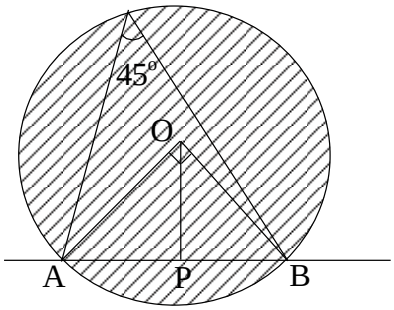
WINDOW TO JEE MAINS

Q.1

$x^2 + y^2 = 1$



$\Rightarrow$ y = mx + 1 subtend angle 90° at origin O (0, 0)

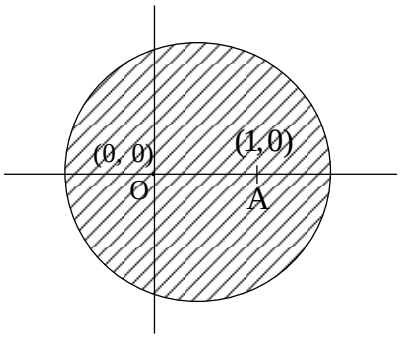
$$\Rightarrow |OP| = \frac{1}{\sqrt{1+m^2}}$$

$$\Rightarrow \therefore |OP|^2 + |AP|^2 = OA^2$$

$$\Rightarrow 2 \cdot \frac{1}{1+m^2} = 1$$

$$\Rightarrow m = \pm 1$$

Q.3 (B)



Equation of family of circles passing through O (0, 0) & A (1, 0) is

$$\Rightarrow \left(x(x-1) + y^2\right) + \lambda(y) = 0$$

$$\Rightarrow x^2 + y^2 - x + \lambda y = 0 \qquad \text{…….(1)}$$

$\Rightarrow C_1\left(\frac{1}{2}, \frac{-\lambda}{2}\right)$

Circle (1) touches $x^2 + y^2 = 9$ internally

$\Rightarrow \therefore C_1C_2 = r_1 + r_2$

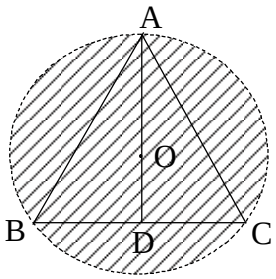
$\Rightarrow \sqrt{\frac{1}{4} + \frac{\lambda^2}{4}} = 3 - \sqrt{\frac{1}{4} + \frac{\lambda^2}{4}}$

$\Rightarrow \lambda^2 + 1 = 9$

$\Rightarrow \lambda = 2\sqrt{2}$

So centre $C_1\left(\frac{1}{2}, -\sqrt{2}\right)$

Q.4 (C)

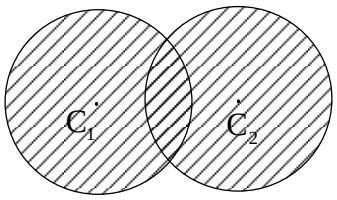


$\Rightarrow AD = 3a$

$\Rightarrow \therefore OA = 2a$

So equation of circle is $x^2 + y^2 = 4a^2$

Q.5 (D)



$\Rightarrow x^2 + y^2 - 8x + 2y + 8 = 0$

$\Rightarrow C_1(4, -1), r_1 = \sqrt{16 + 1 - 8}$

$\Rightarrow r_1 = 3$

$\Rightarrow (x-1)^2 + (y-3)^2 = r^2$

$\Rightarrow C_2(1,3), r_2 = r$

$\Rightarrow \therefore C_1C_2 = \sqrt{9+16} = 5$

$\Rightarrow |r_1 - r_2| < C_1C_2 < r_1 + r_2$

$\Rightarrow |r-3| < 5 < r+3$

Solve to get $2 < r < 8$

Q.6 (B)

$\pi r^2 = 154$

$\Rightarrow \frac{22}{7} r^2 = 154$

$\Rightarrow r = 7$

$\Rightarrow \left.\begin{matrix} 2x - 3y = 5 \\ 3x - 4y = 7 \end{matrix}\right\}$ are diameters

So centre C (1, – 1)

$\Rightarrow \therefore$ Circle's equation is $(x-1)^2 + (y+1)^2 = 49$

$\Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0$

Q.7 (B)

Let centre is (h, k) & passes through (a, b) so

Equation of circle is $(x-h)^2 + (y-k)^2 = (h-a)^2 + (k-b)^2$

$\Rightarrow x^2 + y^2 - 2hx - 2ky + 2ah + 2bk - a^2 - b^2 = 0$ …….(1)

$\Rightarrow x^2 + y^2 = 4$ ……(2)

$\because$ Equation of (1) & (2) are orthogonal circle so

$\Rightarrow 2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\Rightarrow 0 + 0 = \left(2bk + 2ah - a^2 - b^2\right) - 4$

So locus of (h, k) is

$\Rightarrow 2ax + 2by - a^2 - b^2 - 4 = 0$

Q.8 Same question of window to JEE mains (straight lines) Q.13

Q.9 (C)

Circumference $= 10\pi = 2\pi r$

$\Rightarrow r = 5$

diameters equation $2x + 3y + 1 = 0$

$\Rightarrow 3x - y - 4 = 0$

So centre C $(1, -1)$

So equation of circle is

$\Rightarrow (x-1)^2 + (y+1)^2 = 25$

Q.10 (C)

Line $y = x$ & circle $x^2 + y^2 - 2x = 0$

For intersection point of line & circle

$\Rightarrow x^2 + y^2 - 2x = 0$

$\Rightarrow x = 0, 1$

$\Rightarrow y = 0, 1$

So, A (0, 0); B (1, 1)

$\therefore$ equation of circle as AB diamter is

$\Rightarrow (x - 0)(x - 1) + (y - 0)(y - 1) = 0$

$\Rightarrow x^2 + y^2 - x - y = 0$

Q.11 (A)

Circle S_1: $x^2 + y^2 + 2ax + cy + a = 0$ ……(1)

$\Rightarrow$ Circle S_2 : $x^2 + y^2 - 3ax + dy - 1 = 0$ ……(2)

Equation of common chord PQ is $S_1 - S_2 = 0$

$\Rightarrow 5ax + (c - d)y + (a + 1) = 0$ ……(3)

$\Rightarrow 5x + by - a = 0$ ……(4)

Equation (3) & (4) are same to compare

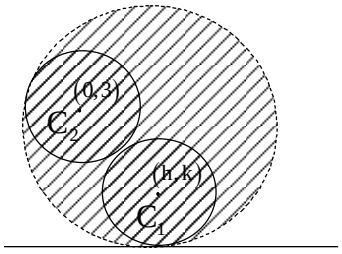
$$\Rightarrow \frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a}$$

$$\Rightarrow \therefore -a^2 = a+1$$

$$\Rightarrow a^2 + a + 1 = 0$$

No real value of a.

Q.12 (C)



radius of circle with centre $C_1(h, k)$ is = k

$$\Rightarrow \therefore C_1C_2 = r_1 \pm r_2$$

$$\Rightarrow \sqrt{h^2 + (k-3)^2} = k \pm 2$$

$$\Rightarrow h^2 + k^2 - 6k + 9 = k^2 + 4 \pm 4k$$

$$\Rightarrow h^2 = 10k - 5 \text{ or } h^2 = 2k - 5$$

Locus is $x^2 = 10y - 5$ or $x^2 = 2y - 5$

$\therefore$ Locus is parabola

Q.13 (C)

Let centre (h, k)

$\Rightarrow \therefore$ Equation of circle $(x-h)^2+(y-k)^2-(h-a)^2+(k-b)^2$

$\Rightarrow x^2+y^2-2xh-2yk-a^2-b^2+2ah+2bh=0$

$\Rightarrow x^2+y^2=p^2$

$\Rightarrow \therefore 2g_1g_2+2f_1f_2=c_1+c_2$ for orthogonality

$\Rightarrow 0+0=-p^2-a^2-b^2+2ah+2bk$

$\therefore$ Locus is $2ax+2ay-(a^2+b^2+p^2)=0$

Q.14 (D)

$3x - 4y - 7 = 0$

$\Rightarrow 2x - 3y - 5 = 0$

Centre is intersection point of given 2 diameter.

$\Rightarrow C\ (+1, -1)$

Also $\pi r^2 = 49\pi$

$\Rightarrow r = 7$

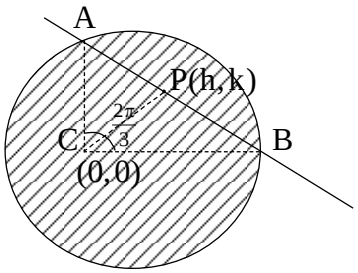
$\therefore$ equation of circle is

$\Rightarrow (x-1)^2+(y+1)^2=49$

$\Rightarrow x^2+y^2-2x+2y-47=0$

Q.15

C (0, 0), r = 3



Equation of circle is $x^2 + y^2 = 9$

Let mid-point of chord P (h, k)

$\therefore$ equation of chord is $T = S_1$

$\Rightarrow xh + ky - 9 = h^2 + k^2 - 9$

$\Rightarrow xh + ky = h^2 + k^2$

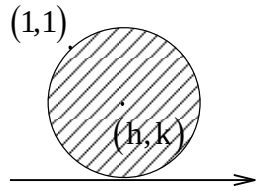
$\Rightarrow CP = \left| \dfrac{0 + 0 - h^2 - k^2}{\sqrt{h^2 + k^2}} \right| = 1$

$\Rightarrow CP = 1 = \sqrt{h^2 + k^2}$

$\Rightarrow h^2 + k^2 = 1$

$\Rightarrow \therefore$ Locus is $x^2 + y^2 = 1$

Q.16 (D)



$\Rightarrow$ Equation of x axis is y = 0

$\Rightarrow$ Equation of circle is $(x-h)^2 + (y-k)^2 = (h-1)^2 + (k-1)^2$

$\Rightarrow x^2 + y^2 - 2xh - 2ky + 2h + 2k - 2 = 0$

This circle touches x axis

So, $g^2 - c = 0$

$\Rightarrow (-h)^2 - 2(2h + 2k - 2) = 0$

$\Rightarrow h^2 - 2h - 2(k-1) = 0$

as h is real so,

$\Rightarrow D \geq 0$

$\Rightarrow 4+8(k-1)\geq 0$

$\Rightarrow 2k\geq 1$

$\Rightarrow k\geq \frac{1}{2}$

Q.17 (D)

$x^2+y^2+2x+4y-3=0$

Centre C (– 1, – 2)

$\Rightarrow$P (1, 0), Q (a, b)

Mid-point of P & Q is C

$\Rightarrow$So, a = – 3, b = – 4

$\Rightarrow \therefore Q(-3,-4)$

Q.18 (B)

$x^2+y^2-4x-8y-5=0$

$\Rightarrow$Centre C (2, 4), $r=\sqrt{4+16+5}=5$

$\Rightarrow$Line 3x – 4y – m = 0

$\Rightarrow$Line intersect if $\left|\frac{3(2)-4(4)-m}{\sqrt{3^2+4^2}}\right|<5$

$\Rightarrow |10+m|<25$

$\Rightarrow -25<10+m<25$

$\Rightarrow -35<m<15$

Q.19 (C)

$x^2+y^2-ax=0$ $\qquad$ $x^2+y^2-c^2=0$

$\Rightarrow C_2\left(\frac{a}{2},0\right)$ $\qquad$ $C_2(0,0)$ $\qquad$ …….(C > 0)

$\Rightarrow r_1 = \frac{a}{2}$ $\qquad\qquad$ $r_2 = C$

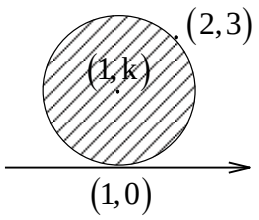
Circle touches each other if

$\Rightarrow C_1C_2 = r_1 + r_2$ $\qquad$ or $\qquad$ $C_1C_2 = |r_1 - r_2|$

$\Rightarrow \left|\frac{a}{2}\right| = C\left|\frac{a}{2}\right|$ $\qquad$ or $\qquad$ $\left|\frac{a}{2}\right| = \left|\frac{a}{2} - C\right|$

$\Rightarrow C = |a|$

Q.20



radius = k

$\Rightarrow \sqrt{(k-3)^2 + 1} = k$

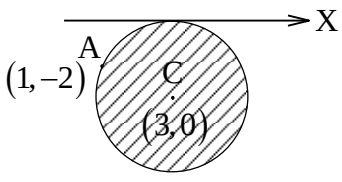
$\Rightarrow k^2 - 6k + 10 = k^2$

$\Rightarrow k = \frac{5}{3}$

$\therefore$ diameter length = 2k

$\Rightarrow \frac{10}{3}$

Q.21 (C)



$\Rightarrow$ Centre $C \equiv (3,-k)$ $\therefore k > 0$

$\Rightarrow CA = |k|$

$\Rightarrow \therefore \sqrt{4+(-k+2)^2} = |k|$

$\Rightarrow 4 + k^2 - 4k + 4 = k^2$

$\Rightarrow k = 2$

$\Rightarrow \therefore \text{centre}(3,-2)$

$\Rightarrow \text{radius} = 2$

Only point (5, – 2) is at distance “2” from the centre.