WINDOW TO JEE MAINS

STRAIGHT LINE

Q.1 [A]

Pair of lines $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

$\Rightarrow$ At y axis $x = 0$

$\Rightarrow$ So $by^2 + 2fy + c = 0$

$\Rightarrow D = 4f^2 + 4bc = 0$

$\Rightarrow f^2 = bc$

$\Rightarrow \Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow (abc) + 2fgh - a(bc) - bg^2 - ch^2 = 0$

$\Rightarrow 2fgh = bg^2 + ch^2$

Q.2 [A]

$3ax^2 + 5xy + \left(a^2 - 2\right)y^2 = 0$ ....(i)

Equation (i) represent homogeneous equation of pair of lines as $y = mx$

So, $m^2\left(a^2 - 2\right) + 5m + 3a = 0$

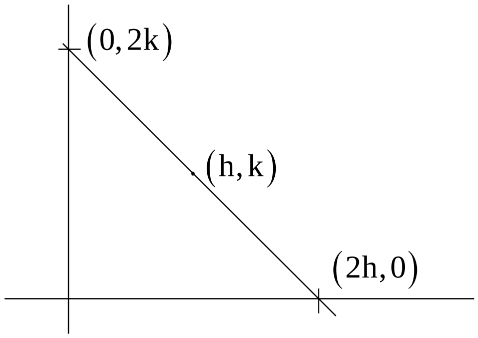
$\Rightarrow m_1m_2 = \frac{3a}{a^2 - 2} = -1$

$\Rightarrow a^2 - 2 + 3a = 0$

$\Rightarrow D = 9 - 4(-2) = 17 > 0$

$\Rightarrow$ So a has 2 real values.

Q.3 [D]



$$\frac{x}{2h}+\frac{y}{2k}=1 \qquad \ldots\ldots\ldots\ldots(i)$$

$$\Rightarrow x\cos a+y\sin a=p \qquad \ldots\ldots\ldots\ldots(ii)$$

Both (i) & (ii) are same so compare

$$\frac{\cos a}{\left(\frac{1}{2}b\right)}=\frac{\sin a}{\left(\frac{1}{2}k\right)}=\frac{p}{1}$$

$$\Rightarrow \cos a=\frac{p}{2h}, \sin a=\frac{p}{2k}$$

$$\Rightarrow \frac{p^2}{4h^2}+\frac{p^2}{4k^2}=1$$

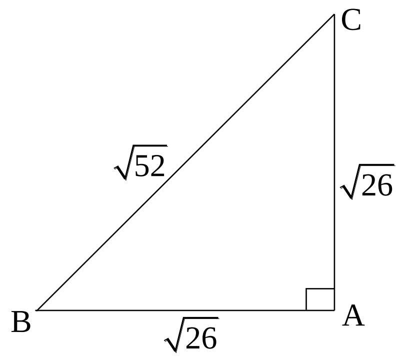
$\Rightarrow$Sp locus of (h, k) is

$$\Rightarrow \frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{p^2}$$

Q.4 [A]

A (4, 0), B (– 1, – 1), C (3, 5)

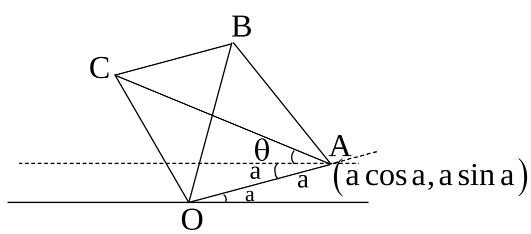
$$\Rightarrow AB=\sqrt{25+1}=\sqrt{26}$$



$\Rightarrow BC = \sqrt{52}$

$\Rightarrow AC = \sqrt{26}$

Q.5 [C]



Diagonal AC makes θ angle with 'x' axis

$\Rightarrow \theta = 45^o - a$

$\Rightarrow \therefore m_{AC} = \tan(\pi - \theta) = -\tan\theta$

$\Rightarrow -\tan(45^o - a) = -\left[\frac{1-\tan a}{1+\tan a}\right]$

$\Rightarrow \therefore$ Equation of AC is $y - a\sin a = \left(\frac{-1+\tan a}{1+\tan a}\right)(x - a\cos a)$

$\Rightarrow$ (sin a + cos a) y – a sina (sin a + cos a)

$\Rightarrow$ (– sin a – cos a) x – a cos a (sin a – cos a)

$\Rightarrow$ y (sin a + cos a) + x (cos a – sin a) = a

Q.6 [C]

$x^2 - 2pxy - y^2 = a$ ……….(i)

$\Rightarrow x^2 - 2qxy - y^2 = 0$ ……….(ii)

(i) & (ii) are angle bisector of each other.

So angle bisector of (i) is $\frac{x^2 - y^2}{1+1} = \frac{xy}{-p}$

$\Rightarrow x^2 - y^2 + \frac{2}{p}xy = 0$ ……….(iii)

$\Rightarrow$ As (ii) & (iii) are same so $-2q = \frac{2}{p} \Rightarrow pq = -1$

Q.7 [A]

A (a cos t; a sin t), B (cosin t, – b cos t), C (1, 0)

Let centroid of ΔABC is (h, k)

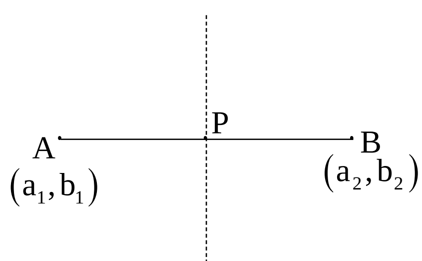
$\Rightarrow h = \frac{a\cos t + b\sin t + 1}{3}, k\frac{a\sin t - b\cos t}{3}$

$\Rightarrow$ So, $(3h-1)^2 + (3k)^2 = a^2 + b^2$

$\Rightarrow$ So locus is $(3x-1)^2 + (3y)^2 = a^2 + b^2$

Q.8 [D]

$A(a_1, b_1)$ $B(a_2, b_2)$



P is mid-point of A & B

$$\Rightarrow \therefore P\left(\frac{a_1+a_2}{2},\frac{b_1+b_2}{2}\right)\text{will satisfy the equation}$$

$$\Rightarrow (a_1-a_2)x+(b_1-b_2)y+c=0$$

$$\Rightarrow \therefore C=\frac{a_1^2-a_2^2+b_1^2-b_2^2}{2}$$

Q.9

A (2, – 3), B (– 2, 1), C (h, k)

$$\Rightarrow \text{Centroid } G\left(\frac{2-2+h}{3},\frac{-3+1+k}{3}\right)\equiv G\left(\frac{h}{3},\frac{-2+k}{3}\right)$$

$\Rightarrow$ G lies on 2x + 3y = 1

$$\Rightarrow \text{So, } 2\left(\frac{h}{3}\right)+3\left(\frac{-2+k}{3}\right)=1$$

$\Rightarrow$ 2h + 3k = 9

So locus is 2x + 3y = 9.

Q.10 [D]

y – 3 = m (x – 4)

$$\Rightarrow \left.\begin{matrix}\text{x intercept } = 4-\left(\frac{3}{m}\right)\\ \text{y intercept } = 3-4m\end{matrix}\right\}\text{given}$$

$$\Rightarrow 4-\frac{3}{m}+3-4m=-1$$

$$\Rightarrow 4m+\frac{3}{m}-8=0$$

$$\Rightarrow 4m^2-8m+3=0$$

$\Rightarrow$ (2m – 3) (2m - 1) = 0

$$\Rightarrow m=\frac{3}{2},\frac{1}{2}$$

$\Rightarrow$ So lines are 3x – 2y = 6 & x – 2y = 2

Q.11 [A]

$x^2 - 2cxy - 7y^2 = 0$

Pair of straight lines passes through origin

$\Rightarrow$ y = mx

So, $7m^2 + 2cm - 1 = 0$

$$\Rightarrow m_1 m_2 = -\frac{1}{7}, m_1 + m_2 = -\frac{2c}{7}$$

$$\Rightarrow \therefore \left(-\frac{2c}{7}\right) = 4\left(-\frac{1}{7}\right)$$

$\Rightarrow$ C = 2

Q.12 [D]

$6x^2 - xy + 4cy^2 = 0$

$\Rightarrow 4cm^2 - m + 6 = 0$

One of the line 3x + 4y = 0

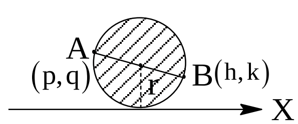
$$\Rightarrow m_1 = -\frac{3}{4}$$

$$\Rightarrow 4c\left(\frac{9}{16}\right) + \frac{3}{4} + 6 = 0$$

$\Rightarrow$ c = – 3

Q.13 [C]

A (p, q), B (h, k)



$$\Rightarrow r = \frac{AB}{2}$$

$$\Rightarrow \left(\frac{q+k}{2}\right) = \frac{\sqrt{(p-h)^2+(k-q)^2}}{2}$$

$$\Rightarrow q^2+k^2+2qk = (h-p)^2+k^2+q^2-2qk$$

$$\Rightarrow (h-p)^2 = 4qk$$

So locus of (h, k) is

$$\Rightarrow (x-p)^2 = 4qy$$

Q.14 [B]

$(ax+2by+3b)+\lambda(bx-2ay-3a)=0$ ……….(1) is family of lines

as (1) is parallel to x – axis so m = 0

$$\Rightarrow (a+\lambda b) = 0$$

$$\Rightarrow \lambda = -\frac{a}{b}$$

So equation is $(ax+2by+3b)-\frac{a}{b}(bx-2ay-3a)=0$

$$\Rightarrow 2(b^2+a^2)y+3(b^2+a^2)=0$$

$$\Rightarrow y = -\frac{3}{2}$$

Q.15

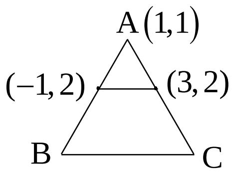
$\frac{2}{b}=\frac{1}{a}+\frac{1}{c}$ ……….(1)

$\Rightarrow$Equation of line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$

$$\Rightarrow \frac{x}{a}+\frac{y}{b}+\left(\frac{2}{b}-\frac{1}{a}\right)=0$$

$\Rightarrow \frac{1}{a}(x-1)+\frac{1}{b}(y+2)=0$; represent a family of lines, passes through (1, – 2)

Q.16 [D]

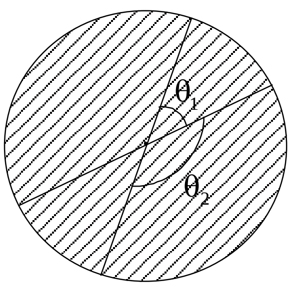


$\Rightarrow$ So, $B(-3,3)$

$\Rightarrow C(5,3)$

$\Rightarrow \therefore G \equiv \left(\frac{1-3+5}{3}, \frac{1+3+3}{3}\right) \equiv \left(1.\frac{7}{3}\right)$

Q.17 [C]



$\theta_1 = \frac{\theta_2}{3}$

$\Rightarrow \theta_1 + \theta_2 = 180^o$

$\Rightarrow \theta_1 = 45^o$

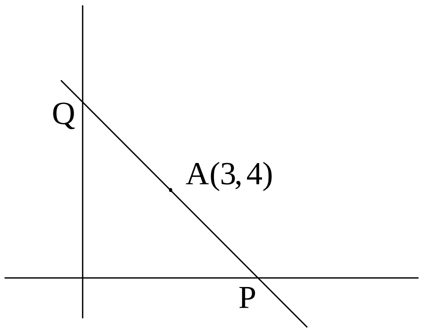
$\Rightarrow$ Angle between pair of lines $ax^2 + 2(a+b)xy + by^2 = 0$ is 45^o

$\Rightarrow$ So, $\tan\theta = \frac{2\sqrt{(a+b)^2 - ab}}{a+b} = 1$

$\Rightarrow (a+b)^2 = 4\left[a^2 + b^2 + ab\right]$

$\Rightarrow 3a^2 + 3b^2 + 2ab = 0$

Q.18 [C]



A is mid-point of P & Q

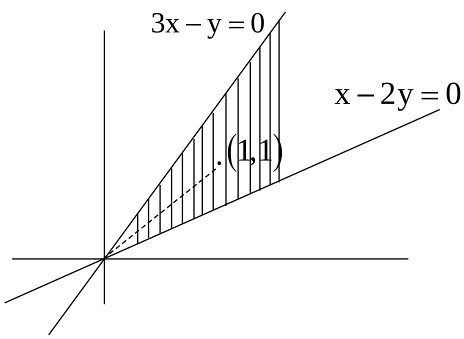
$\Rightarrow$ So, P (6, 0), Q (8, 0)

$\Rightarrow$ So equation of line is

$$\Rightarrow \frac{x}{6} + \frac{y}{8} = 1$$

$\Rightarrow 4x + 3y = 24$

Q.19



$\Rightarrow y = 3x; x > 0$

$$\Rightarrow y = \frac{x}{2}; x > 0$$

Pint (1, 1) lies in the shaded region.

(a, a^2) also lies in the same region then

$\Rightarrow 3(a) - a^2 > 0$

$\Rightarrow a\ (a - 3) < 0$

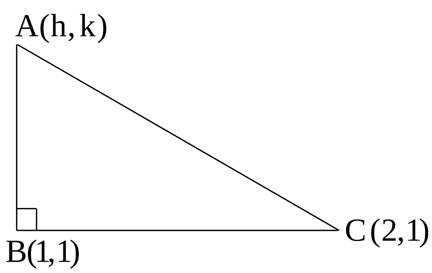
& $a - 2a^2 < 0$

$\Rightarrow a\ (2a - 1) > 0$

$\Rightarrow a \in (0, 3) \wedge a \in (-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)$

$\Rightarrow$ So, $a \in \left(\frac{1}{2}, 3\right)$

Q.20 [A]



As $\angle B = 90^o$

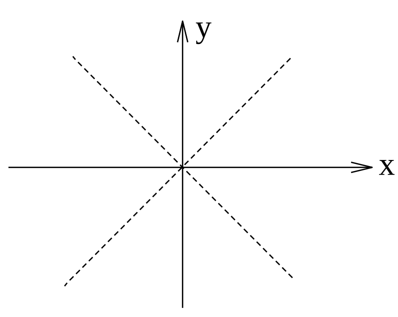
$\Rightarrow$ So, $h = 1$

Area of triangle $= \frac{1}{2}(1)|k - 1| = 1$

$\Rightarrow |k - 1| = 2$

$\Rightarrow k = 3, -1$

Q.22 [A]



$\Rightarrow 3y^2 + \left(1 - m^2\right)xy - mx^2 = 0$

$\Rightarrow m\left(\frac{y}{x}\right)^2 + \left(1 - m^2\right)\left(\frac{y}{x}\right) - m = 0$

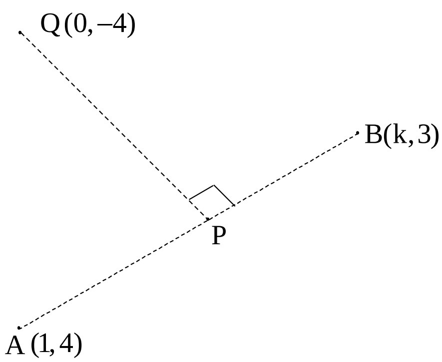
$\Rightarrow$ Scope of the lines are 1 & – 1.

$\Rightarrow$ So, $-\frac{\left(+1 - m^2\right)}{m} = 0 \Rightarrow m = \pm 1$

$\Rightarrow \frac{-m}{m} = -1 \Rightarrow -1 = -1$

$\Rightarrow$ So $m = \pm 1$

Q.23 [A]



$\Rightarrow P\left(\frac{k+1}{2}, \frac{7}{2}\right) \quad Q\left(0, -4\right)$

$\Rightarrow m_{PQ} = \frac{\frac{7}{2} + 4}{\left(\frac{k+1}{2}\right) - 0} = \frac{15}{k+1}$

$\Rightarrow m_{AB} = \frac{4-3}{1-k} = \frac{1}{1-k}$

$\Rightarrow \therefore \left(\frac{15}{k+1}\right)\left(\frac{1}{1-k}\right) = -1$

$\Rightarrow k = \pm 4$

Q.24 [A]

$p\left(p^2+1\right)x-y+q=0$ ....(1)

$\Rightarrow \left(p^2+1\right)^2 x+\left(p^2+1\right)y+2q=0$ ....(2)

Both lines (1) & (2) perpendicular to a given line then (1) & (2) have to be parallel or identical.

$$\Rightarrow \therefore \frac{p\left(p^2+1\right)}{\left(p^2+1\right)^2}=\frac{-1}{\left(p^2+1\right)}$$

$\Rightarrow p=-1$

Q.25 [D]

$\frac{x}{5}+\frac{y}{b}=1$ ....x = 13 & y = 32

$$\Rightarrow \frac{13}{5}+\frac{32}{b}=1$$

$$\Rightarrow \frac{32}{b}=-\frac{8}{5}$$

$\Rightarrow$b = – 20

$\Rightarrow \frac{x}{5}-\frac{y}{20}=1$ ....(1)

$\Rightarrow \frac{x}{c}+\frac{y}{3}=1$ ....(2)

Equation (1) & (2) both are parallel to

$$\Rightarrow \frac{\left(\frac{1}{5}\right)}{\left(\frac{1}{20}\right)}=-\frac{3}{c}$$

$$\Rightarrow 4=-\frac{3}{c}$$

$\Rightarrow c = -\frac{3}{4}$

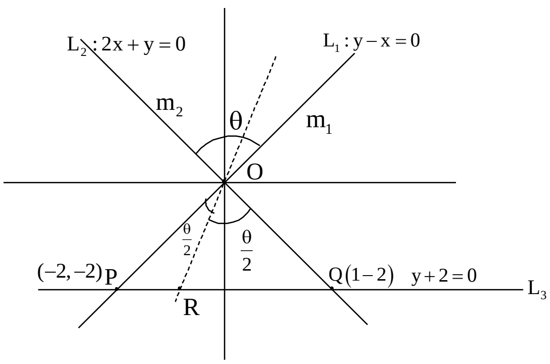
$\Rightarrow$ So, equation (1) is x – y = 20 ....(L)

$\Rightarrow$ Equation (2) is 4x – y = – 3 ....(k)

$\Rightarrow$ Distance between (L) & (k) = $\frac{20-(-3)}{\sqrt{16+1}}$

$\Rightarrow \frac{23}{\sqrt{17}}$

Q.26



$$\Rightarrow \frac{PR}{RQ} = \frac{|OP|}{|OQ|} = \frac{\sqrt{4+4}}{\sqrt{1+4}} = \frac{2\sqrt{2}}{\sqrt{5}}$$

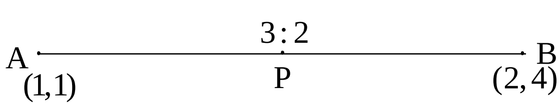
$$\Rightarrow \tan\theta = \frac{m_2 - m_1}{1+m_1m_2} = \frac{(-2)-(1)}{1+(-2)(1)} = \frac{-3}{-1} = 3$$

$\Rightarrow$ So, $\theta < 90^o$

$$\Rightarrow \frac{OP}{OQ} = \frac{PR}{RQ}$$

$\Rightarrow$ So, ΔOPR & ΔORQ are similar triangle.

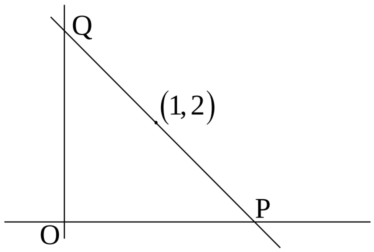
Q.27



$\Rightarrow P\left(\frac{8}{5},\frac{14}{5}\right)$ lies on 2x + y = k

$\Rightarrow$ So, k = 6

Q.28 [C]



Let slop of line = m

So, lines equation is,

$\Rightarrow$ y – 2 = m (x – 1)

$$\Rightarrow \therefore P\left(1-\frac{2}{m},0\right),Q\left(0,2-m\right)$$

$$\Rightarrow \therefore \text{Area of triangle} = \frac{1}{2}\left(1-\frac{2}{m}\right)\left(2-m\right)$$

$$\Rightarrow \frac{1}{2}\left(2-m-\frac{4}{m}+2\right)$$

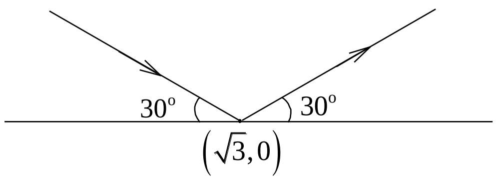
$$\Rightarrow A = \frac{1}{2}\left[4-\frac{4}{m}-m\right]$$

$$\Rightarrow \frac{dA}{dm} = \frac{1}{2}\left[0+\frac{4}{m^2}-1\right] = 0$$

$$\Rightarrow m = \pm 2$$

So, for m = – 2 area is minimum.

Q.29 [B]



$\Rightarrow$ So equation of reflected ray is $y-0=\left(\frac{1}{\sqrt{3}}\right)\left(x-\sqrt{3}\right)$

$\Rightarrow x-\sqrt{3}y-\sqrt{3}=0$

Q.30

(k + 1)x + 8y = 4k ………….(1)

$\Rightarrow$ kx + (k + 3)y = 3k – 1 ………….(2)

for no solution, lines (1) & (2) has to be parallel.

$\Rightarrow \frac{k+1}{k}=\frac{8}{k+3}=\frac{4k}{3k-1}$

So, $\frac{(k+1)}{k}=\frac{8}{k+3}$

$\Rightarrow k^2+4k+3=8k$

$\Rightarrow k^2-4k+3=0$; k = 1, 3

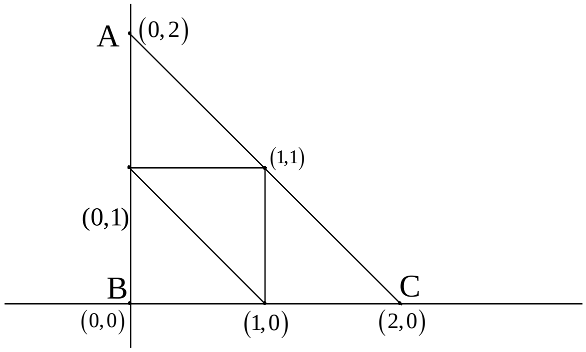
$\Rightarrow \frac{k+1}{k}=\frac{4k}{3k-1}$

$\Rightarrow 4k^2=3k^2+2k-1$

$\Rightarrow k^2-2k+1=0 \Rightarrow$ k = 1

So, only for k = 1 no solution.

Q.31 [B]



$\Rightarrow$ Abscissa of in-centre = $\dfrac{2(0)+2(2)+2\sqrt{2}(0)}{2+2+2\sqrt{2}}$

$$\Rightarrow \frac{2}{2+\sqrt{2}} = \left(2-\sqrt{2}\right)$$