$\Rightarrow (x)^{x\sqrt{x}} = \left(x\sqrt{x}\right)^{x}$ $(\text{here } x \neq 0)$

$\Rightarrow x^{x^{\frac{3}{2}}} = \left(x^{\frac{3}{2}}\right)^{x}$

$\Rightarrow x^{x^{\frac{3}{2}}} = x^{\frac{3}{2}x}$

Take log both sides

$\Rightarrow x^{\frac{3}{2}} \log_x x = \frac{3}{2} x \log_x x$

$\Rightarrow x^{\frac{3}{2}} = \frac{3}{2}x$

$\Rightarrow x^{\frac{1}{2}} = \frac{3}{2}$

$\Rightarrow x = \frac{9}{4}$

23. (C)

$\Rightarrow 5^{x-1} + 5(0.2)^{x-2} = 26$

$\Rightarrow 5^{x-1} + 5\left(\frac{1}{5}\right)^{x-2} = 26$

$\Rightarrow 5^{x-1} + 5^{1-x+2} = 26$

$\Rightarrow 5^{x-1} + 5^{3-x} = 26$

At $x = 1, 3$ above equation satisfy

24. (D)

$\Rightarrow x - \frac{1}{x} = 2$

$\Rightarrow x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2$

$= \left[\left(x - \frac{1}{x}\right)^2 + 2\right]^2 - 2$

$= \left[2^2 + 2\right]^2 - 2$

$= 36 - 2 = 24$

25. (C)

$\Rightarrow \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} . \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} . \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}$

$\Rightarrow x^{\left(\frac{1}{c}-\frac{1}{b}\right)} \times x^{\left(\frac{1}{a}-\frac{1}{c}\right)} \times x^{\left(\frac{1}{b}-\frac{1}{a}\right)}$

$\Rightarrow x^{\frac{1}{c}-\frac{1}{b}+\frac{1}{a}-\frac{1}{c}+\frac{1}{b}-\frac{1}{a}}$

$\Rightarrow x^0 = 1$

26. (C)

$\Rightarrow a^m . a^n = a^{mn}$

$\Rightarrow a^{m+n} = a^{nm}$

$\Rightarrow m + n = mn$ ………(1)

Then $m(n-2) + n(m-2) = ?$

$\Rightarrow 2mn - 2m - 2n$

$\Rightarrow 2(m+n) - 2(m+n)$

27. (B)

$\Rightarrow \log(x+1)+\log(x-1)=\log 3$

$\Rightarrow \log(x+1)(x-1)=\log 3$

$\Rightarrow x^2-1=3$

$\Rightarrow x^2=4$

$\Rightarrow x=\pm 2$

But $x+1>0$ & $x-1>0$

$\Rightarrow x>-1 \& x>1$

So, $x\in(1,\infty)$ is our feasible region

Only $x=2$ lies in the feasible region.

28. (D)

$\Rightarrow f(x)=5-|x-3|$

$\Rightarrow f(x)=5-(x-3); x\in[3,\infty)$

$\Rightarrow 5+(x-3); x\in(-\infty,3)$

$\Rightarrow f(x)=8-x; x\in[3,\infty)$

$=2+x; x\in(\infty,3)$

So, greatest value of function occur at $x=3$

So $f(3)=8-3=5$

29. (B)

$$\Rightarrow \frac{2^{m+3}\times 3^{2m-2n}\times 5^{m+3+n}\times 6^{n+1}}{6^{m+1}\times 10^{n+3}\times 15^{m}}$$

$$\Rightarrow \frac{(2^{m+3})(3^{2m-2})(5^{m+n+3})\times 2^{n+1}\times 3^{n+1}}{(2^{m+1}3^{m+1})(2^{n+3}5^{n+3})(3^{m})(5^{m})}$$

$\Rightarrow 2^{m+3+n+1-(m+1)}3^{2m-n+n+1-(m+1)-m}5^{m+n+3-(n+3)-m}$

$\Rightarrow 2^0 3^0 5^0$

$\Rightarrow 1$

30. (A)

$\Rightarrow x^{\frac{2}{3}}+x^{\frac{1}{3}}=2$

Take cube both side

$$\Rightarrow x^2+x+3x^{\frac{2}{2}}x^{\frac{1}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)=8$$

$\Rightarrow x^2+x+3x(2)=8$

$\Rightarrow x^2+7x-8=0$

$\Rightarrow (x+8)(x-1)=0$

$\Rightarrow x=1,-8$

31. (D)

If $a+b+c=0$

$$\Rightarrow \frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab} \Rightarrow \frac{a^3+b^3+c^3}{abc}$$

$$\Rightarrow \frac{\left\{(a+b)^3-3ab(a+b)\right\}+c^3}{abc} \Rightarrow \frac{\left\{(-c)^3-3ab(-c)\right\}+c^3}{abc}$$

$$\Rightarrow \frac{3abc}{abc}=3$$

32. (C)

$$\Rightarrow 2^x-2^{x-1}=4$$

$$\Rightarrow 2^x-\frac{2^x}{2}=4$$

$$\Rightarrow 2.2^x-2^x=8$$

$$\Rightarrow 2^x=8$$

$$\Rightarrow x=3$$

So, $x^x=3^x=27$

33. (B)

$$\Rightarrow \frac{(\log x-\log y)\left(\log x^2+\log y^2\right)}{\left(\log x^2-\log y^2\right)(\log x+\log y)}$$

$$\Rightarrow \frac{(\log x-\log y)2(\log x+\log y)}{2(\log x-\log y)x(\log x+\log y)}$$

$$=1$$

34. (B)

$$\Rightarrow \log_{10}\left(2x^2+7x+16\right)=1$$

$$\Rightarrow 2x^2+7x+16=10^1$$

$$\Rightarrow 2x^2+7x+6=0$$

$$\Rightarrow 2x^2+4x+3x+6=0$$

$$\Rightarrow (2x+3)(x+2)=0$$

$$\Rightarrow x=-\frac{3}{2}.-2$$

35. (C)

$$\Rightarrow a>0, b>0, c>0$$

$$\Rightarrow \log\left(a^a b^b c^c\right)+\log\left(\frac{1}{abc}\right)$$

$$\Rightarrow \log\left(\frac{a^a b^b c^c}{abc}\right)$$

$$\Rightarrow \log\left(a^{a-1}b^{b-1}c^{c-1}\right)$$

36. (B)

$\Rightarrow \log_{10}\left[\log_{10}\left(\log_{10} x\right)\right] = 0$

$\Rightarrow \log_{10}\left(\log_{10} x\right) = 10^0 = 1$

$\Rightarrow \log_{10} x = 10^1 = 10$

$\Rightarrow x = 10^0$

37. (C)

$\Rightarrow (25)^{x-2} = (125)^{2x-4}$

$\Rightarrow \left(5^2\right)^{x-2} = \left(5^3\right)^{2x-4}$

$\Rightarrow 5^{2x-4} = 5^{6x-12}$

$\Rightarrow 5^{6x-12-(2x-4)} = 1$

$\Rightarrow 5^{4x-8} = 1 = 5^{\circ}$

So, $4x - 8 = 0$

$\Rightarrow x = 2$

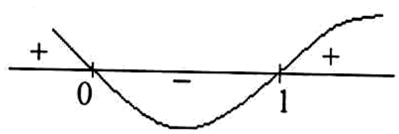
38. (C)

$\Rightarrow \sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = \frac{13}{6}$ here $\frac{x}{1-x} \ge 0; \frac{x}{1-x} \ge 0; \frac{x}{x-1} \le 0$

So, $x \in [0,1)$ is the feasible region for the equation

$\Rightarrow \frac{(x)+(1-x)}{\sqrt{1-x}\sqrt{x6}} = \frac{13}{6}$

$\Rightarrow \frac{1}{\sqrt{x(1-x)}} = \frac{13}{6}$



Taking square both side

$\Rightarrow x(1-x) = \frac{36}{169}$

$\Rightarrow x^2 - x + \frac{36}{169} = 0$

$\Rightarrow \left(x - \frac{9}{13}\right)\left(x - \frac{4}{13}\right) = 0$

$\Rightarrow x = \frac{9}{13}, \frac{4}{13}$

Here values lies in the feasible region

So, $x = \frac{9}{13}, \frac{4}{13}$

39. (D)

$\Rightarrow \sqrt{3y+1} = \sqrt{y-1}$ ……..(1)

$\Rightarrow 3y+1 \geq 0 \;\&\; y-1 \geq 0$

$\Rightarrow y \geq -\frac{1}{3} \;\&\; y \geq 0$

$\Rightarrow y \in [0, \infty)$ is our feasible region

By equation (1), taking square of both side,

$\Rightarrow 3y+1 = y-1$

$\Rightarrow 2y = -2$

$\Rightarrow y = -1$; which does not lie in feasible range of y.

So no solution of y.

40. (D)

$\Rightarrow x = 7+4\sqrt{3}; x = 4+4\sqrt{3}+3$

$\Rightarrow x = (2)^2 + 2.2\sqrt{3} + \left(\sqrt{3}\right)^2$

$\Rightarrow x = \left(2+\sqrt{3}\right)^2$

$\Rightarrow \sqrt{x} = 2+\sqrt{3}$ ……..(1)

$\Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2+\sqrt{3}}$

$\Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3}$

$\Rightarrow \frac{1}{\sqrt{x}} = 2-\sqrt{3}$ ………(2)

$\Rightarrow \sqrt{x} + \frac{1}{\sqrt{x}} = \left(2+\sqrt{3}\right) + \left(\frac{1}{2+\sqrt{3}}\right)$

$= \left(2+\sqrt{3}\right) + \left(2-\sqrt{3}\right)$

$= 4$

41. (A)

$\Rightarrow x^3 - 3x + 9 = 0$

Let the roots are α, α, β

So, $\alpha + \alpha + \beta = 0$

$\Rightarrow 2\alpha + \beta = 0$ ……..(1)

$\Rightarrow \alpha.\alpha + \alpha.\beta + \beta.\alpha = -3$

$\Rightarrow \alpha^2 + 2\alpha\beta = -3$ ……….(2)

$\Rightarrow \alpha^2\beta = -q$

$\Rightarrow q = -\alpha^2\beta$ ………(3)

By equations (2), (1)

$\Rightarrow \alpha^2 + 2\alpha(-2\alpha) = -3$

$\Rightarrow -3\alpha^2 = -3$

$\Rightarrow \alpha^2 = 1$

$\Rightarrow \alpha = \pm 1$ ……….(4)

$\Rightarrow \because q = -\alpha^2\beta$

$= -\alpha^2(-2\alpha)$

$\Rightarrow q = 2\alpha^3$

$\Rightarrow q = \pm 2$

42. (D)

$\Rightarrow \log_{16} x + \log_4 x + \log_2 x = 14$

$\Rightarrow \frac{1}{4}\log_2 x + \frac{1}{2}\log_2 x + \log_2 x = 14$ $\left(\text{by} \log_{a^{k}} {}^{b} = \frac{1}{k}\log_a b\right)$

$\Rightarrow \frac{7}{4}\log_2 x = 14$

$\Rightarrow \log_2 x = 8$

$\Rightarrow x = 2^8 = 256$

43. (C)

$\Rightarrow |4-3x| \le \frac{1}{2}$

Case 1: $4-3x \ge 0 \Rightarrow x \le \frac{4}{3}$ …….(i)

So $4-3x \le \frac{1}{2}$

$\Rightarrow -3x \le -\frac{7}{2}$

$\Rightarrow x \ge \frac{7}{6}$

By intersection of (i) & (ii) $x \in \left[\frac{7}{6}, \frac{4}{3}\right]$ ……..(A)

Case 2: $4-3x \le 0 \Rightarrow x > \frac{4}{3}$ ………(iii)

$\Rightarrow$ So $-(4-3x) \le \frac{1}{2}$

$\Rightarrow -4+3x \le \frac{1}{2}$

$\Rightarrow 3x \le \frac{9}{2}$

$\Rightarrow x \le \frac{3}{2}$ ………(iv)

Taking intersection of (iii) & (iv)

$\Rightarrow x \in \left(\frac{4}{3}, \frac{3}{2}\right]$ ……….(B)

So union of A & B is the solution of the given inequality $x \in \left[\frac{7}{6}, \frac{3}{2}\right]$ ……..(C)

44. (A)

$\Rightarrow x^2 - |x| - 6 = 0$

Case 1: $x \ge 0$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x = 3, -2$

But $x \ge 0$ so $x = 3$ is the only root.

Case 2: $x < 0$

$\Rightarrow x^2 - (-x) - 6 = 0$

$\Rightarrow x^2 + x - 6 = 0$

$\Rightarrow x = 3, -2$

But $x < 0$ so $x = -3$ is the solution.

So multiplication $= 3(-3) = -9$

45. (C)

$\Rightarrow \frac{|x| - 1}{|x| + 2} > 0$

So, $|x| > -2$ or $|x| > 1$

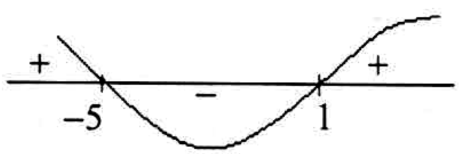
$\because |x| > -2$ holds true for $x \in R$

Now, $|x| > 1$

$\Rightarrow x \in (-\infty, -1) \cup (1, \infty)$

46. (A)

$\Rightarrow |x - 1| + |x + 5| = 6$



This is special case as $(x - 1) - (x + 5) = -6$

So the given expression will hold true if $(x - 1)(x + 5) \le 0$

$\Rightarrow x \in [-5, 1]$

47. (D)

$\Rightarrow \log_{\frac{1}{2}}(x^2 - 1) > 0$

As base of log is less than 1

So, $\log_{\frac{1}{2}}(x^2 - 1) > 0$

$\Rightarrow 0 < x^2 - 1 < 1$

$\Rightarrow 1 < x^2 < 2$

$\Rightarrow x^2 > 1$ and $x^2 < 2$

$\Rightarrow x \in (-\infty, -1) \cup (1, \infty)$ & $x \in \left(-\sqrt{2}, \sqrt{2}\right) \Rightarrow x \in \left(-\sqrt{2}, -1\right) \cup \left(1, \sqrt{2}\right)$

48. (D)

$\Rightarrow \log_2 x + \log_x 2 = \frac{10}{3} = \log_2 y + \log_y 2$

$\Rightarrow \log_2 x + \log_x 2 = \frac{10}{3}$

$\Rightarrow \log_2 x + \frac{1}{\log_2} x = \frac{10}{3}$

Let's take $\log_2 x = a$

So, $a + \frac{1}{a} = \frac{10}{3}$

$\Rightarrow a^2 - \frac{10}{3}a + 1 = 0$

$\Rightarrow \left(a - \frac{9}{3}\right)\left(a - \frac{1}{3}\right) = 0$

$\Rightarrow a = \frac{9}{3}, \frac{1}{3}$

$\Rightarrow \log_2 x = \frac{9}{3}, \frac{1}{3} = 3, \frac{1}{3}$

$\Rightarrow x = 2^3, 2^{\frac{1}{3}}$

$\Rightarrow x = 8, 2^{\frac{1}{3}}$

Similarly, $y = 8, 2^{\frac{1}{3}}$

If $x \neq y$ then $x + y = 8 + 2^{\frac{1}{3}}$

49. (A)

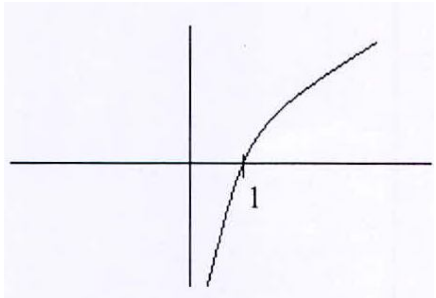
$\Rightarrow \log_{\frac{\sqrt{3}}{2}} \left(x^2 - 3x + 2\right) \geq 2$

$\Rightarrow \log_{\frac{\sqrt{3}}{2}} \left(x^2 - 3x + 2\right) \geq \log_{\frac{\sqrt{3}}{2}} \left(\frac{\sqrt{3}}{2}\right)^2$

$\Rightarrow \log_{\frac{\sqrt{3}}{2}} \left(\frac{x^2 + 3x + 2}{\left(\frac{3}{4}\right)}\right) \geq 0$

$\because$ base is less than 1 so

$\Rightarrow 0 < \left(\frac{x^2 - 3x + 2}{\frac{3}{4}}\right) \leq 1$



$\Rightarrow 0 < x^2 - 3x + 2 \;\&\; \left(x^2 - 3x + 2\right) \le \frac{3}{4}$

$\Rightarrow (x-2)(x-1) > 0 \;\&\; x^2 - 3x + \frac{5}{4} \le 0$

$\Rightarrow x \in (-\infty, 1) \cup (2, \infty)$ …….(1) & $x^2 - \frac{5}{2}x - \frac{1}{2}x + \frac{5}{4} \le 0$

$\& \left(x - \frac{5}{2}\right)\left(x - \frac{1}{2}\right) \le 0$

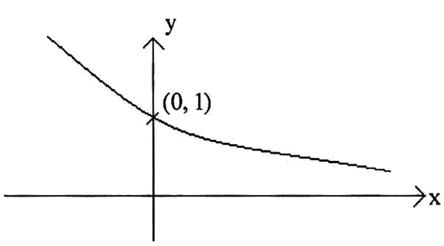
$\& x \in \left(\frac{1}{2}, \frac{5}{2}\right)$ …….(2)

Taking intersection of (i) & (ii)

$\Rightarrow x \in \left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$

50. (A)

$\Rightarrow a^{x^2 - x} \ge a^2; 0 < 9 < 1$



$\Rightarrow \frac{a^{x^2} - x}{a^2} \ge 1$

$\Rightarrow a^{x^2 - x - 2} \ge 1$

$\Rightarrow x^2 - x - 2 \le 0$

$\Rightarrow (x-2)(x+1) \le 0$

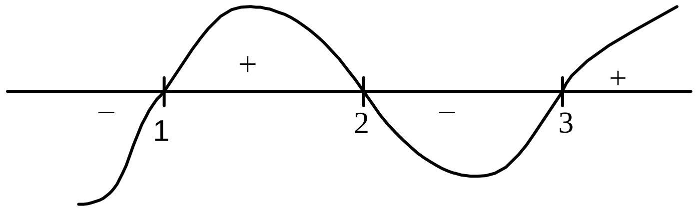
$\Rightarrow x \in [-1, 2]$

51. (C)

$(x-1)\left(x^2 - 5x + 7\right) < (x-1)$

$(x-1)\left[x^2 - 5x + 6\right] < 0$

$(x-1)(x-2)(x-3) < 0$



$x \in (-\infty, 1) \cup (2, 3)$

52. (D)

$||x| - 1| < |1 - x|$

Case I $x \geq 1$

$x - 1 < -(1 - x)$

$x - 1 < -(1 - x)$

$-1 < -1$

No solution

Case II $0 \leq x \leq 1$

$1 - x < 1 - x$

No solution

Case III $-1 \leq x < 0$

$|-x - 1| < 1 - x$

$1 + x < 1 - x$

$x < 0$

$\Rightarrow x \in [-1, 0)$

Case IV: $x < -1$

$|-x - 1| < |1 - x|$

$|1 + x| < 1 - x$

$-(1 + x) < 1 - x$

$-1 - x < 1 - x$

$-1 < 1$

True for all x.

$x \in (-\infty, -1)$

$\therefore x \in (-\infty, -1] \cup [-1, 0)$

$\Rightarrow x \in (-\infty, 0)$

53. (A)

$4^{-x+0.5} - 7.2^{-x} < 4$

$\frac{4^{0.5}}{4^x} - \frac{7}{2^x} < 4$

Let $\frac{1}{2^x} = k$

$\therefore 2.k^2 - 7k < 4$

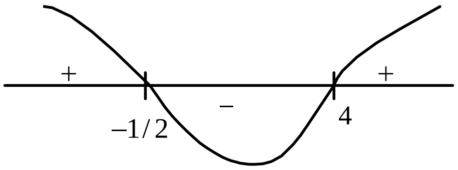
$2k^2 - 7k - 4 < 0$

$2k^2 - 7k - 4 < 0$

$2k^2 - 8k + k - 4 < 0$

$2k(k-4) + (k-4) < 0$

$(2k+1)(k-4) < 0$



$k \in \left(-\frac{1}{2}, 4\right)$

As $k = \frac{1}{2^x}$ it can only be $+ve$

$\therefore 0 < \frac{1}{2^x} < 4$

$\Rightarrow x \in (-2, \infty)$

54. (A)

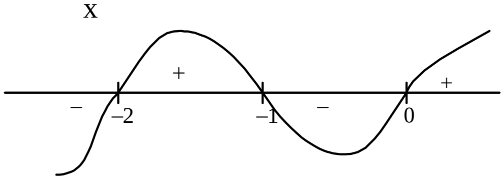
$\left|x + \frac{2}{x}\right| < 3$

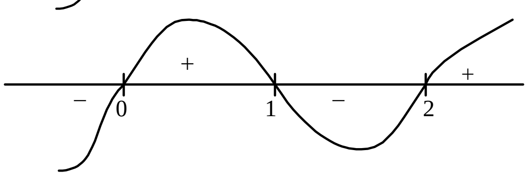
$-3 < x + \frac{2}{x} < 3$

$x + \frac{2}{x} + 3 > 0$, $x + \frac{2}{x} - 3 < 0$

$\frac{x^2 + 3x + 2}{x} > 0$, $\frac{x^2 - 3x + 2}{x} < 0$

$\frac{(x+1)(x+2)}{x} > 0$, $\frac{(x-1)(x-2)}{x < 0}$





$x \in (-2, -1) \cup (0, \infty)$ $x \in (-\infty, 0) \cup (1, 2)$

$\therefore x \in (-2, -1) \cup (1, 2)$

55. (D)

$2^x + 2^{|x|} \geq 2\sqrt{2}$

Case I $x \geq 0$

$2^x + 2^x \geq 2\sqrt{2}$

$2^x \geq \sqrt{2}$

$\Rightarrow x \geq \frac{1}{2}$

Case II $x < 0$

$2^x + \frac{1}{2^x} \geq 2\sqrt{2}$

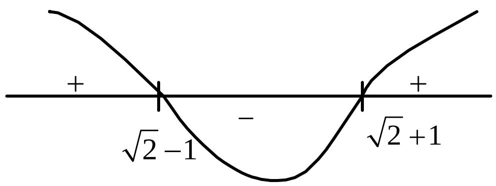
Let $2^x = k$ $\quad k + \frac{1}{k} \geq 2\sqrt{2}$

$k^2 - 2\sqrt{2}k + 1 \geq 1$

$k = \frac{2\sqrt{2} \pm \sqrt{8-4}}{2} = \frac{2\sqrt{2} \pm 2}{2}$

$K = \sqrt{2} \pm 1$

$\left(k - \left(\sqrt{2}+1\right)\right)\left(k - \sqrt{2} - 1\right) \geq 0$



$-\infty < 2^x \leq \sqrt{2} - 1 \,\&\, \sqrt{2} + 1 \leq 2^x < \infty$

$\Rightarrow 0 < 2^x \leq \sqrt{2} - 1 \,\&\, \sqrt{2} + 1 \leq 2^x < \infty$

$x \in \left(-\infty, \log_2\left(\sqrt{2}-1\right)\right] \,\&\, \left[\log_2\left(\sqrt{2}+1\right), \infty\right)$

From case I & II

$x \in \left(-\infty, \log_2\left(\sqrt{2}-1\right)\right] \cup \left[\frac{1}{2}, \infty\right)$

56. (A)

$\log_{1/3}\left(x^2 + x + 1\right) + 1 < 0$

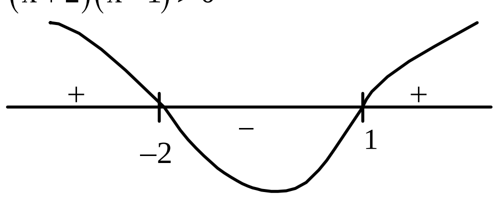
$\log_{1/3}\left(x^2 + x + 1\right) < -1$

$x^2 + x + 1 > \left(\frac{1}{3}\right)^{-1}$

$x^2 + x + 1 > 3$

$x^2 + x - 2 > 0$

$\left(x+2\right)\left(x-1\right) > 0$



$x \in \left(-\infty, -2\right) \cup \left(1, \infty\right)$

57. (B)

$x^2 = |x+2| + x > 0$

Case I $x \geq -2$

$x^2-(x+2)+x>0$

$x^2-2>0$

$x\in(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$

Case II $x\le -2$

$x^2+(x+2)+x>0$

$x^2+2x+2>0$

$(x+1)^2+1>0$

$x\in R$ i.e. $x<-2$

From (1) & (2)

$x\in(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$

58. (C)

$\log_{1/2}x\ge\log_{1/3}x$

$-\log_2 x\ge-\log_3 x$

$\log_2 x\le\log_3 x$

$\frac{\log x}{\log 2}\le\frac{\log x}{\log 3}$

$\log^x\left[\frac{1}{\log 2}-\frac{1}{\log 3}\right]\le 0$

$\log x\le 0\ \left\{\because\frac{1}{\log 2}-\frac{1}{\log 3}>0\right\}$

$\Rightarrow x\in(0,1]$

59. (B)

$x^{\log_3 x^2}+(\log 3x)^2-10=\frac{1}{x^2}$

Clearly one solution is $x=1$

OR

$\log_3 x^2+(\log_3 x)^2-10=-2$

$2\log_3 x+(\log_3 x)^2=8$

Let $\log_3 x=k$

$\therefore 2k+k^2-8=0$

$k^2+2k-8=0$

$(k+4)(k-2)=0$

$\Rightarrow \log_3 x=2$ or $\log_3 x=-4$

$\Rightarrow x=0$ or $x=\frac{1}{81}$

$x\in\left\{1,9,\frac{1}{81}\right\}$

60. (B)

$x=\log_a nc=\frac{\log bc}{\log a}$

$$x+1=\frac{\log bc}{\log a}+1$$

$$\Rightarrow x+1=\frac{\log bc+\log x}{\log a}$$

$$\Rightarrow x+1=\frac{\log abc}{\log a}$$

Similarly

$$y+1=\frac{\log abc}{\log b} \;\&\; z+1=\frac{\log abc}{\log c}$$

$$\therefore \frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}$$

$$\Rightarrow \frac{\log a}{\log abc}+\frac{\log b}{\log abc}+\frac{\log c}{\log abc}$$

$$=\frac{\log abc}{\log abc}=1$$

INEQUATIONS & EQUATION
EXERCISE 1 (B)

1. (D)

$\Rightarrow |3^x - 1| > |3^x - 9|$

Take square both side

$\Rightarrow (3^x - 1)^2 - (3^x - 9)^2 > 0$

$\Rightarrow [(3^x - 1) + (3^x - 9)][3^x - 1 - (3^x - 9)] > 0$

$\Rightarrow [2.3^x - 10][8] > 0$

$\Rightarrow 3^x - 5 > 0 \Rightarrow 3^x > 5$

$x > \log_3 5$

2. (B)

$\Rightarrow \sqrt{25 - 5^x} = \sqrt{4^x - 16}$ ………(1)

Here $25 - 5^x \geq 0$	&	$4^x - 16 \geq 0$
$\Rightarrow 5^x \leq 25$	&	$4^x \geq 4^2$
$\Rightarrow 5^x \leq 5^2$	&	$x \geq 2$
$\Rightarrow x \in (-\infty, 2]$	&	$x \in [2, \infty)$

So only feasible region for given equation is $x = 2$

For $x = 2$, gives equation is satisfied

So no of solutions $= 1$

3. (D)

$\sqrt{4x + 1} + \sqrt{7 - x} = 0$

As square root is always positive so given equation is feasible only if

$\Rightarrow 4x + 1 = 0$	&	$7 - x = 0$
$\Rightarrow x = -\frac{1}{4}$	&	$x = 7$

So no common solution.

4. (C)

$\Rightarrow 4^{x^2+2} - 9.2^{x^2+2} + 8 = 0$

$\Rightarrow 2^{2x^2+4} - 9.2^2 2^{x^2+8} = 0$

$\Rightarrow 16.2^{2x^2} - 36.2^{x^2} + 8 = 0$

$\Rightarrow 4.2^{2x^2} - 9.2x^2 + 2 = 0$

Put $2^{x^2} = a$

so, $4a^2 - 9a + 2 = 0$

$\Rightarrow (4a - 1)(a - 2) = 0$

$\Rightarrow a = \frac{1}{4}, 2$

$\Rightarrow 2^{x^2} = \frac{1}{4}, 2$

$\Rightarrow x^2 = -2, 1$

$\Rightarrow x^2 = -2$ is not possible; $x^2 = 1$

$\Rightarrow x = \pm 1$

5. (A)

$\Rightarrow \sqrt{(x-1)^2} + \sqrt[4]{(2x+1)^4} - \sqrt[3]{\left(x-\frac{1}{2}\right)^3}; x \in (0,1)$

$\Rightarrow |x-1| + (2x+1) - \left(x - \frac{1}{2}\right)$ (as $1+2x>0$ for $x \in (0,1)$)

$\Rightarrow -(x-1) + (2x+1) - \left(x - \frac{1}{2}\right)$

$\Rightarrow \frac{5}{2}$

6. (A)

$\Rightarrow \frac{2^{x-1}}{2^{x+1}+1} < 2$

We can cross multiply $\left(2^{x+1}+1\right)$ as $2^{x+1}+1>0$ for $x \in R$

$\Rightarrow 2^{x-1} - 1 < 2\left(2^{x+1}+1\right)$

$\Rightarrow \frac{2^x}{2} - 1 < 4.2^x + 4$

$\Rightarrow \frac{7}{2}2^x > -5$

$\Rightarrow 2^x > \frac{-10}{7}$

This is true for $x \in R$

7. (C)

$\Rightarrow \sqrt{2^{2x}-7} < 2^x - 1$ ……..(1)

Here $2^{2x} - 7 \geq 0$

$\Rightarrow 2^{2x} \geq 7$

$\Rightarrow 2x \geq \log_2 7$

$\Rightarrow x \geq \frac{1}{2}\log_2 7$ ……..(2) (feasible region)

From feasible region it is clear that $2^x - 1 > 0$

So by taking square of (1)

$\Rightarrow 2^{2x} - 7 < \left(2^x - 1\right)^2$

$\Rightarrow 2^{2x} - 7 < 2^{2x} + 1 - 2.2^x$

$\Rightarrow 2.2^x < 8$

$\Rightarrow 2^x < 4$

$\Rightarrow x < 2$ ………(2)

By taking intersection of (1) & (2)

We get $x \in \left[\frac{1}{2}\log_2 7, 2\right)$

$\Rightarrow x \in [\log_4 7, 2)$

8. (A)

$\Rightarrow |x^3 - 1| \geq 1 - x$

Case I: $x^3 - 1 \geq 0$

$\Rightarrow x \geq 1$

So $(x^3 - 1) \geq 1 - x$

$\Rightarrow x^3 + x - 2 \geq 0$

$\Rightarrow (x-1)(x^2 + x + 2) \geq 0$

$\Rightarrow x \in [1, \infty)$ ……..(1)

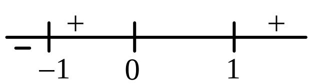
Case 2: $x^3 - 1 < 0$

$\Rightarrow x^3 < 1 \Rightarrow x < 1$

So, $-(x^3 - 1) \geq 1 - x$

$\Rightarrow -x^3 + x \geq 0$

$\Rightarrow x^3 - x \geq 0$



$\Rightarrow x(x-1)(x+1) \leq 0$

$\Rightarrow x \in (-\infty, 1] \cup [0, 1]$ ……….(2)

So take union of (1) & (2)

$\Rightarrow x \in (-\infty, -1] \cup [0, \infty)$

9. (B)

$\Rightarrow \frac{|x+2| - x}{x} < 2$

Case I: $x + 2 \geq 0$

$\Rightarrow x > -2$ ……..(i)

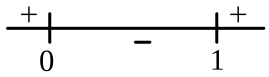
$\Rightarrow \frac{(x+2) - x}{x} < 2$

$\Rightarrow \frac{2}{x} - 2 < 0$

$\Rightarrow \frac{1-x}{x} < 0$

$\Rightarrow \frac{x-1}{x} < 0$

$\Rightarrow x \in (-\infty, 0) \cup (1, \infty)$



Taking intersection of (i) & (ii)

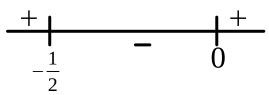
$\Rightarrow x \in [-2, 0) \cup (1, \infty)$ …….(iii)

Case 2: $\Rightarrow x + 2 < 0 \Rightarrow x \in (-\infty, -2)$ ……..(A)

$$\Rightarrow \frac{-(x+2)-x}{x} < 2$$

$$\Rightarrow \frac{-2x-2-2x}{x} < 0$$

$$\Rightarrow \frac{2x-1}{x} > 0$$



$\Rightarrow x \in \left(-\infty, -\frac{1}{2}\right) \cup (0, \infty)$ …….(B)

Taking intersection of A & B

$\Rightarrow x \in (-\infty, -2)$ ………(C)

So by taking union of (iii) & (c)

$\Rightarrow x \in (-\infty, 0) \cup (1, \infty)$

10. (A)

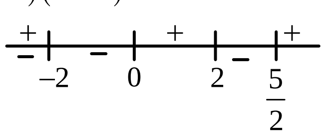
$$\Rightarrow \frac{x^2-5x+4}{x^2-4} \le 1$$

$$\Rightarrow -1 \le \frac{x^2-5x+4}{x^2-4} \le 1$$

Case 1: $\frac{x^2-5x+4}{x^2-4} \ge -1$

$$\Rightarrow \frac{x^2-5x+4+x^2-4}{x^2-4}$$

$$\Rightarrow \frac{x(2x-5)}{(x-2)(x+2)} \ge 0$$



$\Rightarrow x \in (-\infty, -2) \cup [0, 2) \cup \left[\frac{5}{2}, \infty\right)$ ……..(1)

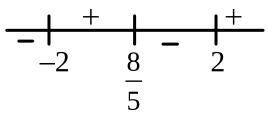
Case 2: $\frac{x^2-5x+4}{x^2-4} \le 1$

$$\Rightarrow \frac{x^2-5x+4-x^{2+4}}{x^2-4} \le 0$$

$$\Rightarrow \frac{8-5x}{(x-2)(x+2)} \le 0$$

$$\Rightarrow \frac{(5x-8)}{(x-2)(x+2)} \ge 0$$

$$\Rightarrow x \in \left(-2, \frac{8}{5}\right] \cup (2, \infty)$$



By taking intersection of (1) & (2)

$$\Rightarrow x \in \left[0, \frac{8}{5}\right] \cup \left[\frac{5}{2}, \infty\right)$$

11. (C)

$$\Rightarrow \frac{x^2-|x|-12}{x-2} \ge 2x$$

Case 1: $x \ge 0$

$$\Rightarrow \frac{x^2-x-12}{x-3} \ge 2x$$

$$\Rightarrow \frac{x^2-x-12}{x-3} - 2x \ge 0$$

$$\Rightarrow \frac{x^2-x-12}{x-3} - 2x \ge 0$$

$$\Rightarrow \frac{x^2-x-12-2x^2+6x}{x-3} \ge 0$$

$$\Rightarrow \frac{-x^2+5x-12}{x-3} \ge 0$$

$$\Rightarrow \frac{x^2-5x+12}{x-3} \le 0$$

as $x^2-5x+12$ is always greater than zero for $x \in R$

so, $\frac{1}{x-3} \le .0$

$$\Rightarrow x \in (-\infty, 3)$$

Case 2: $x < 0$

$$\Rightarrow \frac{x^2+x-12}{x-3} - 2x \ge 0$$

$$\Rightarrow \frac{x^2+x-12-2x^2+6x}{x-3} \ge 0$$

$$\Rightarrow \frac{-x^2+7x-12}{x-3} \ge 0$$

$$\Rightarrow \frac{x^2-7x+12}{x-3} \le 0$$

$$\Rightarrow \frac{(x-4)(x-3)}{(x-3)} \le 0$$

$$\Rightarrow x-4 \le 0$$

$$\Rightarrow x \in (-\infty, 4]$$

But for this case $x < 0$

So we get

$$\Rightarrow x \in (-\infty, 0)$$

Take union of $(1) \& (2)$

$$\Rightarrow x \in (-\infty, 3)$$

12. (C)

$$\Rightarrow \left|\frac{x^2-3x-1}{x^2+x+1}\right| < 3$$

$$\Rightarrow -3\frac{x^3-3x-1}{x^2+x+1} < 3$$

Case 1: $\frac{x^2-3x-1}{x^2+x+1} > -3$

As (x^2+x+1) is always greater than zero

So $x^2-3x-1 > -3(x^2+x+1)$

$$\Rightarrow 4x^2+2 > 0$$

$\Rightarrow x \in R$ …….(1)

Case 2: $\frac{x^2-3x+1}{x^2+x+1} < 3$

$$\Rightarrow x^2-3x-1 < 3x^2+3x+3$$

$$\Rightarrow 2x^2+6x+4 > 0$$

$$\Rightarrow x^2+3x+2 => 0$$

$$\Rightarrow (x+1)(x+2) > 0$$

$\Rightarrow x \in (-\infty, -2) \cup (-1, \infty)$ …….(2)

Take intersection of $(1) \& (2)$

$$\Rightarrow x \in (-\infty, -2) \cup (-1, \infty)$$

13. (D)

$$\Rightarrow \frac{|x+3|+x}{x+2} > 1$$

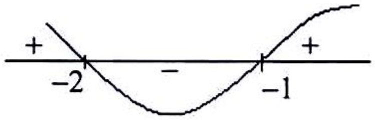
$$\Rightarrow \frac{|x+3|+x}{x+2} - 1 > 0$$

$$\Rightarrow \frac{|x+3|+x-x-2}{x+1>0} > 0$$

$$\Rightarrow \frac{|x+3|-2}{x+2} > 0$$

Case 1: $x+3 \ge x \ge -3$

$$\Rightarrow \frac{x+3-2}{x+2} > 0$$



$$\Rightarrow \frac{x+1}{x+2} > 0$$

$$\Rightarrow x \in (-\infty,-2) \cup (-1,-\infty)$$

But for this case $x \geq -3$

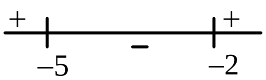
So we get $x \in [-3,-2) \cup (-1,\infty)$

Case 2: $x+3 < 0$

$$\Rightarrow x < -3$$

So, $\frac{-(x+3)}{x+2} > 0$

$$\Rightarrow \frac{x+5}{x+2} < 0$$



$$\Rightarrow x \in (-5,-2)$$

As $x < -3$ so $x \in (-5,-3)$ ……(2)

Take union of $(1) \& (2)$

$$\Rightarrow x \in (-5,-2) \cup (-1,\infty)$$

So least integral value of $x = -4$

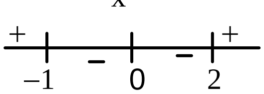
14. (A)

$$\Rightarrow \sqrt{1-\left(\frac{x+2}{x^2}\right)} < \frac{2}{3}$$

So, $1-\left(\frac{x+2}{x^2}\right) \geq 0$

$$\Rightarrow \frac{x^2-x-2}{x^2} \geq 0$$

$$\Rightarrow \frac{(x-2)(x+1)}{x^2} \geq 0$$



So $x \in (-\infty,-1] \cup [2,\infty)$ ……(1)

$$\Rightarrow \sqrt{1-\left(\frac{x+2}{x^2}\right)} < \frac{2}{3}$$

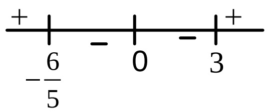
$$\Rightarrow 1-\left(\frac{x+2}{x^2}\right) < \frac{4}{9}$$

$\Rightarrow \frac{5}{9} - \frac{x+2}{x^2} < 0$

$\Rightarrow \frac{5x^2 - 9x - 18}{9x^2} < 0$

$\Rightarrow \frac{(5x+6)(x-3)}{x^2} < 0$

So, $x \in \left(-\frac{6}{5}, 0\right) \cup (0,3)$ ....(2)



By taking $(1) \cap (2)$

$\Rightarrow x \in \left(-\frac{6}{5}, -1\right] \cup [2,3)$

15. $\Rightarrow \sqrt{4 - \sqrt{1-x}} - \sqrt{2-x} > 0$

Here $1 - x \geq 0$

$\Rightarrow 2 - x > 0$

$\Rightarrow x \leq 1$ ....(i)

$\Rightarrow 4 - \sqrt{1-x} \geq 0$

$\Rightarrow 4 \geq \sqrt{1-x}$

$\Rightarrow 16 \geq 1 - x$

$\Rightarrow x > -15$ ....(ii)

So, $(i) \cap (ii)$

$\Rightarrow x \in [-15, 1]$ .... (iii) feasible region

Take intersection of $(iv) \& (iii)$

$\Rightarrow x \in [-2, 1]$

Now

$\Rightarrow \sqrt{4 - \sqrt{1-x}} > \sqrt{2-x}$

Take square both side

$\Rightarrow 4 - \sqrt{1-x} > 2 - x$

$\Rightarrow 2 + x > \sqrt{1-x}$

Here $2 + x \geq 0 \Rightarrow x \geq -2$ ....(iv)

So, $2 + x > \sqrt{1-x}$

Take square $4 + x^2 + 4x > 1 - x$

$\Rightarrow x^2 + 5x + 3 > 0$

$\Rightarrow \left(x - \left(\frac{-5+\sqrt{13}}{2}\right)\right)\left(x - \left(\frac{-5-\sqrt{13}}{2}\right)\right)$

$\Rightarrow x \in \left(-\infty, \frac{-5-\sqrt{13}}{2}\right) \cup \left(\frac{\sqrt{13}-5}{2}, \infty\right)$ ....(2)

$(1)\cap(2)$

$\Rightarrow x \in \left(\frac{\sqrt{13}-5}{2}, 1\right)$

16. (A)

$\Rightarrow \sqrt{4-x^2} + \frac{|x|}{x} \geq 0$

Here $4-x^2 \geq 0$

$\Rightarrow x^2 \leq 4$

$\Rightarrow x \in [-2, 2]$ ……..(i)

Case 1: $x > 0$

$\Rightarrow \sqrt{4-x^2} + \frac{x}{x} \geq 0$ $(x \neq 0)$

$\Rightarrow \sqrt{4-x^2} + 1 \geq 0$

$\Rightarrow \sqrt{4-x^2} \geq -1$ true for $x \in R$

Case 2: $x < 0$

$\Rightarrow \sqrt{4-x^2} + \frac{(-x)}{x} \geq 0$

$\Rightarrow \sqrt{4-x^2} - 1 \geq 0$

$\Rightarrow \sqrt{4-x^2} \geq 1$

Take square $4-x^2 \geq 1 \Rightarrow x^2 \leq 3$

$\Rightarrow x \in \left[-\sqrt{3}, \sqrt{3}\right]$

But $x < 0$

So $x \in \left[-\sqrt{3}, 0\right)$

So case (1) $\cup$ case (2)

$\Rightarrow x \in R$ ……..(iii)

But our feasible region is $x \in [-2, 2]$

So greatest integral $x = 2$

17. (C)

$\Rightarrow \frac{|x+2|-|x|}{\sqrt{4-x^2}} \geq 0$ here $4-x^3 > 0$

$\Rightarrow x^3 < 4 \Rightarrow x \in \left(-\infty, \sqrt[3]{4}\right)$ ………(1)

As $\left(4-x^3\right)$ is greater than zero

$\Rightarrow |x+2|-|x| \geq 0$

$\Rightarrow |x+2| \geq |x|$

$\Rightarrow (x+2)^2 \geq x^2$

$\Rightarrow (x+2-x)(x+2+x) \geq 0$

$\Rightarrow 2(2x+2) \geq 0$

$\Rightarrow x \geq -1$ ………(2)

So $(1) \cap (2)$

$\Rightarrow x \in \left[-1, \sqrt[3]{4}\right)$

18. (B)

$\Rightarrow \log_4\left(\frac{x+1}{x+2}\right) > 0$

$\Rightarrow \frac{x+1}{x+2} > 0$

$\Rightarrow \frac{x+1-x-2}{x+1} > 0$

$\Rightarrow \frac{-1}{x+2} > 0$

$\Rightarrow \frac{1}{x+2} < 0$

$\Rightarrow x \in (-\infty, -2)$

19. (A)

$\Rightarrow x^3 + ax + b = 0$

$\Rightarrow \alpha + \beta + \gamma = 0$

$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = a$

$\Rightarrow \alpha\beta\gamma = -b$

$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 = (\alpha + \beta)\left(\alpha^2 + \beta^2 - \alpha\beta\right) + \gamma^3$

$= (-\gamma)\left((\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta\right) + \gamma^3$

$= (-\gamma)\left(\gamma^2 - 3\alpha\beta\right) + \gamma^3$

$= +3\alpha\beta\gamma = -3b$

$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$\Rightarrow \frac{\alpha^3 + \beta^3 + \gamma^3}{\alpha^2 + \beta^2 + \gamma^2} = \frac{-3b}{-2a} = \frac{3b}{2a}$

20. (A)

$\Rightarrow f(x) = x^3 + 2x^2 + k$

If $(x-1)$ is divisor of $f(x)$

Then $f(1) = 0$

$\Rightarrow f(1) = 1 + 2 + k = 0$

$\Rightarrow k = -3$

21. (D)

$\Rightarrow 4x^3 + 20x^2 - 23x + 6 = 0$

Let's take roots are α, α, β

$\Rightarrow 2\alpha + \beta = \frac{-20}{4} = -5$ ……..(1)

$\Rightarrow \alpha^2 + \alpha\beta + \alpha\beta = \frac{-23}{4}$

$\Rightarrow \alpha^2\beta = \frac{-6}{4}$

$\Rightarrow \alpha^2(-5-2\alpha) = -\frac{6}{4}$

$\Rightarrow 5\alpha^2 + 2\alpha^3 - \frac{6}{4} = 0$

$\Rightarrow \alpha^3 + 20\alpha^3 - 6 = 0$

Which gives $\alpha = \frac{1}{2}$

So $2\left(\frac{1}{2}\right) + \beta = -5 \Rightarrow \beta = -6$

So roots are $-6, \frac{1}{2}, \frac{1}{2}$

22. (D)

$\Rightarrow 2x^3 - 5x^2 + 3x - 1 = 0$

$\Rightarrow \alpha\beta\gamma = \frac{1}{2}$

$\Rightarrow \alpha + \beta + \gamma = \frac{5}{2}$

$\Rightarrow \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\beta\alpha} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{\left(\frac{5}{2}\right)}{\left(\frac{1}{2}\right)} = 5$

23. (A)

$\Rightarrow x^3 - 4x^2 + x + 6 = 0$

Roots are $2\alpha, 3\alpha, \beta$

So, $2\alpha, 3\alpha, \beta = 4$

$\Rightarrow 5\alpha + \beta = 4$ ……….(1)

$\Rightarrow (2\alpha)(3\alpha)(\beta) = -6$

$\Rightarrow \alpha^2\beta = -1$ ………..(2)

By (1) & (2)

$\Rightarrow \alpha^2(4-5\alpha) = -1$

$\Rightarrow 5\alpha^3 - 4\alpha^2 - 1 = 0$

By this we get $\alpha = 1$

So roots are $2, 3, -1$

24. (B)

$\Rightarrow 4x^3 - 12x^2 + 11x + k = 0$

Sum of roots $\left(a-d+a+a+d\right)=\frac{12}{4}=3$

$\Rightarrow a=1$

As a is root of equation so it will satisfy if.

$\Rightarrow 4\left(a\right)^3-12a^2+11a+k=0$

$\Rightarrow 4-12+11+k=0$

$\Rightarrow k-3$

25. (B)

$\Rightarrow x^3-11x^2+37x-35=0$

If root is $3+\sqrt{2}$, second root has to be $3-\sqrt{2}$

Let's take third root α

So, $\left(3+\sqrt{2}\right)+\left(3-\sqrt{2}\right)+\alpha=11$

$\Rightarrow \alpha=5$

26. (B)

As $x+1$ is factor so $x=-1$ will satisfy the expression

$\Rightarrow \left(-1\right)^4+\left(p-3\right)\left(-1\right)^3-\left(3p-5\right)\left(-1\right)^2+\left(2p+q\right)\left(-1\right)+12=0$

$\Rightarrow p=2$

27. (D)

$\Rightarrow x^3-ix+1-i=0$

Roots will be $1+i, 1-i, \alpha$

$\Rightarrow 1+i+1-i+\alpha=0$

$\Rightarrow \alpha=-2$

So equation will roots $\left(1-i\right)\&-2$ is

$\Rightarrow x^2-\left(1-i-2\right)x+\left(1-i\right)\left(-2\right)=0$

$\Rightarrow x^2+\left(1+i\right)x-2\left(1-i\right)=0$

28. (D)

$\Rightarrow 2x^4+9x^3+8x^2+9x+2=0$

29. (A)

$\Rightarrow x^3-1=0$

Roots will be α, β, γ

$\Rightarrow \alpha+\beta+\gamma=0$

$\Rightarrow \sum \alpha\beta=0$

$\Rightarrow \alpha\beta\gamma=1$

$\Rightarrow \sum\left(\alpha+1\right)=\left(\alpha+\beta+\gamma\right)+3=3$

$=\sum\left(\alpha+1\right)\left(\beta+1\right)=\left(\alpha+1\right)\left(\beta+1\right)+\left(\alpha+1\right)\left(\gamma+1\right)\left(\beta+1\right)\left(\gamma+1\right)$

$=\sum \alpha\beta+\left(\alpha+\beta+1\right)+\left(\alpha+\gamma+1\right)+\left(\beta+\gamma+1\right)$

$= 3$

$\Rightarrow (\alpha+1)(\beta+1)(\gamma+1) = (\alpha\beta+\alpha+\beta+1)(\gamma+1)$

$= \alpha\beta\gamma+\alpha\beta+\alpha\gamma+\alpha+\beta\gamma+\beta+\gamma+1 = 2$

So equation with $(\alpha+1),(\beta+1),(\gamma+1)$ as roots

$= x^3-3x^2+3x-2=0$

30. (B)

$\Rightarrow x^3+3x+2=0$

$\Rightarrow a+b+c=0$

$\Rightarrow ab+bc+ca=3$

$\Rightarrow abc=-2$

$\Rightarrow a^3+b^3+c^3-3abx=(a+b+c)(a^2+b^2+c^2+ab+bc+ca)=0$

$\Rightarrow a^3+b^3+c^3=3(-2)=-6$

31. (A)

$\Rightarrow x^3+1=0$

$\Rightarrow \alpha+\beta+\gamma=0$

$\Rightarrow \alpha\beta+\beta\gamma+\gamma\alpha=0$

$\Rightarrow \alpha\beta\gamma=1$

$\Rightarrow (\alpha+\beta+\gamma)^2=\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$\Rightarrow \alpha^2+\beta^2+\gamma^2=0$

$\Rightarrow (\alpha^2+\beta^2+\gamma^2)^2=\alpha^4+\beta^4+\gamma^4+2(\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2)$

$\Rightarrow 0=\alpha^4+\beta^4+\gamma^4+2\left[(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2(\alpha\beta.\beta\gamma+\beta\gamma.\gamma\alpha+\gamma\alpha.\alpha\beta)\right]$

$=\alpha^4+\beta^4+\gamma^4+2\left[0-2(\beta+\gamma+\alpha)\right]$ $\quad (\because \alpha\beta\gamma=1)$

$\Rightarrow 0=\alpha^4+\beta^4+\gamma^4+2\left[0-2(0)\right]$

$\Rightarrow \alpha^4+\beta^4+\gamma^4=0$

32. (A)

$\Rightarrow x^3+3x^2+2x+4=0$

$\Rightarrow \alpha\beta\gamma=-3$

$\Rightarrow \sum\alpha\beta=2$

$\Rightarrow \alpha\beta\gamma=-4$

$\Rightarrow x^3-(2\alpha+2\beta+2\gamma)x^2+\left(\sum\alpha 2\beta\right)x-(2\alpha.2\beta.2\gamma)=0$

$\Rightarrow x^3+6x^2+8x+32=0$

33. (A)

As α,β,γ satisfy cubic equation $x^3-5x^2+5x-3=0$

So, α,β,γ are roots of this equation

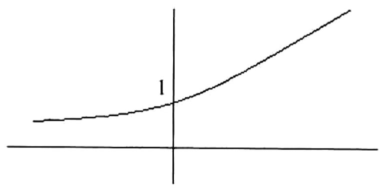
$\Rightarrow \alpha+\beta+\gamma=5$

34. (C)

$\Rightarrow 5^{x+2} > \left(\frac{1}{25}\right)^{\frac{1}{x}}$

$\Rightarrow 5^{x+2} > \frac{1}{5^{\frac{2}{x}}}$

$\Rightarrow 5^{x+2}.5^{\frac{2}{x}} > 1$



$\Rightarrow 5^{x+\frac{2}{x}+2} > 5^{\circ}$

So $x+\frac{2}{x}+2>0$

$\Rightarrow \frac{x^2+2x+2}{x} > 0$

Numerator is always >0

So $\frac{1}{x}>0$

$\Rightarrow x \in (0,\infty)$

35. (B)

$\Rightarrow \frac{x^2-5x+6}{|x|+7} < 0$

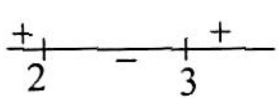
$\Rightarrow \frac{(x-3)(x-2)}{|x|+7} < 0$

As $7+|x| \geq 7$

So we can cross multiply $7+|x|$

$\Rightarrow (x-3)(x-2) < 0$

$\Rightarrow x \in (2,3)$



36. (D)

$\Rightarrow \left|\frac{2x-1}{x-1}\right| > 2$

$\Rightarrow \frac{2x-1}{x-1} < -2$ or $\frac{2x-1}{x-1} > 2$

$\Rightarrow \frac{2x-1+2x-2}{x-1} < 0$ or $\frac{2x-1-2x+2}{x-1} > 0$

$\Rightarrow \frac{4x-3}{x-1}$ or $\frac{1}{x-1} > 0$

$\Rightarrow x \in \left(\frac{3}{4}, 1\right)$ 0r $x > 1$

So, $x \in \left(\frac{3}{4}, \infty\right) - \{1\}$

37. (A)

$\Rightarrow \frac{1}{|x|-3} < \frac{1}{2}$

Case 1: $x \geq 0$

$\Rightarrow \frac{1}{x-3} - \frac{1}{2} < 0$

$\Rightarrow \frac{5-x}{2(x-3)} < 0$

$\Rightarrow \frac{x-5}{(x-3)} > 0$

$\Rightarrow x \in (-\infty, 3) \cup (5, \infty)$

But $x \geq 0$

So, $x \in [0.3) \cup (5, \infty)$ ……..(1)

Case 2: $x < 0$

$\Rightarrow \frac{1}{-x-3} - \frac{1}{2} < 0$

$\Rightarrow \frac{1}{x+3} + \frac{1}{2} > 0$

$\Rightarrow \frac{5+x}{x+3} > 0$

$\Rightarrow x \in (-\infty, -5) \cup (-3, \infty)$

But $x < 0$

So, $x \in (-\infty, -5) \cap (-3, 0)$ ……..(2)

So (1) $\cup$ (2)

$\Rightarrow x \in (-\infty, -5) \cup (-3, \infty)$

So least positive integer value $= 1$

38. (C)

$\Rightarrow (x-1)\sqrt{x^2-x-2} \geq 0$

$\Rightarrow (x^2-x-2) \geq 0$ & $(x-1) \geq 0$

$\Rightarrow (x-2)(x+1) \geq 0$ & $x \geq 1$

$\Rightarrow x \in (-\infty, -1] \cup [2, \infty)$ & $x \in [1, \infty)$

So, $x \in [2, \infty)$

39. (B)

$\Rightarrow (x^2 - 1)\sqrt{x^2 - x - 2} \ge 0$

$\Rightarrow (x^2 - 1) \ge 0$ & $x^2 - x - 2 \ge 0\,xh2$

$\Rightarrow x \in (-\infty, -1] \cup [1, \infty)$ ……(1) & $(x-2)(x+1) \ge 0$

$x \in (-\infty, -1] \cup [2, \infty)$ ……(2)

Take $(1) \cap (2)$

So, $x \in (-\infty, -1] \cup [2, \infty)$

Least positive integer $= 2$

40. (C)

$\Rightarrow 49^x + 7^{x+1} - 98 < 0$

$\Rightarrow 7^{2x} + 7.7^x - 98 < 0$

$\Rightarrow 7^x = a; a^2 + 7a - 78 < 0$

$\Rightarrow (a+14)(a-7) < 0$ $(\because a + 14 = 7^x + 14 > 0 \text{ for } x \in R)$

$\Rightarrow 7^x < 7$

So, $x < 1$

41. (B)

$\alpha, \beta, \gamma \to x^3 + 2x^2 - 3x - 1 = 0$

The equation whose roots are $\frac{1}{\alpha^2}, \frac{1}{\beta^2}, \frac{1}{\gamma^2}$

$x = \frac{1}{\alpha^2}$

$\Rightarrow \alpha = \frac{1}{\sqrt{x}}$

$\left(\frac{1}{x}\right)^{\frac{3}{2}} + \frac{2}{x} - \frac{2}{x^{\frac{1}{2}}} - 1 = 0$

$\Rightarrow 1 + 2\sqrt{x} - 3x - x^{\frac{3}{2}} = 0$

$\Rightarrow 1 - 3x = x^{\frac{3}{2}} - 2x^{\frac{1}{2}}$

$\Rightarrow 1 + 9x^2 - 6x = x^3 + 4x - 4x^2$ (squaring)

$\Rightarrow x^3 - 13x^2 + 10x - 1 = 0$

Hence, $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4} = S_1^2 - 2S_2$

$= 169 - 20 = 149$

42. (C)

Let roots one diminished by k

So, $\alpha - k + \beta - k + \gamma - k = 0$

$\Rightarrow (\alpha + \beta + \gamma) - 3k = 0$

$\Rightarrow k = \frac{9/2}{3} = \frac{3}{2}$

43. (A)

$\alpha + \beta, \beta + \gamma, \gamma + \alpha$

$\alpha + \beta + \gamma = p$

Hence roots of the equations are $p - \alpha, p - \beta, p - \gamma$

$x = p - \alpha \Rightarrow \alpha = p - x$

Equation is

$(p - x)^3 - p(p - x)^2 + q(p - x) - r = 0$

$\Rightarrow x^3 - 3px^2 + 3p^2x - p^3 + p^3 - 2xp^2 + px^2 - qp + qx + r = 0$

$\Rightarrow x^3 - 2px^2 + 2(p^2 + q)x + r - pq = 0$

44. (B)

$x_1 = \beta\gamma + \frac{1}{\alpha} \quad = \frac{\alpha\beta\gamma + 1}{\alpha} = \frac{3+1}{\alpha} = \frac{4}{\alpha}$

Hence, $x_1 = \frac{4}{\alpha} \Rightarrow \alpha = \frac{4}{x_1}$

Equation

$\frac{64}{x_1^3} - \frac{2.16}{\alpha_1^2} + \frac{5.4}{\alpha_1} - 3 = 0$

$\Rightarrow 3x_1^3 - 20x_1^2 + 32x_1 - 64 = 0$

45. (B)

$x = \sqrt{3} - \sqrt{2}$

$x + \sqrt{2} = \sqrt{3}$

$\Rightarrow x^2 + 2\sqrt{2}x + 2 = 3$ $\quad$ $\left(\text{roots are} - \sqrt{2} \pm \sqrt{3}\right)$

$\Rightarrow x^2 - 1 = -2\sqrt{2}\,x$

$\Rightarrow x^2 - 2x^2 + 1 = 8x^2$

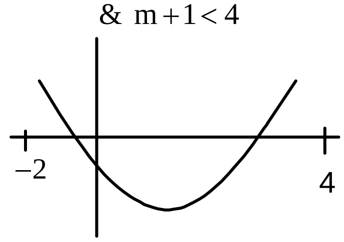
$\Rightarrow x^4 - 10x^2 + 1 = 0$ $\quad$ (roots are $\pm\sqrt{2} \pm \sqrt{3}$)

46. (B)

$(x - m)^2 = 1$

$= m - 1, m + 1$

$m - 1 > -2$ $\quad$ & $m + 1 < 4$



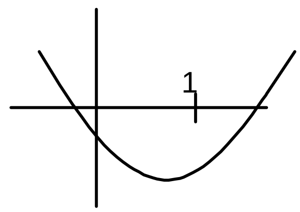
$m > -1$ $\qquad\qquad$ $m < 3$

$(-1, 3)$

47. (A)

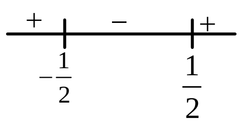
1 lies in between the roots

So, $a.f(1) < 0$



$\Rightarrow (2k+1).[2k+1-k+k-2] < 0$

$(2k+1)(2k-1) < 0$



$k \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

48. (C)

$(r_1+2)(r_2+2)(r_3+2)$

Equation whose roots are $(r_1+2), (r_2+2) \& (r_3+2)$ is

$(x-2)^3 - 2(x-2)^2 + 4(x-2) + 5074 = 0$

$\Rightarrow x^3 - 6x^2 + 12x - 8 - 2x^2 + 8x - 8 + 4x - 8 + 5074 = 0$

$\Rightarrow 1x^3 - 8x^2 + 24x + 5050 = 0$

Product $= -5050$

49. (A)

$P(x) = (x-2)(4x-1)(x) + Ax + B$

$2A + B = 1$

$\frac{A}{4} + B = 2$

$\frac{7A}{4} = -1 \qquad \Rightarrow A = -\frac{4}{7}, B = 1 + \frac{8}{7} = \frac{15}{7}$

Hence, $-\frac{4x+15}{7}$

50. (D)

$$\frac{1}{\log_3(2^{2x}-1)} > \frac{1}{\log_3(2^x+1)}$$

Domain $x > 0, x \neq \frac{1}{2}$

$\log_3 2^x + 1$ is always positive

Hence, $\log_3\left(2^{2x}-1\right) > 0$

$$x > \frac{1}{2}$$

And $\log_3\left(2^{2x}-1\right) < \log_3\left(2^x+1\right)$

$\Rightarrow 2^{2x} - 1 < 2^x + 1$

$\Rightarrow t^2 - t - 2 < 0$ $\qquad\qquad\qquad\qquad (t = 2x)$

$\Rightarrow t \in (-1, 2)$

$\Rightarrow x \in (-\infty, 1)$

Hence solution is $\left(\frac{1}{2}, 1\right)$

51. (B)

$A = 12^{300}$

$\log_{10} A = 300\left[\log_{10} 12\right]$

$= 300\left[0.6010 + 0.4771\right]$

$= 300 \times 1.0781 = 323.43$

$\Rightarrow A = 10^{323.43}$

Hence 324 digits

52. (A)

$\log_{10} 2 = 0.3010$

$\log_5 64 = \dfrac{\log_{10} 64}{\log_{10} 5} = \dfrac{6\log_{10} 2}{\log_{10} 10 - \log_{10} 2}$

$= \dfrac{6 \times 0.3010}{1 - 0.3010}$

$= \dfrac{1.8060}{0.6990}$

$= \dfrac{602}{233}$

53. (B)

$2^{\log_{10} 3\sqrt{3}} = 3^{k \log_{10} 2}$

$\Rightarrow 2^{\log_{10} 3\sqrt{3}} = 2^k \log_{10} 3$

$\Rightarrow \log_{10} 3\sqrt{3} = \log_{10} 3^k$

$\Rightarrow k = 3^{3/2}$

$\Rightarrow k = 3/2$

54. (A)

$x = \log_5(1000) = \log_5 125 + \log_5 8 > 4$

$y = \log_{57}(2056) = \log_7 343 + \log_7 6 < 4$

Hence $x > 4$

55. (B)

For compare $\frac{b}{a}$. Sum of the roots ≥ 3

Hence $\frac{-b}{a} \geq 3$ and $-\frac{b}{2a} \geq 2$

$\Rightarrow \left(\frac{b}{a}\right) \geq 3$ $\Rightarrow |b| \geq 4|a|$

$\Rightarrow |b| \geq 3|a|$ Hence $|b| \geq 4a$

56. (A)

$x = \frac{1+\alpha}{1-\alpha}$

$\Rightarrow \alpha = \frac{x-1}{x+1}$

$\left(\frac{x-1}{x+1}\right)^3 - \left(\frac{x-1}{x+1}\right) - 1 = 0$

$\Rightarrow (x-1)^3 - (x-1)(x+1)^2 - (x+1)^3 = 0$

$\Rightarrow -6x^2 - 2 - x^3 - x^2 + x + 1 = 0$

$\Rightarrow x^3 + 7x^2 - x + 1 = 0$

$\Rightarrow x^3 + 7x^2 - x + 1 = 0$

57. (D)

$x = \alpha(\beta + r) = \alpha\beta + \alpha v$

$= 3 - \beta r$

$= 3 - \frac{\beta r \alpha}{\alpha}$

$= 3 - \frac{(-2)}{\alpha} = 3 + \frac{2}{\alpha}$

$\Rightarrow x = 3 + \frac{2}{\alpha}$

$\Rightarrow \alpha = \frac{2}{x-3}$

58. (A)

$ax + by = 1$

$px^2 + qy^2 = 1$

$\Rightarrow px^2 + q\left(\frac{ax-1}{b}\right)^2 = 1$

$\Rightarrow b^2px^2 + qa^2x^2 - 2aqx + q - b^2 = 0$

Has one root

$\Rightarrow 4a^2q^2 - 4\left(b^2q + a^2q\right)\left(q - b^2\right) = 0$

$\Rightarrow 4\left(a^2/q^2 - b^2pq - a^2/q^2 + b^4p + b^2a^2q\right) = 0$

$\Rightarrow b^2\left(-pq + b^2p + a^2q\right) = 0$

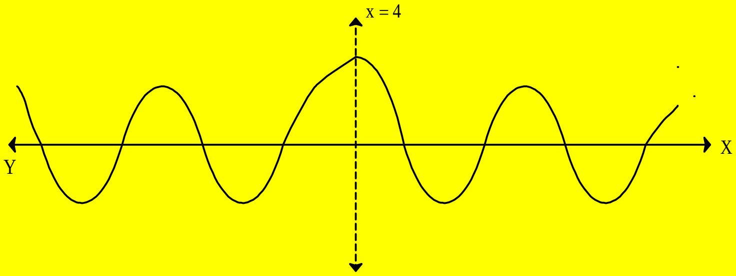
Hence $\frac{b^2}{q} + \frac{a^2}{p} = 1$

59. (C)

Express is symmetrical about $x = 4$

So, roots will be $4 \pm \alpha, 4 \pm \beta, 4 \pm v,$

Hence sum $= 8 \times 4 = 32$



60. (D)

$x^2 + 2xy + 3y^2 - 6x + 6y \geq k$ (where k is min unvalue)

$\Rightarrow x^2 + (2y - 6)x + 3y^2 + 6y - k \geq 0$

Should have all real x

Hence $4(y-3)^2 - 4\left(3y^2 + 6y - k\right) \leq 0$

$\Rightarrow 4\left[y^2 + 9 - 6y - 3y^2 - 6y + k\right] \geq 0$

$\Rightarrow \left[2y^2 + 12y - (9+k)\right] \leq (\forall y \in R)$

Hence, $144 + 8(9+k) \leq 0$

$\Rightarrow k + 9 \leq -18$

$\Rightarrow k \leq -27$

Hence $k = -27$

EXERCISE – 2 (A)

1. (AC)

$\Rightarrow 18^{4x-3} = \left(54\sqrt{2}\right)^{3x-4}$

$\Rightarrow 18^{4x-3} = (18)^{3x-4}\left(3\sqrt{2}\right)^{3x-4}$

$\Rightarrow 18^{x+1} = \left(3\sqrt{2}\right)^{3x-4}$

$\Rightarrow \left[\left(3\sqrt{2}\right)^{2}\right]^{x+1} = \left(3\sqrt{2}\right)^{3x-4}$

$\Rightarrow \left(3\sqrt{2}\right)^{2x+2} \times \left(3\sqrt{2}\right)^{-3x+4} = 1$

$\Rightarrow \left(3\sqrt{2}\right)^{-x+6} = 1$

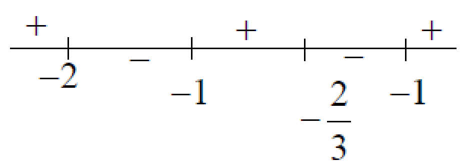
So $-x = 6 \Rightarrow x = 6$

2. (BC)

$\Rightarrow \dfrac{2x}{2x^2+5x+2} > \dfrac{1}{x+1} \Rightarrow \dfrac{2x}{2x^2+5x+2} - \dfrac{1}{x+1} > 0$

$\Rightarrow \dfrac{2x^2+2x-2x^2-5x+2}{\left(2x^2+5x+2\right)(x+1)} > 0$

$\Rightarrow \dfrac{3x+2}{(2x+1)(x+2)(x+1)} < 0$



So, $x \in (-2,-1) \cup \left(-\dfrac{2}{3},-1\right)$

3. (ABC)

$\Rightarrow$ 2576a456b

$\Rightarrow$ b has to 0 or 5

Sum of digits has to be multiple of 3, to divide it by 3.

$\Rightarrow$ 2 + 5 + 7 + 6 + a + 4 + 5 + 6 + b = 35 + a + b

If a take value of 4, then b will also take 5.

If a take value of 4, then b has to be zero.

If take 6, then 35 + a + b can't become multiple of 3, for b = 0.5

4. (ABD)

$\Rightarrow x^2 - 6x - 5|x-3| - 5 = 0$

Case – I : $x \geq 3$

$\Rightarrow^2 -6x - 5|x-3| - 5 = 0$

$\Rightarrow x^2 - 11x + 10 = 0$

$\Rightarrow$ x = 1, 10

$\Rightarrow$ x = 1 not a solution for $x \geq 3$

$\Rightarrow$ x = 10

Case – 2: x < 3

$\Rightarrow x^2 - 6x + 5x - 15 - 5 = 0$

$\Rightarrow x^2 - x - 20 = 0$

$\Rightarrow$ (x – 5) (x + 4) = 0

For x < 3, x = 1 – 4

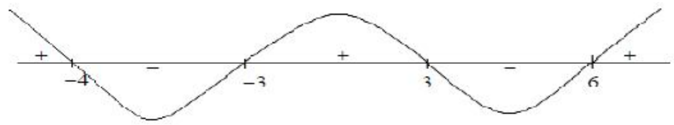
So, $l = 1$, m = 1

5. (BCD)

$\Rightarrow \frac{2x+3}{x^2+x-12} \le \frac{1}{2}$

$\Rightarrow \frac{2(2x+3)-(x^2+x-12)}{2(x^2+x-12)} \le 0$

$\Rightarrow \frac{-x^2+3x+18}{(x+4)(x-3)} \le 0$



$\Rightarrow \frac{(x-6)(x+3)}{(x+4)(x-3)} \ge 0$

$\Rightarrow \in (-\infty,-4) \cup [-3,3) \cup [6,\infty)$

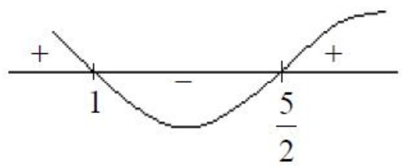
6. (AD)

$\Rightarrow |x-1| + |5-2x| = |3x-6|$

As (x – 1) – (5 – 2x) = 3x – 6

So, $(x-1)(5-2x) \le 0$

$\Rightarrow (x-1)(2x-5) \ge 0$



$\Rightarrow x \in (-\infty,1] \cup \left[\frac{5}{2},\infty\right)$

7. (AC)

$\Rightarrow \log_2 3 > 1$

$\Rightarrow \log_{12} 5 < 1$

So, $\log_2 3 > \log_{12} 5$

Similarly, $\log_6 5 < 1$

$\Rightarrow \log_7 11 > 1$

$\Rightarrow \log 82 > \log_3 3^4$

$\Rightarrow \log_3 81 > 4$

$\Rightarrow \log_2 15 < \log_2 16$

$\Rightarrow \log_2 15 < \log_2 2^4$

$\Rightarrow \log_2 15 < 4$

$\Rightarrow \log_{16} 15 < 1$

$\Rightarrow \log_{10} 11 > 1$

$\Rightarrow \log_7 6 < 1$

8. (BC)

$\Rightarrow 2x^2 + 6\sqrt{2}x + 1 = 0$

$\Rightarrow x = \frac{-6\sqrt{2} \pm \sqrt{72-8}}{4}$

$\Rightarrow x = \frac{-3\sqrt{2}}{2} + 2, \frac{-3\sqrt{2}}{2} - 2$

9. (BD)

$\Rightarrow \log_{x+1}(x-0.5) = \log_{x-0.5}(x+1)$

$\Rightarrow$ x – 5 > 0; x – 0.5 > 0; $x - 0.5 \neq 1$

$\Rightarrow$ x > 0.5; x > 0.5; $x \neq 1.5$

$\Rightarrow$ x + 1 > 0; x + 1 > 0

$\Rightarrow$ x > –1; x > –1

So, $x \in (0.5, 1.5) \cup (1.5, \infty)$ is feasible reason

$$\Rightarrow \frac{\log(x-0.5)}{\log(x+1)} = \frac{\log(x+1)}{\log(x-0.5)}$$

$\Rightarrow \log^2 (x - 0.5) - \log^2 (x + 1) = 0$

$$\Rightarrow \log\left(\frac{x-0.5}{x+1}\right)\log\{(x-0.5)(x+1)\} = 0$$

So, $\frac{x-0.5}{x+1} = 1$ or (x – 0.5) (x + 1) = 1

$\Rightarrow$ x – 0.5 = x + 1 (no solution)

Or $x^2 + 0.5x - 1.5 = 0$

$\Rightarrow 2x^2 + x - 3 = 0$

$\Rightarrow 2x^2 + 3x - 2x - 3 = 0$

$\Rightarrow$ x = 1

As $x \neq \frac{-3}{2}$

So x = 1 is the only solution.

10. (CD)

$$\Rightarrow \log\left[(1)^{\frac{1}{5}} + (32)^{\frac{1}{5}} + (243)^{\frac{1}{5}}\right]$$

$\Rightarrow \log\{1+2+3\}$

$\Rightarrow \log 6$

$$\Rightarrow \frac{1}{5}\{\log 1 + \log 32 + \log 243\} = \frac{1}{5}\log\{1 \times 32 \times 243\}$$

$$= \log\left\{(1)^{\frac{1}{5}}(32)^{\frac{1}{5}}(243)^{\frac{1}{5}}\right\}$$

= log 6

11. (BCD)

$\Rightarrow \alpha, \beta, \gamma$ are roots of $x^3 - 2x + 3 = 0$

$\Rightarrow \alpha + \beta + \gamma = 0$

$\Rightarrow \sum \alpha\beta = -2$

$\Rightarrow \alpha\beta\gamma = -3$

Equation with roots $-\alpha, -\beta, -\gamma$ is $x^3 - (0)\,x^2 + (-2)\,x - (3) = 0$

$\Rightarrow x^3 - 2x - 3 = 0$

$$\Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\sum \alpha\beta}{\alpha\beta\gamma} = \frac{-2}{-3} = \frac{2}{3}$$

$$\Rightarrow \sum \frac{1}{\alpha}.\frac{1}{\beta} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = 0$$

$\Rightarrow \frac{1}{\alpha}.\frac{1}{\beta}.\frac{1}{\gamma} = -\frac{1}{3}$

So, equation $x^3 - \left(\frac{2}{3}\right)x^2 + (0)x - \left(-\frac{1}{3}\right) = 0$

$\Rightarrow 3x^3 - 2x^2 + 1 = 0$

$\Rightarrow \sum(\alpha+\beta) = 0$

$\Rightarrow \sum(\alpha+\beta)(\beta+\gamma) = (\alpha\beta+\beta\gamma+\gamma\alpha+\beta^2)+(\alpha\beta+\beta\gamma+\gamma\alpha+\alpha^2)+(\alpha\beta+\beta\gamma+\gamma\alpha+\gamma^2)$

$= 3(-2)+(\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha s)$

$= 3(-2) + 0 - 2(-2)$

$\Rightarrow \sum(\alpha+\beta)(\beta+\alpha) = -2$

$\Rightarrow (\alpha\beta+\beta\gamma+\gamma\alpha) = (\alpha\beta+\beta\gamma+\gamma\alpha+\beta^2)(\gamma+\alpha)$

$= (-2+\beta^2)(-\beta)$

$= 2\beta - \beta^3$

$= 3$ $\qquad (\because \beta^3 - 2\beta + 3 = 0)$

So equation $x^3 - 0.x^2 + (-2)x - (3) = 0$

$\Rightarrow x^3 - 2x - 3 = 0$

$\Rightarrow \sum\frac{\alpha+\beta}{\gamma+1} = \frac{-\gamma}{\gamma+1} - \frac{\alpha}{\alpha+1} - \frac{\beta}{\beta+1} = -3 + \left[\frac{1}{\gamma+1} + \frac{1}{\alpha+1} + \frac{1}{\beta+1}\right]$

$= -3 + \left[\frac{\sum(\alpha+1)(\beta+1)}{(\alpha+1)(\beta+1)(\gamma+1)}\right]$

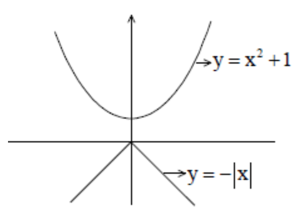
$= -3 - \frac{1}{4} = \frac{-13}{4}$

12. (B)

$\Rightarrow S_1 : x^2 + |x| + 1 = 0$

$\Rightarrow x^2 + 1 = -|x|$

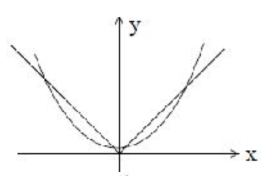
No solution



$\Rightarrow S_2 : x^2 - 5|x| + 6 = 0$

$\Rightarrow x^2 + 6 = 5|x|$

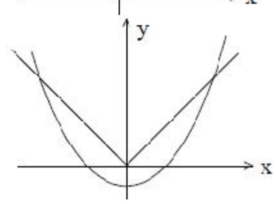
$\Rightarrow$ 4 solution.



$\Rightarrow S_3 : x^2 - |x| - 2 = 0$

$\Rightarrow x^2 - 2 = |x|$

$\Rightarrow$ 2 solution



13. (BC)

$\Rightarrow x^{1-og_5 x} = 0.04$

$\Rightarrow x > 0$

$\Rightarrow x.x^{\log_5 \frac{1}{x}} = 0.04$

Only $x = 25, \frac{1}{5}$ satisfy the equation

14. (AD)

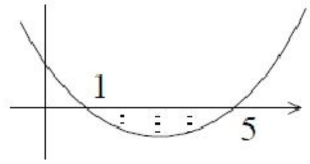
$\Rightarrow f(x) = x^2 - 6x + 5$

Least value at $x = \frac{6}{2} = 3$

Least value = $(3)^2 - 6(3) + 5 = -4$

$\Rightarrow$ f (2) = $(2)^2 - 6(2) + 5 = -3$

$\Rightarrow$ f (4) = $(4)^2 - 6(4) + 5 = -3$



15. (AB)

$\Rightarrow 10^{\frac{2}{x}} + 25^{\frac{1}{x}} = \frac{17}{4}(50)^{\frac{1}{x}}$

$\Rightarrow 100^{\frac{1}{x}} + 25^{\frac{1}{x}} = \frac{17}{4}(50)^{\frac{1}{x}}$

$\Rightarrow 4^{\frac{1}{x}} + 1 = \frac{17}{4}2^{\frac{1}{x}}$

$\Rightarrow 2^{\frac{2}{x}} - \frac{17}{4}2^{\frac{1}{x}} + 1 = 0$

$\Rightarrow \left(2^{\frac{1}{x}} - 4\right)\left(2^{\frac{1}{x}} - \frac{1}{x}\right) = 0$

$\Rightarrow 2^{\frac{1}{x}} = 4, \frac{1}{4}$

$\Rightarrow x = \frac{-1}{2}, \frac{1}{2}$

16. (BC)

$\Rightarrow \left|x^2 + 4x + 3\right| + 2x + 5 = 0$

$\Rightarrow \left|(x+1)(x+3)\right| + 2x + 5 = 0$

Case 1: $x \in (-\infty, -3] \cup [-1, \infty)$

$\Rightarrow x^2 + 4x + 3 + 2x + 5 = 0$

$\Rightarrow$ $x^2 + 6x + 8 = 0$

$\Rightarrow$ x = –4, –2

So x = –4

Case 2: $x \in (-3, -1)$

$\Rightarrow - (x^2 + 4x + 3) + 2x + 5 = 0$

$\Rightarrow$ $x^2 + 2x - 2 = 0$

$\Rightarrow x = \frac{-2 \pm \sqrt{12}}{2}$

$\Rightarrow x = -1 + \sqrt{3}, -1 - \sqrt{3}$

So, $x = -1 - \sqrt{3}$

17. (BC)

$\Rightarrow 4x^2 + 4x + 12\lambda^2 \geq 47 \qquad \forall x \in R$

$\Rightarrow 4\left(x^2 + x + 3\lambda^2\right) \geq 47$

$\Rightarrow x^2 + x + 3\lambda^2 - \frac{47}{4} \geq 0$

So, $8 \leq 0$

$\Rightarrow 1 - 4\left(3\lambda^2 - \frac{47}{4}\right) \leq 0$

$\Rightarrow 4 - \lambda^2 \leq 0$

$\Rightarrow \lambda^2 \geq 4$

$\Rightarrow \lambda \in (-\infty, -2] \cup [2, \infty)$

18. (ABD)

$\Rightarrow \frac{1}{2} \leq \log_{0.1} x \leq 2$; $x > 0$

$\Rightarrow \frac{1}{2} \leq -\log_{10} x \leq 2$

$\Rightarrow -\frac{1}{2} \geq \log_{10} x \geq -2$

$\Rightarrow \log_{10} x \leq -\frac{1}{2}$ & $\log_2 x \geq -2$

$\Rightarrow x \leq (10)^{-\frac{1}{2}}$ & $x \geq 10^{-2}$

$\Rightarrow x \leq \frac{1}{\sqrt{10}}$ …(1) $x \geq \frac{1}{100}$ …(2)

(1) $\cap$ (2)

$\Rightarrow x \in \left[\frac{1}{100}, \frac{1}{\sqrt{10}}\right]$

19. (A)

$\Rightarrow \log_2\left(3^{2x+2} + 7\right) = 2 + \log_2\left(3^{x-1} + 1\right)$

$\Rightarrow \log_2\left(\frac{9.3^{2x} + 7}{\frac{3^x}{3} + 1}\right) = 2$

$\Rightarrow \frac{3\left(9.3^{2x} + 7\right)}{3^x + 3} = 4$

Put $3^x = a$

$\Rightarrow 27a^2 + 21 = 4a + 12$

$\Rightarrow 21a^2 - 4a + a = 0$

Which is not possible for values of a.

So no solution.

20. (AB)

$\Rightarrow \log_{a_1 a_2} a_1 = 4$

$\Rightarrow \log a_1 = 4\left[\log_{a_1} + \log_{a_2}\right] 3\log_{a_1} + 4\log_{a_2} = 0$

$$\Rightarrow \log_{a_1a_2}\frac{(a_1)^{\frac{1}{3}}}{(a_2)^{\frac{1}{2}}} \Rightarrow \log_{a_1a_2}(a_1)^{\frac{1}{3}} - \log_{a_1a_2}(a_2)^{\frac{1}{2}}$$

$$\Rightarrow \frac{1}{3}[4] - x$$

$$\Rightarrow x = \frac{1}{2}\left[\frac{\log(a_2)}{\log a_1a_2}\right] = \frac{1}{2}\left[\frac{\log a_1a_2}{\log a_1a_2}\right]$$

$$= \frac{1}{2}\left[1 - \log_{a_1a_2} a_1\right]$$

$$= \frac{1}{2}[1-4] = -\frac{3}{2}$$

So, $\log_{a_1a_2}\frac{(a_1)^{\frac{1}{3}}}{(a_2)^{\frac{1}{2}}} = \frac{4}{3} + \frac{3}{2} = \frac{17}{6}$

21. (AD)

$f(x) = x^3 - 3x^2 + 4 = 0$

clearly $x = -1$ is a root of given equation.

$$\frac{x^3 - 3x^2 + 4}{x+1} = (x-2)^2$$

$x = 2$ is a repeated root & $x = -1$ is a root

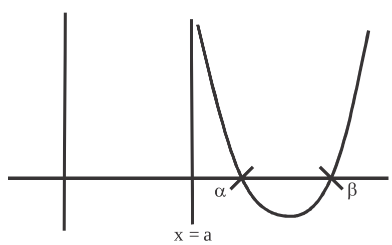
22. (AD)

Given $a < \alpha, \beta$

(i) f (a) > 0

(ii) D > 0

(iii) $-\frac{B}{2A} > a$



(i) $f(a) = a^2 + a + a = a^2 + 2a > 0$

$a \in (-\infty, -2) \cup (0, \infty)$ … (1)

(ii) D > 0

$1 - 4a > 0$

$a < 1/4$ …(2)

(iii) $-\frac{B}{2A} > a$

For $x^2 + x + a$, A = 1 & B = 1

$-\frac{1}{2} > a$ …(3)

From (1), (2), (3)

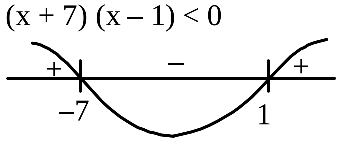
$a < -2$

23. (CD)

$\tan x \geq 3\cot x, x \in \left(-\pi/2, \pi/2\right)$ & $x \neq 0$

{$\because$ cos x > 0 for given integral}
(i) for x > 0
$\tan x \ge \frac{3}{\tan x}$ {here tan x > 0}
$\tan^2 x \ge 3$
$\Rightarrow \tan x \ge \sqrt{3}$
$x \in \left[\pi/3, \pi/2\right]$
(ii) for x < 0
$\tan x \ge \frac{3}{\tan x}$ {here tan x < 0}
$\tan^2 x \le 3$
$\tan x \le -\sqrt{3}$
$\Rightarrow x \in \left(-\pi/3, 0\right]$

24. (AD)
$\frac{3}{\sqrt{2-x}} - \sqrt{2-x} < 2$
$3-(2-x) < 2\sqrt{2-x}$
$1+x < 2\sqrt{2-x}$
Here 2 – x > 0
$\Rightarrow$ x < 2
$(1 + x)^2 < 4(2 – x)$
$1 + x^2 + 2x < 8 – 4x$
$x^2 + 6x – 7 < 0$
$(x + 7)(x – 1) < 0$



$x \in (-7, 1)$

25. (ABC)
$xy + 3y^2 – x + 4y – 7 = 0$
& $2xy + y^2 – 2x – 2y + 1 = 0$
$xy + 3y^2 – x + 4y – 7 = 0$
$xy – x + 3y^2 – 3 + 4y – 4 = 0$
$x(y – 1) + 3(y – 1)(y + 1) + 4(y – 1) = 0$
$(y – 1)[x + 3(y + 1) + 4] = 0$
$\Rightarrow$ either y = 1 or x + 3y + 7 = 0 …(i)
$2xy + y^2 – 2x – 2y + 1 = 0$
$2xy – 2x + y^2 – 2y + 1 = 0$
$2x(y – 1) + (y – 1)^2 = 0$
$(y – 1)[2x + y – 1] = 0$
Either y = 1 or 2x + y – 1 = 0

26. (ABC)
$kx^2 + kx – (2k + 1) > 0$
If true for at least one real x

$\Rightarrow$ D > 0

$k^2 - 4k \times (-1)(2k+1) > 0$

$k^2 + 8k^2 + 4k > 0$

k (9k + 4) > 0

$\Rightarrow k \in \left(-\infty, -4/9\right) \cup (0, \infty)$

27. (BC)

$\log_x 2 \,.\, \log_{2x} 2 = \log_{4x} 2$

$\frac{\log 2}{\log x} \cdot \frac{\log 2}{\log^{2x}} = \frac{\log 2}{\log^{4x}}$

log 2 (log 4x) = log x . log 2x

log 2 (2 log 2 + log x) = log x (log 2 + log x)

$2(\log 2)^2 + \log 2.\log x = \log 2.\log x + (\log x)^2$

$\log x = \pm\sqrt{2}\log 2$

$x = 2^{\pm\sqrt{2}}$

28. (a)

$\log_{(2x+3)}\left(6x^2 + 23x + 21\right)$

$= 4 - \log_{(3x+7)}\left(4x^2 + 12x + 9\right)$

$\Rightarrow \log_{(2x+3)}\left[(2x+3)(3x+7)\right]$

$= 4 - \log_{(3x+7)}\left[(2x+3)^2\right]$

$\Rightarrow 1 + \frac{\log(3x+7)}{\log(2x+3)} = 4 - \frac{2\log(2x+3)}{\log(3x+7)}$

Let $\frac{\log(3x+7)}{\log(2x+3)} = k$

$\Rightarrow 1 + k = 4 - \frac{2}{k}$

$k + \frac{2}{k} - 3 = 0$

$k^2 - 3k + 2 = 0$

(k – 1) (k – 2) = 0

$\frac{\log(3x+7)}{\log(2x+3)} = 1, 2$

3x + 7 = 2x + 3 or $3x + 7 = (2x + 3)^2$

x = –4 or $3x + 7 = 4x^2 + 9 + 12x$

$4x^2 + 9x + 2 = 0$ $\quad$ $4x^2 + 8x + x + 2 = 0$

4x (x + 2) + (x + 2) = 0

x = –2 or $x = -1/4$

So, $x = -1/4, -2, -4$

Also 2x + 3 > 0 & 3x + 7 > 0

$x > -3/4$ & $x > -7/4$

$\Rightarrow$ only value of $x = -\frac{1}{4}$

29. (BD)

$2^{\log_{\sqrt{2}}(x-1)} > x+5$

$2^{\log_2 (x-1)^2} > x+5$

$(x-1)^2 > x+5$

$x^2 + 1 - 2x > x + 5$

$x^2 - 3x - 4 > 0$

$(x-4)(x+1) > 0$

$x \in (-\infty, -1) \cup (4, \infty)$

30. (BCD)

Given $\frac{1}{3} < \frac{x^2 - 2x + 4}{x^2 + 2x + 4} < 3$

$\Rightarrow \frac{1}{3} < \frac{x^2 + 2x + 4}{x^2 - 2x + 4} < 3$

$\frac{9.3^{2x} + 6.3^x + 4}{9.3^{2x} - 6.3x + 4}$

$\frac{3^{2x+2} + 2.3.3^x + 4}{3^{2x+2} - 2.3.3x + 4}$

$\frac{\left(3^{x+1}\right)^2 + 2.\left(3^{x+1}\right) + 4}{\left(3^{x+1}\right)^2 - 2.\left(3^{x+1}\right) + 4}$

Let $3^{x+1} = y$

$\Rightarrow \frac{y^2 + 2y + 4}{y^2 - 2y + 4}$

It will also lie between (1/3, 3)

31. (A)

$\log_2 x + \log_x 2 = \frac{10}{3} = \log_2 y + \log_y 2$

Also $x \neq y$

$\frac{\log x}{\log^2} + \frac{\log 2}{\log x} = \frac{10}{3}$

Solving we get

$\frac{\log x}{\log 2} = 3, \frac{1}{3}$

x = 8 or $x = 2^{1/3}$

If x = 8 $\Rightarrow y = 2^{1/3}$

& if $x = 2^{1/3} \Rightarrow$ y = 8

$\therefore \frac{x}{y} = \frac{2^3}{2^{1/3}}$ or $\frac{2^{1/3}}{2^3}$

$\frac{x}{y} = 2^{8/3}$ or $2^{-8/3}$

32. (ACD)

$2x^2 - 2(2a+1)x + a(a+1) = 0$

(i) f (a) < 0

(ii) D > 0

(i) $f(a) = 2a^2 - 2(2a + 1)a + a(a + 1)$

$= 2a^2 - 4a^2 - 2a + a^2 + a$

$= -a^2 - a < 0$

$= a(a + 1) > 0$

$a \in (-\infty, -1) \cup (0, \infty)$

(ii) D > 0

$4(2a + 1)^2 - 4.\ 2.\ a(a + 1) > 0$

$4a^2 + 1 + 4a - 2a^2 - 2a > 0$

$2a^2 + 2a + 1 > 0$

$\Rightarrow a \in R$

$\therefore a \in (-\infty, -1) \cup (0, \infty)$

33. (D)

$2^x + 2^{|x|} \geq 2\sqrt{2}$

Case – 1: $x \geq 0$

$2^x + 2^x \geq 2\sqrt{2}$

$2^x \geq \sqrt{2}$

$\Rightarrow x \geq 1/2$

Case – II: x < 0

$2^x + 2^{-x} \geq 2\sqrt{2}$

Let $2^x = k$

$k + 1/k \geq 2\sqrt{2}$

$k^2 - 2\sqrt{2}k + 1 \geq 0$

$\left(k - \left(\sqrt{2}+1\right)\right)\left(k - \left(\sqrt{2}-1\right)\right) \geq 0$

$-\infty < 2^x \leq \sqrt{2} - 1 \,\&\, \sqrt{2} + 1 \leq 2^x < \infty$

$\Rightarrow 0 < 2^x < \sqrt{2} - 1 \,\&\, \sqrt{2} + 1 \leq 2^x < \infty$

$x \in \left(-\infty, \log_2\left(\sqrt{2}-1\right)\right) \& \left(\log_2\left(\sqrt{2}+1\right), \infty\right)$

From case I & II

$x \in \left(-\infty, \log_2\left(\sqrt{2}-1\right)\right] \cup \left[1/2, \infty\right)$

34. (AC)

$(3 - x)^4 + (2 - x)^4 = (5 - 2x)^4$

$(3 - x)^4 + (2 - x)^4 = (3 - x + 2 - x)^4$

Let 3 – x =a & 2 – x = b

$a^4 + b^4 = (a + b)^4$

clearly either a = 0 or b = 0

$\Rightarrow$ either x = 3 or x = 2

Now

$a^4 + b^4 = (a^2 + b^2 + 2ab)^2$

$a^4 + b^4 = a^4 + b^4 + 4a^2 b^2 + 2a^2 b^2 + 4ab^3 + 4a^3b$

$\Rightarrow 4ab^3 + 6a^2b^2 + 4a^3b = 0$

$2ab\,[2b^2 + 3ab + 2a^2] = 0$

Either a = 0 or b = 0

Or $2a^2 + 3ab + 2b^2 = 0$

$2\left(\frac{a}{b}\right)^2 + 3\frac{a}{b} + 2 = 0$

Here D < 0

$\Rightarrow$ two real roots & two non real roots

35. (AB)

$4^x - (a-3)2^x + a - 4 = 0$

Roots are non positive

$x \le 0$

$\Rightarrow 0 < 2^x \le 1$

$\left(2^x\right)^2 - (a-3)2^x + a - 4 = 0$

$k^2 - (a-3)k + (a-4) = 0$

(i) f (0) > 0

(ii) $f(1) \ge 0$

(iii) D > 0

(i) f (0) = a – 4 > 0

a > 4

(ii) $f(1) \ge 0$

$1 - (a-3) + a - 4 \ge 0$

(iii) D > 0

$(a-3)^2 - 4(a-4) > 0$

$a^2 + 9 - 6a - 4a + 16 > 0$

$a^2 - 10a + 25 > 0$

$(a-5)^2 > 0$

$a \in R$

(iv) $0 < -\frac{B}{2A} \le 1$

$0 < -\frac{\left(-(a-3)\right)}{2\times 1} \le 1$

$0 < \frac{a-3}{2} \le 1$

$3 < a \le 5$

From (i), (ii), (iii), (iv)

$a \in (4,5]$

36. (BCD)

$27^x + 2\cos 3y + 8 + 6\cos y = 4.3^{x+1}.\cos y$

$3^{3x} + 2\cos 3y + 8 + 6\cos y = 4.3^{x+1}.\cos y$

$\left(3^x\right)^3 + 8\cos^3 y - \cancel{6\cos y} + 8 + \cancel{6\cos y} = 3.2.3^x.2\cos y$

$\left(3^x\right)^3 + (2\cos y)^3 + 2^3 = 3.\left(3^x\right)(2\cos y)(2)$

$3^x + 2\cos y + 2 = 0$

Or $3^x = 2\cos y = 2$

$x = \log_3 2, \cos y = 1$

37. (ABD)

$x + y = \sqrt{3}$ …(1)

$y + z = \sqrt{5}$ …(2)

$z - x = \sqrt{7}$ …(3)

(1) & (2) we get

$x - z = \sqrt{3} - \sqrt{5}$

Add in (3) we get

$0 = \sqrt{3} - \sqrt{5} + \sqrt{7}$

Not possible

No real values of x, y, z

38. (ABCD)

$y = \dfrac{x^2 + 2x - 7}{x^2 + 2x + 7}$

Range in given D

For $x = 1$, $y = -2/5$

$x = -1$ $y = -4/3$

$x = 0$ $y = -1$

$x = 2$ $y = -1$

39. (CD)

$\log_2 x = \log_4 y + \log_4 (4 - x)$

$4 - x > 0 \Rightarrow x < 4$

& x, y > 0

$\log_2 x = \dfrac{1}{2}\log_2 (y.(4 - x))$

$x^2 = (4 - x).y$ …(1)

$\log_3 (x + y) = \log_3 x - \log_3 y$

x + y > 0 & x > 0 & y > 0

$x + y = x/y$

$y = \dfrac{x}{y} - x$

$y = \left(\dfrac{1}{y} - 1\right)^x$ … (2)

$x = 4/3, y = 2/3$

40. (AC)

$2\cos^2 x > 3/2$

$\left(\cos x - \sqrt{3}/2\right)\left(\cos x + \sqrt{3}/2\right) > 0$

$-1 \le \cos x < -\sqrt{3}/2 < \cos x \le 1$

41. (ABCD)

$f(x) = x^8 - x^5 + x^2 - x + 1$

$f(x) = x^5(x^3 - 1) + x(x - 1) + 1$

$f(x) = x(x - 1)[x^4(x^2 + x + 1) + 1] + 1$

$x^4(x^2 + x + 1) > 0$

Also $x(x - 1) > 0$ for $n \in (-\infty, 0) \cup (1, \infty)$

Now for $x \in [0,1]$

$x^8 + x^2 + 1 - x^5 - x > 0$

$\therefore f(x) > 0 \Rightarrow x \in R$

42. (ABC)

$x^2 + y^2 - xy - x - y + 1 \ge k$

$x^2 - (y+1)x + y^2 - y + 1 = k \ge 0$

$D \le 0 (y+1)^2 - 4y^2 + 4y - 4 + 4k \le 0$

$3y2 - 6y + 3 - 4k \ge 0 (\forall y \in R)$

$(D \le 0) 36 - 36 + 48k \le 0$

Minimum value of expected = 0

$k \le 0$

Have minimum value is 0

In–equation has solution $x, y \in R$

$\therefore$ All equations with real values if x & y are true.

43. (ABCD)

$x^2 + \left(a \log\left(1 - a^2\right)\right)x - \left(a^2 - 1\right) = 0$

Roots are opposite in sign

$\frac{C}{A} <)$

$-\left(a^2 - 1\right) < 0$

$a \in (-\infty, -1) \cup (1, \infty)$ …(1)

As we have tan log $(1 - a^2)$

$\therefore a^2 < 1$ …(2)

From (1) & (2) no value of a for which roots are opposite in sign

44. (CD)

$x^4 - 4x^3 + ax^2 - bx + 1 = 0$

$\alpha + \beta + \gamma + \delta = 4$

$\alpha.\beta.\gamma.\delta = 1$

Now by $AM \ge am$

$AM = \frac{\alpha + \beta + \gamma + \delta}{4} = 1$

$GM = (\alpha.\beta.\gamma.\delta)^{1/4} = 1$

Now, $\because$ AM = GM

$\Rightarrow \alpha = \beta = \gamma = \delta = 1$

Now, $a = \alpha\beta + \beta\gamma + \gamma\delta + \delta\gamma + \alpha\gamma + \beta\delta$

$\Rightarrow$ b = 4

45. (BC)

$\log_{(a)} x \le \log_a x^2$

For 0 < a < 1

$x \ge x^2$

$x \in (0,1]$ $\{\text{as } x \ne 0\}$

For 1 < a

$x \le x^2$

$x(x-1) \ge 0$

$x \in [1, \infty)$

46. (AC)

$\log_3(x+2) > \log_{x+2} 81$

$$\frac{\log(x+2)}{\log 3} > \frac{4\log 3}{\log(x+2)}$$

(i) for x > –1

log(x + 2) > 0

$\therefore$ [log (x + 2)]^2 > 4 (log 3)^2

(log (x + 2) – 2 log 3) (log (x + 2) + 2 log 3) > 0

$\Rightarrow$ log (x + 2) < – 2 log 2 & log (x + 2) > 2 log 3

& log (x + 2) > 2 log 3

$x < \frac{1}{9} - 2$ & $x > 7$

$x < \frac{-17}{9}$

(ii) for x < –1

$\log(x+2)^2 \le 4(\log 3)^2$

$-2\log 3 < \log(x+2) < 2\log 3$

$\Rightarrow \frac{-17}{9} < x < 7$

47. (BC)

$$\frac{1}{\log_4\left(\frac{x+1}{x+2}\right)} \le \frac{1}{\log_4(x+3)}$$

$$\frac{x+1}{x+2} > 1$$

$$\frac{x+1-x-2}{x+2} > 0$$

$-\frac{1}{x+2} > 0$

$\frac{1}{x+2} < 0$

$\Rightarrow x < -2$ … (1)

$\frac{x+1}{x+2} > 0$

$\Rightarrow x > -1$ & $x < -2$ … (2)

$\Rightarrow 0 < \frac{x+1}{x+2} < 1$

For $x > -1$

Case I take $x > -1$

$\frac{1}{\log_4\left(\frac{x+1}{x+2}\right)} \le \frac{1}{\log_4(x+3)}$

$\log_4(x+3) \ge \log_4\left(\frac{x+1}{x+2}\right)$

$\left\{\text{as } \log_4\left(\frac{x+1}{x+2}\right) < 0 \text{ for } x > -1\right\}$

$x+3 \ge \frac{x+1}{x+2}$

$x+3-\left(\frac{x+1}{x+2}\right) \ge 0$

$\Rightarrow x > -2$

{we have taken $x > -1$}

i.e. $x > -1$

case II: $-3 < x < -2$

$x+3 = \frac{x+1}{x+2} \quad \frac{x^2+4x+5}{x+2} \ge 0$

$x > -2$

no solution

48. (ABC)

$n = x(x+1)(x+2)(x+3)$

$= x^4 + 6x^3 + 11x^2 + 6x$

Now $(x^2 + 3x + 1)^2 = x^4 + 6x^3 + 11x^2 + 6x + 1$

$\Rightarrow$ n + 1 is a perfect square

n is a never a perfect square.

Also in four consecutive integers there is a factor of 2, a factor of 4, a factor of 3

$\therefore$ n is always divisible by $2 \times 4 \times 3 = 24$

49. (BC)

Possible values 10 & 12 from options

50. (C)

$28x + 30y + 31z = 365$

One of solution is (1, 4, 7) for (x, y, z)

$z - 2x = 5$

INEQUATION & EQUATION
EXERCISE 2 (B)

1. (D)

$\left||x+1|-2\right|=1$

$\Rightarrow |x-1|=3 \quad or \quad |x-1|=1$

$\Rightarrow (x-1)^2=9 \qquad x-1=\pm 1$

$\Rightarrow (x-4)(x+2)=0 \quad x = 2, 0$

$\Rightarrow x = -2, 4$

2. (B)

$\left||x-2|-3\right|=4$

$\Rightarrow |x-2|=7$

$\Rightarrow (x-2)^2=7^2$

$\Rightarrow (x-9)(x-5)=0$

$\Rightarrow x = 5, 9$

3. (C)

$|x-2|=10, 0$

$x = 2, 12, -8$

4. (A)

$A=2^{\log_2 4}+3^{\log_2 4}+4^{\log_2 2}-4^{\log_2 3}$

$\Rightarrow A=2^2+3^{\log_2 4}+4^1-3^{\log_2 4}=8$

$D=(\log_5 49)(\log_7 125)$

$\Rightarrow D=\frac{\log 49}{\log 5}\times\frac{\log 125}{\log 7}=\frac{2\log 7}{\log 5}\times\frac{3\log 5}{\log 7}$

$\Rightarrow$ D = 6

$\Rightarrow a=\log_A D=\log_8 6$

$\Rightarrow a=\frac{\log_2 6}{3}$

Now $\log_6 12 = \log_6 6 + \log_6 2$

$\Rightarrow \log_6 12=1+\frac{1}{\log_2 6}$

$\Rightarrow \log_6 12=1+\frac{1}{3a}=\frac{1+3a}{3a}$

5. (A)

$N=7^{\log_{49} 900}$

$\Rightarrow N=7^{\log_{7^2} 30^2}=7^{\log_7 30}=30$

$\log_6 12=\frac{1+ma}{ma}\Rightarrow m=n=3$

$\log_N m=\log_{30} 3$

$\log_m N=\log_3 30$

$\log_n N=\log_3 30$

Clearly $\log_{30}3 < \log_3 30$

$\therefore \log_N m < \log_m N = \log_n N$

6. (B)

$\log_{\left(A-\frac{N}{10}\right)}|N+A+D+m+n| - \log_5 2 = \log_{\left(8-\frac{30}{10}\right)}|30+8+6+3+3| - \log_5 2$

$= \log_5 50 - \log_5 2 = \log_5 25 + \log_5 2 - \log_5 2$

$= \log_5 5^2 = 2$

7. (A)

$y = 4 - |4x^2 - 9| \Rightarrow |4x^2 - 9| = 4 - y \ge 0$

$\Rightarrow y \le 4$

8. (A)

$\Rightarrow (4x^2 - 9)^2 = 7^2$

$\Rightarrow (4x^2 - 16)(4x^2 - 2) = 0$

$\Rightarrow x = \pm 2, \pm \frac{1}{\sqrt{2}}$

9. (B)

$|Z| + 4 = 4 - |4x^2 - 9| \Rightarrow |Z| + |4x^2 - 9| = 0$

$\Rightarrow 4x^2 - 9 = 0, Z = 0$

$\Rightarrow x = \pm \frac{3}{2}, Z = 0$

10. (D)

$y = \log_x (4-x) + \log_3 (2+x)$

$x > 0, x \ne 1, 4 - x > 0 \,\&\, 2 + x > 0$

$x \in (0,1) \cup (1,4)$

11. (A)

$\Rightarrow \frac{x}{x+3} \ge 1 \Rightarrow \frac{3}{x+3} \le 0$

$\Rightarrow x < -3$

12. (C)

$y = \log_{\frac{1}{2}}\left(\frac{x+4}{x+1}\right) + \sqrt{x-1}$

$\Rightarrow \frac{x+4}{x+1} > 0 \,\&\, x - 1 \ge 0$

$\Rightarrow x < -4$ or $x > -1$ & $x \ge 1$

$\Rightarrow (-\infty, -4) \cup [1, \infty)$

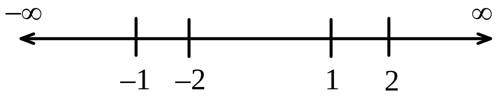
13. (A)

$f(x) = |x-2| + |x-1| + |x+1| + |x+2|$

$f(-x) = f(x)$ then given functions is symmetric about y – axis

14. (B)

$|x-1|+|x-2|+|x+1|+|x+2|=6$



Case I: x < –2

$\therefore \cancel{1}-x+\cancel{2}-x-n-\cancel{1}-n-\cancel{2}=6$

– 4x = 6

$x=-3/2$

Case II: $-2 \le x < -1$

$\cancel{1}-n+\cancel{2}-n-\cancel{n}-\cancel{1}+\cancel{x}+2=6$

$-2x=2 \qquad \therefore\ x=-1$

Case III: $-1 \le x < 1$

$1-\cancel{x}+2-\cancel{x}+\cancel{x}+1+\cancel{x}+2=6$

6 = 6

Case IV: $1 \le x < 2$

$\cancel{x}+1+2-\cancel{x}+n+1+n+2=6$

2x = 0 $\qquad \therefore$ n = 0

Case V: $x \ge 2$

4x = 6 $\qquad \therefore x=3/2$

$\therefore x \in [-1,1]$

15. (C)

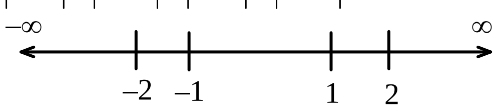
$f(n)=|x-2|+|x-1|+|x+1|+|x+2|$

$f(x)_{\min}=6$

$\therefore k=3$

16. (C)

$|x-1|+|x-2|+|x+1|+|x+2| \le 20$



Case I: x < –2

$\cancel{1}-x+\cancel{2}-x-x-\cancel{1}-x-\cancel{2} \le 20$

$-4x \le 20, \qquad x \ge -5$

$\therefore x \in [-5,-2]$

Case II: $-2 \le x < -1$

$\cancel{1}-x+\cancel{2}-x-\cancel{x}-\cancel{1}+\cancel{x}+2 \le 20$

$-2x \le 16 \qquad \therefore x \ge -8$

$x \in [-2,-1]$

Case III: $-1 \le x < 1$

$1-\cancel{x}+2-\cancel{x}+\cancel{n}+1+\cancel{n}+2 \le 20$

$6 \le 20$

Case IV: $1 \le x < 2$

$\cancel{x}-\cancel{1}+2-\cancel{x}+n+\cancel{1}+x+2 \le 20$

$2x \le 16 \qquad x \le 8$

Case V: $x \geq 2$

$4x \leq 20 \qquad x \leq 5$

$x \in [-5, 5]$

17. (D)

If $x \in (-\infty, -1), y \in (0, 1)$

18. (B)

If $x \in (-1, 2)$ then $y \in (-\infty, 0)$

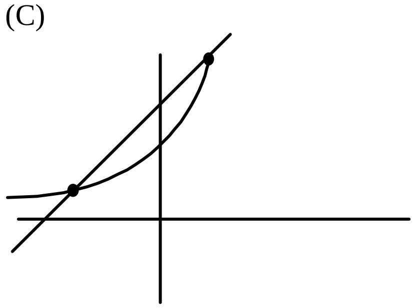
19. (C)

If $n \in (2, 4)$, then $y \in (-\infty, \infty)$

20. (A)

$f(x) = k$ has root in $(4, \infty)$ then $k \in (1, \infty)$

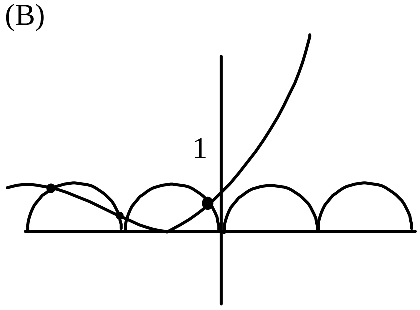
21. (C)



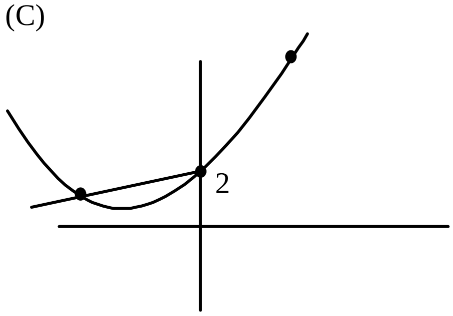
$2^x + 3^x - 1 = 2x + 3$

$2^x + 3^x = 2x + 4$

22. (B)



23. (C)



$2^x + 3^x - 1 = 2x^2 - 2$

$-2^x - 3$

$\Rightarrow 2^x + 3^x = 2(x^2 + x + 1)$

For 24 – 26

$\tan\left(\theta+\frac{\pi}{4}\right)=3\tan 3\theta$

$\frac{1+t\theta}{1-t\theta}=\frac{3\left(3t\theta-t^3\theta\right)}{\left(1-3\tan^2\theta\right)}$

Get tan θ = t $(1 + t)(1 - 3t^2) = 3(1 - t)(3t - t^3)$

$1-3t^2+t-\cancel{3t^3}=9t-\cancel{3t^3}-9t^2+3t^4$

$3t^4 - 6t^2 + 8t - 1 = 0$

24. (D)

$\sum \tan\alpha = 0$

25. (A)

$\pi\tan\alpha=-\frac{1}{3}$

26. (B)

$\frac{1}{t\alpha}+\frac{1}{t\beta}+\frac{1}{t\gamma}+\frac{1}{t\delta}=\frac{\sum t\alpha\, t\beta \tan\gamma}{t\alpha\, t\beta\, t\gamma\, t\delta}$

$=\frac{-8/3}{-1/3}=8$

27. (A)

Statement – 1 is correct as 13 doesn't $x^4-5x^3+56|x-3|=0$

Statement – 2: $a_0x^n+a_1x^{n-1}+...+a_{n-1}x+a_n=0$ where $a_0,a_1,....,a_n\in I, a_0\neq 0, a_n\neq 0$

$\Rightarrow\left(a_0x^{n-1}+a_1x^{n-2}+...+a_{n-1}\right)=-\frac{a_n}{x}$

Clearly for an integral root an must be divisible by x.

Hence statement –2 is also correct & is a correct explanation of statement – 1

28. (D)

For common root of equations $x^2-4x+1=0$ and $x^2 - ax + b = 0$, if $a = 4$ & $b = 1$, then both the roots will be common.

If quadratic equations $ax^2+bx+c=0$ and $a_1x^2+b_1x+c_1=0$ with rational coefficient have both the roots common, then we must have $a=a_1k$, $b=b_1k$ and $c=c_1k$, for some real number $k\neq 0$.

29. (D)

Let the polynomial be P (x) = (x – 1) (x – 2) Q (x) + ax + b

P (1) = –1 ⇒ a + b = –1

P (2) = 1 ⇒ 2a + b = 1

⇒ a = 2, b= –3

⇒ remainder is 2x – 3

Statement –1 is false.

Statement – 2 is correct.

30. (D)

As $x^2+1 \ge 1$ hence $\log_3(x^2+1) \ge 0$, also $\left|\log_4 y^2\right| \ge 0$

Hence $\log_3(x^2+1)+\left|\log_4 y^2\right| = 0 \Rightarrow \log_3(x^2+1)=0$ & $\left|\log_4 y^2\right| = 0$

$\Rightarrow x^2 + 1 = 1$ & $y^2 = 1$

Statement –1 is false

Statement –2 is correct

31. (A)

$\log_{0.1}\left(\sqrt{14}-\sqrt{13}\right) = \log_{10}\left(\sqrt{14}+\sqrt{13}\right)$

Also $\sqrt{13}-\sqrt{12} < \sqrt{14}+\sqrt{13} \Rightarrow \log_{0.1}\left(\sqrt{14}-\sqrt{13}\right) > \log_{10}\left(\sqrt{13}-\sqrt{12}\right)$

Statement –1 is true

Statement –2 is also correct & correct explanation of statement –1

32. (C)

Statement –1 $\log_{\sin^2\theta} b < \log_{\sin^2\theta} C$

$\Rightarrow$ b > c

As $\log_x b < \log_x c$

If $x \in (0,1)$ b > c

& if x > 1 b < c

True

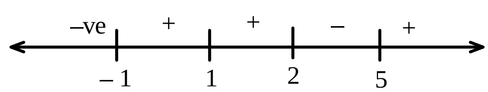
$\Rightarrow$ statement 2 is false. It's not always true.

33. (A)

34. (A) $\dfrac{(x-1)^2(x+1)(x-2)}{(x-5)} \ge 0$

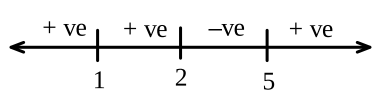
x = 1, 1, –1, 2 (roots)

x = 5 (pole)



$x \in [-1,2] \cup (5,\infty)$

(B) $\dfrac{|x|(x-5)}{(x-2)} \ge 0$



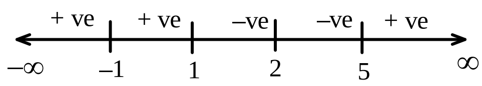
$x \in (-\infty,2] \cup [5,\infty)$

(c) $\log_{10}$ x

If x is two digit number then Q = [1, 2]

P = $\log_{10}$ x, x is four digit number then $P = [3,4)$

(d) $\frac{(x-1)(x+1)(x-2)^2}{(x-5)} \le 0$



$x \in [1,5]$

35. (A) – (r, s)

$\Rightarrow \log_{49} 7 = 1/2$

$\Rightarrow \log_3 \left(5 + 8\log_{49}\left(5 + 4\log_{49} 7\right)\right)$

$= \log_3 \left(5 + 8\log_{49} 7\right)$

$= \log_3 \left(5 + 8/2\right)$

$= \log_3 9 = 2$

$|k| = 2 \Rightarrow k = \pm 2$

(B) $\sqrt{\left(\log_{2^{-1}} .2^2\right)^2} = \sqrt{(-2)^2} = |-2| = 2$

(C) $\log_x \left(x^2 - 1\right) = 0$

$\therefore x^2 - 1 = 1$

$x^2 = 2$

$x = \pm\sqrt{2}$

But $x = -\sqrt{2}$ (rejected)

$\therefore$ only one solution

(D) $\frac{1}{x} = \frac{\sqrt{9-\sqrt{77}}}{\sqrt{81-77}}$ $\quad \therefore \quad$ $\frac{2}{x} = \sqrt{9-\sqrt{77}}$

$x + \frac{2}{x} = \sqrt{9+\sqrt{77}} + \sqrt{9-\sqrt{77}}$

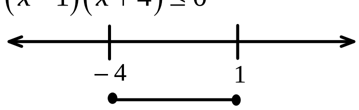
$\left(x + \frac{2}{x}\right)^2 = 18 + 2 \times 2 = 22$

$\frac{1}{11}\left(x + \frac{2}{x}\right)^2 = 2$

36. (A) $x^2 + 4x - x - 4 \le 0$

$x(x+4) - (x+4) \le 0$

$(x-1)(x+4) \le 0$



$x \in [-4,1]$

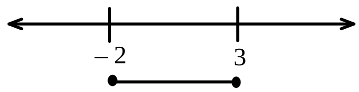
(B) $x^2 - 2x + 2 > 0$

$\therefore x \in R$

(C) $x^2 - 3x + 2x - 6 < 0$

$x(x-3) + 2(x-3) < 0$

$(x+2)(x-3) < 0$

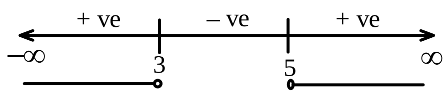


$x \in (-2, 3)$

(D) $x^2 - 3x - 5x + 15 > 0$

$x(x-3) - 5(x-3) > 0$

$(x-3)(x-5) > 0$



$x \in (-\infty, 3) \cup (5, \infty)$

37. (A) – (QR)

$1 - x^2 > 0$ $\quad\therefore\ x^2 < 1$

$2 - x^2 > 2\sqrt{1 - x^2}$ $\quad |x| < 1$

$4 + x^4 - \cancel{4x^2} > 4 - \cancel{4x^2}$

$x^4 > 0$ $\quad\therefore x \in (-1, 1)$

(B) – (PQRS)

$x^8 - x^5 + x^2 - x + 1 > 0 \ x \in R$

For $x < 0$ (time)

For $x \in (011)$

$$\frac{x^8}{+\text{ve}} + \frac{(x^2 - x^5)}{+\text{ve}} + \frac{(x)}{+\text{ve}} > 0$$

For $x > 1$

$$\frac{(x^8 - x^5)}{+\text{ve}} + \frac{(x^2 - \alpha)}{+\text{ve}} + \frac{1}{+\text{ve}} > 0$$

(C) – (PQRS)

$x^{12} - x^9 + x^4 - x + 3 > 0$ (same)

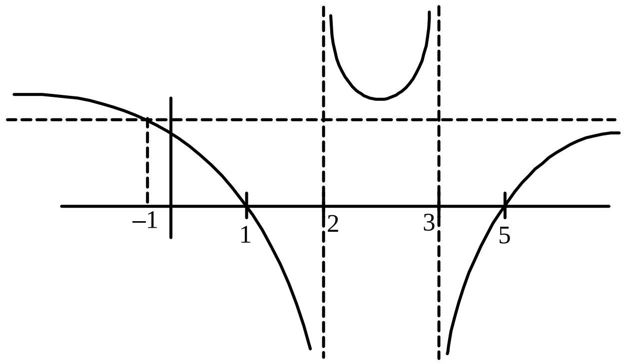
(D) – (PQRS)

$2x^2 + 1 > \sqrt{4x^2 + 1}$

$4x^4 + 4x^2 + 1 > 4x^2 + 1$

$x^4 > 0 \ \ n \in (-\infty, \infty) - \{0\}$

38. $$\frac{(x-1)(x-5)}{(x-2)(x-3)}$$



(A) – (PRS)
$-1 < x < 1$, $f(x) \in (0,1)$

(B) – (QS)
1 < x < 2, $f(x) \in (-\infty, 0)$

(C) – (QS)
3 < x < 5, $f(x)(-(-\infty, 0))$

(D) – (PRS)
x > 5, $f(x) \in (0,1)$

39. (A) – (R)
$|x-2| < 5$
$-5 < x-2 < 5$
$-3 < x < 7$

(B) – (P)
$|x-1| + |x-2| < 6$
Case (i) $x \geq 2$
2x – 3 < 6
x < 9.5
Case (ii) $1 \leq x < 2$
x – 1 – x + 2 < 6
1 < 6
Case (iii) $x \leq 1$
– x + 1 – x + 2 < 6
3 < 6 + 2x
x > –3

(C) – (S)
$|4-3x| \leq 1/2$
$-1/2 \leq 4-3x \leq 1/2$
$-9/2 \leq -3x \leq -7/2$
$3/2 \geq x \geq 7/6$

(D) – (Q)
$|x^2 - 2| < 7$

$-7 < x^2 - 2 < 7$
$-5 < x^2 < 9$
$0 < x^2 < 9$
$-3 < x < 3$

40. (A) – (QS)

$x^{\log_{10} x} = 100x$

$\Rightarrow (\log_{10} \lambda)^2 = 2(\log_{10} \alpha)$

$\Rightarrow \log_{10} x = 2, -1$

$x = 100, \frac{1}{10}$

$x_1 . x_2 = 10$

(B) – (QS)

$\log_2 (9 - 2^x) = 3 - x$

$\Rightarrow 9 - 2^x = \frac{8}{2^x}$

$\Rightarrow (2^x)^2 - 9.(2^x) + 8 = 0$

$2^x = 1, 2^x = 8$

$x = 0, x = 3$

$\lambda_1^{\ 2} + \alpha_2^{\ 2} = 9$

(C) – (P)

$\log_{1/8} \log_{1/4} \log_{1/2} (x) = \frac{1}{3}$

$\Rightarrow \log_{1/4} \log_{1/2} x = \frac{1}{2}$

$\Rightarrow \log_{1/2} x = \left(\frac{1}{4}\right)^{1/2} = 1/2$

$\Rightarrow x = (1/2)^{1/2} = \frac{1}{\sqrt{2}}$

(D) – (QR)

$\log_b a = 3 \qquad \log_b c = -4$

$a = b^3, \qquad c = \frac{1}{b^4}$

$b^{9x} = \frac{1}{b^{4x-4}}$

$\Rightarrow b^{9x} = b^{-4x+4}$

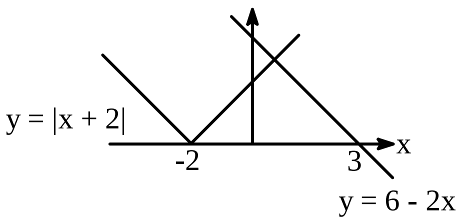
$9x = -4x + 4$

$\Rightarrow x = 4/13$

$p + q = 17$

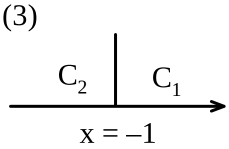
INEQUATION & EQUATION
EXERCISE – 2(C)

1. (1)

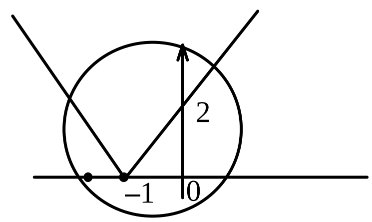


2. (2)
Same as above.

3. (3)



C_1: If $x \geq 1$ … (a)
Then, $2(x + 1) > x + 4$
$x > 2$ … (b)
(a) n (b)
$x \in (2, \infty)$ (C – 1)



C_2: If $x < -1$ … (a)
Then, $-2(x + 1) > x + 4$
$\Rightarrow x < -3$ … (b)
(a) n (b)
$x \in (-\infty, -3) \cup (2, \infty)$
$C-1 \cup C-2$
So required x = 3

4. (2)
Same as above take three cases.

5. (5)
$(|x| - 2)(|x| - 3) < 0 \Rightarrow 2 < |x| < 3$
$\Rightarrow x \in (-3, -2) \cup (2, 3)$
So (a + b) = 5

6. (4)
$-5 < x^2 - 4x < 5$
(1) (2)
(1) $x^2 - 4x + 5 > 0 \Rightarrow x \in R\,(\therefore D < 0)$ …(1)
(2) $x^2 - 4x - 5 < 0 \Rightarrow (x-5)(x+1) < 0 \Rightarrow -1 < x < 5$ …(2)
(1) n (2)
$x \in (-1, 5)$
So, m = 0, n = 4 $\Rightarrow$ (n – m) = 4

7. (4)
Same as question 6.

8. (7)

Domain $x \ge 3$

So $x \in (3,10]$

So number of integer n = 7

9. (2)

$0 \le x^2 + 2x - 3 < 1$

(1) (2)

(1) $(x+3)(x-1) \ge 0 \Rightarrow x \in (-\infty,3] \cup [1,\infty)$ … (1)

(2) $x^2 + 2x - 4 < 0 \Rightarrow x = -1 \pm \sqrt{5} - 1 - \sqrt{5} < x < -1 + \sqrt{5}$ …(2)

(1) n (2)

$x \in \left(-1-\sqrt{5}, -3\right] \cup \left[1, \sqrt{5}-1\right)$

So integer $x = \{-3, 1\}$

10. (1)

$0 \le \frac{x^2 - x - 2}{x^2} < \frac{4}{3}$ & solve as above

11. (9)

$\frac{\sqrt{2x^2 + 15x - 17}}{10 - x} \ge 0$

$2x^2 + 15x - 17 \ge 0$

$2x^2 + 17x - 2x - 17 \ge 0$

$x(2x+17) - (2x+17) \ge 0$

$(x-1)(2x+17) \ge 0$

$x \in (-\infty, -8.5] \cup [1, \infty)$

Also x < 10

$\therefore$ No. of integers positive are

{1, 2, 3, 4, 5, 6, 7, 8, 9}

No. of positive integers are 9.

12. (6)

Domain: $x \in [1,9]$ & on solving $x \in [1,5]$

So (m + n) = 6

13. (5)

Domain (a) $-x^2 + 2x + 2y \ge 0 \Rightarrow x^2 - 2x - 2y \le 0$

$\Rightarrow (x-6)(x-4) \le 0 \Rightarrow -4 \le x \le 6$ … (a)

(b) $8x - x^2 \ge 0 \Rightarrow x^2 8x \le 0 \Rightarrow x \in [0,8]$ …(b)

(a) n (b) $x \in [0,6]$ …(1)

(b) $-x^2 + 2x + 2y \ge 8x - x^2 \Rightarrow x \le y$ … (2)

(1) & (2) $x \in [0,4]$

So answer is (5)

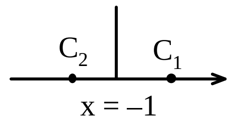
14. (1)

As function is increasing so equation change at most one solution

15. (1)
Same as Q. 13

16. (1)
Same as above

17. (2)
$\left|2^{x+1}-1\right|+\left|2^{x+1}+1\right|=2^{|x+1|}; x\in R$



C_1 : if $x \geq -1$ …(a)
$2.2^x - 1 + 2.2^x + 1 \Rightarrow 2^x.2$
$\Rightarrow 2^x = 0 \Rightarrow x \in \phi$ … (b)
(a) & (b) $x \in \phi$ … C_1
C_2: If $x < -1$
$1 - 2.2^x + 2.2^x + 1 = \frac{1}{2}.2^{-x}$
$\Rightarrow 2^{-x} = 4 \Rightarrow x = -2$ … (b)
(a) & (b) x = –2 … C_2
$C-1 \cup C-2$
$x = -2 \Rightarrow |x| = 2$

18. (7)
(x – 2) = 0 or 1 $\Rightarrow x = 2, 3$
or $\log_2 x^3 - 3\log_x{}^4 = 3 \Rightarrow 3\log_2{}^x - \frac{6}{\log_2 x} = 3$
$\Rightarrow \log_2^2 x - \log_2 x - 2 = 0 \Rightarrow (\log_2 x - 2)(\log_2 x + 1) = 0$
$\log_2 x = -1, 2 \Rightarrow x = \frac{1}{2}, 4$
So sum = 9.5

19. (8)
As function is increasing so only one solution
$x = (16)^2 = 2^8$

20. (2)
$\sqrt{\log_2^x} + \sqrt[3]{\log_2^x} = 2$
x = 2

21. (0)
$(|x| - 3)(|x| + 2) = 0 \Rightarrow |x| = -2, 3 \Rightarrow x = \pm 3$

22. (0)
$\log(x-2)(1-x) \geq -1$
$x \in \phi \Rightarrow$ No solution

23. (0)

$(x+1)(x-1)-2\sqrt{x^2-1}=4x-1$

$\Rightarrow -2\sqrt{x^2-1}=(2x-1)$

$\Rightarrow 4(x^2-1)=4x^2+1-4x \Rightarrow x=5/4$

But $x=\frac{5}{4}$. Doesn't satisfy so no solution

24. (2)

Same as Q. 18

25. (4)

$D \ge 0 \Rightarrow 16-\log_2 n \ge 0 \Rightarrow \log_2 n \le 2$

$(0<n\le 4)$

26. (1)

$(2x)^2 = x^2+75 \Rightarrow x^2=25 \Rightarrow x=\pm 5$

Checking at $x=\pm 5$ then only x = 5 possible

27. (2)

Same as above

28. (7)

(1) Domain: a) $x>0, x\ne \frac{1}{2}$ …(a)

b) $x^2-5x+6>0$

$\Rightarrow (x-2)(x-3)>0 \Rightarrow x \in (-\infty,2)\cup(3,\infty)$ … (b)

(a) & (b) $x\in(0,2)\cup(3,\infty)-\left\{\frac{1}{2}\right\}$ … (1)

(2) C–1: If $x>\frac{1}{2}$ … (a)

$x^2 - 5x + 6 > 2x$

$\Rightarrow x^2 - 7x + 6 > 0 \Rightarrow (x-6)(x-1) > 0$

$\Rightarrow x\in(-\infty,1)\cup(6,\infty)$ …(b)

(a) & (b) $x(6,\infty)$ … (C–1)

C–2 if $x<\frac{1}{2}$ … (a)

$x^2-5x+6<2x$

$x \le (1,6)$ … (b)

n $x\in\phi$ … (C–2)

$(C-1)\cup(C-2)$

$x\in(6,\infty)$ …(2)

(1) n (2) $x\in(6,\infty)$

29. (2)

$y^{(6-y)^2+7(6-y)+12}=1$

$\Rightarrow y^{y^2-19y+90}=1\Rightarrow^{(y-10)(y-9)}=1$

So y = 1, 9, 10, –1 $\Rightarrow$ x = 5, –3, –4, 7

So $(x,y)\equiv(5,1)$ or $(7,-1)$

30. (1)

$3+|x+2|=\sqrt{9-y^2}$

So x = –2, y = 0

So (–2, 0)

31. (2)

$\log(-x)=\sqrt{\log|x|}$

Domain: x < 0

$\log^2(-x)=\log(-x)$

log (–x) = 0, 1

–x = 1, 10 $\Rightarrow$ x = –1, –10

i.e. two solutions

32. (2)

Let $\log_a^x=t$

$$\frac{2}{t}+\frac{1}{1+t}+\frac{3}{2+t}=0$$

$\Rightarrow$ 2(1 + t) (2 + t) + t (2 + t) + 3t (1 + t) = 0

$\Rightarrow 6t^2 + 11t + 2 = 0$

$$t=\frac{-11\pm\sqrt{121-48}}{12}=\frac{-11\pm\sqrt{73}}{12}$$

i.e. two values

33. (3)

$N=\log_3(3^3.5).\log(5.3)-\log_3{}^5.\log_3(3^4.5)$

$=(3+\log_3{}^5)(1+\log_3{}^5)-(\log_3{}^5)(4+\log_3{}^5)$

= 3

34. (0)

log (x – 3) (x – 1)

$\left(\frac{1}{10}\right)(x-3)\geq 1$

log (x – 3) (x + 1)

$\Rightarrow 10(x-3)\leq 1$

Domain: $x\in(3,\infty)-\{4\}$

$10^{1+\log_{(x-3)}^{(x-1)}}\leq 1$

$\Rightarrow \log_{(x-3)}^{(x-1)}\leq -1$

Not possible as x > 3 so no solution

35. (2)

Same as Q. 32

36. (2)

$\sqrt{(x-1)+4-2.\sqrt{4}\sqrt{2-1}}+\sqrt{(x-1)+9-2\sqrt{9}\sqrt{-1}}=1$

$\Rightarrow \left|\sqrt{x-1}-2\right|+\left|\sqrt{x-1}-3\right|=1$

C_1: If $x \geq 4$ … (a)

$\left(\sqrt{x-1}-2\right)+\left(\sqrt{x-1}-3\right)=1 \Rightarrow \sqrt{x-1}=3$

x = 10 … (b)

(a) n (b) x = 10 … (C–1)

C_2: If $x \in (3,4)$ … (a)

$\left(\sqrt{x-1}-2\right)-\left(\sqrt{x-1}-3\right)=1 \Rightarrow 1=1 \Rightarrow x \in R$ …(b)

(a) n (b) $x \in (3,4)$ …(C – 2)

C_3: If $x \leq 3$ … (a)

$-\left(\sqrt{x-1}-2\right)-\left(\sqrt{x-1}-3\right)=1$

$\Rightarrow \left(\sqrt{x-1}-2\right)\left(\sqrt{x-1}-3\right)=1$

$\Rightarrow \sqrt{x-1}=2 \Rightarrow x=5$ … (b)

(a) n (b) $x \in \phi$ … (C – 3)

$(C-1)\cup(C-2)\cup(C-3)$

$x \in (3,4)\cup\{10\}$

So, x = 10

37. (0)

$2x^2\left(2^x\right)+4.2^{|x-3|}=16x^2.2^{|x-3|}+\frac{2^x}{2}$

$\Rightarrow 4x^2.\left(2^x\right)+8.2^{|x-3|}=32x^2.2^{|x-3|}+2^x$

$\Rightarrow 4x^2\left(2^x-8.2^{|x-3|}\right)=\left(2^x-8.2^{|x-3|}\right)$

So, $4x^2 = 1$ or $2^x=2^{|x-3|+3}$

$x=\pm\frac{1}{2}$ or $x=|x-3|+3$

$|x-3|=(x-3)\ x\geq 3$

So negative integral $x=\phi$

38. (0)

Domain: $x \geq -1$

$(x+1)+(x+4)+2\sqrt{(x+1)(x+4)}=(x+2)+(x+3)+2\sqrt{(x+2)(x+3)}$

$\Rightarrow$ (x + 1) (x + 4) = (x + 2) (x + 3)

$\Rightarrow$ y = 6 $\Rightarrow x \in \phi$

39. (2)

$x^{x-y}=y^{x+y}, \left(\sqrt{x}\right)y=1$

$x^{x-y}=\left[\left(\frac{1}{x}\right)^{1/2}\right]^{x+y}$

$$x - y = -\frac{(x+y)}{2}$$
$\Rightarrow 3x = y$
$\Rightarrow (3x).\sqrt{x} = 1$
$\Rightarrow x = \left(\frac{1}{3}\right)^{2/3}$
Also x = y = 1 is a possible solution
So two pairs possible

40. (1)
$2^{x+1} = y^2 + 4$ & $2^{x-1} \le y$
$\Rightarrow 2^x = \frac{y^2+4}{2}$ & $2^x \le 2y$
$\Rightarrow \frac{y^2+4}{2} \le 2y$
$\Rightarrow (y-2)^2 \le 0$
$\Rightarrow y = 2$
So no. of values is 1

41. (6)
$|x| + |y| = l$, k > 0
xy (x + y) = 0
$\Rightarrow$ x = 0 or y = 0 or x + y = 0
(i) If x = 0 $\Rightarrow |y| = k$
$\Rightarrow y = \pm k$
(ii) If y = 0 $\Rightarrow |x| = k$
$\Rightarrow x = \pm k$
(iii) If x + y = 0 $\Rightarrow x = -y$
$\Rightarrow |x| = |y|$
$\Rightarrow |x| + |x| = k$
$\Rightarrow x = \pm\frac{k}{2} = y$
Hence total 6 solutions

42. (7)
$x^2 - mx + 2m = 0$
$x_1 + x_2 = 1m$ … (1)
$x_1x_2 = 2m$ … (2)
Now, $x_1^3 + x_2^3 = x_1^2 + x_2^2$
$(x_1 + x_2)^3 - 3x_1x_2(x_1 + x_2) = (x_1 + x_2)^2 - 2x_1x_2$
$(m)^3 - 3.2m.\ m = (m)^2 - 2.2m$
$m^3 - 6m^2 = m^2 - 4m$
$m^3 - 7m^2 + 4m = 0$
$\Rightarrow m_1 = 0$ & $m_2 + m_3 = 7$
$\Rightarrow m_1 + m_2 + m_3 = 7$

43. (2)

$\left|x^2+3x\right|+x^2-2=0$

Case – I

$x^2+3x\ge 0$

$x(x+3)\ge 0$

$x\in(-\infty,-3)\cup(0,\infty)$

Now, $x^2+3x+x^2-2=0$

$2x^2+3x-2=0$

$x=1/2$ & $x=-2$

$\therefore x=1/2$ is the solution

Case – II

$x^2+3x<0$

$x(x+3)<0$

$x\in(-3,0)$

Now, $-(x^2+3x)+x^2-2=0$

$-3x-2=0$

$\Rightarrow x=-2/3$

$\therefore x=-2/3$ is the solution

$\Rightarrow$ Total 2 solutions

44. (1)

$4^{\frac{x}{y}+\frac{y}{x}}=32$

$2^{2\left(x/y+y/x\right)}=2^5$

$\Rightarrow 2\left(\frac{x}{y}+\frac{y}{x}\right)=5$ …(1)

$\log_3(x-y)=1-\log_3(x+y)$

$\Rightarrow \log_3\left(x^2-y^2\right)=1$

$\Rightarrow x^2-y^2=3$

$\Rightarrow x=\pm\sqrt{3+y^2}$ … (2)

Solving (1) & (2) we get

$x=\pm 2$ & $y=\pm 1$

Also $x-y>0$ & $x+y>0$

Only solution which satisfy is

$x=2$ & $y=1$

So one solution

45. (0)

$x_1+x_2-x_3-x_4=1$ … (1)

$x_1+2x_2+3x_3-x_4=2$ … (2)

$3x_1+5x_2+5x_3-3x_4=6$ … (3)

Here (1) & (2) gives

$x_2+4x_3=1$ … (4)

Also $2\times(2)-(3)$ gives

$x_2+4x_3=0$ … (5)

$\therefore$ From (4) & (5)

No solution