WINDOW TO JEE MAIN

QUADRATIC EQUATIONS

Q.1 [B]

$\Rightarrow x^2 - 5x - 16 = 0$ α, β are roots ……….(1)

$\Rightarrow x^2 + px + q = 0$ $\alpha + \beta + 2, \frac{\alpha\beta}{2}$ are roots ……….(2)

From (1)

$\Rightarrow \alpha + \beta = +5$ and $\alpha\beta = -16$

Form (2)

$\Rightarrow \alpha + \beta + 2 + \frac{\alpha\beta}{2} = -p$ and $(\alpha + \beta + 2)\left(\frac{\alpha\beta}{2}\right) = q$

$\Rightarrow 5 + 2 - \frac{16}{2} = -p$ and $(5 + 2)\left(-\frac{16}{2}\right) = q$

$\Rightarrow$ p = 1 and – 56 = q

So, p = 1 and q = – 56

Q.2 [C]

(x – a) (x – b) = c.

$\Rightarrow x^2 - ax - bx + ab = c$

$\Rightarrow x^2 - (a + b)x + ab - c = 0$

$\Rightarrow \alpha, \beta$ are roots.

$\Rightarrow \alpha + \beta = a + b$ and $\alpha\beta = ab - c$

Now in equation $(x - \alpha)(x - \beta) + c = 0$

$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta + c = 0$

$\Rightarrow x^2 - (a + b)x + ab = 0$

$\Rightarrow \therefore$ a and b are the roots.

Q.3 [A]

The equation having α and β as its roots is $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

Since $\alpha^2 = 5\alpha - 3 \Rightarrow \alpha^2 - 5\alpha + 3 = 0$

and $\beta^2 = 5\beta - 3 \Rightarrow \beta^2 - 5\beta + 3 = 0$

These two equations shows that α and β are the roots of the equation

$\Rightarrow x^2 - 5x + 3 = 0$

$\Rightarrow \therefore \alpha + \beta = 5$ and $\alpha\beta = 3.$

$$\Rightarrow \text{Now, } \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{25-6}{3} = \frac{19}{3}$$

Q.4 [D]

Let α and β be the roots of $ax^2 + bx + c = 0$ then $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$\Rightarrow \alpha + \beta = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$$

$$\Rightarrow -\frac{b}{a} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}} \Rightarrow \frac{-b}{a} = \frac{b^2 - 2ac}{c^2}$$

$\Rightarrow -bc^2 = b^2a - 2a^2c$

$\Rightarrow 2a^2c = b^2a + bc^2$

$$\Rightarrow \frac{2a}{b} = \frac{b}{c} + \frac{c}{a}$$

$\Rightarrow \therefore \frac{b}{c}, \frac{a}{b}, \frac{c}{a}$ are in A.P

$\Rightarrow \frac{c}{b}, \frac{b}{a}, \frac{a}{c}$ are in H.P

Q.5 [B]

$\left(a^2 - 5a + 3\right)x^2 + (3a - 1)x + 2 = 0$

Let α and 2α are the roots of this equation, then

$$\Rightarrow \alpha + 2\alpha = -\frac{(3a-1)}{\left(a^2 - 5a + 3\right)}$$

$$\Rightarrow 3\alpha = -\frac{(3a-1)}{\left(a^2 - 5a + 3\right)} \Rightarrow \alpha = \frac{-(3a-1)}{3\left(a^2 - 5a + 3\right)} \qquad \ldots\ldots\ldots\ldots.(1)$$

$$\text{and } \alpha \cdot 2\alpha = \frac{2}{a^2 - 5a + 3} \Rightarrow 2a^2 = \frac{2}{a^2 - 5a + 3} \qquad \ldots\ldots\ldots\ldots.(2)$$

$$\Rightarrow \therefore 2\left[\frac{-(3a-1)}{3\left(a^2 - 5a + 3\right)}\right] = \frac{2}{\left(a^2 - 5a + 3\right)} \qquad \text{[from (1) and (2)]}$$

$$\Rightarrow \frac{(3a-1)^2}{9\left(a^2 - 5a + 3\right)^2} = \frac{1}{a^2 - 5a + 3}$$

$$\Rightarrow (3a-1)^2 = 9\left(a^2 - 5a + 3\right)$$

$$\Rightarrow 9a^2 + 1 - 6a = 9a^2 - 45a + 27$$

$$\Rightarrow 45a - 6a = 27 - 1$$

$$\Rightarrow a = \frac{2}{3}$$

Q.6 [C]

$x^2 - 3|x| + 2 = 0$

$\Rightarrow$ If $x > 0$, then $|x| = x$

$\Rightarrow \therefore x^2 - 3x + 2 = 0$

$\Rightarrow (x - 1)(x - 2) = 0$

$\Rightarrow x = 1, 2$

$\Rightarrow$ If $x < 0$ then $|x| = -x$

$\Rightarrow x^2 + 3x + 2 = 0$

$\Rightarrow (x + 1)(x + 2) = 0$

$\Rightarrow x = -1, -2$

four solution possible

Q.7 [C]

(1 – P) is one root of $x^2 + px + (1-p) = 0$

$\Rightarrow \therefore (1-p)^2 + p(1-p) + (1-p) = 0$

$\Rightarrow 1 + p^2 - 2p + p - p^2 + 1 - p = 0$

$\Rightarrow -2p + 2 = 0$

$\Rightarrow p = 1$

putting value of p in equation

$\Rightarrow x^2 + x = 0$

$\Rightarrow x(x + 1) = 0$

$\Rightarrow x = 0, -1$

Q.8 [A]

$x^2 + px + 12 = 0$; 4 is root

$\Rightarrow \therefore 16 + 4p + 12 = 0 \Rightarrow p = -7$

For equation $x^2 - 7x + q = 0$ whose roots are equal.

Let the roots be α and α

$\Rightarrow \therefore \alpha + \alpha = \frac{7}{1} \Rightarrow 2\alpha = 7 \Rightarrow \alpha = \frac{7}{2}$

and $\alpha - \alpha = q \Rightarrow \alpha^2 = q$

$\Rightarrow \left(\frac{7}{2}\right)^2 = q \Rightarrow q = \frac{49}{4}$

Q.9 [A]

$x^2 - (a-2)x - a - 1 = 0$

Let α and β be the roots.

Then, $\alpha + \beta = a - 2$

and $\alpha\beta = -a - 1$

Now sum of squares of roots is

$\Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow (a-2)^2 - 2(-a-1)$

$\Rightarrow a^2 + 4 - 4a + 2a + 2$

$\Rightarrow a^2 - 2a + 6$

$\Rightarrow a^2 - 2a + 1 + 5$

$\Rightarrow (a - 1) + 5$ …………(1)

for a = 1, (1) will obtain minimum values.

Q.10 [D]

$x^2 - bx + c = 0$

Let the roots be α and $\alpha + 1$

Then, $\alpha + \alpha + 1 = +b$

$\Rightarrow 2\alpha + 1 = b$

$\Rightarrow \alpha(\alpha + 1) = c \Rightarrow \alpha^2 + \alpha = c$

$\Rightarrow b^2 - 4c = (2\alpha + 1)^2 - 4(\alpha^2 + \alpha)$

$\Rightarrow 4\alpha^2 + 1 + 4\alpha - 4\alpha^2 - 4\alpha$

$\Rightarrow 1$

Q.11 [B]

$\angle P + \angle Q + \angle R = 100^o$

$\Rightarrow \text{As } \angle R = \frac{\pi}{2} \Rightarrow \angle P + \angle Q = \frac{\pi}{2} \Rightarrow \frac{\angle P}{2} + \frac{\angle Q}{2} = \frac{\pi}{4}$

$\Rightarrow \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = \frac{\tan\frac{P}{2} + \tan\frac{Q}{2}}{1 - \tan\frac{P}{2}\tan\frac{Q}{2}}$ ....(1)

$\Rightarrow ax^2 + bx + c = 0 \text{ and } \tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right) \text{are roots}$

$\Rightarrow \text{then } \tan\frac{P}{2} + \tan\frac{Q}{2} = -\frac{b}{2}$

$\Rightarrow \text{and } \tan\frac{P}{2} - \tan\frac{Q}{2} = \frac{c}{a}$

substituting these value in (1)

$\Rightarrow \tan\left(\frac{\pi}{4}\right) = \frac{\frac{-b}{a}}{1 - \frac{c}{a}}$

$\Rightarrow 1 = \frac{-b}{a - c} \Rightarrow a - c = -b \Rightarrow a + b = c$

Q.12

$x^2 - 2kx + k^2 - k - 5 = 0$

Both roots are then 5 when

$\Rightarrow (1)\ \frac{-b}{2a} < 5$

$$\Rightarrow \frac{2k}{2\left(k^2+k-5\right)}<5$$

$$\Rightarrow \frac{k}{k^2+k-5}<5 \Rightarrow k < 5.$$

$\Rightarrow$(2) f (5) > 0

$$\Rightarrow 25-10k+k^2+k-5>0$$

$$\Rightarrow k^2-9k+20>0$$

$$\Rightarrow k \in \left(-\infty,4\right)\cup\left(5,\infty\right) \qquad \text{……..(2)}$$

$\Rightarrow$(3) $D \geq 0$

$$\Rightarrow 4k^2-4\left(k^2+k-5\right)\geq 0$$

$$\Rightarrow k \leq 5$$

From results of (1), (2) and (3)

$$\Rightarrow k \in \left(-\infty,4\right)$$

Q.13 [A]

x^2+px+q

$\Rightarrow \tan 30^o$ and $\tan 15^o$ are roots

$\Rightarrow$then $\tan 30^o+\tan 15^o=-p$

$\Rightarrow$and $\tan 30^o \times \tan 15^o = q$

$\Rightarrow$as $30^o+15^o=45^o$

taking tan both sides

$$\Rightarrow \tan\left(30^o+15^o\right)=\tan 45^o$$

$$\Rightarrow \frac{\tan 30^o+\tan 15^o}{1-\tan 30^o \tan 15^o}=1$$

$\Rightarrow \frac{-p}{1-q} = 1$

$\Rightarrow - p = 1 - q \Rightarrow q - p = 1$

Now, 2 + q – p = 2 + 1 = 3.

Q.14 [B]

Both roots of equation $x^2 - 2mx + m^2$ are greater then – 2 and less than 4 when

(1) $D \geq 0 \Rightarrow 4m^2 - 4m^2 + 4 \geq 0 \Rightarrow m \in R$

(2) $-2 < -\frac{b}{2a} < 4 \Rightarrow -2 < \frac{2m}{2 \cdot 1} < 4$

$\Rightarrow - 2 < m < 4 \Rightarrow m \in (-2, 4)$

(3) $f(4) > 0 \Rightarrow 16 - 8m + m^2 - 1 > 0$

$\Rightarrow m \in (-\infty, 3) \cup (-5, \infty)$

(4) $f(-2) > 0 \Rightarrow 4 + 4m + m^2 - 1 > 0$

$\Rightarrow m \in (-\infty, 3) \cup (-1, \infty)$

From results of (1), (2), (3) and (4)

$\Rightarrow m \in (-1, 3)$

Q.15 [A]

$$\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7} = 1 + \frac{10}{3\left(x^2 + 3x + \frac{7}{3}\right)}$$

$$\Rightarrow 1 + \frac{10}{3\left[\left(x + \frac{3}{2}\right)^2 + \frac{1}{12}\right]}$$

$\Rightarrow \therefore$ Maximum value is at $x = -\frac{3}{2}$

$\Rightarrow$ And minimum value is $=1+\dfrac{10}{3\left(\dfrac{1}{12}\right)}=1+40=41$

Q.16 [A]

$x^2+ax+1=0$

Let α,β are the roots.

Then $\alpha+\beta=-a$ and $\alpha\beta=1$

$\Rightarrow |\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$

$\Rightarrow \sqrt{a^2-4}$

Now, $|\alpha-\beta|<\sqrt{5}\Rightarrow\sqrt{a^2-4}<\sqrt{5}$

$\Rightarrow a^2-4<5\Rightarrow a^2-a<0$

$\Rightarrow a\in(-3,3)$

Q.17 [C]

$x^2-6x+a=0$

Let α and 4β are the roots

Then, $\alpha+4\beta=6$ and $4\alpha\beta=a$ ……….(1)

$\Rightarrow x^2-cx+6=0$ Let α and 3β are the roots

Then $\alpha+3\beta=0$ and $3\alpha\beta=6$ ……….(2)

From (1) and (2)

We get $\alpha\beta=2\Rightarrow$ a = 8

So first equation is $x^2 - 6x + 8 = 0$

And roots are 2,4

If $\alpha=2$ then $4\beta=4\Rightarrow 3\beta=3$

If $\alpha = 4$ then $4\beta = 2 \Rightarrow 3\beta = \frac{3}{2}$ (non integer)

Therefore common roots is α i.e. 2

Q.18 [A]

$f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$

$\Rightarrow f'(x) = x^6 + 70x + 48x^2 + 30 > 0 \ \ \forall x$

$\Rightarrow \therefore$ f (x) is an increasing function and cut the x – axis only once.

Q.19 [C]

As $bx^2 + cx + a = 0$ has imaginary roots

So, $c^2 < 4ab$.

Now, $3b^2x^2 + 6bcx + 2c^2$

$\Rightarrow 3(bx + c)^2 - c^2 \geq -c^2 \geq -4ab$

Q.20 [A]

Le the correct equation is $ax^2 + bx + c = 0$ sachin writes constant term as c'

$\Rightarrow \therefore 4 + 3 = \frac{-b}{a}$ ....(1) and $4 \cdot 3 = \frac{c'}{a}$ ....(2)

Rahul writes coefficient of x as b'

$\Rightarrow$ Then $3 + 2 = \frac{-b'}{a}$ ....(3) and $3 \cdot 2 = \frac{c}{a}$ ....(4)

From (1) and (4)

$\Rightarrow -\frac{b}{a} = 7$ and $\frac{c}{a} = 6$

$\Rightarrow \therefore$ correct equation is $x^2 + 7x + 6 = 0$ and roots are 6, 1