PHYSICS BOOKLET SOLUTION
VECTORS

EX – 1

1. Let forces be $\vec{A}$ and $\vec{B}$

Let $\left(\vec{A}\right) = x$, then $\left(\vec{B}\right) = 16 - x$

Now $R = \sqrt{x^2 + (16-x)^2 + 2x(16-x)\cos\theta}$

Also $\tan\left(90^o\right) = \dfrac{(16-x)\sin\theta}{x+(16-x)\cos\theta}$

$\therefore \quad x + (16-x)\cos\alpha = 0$

$\therefore \quad (16-x)\cos\theta = -x$

$\therefore \quad R = \sqrt{x^2 + (16-x)^2 + 2\ x\ (-x)} = 8$

$\Rightarrow (16-x)^2 - x^2 = 64 = 10^2 - 6^2$

$= (16-6)^2 - 6^2$

$\therefore \ x = 6$

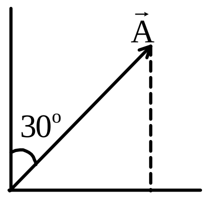
Forces are 6 N and 10 N

2. Component is equal to or smaller than the magnitude of vector

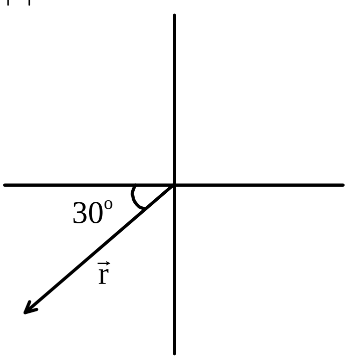
3. $A\cos 60^o = 10$

$\therefore \ A\dfrac{1}{2} = 10$

$\therefore \ A = 20$



4. $\left|\vec{r}\right| = 25$



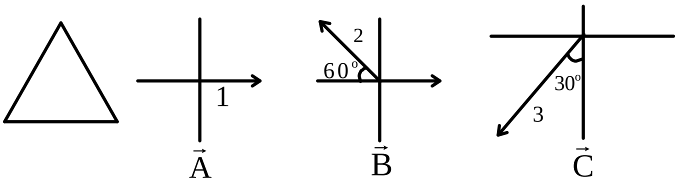
γ component = - 25 sin 30^o

$= -\dfrac{25}{2}$

= - 12.5

5. **(B)**

6.



$\vec{A} = 1\hat{i}$

$\vec{B} = -2\cos 60^\circ\,\hat{i} + 2\sin 60^\circ\,\hat{j}$

$\vec{C} = -3\sin 30^\circ\,\hat{i} - 3\cos 30^\circ\,\hat{j}$

Adding, $\vec{R} = \left(1 - 2\cos 60^\circ - 3\sin 30^\circ\right)\hat{i} + \left(2\sin 60^\circ - 3\cos 30^\circ\right)\hat{j}$

$$= \left(-\frac{3}{2}\right)\hat{i} + \left(2\frac{\sqrt{3}}{2} - 3\frac{\sqrt{3}}{2}\right)\hat{i}$$

$$= -\frac{3}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}$$

Magnitude $= \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3}$

7. $\tan\theta = \frac{\sqrt{3}/2}{3/2} = \frac{1}{\sqrt{3}}$

$\therefore \theta = 30^\circ$

So angle with 1 st

Vector $= 180^\circ + 30^\circ = 210^\circ$

8. Displacement $\vec{r} = 10\hat{j}$

$\vec{F} = -2\hat{i} + 15\hat{j} + 6\hat{k}$

$\therefore$ work = $\vec{F}\cdot\vec{r} = \left(-2\hat{i} + 15\hat{j} + 6\hat{k}\right)\left(10\hat{j}\right)$

= 150 joules

9. $\vec{A} = \hat{i} + \hat{j}$,

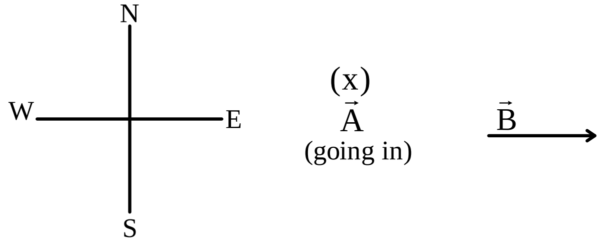
$\vec{B} = \hat{i} - \hat{j}$

Perpendicular unit vector $\rightarrow \pm \frac{\left(\hat{i} + \hat{j}\right)\times\left(\hat{i} - \hat{j}\right)}{\left|\left(\hat{i} + \hat{j}\right)\times\left(\hat{i} - \hat{j}\right)\right|}$

$$= \pm \frac{-\hat{k} - \hat{k}}{\left|-2\hat{k}\right|} = \pm\,\hat{k}$$

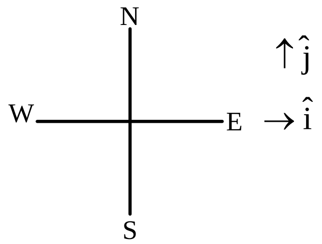
Vector of magnitude 3 perpendicular to $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j} \rightarrow \pm\, 3\,\hat{k}$

10. Top – view



$\vec{A}\times\vec{B}$ will be in south (using right hand thermal rule)

11. top – view



$\vec{V}_{in} = 5\hat{i}$

$\vec{v}_i = 5\hat{j}$

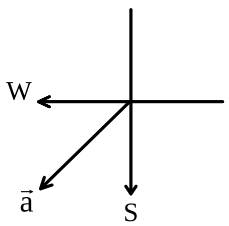
$\overrightarrow{\Delta v} = \overrightarrow{V_f} - \hat{V}$

$= 5\hat{j} - 5\hat{i}$

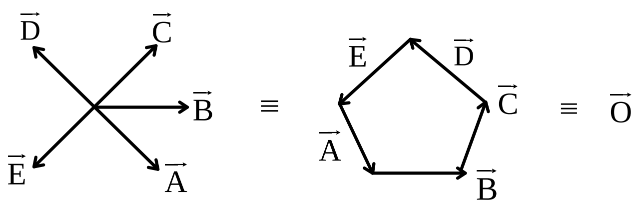
$\vec{a}_{arey} = \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{5\hat{j} - 5\hat{i}}{10} = \frac{\hat{j} - \hat{i}}{2}$

Magnitude $= \frac{1}{\sqrt{2}}$

South – west



12.



13. unit vector along AB

$= \frac{(4-1)\hat{i} + 4\hat{j} + 12\hat{k}}{\sqrt{3^2 + 4^2 + 12^2}}$

$= \frac{3\hat{i} + 4\hat{j} + 12\hat{k}}{13}$

Speed = 65m/s,

Time = 2 s

Distance = 130 m along AB

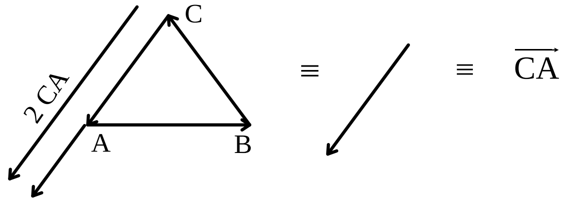
$\therefore \quad \overrightarrow{\text{Displacement}} = \frac{3\hat{i} + 4\hat{j} + 12\hat{k}}{13} \times 130$

$= 30\hat{i} + 40\hat{j} + 120\hat{k}$

$\therefore$ triangle position $\equiv (30+1)\hat{i} + 1 + 0\hat{j} + 120\tilde{\hat{k}}$

$= 31\hat{i} + 40\hat{j} + 120\hat{k}$

14.



15. $\sqrt{(3P)^2+(2P)^2+2.3P.2P\cos\theta}=R$ …(1)

$\sqrt{(6P)^2+(2P)^2+2.6P.2P\cos\theta}=2R$ …(2)

From (1) $13P^2+12P^2\cos\theta=R^2$

$\therefore\ 40P^2+24P^2\cos\theta=4\left(13P^2+12P^2\cos\theta\right)$

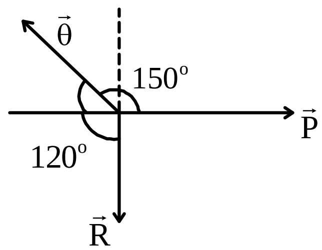
$\therefore\ 40+24\cos\theta=52+48\cos\theta$

$\Rightarrow\ -24\cos\theta=12$

$\cos\theta=-\frac{1}{2}$

$\therefore\ \theta=120^o$

16.



$\vec{P}=P\hat{i}$

$\vec{Q}=-\theta\cos30^o\,\hat{i}+\theta\sin30^o\,\hat{j}$

$\vec{R}=-R\,\hat{j}$

For equilibrium, sum = 0

$\therefore\ P-\theta\cos30^o=0$ $\quad\therefore\ \frac{P}{\theta}=\cos30^o=\frac{\sqrt{3}}{2}$

and $\theta\sin30^o-R=0$ $\quad\therefore\ \frac{\theta}{2}=R$

$\therefore\ \frac{\theta}{R}=\frac{2}{1}$

$\therefore\ P:Q:R=\sqrt{3}:2:1$

17. $\tan90^o=\frac{v\sin\theta}{u+v\cos\theta}$

$\therefore\ u+v\cos\theta=0$

$R=\sqrt{u^2+v^2+2uv\cos\theta}=\frac{1}{2}v$

$\Rightarrow\ (-v\cos\theta)^2+v^2+2(-v\cos\theta)v\cos\theta=\frac{v^2}{4}$

$\Rightarrow v^2\cos^2\theta+v^2-2v^2\cos^2\theta=\frac{v}{4}$

$\Rightarrow 1-\cos^2\theta=\frac{1}{4}$

$\Rightarrow \sin^2\theta=\frac{1}{2}$ $\quad\quad \sin\theta=\pm\frac{1}{2}$

$\theta=30^o,150^o$

18. **()**

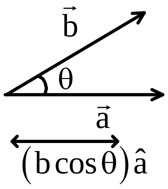
19. max $\rightarrow$ 6 + 8 = 14
min $\rightarrow$ 8 – 6 = 2
value can be 2 and 14
10 N

20. (a) [min, max] $\equiv$ [0, 4]
(b) [min, max] $\equiv$ [2, 6]
(c) [min, max] $\equiv$ [4, 8]
(d) [min, max] $\equiv$ [6, 10]

21. $\vec{a}=\hat{i}+2\hat{j}+2\hat{k}$

$\vec{b}=2\hat{i}+\hat{j}+2\hat{k}$

$b\cos\theta\ \hat{a}$



$=b\left(\frac{\vec{a}\cdot\vec{b}}{\vec{a}\ \vec{b}}\right)\hat{a} \quad = \frac{\vec{a}\cdot\vec{b}}{a}\hat{a}$

$=\frac{2+2+4}{3} \quad \frac{\hat{i}+2\hat{j}+2\hat{j}}{3} \quad = \frac{8}{9}\left(\hat{i}+2\hat{j}+2\hat{k}\right)$

22. $\overrightarrow{F_1}+\overrightarrow{F_2}+......\overrightarrow{F_{x-1}}=\overrightarrow{O}-\overrightarrow{F_n}$

$=\overrightarrow{F_n}$

Magnitude $=\left|-\overrightarrow{F_n}\right|=F$

23. $\vec{F}=\frac{\overrightarrow{J\,P}}{J\,t}$

Also $\vec{P}=2\cos t\,\hat{i}+2\sin t\,\hat{j}$

$\therefore\ \vec{F}=-2\sin t\ \hat{i}+2\cos+\hat{j}$

$\vec{F}.\vec{P}=-4\cos t\sin t+\sin t\cos t=0$

$\therefore$ Angle b/w $\vec{F}$ and $\vec{P}$ is 90^o

24. **(ABD)**

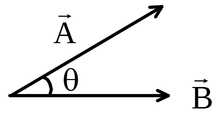
25. $\vec{r}=\left(a\cos w\,t\right)\hat{i}+\left(a\sin w\,t\right)\hat{j}$

$\vec{v}=-a\,w\sin w\,t\ i+a\,w\cos 40t\ \ \hat{j}$

Dot product is o so (6)

EX – 2
MULTIPLE CORRECT

1. $\vec{A} = 3\hat{i} + 4\hat{j}$, $\vec{B} = \hat{i} + \hat{j}$



Component of $\vec{A}$ along
$\vec{B}$ is A $\cos\theta$ $\hat{B}$

$= \left|\vec{A}\right| \cos\theta \ \frac{\hat{i}+\hat{j}}{\sqrt{2}}$

Component of $\vec{A}$ perpendicular to $\vec{B}$
Will be along
$\left(\hat{i}-\hat{j}\right)$ because $\left(\hat{i}-\hat{j}\right)$ is perpendicular to $\hat{i}+\hat{j}$
So, the other resolute component is

$\left(\vec{A}\right)\sin\theta\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$

2. $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$

$= \hat{i}(1-1) - \hat{j}(2-1) + \hat{k}(2-1) = -\hat{j}+\hat{k}$

Limit vector $\perp$ to $\vec{A}$ and $\vec{B}$ is $\left(\frac{-\hat{j}+\hat{k}}{\sqrt{2}}\right)$

So, (a) and (c) also (B)

3. $\left(\vec{v}_1+\vec{v}_2\right)\ \left(\vec{v}_1-\vec{v}_2\right) = 0$

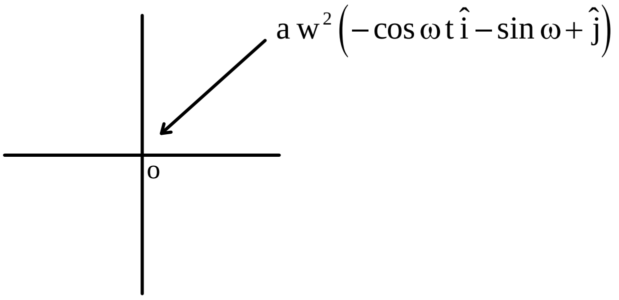
$\therefore\ \vec{v}_1\cdot\vec{v}_1 - \vec{v}_1\cdot\vec{v}_2 + \vec{v}_2\cdot\vec{v}_1 - \vec{v}_2\cdot\vec{v}_2 = 0$

$\Rightarrow \vec{v}_1\cdot\vec{v}_1 = \vec{v}_2\cdot\vec{v}_2$

$\Rightarrow v_1{}^2 = v_2{}^2$ or $\left|\vec{v}_1\right| = \left|\vec{v}_2\right|$

4. $\vec{A}+\vec{B}+\vec{C} = \vec{O}$ only if $\vec{A},\vec{B},\vec{C}$ form a triangle
Only vectors in same plane from a triangle

5. b and c are not meaning full because taking lot product of a scalar and a vector is meaningless.



6. $\vec{a}\cdot\vec{b} = 8$

$a\,b\ \cos\theta = 8$ …(1)

$\left|\vec{a}\times\vec{b}\right| = 8\sqrt{3}$

$a\,b\ \ \sin\theta = \pm 8\sqrt{3}$ …(2)

form (1) and (2),

$\frac{a\,b\sin\theta}{a\,b\cos\theta} = \pm\frac{8\sqrt{3}}{8} = \pm\sqrt{3} = 0$

$\therefore\ \ \theta = 60^o\ \text{ or }\ 120^o$

7. **(ACD)**

8. **()**

9. **(AB)**

$\sin\theta \le AB$

EX – 3

1. $\vec{A} = 4\hat{i} + 4\hat{j}$, $\vec{B} = 4\hat{i} - 4\hat{j}$

$\vec{A} + \vec{B} = 8\hat{j}$

$\left(\vec{A} + \vec{B}\right) = 8$

$\vec{A} - \vec{B} = 8\hat{i}$

$\left(\vec{A} - \vec{B}\right) = 8$

$\vec{A}\cdot\vec{B} = \left(4\hat{i} + 4\hat{j}\right)\cdot\left(4\hat{i} - 4\hat{j}\right)$

$= 16 - 16 = 0$

$\left|\vec{A}\times\vec{B}\right| = \left|-16\,\hat{k} - 16\,\hat{k}\right|$

$= \left|-32\,\hat{k}\right| = 32$

2. $\left|\vec{A}\right| = 1$, $\left|\vec{B}\right| = 2$, $\theta = 90^o$

$\vec{A}\cdot\vec{B} = 0$

$\left|\vec{A}\times\vec{B}\right| = AB\ \sin\theta\ = 1.2\ \ \sin 90^o$

$= \ 2$

$\left|\vec{A} + \vec{B}\right| = \sqrt{A^2 + B^2 + 2AB\cos 90^o}$

$= \sqrt{A^2 + B^2} = \sqrt{5}\ ;\ \left|\vec{A} - \vec{B}\right| = \sqrt{5}$

SUBJECTIVE

2. Apply right hand thumb rule

3. (a) $\vec{a} = 5\hat{i} + 4\hat{j} - 6\,\hat{k}$

$\vec{b} = -2\hat{i} + 2\,\hat{j} + 3\,\hat{k}$

$\vec{c} = 4\hat{i} + 3\hat{j} + 2\,\hat{k}$

$\vec{r} = \vec{a} - \vec{b} + \vec{c}$

$\vec{r} = 11\,\hat{i} + 5\hat{j} - 7\hat{k}$

(b) + ve 2 axis unit vector $= \hat{k}$

Angle b/w 2 axis and $\hat{k}$

$$\cos\theta = \frac{\left(11\,\hat{i} + 5\,\hat{j} - 7\,\hat{k}\right)\cdot\hat{k}}{\sqrt{11^2 + 5^2 + 7^2}\cdot 1}$$

$$= \frac{-7}{\sqrt{195}}$$

(c) $$\frac{\vec{a}\cdot\vec{b}}{\left|\vec{b}\right|} = \frac{-10 + 8 - 18}{\sqrt{2^2 + 2^2 + 3^2}}$$

$$= \frac{-20}{\sqrt{17}}$$

4. (a) $\vec{a} = 3\hat{i} + 5\hat{j}$

$\vec{b} = 2\hat{i} + 4\hat{j}$

$\vec{a}\times\vec{b} = \left(3\hat{i} + 5\hat{j}\right)\times\left(2\hat{i} + 4\hat{j}\right)$

$= 0 + 12\left(\hat{i}\times\hat{j}\right) + 10\left(\hat{j}\times\hat{i}\right)$

$= 12\,\hat{k} - 10\,\hat{k}$

$= 2\,\hat{k}$

(b) $\vec{a}\cdot\vec{c}$

$= \left(3\hat{i} + 5\hat{j}\right)\cdot\left(2\hat{i} + 4\hat{j}\right)$

$= 6 + 20 = 26$

(c) $\left(\vec{a} + \vec{b}\right)\cdot\vec{b}$

$\left(5\hat{i} + 9\hat{j}\right)\cdot\left(2\hat{i} + 4\hat{j}\right)$

$= 10 + 36 = 46$

(d) $$\frac{\vec{a}\cdot\vec{b}}{\left|\vec{b}\right|} = \frac{6 + 20}{\sqrt{2^2 + 4^2}} = \frac{26}{\sqrt{20}} = \frac{13}{\sqrt{5}}$$

6. $\left|\vec{B}\right| = B = 8$

$\vec{A} = A\,\hat{i}$

C = 2A

To find,

$\vec{A} + \vec{B} = \vec{C} = C\,\hat{j}$

$\vec{B} = C\hat{j} - A\hat{i}$

$= 2A\,\hat{j} - A\hat{i}$

Also $\left|\vec{B}\right| = 8$

$\therefore\ \sqrt{(2A)^2 + A^2} = 8$

$\Rightarrow \sqrt{5A^2} = 8$

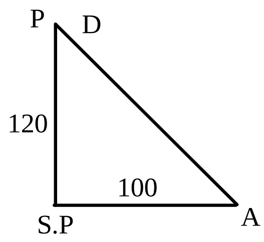
$$\therefore A^2 = \frac{6^4}{5}$$

$A = \frac{8}{\sqrt{5}}$

7. S.P $\equiv$ Starting point

D $\equiv$ Destination

A $\equiv$ The point to which ship was shown away



(a) $AD = \sqrt{(100)^2 + (120)^2} = 156.2$

(b) If last $\equiv \hat{i}$, north $\equiv \hat{j}$

$\overrightarrow{AD} = -100\hat{i} + 120\hat{j}$

8. $\vec{a} - \vec{b} = 2\vec{c}$ …(1)

$\vec{a} + \vec{b} = 4\vec{c}$ …(2)

$\vec{c} = 3\hat{i} + 4\hat{j}$

Adding (1) and (2),

$2\,\vec{a} = 6\,\vec{c}$

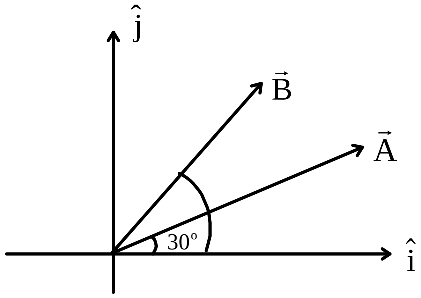
$\therefore\ \vec{a} = 3\hat{i} = 9\hat{i} + 12\hat{j}$

Subtracting (1) from (2),

$2\vec{b} = 2\vec{c}$

$\therefore \vec{b} = \vec{c} = 3\hat{i} + 4\hat{j}$

(a)



$\vec{A} = 10\cos 30\hat{i} + 10\sin 30\hat{j}$

$= 5\sqrt{3}\,\hat{i} + 5\hat{j}$

$\vec{B} = 10\cos 60\,\hat{i} + 1 \Rightarrow \sin 60\hat{j}$

$5\hat{i} + 5\sqrt{3}\,\hat{j}$

$\vec{R} = \vec{A} + \vec{B} = \left(5\sqrt{3} + 5\right)\hat{i} + \left(5 + 5\sqrt{3}\right)\hat{j}$

10. $\overrightarrow{AB} = (9-2)\hat{i} + (25-1)\hat{j}$

$= 7\hat{i} + 24\hat{j}$

$\widehat{AB} = \frac{7\hat{i} + 24\hat{j}}{25}$

Velocity is along AB.

So $\hat{V} = \frac{7\hat{i} + 24\hat{j}}{25}$

$\vec{V} = V \cdot \hat{V}$

$= 50\frac{7\hat{i} + 24\hat{j}}{25}$

$= 14\,\hat{i} + 48\hat{j}$

11. let vectors be $\vec{A}$ and $\vec{B}$

Then $|A - B| = 10$

$\sqrt{A^2 + B^2 + 2AB\cos 90^o} = 50$

$\Rightarrow \sqrt{A^2 + B^2} = 50$

Let A has high magnitude

Then, A – B = 10

Or A = B + 10

Solving these equations,

A = 40, B = 30

Check : A – B = 40 – 30 = 10

$\sqrt{A^2 + B^2} = \sqrt{40^2 + 30^2} = 50$

12. (a) No, it should have a fired spatial direction and magnitude.

(b) No, yes

(c) yes, When $\vec{b} = \vec{0}$

(d) $\vec{b} + b = \vec{c}$

$\therefore |\vec{a} + \vec{b}| = |\vec{c}|$

$\Rightarrow \sqrt{a^2 + b^2 + 2ab\cos\theta} = C$

Also a + b = c

$\Rightarrow \sqrt{a^2 + b^2 + 2a\ b\cos\theta} = a + b$

$\Rightarrow a^2 + b^2 + 2ab\cos\theta = a^2 + b^2 + 2ab$

$\therefore \cos\theta = 1$ or vectors are parallel

(e) 90^o

(f) No, time has not got a spatial direction.

(g) $\underbrace{\vec{a} \times \vec{b}}_{\downarrow} = \underbrace{\vec{a} \cdot \vec{b}}_{\downarrow}$

vector scalar

(h) No.

13. $|\vec{A}| = 2$

$|\vec{B}| = 3$

$\theta = 60^o$

(a) $\vec{A} \cdot \vec{B} = AB\cos\theta$

$2.3\cos 60^o \quad = 2.3.\frac{1}{2}$

$= 3$

(b) $|\vec{A} \times \vec{B}| = A\ B\sin\theta$

$= 2.3.\frac{\sqrt{3}}{2} = 3\sqrt{3}$