Binomial Theorem

Exercise – 1

Q.1 [B]

$$\sum_{m=0}^{100} {}^{100}C_m \left(x-3\right)^{100-m} 2^m = \left(x-3+2\right)^{100} = \left(x-1\right)^{100} = \left(1-x\right)^{100}$$

$\Rightarrow$ The coefficient of $x^{53} = -{}^{100}C_{53}$

Q.2 [B]

$$\left.\begin{aligned}\left(1+\sqrt{5}\right)^5 &= 1+{}^nC_1\left(\sqrt{5}\right)^1+{}^nC_2\left(\sqrt{5}\right)^2+{}^nC_3\left(\sqrt{5}\right)^3+\dots\dots\dots\\ \left(1-\sqrt{5}\right)^5 &= 1-{}^nC_1\left(\sqrt{5}\right)^1+{}^nC_2\left(\sqrt{5}\right)^2-{}^nC_3\left(\sqrt{5}\right)^3+\dots\dots\dots\end{aligned}\right\}\left(n=5\right)$$

Adding,

$$\Rightarrow \left(1+\sqrt{5}\right)^5+\left(1-\sqrt{5}\right)^5 = 2\left[1+{}^5C_2\left(5\right)+{}^5C_4\left(25\right)\right]$$

$\Rightarrow 352$

Q.3 [A]

3r + r + 2 = 2n = n

$$\Rightarrow \frac{n}{2} = r$$

Q.4 [C]

General term,

$$\Rightarrow T_{r+1} = {}^9C_r\left(\frac{4}{3}x^2\right)^{9-r}\left(\frac{-3}{2x}\right)^r$$

$\Rightarrow 2\,(9 - r) - r = 0$

$\Rightarrow r = 6$

$\Rightarrow 7^{th}$ term.

Q.5 [B]

3^{rd} term = $\frac{-1}{8}x^2 = \frac{m(m-1)}{2}x^2$

$\Rightarrow m = \frac{1}{2}$

Q.6 [D]

Last term $\left(-2^{-\frac{1}{2}}\right)^n = \left(3^{\frac{-5}{3}}\right)^{\log_3^8} = (8)^{\log_3\left(3^{\left(\frac{-5}{3}\right)}\right)} = 8^{-\frac{5}{3}} = \frac{1}{32}$

$\Rightarrow n = 10$

Hence, 5^{th} term from beginning $= {}^{10}C_4\left(2^{\frac{1}{3}}\right)^6\left(-2^{\frac{-1}{2}}\right)^4$

$\Rightarrow {}^{10}C_4$

Q.7

$T_5 + T_6 = 0$

$\Rightarrow {}^nC_4 b^4 a^{n-4}(-1)^4 + {}^nC_5 b^5 a^{n-5}(-1)^5 = 0$

$\Rightarrow \frac{a}{b} = \frac{n-4}{5}$

Q.8 [A]

$(1+ix)^{4n-2} = {}^nC_0 + {}^nC_1(ix)^1 + {}^nC_2(ix)^3 + ...(ix)^{4n-2}$

Negative real terms are corresponding to the exponents

$\Rightarrow 4t-2, t \in N, 1 < t < n$

Q.9 [A]

The coefficient of t^{32} in the expansion of

$\Rightarrow (1+t^2)^{12}(1+t^{12}+t^{24}+t^{36})$ is

$\Rightarrow {}^{12}C_4 + {}^{12}C_{10} = {}^{12}C_6 + 2$

Q.10 [A]

Coefficient of x $= (-2)\,{}^8C_3 + 3\times {}^8C_5$

$\Rightarrow (-2)\dfrac{8\times7\times6}{6} + \dfrac{3\times8\times7\times6}{6}$

$\Rightarrow 56$

Q.11 [A]

Every term in the expansion is irrational, since each term in the expansion consists of an even and an odd exponent.

Q.12 [A]

Terms which do not have fractional power of variable are

$\Rightarrow \left(x^{\frac{1}{5}}\right)^{55}, {}^{55}C_{10}\left(x^{\frac{1}{5}}\right)^{45}\left(y^{\frac{1}{10}}\right)^{10}, {}^{55}C_{20}\left(x^{\frac{1}{5}}\right)^{35}\left(y^{\frac{1}{10}}\right)^{50}, {}^{55}C_{30}\left(x^{\frac{1}{5}}\right)^{25}\left(y^{\frac{1}{10}}\right)^{30},$

${}^{55}C_{40}\left(x^{\frac{1}{5}}\right)^{15}\left(y^{\frac{1}{10}}\right)^{40}$ and ${}^{55}C_{50}\left(x^{\frac{1}{5}}\right)^{5}\left(y^{\frac{1}{10}}\right)^{50}$.

Q.13

5^{th} term + 7^{th} term = 2 (6^{th} term)

$\Rightarrow {}^nC_4 + {}^nC_6 = 2\times {}^nC_5$

$\Rightarrow \dfrac{n!}{(n-4)!\,4!} + \dfrac{n!}{(n-6)!6!} = \dfrac{2\times n!}{(n-5)!\,5!}$

$\Rightarrow \dfrac{1}{(n-4)(n-5)} + \dfrac{1}{5\times6} = \dfrac{2}{(n-5)\times5}$

Q.14 [A]

${}^5C_2(x)^2\left(x^{\log_{10}^{x}}\right)^3 = 10^6$

$\Rightarrow (x)^{2+3\log_{10}^{x}} = 10^5$

Taking log with base = 10 on both sides.

$$\Rightarrow \left(2+3\log_{10}^{x}\right)\left(\log_{10}^{x}\right)=5$$

$$\Rightarrow 3\left(\log_{10}^{x}\right)^{2}+2\left(\log_{10}^{x}\right)-5=0$$

$$\Rightarrow \log_{10}^{x}=\frac{-2\pm\sqrt{4+60}}{6}=1,\frac{-5}{3}$$

Hence, $x=10^{1}=10$

Q.15

$$^{8}C_{5}\left(x^{-\frac{8}{3}}\right)^{3}\left(x^{2}\log_{10}^{x}\right)^{5}=5600$$

$$\Rightarrow x^{2}\left(\log_{10}^{x}\right)^{5}=\frac{5600\times 6}{8\times 7\times 6}=100$$

$$\Rightarrow \log_{10}^{x}=\left(\frac{100}{x^{2}}\right)^{\frac{1}{5}}=\left(\frac{10}{x}\right)^{\frac{2}{5}}$$

Q.16 [A]

$$^{n}C_{1}+^{n}C_{2}=36$$

$$\Rightarrow {}^{n+1}C_{2}=36$$

$$\Rightarrow \frac{(n+1)!}{(n-1)!2!}=36$$

$\Rightarrow$ (n + 1) n = 72

$\Rightarrow$ n = 8 [as n = – 9 is rejected, – 9 not being a natural number]

Now, $\dfrac{^{8}C_{2}\left(2^{x}\right)^{6}\left(2^{-2x}\right)^{2}}{^{8}C_{1}\left(2^{x}\right)^{7}\left(2^{-2x}\right)^{1}}=7$

$$\Rightarrow x=-\frac{1}{3}$$

Q.17 [C]

Middle terms are T_5 & T_6

$$\Rightarrow T_5 = {}^9C_4 (2a)^4 \left(\frac{-a^2}{4}\right)^5 = \frac{-63}{32} a^{14}$$

$$\Rightarrow T_6 = {}^9C_5 (2a)^5 \left(\frac{-a^2}{4}\right)^4 = \frac{63}{4} a^{13}$$

Q.18 [A]

$T_4 > T_3$ & $T_4 > T_3$

$$\Rightarrow {}^{10}C_3 2^7 \left(\frac{3|x|}{8}\right)^3 > {}^{10}C_2 2^8 \left(\frac{3|x|}{8}\right)^2 \quad \& \quad {}^{10}C_3 2^7 \left(\frac{3x}{8}\right)^3 > {}^{10}C_4 2^6 \left(\frac{3x}{8}\right)^4$$

$$\Rightarrow \frac{8}{3\times 2}\left(\frac{3x}{8}\right) > 1 \quad \& \quad 1 > \frac{7}{4}\frac{3|x|}{8\times 2}$$

$$\Rightarrow |x| > 2 \quad \& \quad |x| < \frac{64}{21}$$

$$\Rightarrow x < -2 \text{ or } x > 2 \quad \& \quad \frac{-64}{21} < x < \frac{64}{21}$$

Hence, $x \in \left(\frac{-64}{21}, -2\right) \cup \left(2, \frac{64}{21}\right)$

Q.19 [A]

$T_4 > T_5$ & $T_4 > T_3$

$$\Rightarrow {}^{10}C_3 \; 2^7 \left(\frac{3|x|}{8}\right)^3 > {}^{10}C_4 2^6 \left(\frac{3|x|}{8}\right)^4 \quad \& \quad {}^{10}C_3 2^7 \left(\frac{3|x|}{8}\right)^3 > {}^{10}C_2 2 \left(\frac{3|x|}{8}\right)^2$$

$$\Rightarrow 2 > \frac{7}{4} \times \frac{3|x|}{8}$$

$$\Rightarrow |x| < \frac{64}{21} \quad \& \quad \frac{8}{3\times 2}\frac{3|x|}{8} > 1$$

$\Rightarrow |x| < \frac{64}{21}$ & $|x| > 2$

Q.20 [C]

Term independent of x $\Rightarrow {}^{10}C_r (x \sin\alpha)^{10-r} \left(\frac{\cos\alpha}{x}\right)^r$ is corresponding to 10 – 2r = 0

$\Rightarrow$ r = 5

Hence, the term is ${}^{10}C_5 (\sin\alpha)^5 (\cos\alpha)^5$

$$\Rightarrow \frac{{}^{10}C_5}{2^5}(\sin 2\alpha)^5$$

Greater value of term independent of x = $\frac{(10)!}{5!\,5!2^5}$

Q.22 [A]

${}^{n_1+n_2}P_2 = 90$

$\Rightarrow (n_1+n_2)(n_1+n_2-1) = 90$ & $3 \times {}^{n_1-n_2}P_2 = 90$

Let, $n_1+n_2 = t$ & $(n_1-n_2)(n_1-n_2-1) = 30$

$\Rightarrow t^2 - t - 90 = 0$

$\Rightarrow$ t = 10, – 9 & $n_1 - n_2 = 6, -5$ ………(ii)

$\Rightarrow \therefore n_1 + n_2 = 10$ ……(i)

Hence, $(n_1 n_2) = (8, 2)$

Q.23 [D]

$$\frac{(n-1)!}{(n-r-1)!\,r!} = \frac{n!}{(n-r-1)!\,(r+1)!}(k^2-3)$$

$$\Rightarrow (k^2-3) = \frac{(r+1)}{n}$$

Now, $n \geq r+1$

$\Rightarrow \frac{n}{r+1} \geq 1$

$\Rightarrow k^2 - 3 \leq 1$

$\Rightarrow k \in [-2, 2]$ ……….(i)

Also,

$\Rightarrow k^2 - 3 > 0$

$\Rightarrow k \in \left(-\infty, -\sqrt{3}\right) \cup \left(\sqrt{3}, \infty\right)$

Q.24 [C]

${}^{n}P_r = 5040\ {}^{n}C_r$

$\Rightarrow r! = 5040 = 7!$

$\Rightarrow r = 7$

Q.25 [A]

${}^{35}C_8 + \sum_{r=1}^{7} {}^{42-r}C_7 + \sum_{r=1}^{5} {}^{47-r}C_7$

$\Rightarrow {}^{35}C_8 + {}^{35}C_7 + {}^{36}C_7 + {}^{37}C_7 + {}^{38}C_7 + {}^{39}C_7 + {}^{40}C_7 + {}^{41}C_7 + {}^{42}C_7 + {}^{43}C_7 + {}^{44}C_7 + {}^{45}C_7 + {}^{46}C_7$

$\Rightarrow {}^{47}C_8$ (using ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$)

Q.26 [D]

$(1+x)^{131}\left(x^2 - x + 1\right)^{130} = (1+x)\left(1+x^3\right)^{130}$

$\Rightarrow$ Coefficient of $x^{65} = 0$.

Q.31

$\frac{1}{n!}\left[{}^{n}C_1 + {}^{n}C_2 + {}^{n}C_3 + \dots\dots\dots + {}^{n}C_n\right] = \frac{1}{n!}\left[2^n - 1\right]$

Q.35 [A]

$$T_4 = \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{6}\left(\frac{3x}{2}\right)^3$$

$$\Rightarrow \frac{27}{128}x^3$$

Q.36 [A]

$$E = \frac{(1+x)^{n+1}-1}{x}$$

$\Rightarrow$ Hence, the coefficient of x^k in $E = {}^{n+1}C_{k+1}$

Q.37 [D]

$$S = C_0 + 3C_1 + 5C_2 + \text{.............} + (2n+1)C_n$$
$$S = (2n+1)C_n + (2n-1)C_{n-1} + \text{.......} + C_o$$
$$2S = (2n+2)(C_0 + C_1 + C_2 + \text{.........} + C_n)$$

$$\Rightarrow S = (n+1)2^n$$

Q.38 [C]

$$S = C_1^2 + C_3^2 + C_5^2 + \text{...............} + C_{n-2}^2 + C_n^2$$

$$\Rightarrow S = C_{n-1}^2 + C_{n-3}^2 + C_{n-5}^2 + \text{...........} + C_2^2 + C_0^2$$

$$\Rightarrow 2S = \left(C_0^2 + C_1^2 + C_2^2 + C_3^2 + C_4^2 + \text{...........} + C_n^2\right) = {}^{2n}C_n$$

$$\Rightarrow S = \frac{1}{2}\left({}^{2n}C_n\right) = \frac{(2n)!}{(n!)^2\,2}$$

Q.39 [A]

$$S = {}^{n}C_0 + {}^{n}C_1 + {}^{n+1}C_2 + {}^{n+2}C_3 + \text{..............} + {}^{n+r-1}C_r$$

$$\Rightarrow {}^{n+1}C_1 + {}^{n+1}C_2 + {}^{n+2}C_3 + \text{.......} + {}^{n+r-1}C_r$$

$$\Rightarrow {}^{n+2}C_2 + {}^{n+2}C_3 + \text{..........} + {}^{n+r-1}C_r$$

$$\Rightarrow {}^{n+3}C_3 + {}^{n+3}C_4 + \text{............} + {}^{n+r-1}C_r$$

$$\Rightarrow \left({}^{n+r-1}C_{r-1}\right)+\left({}^{n+r-1}C_{r}\right)$$

$$\Rightarrow {}^{n+r}C_{r} \qquad \left[\because {}^{n}C_{r}+{}^{n}C_{r-1}={}^{n+1}C_{r}\right]$$