WINDOW TO JEE MAIN

SEQUENCES & SERIES

Q.1 [C]

$$\Rightarrow 1^3 - 2^3 + 3^3 - .... + 9^3 = 1^3 + 2^3 + 3^3 + ...9^3 - 2\left(2^3 + 4^3 + 6^3 + 8^3\right)$$

$$\Rightarrow \left(\frac{n\left(n+1\right)}{2}\right)^2 - 2\left(2^3 \cdot 1^3 + 2^3 \cdot 2^3 + 2^3 \cdot 3^3 + 2^3 \cdot 4^3\right)$$

$$\Rightarrow \left(\frac{n\left(n+1\right)}{2}\right)^2 - 2 \cdot 2^3\left(\frac{n^1\left(n^1+1\right)}{2}\right) \qquad ....(1)$$

where n = 9 and $n^1 = 4$

$$\Rightarrow (1) = \left(\frac{9\left(9+1\right)}{2}\right)^2 - 2 \cdot 2^3\left(\frac{4\left(4+1\right)}{2}\right)^2$$

$\Rightarrow 425$

Q.2 [C]

$$\Rightarrow S_\infty = 20 = a + ar + ar^2 + ... = \frac{a}{1-r} \qquad ....(1)$$

Where $|r| < 1$

$$\Rightarrow S_\infty^1 = a^2 + a^2r^2 + a^2r^4 + ... = \frac{a^2}{1-r^2} = 100 \qquad ....(2)$$

$$\Rightarrow \text{From (1) } 1 - r = \frac{a}{20} \qquad ....(3)$$

Substituting this in (2) we get

$$\Rightarrow \frac{a^2}{\left(1-r\right)\left(1+r\right)} = 100$$

$$\Rightarrow \frac{a}{\frac{a}{20}\left(1+r\right)} = 100$$

$$\Rightarrow \frac{a\,20}{(1+r)}=100$$

$$\Rightarrow \frac{a}{5}=1+r \qquad \ldots\ldots\ldots\ldots\ldots(4)$$

Adding (3) and (4)

$$\Rightarrow 2=\frac{a}{20}+\frac{a}{5}$$

$$\Rightarrow 2=\frac{5a}{20}$$

$\Rightarrow a = 8$

$$\Rightarrow \therefore r=\frac{3}{5}$$

Q.3 [B]

$\Rightarrow T_3 = 7$

$\Rightarrow a + 2d = 7 \qquad \ldots\ldots\ldots\ldots(1)$

Now $T_7+2=3T_3$

$\Rightarrow a + 6d – 2 = 3.7$

$\Rightarrow a + 6d = 23 \qquad \ldots\ldots\ldots..(2)$

solving (1) and (2)

$$\begin{array}{l} a+2d=7 \\ a+6d=23 \\ - \;\; - \\ \hline -4d=-16 \end{array}$$

$\Rightarrow d = 4$

from (1)

$\Rightarrow a = – 1$

$\Rightarrow$ Now, $S_{20} = \frac{20}{2}\left[-2\cdot 1 + (20-1)4\right]$

$\Rightarrow$ 10 [3 + 76] = 730.

Q.4 [B]

Let α and β are the roots of the equation

$\Rightarrow x^2 - (\alpha+\beta)x + \alpha\beta = 0$

A.M. of α and β is $\frac{\alpha+\beta}{2} = 9 \Rightarrow \alpha+\beta = 18$

G.M. of α and β is $\sqrt{\alpha\beta} = 4 \Rightarrow \alpha\beta = 4$

Therefore equation is $x^2 - 18x + 16 = 0$

Q.5 [A]

Given that $T_m = \frac{1}{n}$

$\Rightarrow a + (m-1)d = \frac{1}{n}$ ………..(1)

$\Rightarrow$ and $T_n = \frac{1}{m}$

$\Rightarrow a + (n-1)d = \frac{1}{m}$ ………(2)

Solving (1) and (2); we get

$\Rightarrow a = \frac{1}{mn}$ and d = mn

$\Rightarrow \therefore$ a – d = 0

Q.6 [B]

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 .... = \frac{n(n+1)^2}{2}$

When n is even

Let n is odd i.e. n = 2m + 1

Then $S_{2m+1} = S_{2m} + (2m+1)^{\text{th}}$ term

$$\Rightarrow \frac{2m(2m+1)^2}{2} + (2m+1)^2$$

$$\Rightarrow \frac{(n-1)n^2}{2} + n^2$$

$$\Rightarrow n^2\left[\frac{n-1+2}{2}\right]$$

$$\Rightarrow \frac{n^2(n+1)}{2}$$

Q.7 [D]

$$x = \sum_{n=0}^{n} a^n = 1 + a + a^2 + ...$$

$$\Rightarrow x = \frac{1}{1-a}$$

Similarly $y = \frac{1}{1-b}$ and $z = \frac{1}{1-c}$

Now, a, b, c are in A.P.

$\Rightarrow$ – a, – b, – c are in A.P.

$\Rightarrow$ 1 – a, 1 – b, 1 – c are in A.P.

$\Rightarrow \frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c}$ are in H.P.

Q.8 [C]

$$\frac{a_1 + a_2 + ... + a_p}{a_1 + a_2 + ... + a_q} = \frac{p^2}{q^2}$$

$$\Rightarrow \frac{\frac{p}{2}\left[2a_1+(p-1)d\right]}{\frac{q}{2}\left[2a_1+(q-1)d\right]}=\frac{p}{q}$$

$$\Rightarrow \frac{(2a_1-d)+pd}{(2a_1-d)+qd}=\frac{p}{q}$$

$$\Rightarrow (2a_1-d)(p-q)=0$$

$$\Rightarrow a_1=\frac{d}{2}$$

Now, $\frac{a_6}{a_{21}}=\frac{a_1+5d}{a_1+20d}=\frac{\frac{d}{2}+5d}{\frac{d}{2}+20d}$

$$\Rightarrow \frac{11d}{41d}=\frac{11}{41}$$

Q.9 [C]

$a_1, a_2, a_3 ....a_n$ are in H.P

$\Rightarrow \therefore \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3} .... \frac{1}{a_n}$ are in A.P

Let d be the common difference of A.P

$$\Rightarrow \therefore \frac{1}{a_2}-\frac{l}{a_1}=d \Rightarrow a_1-a_2=a_1a_2d$$

Similarly, $a_3-a_2=a_2a_3d$

$\vdots$

$a_{n-1}-a_n=a_{n-1}a_nd$

On adding all of them, we get.

$\Rightarrow a_1-a_n=d\left(a_1a_2+a_2a_3+..........+a_{n-1}a_n\right)$ ……….(1)

and $T_n=\frac{1}{a_n}=\frac{1}{a_1}+(n-1)d$

$$\Rightarrow \therefore d = \frac{a_1 - a_n}{a_1 a_n (n-1)}$$

Substituting this in (1) we get

$$\Rightarrow a_1 - a_n = \frac{a_1 - a_n}{a_1 a_n (n-1)}\left(a_1 a_2 + \text{..........} a_{n-1} a_n\right)$$

$$\Rightarrow a_1 a_2 + \text{.........} a_{n-1} a_n = a_1 a_n (n-1)$$

Q.10 [D]

$$a = ar + ar^2$$

$$\Rightarrow a = ar\,(1 + r)$$

$$\Rightarrow 1 = r + r^2$$

$$\Rightarrow r^2 + r - 1 = 0$$

$$\Rightarrow r = \frac{-1 \pm \sqrt{1+4}}{2}$$

$$\Rightarrow \frac{-1 \pm \sqrt{5}}{2}$$

$$\Rightarrow r = \frac{\sqrt{5} - 1}{2}$$

Q.11 [B]

$$s = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \text{.....}$$

$$\frac{s}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \text{........}$$

$$- \quad - \qquad\quad -$$

$$\frac{2}{3}s = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^4} + \text{........}$$

$$\Rightarrow \frac{4}{3} + \frac{\frac{4}{3^2}}{1 - \frac{1}{3}} = 2$$

$\Rightarrow$ S = 3

Q.12

Let it take p minutes for to count all the notes.

Clearly p > 10.

$$\Rightarrow \therefore \sum_{i=1}^{p} a_i = \sum_{i=1}^{10} a_i + \sum_{i=11}^{p} a_i = \sum_{i=1}^{10} 150 + \sum_{i=11}^{p} a_i \left[150 - (i-10)2\right]$$

$$\Rightarrow \sum_{i=1}^{10} 150 + \sum_{i=11}^{p} (170 - 2i)$$

$\Rightarrow$ 4500 = – (p + 1) + 170p – 90

$$\Rightarrow p^2 - 169p + 4590 = 0$$

$\Rightarrow$ (p – 135) (p – 34) = 0

$\Rightarrow$ p = 34 as (170 – 2i) format be negative.

Q.13 [D]

Let it happened after m months.

$$\Rightarrow 2 \times 300 + \frac{m-3}{2}\left(2 \times 240 + (m-4) \times 40\right) = 11040.$$

$$\Rightarrow m^2 + 5m - 546 = 0$$

$\Rightarrow$ (m + 26) (m – 21) = 0

$\Rightarrow$ m = 21

Q.14 [B]

$$S_1 = 1 + (1+2+4) + (4+6+9) + (9+12+16) + \ldots.. + (361+380+400)$$

Number of terms in sequin all 20

$$\Rightarrow S_2 = \sum_{k=1}^{20} \left(k^3 - (k-1)^3\right) = n^3 = (20)^3 = 8000$$

Q.15 [D]

$$100\left(T_{100}\right)=50\left(T_{50}\right)$$

$$\Rightarrow 100\left(a+99d\right)=50\left(a+49d\right)$$

$\Rightarrow$ 2 (a + 99d) = a + 49d

$\Rightarrow$ a + 149d = 0

$$\Rightarrow T_{50}=0$$

Q.16 [C]

$$S=0.7+0.77+0.777+.......$$

$$\Rightarrow \frac{7}{10}+\frac{77}{100}+\frac{777}{10^3}$$

$$\Rightarrow 7\left[\frac{1}{10}+\frac{11}{10^2}+\frac{111}{10^3}+.......\right]$$

$$\Rightarrow \frac{7}{9}\left[\frac{9}{10}+\frac{99}{10^2}+\frac{999}{10^3}+.......\right]$$

$$\Rightarrow \frac{7}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10^2}\right)+\left(1-\frac{1}{10^3}\right)+......\right]$$

$$\Rightarrow \frac{7}{9}\left[20-\frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^{20}\right)}{1-\frac{1}{10}}\right]$$

$$\Rightarrow \frac{7}{9}\left[20-\frac{1}{9}\left(1-\left(\frac{1}{10}\right)^{20}\right)\right]$$

$$\Rightarrow \frac{7}{9}\left[\frac{179}{9}+\frac{1}{9}\left(\frac{1}{10}\right)^{20}\right]$$

$$\Rightarrow \frac{7}{81}\left[179+\left(10\right)^{20}\right]$$

Q.17 [A]

$2y = x + z$

$\Rightarrow 2\tan^{-1} y = \tan^{-1} x + \tan^{-1} z$

$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+2}{1-xy}\right)$

$\Rightarrow \frac{2y}{1-y^2} = \frac{x+z}{1-xy}$

$\Rightarrow \frac{x+z}{1-y^2} = \frac{x+z}{1-xy}$

$\Rightarrow y^2 = xz$ or $z + x = 0$

$\Rightarrow x = y = z$