WINDOW TO JEE MAINS

Q.1 (C)

$\left(1+10^{-4}\right)10^{4} = 1+10^{4}\left(10^{-4}\right)+............$

$\Rightarrow$ 2 + a + ve real number less than 1

Q.2 (C)

r + 2 + 3r = 2n + 2

$\Rightarrow$ n = 2r

Q.3 (A)

coeff of x^{p} = coeff of $x^{q} = {}^{p+q}C_{p} = {}^{p+q}C_{q}$

Q.4 (C)

sum of coefficients = 2^{n} = 4096

$\Rightarrow$ n = 12

$\Rightarrow$ Greatest coefficient in the expansion = ${}^{12}C_{6} = 924$

Q.5 (D)

First negative term will occur when $\left(\frac{27}{5}-r\right)$ will become negative, which is for r = 6 & for 7th term

Q.6 (A)

General term ${}^{256}C_{r}\left(3^{\frac{1}{2}}\right)^{256-r}\left(5^{\frac{1}{8}}\right)^{r}$ will be integral, if r = 8k, $k \in w$

Q.7 (A)

${}^{4}C_{2}\alpha^{2} = {}^{6}C_{3}\left(-\alpha\right)^{3}$

$\Rightarrow \alpha = -\frac{3}{10}$

Q.8 (B)

Coefficient of $x^n = (-1)^n + n(-1)^{n-1}$

$\Rightarrow (-1)^n [1-n]$

Q.9 (C)

$$T_n = \frac{0}{{}^nC_0} + \frac{1}{{}^nC_1} + \frac{2}{{}^nC_2} + \ldots\ldots\ldots\ldots + \frac{n}{{}^nC_n}$$

$$\Rightarrow T_n = \frac{n}{{}^nC_n} + \frac{n-1}{{}^nC_{n-1}} + \ldots\ldots\ldots\ldots + \frac{0}{{}^nC_0}$$

$$\Rightarrow 2T_n = n\left(\frac{1}{{}^nC_0} + \frac{1}{{}^nC_1} + \ldots\ldots\ldots\ldots + \frac{1}{{}^nC_n}\right) = nS_n.$$

$$\Rightarrow \frac{T_n}{S_n} = \frac{n}{2}$$

Q.10 (C)

$$T_{r+1} = {}^{11}C_r \left(ax^2\right)^{11-r} \left(\frac{1}{b}x^{-1}\right)^r = \text{coeff. of } x^7$$

$\Rightarrow 22 - 3r = 7$

$\Rightarrow r = 5$

$$\Rightarrow T_{r+1} = \text{coeff. of } x^{-7} \text{in} \left(ax - \frac{1}{bx^2}\right)^{11}$$

General term in the expansion,

$$\Rightarrow {}^{11}C_r (ax)^{11-r} \left(\frac{-1}{b}x^{-2}\right)^r = \text{coeff of } x^{-7}$$

$\Rightarrow 11 - 3r = -7$

$\Rightarrow r = 6$

Now, ${}^{11}C_5 a^6 b^{-5} = {}^{11}C_6 a^5 b^{-6}$

$\Rightarrow ab = 1$

Q.11 (D)

$$\left[(1+x)^{\frac{3}{2}}-\left(1+\frac{x}{2}\right)^{3}\right]\left[(1-x)^{-\frac{1}{2}}\right]$$

$$\Rightarrow\left[\left(1+\frac{3x}{2}+\frac{\frac{3}{2}\times\frac{1}{1}}{2}x^{2}+........\right)-\left(1+\frac{3x}{2}+\frac{3\times 2}{2}\frac{x^{2}}{4}+.....\right)\right]\left[1+\frac{x}{2}+\frac{\frac{1}{2}\times\frac{3}{2}}{2}x^{2}+.....\right]$$

$$\Rightarrow\left[-\frac{3}{8}x^{2}+........\right]\left[1+\frac{x}{2}+.....\right]$$

$$\Rightarrow-\frac{3}{8}x^{2}+............\text{higher powers}$$

$$\Rightarrow\approx\frac{-3}{8}x^{2}$$

Q.12 (C)

$$e^{\frac{1}{2}}=1+\frac{1}{2}+\frac{\left(\frac{1}{2}\right)^{2}}{2!}+\frac{\left(\frac{1}{2}\right)^{3}}{3!}+\frac{\left(\frac{1}{2}\right)^{4}}{4!}+.........$$

$$\Rightarrow e^{-\frac{1}{2}}=1-\frac{1}{2}+\frac{\left(\frac{1}{2}\right)^{2}}{2!}-\frac{\left(\frac{1}{2}\right)^{3}}{3!}+\frac{\left(\frac{1}{2}\right)^{4}}{4!}-.........$$

$$\Rightarrow\frac{e^{\frac{1}{2}}+e^{-\frac{1}{2}}}{2}=1+\frac{1}{(4\cdot 2!)}+\frac{1}{16\cdot(4!)}+\frac{1}{64\cdot(6!)}+...........$$

$$\Rightarrow\frac{e+1}{2\sqrt{e}}$$

Q.13 (D)

$$(1-ax)^{-1}(1-bx)^{-1}=\left(1+ax+a^{2}x^{2}+a^{3}x^{3}+........\right)\left(1+bx+b^{2}x^{2}+b^{3}x^{3}+.........\right)$$

$$\Rightarrow a_{n}=\text{coeff of } x^{n}=b^{n}+ab^{n-1}+a^{n}b^{n-2}+.........+ba^{n-1}+a^{n}$$

$$\Rightarrow a^n\left[1+\frac{b}{a}+\frac{b^2}{a^2}+\frac{b^3}{a^3}+........+\left(\frac{b}{a}\right)^n\right]$$

$$\Rightarrow \frac{a^n\left[\left(\frac{b}{a}\right)^{n+1}-1\right]}{\left(\frac{b}{a}-1\right)}=\frac{b^{n+1}-a^{n+1}}{b-a}$$

Q.14 (D)

$a_1 = n - m = 10$ ....(i)

$\Rightarrow a_2 = {}^mC_2 + {}^nC_2 - {}^mC_1{}^nC_1 = 10$ ....(ii)

From (i) & (ii)

$\Rightarrow$ m = 35

$\Rightarrow$ n = 45

$\Rightarrow (m, n) \equiv (34, 45)$

Q.15 (B)

$T_5 + T_6 = 0$

$\Rightarrow {}^nC_4 a^{n-4}(-b)^4 + {}^nC_5\, a^{n-5}(-b)^5 = 0$

$\Rightarrow \frac{a}{b} = \frac{n-4}{5}$

Q.16 (C)

$$\sum_{r=0}^{n} e\,{}^nC_r + \sum_{r=0}^{n} {}^nC_r = n \cdot 2^{n-1} + 2^n = (n+2)2^{n-1}$$

Hence, statement (1) is true.

$$\Rightarrow \sum_{r=0}^{n} e\,{}^nC_r x^r + \sum_{r=0}^{n} {}^nC_r x^r$$

$$\Rightarrow nx(1+x)^{n-1} + (1+x)^n$$

Q.17 (A)

$$64^n - (62)^{2n+1}$$

$$\Rightarrow (1+63)^n + (1-63)^{2n+1}$$

$$\Rightarrow \left[1 + 63x\,{}^nC_1 + (63)^2\,{}^nC_2 + \ldots\ldots\ldots\ldots\right] + \left[1 + {}^{2n+1}C_1(-63) + {}^{2n+1}C_2(-63)^2 + \ldots\ldots\ldots\right]$$

$$\Rightarrow (1+63\alpha) + (1+63\beta) = 2 + 63(\alpha+\beta) = 2 + 9k$$

Q.18 (A)

$$S_2 = \sum_{j=1}^{10} j \frac{10!}{(10-j)!\,j!} = 10\left(\sum_{j=1}^{10} \frac{9!}{(10-j)!\,(j-1)!}\right) = 10\left(\sum_{j=1}^{10} {}^9C_{j-1}\right) = 10 \times 2^9$$

$$\Rightarrow S_3 = \sum_{j=1}^{10} j \frac{10!}{(10-j)!\,(j-1)!} = 10\left(\sum_{j=1}^{10} j \cdot {}^9C_{j-1}\right)$$

$$\Rightarrow S_3 = 10\left[1\,{}^9C_0 + 2\cdot{}^9C_1 + 3\cdot{}^9C_2 + \ldots\ldots\ldots\ldots + 10\cdot{}^9C_9\right]$$

$$\Rightarrow S_3 = 10\left[10\cdot{}^9C_9 + 9\cdot{}^9C_8 + 8\cdot{}^9C_7 + \ldots\ldots\ldots\ldots + 1\cdot{}^9C_0\right]$$

$$\Rightarrow 2S_3 = 10 \times 11 \times \left[{}^9C_0 + {}^9C_1 + \ldots\ldots\ldots + {}^9C_9\right] = 10 \times 11 \times 2^9$$

$$\Rightarrow S_3 = 5 \times 10 \times 2^9$$

$$\Rightarrow S_1\left(\sum_{j=1}^{10} j^2\,{}^{10}C_j\right) - \left(\sum_{j=1}^{10} j\,{}^{10}C_j\right) = S_3 - S_2 = 45 \times 2^9 = 90 \times 2^8$$

Q.19 (D)

$$(1 - x - x^2 + x^3)^6 = (1-x)^6(1-x^2)^6$$

$$\Rightarrow \text{Coeff. of } x^7 = {}^6C_1 \cdot {}^6C_3 - {}^6C_2 \cdot {}^6C_3 - {}^6C_1 \cdot {}^6C_5$$

$$\Rightarrow -144$$

Q.20 (A)

$$\left(1+\sqrt{3}\right)^{2n} = {}^{2n}C_0 + {}^{2n}C_1\left(\sqrt{3}\right)^1 + {}^{2n}C_2\left(\sqrt{3}\right)^2 + {}^{2n}C_3 + \ldots\ldots\ldots\ldots$$

$$\Rightarrow \left(1-\sqrt{3}\right)^{2n} = {}^{2n}C_0 - {}^{2n}C_1\left(\sqrt{3}\right)^1 + {}^{2n}C_2\left(\sqrt{3}\right)^2 - {}^{2n}C_3 + \ldots\ldots\ldots$$

Hence, $\left(1+\sqrt{3}\right)^{2n} - \left(1-\sqrt{3}\right)^{2n}$

$$\Rightarrow 2\left[{}^{2n}C_1\left(\sqrt{3}\right)^1 + {}^{2n}C_3\left(\sqrt{3}\right)^3 + {}^{2n}C_5\left(\sqrt{5}\right)^5 + \ldots\ldots\ldots\right]$$

Q.21 (C)

$$\left[\frac{\left(x^{\frac{1}{3}}\right)^3 + (1)^3}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{\left(x^{\frac{1}{2}}\right)^2 - (1)^2}{\left(x^{\frac{1}{2}}\right)\left[x^{\frac{1}{2}} - 1\right]}\right]^{10}$$

$$\Rightarrow \left[\left(x^{\frac{1}{3}} + 1\right) - \frac{\left(x^{\frac{1}{2}} + 1\right)}{x^{\frac{1}{2}}}\right]^{10}$$

$$\Rightarrow \left[x^{\frac{1}{3}} + 1 - 1 - x^{-\frac{1}{2}}\right]^{10} \Rightarrow \left(x^{\frac{1}{3}} - x^{-\frac{1}{2}}\right)^{10}$$

Now, General term in this expansion = ${}^{10}C_r\left(x^{\frac{1}{3}}\right)^{10-r}\left(-x^{-\frac{1}{2}}\right)^r$

Term independent of x = $\frac{10-r}{3} - \frac{r}{2} = 0$

$\Rightarrow$ r = 4

Hence, the required term = ${}^{10}C_4 = 210$.